電子散乱の原子核構造関数と原子核構造
(Nuclear structure functions in electron scattering and nuclear structure)

Shunzo Kumano
Japan Women's University
High Energy Accelerator Research Organization (KEK)
http://research.kek.jp/people/kumanos/

第23回高エネルギーQCD核子構造勉強会： 短距離離核子相関や原子核構造とEICにおける展望
(High-energy QCD and nucleon structure:
Short-range nucleon correlations and nuclear cluster structure at EIC)
Riken, Wako, Japan
https://indico2.riken.jp/event/5050/

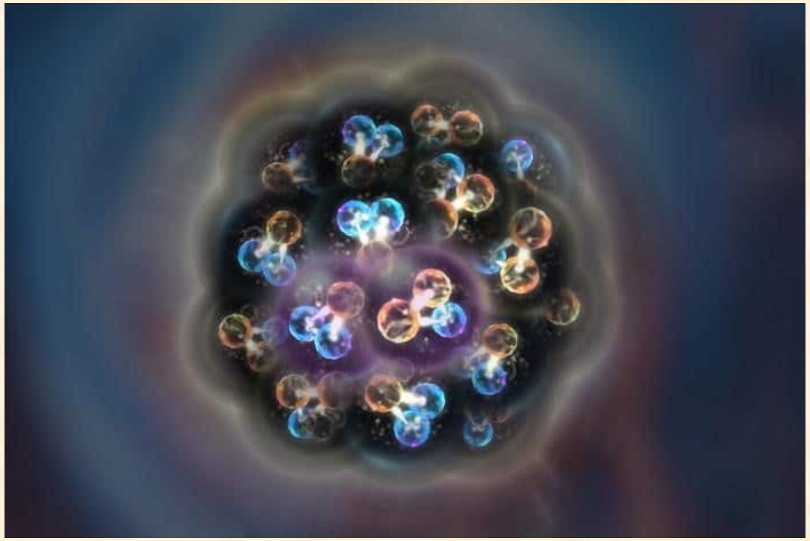
December 23, 2024

One of top 10 breakthroughs in physics world in 2024

https://physicsworld.com/a/top-10-breakthroughs-of-the-year-in-physics-for-2024-revealed/

https://physicsworld.com/a/two-distinct-descriptions-of-nuclei-unified-for-the-first-time/

Modification of Quark-Gluon Distributions in Nuclei by Correlated Nucleon Pairs, A.W. Denniston *et al*., Phys. Rev. Lett. 133 (2024) 152502.



The team developed a unified framework that integrates both the partonic structure of nucleons and the interactions between nucleons in atomic nuclei. This approach is particularly useful for studying SRC nucleon pairs, whose interactions have long been recognized as crucial to understanding the structure of nuclei, but they have been notoriously difficult to describe using conventional theoretical models.

By combining particle and nuclear physics descriptions, the researchers were able to derive PDFs for SRC pairs, providing a detailed understanding of how quarks and gluons behave within these pairs.

The new model can be further tested using data from future experiments, such as those planned at the Jefferson Lab and at the Electron-Ion Collider at Brookhaven National Laboratory. These facilities will allow scientists to probe quark-gluon dynamics within nuclei with even greater precision, providing an opportunity to validate the predictions made in this study.

KEK workshop in 2009 (but not continued ... at KEK)

Workshop on Short-Range Correlations and Tensor Structure at J-PARC,
KEK, Tsukuba, Japan, 2009.9.25,
https://www-conf.kek.jp/hadron1/j-parc-src09/

KEK theory center workshop on
Short-range correlations and tensor structure at J-PARC

September 25, 2009 (Friday)
KEK, Building 4, Room 345

Speakers

Claudio Ciofi degli Atti (Perugia Univ)	Short Range Correlations and their impact on nuclear and particle physics and astrophysics: recent advances and possible studies at J-PARC
Shin'ya Sawada (KEK)	Status of J-PARC facility and hadron hall
Chiara Benedetta Mezzetti (Perugia Univ)	Two- and three-nucleon correlations and inclusive electron scattering
Hiko Morita (Sapporo Gakuin Univ)	Nulcear spectral function based on the two-nucleon correlation model
Makoto Sakuda (Okayama Univ)	Effect of spectral function in neutrino-nucleus interactions in the MeV-GeV region: Experimental View
Ryozo Tamagaki (Kyoto Univ)	Nucleon correlations and neutron star physics
Noriyoshi Ishii (Tokyo Univ)	Short-range nuclear force in lattice QCD
Hooi Jin Ong (RCNP)	Search for direct evidence of tensor interaction via (p,d) reaction
Choki Nakamoto (Suzuka Tech)	Short-range NN and YN interactions in the SU_6 quark model
Takayuki Myo (Osaka Tech)	Tensor-optimized shell model using bare interaction for light nuclei
Toshiaki Shibata (Tokyo Tech)	Tensor structure function b_1 of the deuteron measured with high-energy electron scattering by HERMES
Shunzo Kumano (KEK)	Tensor structure in high-energy proton-deuteron reactions

Orgainzers: Shunzo Kumano (KEK), Hiko Morita (Sapporo Gakuin Univ), Shin'ya Sawada (KEK)
Secretary: Reiko Kusama (KEK)
E-mail: hadron-ns09(AT)ml.post.kek.jp [(AT) --> @]
(Email to Kusama, Kumano, Morita, and Sawada)

Talk slides are available even now.

Contents

1. Introduction to lepton deep inelastic scattering for beginners

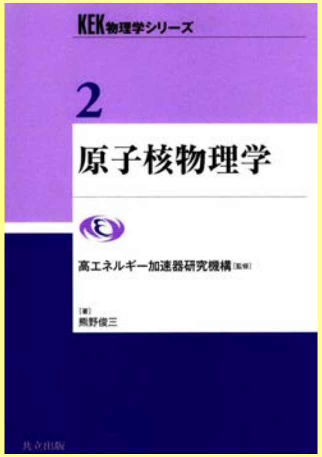
may skip some slides

2. Nuclear modifications of structure functions
3. Cluster structure in nuclear structure functions
4. Short-range correlations in electron scattering
5. Summary

note: Long history of short-range correlation studies in Japan
even at INS (Institute of Nuclear Studies) of Tokyo in 1990's:

T. Emura *et al*., PRL 73 (1994) 404; PRC 49 (1994) R597;
K. Maruyama *et al*., Nucl. Instr. and Meth. A 376 (1996) 335;
K. Maruyama and T. Suda, J. Phys. Soc. Japan, 52 (1997) 103.

Introduction to lepton deep inelastic scattering for beginners



原子核物理学、熊野俊三
共立出版(2015)
Nuclear Physics, S. Kumano
Kyoritsu Shuppan (2015)

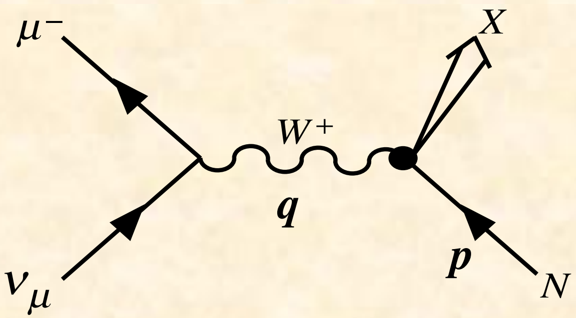
Deep inelastic scattering

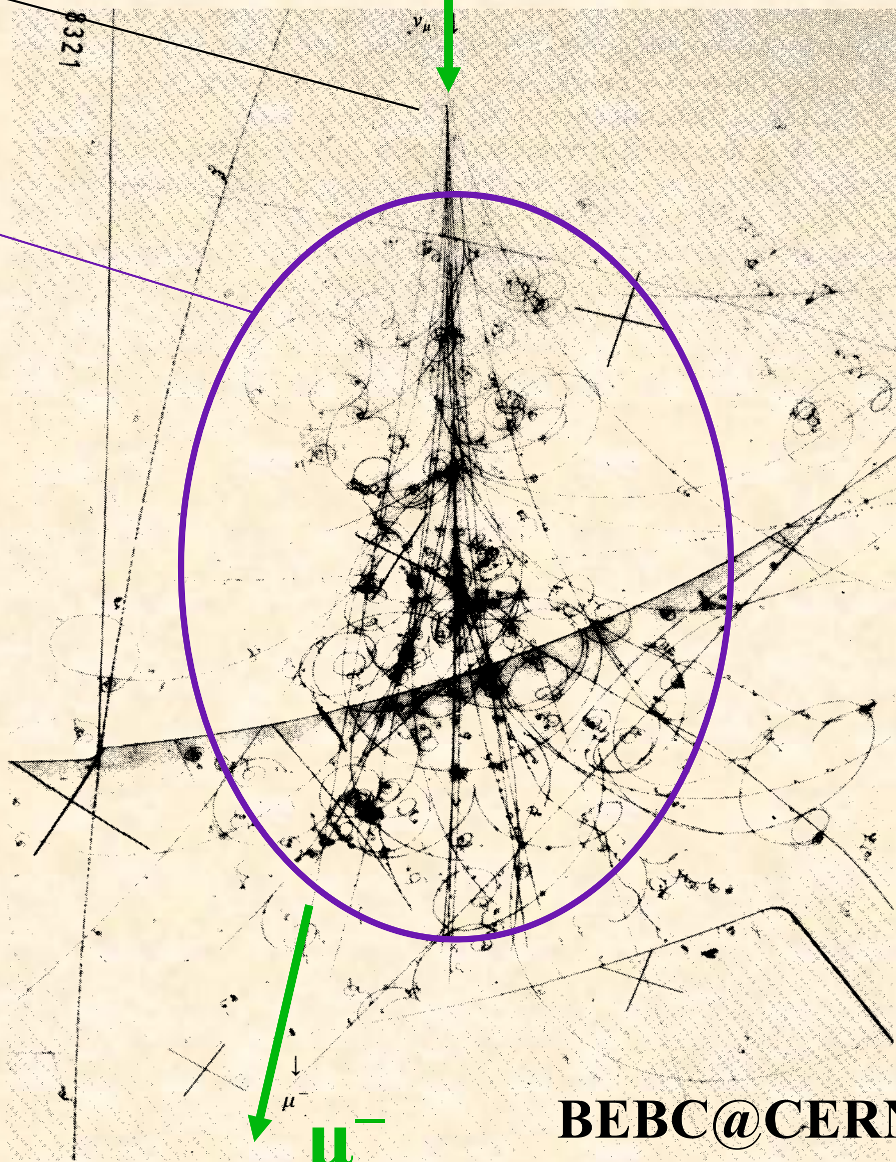
ν_μ $\sim$ 200 GeV

A nucleon is broken up by a high-energy neutrino.

Hadrons are produced; however, these are not usually measured. (inclusive reaction)

$$\nu_\mu + N \to \mu^- + X$$





μ^-

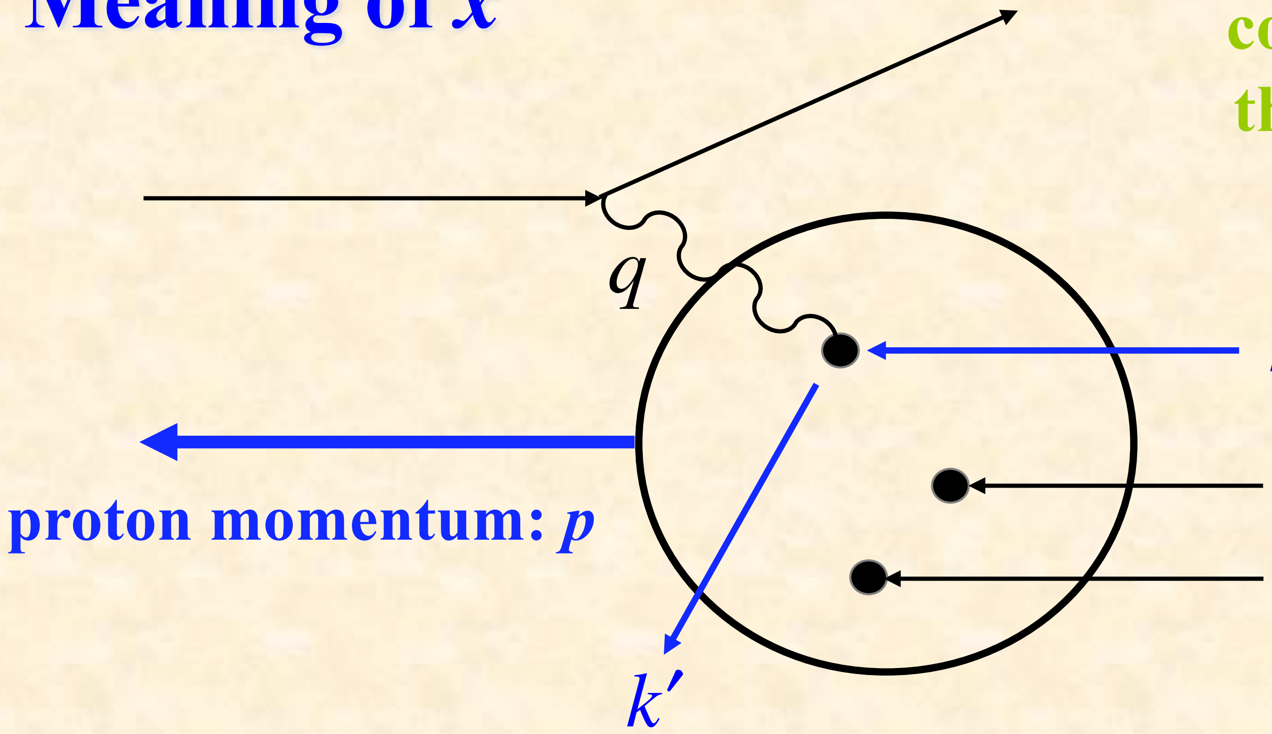
BEBC@CERN

Momentum transfer: $q^2 = (k - k')^2 = -Q^2$

Bjorken scaling variable: $x = \dfrac{Q^2}{2p \cdot q}$

Invariant mass: $W^2 = p_X^2 = (p+q)^2$

Meaning of x



consider the frame where the proton is moving fast

$$(k+q)^2 = k'^2$$

$$(k+q)^2 = m_q^2 + 2k\cdot q - Q^2$$

$$k'^2 = m_q^2$$

$$\longrightarrow 2k\cdot q = Q^2$$

if $k = \xi p,\ \ 2\xi p\cdot q = Q^2$

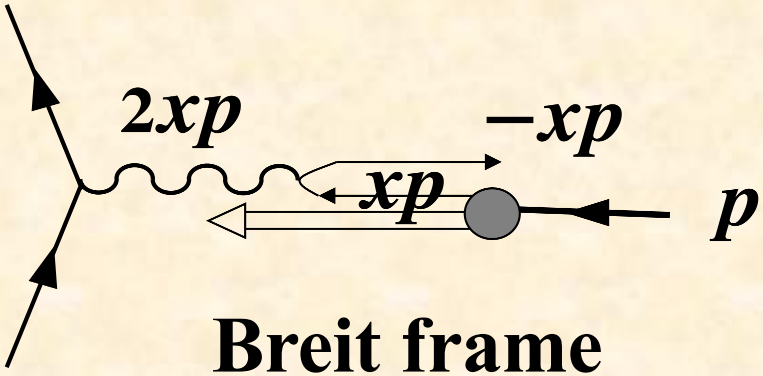
$$\xi = \frac{Q^2}{2p\cdot q} = x$$

x = momentum fraction carried by the struck parton

For example, x=0.5 means that the struck parton carries 50% momentum of the proton.

Meaning of Q^2

Breit frame is defined as the frame in which exchanged boson is completely spacelike: $q = (0, 0, 0, q)$.



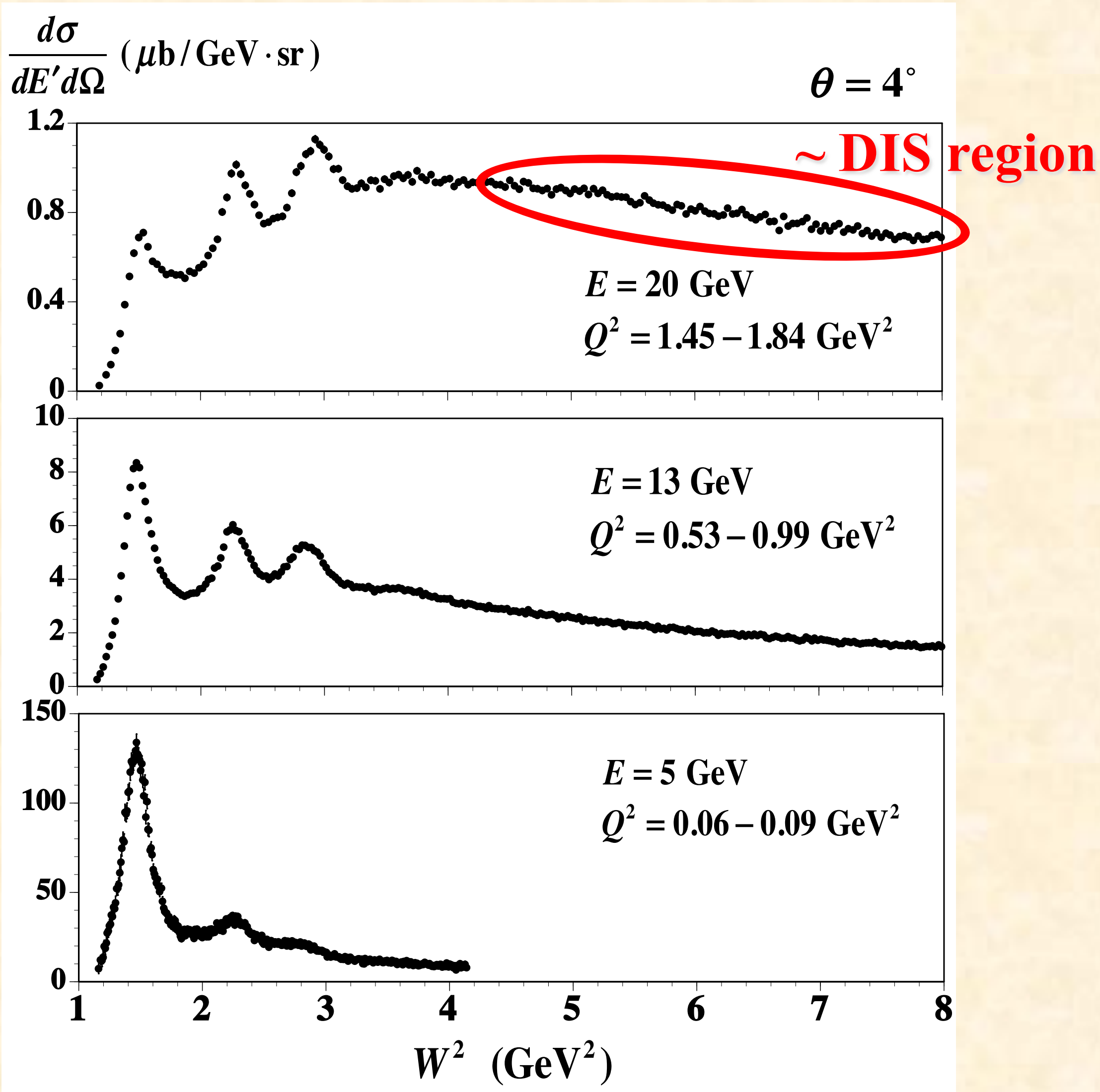
Breit frame

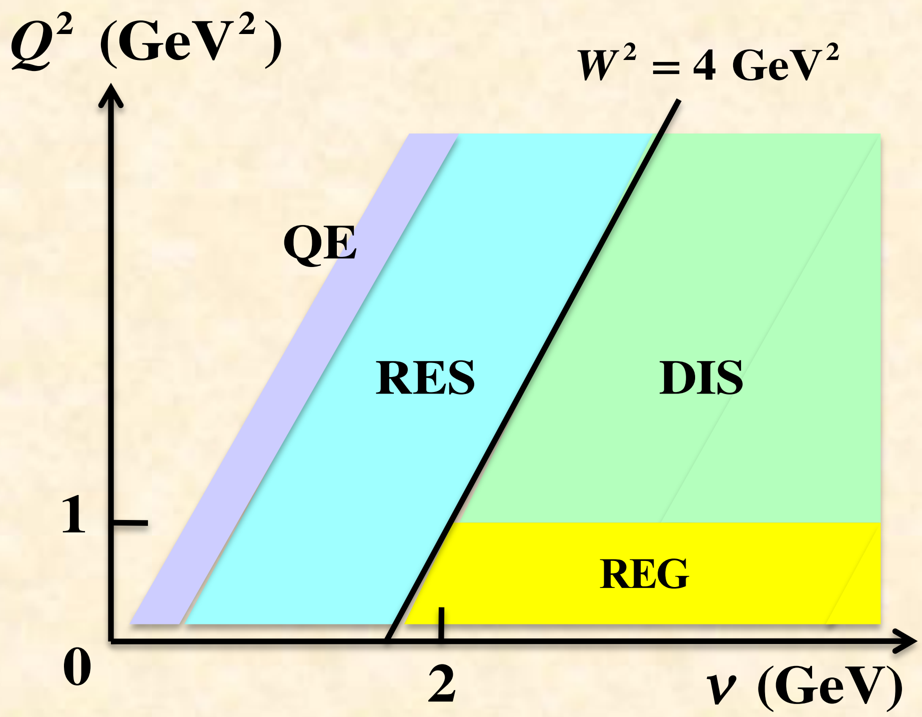
$q^0=0$: photon does not transfer any energy

Spatial resolution = reduced wavelength $\lambdabar = \dfrac{1}{|\vec{q}|} = \dfrac{1}{\sqrt{Q^2}}$

Q^2 corresponds to the "spatial resolution".

Lepton-nucleon/nucleus scattering





Depending on the lepton beam energy, different physics mechanisms contribute to the cross section.

- **QE (Quasi elastic)**
- **RES (Resonance)**
- **DIS (Deep inelastic)**
- **REG (Regge)**

S. X. Nakamura *et al.*,
Rept. Prog. Phys. 80 (2017) 056301.

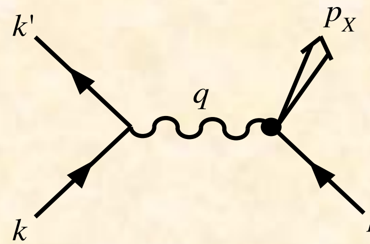
Cross section

$$d\sigma = \frac{1}{4\sqrt{(k\cdot p)^2 - m^2 M_N^2}} \overline{\sum_{pol}} \sum_X (2\pi)^4 \delta^4(k+p-k'-p_X)\,|M|^2 \frac{d^3k'}{(2\pi)^3 2E'}, \qquad M = e\,\bar{u}(k',\lambda')\gamma_\mu u(k,\lambda)\,\frac{g^{\mu\nu}}{q^2}\,\langle X|\,e J_\nu^{em}(0)|\,p,\lambda_N\rangle$$

$$d\sigma = \frac{2M_N}{s-M_N^2}\frac{\alpha^2}{Q^4} L^{\mu\nu} W_{\mu\nu} \frac{d^3k'}{E'}, \qquad L^{\mu\nu} = \overline{\sum_{\lambda,\lambda'}} \left[\bar{u}(k',\lambda')\gamma^\mu u(k,\lambda)\right]^* \left[\bar{u}(k',\lambda')\gamma^\nu u(k,\lambda)\right] = 2\left[k^\mu k'^\nu + k'^\mu k^\nu - (k\cdot k' - m^2) g^{\mu\nu}\right]$$

$$W_{\mu\nu} = \frac{1}{4\pi M_N} \overline{\sum_{\lambda_N}} \sum_X (2\pi)^4 \delta^4(k+p-k'-p_X)\,\langle p,\lambda_N|\,J_\mu^{em}(0)|\,X\rangle\langle X|\,J_\nu^{em}(0)|\,p,\lambda_N\rangle$$

$$= \frac{1}{4\pi M_N} \overline{\sum_{\lambda_N}} \int d^4\xi\, e^{iq\cdot\xi} \langle p,\lambda_N|\left[J_\mu^{em}(\xi), J_\nu^{em}(0)\right]|\,p,\lambda_N\rangle$$



Lightcone dominance

Dominant contribution to $W_{\mu\nu}$ comes from the light-cone region: $0 < \xi^2 < O(1/Q^2)$.

$$q = (\nu,\ 0,\ 0,\ -\sqrt{\nu^2+Q^2}),\quad q^+ = -\frac{Mx}{\sqrt{2}},\quad q^- = -\frac{2\nu+Mx}{\sqrt{2}} \simeq \sqrt{2}\nu \gg M$$

$$q\cdot\xi = q^+\xi^- + q^-\xi^+ - \vec{q}_T\cdot\vec{\xi}_T = q^+\xi^- + q^-\xi^+,\quad \xi^\pm = \frac{\xi^0 \pm \xi^3}{\sqrt{2}}$$

$$\int d^4\xi\, e^{iq\cdot\xi} \langle p,\lambda_N|\left[J_\mu^{em}(\xi), J_\nu^{em}(0)\right]|\,p,\lambda_N\rangle \simeq \int d^4\xi\, \underline{e^{i(q^+\xi^- + q^-\xi^+)}} \langle p,\lambda_N|\underline{\left[J_\mu^{em}(\xi), J_\nu^{em}(0)\right]}|\,p,\lambda_N\rangle$$

Rapidly oscillating integral: Significant contribution to the integral comes only from the region, $q^+\xi^- + q^-\xi^+ < 1$.

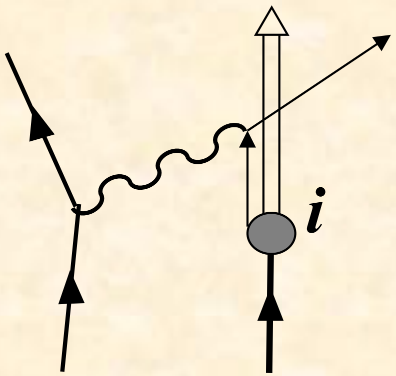
(Note: Riemann-Lebesgue theorem)

$$|\xi^+| \lesssim \left|\frac{1}{q^-}\right| \simeq \frac{1}{\sqrt{2}\nu},\quad |\xi^-| \lesssim \left|\frac{1}{q^+}\right| = \frac{\sqrt{2}}{Mx} \quad \to \quad \xi^2 \simeq 2\xi^+\xi^- \lesssim \frac{2}{Mx\nu} = O\left(\frac{1}{Q^2}\right)$$

Parton Model: validity of incoherent assumption

$$d\sigma \sim |M|^2 = \left|\sum_i M_i\right|^2 \simeq \sum_i |M_i|^2$$

incoherent assumption
(impulse approximation)

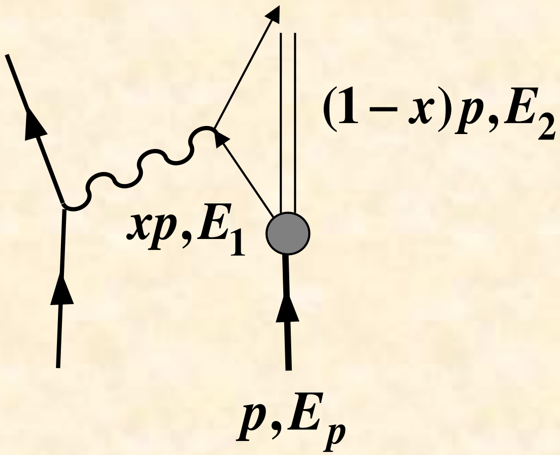


This approximation is valid if the γ interaction time is much shorter than the *qq* interaction time.

Consider the c.m. frame of *ep* [≈ infinite momentum frame ($p\to\infty$)]

$$k = (E_e,\, 0, 0, -p),\;\; p = (E_p,\, 0, 0, +p),\;\; q = (q^0,\, \vec{q}_T, -q_z)$$

γ interaction time: $$\tau \approx \frac{1}{q^0} = \frac{4|\vec{p}|}{2M_N\nu - (Q^2 - \vec{q}_T^{\,2})/y} \approx \frac{4|\vec{p}|}{2M_N\nu - Q^2}$$



$$E_N = \sqrt{M_N^2 + \vec{p}^{\,2}}, \quad E_1 = \sqrt{m^2 + \vec{p}_\perp^{\,2} + x^2\vec{p}^{\,2}}, \quad E_2 = \sqrt{m'^2 + \vec{p}_\perp^{\,2} + (1-x)^2\vec{p}^{\,2}}$$

Time of virtual state: $$T \approx \frac{1}{\Delta E} = \frac{1}{|E_1 + E_2 - E_p|} = \frac{2|\vec{p}|}{\left|\dfrac{m^2 + \vec{p}_\perp^{\,2}}{x} + \dfrac{m'^2 + \vec{p}_\perp^{\,2}}{1-x} - M_N^2\right|}$$

$$\approx \frac{2|\vec{p}|}{M_N^2}$$

$$\boxed{\tau \sim \frac{4|\vec{p}|}{2M_N\nu - Q^2}, \quad T \sim \frac{2|\vec{p}|}{M_N^2} \quad \to \tau \ll T}$$

The γ interaction time is much shorter than the *qq* interaction time.

$L^{\mu\nu}$ is symmetric under $\mu \leftrightarrow \nu$ $\longrightarrow$ Antisymmetric part of $W_{\mu\nu}$ does not contribute to the cross section.

Before discussing models for $W_{\mu\nu}$, we try to find its general expression.

$$W_{\mu\nu} = \frac{1}{4\pi M_N} \overline{\sum_{\lambda_N}} \int d^4\xi \; e^{iq\cdot\xi} \langle p,\lambda_N | \left[J_\mu^{em}(\xi), J_\nu^{em}(0) \right] | p,\lambda_N \rangle$$

Expand it by Lorentz vectors

Available quantities: $g_{\mu\nu}$, p_μ, q_μ

Neutrino reactions

$$W_{\mu\nu} = -W_1 g_{\mu\nu} + W_2 \frac{p_\mu p_\nu}{M_N^2} + W_4 \frac{q_\mu q_\nu}{M_N^2} + W_5 \frac{p_\mu q_\nu + q_\mu p_\nu}{M_N^2} \quad \cancel{\left(+ \frac{i}{2M_N^2} W_3 \varepsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma \right)}$$

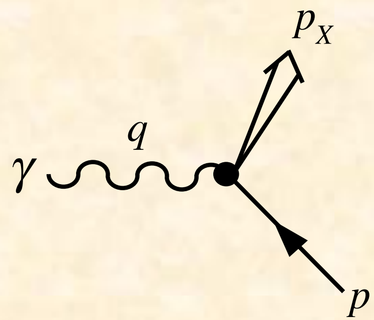
current conservation: $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -W_1 \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + W_2 \frac{1}{M_N^2} \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p\cdot q}{q^2} q_\nu \right)$$

Combining $W_{\mu\nu}$ with $L^{\mu\nu}$, we obtain the cross section:

$$\frac{d\sigma}{d\Omega\, dE} = \frac{\alpha^2}{4E^2 \sin^4(\theta/2)} \left[2W_1(\nu,Q^2)\sin^2\frac{\theta}{2} + W_2(\nu,Q^2)\cos^2\frac{\theta}{2} \right]$$

Longitudinal and transverse cross sections of $\gamma^*+N\to X$ and their relations to W_1 and W_2



$$\sigma_\lambda^{tot} = \frac{1}{2M_N 2K}\overline{\sum_{\lambda_N}}\sum_X (2\pi)^4\delta^4(q+p-p_X)\,|M|^2, \quad M=\varepsilon^\mu(\lambda,\mathrm{q})\langle X|eJ_\mu^{em}(0)|p,\lambda_N\rangle$$

λ = helicity, photon polarization vector: $\varepsilon^\mu(\lambda,q)$

transverse: $\lambda=\pm1$: $\varepsilon_\pm = \mp\frac{1}{\sqrt{2}}(0,1,\pm i,0)$

longitudinal: $\lambda=0$: $\varepsilon_0 = \frac{1}{\sqrt{-q^2}}\left(\sqrt{\nu^2-q^2},0,0,\nu\right)$

For real photon: $K=\nu$, $W^2=(p+q)^2=M_N^2+2M_NK$

For virtual photon, the factor $(2K)$ is arbitrary

- Hand convention: $K=\frac{W^2-M_N^2}{2M_N}=\nu+\frac{q^2}{2M_N}$
- others: $K=\nu$, $K=|\vec{q}|$

$$\sigma_\lambda^{tot} = \frac{e^2}{4M_NK}\overline{\sum_{\lambda_N}}\sum_X (2\pi)^4\delta^4(q+p-p_X)\langle p,\lambda_N|\,\varepsilon^{\mu*}J_\mu^{em}(0)|X\rangle\langle X|\,\varepsilon^\nu J_\nu^{em}(0)|p,\lambda_N\rangle = \frac{4\pi\alpha}{4M_NK}\varepsilon^{\mu*}\varepsilon^\nu 4\pi M_N W_{\mu\nu}$$

$$= \frac{4\pi^2\alpha}{K}\varepsilon^{\mu*}\varepsilon^\nu W_{\mu\nu}$$

$$\sigma_T \equiv \frac{\sigma_{+1}^{tot}+\sigma_{-1}^{tot}}{2} = \frac{4\pi^2\alpha}{K}W_1, \quad \sigma_L \equiv \sigma_0^{tot} = \frac{4\pi^2\alpha}{K}\left[(1-\nu^2/q^2)W_2 - W_1\right]$$

W_1 = transverse, W_2 = transverse + longitudinal

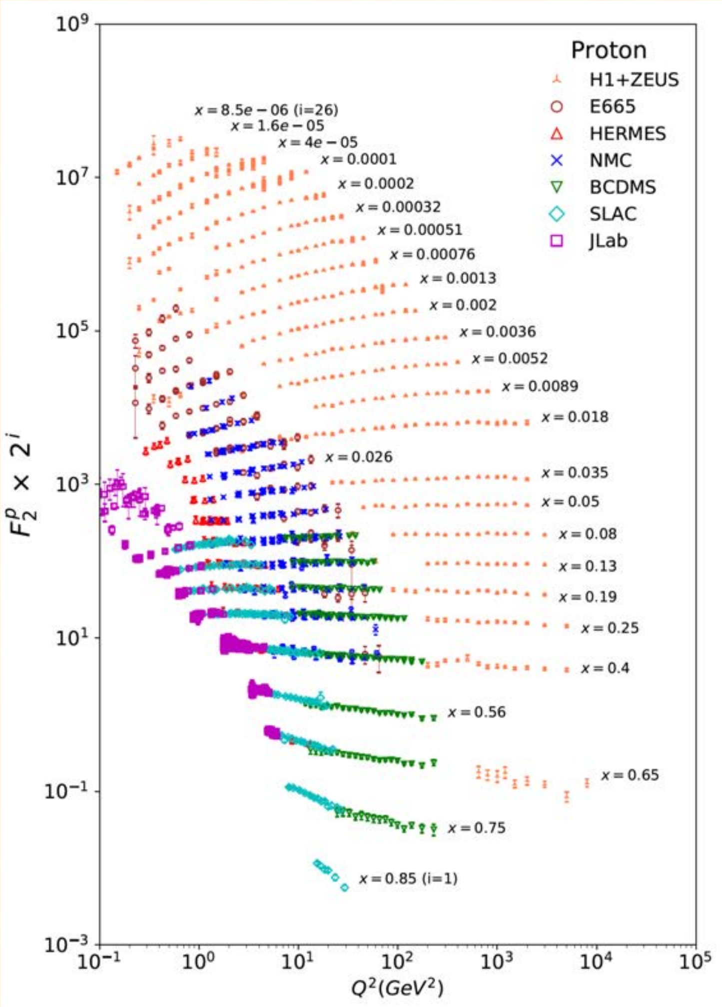
$(1-\nu^2/q^2)W_2 - W_1 \equiv W_L$ = longitudinal

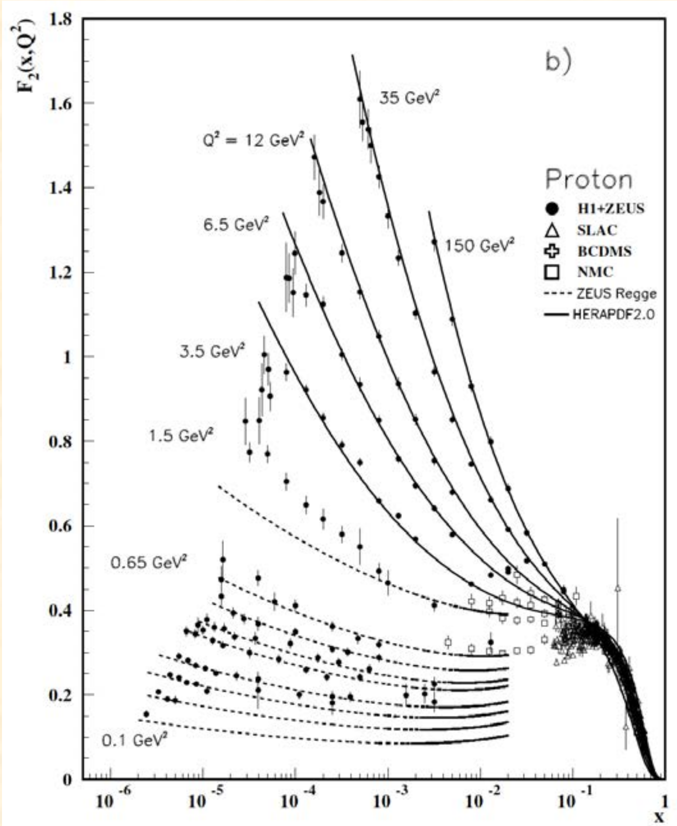
Instead of W_1, W_2 and W_L, the functions F_1, F_2 and F_L are usually used for showing experimental results.

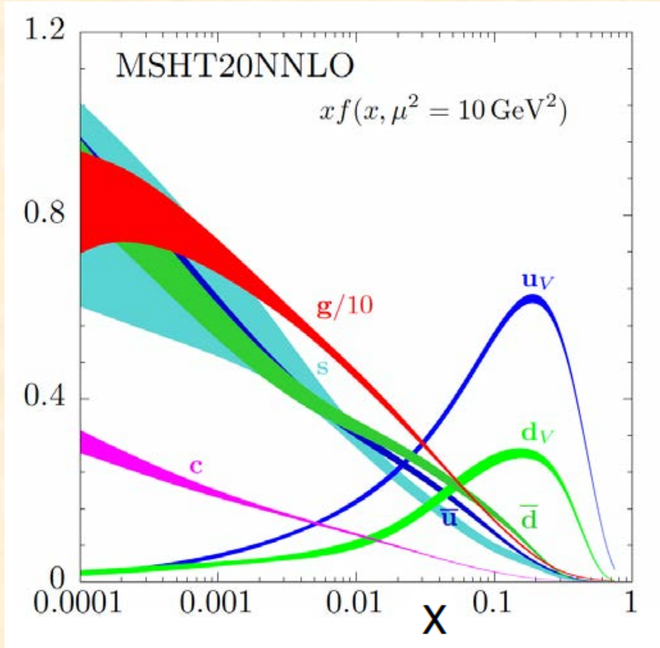
$$F_1 = M_N W_1, \quad F_2 = \nu W_2, \quad F_L = \frac{Q^2}{\nu}W_L = \left(1+\frac{Q^2}{\nu^2}\right)F_2 - 2xF_1$$

Structure functions and parton distribution functions (PDFs) of the proton

from Particle Data Group 2024

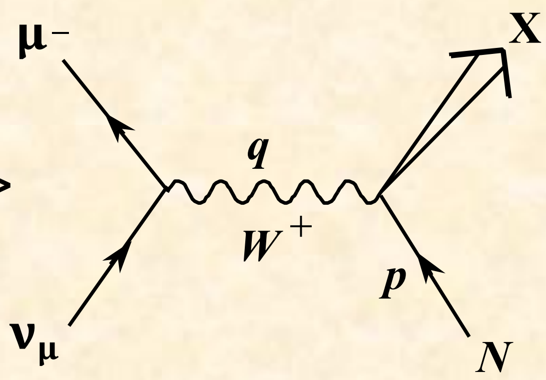






Neutrino deep inelastic scattering (CC: Charged Current)

$$d\sigma = \frac{1}{4k\cdot p}\,\frac{1}{2}\sum_{spins}\sum_{X}(2\pi)^4\,\delta^4(k+p-k'-p_X)\,|M|^2\,\frac{d^3k'}{(2\pi)^3\,2E'}$$



$$M = \frac{1}{1+Q^2/M_W^2}\,\frac{G_F}{\sqrt{2}}\,\bar{u}(k',\lambda')\,\gamma^\mu\,(1-\gamma_5)\,u(k,\lambda)\,<X\,|\,J_\mu^{CC}\,|\,p,\lambda_p>$$

$$\frac{d\sigma}{dE'd\Omega} = \frac{G_F^2}{(1+Q^2/M_W^2)^2}\,\frac{k'}{32\pi^2 E}\,L^{\mu\nu}\,W_{\mu\nu}$$

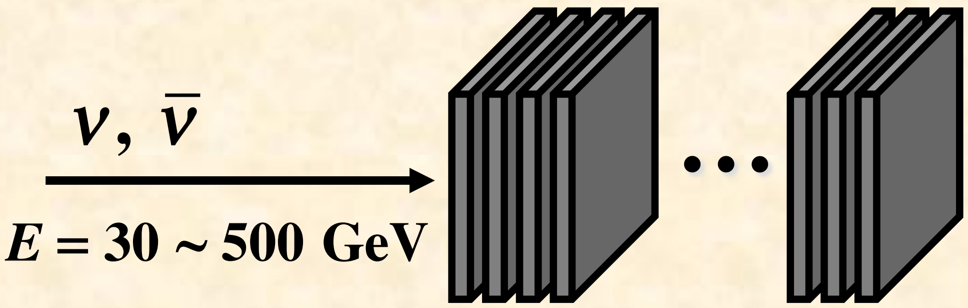
$$L^{\mu\nu} = 8\left[k^\mu k'^\nu + k'^\mu k^\nu - k\cdot k' g^{\mu\nu} + \underline{i\varepsilon^{\mu\nu\rho\sigma}k_\rho k'_\sigma}\right], \quad \varepsilon_{0123} = +1$$

$$W_{\mu\nu} = -W_1\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2\frac{1}{M^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) + \frac{i}{2M^2}\underline{W_3\varepsilon_{\mu\nu\rho\sigma}p^\rho q^\sigma}$$

$$MW_1 = F_1\,,\quad \nu W_2 = F_2\,,\quad \nu W_3 = F_3\,,\quad x = \frac{Q^2}{2p\cdot q}\,,\quad y = \frac{p\cdot q}{p\cdot k}$$

$$\frac{d\sigma^{CC}_{\nu,\bar{\nu}}}{dx\,dy} = \frac{G_F^2\,(s-M^2)}{2\pi\,(1+Q^2/M_W^2)^2}\left[x\,y^2 F_1^{CC} + \left(1-y-\frac{M\,x\,y}{2\,E}\right)F_2^{CC} \underline{\pm x\,y\left(1-\frac{y}{2}\right)F_3^{CC}}\right]$$

Neutrino DIS experiments

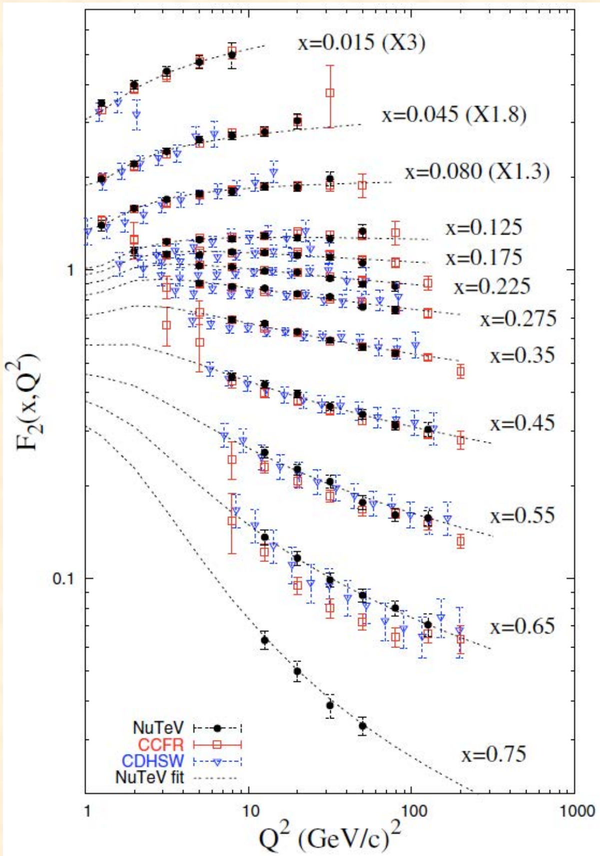


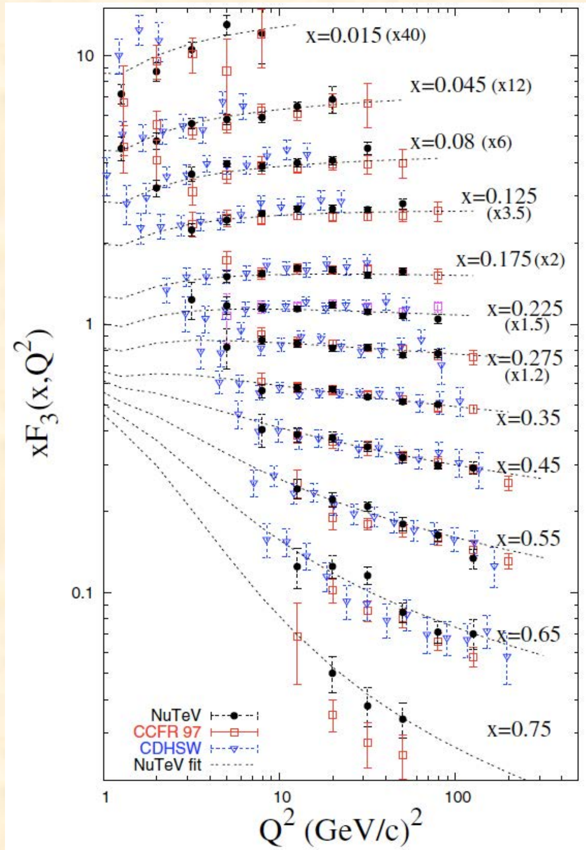
Huge Fe target (690 ton)

Experiment	Target	ν energy (GeV)
CCFR	Fe	30-360
CDHSW	Fe	20-212
CHORUS	Pb	10-200
NuTeV	Fe	30-500

MINERνA (He, C, Fe, Pb), …

M. Tzanov *et al.* (NuTeV), PRD74 (2006) 012008.





Nuclear modifications of parton distribution functions

Kinematical range of x : $0 \le x \le 1$ for the nucleon

(note: $0 \le x \lesssim A$ for a nucleus)

$$x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M_N \nu}$$

$Q^2 \ge 0$: spacelike
(e^+e^- annihilation, $Q^2 \le 0$: timelike)

$$Q^2 = \vec{q}^{\,2} - \nu^2 = (\vec{k} - \vec{k}')^2 - (E - E')^2$$

$$\simeq 2|\vec{k}||\vec{k}'|(1 - \cos\theta) \ge 0 \quad \text{for } m \ll E$$

$$\nu = E - E' \quad \text{in the rest frame of N}$$
$$\ge 0$$

$$x \ge 0$$

$$W^2 = (p+q)^2 = M_N^2 + 2M_N\nu + q^2 \ge M_N^2$$

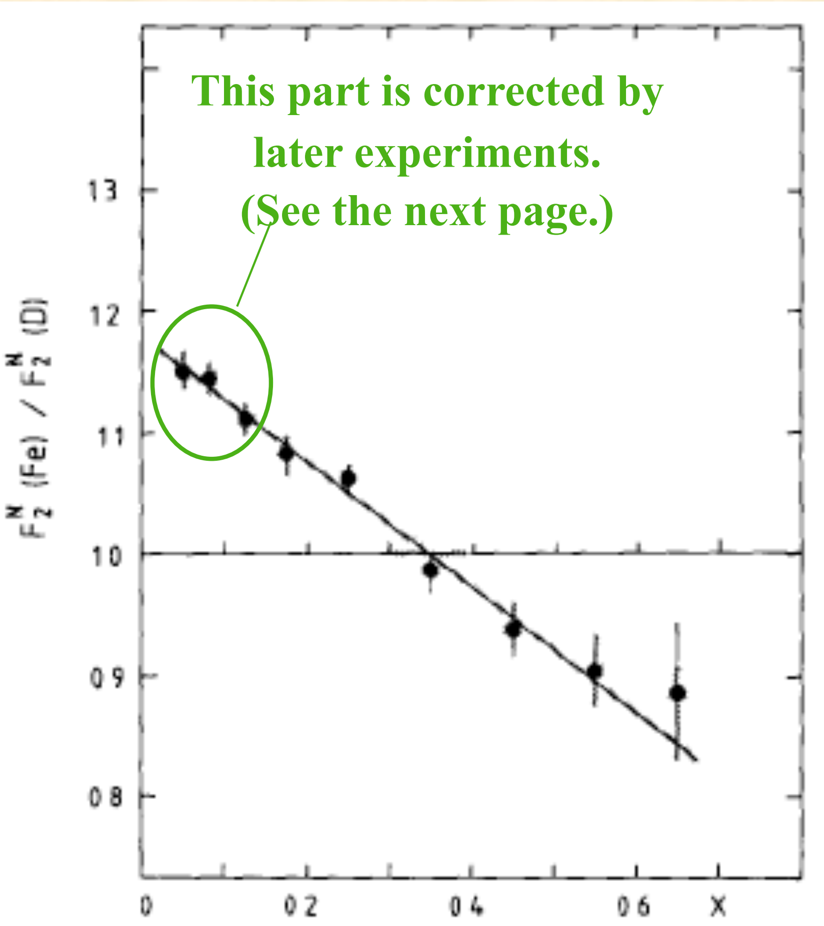
$$\longrightarrow \quad 2M_N\nu + q^2 \ge 0 \quad \longrightarrow \quad x = \frac{-q^2}{2M_N\nu} \le 1$$

In the same way for a nucleus

$$0 \le x_A = \frac{Q^2}{2p_A \cdot q} = \frac{Q^2}{2M_A\nu} = \frac{M_N}{M_A} x \le 1 \quad \rightarrow \quad 0 \le x \lesssim A$$

EMC (European Muon Collaboration) effect

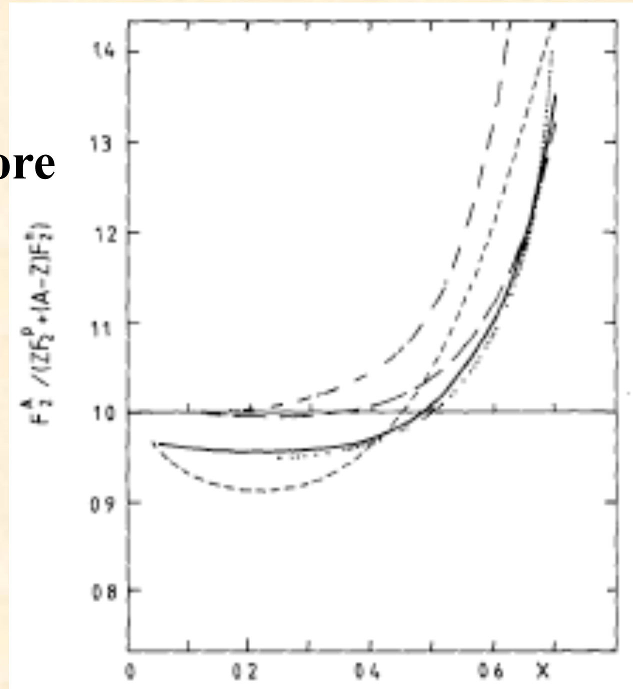
J. J. Aubert et al. (EMC),
Phys. Lett. B123 (1983) 275.



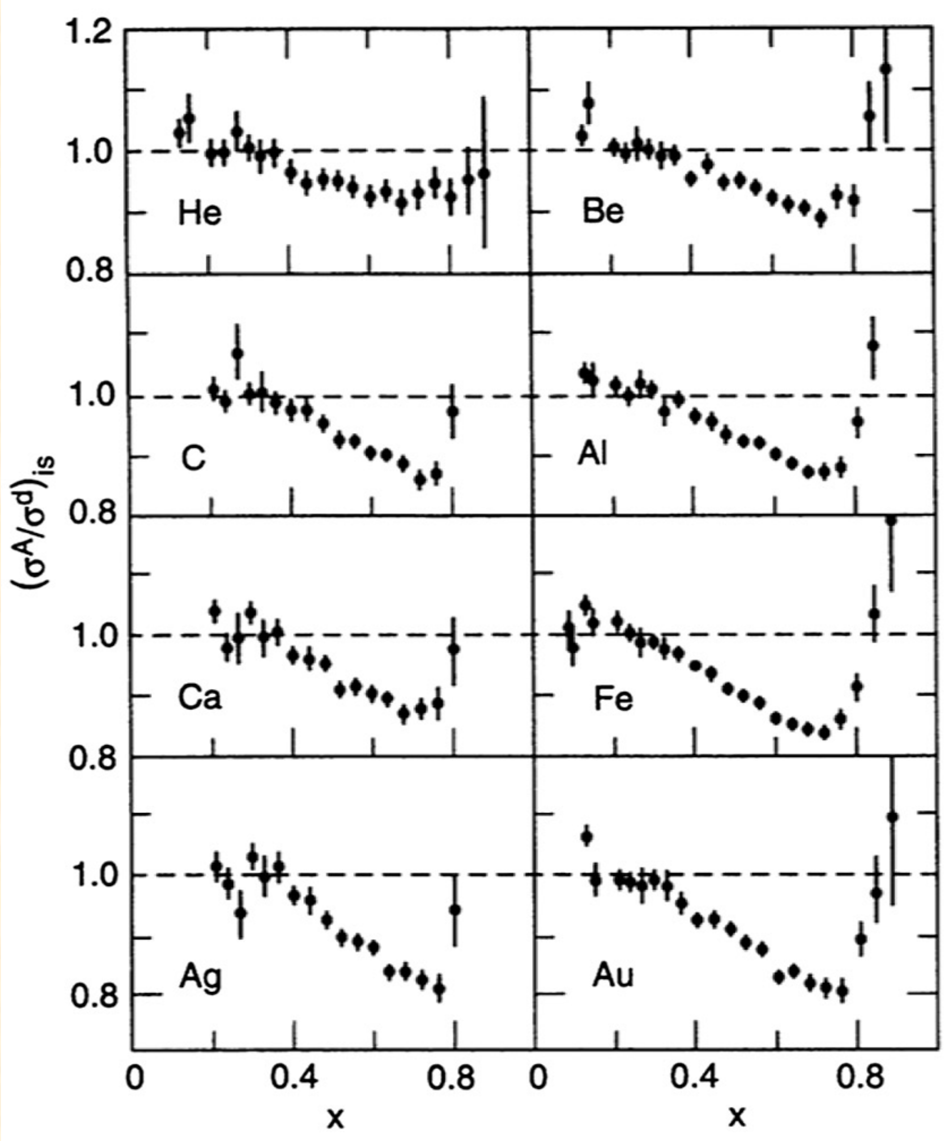
In the EMC paper of 1983, they to pointed out that nuclear modifications exist in a deep Inelastic structure function F_2.

In general, nuclear binding energies are negligible in comparison with typical DIS energies (Q, ν), so that such modifications were expected to be small.

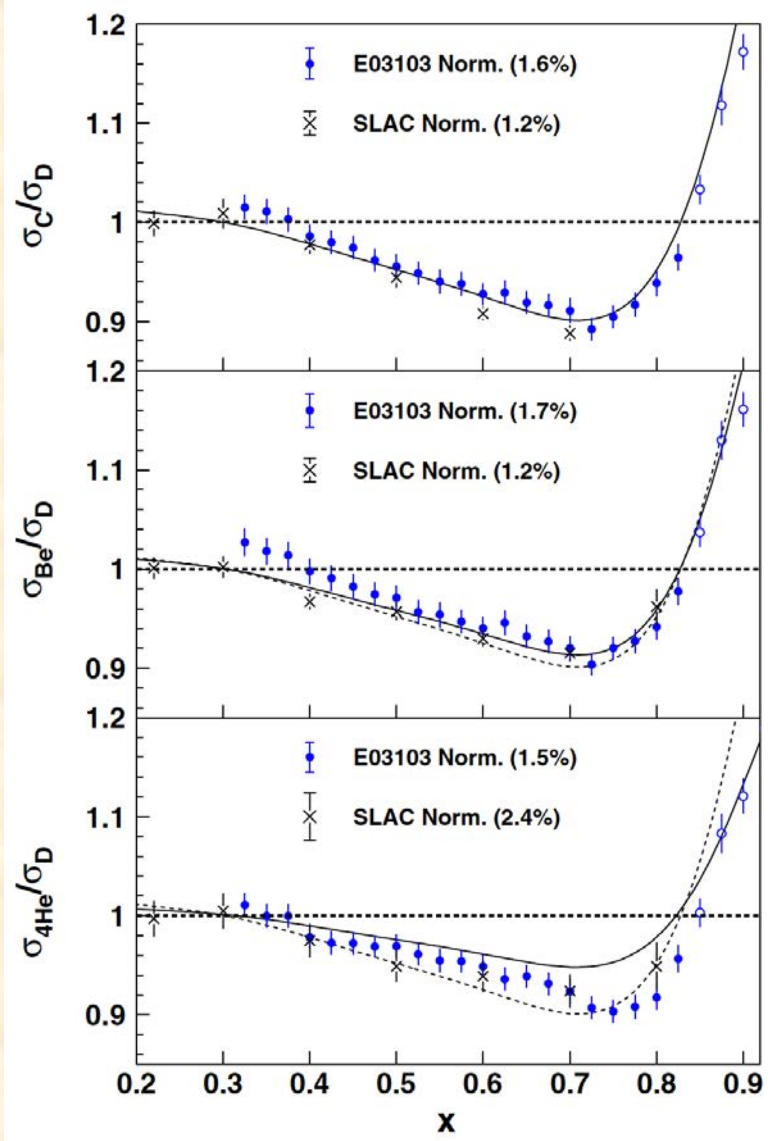
Fermi motion effects were theoretically calculated before the EMC publication. →



Nuclear modifications at SLAC and JLab

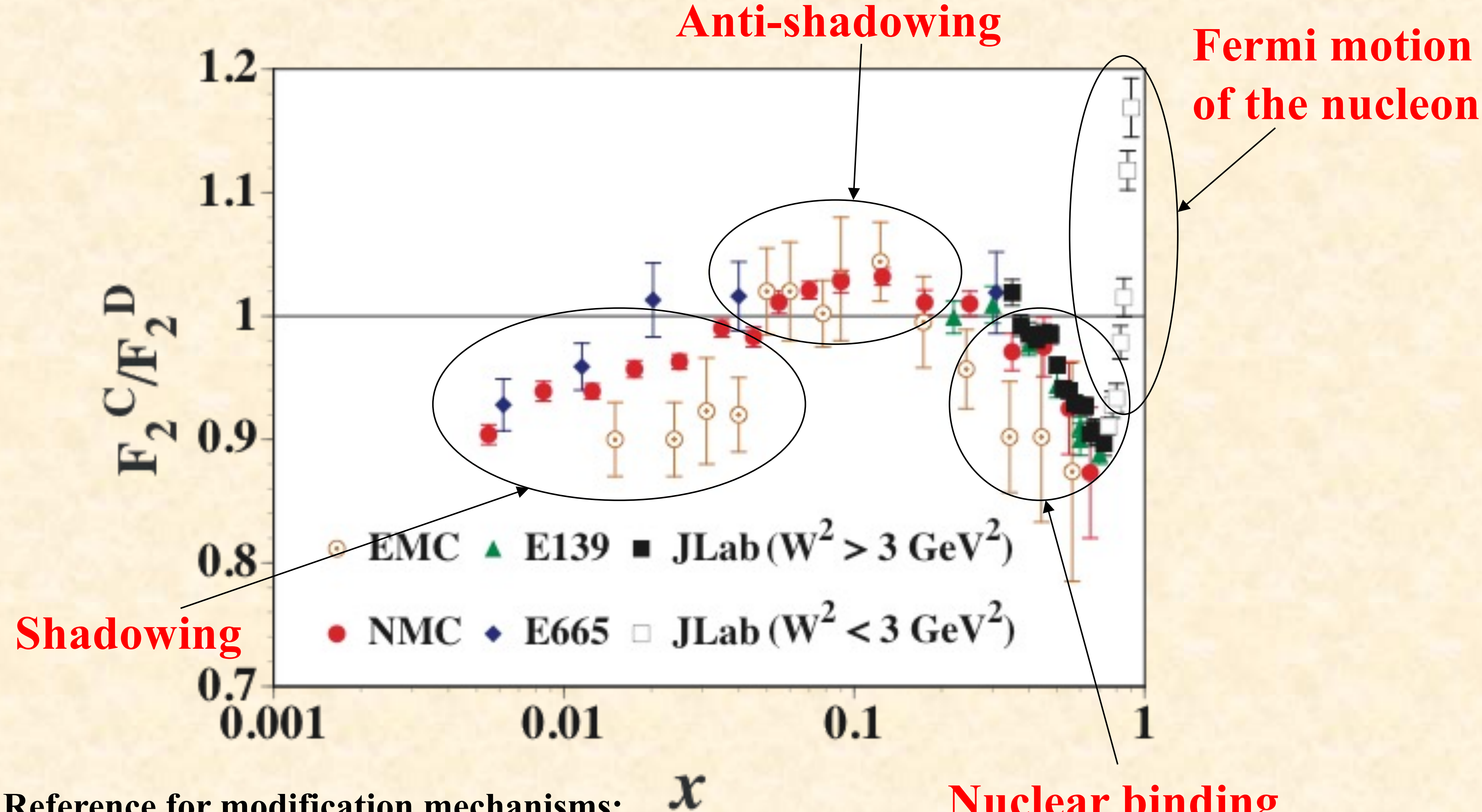


SLAC: J. Gomez *et al.*, Phys. Rev. D 49 (1994) 4348.



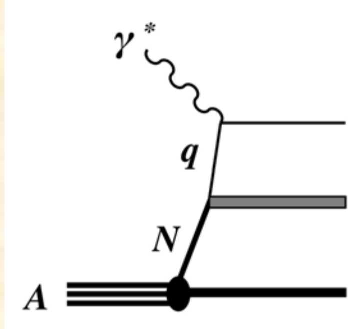
JLab: J. Seely *et al.*, Phys. Rev. Lett. 103 (2009) 202301.

Nuclear modifications of structure function F_2



Reference for modification mechanisms:
D. F. Geesaman, K. Saito, A. W. Thomas, Ann. Rev. Nucl. Part. Sci. 45 (1995) 337.

Binding and Fermi motion



Convolution: $W_{\mu\nu}^{A}(p_A, q) = \int d^4p\, S(p)\, W_{\mu\nu}^{N}(p_N, q)$

$S(p)$ = Spectral function = nucleon momentum distribution in a nucleus

In a simple shell model: $S(p) = \sum_i \left|\phi_i(\vec{p})\right|^2 \delta(p_0 - M_N - \varepsilon_i)$

Separation energy: ε_i

$$\hat{P}_2^{\mu\nu} = -\frac{M_N^2 \nu}{2\tilde{p}^2}\left(g^{\mu\nu} - \frac{3\tilde{p}^{\mu}\tilde{p}^{\nu}}{\tilde{p}^2}\right)$$

$$\hat{P}_2^{\mu\nu} W_{\mu\nu} = F_2$$

Projecting out F_2: $F_2^A(x, Q^2) = \sum_i \int dz\, f_i(z)\, F_2^N(x/z, Q^2)$

$z = \dfrac{p \cdot q}{M_N \nu} \simeq \dfrac{p \cdot q}{p_A \cdot q / A} \simeq \dfrac{p^+}{p_A^+ / A}$ lightcone momentum fraction

$$a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$

$$q = (\nu,\ 0,\ 0,\ -\sqrt{\nu^2 + Q^2})$$

$$q^+ = -\frac{M x}{\sqrt{2}},\quad q^- = \frac{2\nu + M x}{\sqrt{2}} = \sqrt{2}\nu \gg M$$

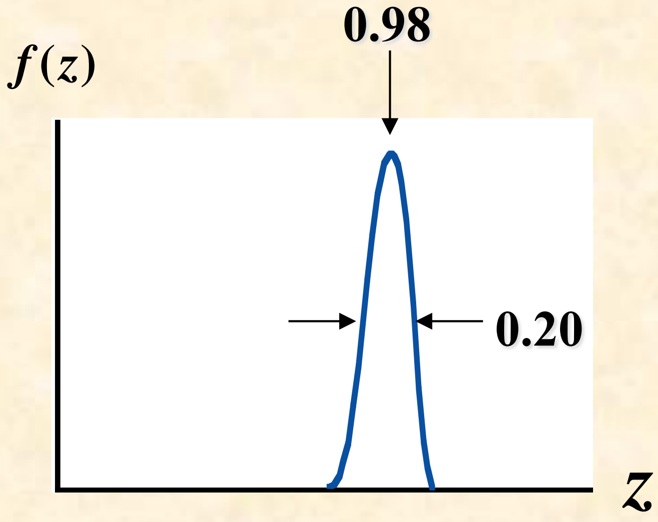
$$p \cdot q = p^+ q^- + p^- q^+ - \vec{p}_T \cdot \vec{q}_T \simeq p^+ q^-$$

$f_i(z) = \int d^3p\, z\, \delta\left(z - \dfrac{p \cdot q}{M_N \nu}\right) \left|\phi_i(\vec{p})\right|^2$ lightcone momentum distribution for a nucleon i

$$F_2^A(x,Q^2) = \sum_i \int dz\, f_i(z)\, F_2^N(x/z,Q^2) \qquad f_i(z) = \int d^3p\, z\, \delta\!\left(z - \frac{p\cdot q}{M_N \nu}\right) |\phi_i(\vec{p})|^2$$

$$z = \frac{p\cdot q}{M_N \nu} = \frac{p^0 \nu - \vec{p}\cdot\vec{q}}{M_N \nu} = 1 - \frac{|\varepsilon_i|}{M_N} - \frac{\vec{p}\cdot\vec{q}}{M_N \nu} \approx 1.00 - 0.02 \pm 0.20 \ \text{ for a medium-size nucleus}$$

If $f_i(z)$ were $f_i(z) = \delta(z-1)$, there is no nuclear modification: $F_2^A(x,Q^2) = F_2^N(x,Q^2)$.

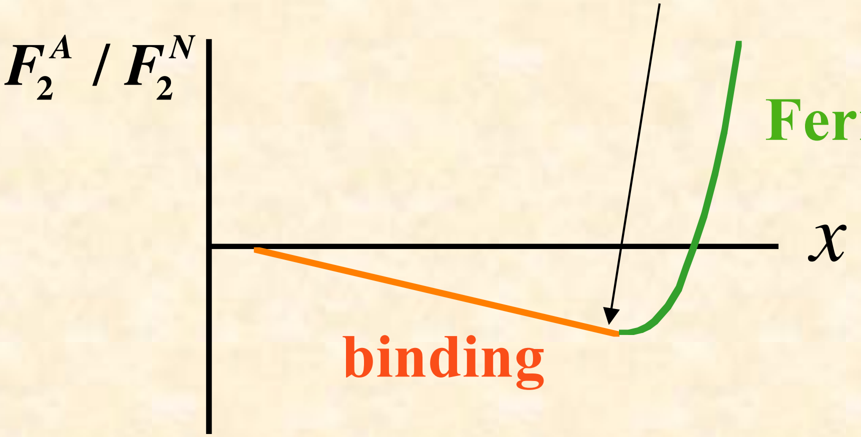


Because the peak shifts slightly (1→ 0.98), nuclear modification of F_2 is created.

$$F_2^A(x,Q^2) \simeq F_2^N(x/0.98,Q^2)$$

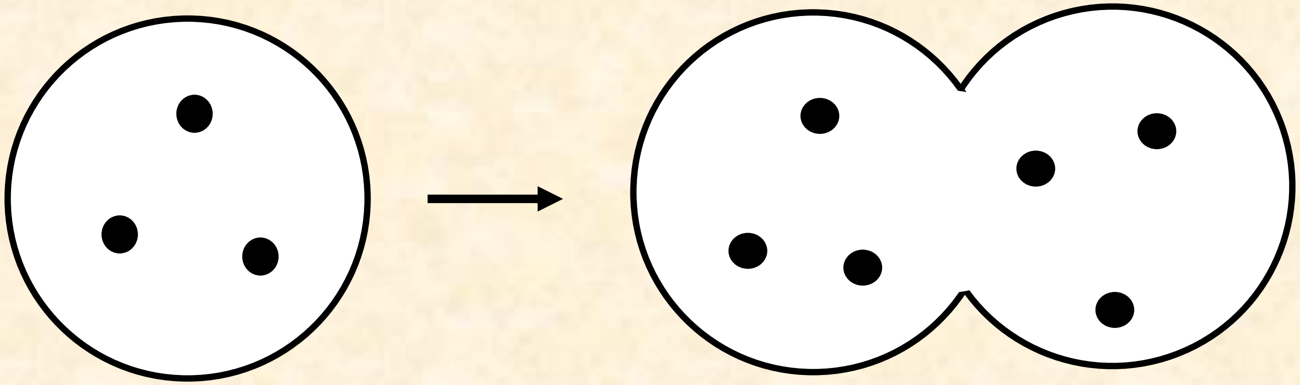
For $x = 0.60$, $x/0.98 = 0.61$

$$\frac{F_2^N(x=0.61)}{F_2^N(x=0.60)} = \frac{0.021}{0.024} = 0.88$$



Fore more details, see *e. g.*
K. Saito, A. Michels, A. W. Thomas, PRC 46 (1992) R2149;
K. Saito, A. W. Thomas, NPA 574 (1994) 659;
H. Mineo, W. Bentz, N. Ishii, A. W. Thomas, K. Yazaki, NPA 735 (2004) 482.

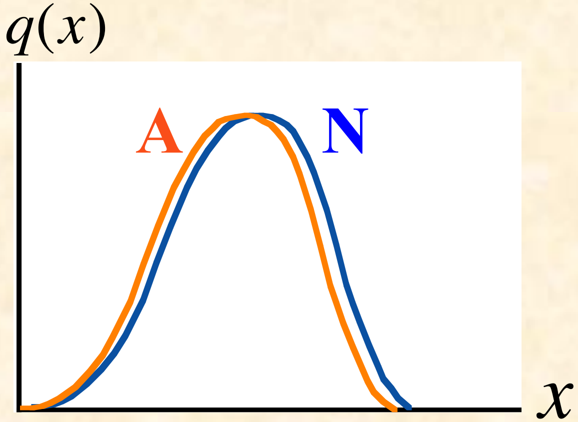
Theoretical ideas at medium x: Q^2 Rescaling Model



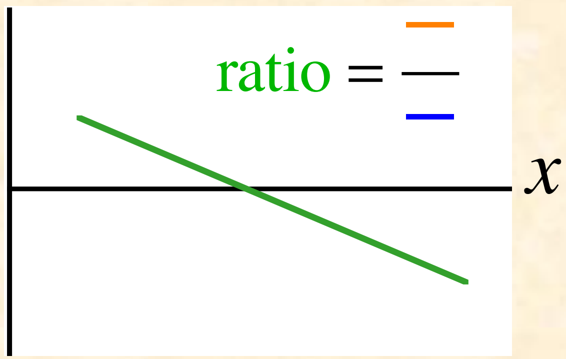
Free nucleon **Nucleon may overlap in a nucleus.**

Average nucleon separation (2 fm) ≈ Nucleon diameter

Confinement radius changes: $\lambda_A > \lambda_N$



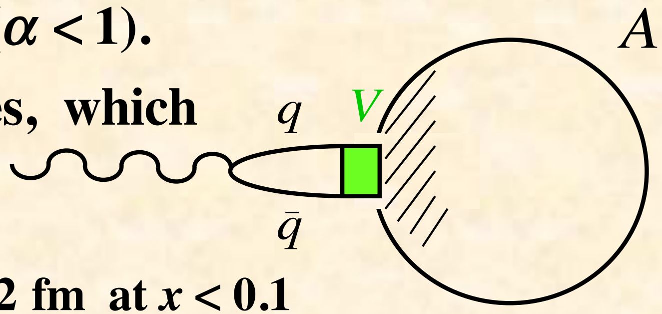
Quark momentum distribution changes.



Ratio $q_A(x)/q_N(x)$ is similar to the observed EMC data in 1983.

Shadowing

- Shadowing means that internal constituents are shadowed due to the existence of nuclear surface ones, so that the cross section is smaller than the each nucleon contribution: $\sigma_A = A^{\alpha}\sigma_N \ (\alpha < 1)$.
- A virtual photon transforms into vector meson (or $q\bar{q}$) states, which then interact with a target nucleus.

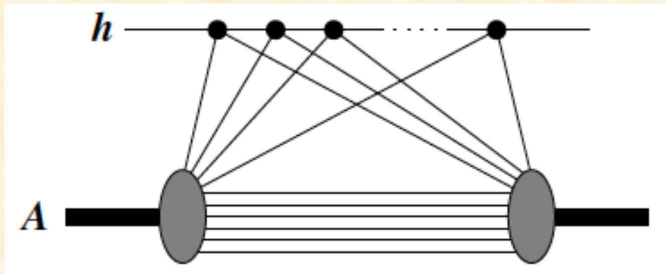


Propagation length of V ($q\bar{q}$): $\lambda = \dfrac{1}{|E_V - E_\gamma|} = \dfrac{2\nu}{M_V^2 + Q^2} = \dfrac{0.2\ \text{fm}}{x} > 2\ \text{fm}$ at $x < 0.1$

At small x, the virtual photon interacts with the target nucleus as if it were a vector meson (or $q\bar{q}$).

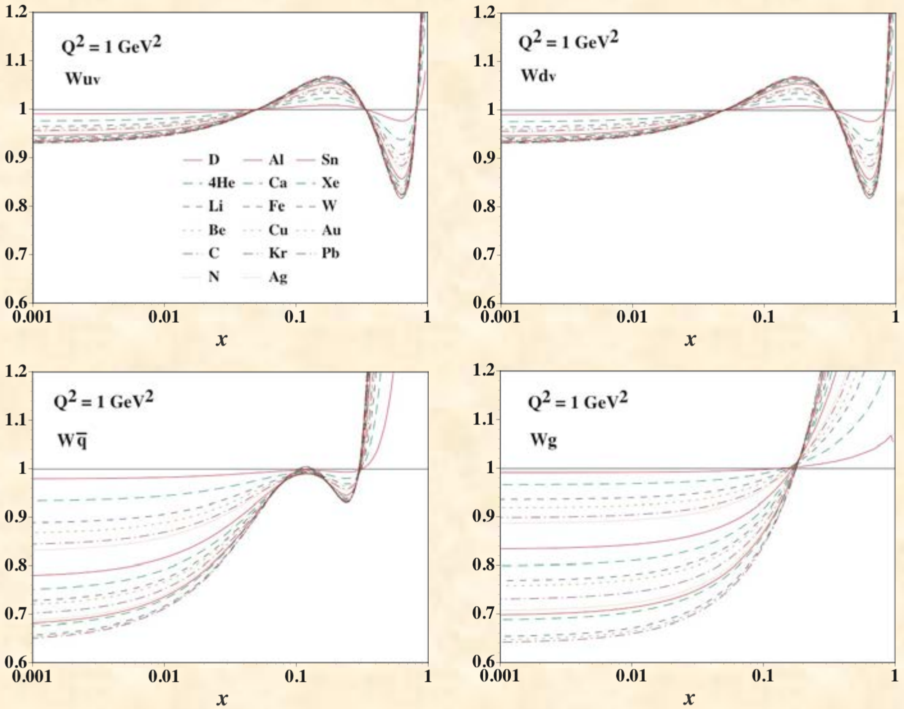
- Shadowing takes place due to multiple scattering.

For example, the vector meson interacts elastically with a surface nucleon and then interacts inelastically with a central nucleon.
Because this amplitude is opposite in phase to the one-step amplitude for an inelastic interaction with the central nucleon, the nucleon sees a reduced hadronic flux (namely the shadowing).

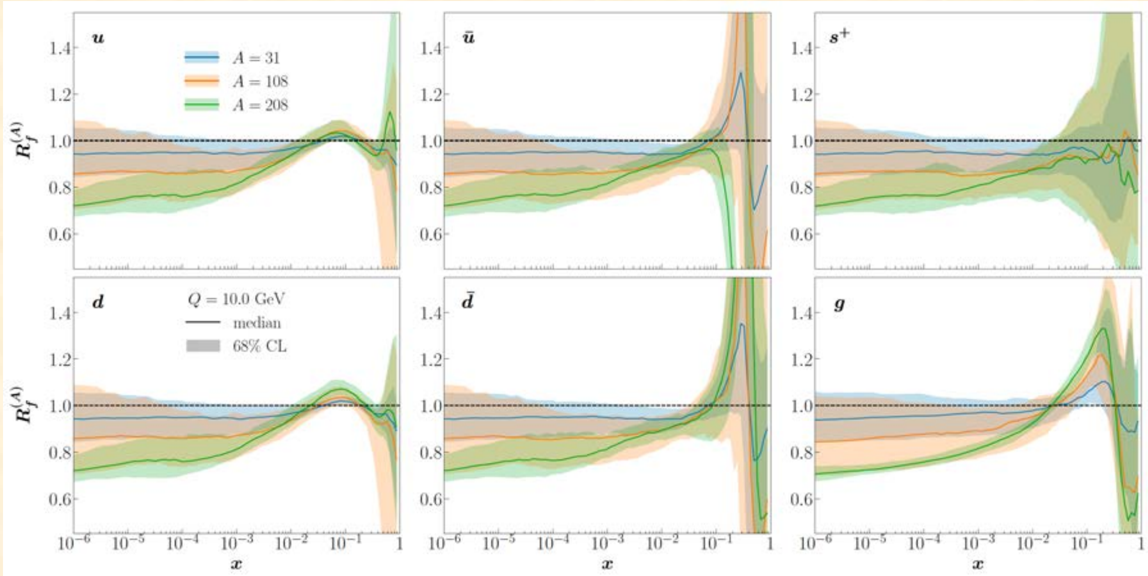


Nuclear PDFs

M. Hirai, SK, T.-H. Nagai, PRC 76 (2007) 065207.



For a recent update, for example, see nNNPDF3.0
R. A. Khalek *et al*., arXiv:2201.12363.



Cluster structure in nuclear structure functions

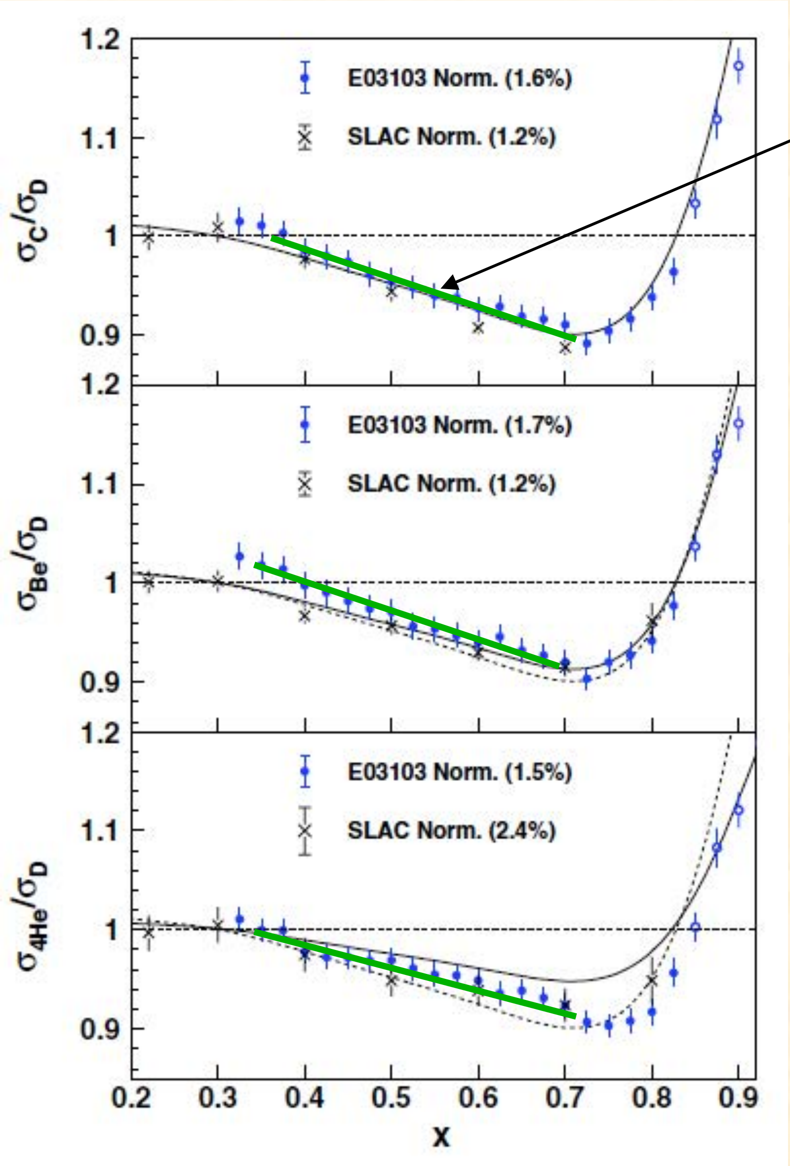
J. Seely *et al*., PRL 103 (2009) 202301;

M. Hirai, SK, K. Saito, and T. Watanabe, PRC 83 (2011) 035202.

JLab "anomaly" on ^{9}Be

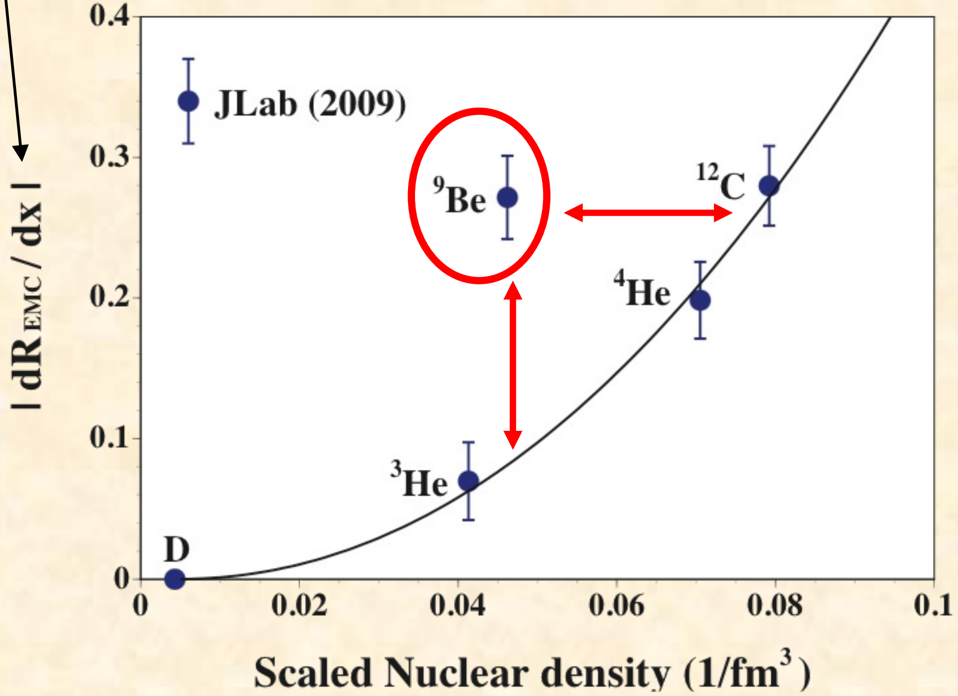
anomalous at the stage of 2009 but it is not now

J. Seely *et al.*,
Phys. Rev. Lett. 103 (2009) 202301.



Slope: $\dfrac{dR_{EMC}}{dx}$, $R_{EMC} = \dfrac{\sigma_A}{\sigma_D}$

^{9}Be anomaly = EMC slope is too large
to be estimated from its nuclear density



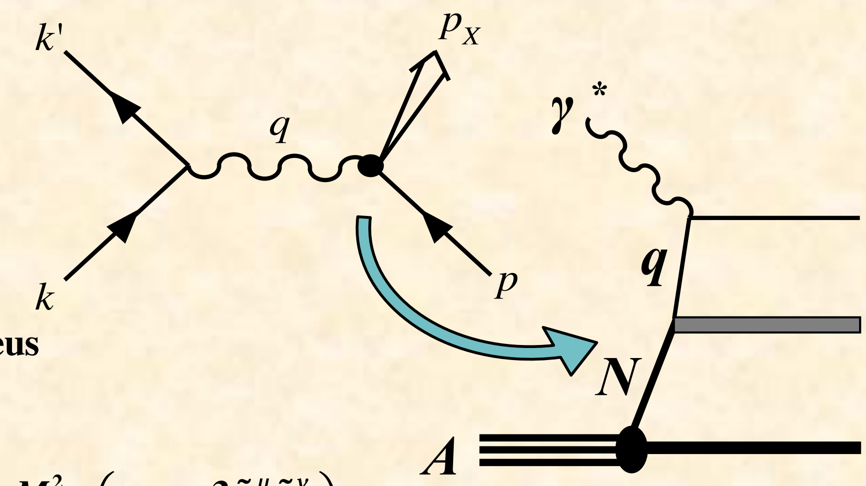
Convolution formalism

M. Ericson and SK,
PRC 67 (2003) 022201.

Charged-lepton deep inelastic scattering from a nucleus

$d\sigma \sim L^{\mu\nu} W^{A}_{\mu\nu}, \quad L^{\mu\nu}$ = Lepton tensor,

Hadron tensor: $W_{\mu\nu} = \dfrac{1}{4\pi} \int d^4\xi \; e^{iq\cdot\xi} \langle p | \left[J^{em}_{\mu}(\xi), J^{em}_{\nu}(0) \right] | p \rangle$



Convolution: $W^{A}_{\mu\nu}(p_A, q) = \int d^4 p \, S(p) \, W^{N}_{\mu\nu}(p_N, q)$

$S(p)$ = Spectral function = nucleon momentum distribution in a nucleus

In a simple model: $S(p_N) = |\phi(\vec{p}_N)|^2 \, \delta\left(p^0_N - M_A + \sqrt{M^2_{A-1} + \vec{p}^{\,2}_N} \right)$

F_2 needs to be projected out from $W_{\mu\nu}$ by the projection operator $\hat{P}^{\mu\nu}_2 = -\dfrac{M_N^2 \nu}{2\tilde{p}^2} \left(g^{\mu\nu} - \dfrac{3\tilde{p}^{\mu}\tilde{p}^{\nu}}{\tilde{p}^2} \right)$:

$$W_{\mu\nu} = -F_1 \frac{1}{M_N} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) + F_2 \frac{\tilde{p}_\mu \tilde{p}_\nu}{M_N^2 \nu}, \quad \tilde{p}_\mu = p_\mu - \frac{p \cdot q}{q^2} q_\mu; \qquad \hat{P}^{\mu\nu}_2 W_{\mu\nu} = F_2$$

$$F^A_2(x, Q^2) = \hat{P}^{\mu\nu}_2(A) W^A_{\mu\nu}(p_A, q) = \int d^4 p \, S(p) \, \hat{P}^{\mu\nu}_2(A) W^N_{\mu\nu}(p_N, q)$$

already explained, skip this page

We obtain $F^A_2(x, Q^2) = \int dy \, f(y) F^N_2(x/y, Q^2), \qquad f(y) = \int d^3 p_N \, y \, \delta\left(y - \dfrac{p_N \cdot q}{M_N \nu} \right) |\phi(\vec{p}_N)|^2$

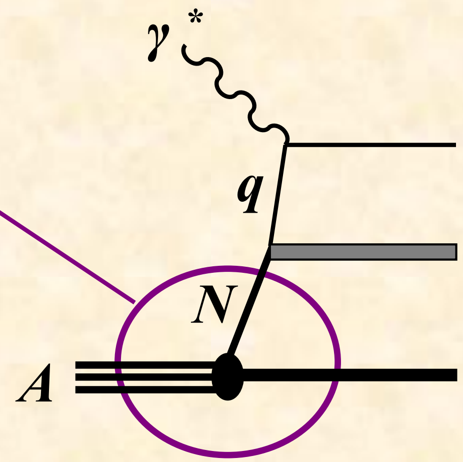
$f(y)$ = lightcone momentum distribution for a nucleon

$y = \dfrac{p_N \cdot q}{M_N \nu} = \dfrac{p^0_N \nu - \vec{p}_N \cdot \vec{q}}{M_N \nu} \simeq \dfrac{p_N \cdot q}{p_A \cdot q / A} \simeq \dfrac{p^+_N}{p^+_A / A} \simeq$ lightcone momentum fraction, $\quad p^{\pm} = \dfrac{p^0 \pm p^3}{\sqrt{2}}$

We estimate $F^A_2(x, Q^2) = \int dy \, f(y) F^N_2(x/y, Q^2), \qquad f(y) = \int d^3 p_N \, y \, \delta\left(y - \dfrac{p_N \cdot q}{M_N \nu} \right) \rho(p_N)$

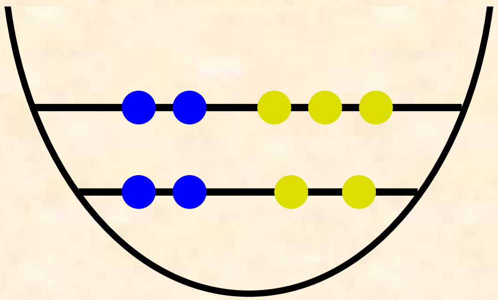
by calculating the nucleon momentum distribution $\rho(p_N)$ by a shell model and anti-symmetrized molecular dynamics (AMD).

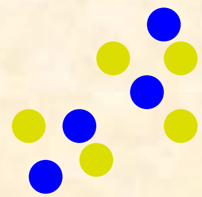
Two theoretical models



$$F_2^A(x,Q^2)=\int dy\, f(y) F_2^N(x/y,Q^2), \quad f(y)=\int d^3p_N\, y\, \delta\left(y-\frac{p_N\cdot q}{M_N \nu}\right)\rho(p_N)$$

Nuclear density $\rho(p_N)$ is calculated by
(1) Simple shell model
(2) Anti-symmetrized molecular dynamics (AMD)
or Fermionic molecular dynamics (FMD)





AMD / FMD: Variational method with effective NN potentials

Review: (AMD) Y. Kanada-En'yo, M. Kimura, H. Horiuchi, C. R. Physique 4 (2003) 497;
(FMD) H. Feldmeier and J. Schnack, Rev. Mod. Phys. 72 (2000) 655.

Simple shell model

$$\psi_{nlm}(r,\theta,\phi)=R_{nl}(r)Y_{lm}(\theta,\phi)$$

$$R_{nl}(r)=\sqrt{\frac{2\kappa^{2\ell+3}(n-1)!}{[\Gamma(n+\ell+1/2)]^3}}\times r^{\ell} e^{-\frac{1}{2}\kappa^2 r^2} L_{n-1}^{\ell+1/2}(\kappa^2 r^2)$$

$$\kappa^2 \equiv M_N\omega, \qquad V=\frac{1}{2}M_N\omega^2 r^2$$

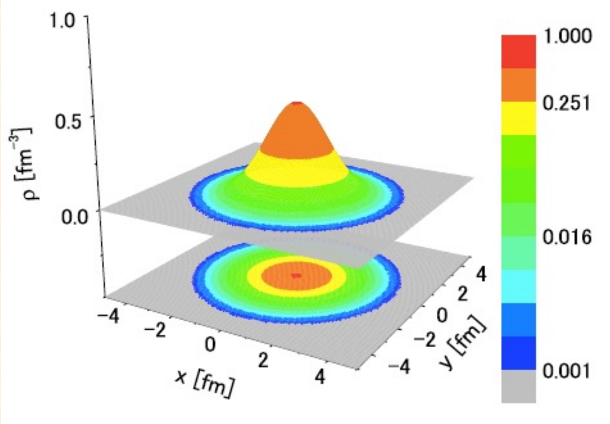
Slater determinant: $$\left|\Phi(\vec r_1,\vec r_2,\cdots,\vec r_A)\right\rangle=\frac{1}{\sqrt{A!}}\begin{vmatrix}\varphi_1(\vec r_1) & \varphi_1(\vec r_2) & \cdots & \varphi_1(\vec r_A)\\ \varphi_2(\vec r_1) & \varphi_2(\vec r_2) & \cdots & \varphi_2(\vec r_A)\\ . & . & \cdots & .\\ \varphi_A(\vec r_1) & \varphi_A(\vec r_2) & \cdots & \varphi_A(\vec r_A)\end{vmatrix}$$

Single-particle wave function: $$\varphi_i(\vec r_j)=\phi_i(\vec r_j)\chi_i\tau_i, \quad \phi_i(\vec r_j)=\left(\frac{2\nu}{\pi}\right)^{3/4}\exp\left[-\nu\left(\vec r_j-\frac{\vec Z_i}{\sqrt{\nu}}\right)^2\right]$$

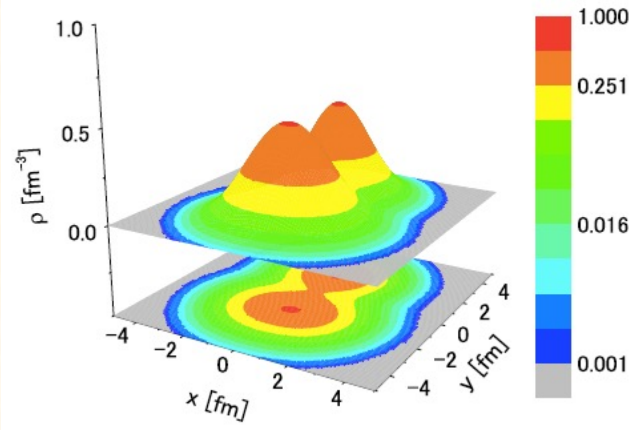
Parameters are determined by a variational method with effective NN potentials with parameters mainly from Y. Kanada En'yo, H. Horiuchi, and A. Ono, PRC 52 (1995) 628.

Cluster structure in ^{9}Be

Density distributions
in ^{4}He and ^{9}Be
by AMD



^{4}He



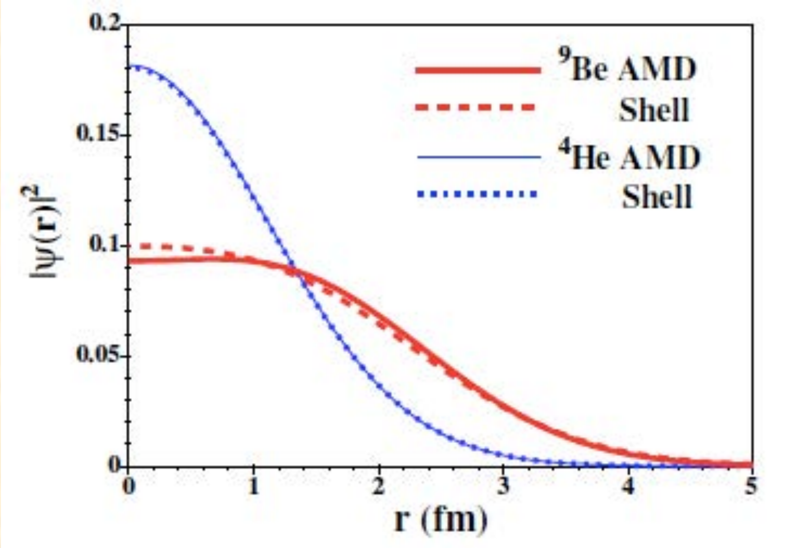
^{9}Be (~ ^{4}He + ^{4}He + n)

Two models:

(1) Shell model

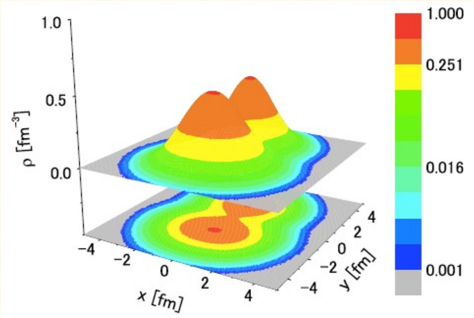
(2) AMD (antisymmetrized molecular dynamics) to describe clustering structure

Space (r) distributions

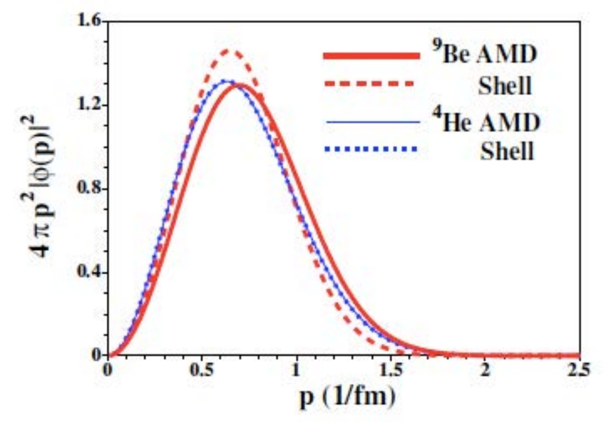


However, if the densities are averaged over the polar and azimuthal angles, differences from shell structure are not so obvious although there are some differences in ^{9}Be in comparison with ^{4}He.

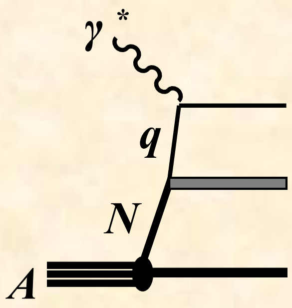
EMC (European muon collaboration) effect



Momentum distributions

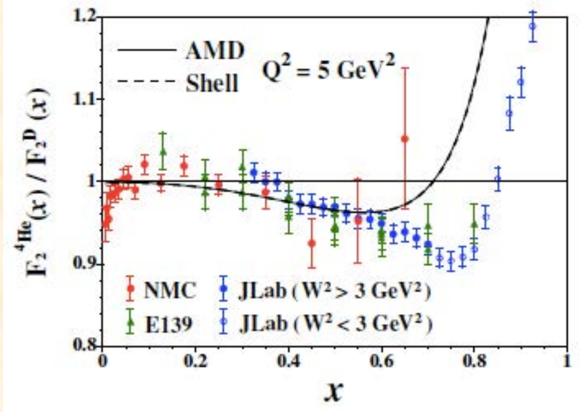


Convolution model

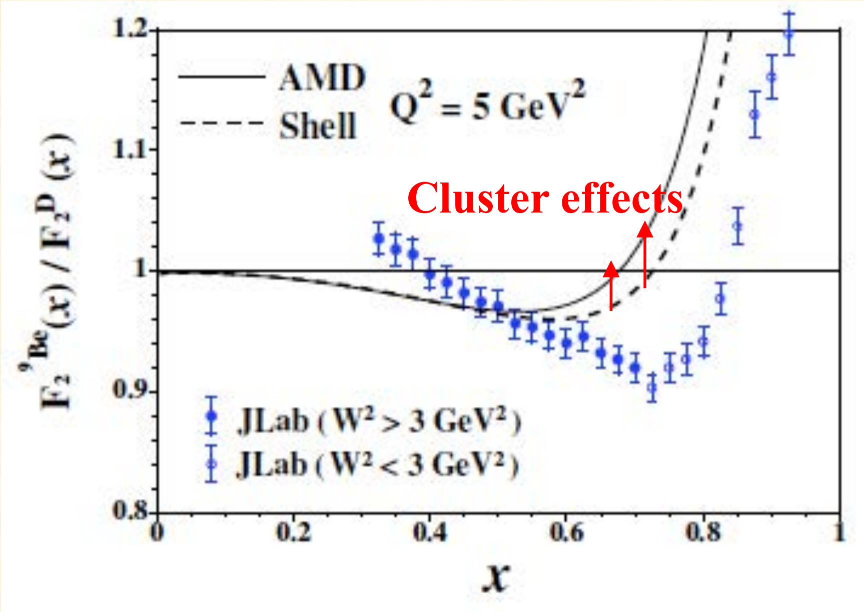


$$F_2^A(x, Q^2) = \int_x^A dy\, f(y)\, F_2^N(x/y, Q^2)$$

^{4}He



^{9}Be

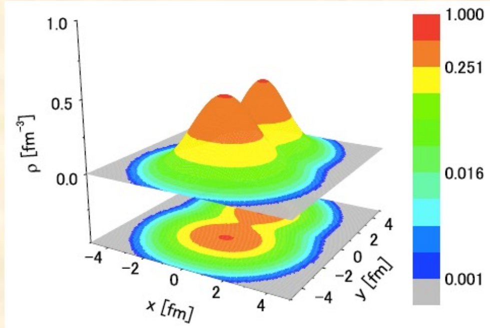


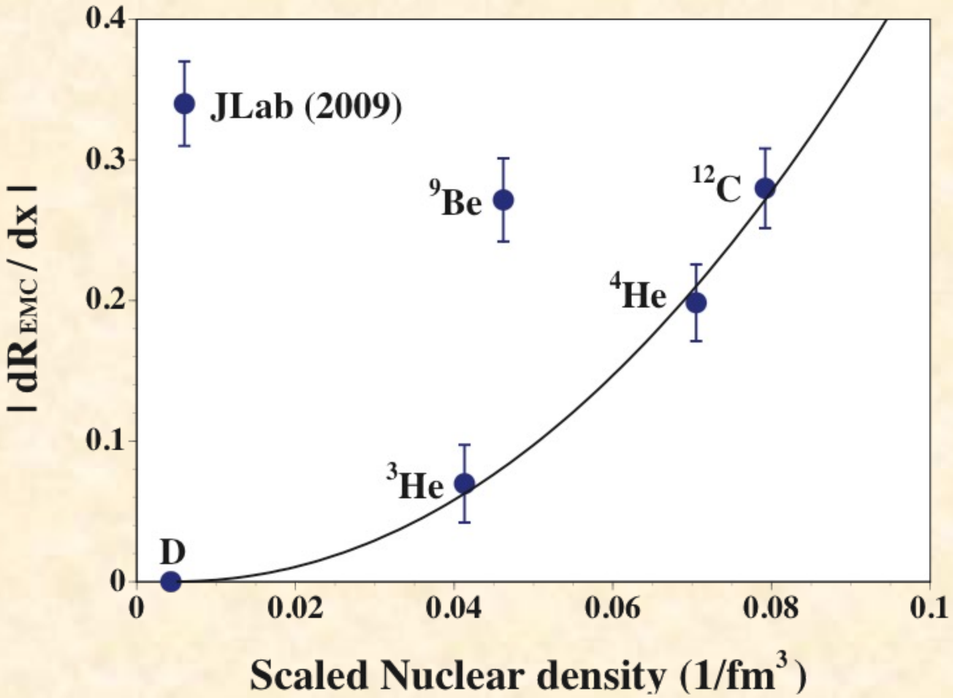
It seems that the mean conventional part cannot explain the large modification of ^{9}Be.

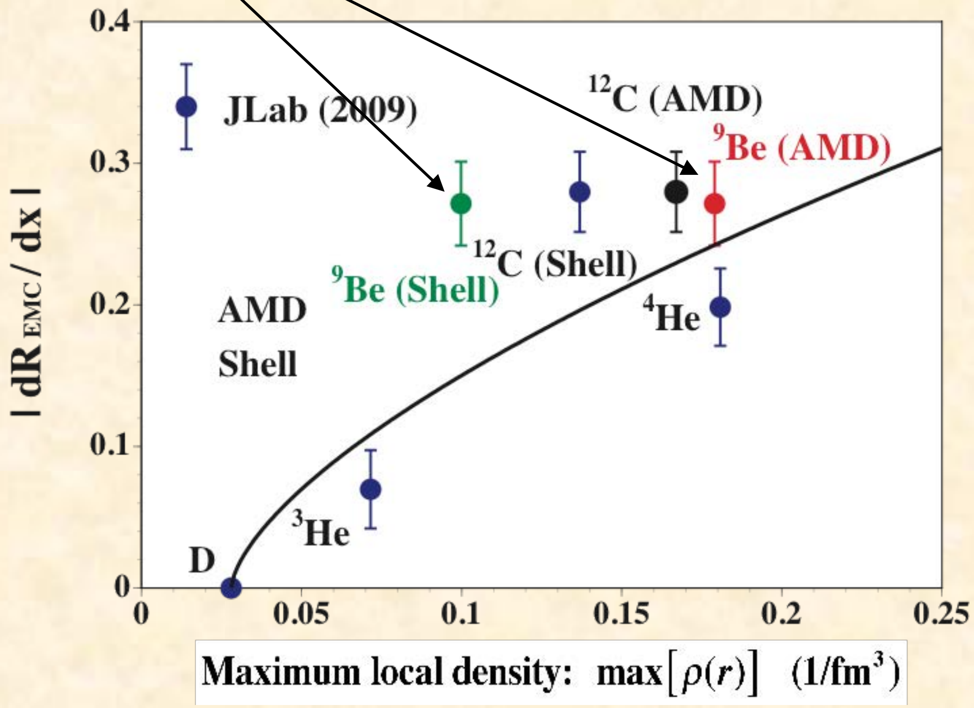
→ Plot the data by the maximum local density created by the cluster formation in ^{9}Be.

EMC slopes plotted by maximum local densities

The ^{9}Be anomaly can be explained by the high-densities, which are created by clustering in the ^{9}Be nucleus.







Original JLab figure → **Plotted by the maximum local densities**

It could be understood by the two-nucleon correlation factor a_2 (shown later).

Our results indicate

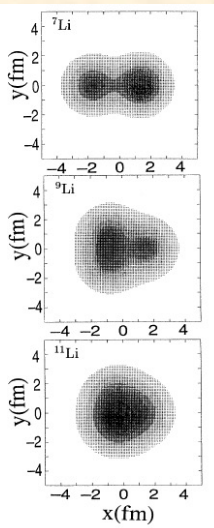
$F_2^A =$ (mean part) + (part created by large densities due to cluster formation)

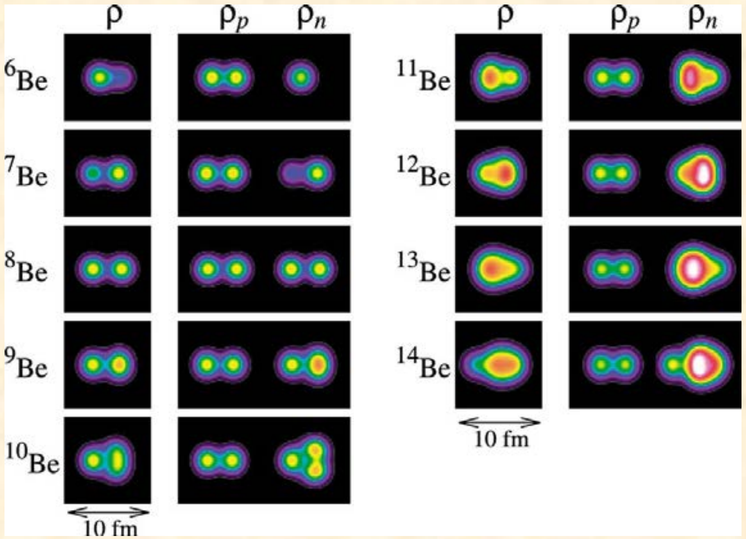
Convolution model indicates clustering effects are small in this term.

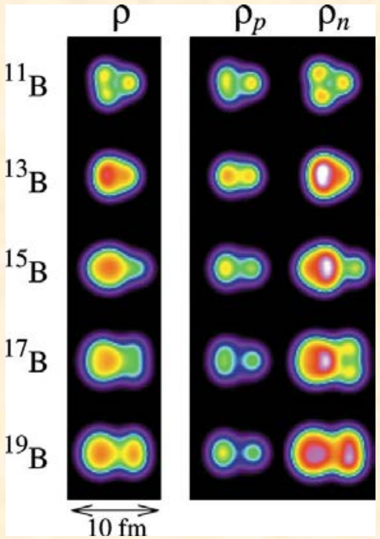
JLab data could be related to this effect due to the nuclear cluster.

In addition to ^{9}Be, ^{6}Li, ^{7}Li, ^{10}B, ^{11}B, … could be measured.

For example, Y. Kanda En'yo, H. Horiuchi, and A. Ono, PRC 52 (1995) 628;
A. Dote, H. Horiuchi, and Y. Kanda En'yo, PRC 56 (1997) 1844;
Y. Kanda En'yo, M. Kimura, and H. Horiuchi, C. R. Physique 4 (2003) 497.







Cluster physicists have a good opportunity to play a major role in this research!

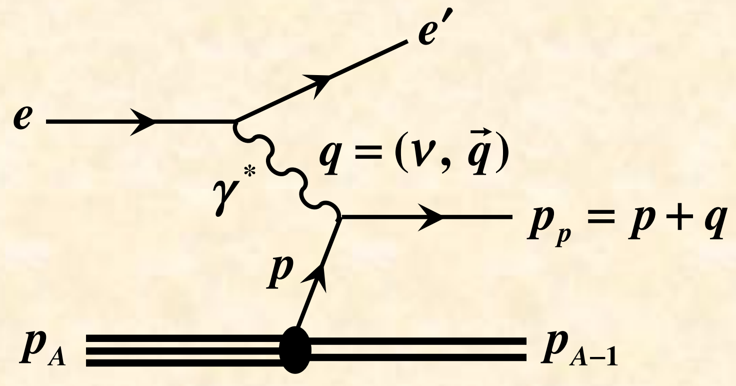
Short-range correlations
in electron scattering

Spectral function in electron scattering $A(e,e'p)$

$$\frac{d\sigma}{d\nu d\Omega_e dE d\Omega_p} = \frac{E_p p_p}{(2\pi)^3}\sigma_{ep} S(E,\vec{p})$$

$$E = \nu - T_p - T_{A-1}, \quad \vec{p} = \vec{q} - \vec{p}_p$$

T. de Forest, Nucl. Phys. A 392 (1983) 232;
J. J. Kelly, Adv. Nucl. Phys. 23 (1996) 75.

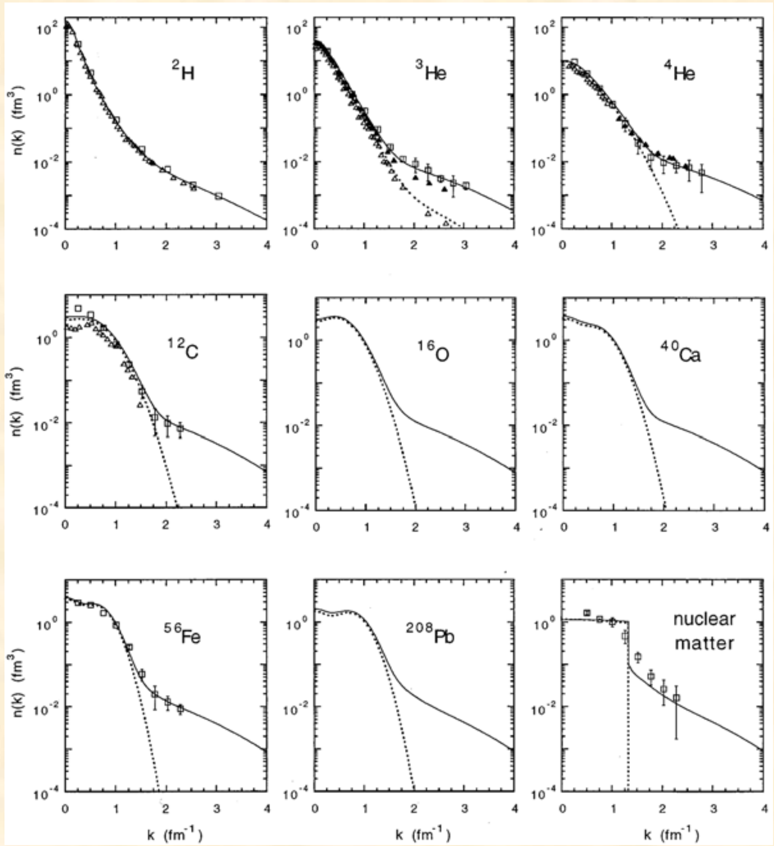


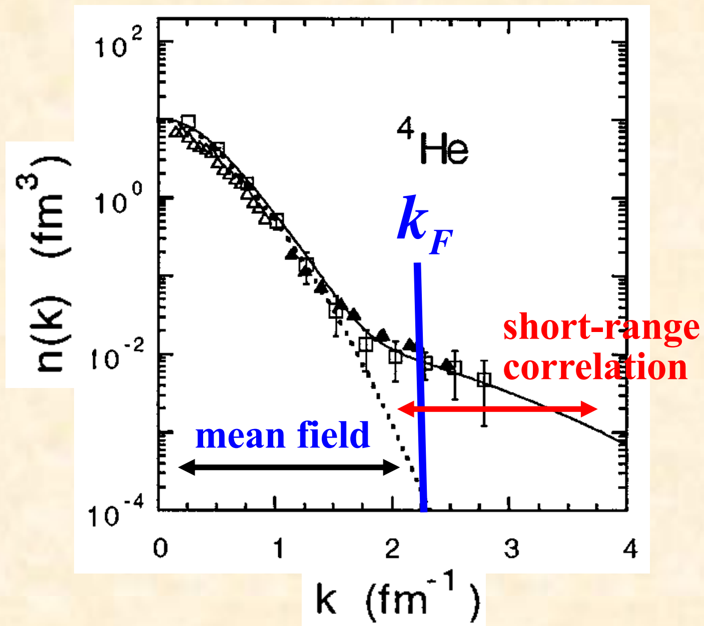
$S(E,\vec{p})$ = spectral function:

= probability of finding a nucleon in the nucleus with the momentum $\vec{p}$ and the separation energy E

$$n(k) = 4\pi \int dE\ S(E,k)$$

C. Ciofi degli Atti and S. Simula, PRC 53 (1996) 1689.

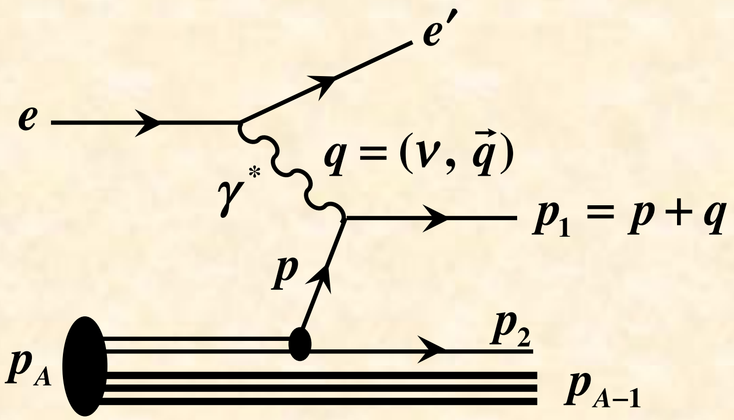


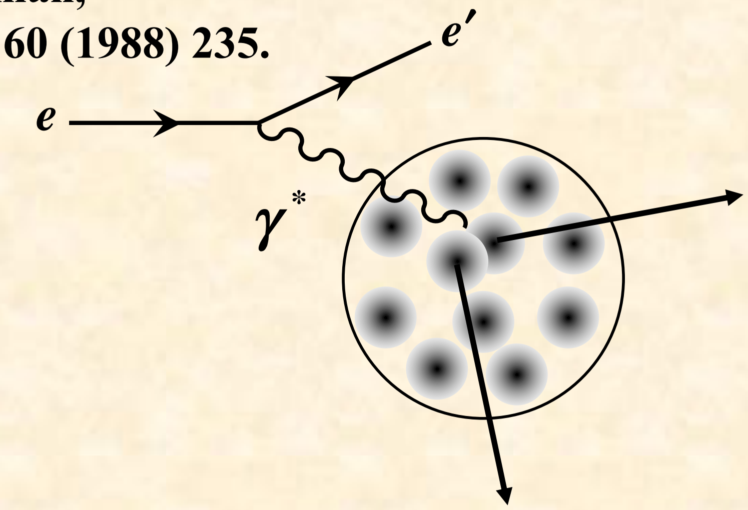


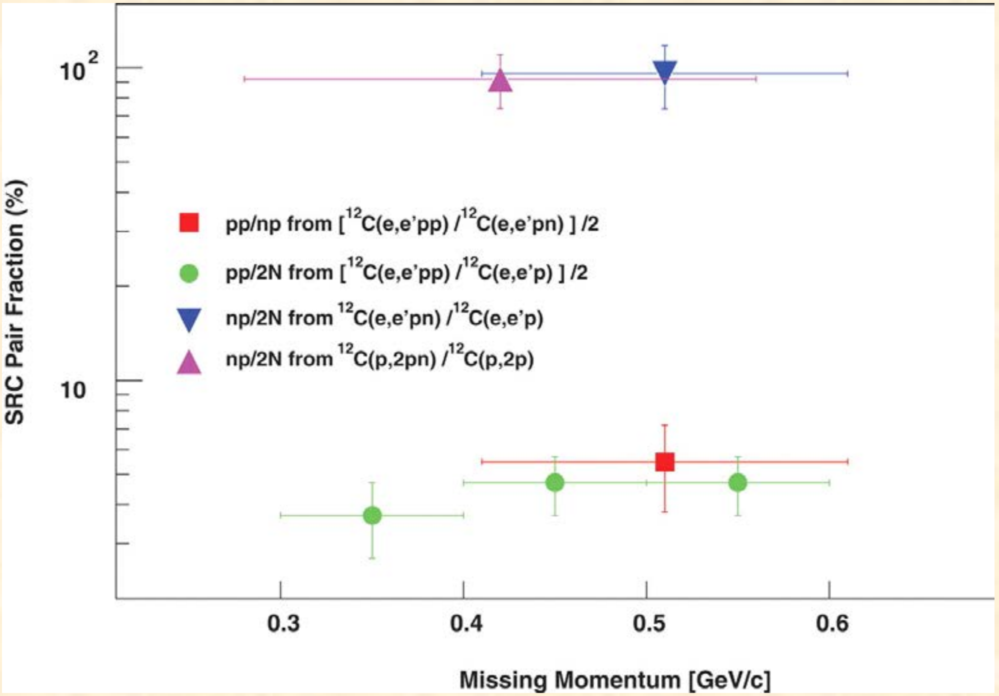
Isospin dependence of short-range correlation

two-nucleon knockout

L. Frankfurt and M. Strikman,
Phys. Rep. 76 (1981) 214; 160 (1988) 235.

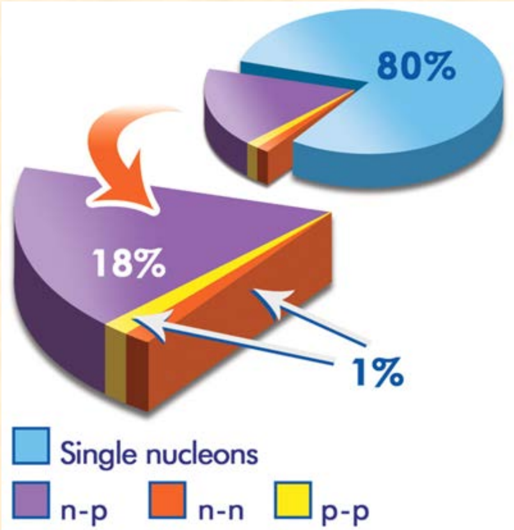






n-p ≫ p-p, n-n

R. Subedi *et al.*,
Science 320 (2008) 1476.



Fraction of nucleons in ^{12}C

Short-range correlation and tensor force

Tensor operator: $S_{12}(r) = \dfrac{3\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r}}{r^2} - \vec{\sigma}_1\cdot\vec{\sigma}_2 = \dfrac{1}{2}\left[\dfrac{3(\vec{S}\cdot\vec{r})^2}{r^2} - \vec{S}^2\right],\ \ \vec{S} = \vec{s}_1 + \vec{s}_2$

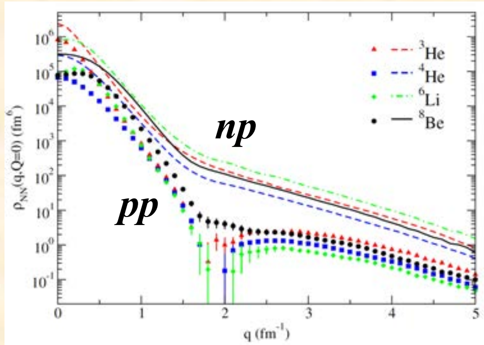
$S_{12}\chi(\text{spin singlet}) = 0,\ \ S_{12}\chi(\text{spin triplet}) \neq 0$

$\hat{P}(T=0,\, L_{\text{even}})$: projection into even parity state with $T=0$

$\hat{P}(T=1,\, L_{\text{odd}})$: projection into odd parity state with $T=1$

$$V_{\text{T}}(r) = \left[V_{\text{T}}^{\text{even}}(r)\,\hat{P}(T=0,\, L_{\text{even}}) + V_{\text{T}}^{\text{odd}}(r)\,\hat{P}(T=1,\, L_{\text{odd}})\right] S_{12}(r)$$

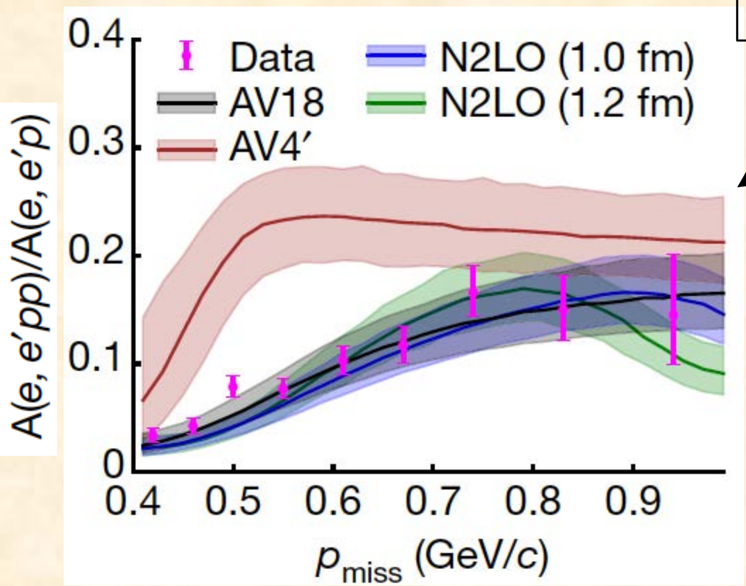
Probability of finding two nucleons with relative momentum q



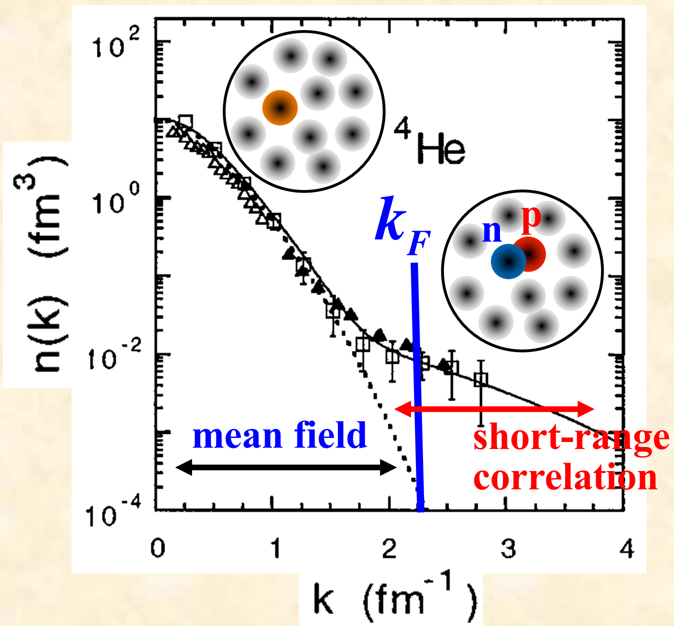
R. Schiavilla *et al*., PRL 98 (2007) 132501

AV4′ : no tensor force
AV18: with tensor force

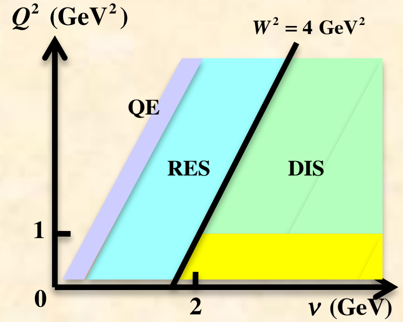
Strong isospin dependence is due to the tensor force.



A. Schmidt *et al*., Nature 578 (2020) 540.

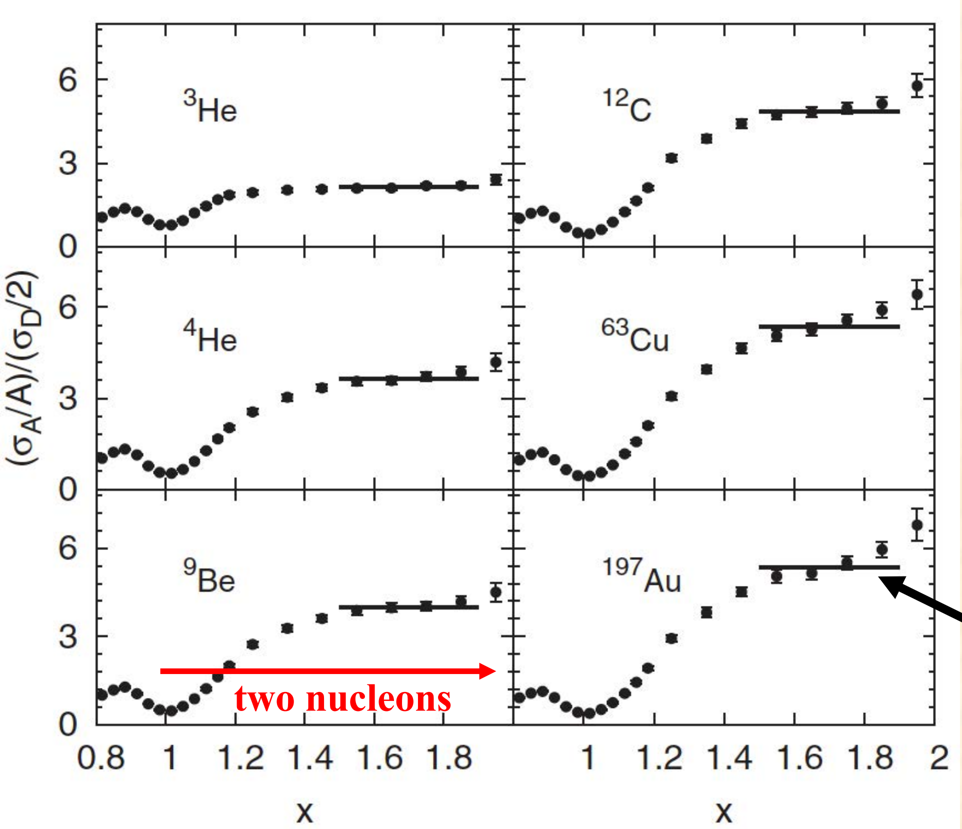


Inclusive quasi-elastic cross section



N. Fomin, *et al.*, PRL 108 (2012) 092502.

$x = \dfrac{Q^2}{2M_N \nu}$
large Q^2 (=2.7 - 4.8 GeV^2)
and small ν



Quasi-elastic cross section (D. B. Day *et al.*, PRL 59 (1987) 427)

$$\frac{d\sigma}{d\nu d\Omega} = F(y, Q^2)(Z\sigma_p + N\sigma_n)\frac{q}{\sqrt{M^2 + (y+q)^2}}$$

$\sigma_{p,n}$: elastic cross sections

$F(y, Q^2)$: probability to find a nucleon momentum $p_\parallel = y$

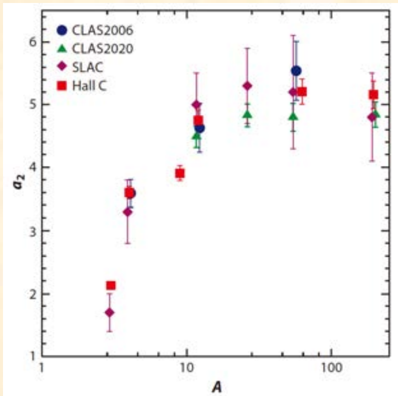
Nucleon momentum distribution: $n(p) = -\dfrac{1}{2\pi p}\dfrac{dF(y)}{dy}$

Cross-section ratio: $\dfrac{\sigma_{A_1}(x, Q^2)}{\sigma_{A_2}(x, Q^2)} \approx \dfrac{n_{A_1}(p)}{n_{A_2}(p)}$

Expand by j-nucleon correlations: $\sigma(x, Q^2) = \sum_{j=2}^{A} a_j(A)\sigma_j(x, Q^2)$

$\sigma_j(x, Q^2) = 0$ for $x > j$

Two-nucleon correlation factor: $a_2(A) = \dfrac{\sigma_A(x, Q^2)/A}{\sigma_D(x, Q^2)/2}$



J. Arrington *et al.*,
Annu. Rev. Nucl. Part. Sci. 72 (2022) 307.

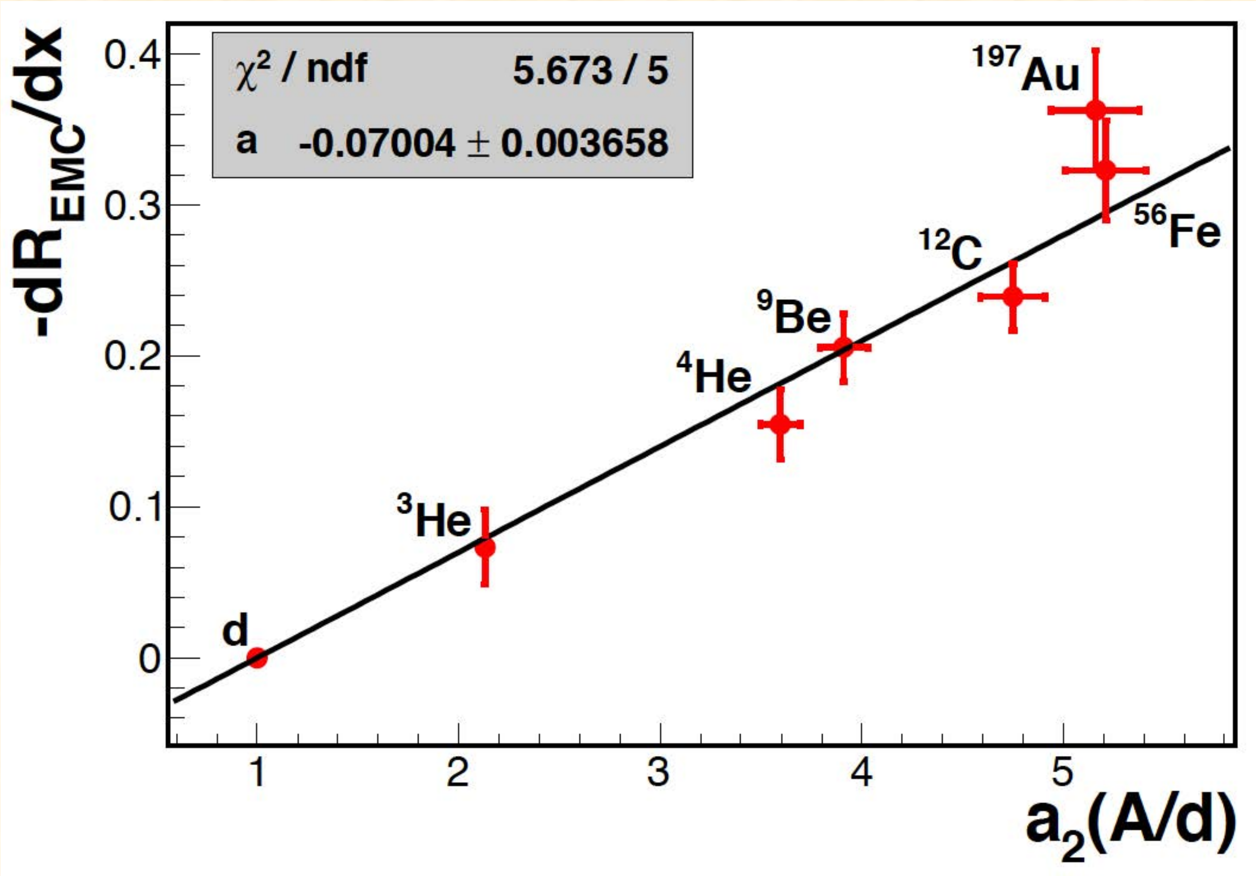
Scattering from a single nucleon: $x_{\max} = 1$

Scattering from two nucleons: $x_{\max} = 2$

Scattering from three nucleons: $x_{\max} = 3$

…

EMC slopes and two-nucleon short-range correlation factor a_2



O. Hen *et al*., Int. J. Mod. Phys. E 22 (2013) 1330017.

One of top 10 breakthroughs in physics world: Short-range correlations

https://physicsworld.com/a /top-10-breakthroughs-of-the-year-in-physics-for-2024-revealed/

Nuclear PDFs with the short-range correlations

A.W. Denniston *et al.*,
PRL 133 (2024) 152502.

$$S_A(E,k) = S_A^{\mathrm{MF}}(E,k) + S_A^{\mathrm{SRC}}(E,k) = \text{mean field} + \text{short-range correlation}$$

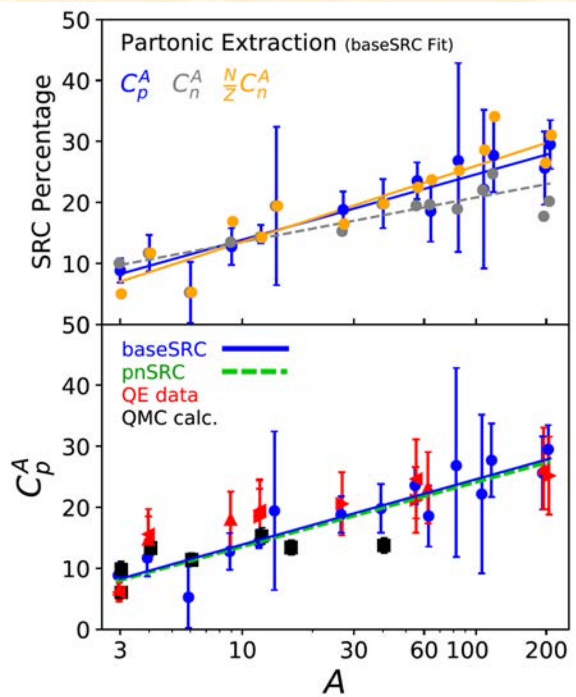
$$\approx S_A^{\mathrm{MF}}(E,k) + \frac{Z}{A} C_p^A S_p^{\mathrm{MF}}(E,k) + \frac{N}{A} C_n^A S_n^{\mathrm{MF}}(E,k), \quad C_{p,n}^A = \text{fraction of nucleons in SRC pairs (pn, pp, nn)}$$

Global nuclear PDF analyses

traditional: $f_i^{\,A}(x,Q^2) = \frac{Z}{A} f_i^{\,p}(x,Q^2) + \frac{N}{A} f_i^{\,n}(x,Q^2)$

baseSRC: $f_i^{\,A}(x,Q^2) = \frac{Z}{A}\left[(1-C_p^A) f_i^{\,p}(x,Q^2) + C_p^A f_i^{\,\mathrm{SRC}\,p}(x,Q^2)\right] + \frac{N}{A}\left[(1-C_n^A) f_i^{\,n}(x,Q^2) + C_n^A f_i^{\,\mathrm{SRC}\,n}(x,Q^2)\right],\ C_{p,n}^A = \text{free}$

pnSRC: $f_i^{\,A}(x,Q^2) = \frac{Z}{A}\left[(1-C_p^A) f_i^{\,p}(x,Q^2) + C_p^A f_i^{\,\mathrm{SRC}\,p}(x,Q^2)\right] + \frac{N}{A}\left[(1-C_n^A) f_i^{\,n}(x,Q^2) + C_n^A f_i^{\,\mathrm{SRC}\,n}(x,Q^2)\right],$ p-n dominance $C_p^A = \frac{N}{Z} C_n^A \equiv C^A$

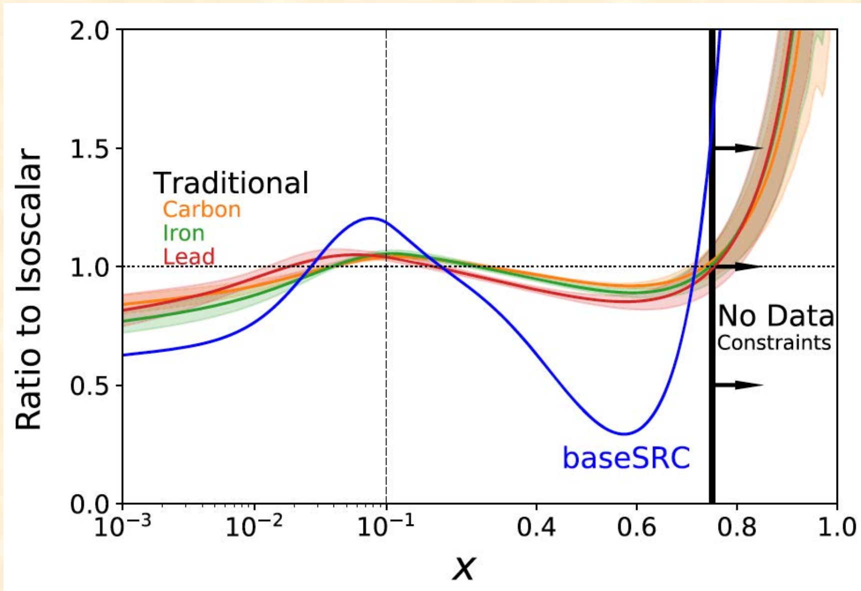


$C_{p(n)}^A$ = fraction of SRC pairs by taking sum over pn and pp (nn and np) pairs

$a_2(A/D) = \frac{a_{2N}(A)}{a_{2N}(D)}, \quad a_{2N}(A) = C_N^A$

$a_{2N}(D)$ = probability of deuteron SRC with $k > 275$ MeV

χ^2/N_{data}	DIS	DY	W/Z	JLab	χ^2_{tot}	$\chi^2_{\mathrm{tot}}/N_{\mathrm{d.o.f.}}$
TRADITIONAL	0.85	0.97	0.88	0.72	1408	0.85
baseSRC	0.84	0.75	1.11	0.41	1300	0.80
pnSRC	0.85	0.84	1.14	0.49	1350	0.82
N_{data}	1136	92	120	336	1684	



First global nuclear PDF analysis with short-range correlation effects
(“Two distinct descriptions of nuclei unified for the first time”, Physics World)

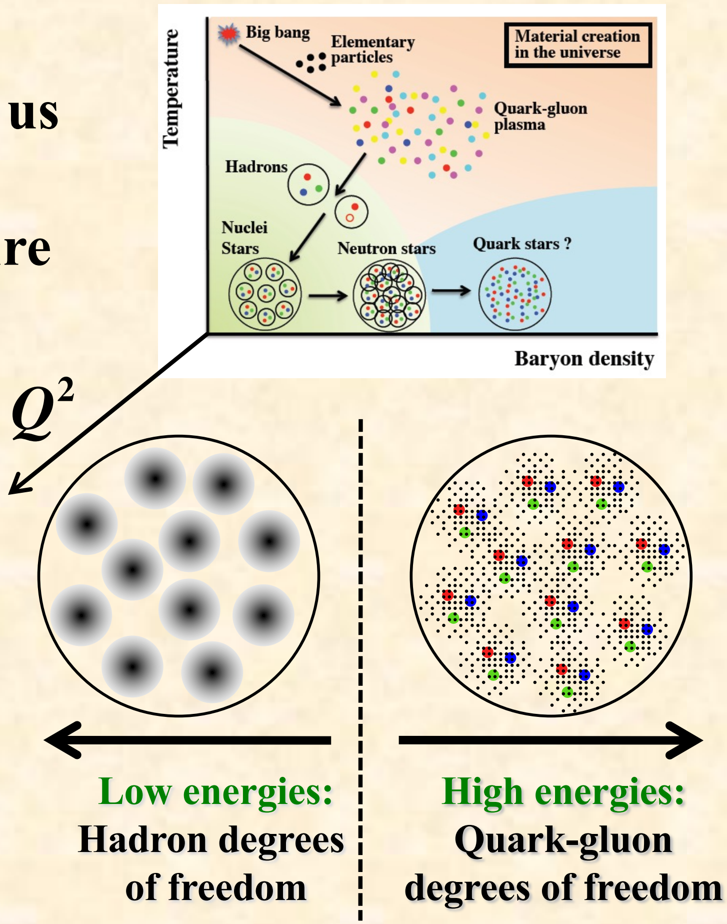
Summary

There are studies on nuclear cluster and short-range nucleon correlations at charged-lepton accelerator facilities, mainly at the JLab in recent years.

Such studies will be done also at future the EIC.

Since there are many scientists on low-energy nuclear structure physics in Japan, it is a good opportunity for us to develop nuclear structure physics at high energies by collaborations between high-energy nucleon structure physicists and low-energy nuclear structure physicists toward success of the EIC project in 2030's.

From these studies, we will establish nuclear physics
from low- to high-energies,
from low- to high-densities, and
from low- to high-temperatures
in terms of quark and gluon degrees of freedom.



The End

The End