A Snapshot of Preheating:

success, challenges and possibilities

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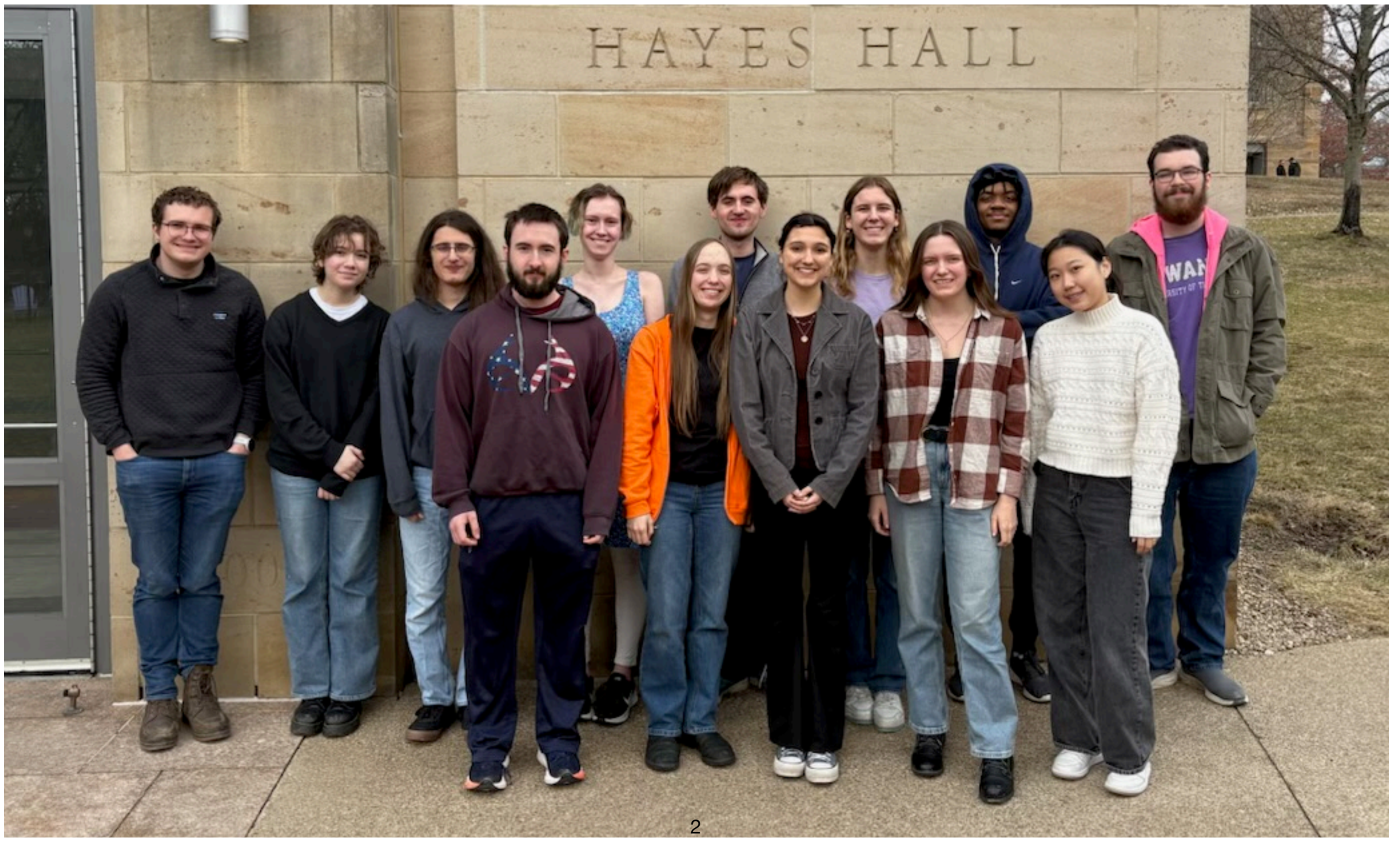
RIKEN Interdisciplinary Theoretical and Mathematical Sciences Program
Saitama, Japan
March 4, 2025

My Group At Kenyon

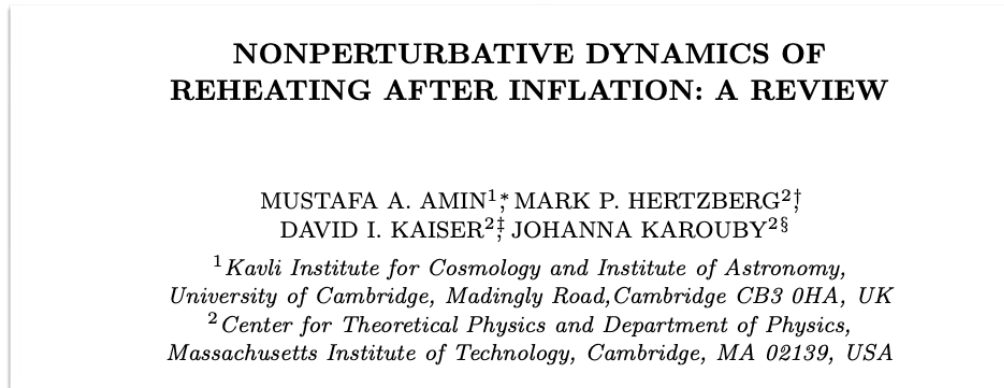


Successes: why preheating?

My apologizes for missing citations

…the literature is wide, varied and excellent

- A good (but now little old) and comprehensive review, 1410.3808
- For GWs, there's a newer review, Caprini and Figueroa 1801.04268
- A (much) older review from Allahverdi, Brandenberger, Cyr-Racine, Mazumdar, 1001.2600
- astro-ph/0507632, 0705.0164



NONPERTURBATIVE DYNAMICS OF REHEATING AFTER INFLATION: A REVIEW

MUSTAFA A. AMIN[1,*], MARK P. HERTZBERG[2,†], DAVID I. KAISER[2,‡], JOHANNA KAROUBY[2,§]

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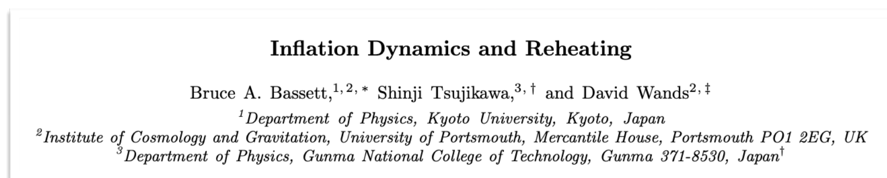
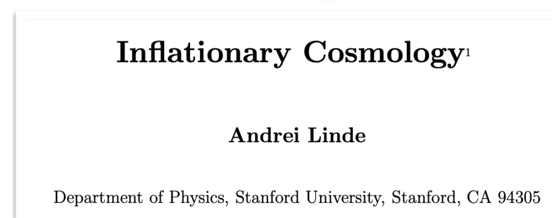
[2] *Center for Theoretical Physics and Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA*



Inflation Dynamics and Reheating

Bruce A. Bassett,[1,2,*] Shinji Tsujikawa,[3,†] and David Wands[2,‡]

[1] *Department of Physics, Kyoto University, Kyoto, Japan*
[2] *Institute of Cosmology and Gravitation, University of Portsmouth, Mercantile House, Portsmouth PO1 2EG, UK*
[3] *Department of Physics, Gunma National College of Technology, Gunma 371-8530, Japan*[†]



Inflationary Cosmology[1]

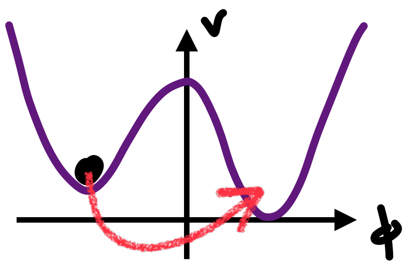
Andrei Linde

Department of Physics, Stanford University, Stanford, CA 94305

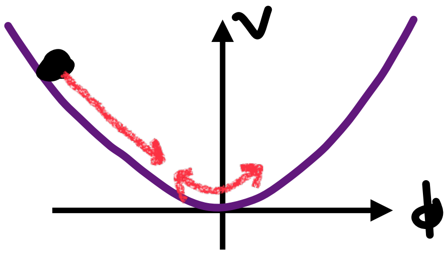
The Graceful Exit Problem

…if you need inflation, you need inflation to end

Old inflation



New Inflation



Inflation ends with
Bubble Collisions

Inflation ends when
slow roll ends

So we need Reheating

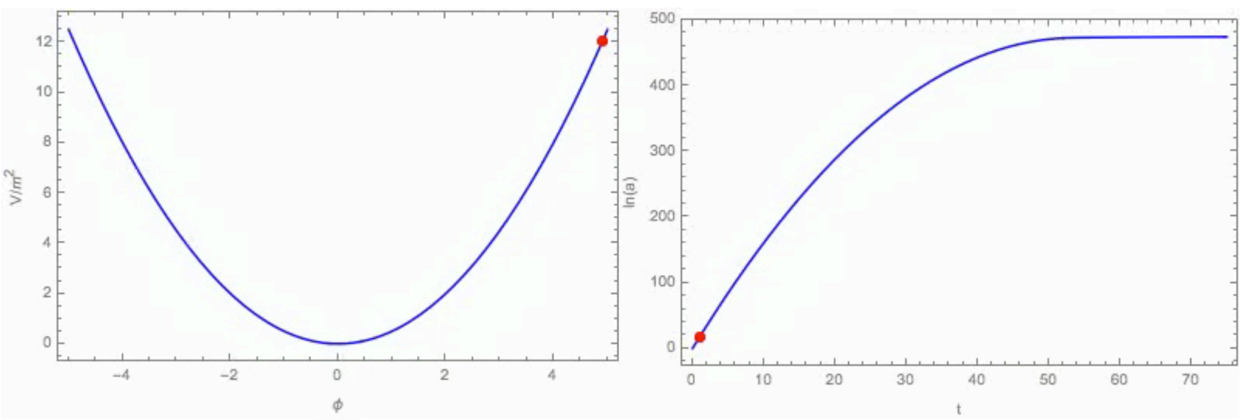
…which might be a problem?

- The strictest (theoretical) limit comes from BBN - requiring that the reheat temperature reach 5 MeV (see, e.g. 2502.08719)
- Couplings between the inflation and other degrees of freedom *must be small*, as not to correct the inflation potential during inflation
- For GUT-scale (or high-scale) inflation, the Old Theory of Reheating has time to reheat using perturbative decay
- For low-scale inflation, getting to a radiation-dominated 5 MeV is more of a challenge

(Vanilla) Preheating

As introduced in Traschen & Brandenburger (PRD 42.2491)

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m^2\phi^2 - \frac{g^2}{2}\phi^2\chi^2$$



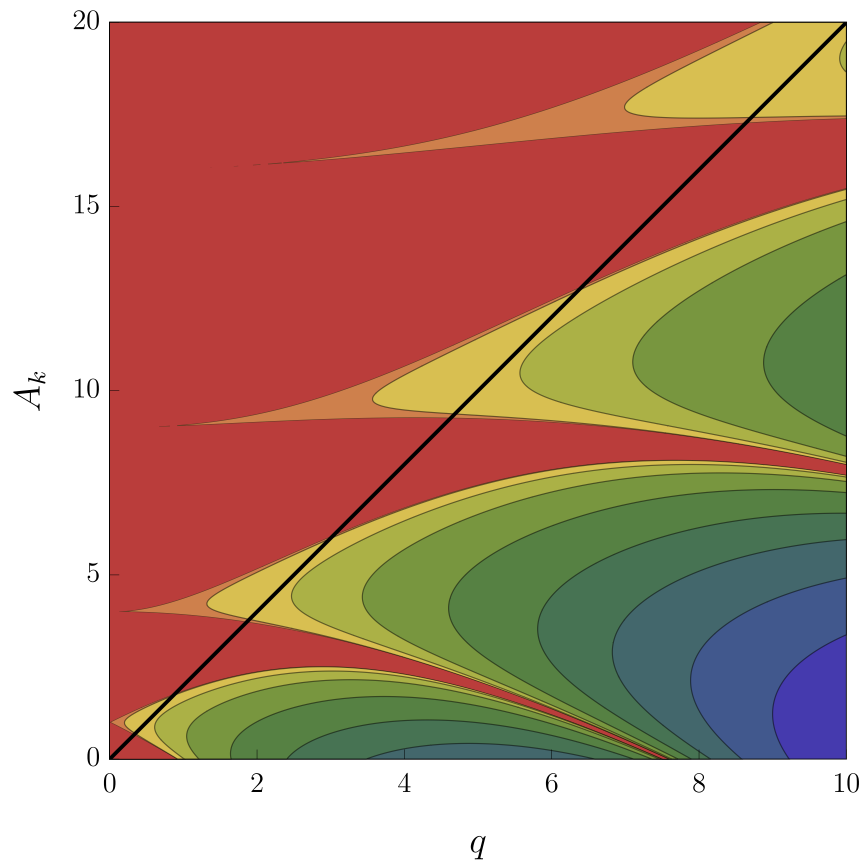
What can we see from this?

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \frac{k^2}{a^2}\chi_k = -\frac{\partial V}{\partial \chi}$$

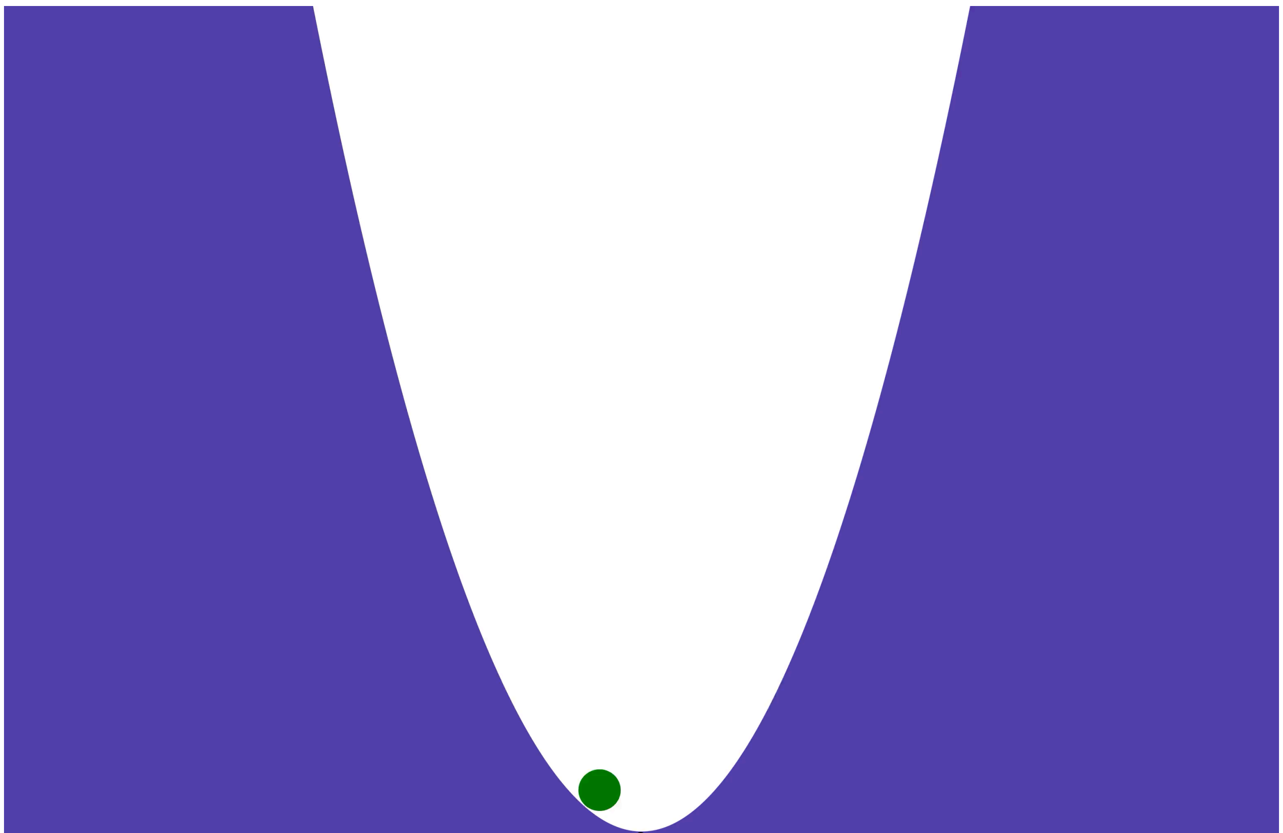
$$g^2\langle\phi^2\rangle\chi$$

time dependent mass

parametric resonance

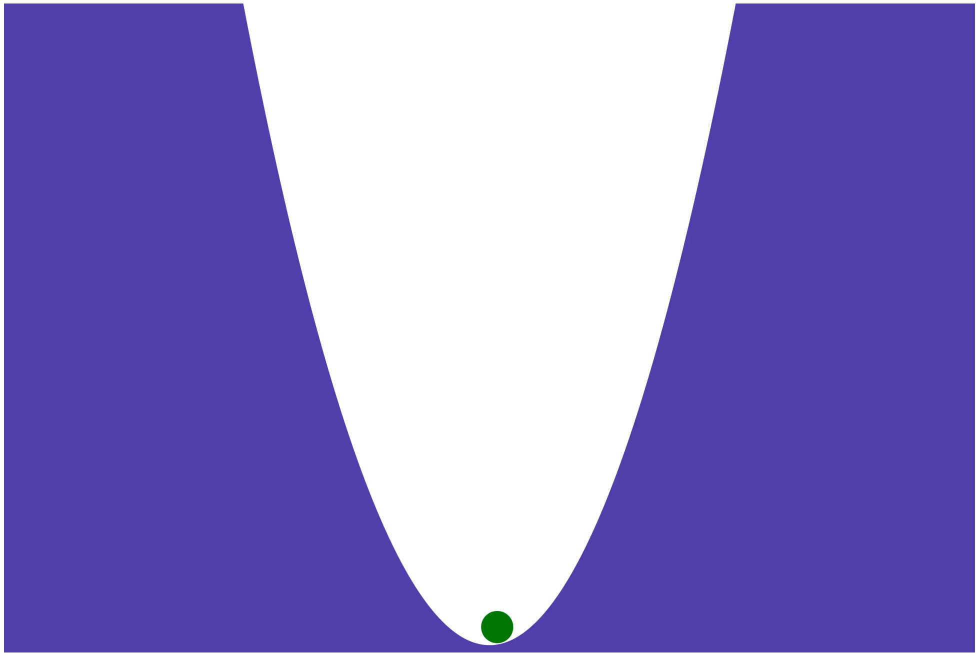


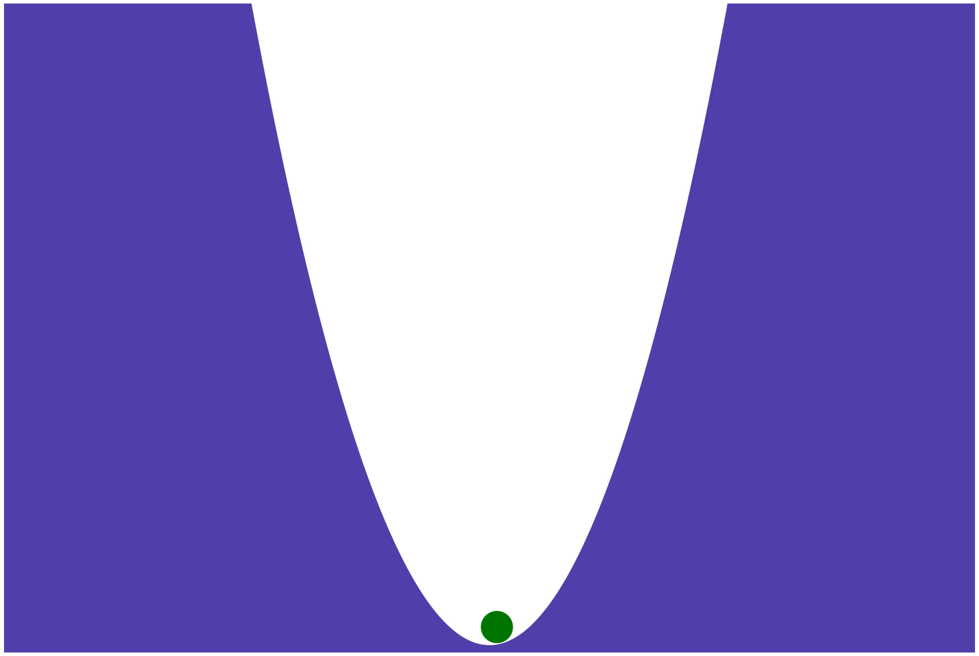
The time-varying effective mass



Movie Credit:
Zach Weiner

Modes on- and off-resonance





Movie Credit:
Zach Weiner

Of Course, Floquet Analysis

…isn't perfect

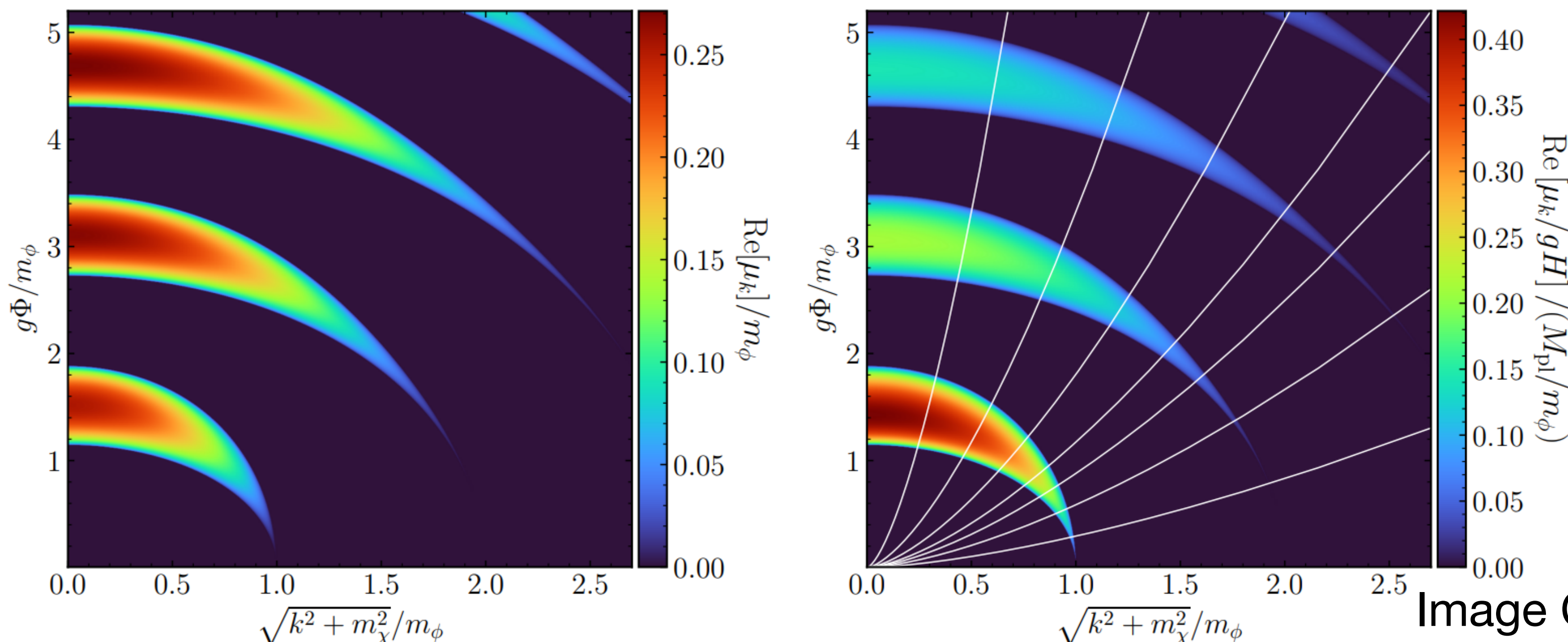
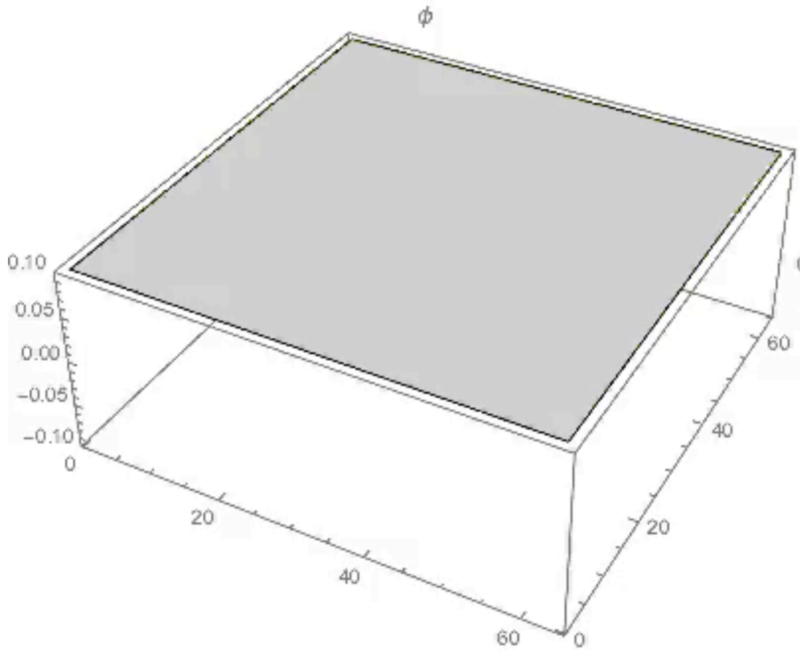
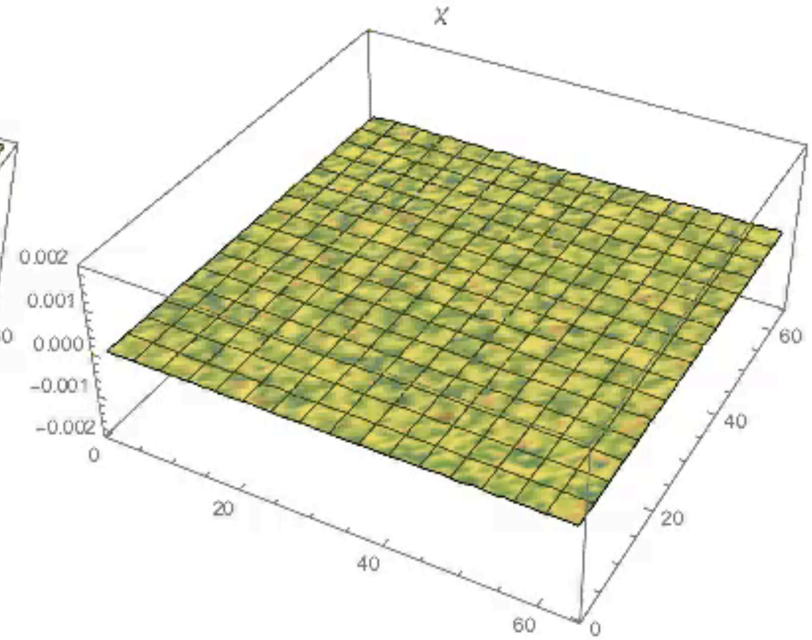
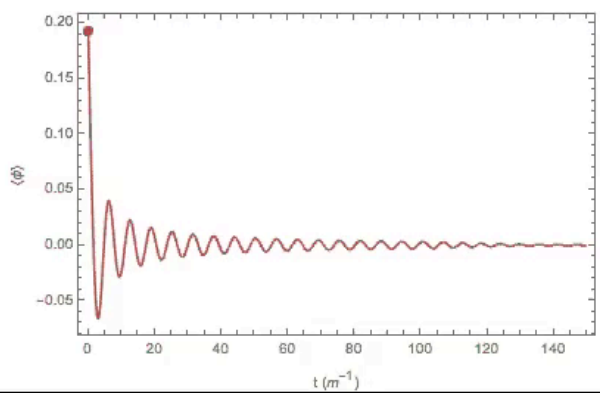


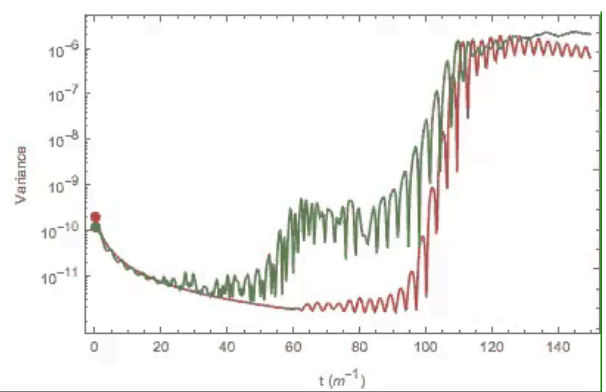
Image Credit:
Zach Weiner

So we do it numerically.


ϕ
0.10
0.05
0.00
−0.05
−0.10
0
20
40
60
0
20
40
60


χ
0.002
0.001
0.000
−0.001
−0.002
0
20
40
60
0
20
40
60


⟨ϕ⟩
0.20
0.15
0.10
0.05
0.00
−0.05
0
20
40
60
80
100
120
140
t (m⁻¹)


Variance
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
10⁻¹¹
0
20
40
60
80
100
120
140
t (m⁻¹)

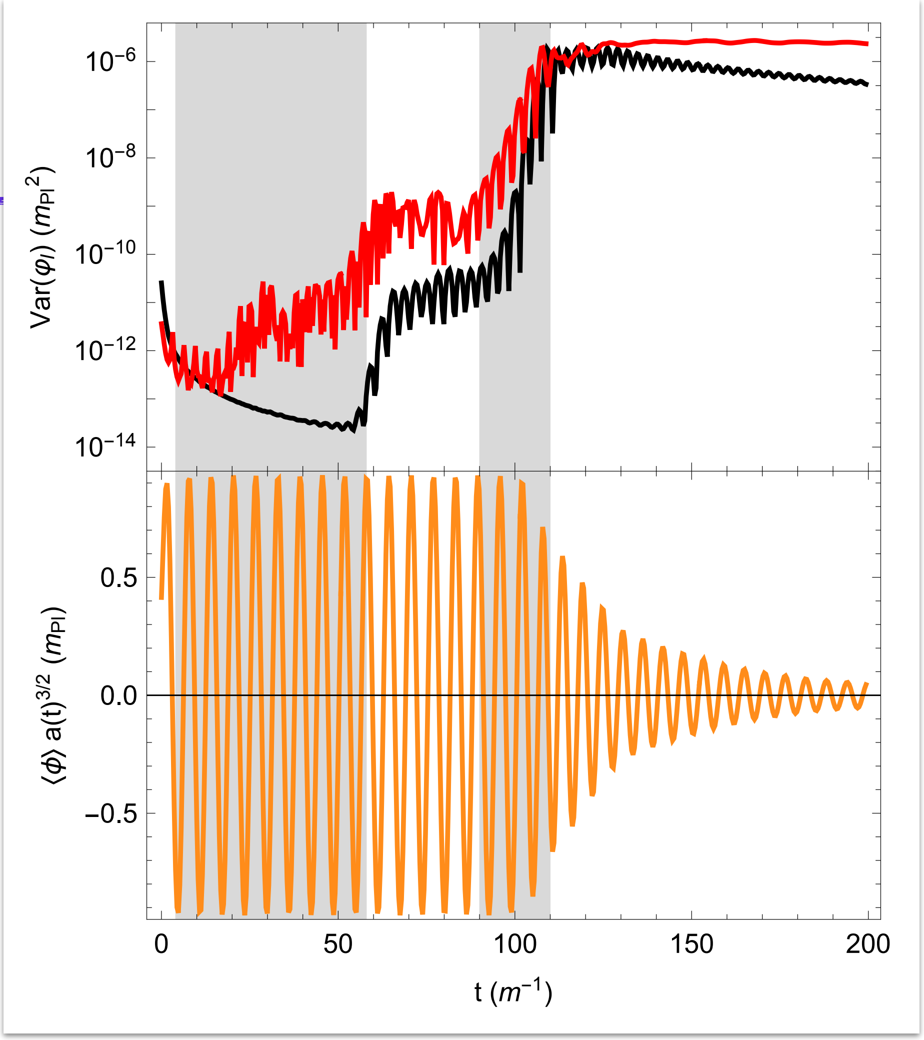
Stages of preheating

1. *Resonance* amplifies modes with little back-reaction onto the inflation condensate

2. The newly created particles interact with the background

3. As interactions continue, the particles fragment the background, causing the condensate to decay

4. Thermalization



When does preheating occur?

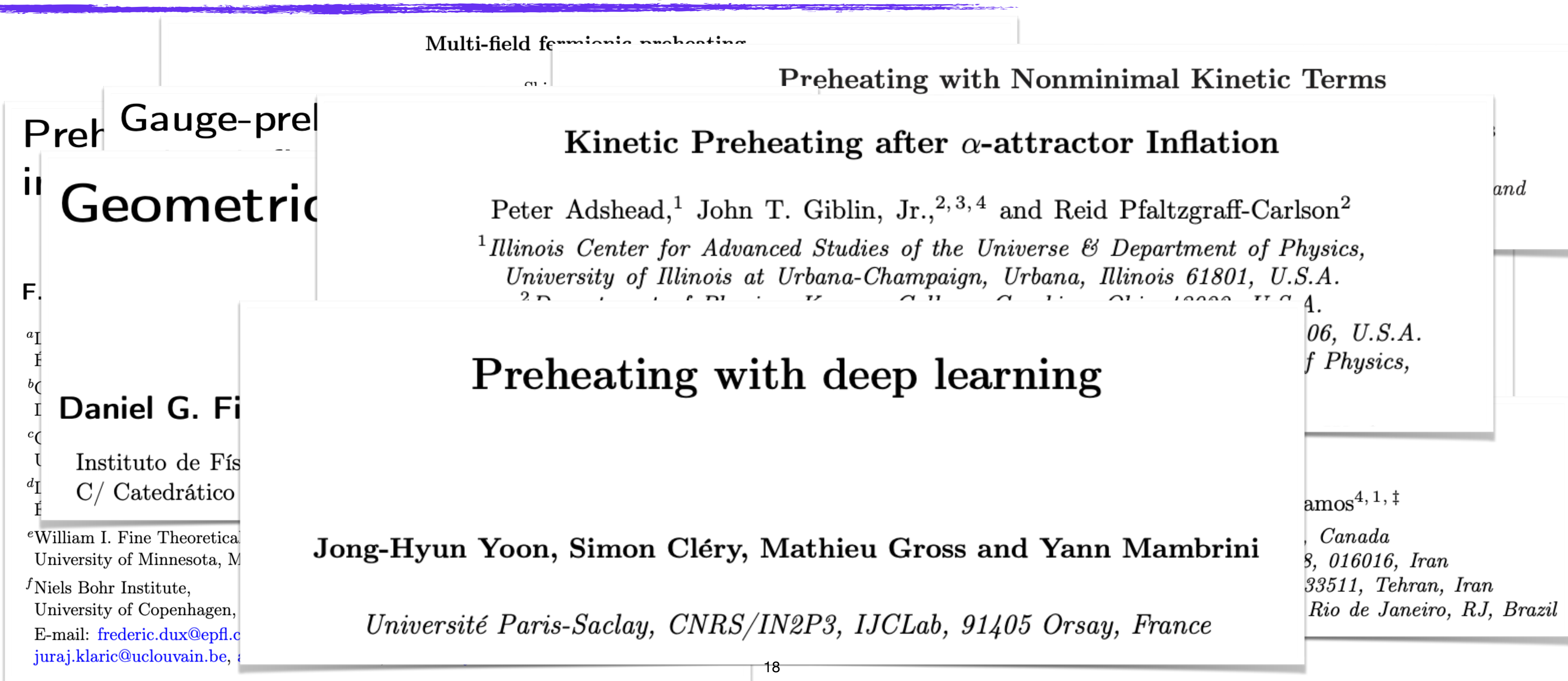
Preheating:
A nonlinear process that follows inflation during which energy is transferred to specific modes which will later thermalize.

Preheating:
A nonlinear process that follows inflation during which energy is transferred to specific modes which will later thermalize.

More on this later this morning?

Of course, there's more than vanilla preheating

…and the list keeps growing



It's (almost) always about the effective mass

- For canonical, minimally coupled fields,

$$\ddot{\phi} + 3H\dot{\phi} + \left(\frac{k^2}{a^2} + m_{\rm eff}^2\right) = 0$$

$$m_{\rm eff}^2(\langle\bar{\phi}(t)\rangle) \sim \frac{k^2}{a^2} \qquad\qquad m_{\rm eff}^2(\langle\phi\rangle) < -\frac{k^2}{a^2}$$

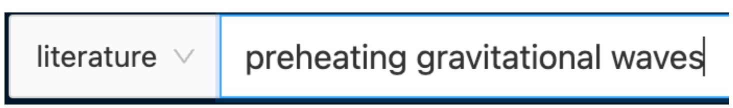
- For kinetically coupled fields, nonmimimally coupled, etc, you can get new terms in the equations of motion that induce tachyonic (or parametric) instabilities, e.g. Kinetic Preheating

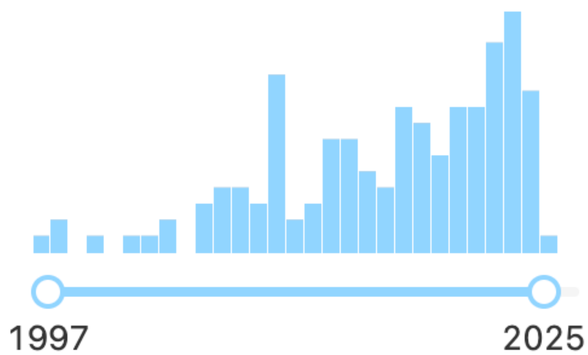
$$\ddot{\chi} = -3\frac{\dot{a}}{a}\dot{\chi} + \frac{\nabla^2\chi}{a^2} - \frac{2}{\mu}\left(\dot{\chi}\dot{\phi} - \frac{(\nabla\chi)(\nabla\phi)}{a^2}\right) \qquad \ddot{\varphi}_k + \left(\frac{k^2}{a^2} - \left(\frac{3}{2}H + \frac{\dot{\phi}}{\mu}\right)^2 - \left(\frac{3}{2}\dot{H} + \frac{\ddot{\phi}}{\mu}\right)\right)\varphi_k = 0.$$

Where can we look for evidence of preheating?

Stochastic Gravitational Waves

- These source stochastic gravitational waves:
  - Phys. Rev. D 56, 653 (1997)
- In 07(ish) a rejuvenation of this
  - astro-ph/0612294 JTG, Easther, Lim
  - astro-ph/0701014 Garcia-Bellido, Figueroa, Sastre
- Has become a *very hot area* for prediction

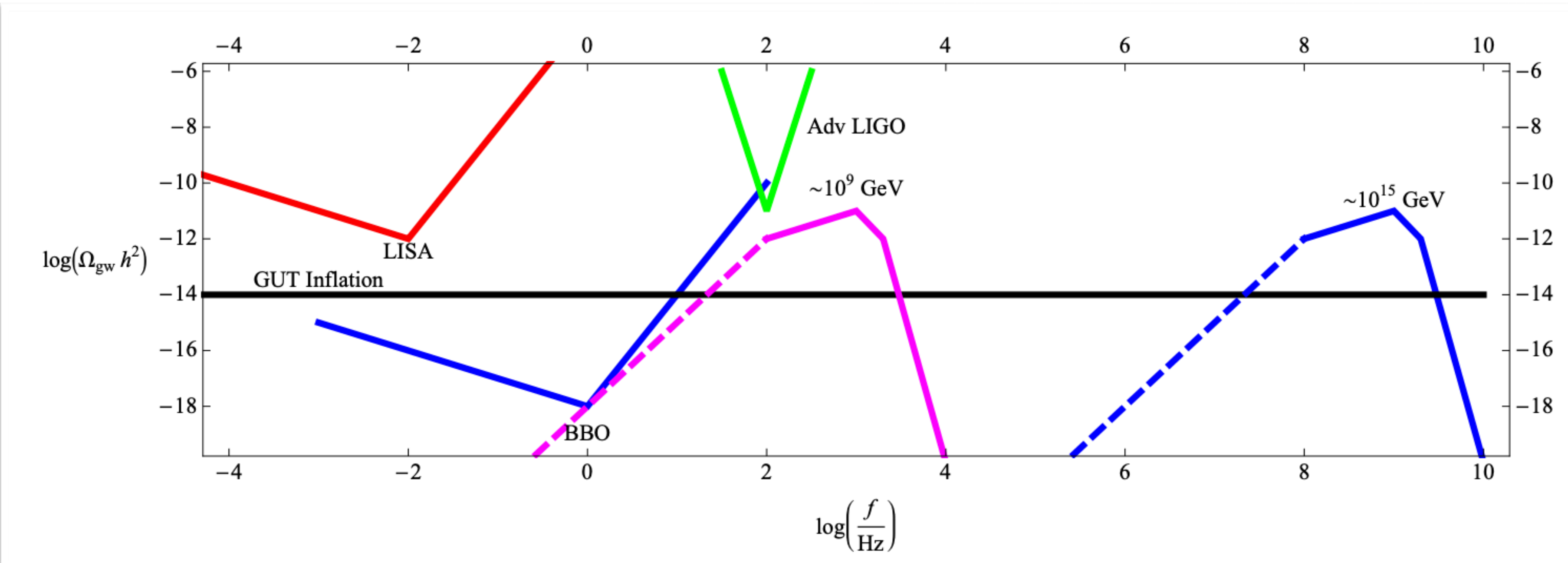




Stochastic Gravitational Waves

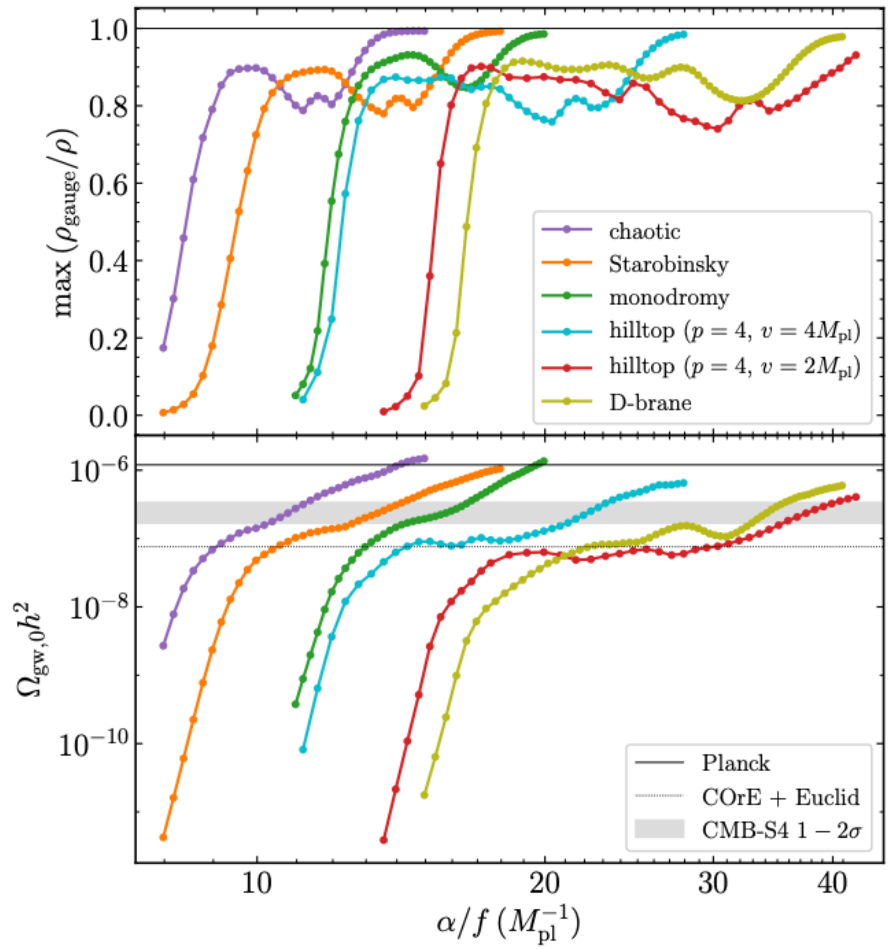
…and it should be!

- These source stochastic gravitational waves:



Which can be informed by $N_{\rm eff}$ bounds

- This looks at gauge-preheating (more on that later)
- Where the *integrated* gravitational wave energy can be constrained by the CMB
- This can be extended to many models — during preheating (or during similar cosmological processes



1909.12843

Primordial non-gaussianity

…a short review

- Enqvist, Jokinen, Mazumdar, Multamäki, Väihkönen, astro-ph/0411394
  - Non-gaussian, $F_{NL} \sim 10^3$
- Barnaby and Cline, astro-ph/0611750
  - Tachyonic preheating, limits on inflationary scale for certain models
- Chambers, Rajantie, 0710.4133
  - “Toward” numerical simulations
- Bond, Frolov, Huang, Kofman, 0903.3407
  - From numerical simulations

Primordial Black Holes?

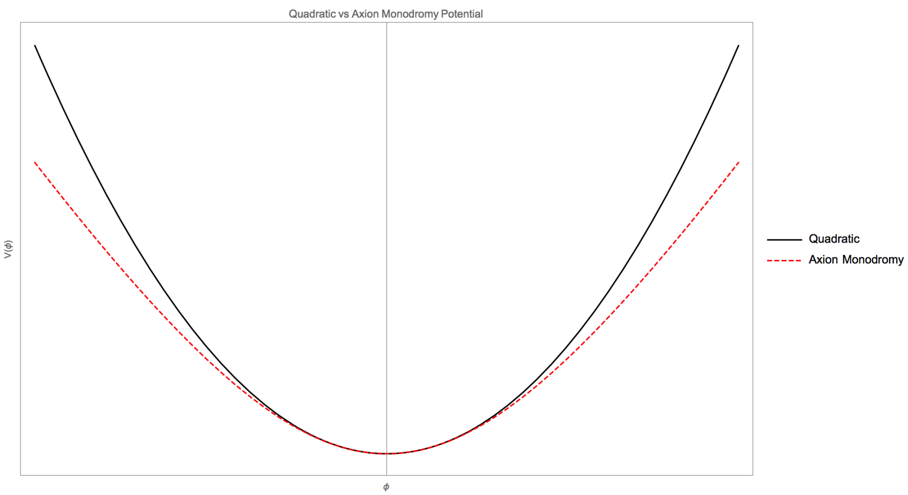
(Much more on this later)

- The process never happens when you take gravity into account: the universe fragments immediately and black holes form. The end of times begin shortly after inflation
- The resonance process itself is strong enough to create primordial black holes.

Inflaton Fragmentation

…or oscillons!

- Copeland, Gleiser, Muller, hep-ph/9503217
- Amin, Easther 1009.2505; Amin, Easther, Finkel, Flauger, Hertzberg, 1106.3335



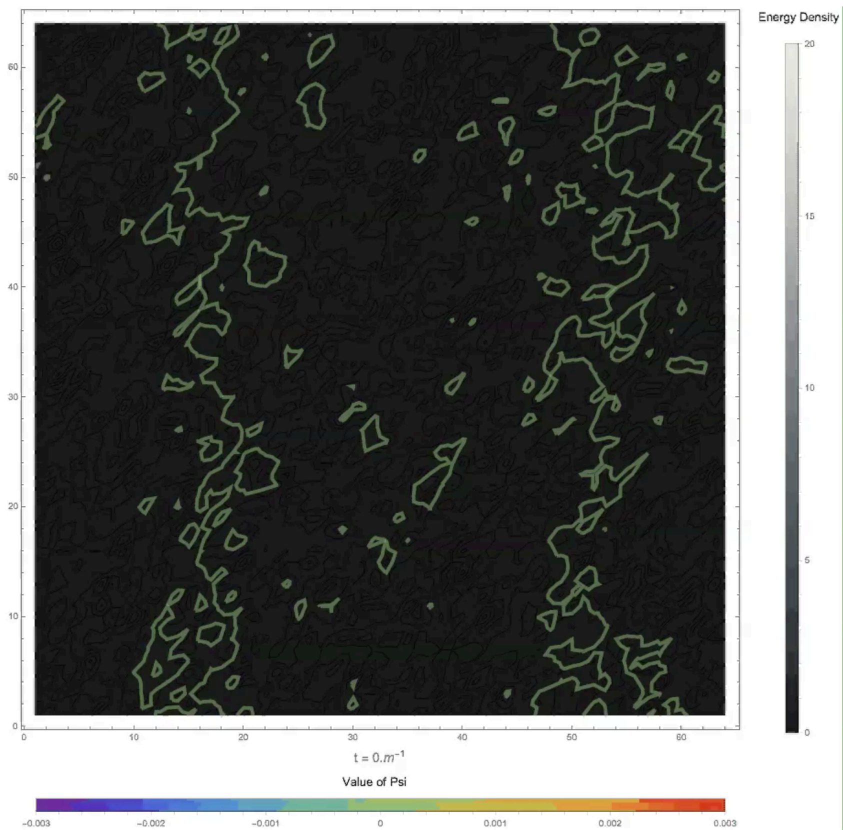
$$V(\phi) = \frac{m^2 M^2}{2\alpha}\left[\left(1 + \frac{\phi^2}{M^2}\right)^{\alpha} - 1\right]$$

$$\alpha = \tfrac{1}{2}.$$

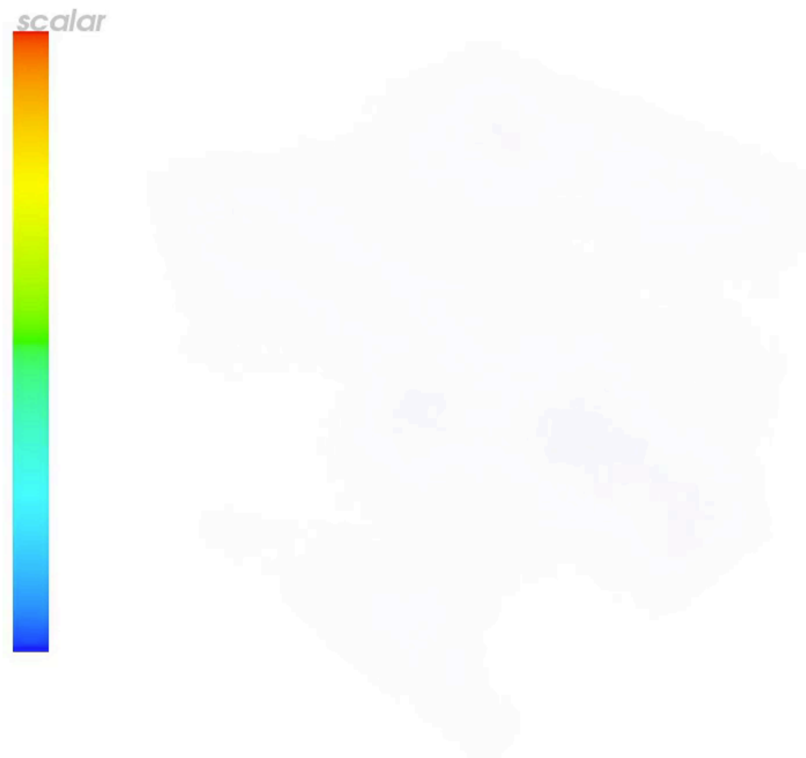
$$m_{\rm pr} = \frac{m}{m_{\rm pl}} = 2.2 \times 10^{-5},$$

$$M_{\rm pr} = \frac{M}{m_{\rm pl}} = 4 \times 10^{-3}.$$

Oscillons



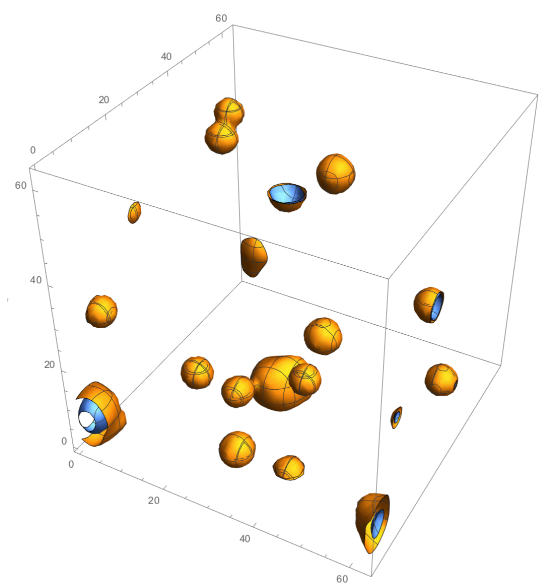
Energy Density
20
15
10
5
0
60
50
40
30
20
10
0
0
10
20
30
40
50
60
t = 0.m⁻¹
Value of Psi
−0.003
−0.002
−0.001
0
0.001
0.002
0.003

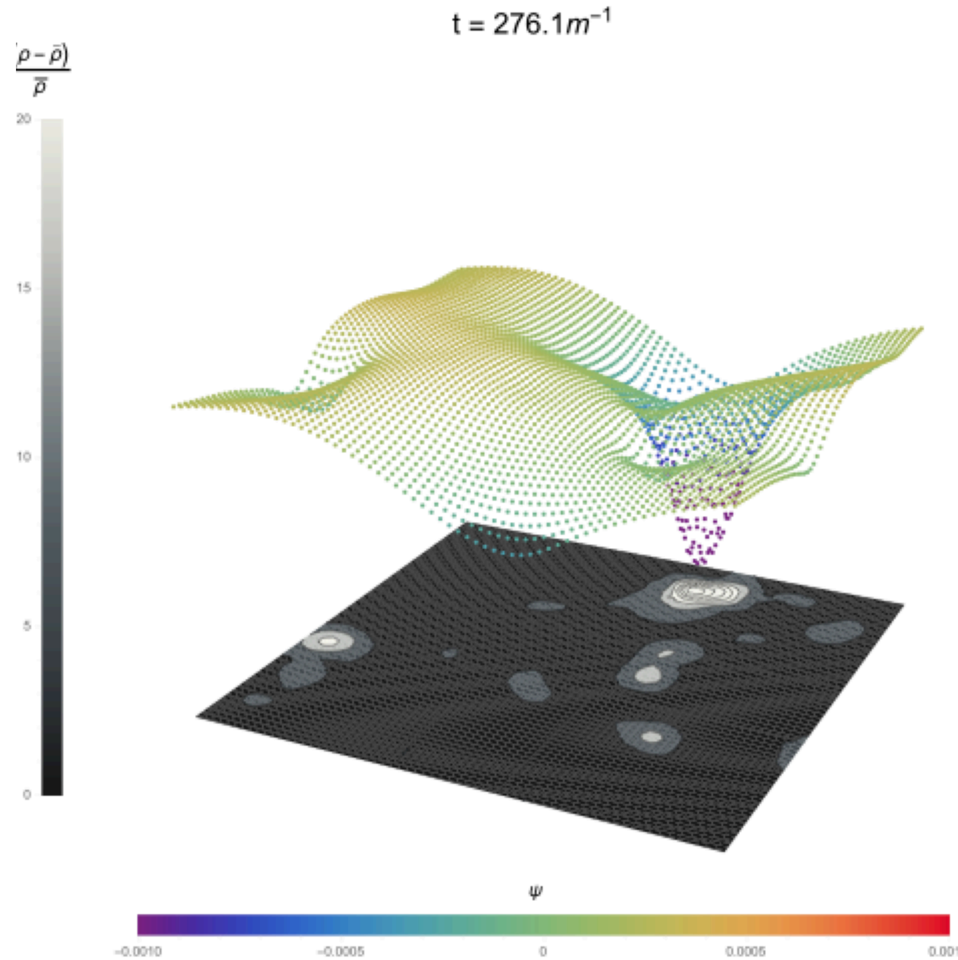


scalar

Oscillons

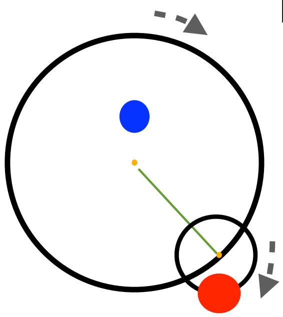
- Compact, non-topological, long-lived structures
- Produce gravitational wells





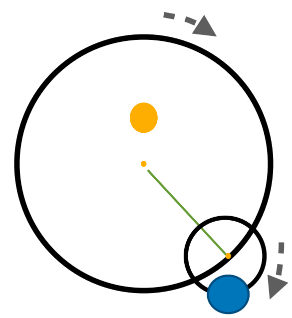
Possibilities?

Does Gravity Matter During Preheating?



Ptolemaic Gravity

Copernican Gravity



Newtonian Gravity

$$\nabla^2\Phi = 4\pi G\,\rho$$

General Relativity

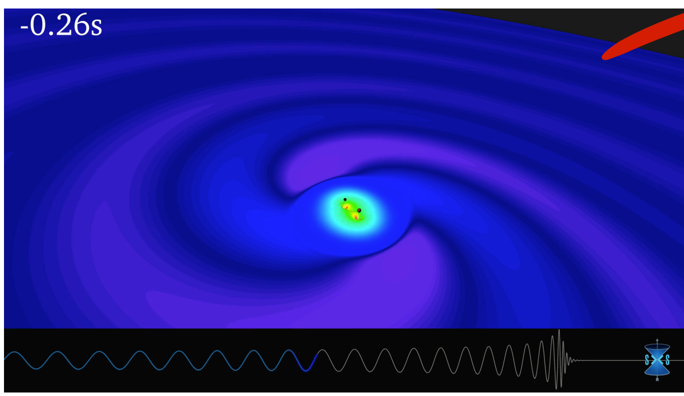
$$G_{\mu\nu} = 8\pi G\,T_{\mu\nu}$$

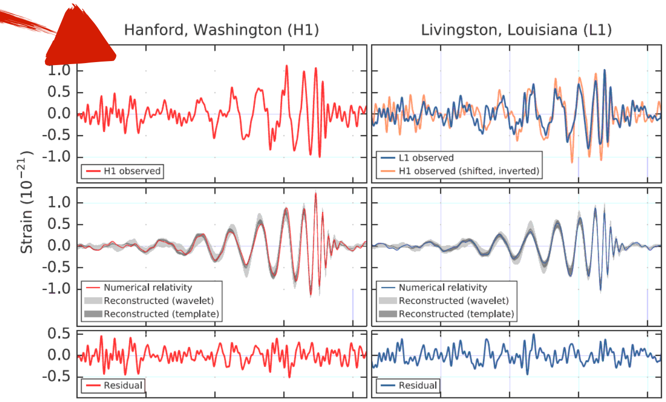
Gravity

- General Relativity appears to be one heck of a theory

Gravity

- An example:
  - Two black holes collide*
  - General Relativity *predicts*** a signal
  - We measure the signal***

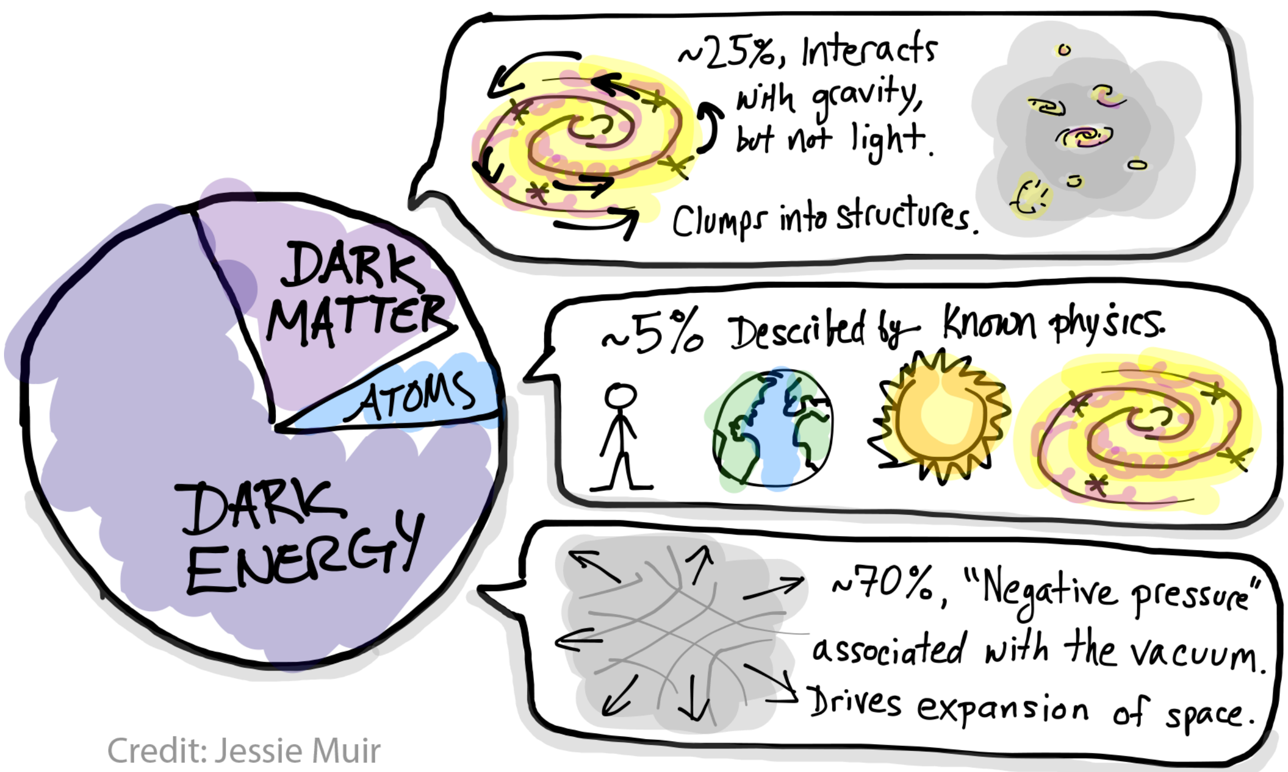




*where'd they come from?
**Many contributors, this analysis from Simulating Extreme Space-time (not me)
**LIGO: Phys. Rev. Lett. 116, 061102 (absolutely not me)

Unfortunately

…no one told the Universe



Credit: Jessie Muir

According to Concordance Cosmology here's what happened (mathematically speaking)

Dark Energy Dominated Universe (expansion of the universes seems to be accelerating)

The Universe today is a combination of Matter and Radiation (mostly matter)

The Universe cools enough to be transparent

Because matter dilutes slower than radiation, the earlier Universe was more radiation than matter

In the distant past, the Universe was very very dense

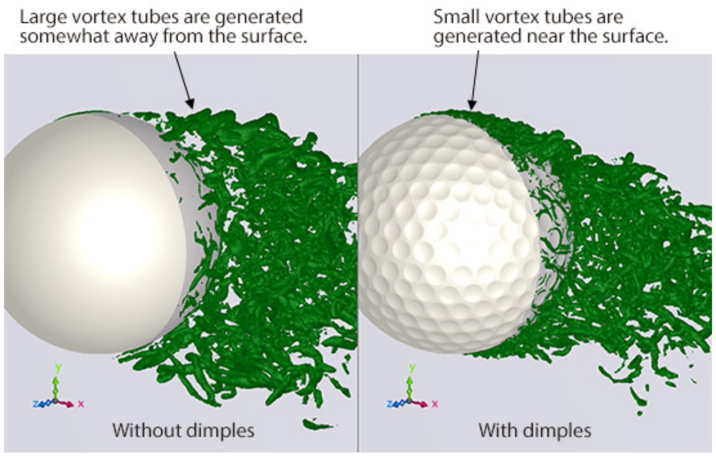
Inflation? Ekpyrosis? Bubbles? Gnomes?

PLUS Dark matter

Gravity is Non-Linear

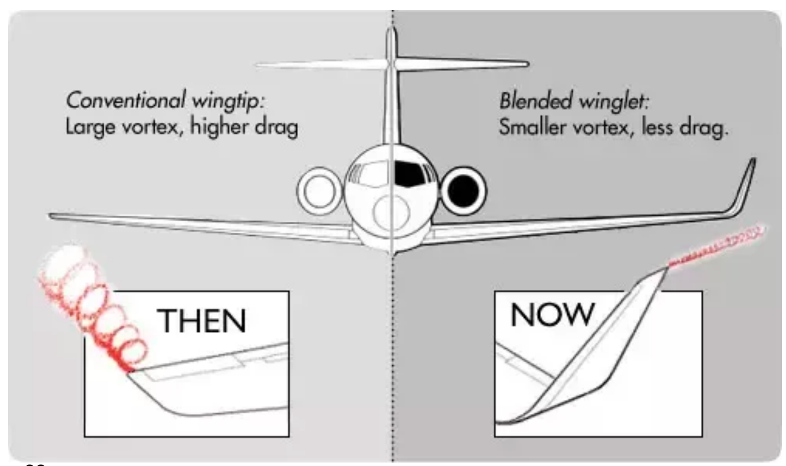
- Being “non-linear” is *more* that just “not being small”
- We like to *separate scales* when doing physics problems (e.g. what happens here, stays here)
    - Non-linear physics can mix up scales - power transferred between scales through *cascades* or *inverse-cascades*

Sometimes things that look like Perturbations



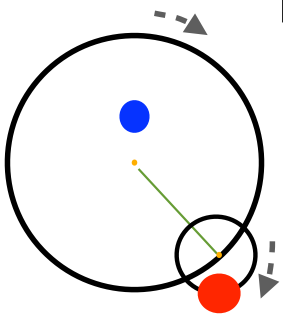
Takao Itami
https://www.cradle-cfd.com/

Anas Maaz
https://www.quora.com/

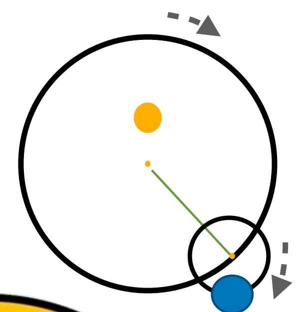


What do Cosmologists do?

Ptolemaic Gravity



Copernican Gravity



Newtonian Gravity

$$\nabla^2\Phi = 4\pi G \, \rho$$

ity

$$\nu = 8\pi G \, T_{\mu\nu}$$

The Main Question:
For the Universe, *does it matter*?

Averaging

in the late Universe

- Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-3} \sim (4000\,\mathrm{Mpc})^3$$

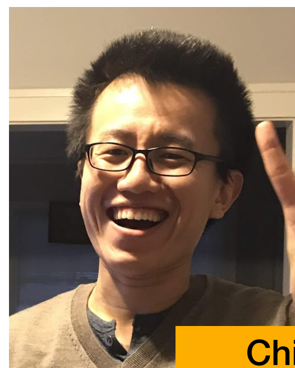
- Yet there is structure at (just) smaller scales
  - Galaxy Clusters $\sim 1 - 10\,\mathrm{Mpc}$
  - Inter-Cluster Distances $\sim 50\,\mathrm{Mpc}$

Work With

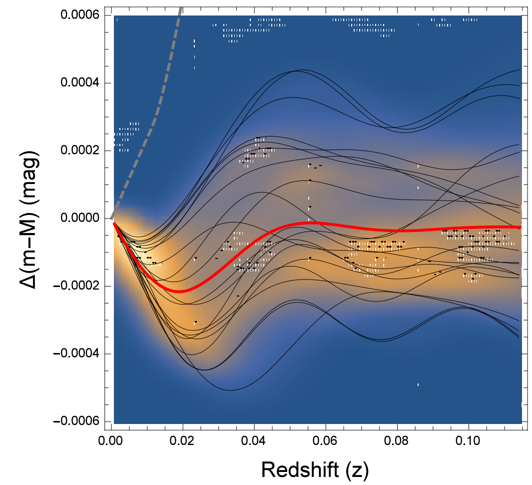
…for late Universe work

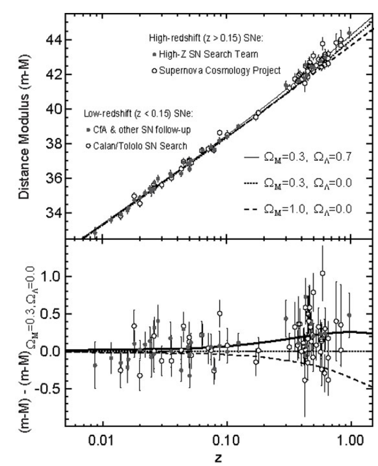
Jim Mertens
Toronto

Glenn Starkman
Case Western



Chi Tian
Anhui University





Scales at Reheating

- Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-1} \propto \mathcal{O}\left(1\right) \times \frac{m_{\rm pl}^2}{m^2\phi_0^2} = \mathcal{O}\left(1\right) \times m^{-1}$$

- YET: we talk about things at scales around this
  - Oscillons
  - Tachyonic/Parametric Resonance

$$\left.\begin{matrix}\text{Oscillons}\\ \text{Tachyonic/Parametric Resonance}\end{matrix}\right\} k \propto \mathcal{O} \times m^{-1}$$

What role does *nonlinear gravity* play in early Universe physics?

What you would *like* to do

- Write down the most general form of the metric,

$$g_{\mu\nu} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{01} & g_{11} & g_{12} & g_{13} \\ g_{02} & g_{12} & g_{22} & g_{23} \\ g_{03} & g_{13} & g_{23} & g_{33} \end{pmatrix}$$

- Plug it into Einstein's Equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- Solve the system of second order differential equations (subject to your gauge-constraints)

```
GREAT functions are: IMetric, Christoffel,
  Riemann, Ricci, SCurvature, EinsteinTensor, SqRicci, SqRiemann.

Enter 'helpGREAT' for this list of functions
```

```
In[11]:= (metric = {{g00[x0, x1, x2, x3], g01[x0, x1, x2, x3], g02[x0, x1, x2, x3],
       g03[x0, x1, x2, x3]}, {g01[x0, x1, x2, x3], g11[x0, x1, x2, x3],
       g12[x0, x1, x2, x3], g03[x0, x1, x2, x3]},
      {g02[x0, x1, x2, x3], g12[x0, x1, x2, x3], g22[x0, x1, x2, x3],
       g23[x0, x1, x2, x3]}, {g03[x0, x1, x2, x3], g13[x0, x1, x2, x3],
       g23[x0, x1, x2, x3], g33[x0, x1, x2, x3]}}) // MatrixForm
```

```
Out[11]//MatrixForm=
 g00[x0, x1, x2, x3]  g01[x0, x1, x2, x3]  g02[x0, x1, x2, x3]  g03[x0, x1, x2, x3]
 g01[x0, x1, x2, x3]  g11[x0, x1, x2, x3]  g12[x0, x1, x2, x3]  g03[x0, x1, x2, x3]
 g02[x0, x1, x2, x3]  g12[x0, x1, x2, x3]  g22[x0, x1, x2, x3]  g23[x0, x1, x2, x3]
 g03[x0, x1, x2, x3]  g13[x0, x1, x2, x3]  g23[x0, x1, x2, x3]  g33[x0, x1, x2, x3]
```

```
In[12]:= coords = {x0, x1, x2, x3}

Out[12]= {x0, x1, x2, x3}

In[13]:= EinsteinTensor[metric, coords]
```

What can we do?

- You can do a little better by making gauge choices that reduce the number of parameters or (re)parameterize so that you have nice equations for.. some.. of them…
- Even then they are extremely difficult to numerically stabilize

Numerical Relativity and Compact Binaries

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Abstract

Numerical relativity is the most promising tool for theoretically modeling the inspiral and coalescence of neutron star and black hole binaries, which, in turn, are among the most promising sources of gravitational radiation for future detection by gravitational wave observatories. In this article we review numerical relativity approaches to modeling compact binaries. Starting with a brief introduction to the 3+1 decomposition of Einstein's equations, we discuss important components of numerical relativity, including the initial data problem, reformulations of Einstein's equations, coordinate conditions, and strategies for locating and handling black holes on numerical grids. We focus on those approaches which currently seem most relevant for the compact binary problem. We then outline how these methods are used to model binary neutron stars and black holes, and review the current status of inspiral and coalescence simulations.

Key words:

Contents

What we have to do…

- Luckily there are a set of new approaches. We use the most common of these: the BSSN formalism.

- It is based on the ADM metric decomposition

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{kl}\beta^k\beta^l & \beta_j \\ \beta_i & \gamma_{ij} \end{pmatrix}$$

- We we introduce more parameters than (minimally) necessary so that the equations are easier to solve

In Cosmology

- We want to think of the two spatial tensors as the dynamic variables and conjugate momenta
- We can then track the spatial 3-metric

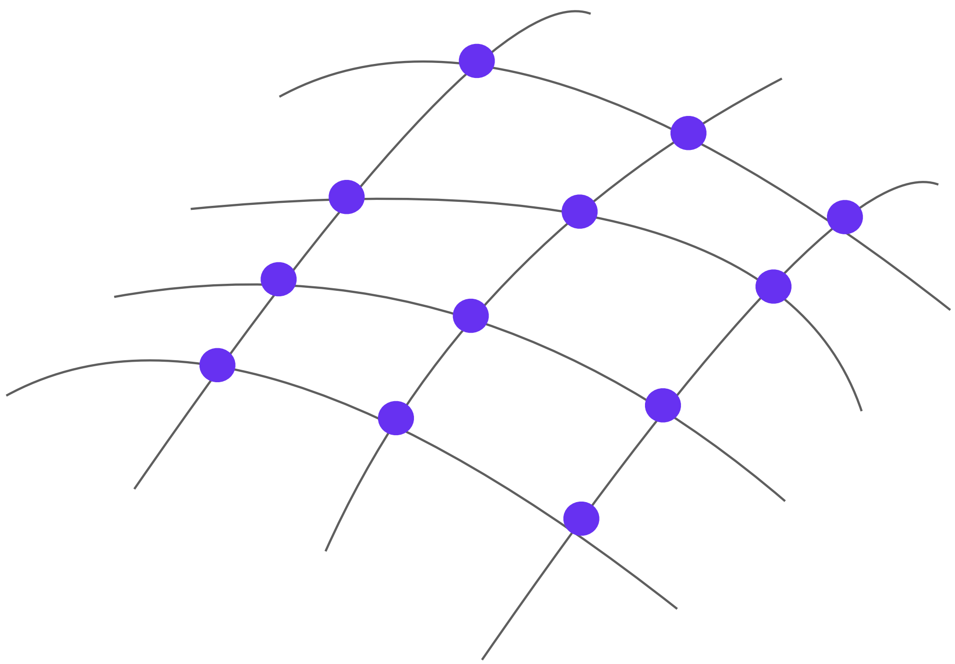
$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

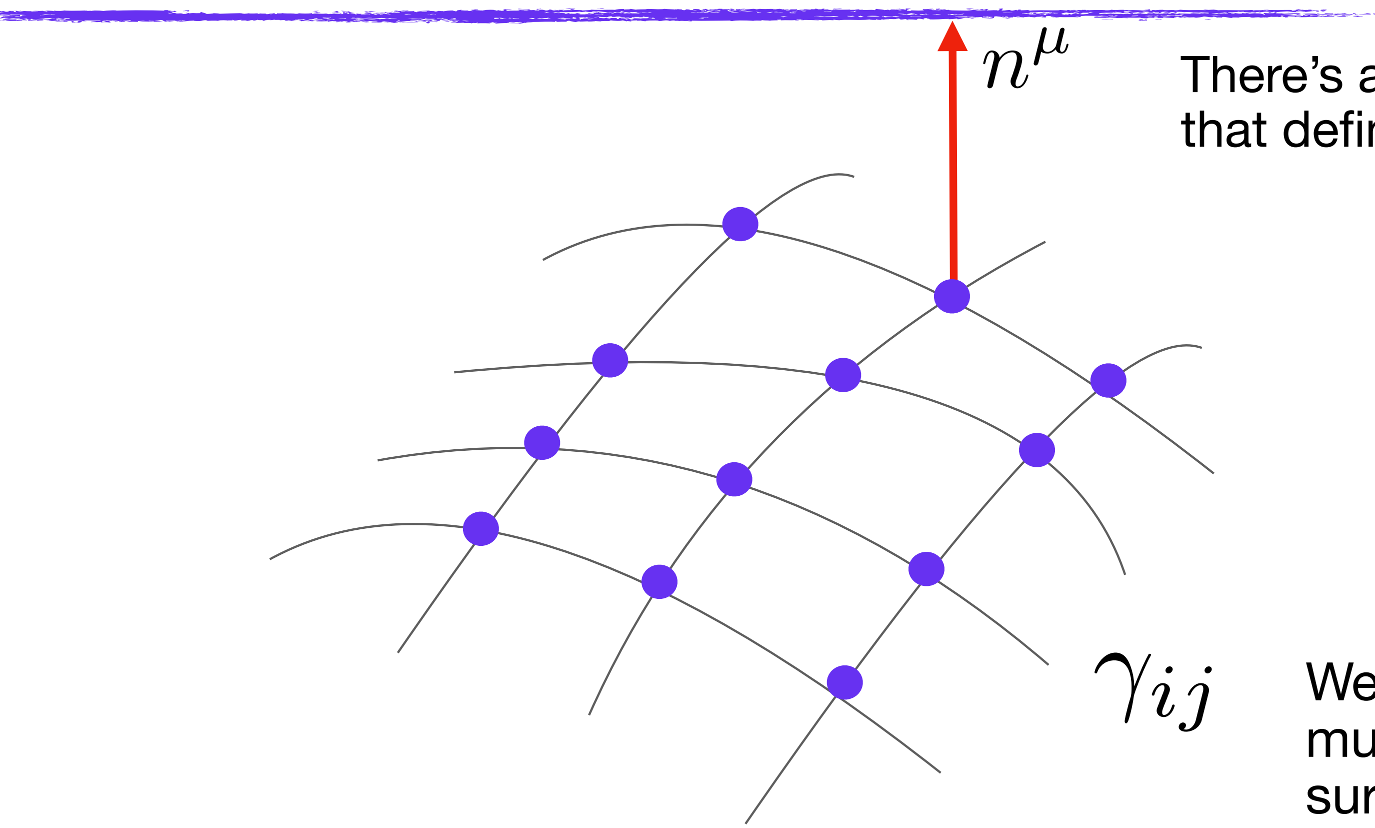
$$K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

How we do a 3+1 Decomposition

This is a 3D slice through space



How we do a 3+1 Decomposition



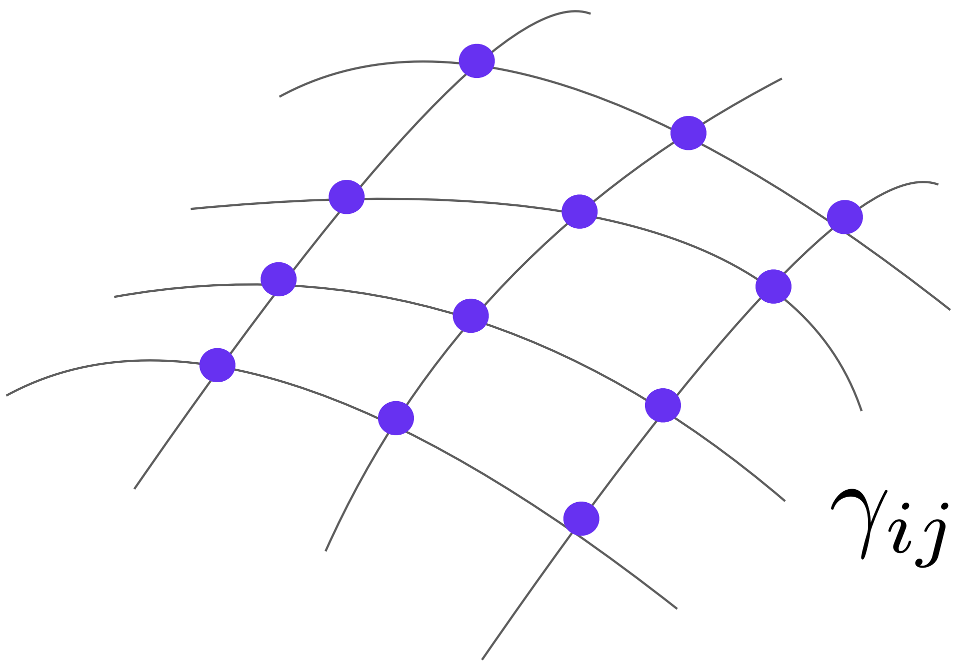
There's a normal vector that defines the surface

We must avoid putting TOO much meaning on these surfaces (at least not yet)

How we do a 3+1 Decomposition

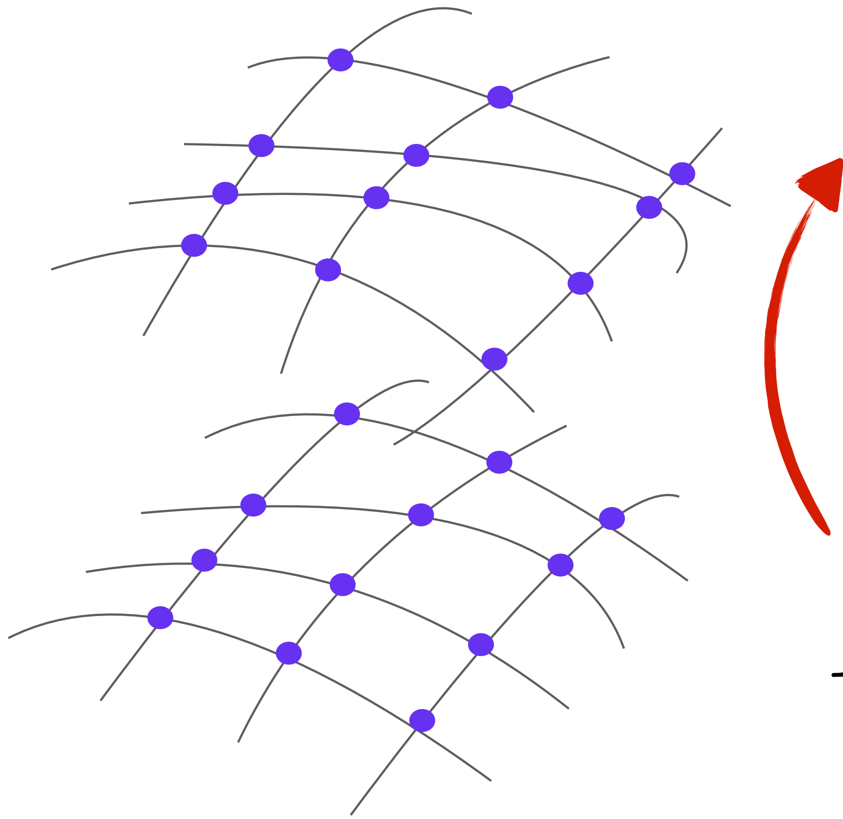
The extrinsic curvature
encodes how the metric is changing.

$$K_{ij}$$



$$\gamma_{ij}$$

How we do a 3+1 Decomposition

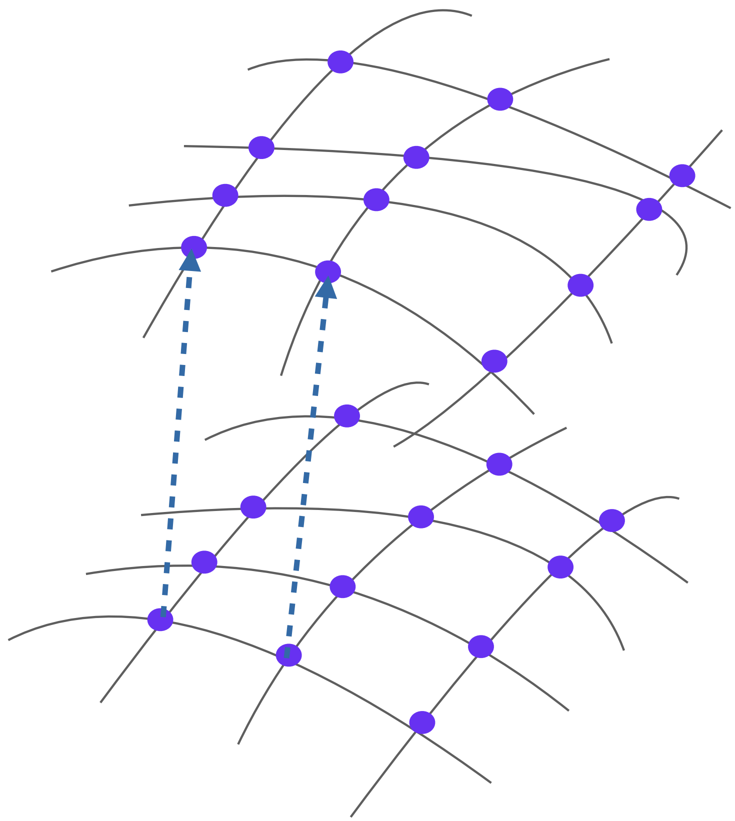


K'_{ij} γ'_{ij}

Einstein's Equations determine the evolution equation for these quantities

K_{ij} γ_{ij}

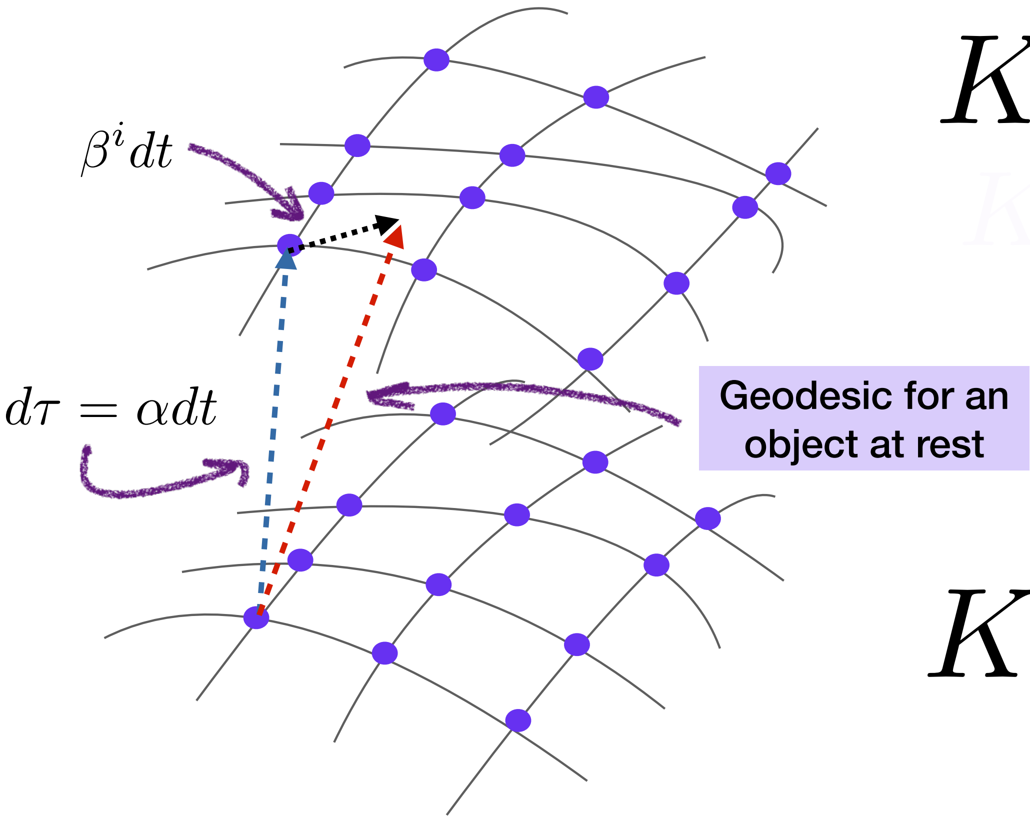
How we do a 3+1 Decomposition



$$K'_{ij} \quad \gamma'_{ij}$$

$$K_{ij} \quad \gamma_{ij}$$

How we do a 3+1 Decomposition



K'_{ij} γ'_{ij}

K_{ij} γ_{ij}

The art of the science

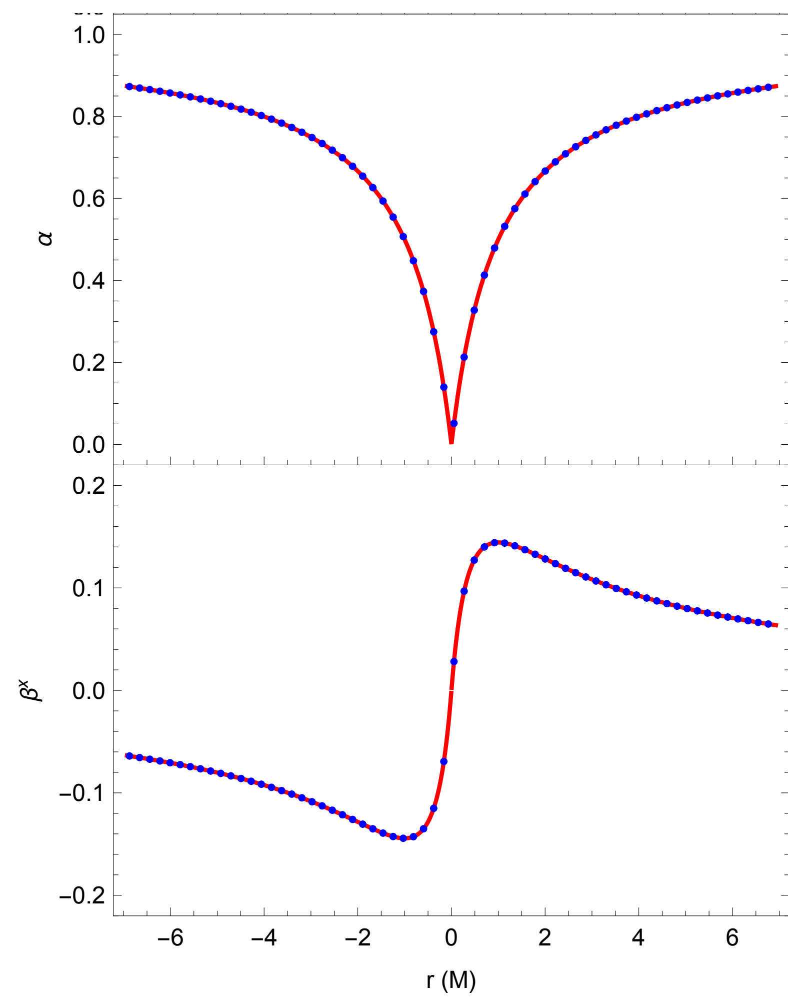
- You get to *choose* the lapse and shift, or put conditions on them

$$\partial_t \alpha \qquad \partial_t \beta^i$$

- So that your surfaces stay purely spatial!

Black Holes?

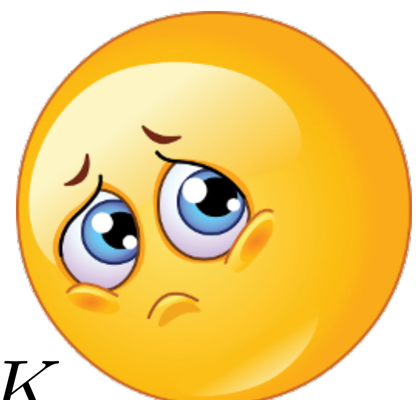
The *lapse goes to zero* which is the first indication of the formation of an horizon



In Cosmology

- We want to think of the two spatial tensors as the dynamic variables and conjugate momenta
- We can then track the spatial 3-metric

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$



- as well as the extrinsic curvature

$$K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

In Cosmology

- We want to think of the two spatial tensors as the dynamic variables and conjugate momenta
- We can then track the spatial 3-metric

Think of this as keeping track of the size of local volumes

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij}$$

- as well as the extrinsic curvature

Think of this as measuring the local expansion rate

$$K_{ij} = e^{4\phi}\tilde{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

Importantly

These variables have well-behaved differential equations and *are a complete description of GR without dimensional reductions or simplifications*

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \bar{\gamma}_{ij} + \bar{\gamma}_{ij}\partial_j \beta^k + \bar{\gamma}_{ij}\partial_i \beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k$$

$$\partial_t K = \gamma^{ij} D_j D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}\left(-(D_i D_j \alpha)^{TF} + \alpha(R_{ij}^{TF} - 8\pi S_{ij}^{TF})\right) + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l{}_j$$

$$+ \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{kj}\partial_i \beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right)$$

$$+ \beta^j \partial_j \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{il}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

Importantly

These variables have well-behaved differential equations and *are a complete description of GR without dimensional reductions or simplifications*

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

$$\bar{\Gamma}^i \equiv -\bar{\gamma}^{jk}\bar{\Gamma}^i_{jk}$$

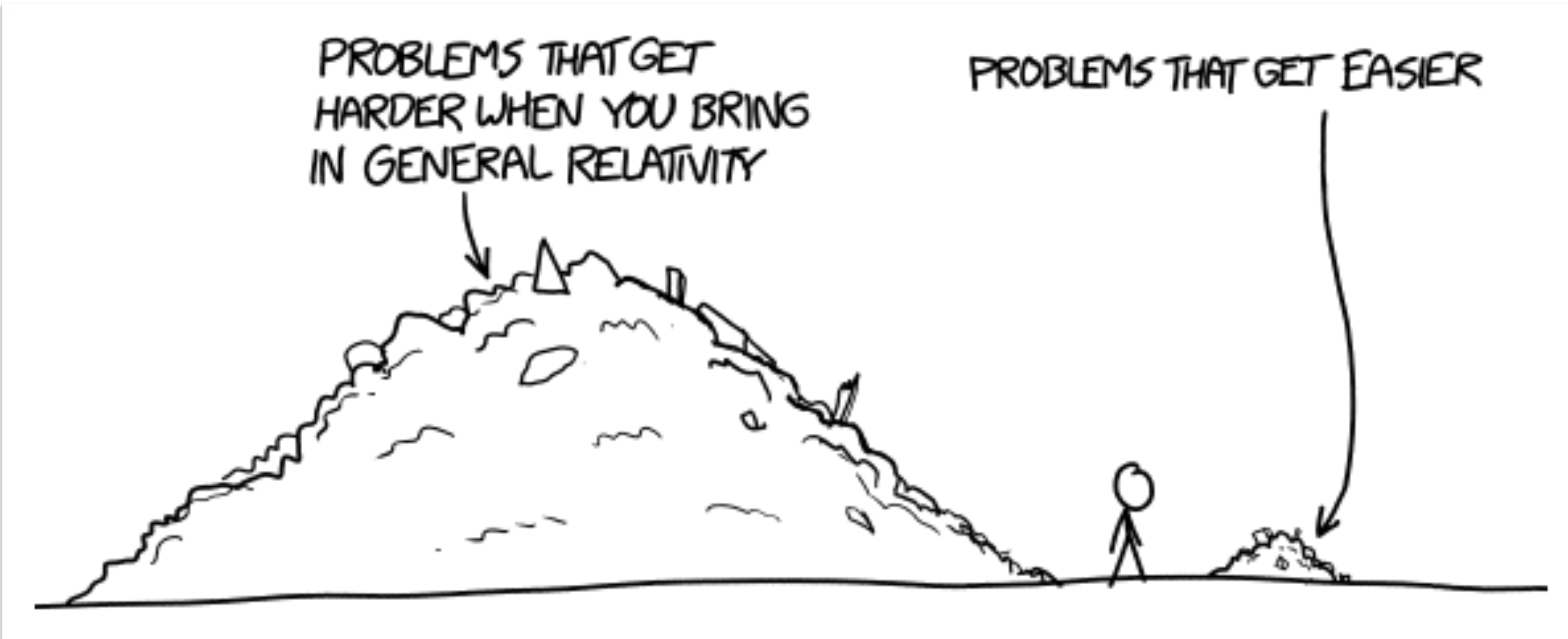
The **POWER is in the redundancy of the equations of motion**

$$\partial_t \bar{\Gamma}^i = -\,2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\left(\bar{\Gamma}^i_{jk}\tilde{A}^{ki} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - \frac{8\pi}{m_{\rm pl}^2}\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\right) + \beta^j \partial_j \bar{\Gamma}^i - \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{li}\partial_k \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

$$+ \beta^j \partial_j \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{li}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

Importantly

These variables have well-behaved differential equations and *are a complete*
c



$$+ \beta^j \partial_j \Gamma^j \partial_j \beta^i + \frac{}{3} \Gamma^i \partial_j \beta^j + \frac{}{3} \bar{\gamma}^{li} \partial_l \partial_j \beta^j + \bar{\gamma}^{ij} \partial_j \partial_l \beta^i$$

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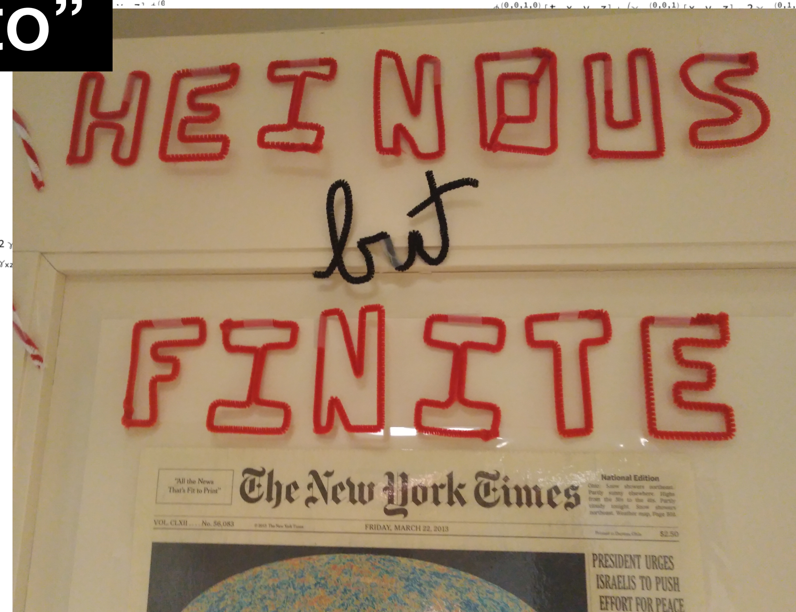
desc

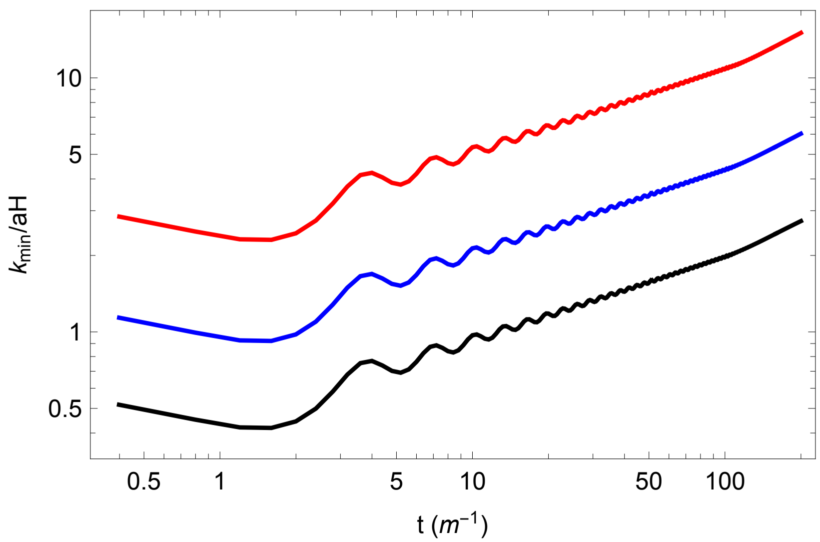
Sort of our "Group Motto"

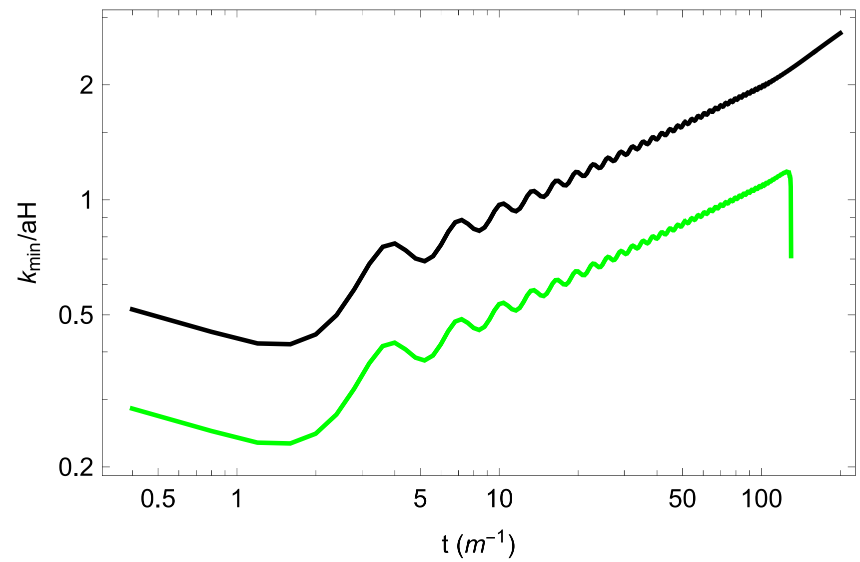


Why do we look for PBH from Preheating?

…because of the breakdown of perturbation theory

Box Sizes
red: 2 m^{-1}
blue: 5 m^{-1}
black: 11 m^{-1}
green: 20 m^{-1}





BOOM! (?)

1907.10601

The linearized Einstein Equation

(asking for a friend)

- when you *linearize* the full Einstein Equations you end up with a set of constraints, e.g.

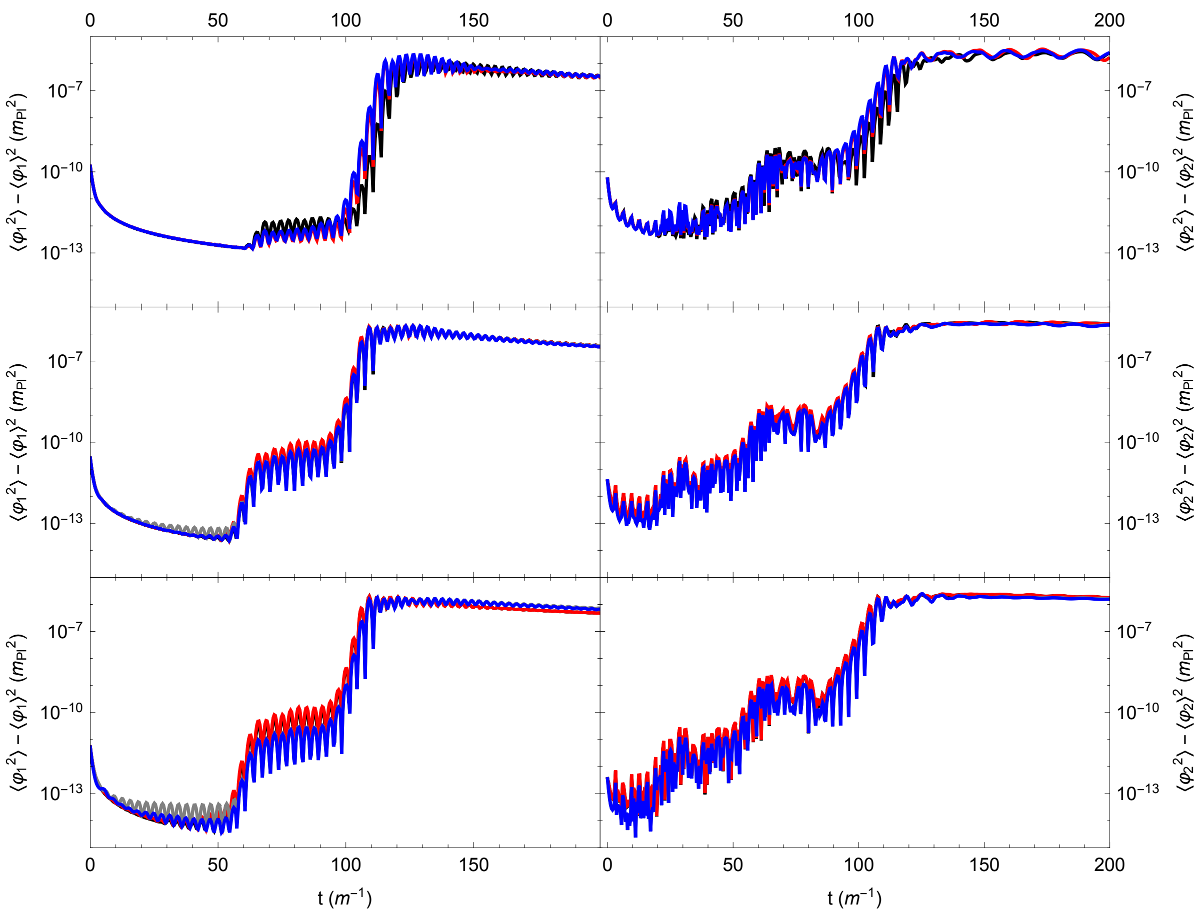
$$ds^2 = -(1+2\Phi)dt^2 + 2a(t)B_{,i}dx^i dt + a^2(t)\left[(1-2\Psi)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j$$

$$\mathcal{G} \equiv 8\pi G a^2 \pi^s + \Phi + \Psi - a^2\ddot{E} - 3a\dot{a}E + 2a\dot{B} + 4\dot{a}B = 0$$

- here we're writing it in terms of the scalar modes only

where $\delta T^i_i = \delta_{ij}\delta p + \partial_i\partial_j\pi^s$

$\langle\phi^2\rangle - \langle\phi\rangle^2$ $\qquad$ $\langle\chi^2\rangle - \langle\chi\rangle^2$



$L = 2\,m^{-1}$

$L = 5\,m^{-1}$

$L = 11\,m^{-1}$

Red = FLRW, Grey = Perturbative, Blue = BSSN

1907.10601

For the big box

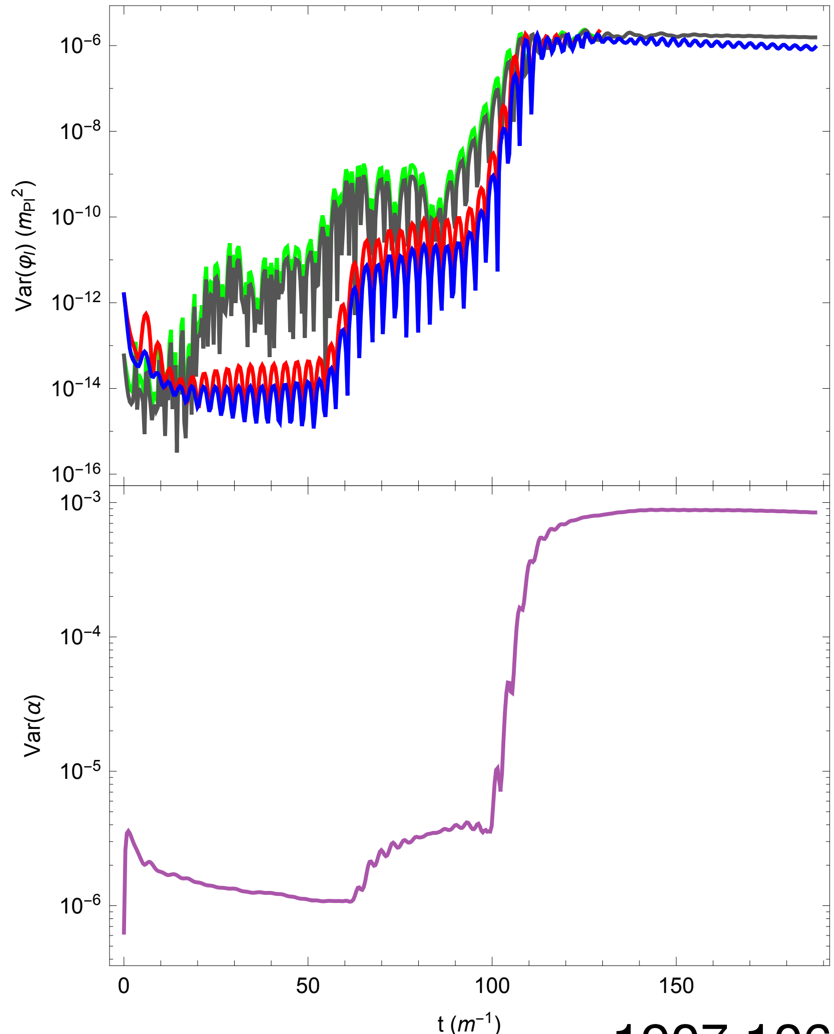
Red: inflaton Perturbative

Blue: inflaton BSSN

Green: decay field Perturbative

Black: decay field BSSN

The *variance* of the lapse does not show departures from homogeneity that indicate back hole formation



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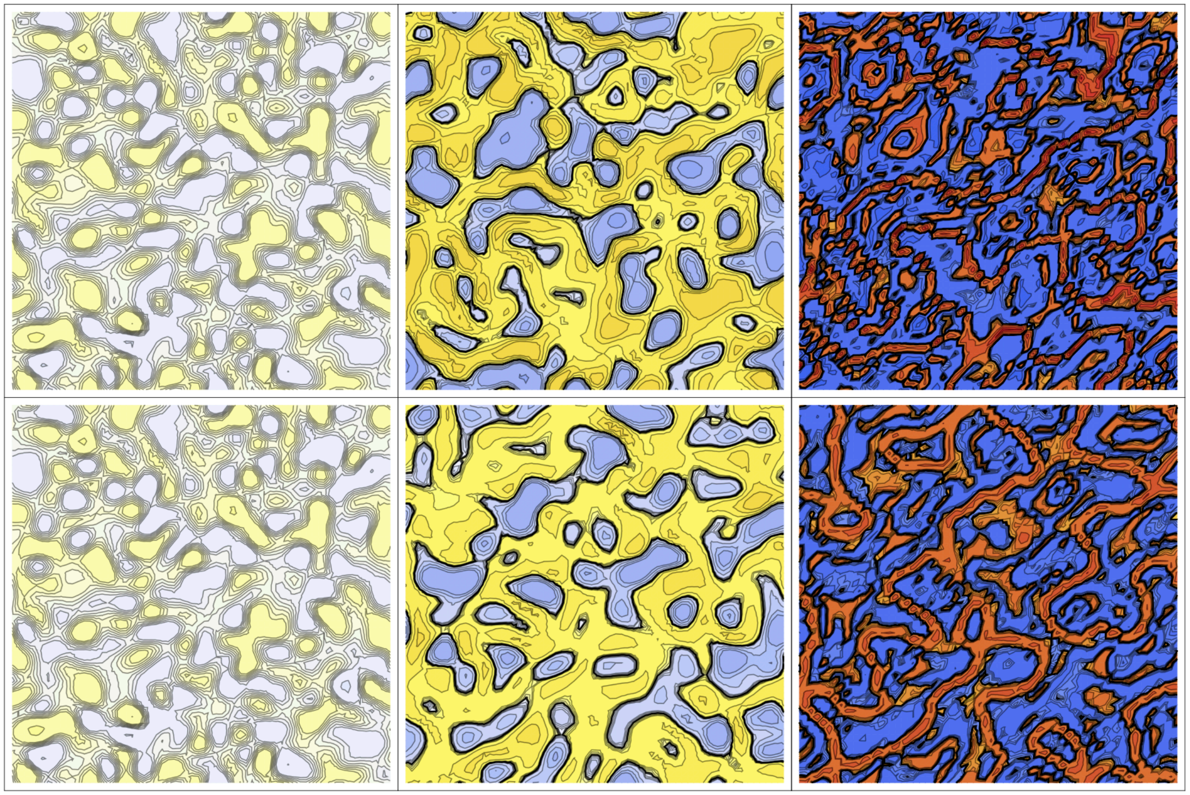
How do they look



Perturbative

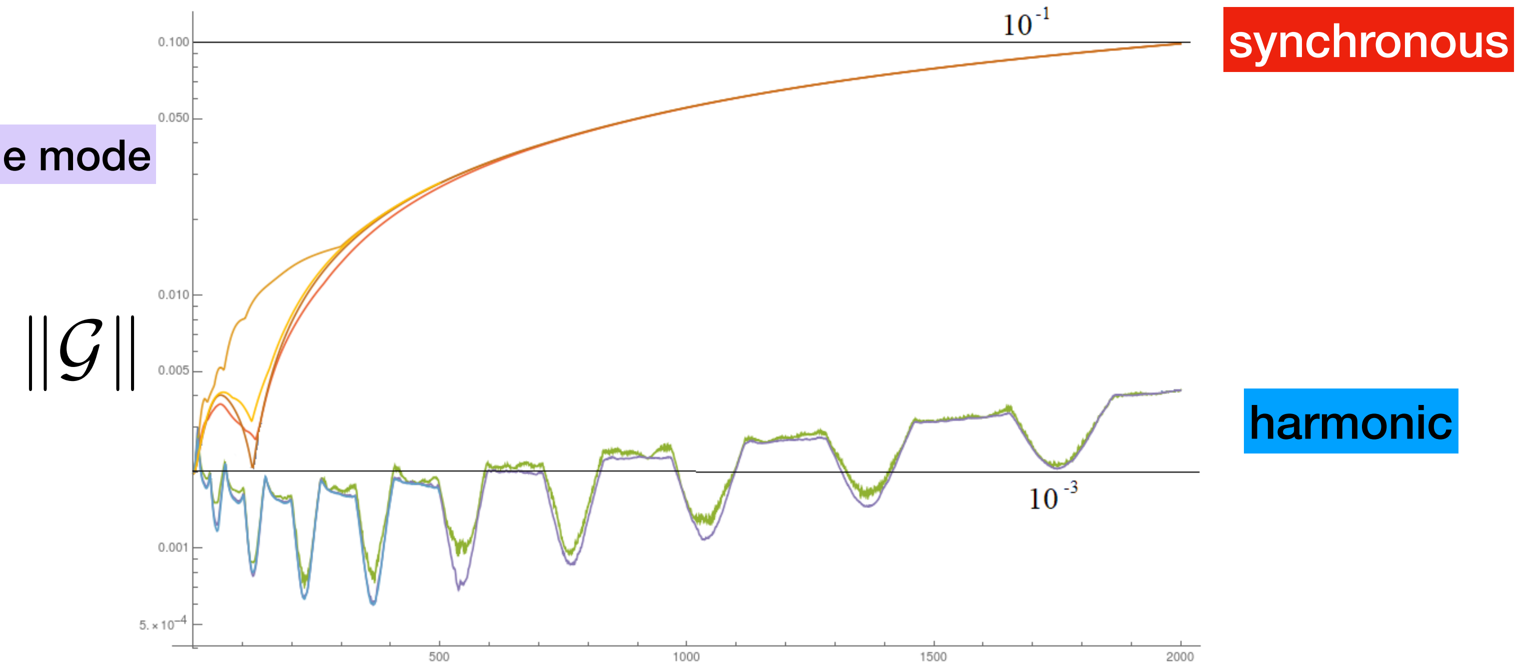
BSSN

1907.10601

Lesson #1:
Violating the linearized Einstein Equations is easy!

Violation of the the *linearized* Einstein Equation

For just a single mode



$$\mathcal{G} \equiv 8\pi G a^2 \pi^s + \Phi + \Psi - a^2 \ddot{E} - 3a\dot{a}E + 2a\dot{B} + 4\dot{a}B = 0$$

1810.05203

Time to get more violent

Gauge-Preheating

- There is a history of incorporating couplings of the inflation to gauge fields, generally with *charged* inflation fields (often in the context of Higgs inflation)
  - Coupling to U(1) fields by A. Rajantie , E. J. Copeland, and S. Pascal
  - Coupling to SU(2) fields by J. Garcia-Bellido et. al., Saffin et al.
- However using uncharged *scalar (or pseudo-scalar)* degrees of freedom were technically a bit more challenging

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- The “normal” Maxwell Stress-Tensor
- (but not for “normal” E/M)

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- W is a *dilatonic* coupling that vanishes as the inflation decays to zero
- Possible generation of long-wavelength magnetic fields during inflation, e.g. Caldwell, Motta, Kamionkowski Phys. Rev. D 84, 123525 (2011).

$$W(\phi) = e^{-\phi}M$$

$$X(\phi) = 0$$

1305.7226

Gauge-Preheating

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{W(\phi)}{4}F_{\mu\nu}F^{\mu\nu} - \frac{X(\phi)}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- X is a *Chern Simons* coupling that couples the inflation to the curl of the vector field
- A coupling consistent with a shift-symmetric inflaton
- Also possible generation of polarized magnetic fields during inflation, e.g. Garretson, Field and Carroll, Phys. Rev. D 46 5346 (1992)

$$W(\phi) = 1 \qquad X(\phi) = \frac{\alpha_g}{f}\phi$$

1502.06506

A Chern-Simons Coupling

$$\mathcal{L} = -\frac{X(\phi)}{4} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

- The form of the interaction begs for a decomposition of the gauge field onto its two helicity states,

$$\tilde{A}_{\mathbf{k}} = \sum_{\lambda=\pm} A_{\mathbf{k}} \vec{\varepsilon}^{\lambda}(\mathbf{k})$$

where

$$k_i \varepsilon_i^{\pm}(\vec{k}) = 0 \qquad \epsilon_{ijk} k_j \varepsilon_k^{\pm}(\vec{k}) = \mp i k \varepsilon_i^{\pm}(\vec{k})$$

$$\varepsilon_i^{\pm}(-\vec{k}) = \varepsilon_i^{\pm}(\vec{k})^* \qquad \varepsilon_i^{\lambda}(\vec{k})\varepsilon_i^{\lambda'}(-\vec{k})^* = \delta^{\lambda\lambda'}$$

1502.06506

A Chern-Simons Coupling

$$\mathcal{L} = -\frac{X(\phi)}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

- The form of the interaction begs for a decomposition of the gauge field onto its two helicity states,

$$\tilde{A}_{\mathbf{k}} = \sum_{\lambda=\pm} A_{\mathbf{k}} \vec{\varepsilon}^{\lambda} (\mathbf{k})$$

so we see the two polarizations couple (with opposite sign) to the velocity of the inflation)

$$\left(\partial_t^2 + k^2 \pm \frac{\alpha}{f} \frac{\dot{\phi}}{H} \frac{k}{\tau} \right) A_{\pm}(\vec{k}) = 0$$

1502.06506

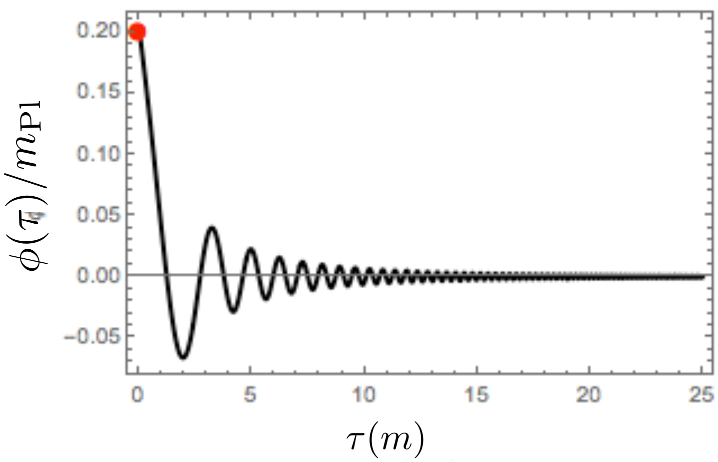
A Chern-Simons Coupling

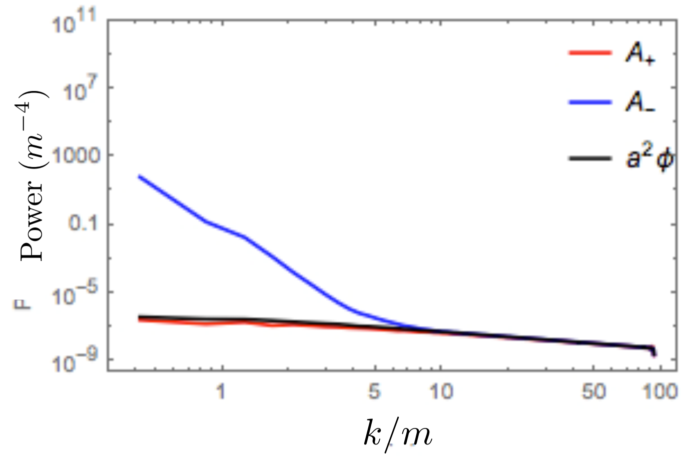
- For large values of the velocity of the inflation (relative to small momenta), there is a clear tachyonic instability
- This instability should be polarized and present during each oscillation of the field

$$\left(\partial_t^2 + k^2 \pm \frac{\alpha}{f}\frac{\dot{\phi}}{H}\frac{k}{\tau}\right) A_{\pm}(\vec{k}) = 0$$

1502.06506

A Small Chern-Simons Coupling





$$\frac{\alpha_g}{f} = 35\, m_{\rm pl}^{-1}$$

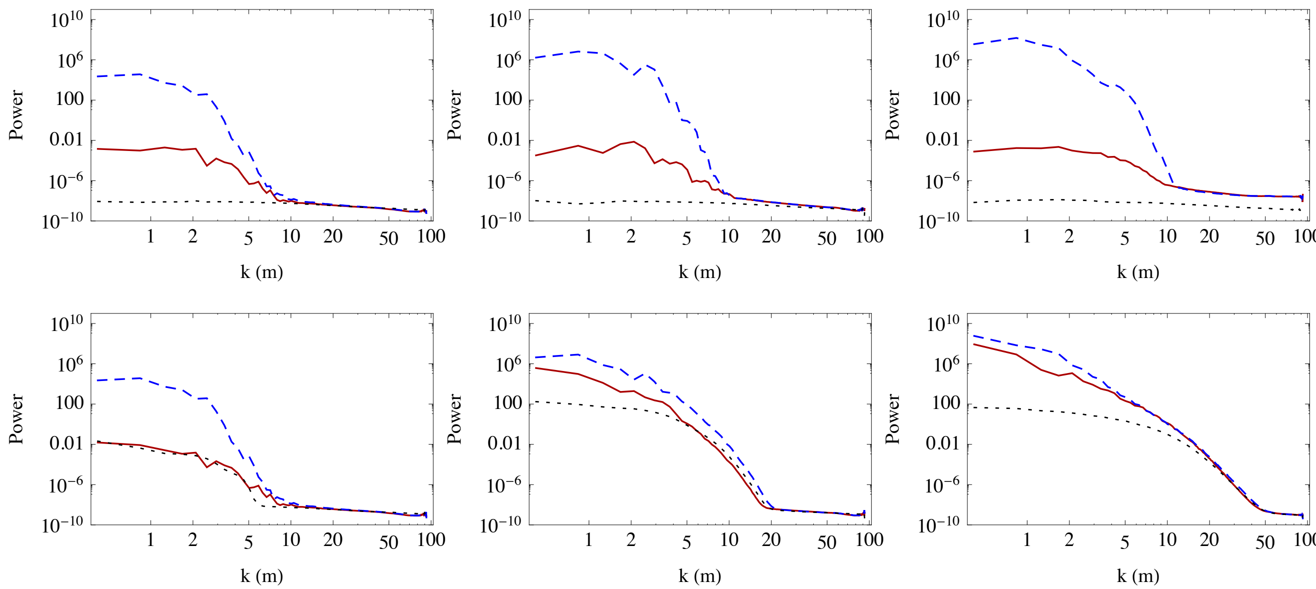
1502.06506

The Final State

$$\frac{\alpha_g}{f} = 35\, m_{\text{pl}}^{-1} \qquad \frac{\alpha_g}{f} = 45\, m_{\text{pl}}^{-1} \qquad \frac{\alpha_g}{f} = 60\, m_{\text{pl}}^{-1}$$

NO back-reaction

WITH back-reaction



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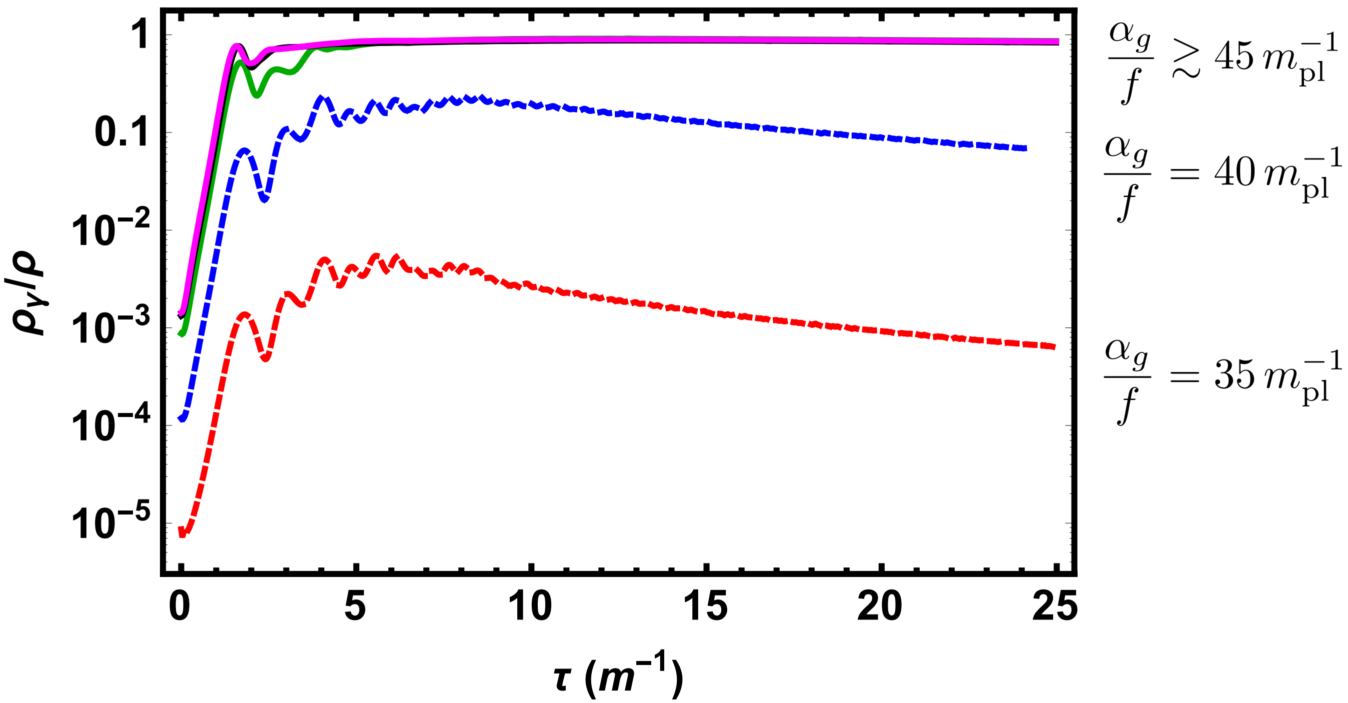
A Chern-Simons Coupling



$$X = \frac{\alpha_g}{f}\phi$$

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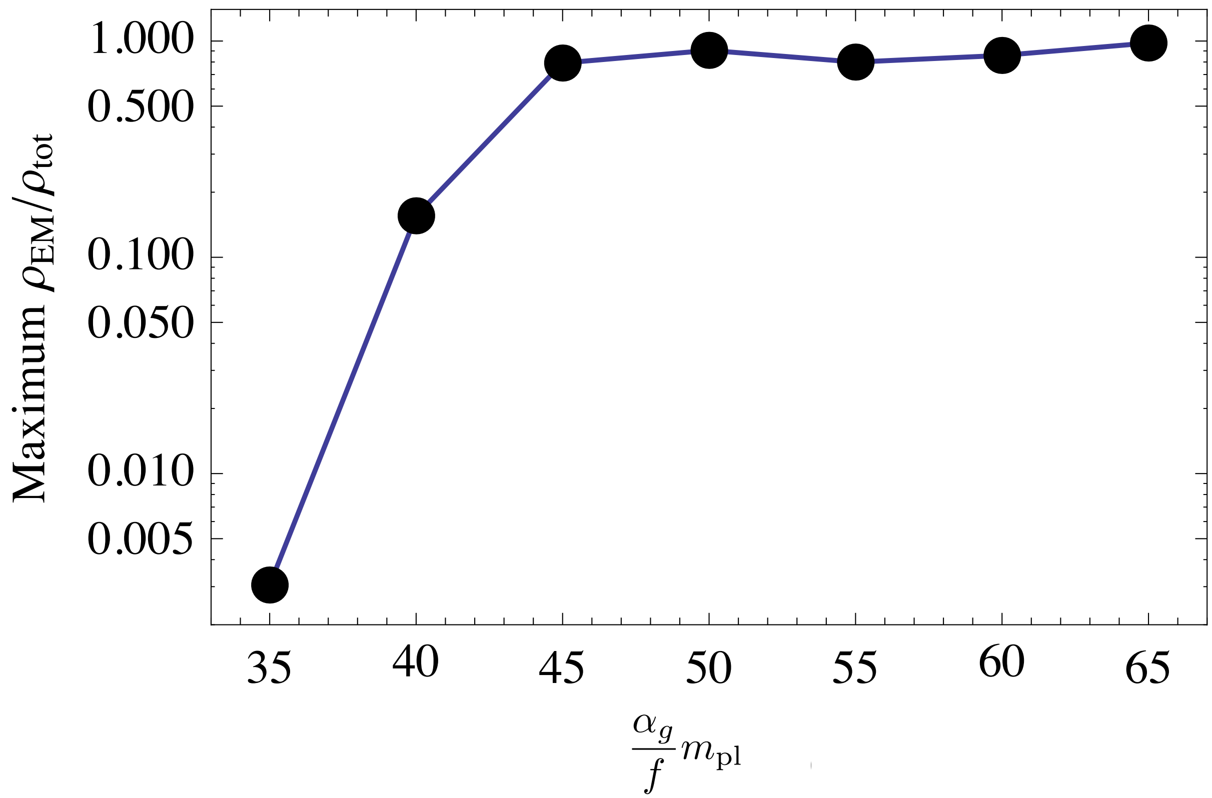
A Chern-Simons Coupling



$$X = \frac{\alpha_g}{f}\phi$$

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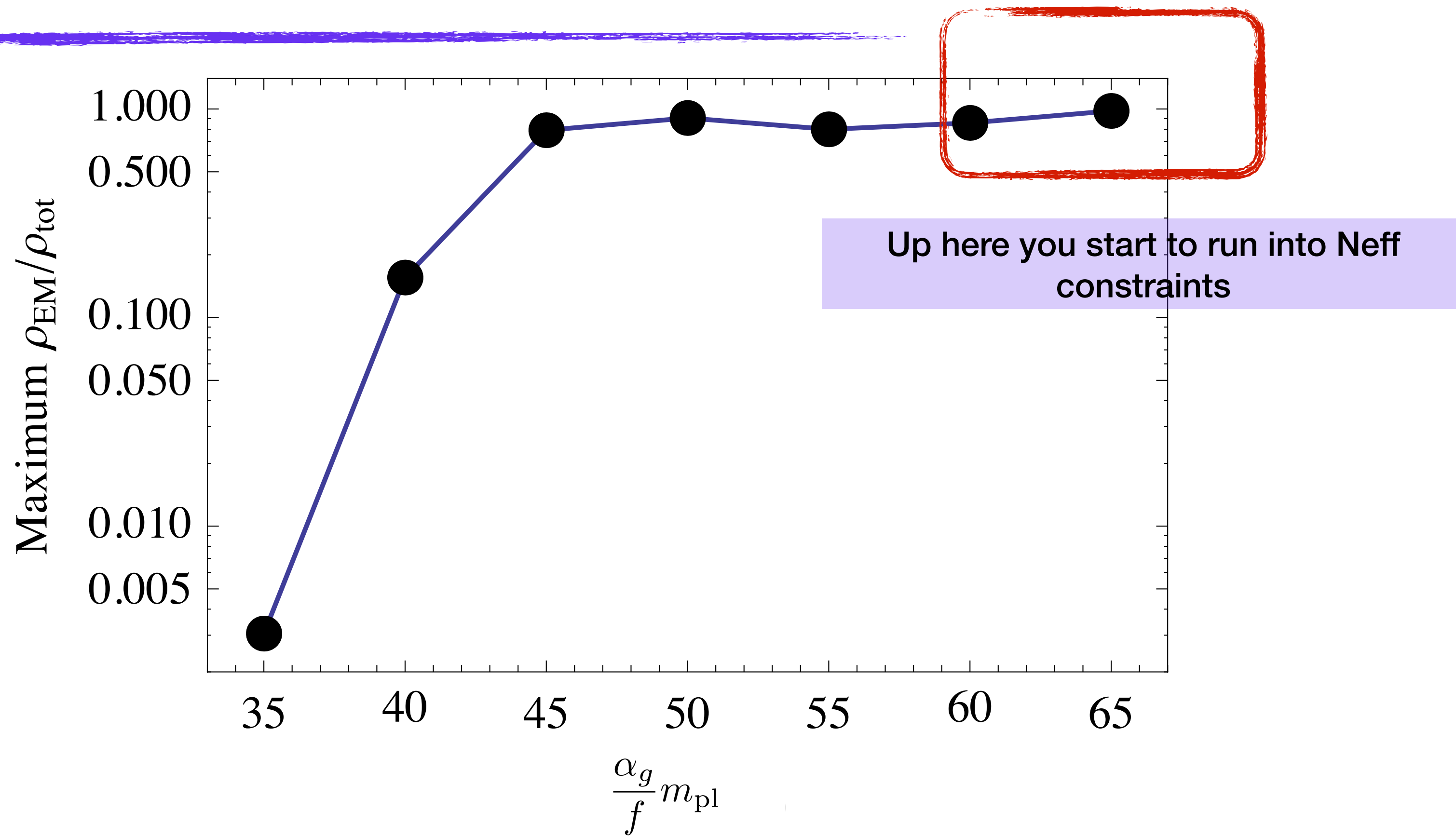
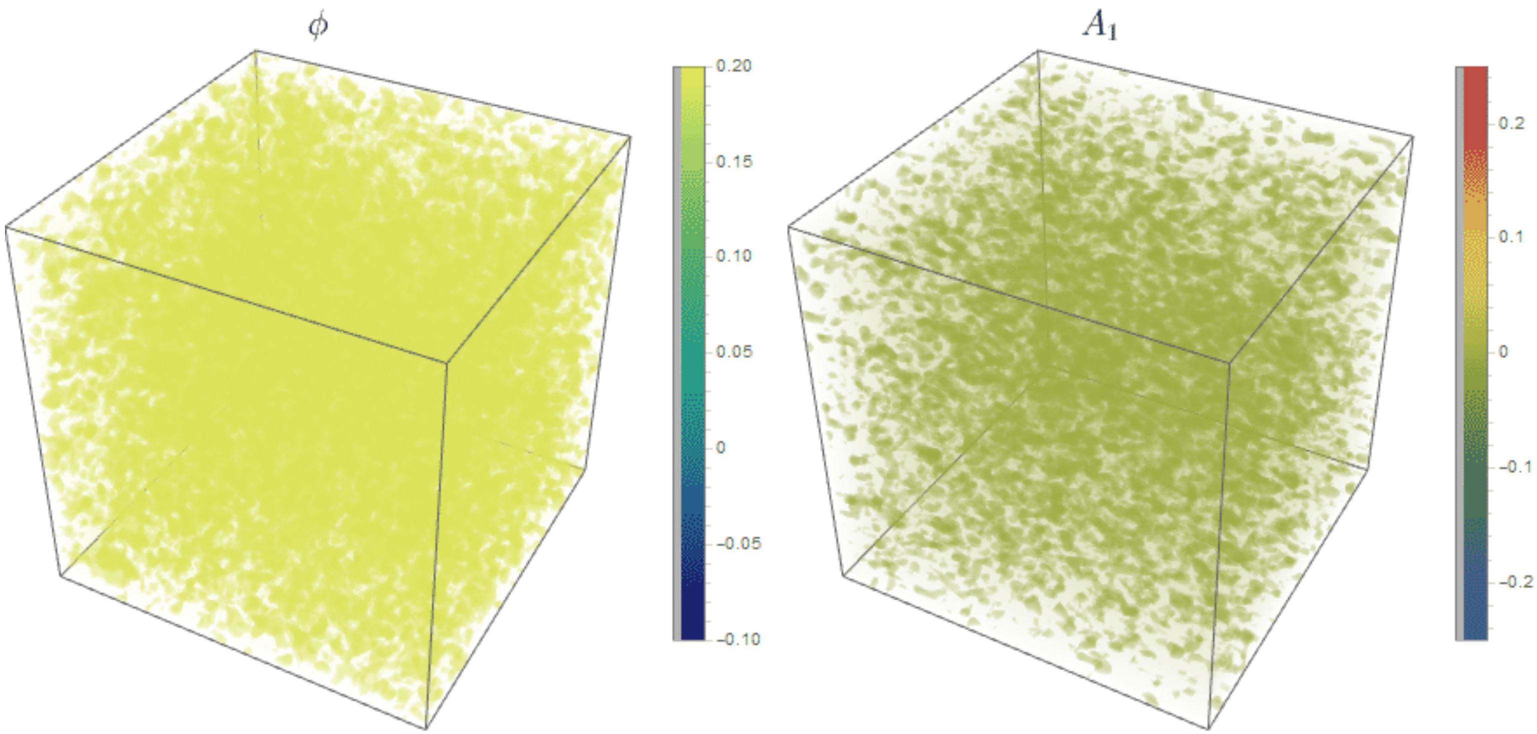
A Chern-Simons Coupling



$$X = \frac{\alpha_g}{f}\phi$$

1502.06506

But… we get structure



2311.01504

The BSSN version

We can write down a set of evolution equations,

$$\partial_t E^m = \beta^o \partial_o E^m - E^o \partial_o \beta^m + \alpha \left(K E^m + \epsilon^{mno} D_n B_o - \mathcal{J}^m \right) + \epsilon^{mno} D_n \alpha B_o$$

$$\partial_t \mathcal{A}_m = \beta^o \partial_o \mathcal{A}_m + \mathcal{A}_o \partial_m \beta^o - \alpha \left(E_m + D_m \mathcal{A} \right) - \mathcal{A} D_m \alpha$$

$$\partial_t \mathcal{A} = \beta^o D_o \mathcal{A} + \alpha \left(K \mathcal{A} - D^m \mathcal{A}_m \right) - \mathcal{A}^m D_m \alpha$$

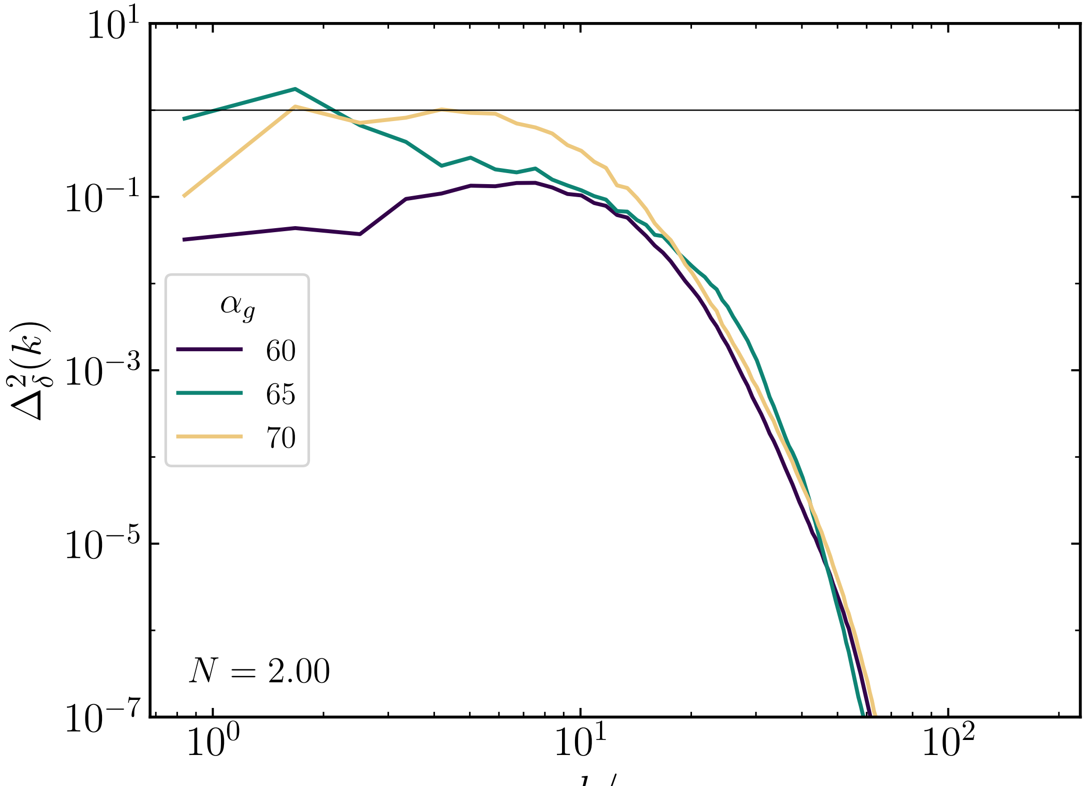
with…

$$\mathcal{J} = -\frac{1}{W(\varphi)} \left(W'(\varphi) E^m D_m \varphi - X'(\varphi) B^m D_m \varphi \right)$$

$$\mathcal{J}^m = \frac{1}{W(\varphi)} \left(W'(\varphi) \left[\Pi E^m - \epsilon^{mno} D_n \varphi B_o \right] - X'(\varphi) \left[\Pi B^m + \epsilon^{mno} D_n \varphi E_o \right] \right)$$

$$B^m = \epsilon^{mno} D_n \mathcal{A}_o = \epsilon^{mno} \partial_n \mathcal{A}_o, = e^{-6\phi} \varepsilon^{mno} \partial_n \mathcal{A}_o$$

We see very BIG density contrasts!



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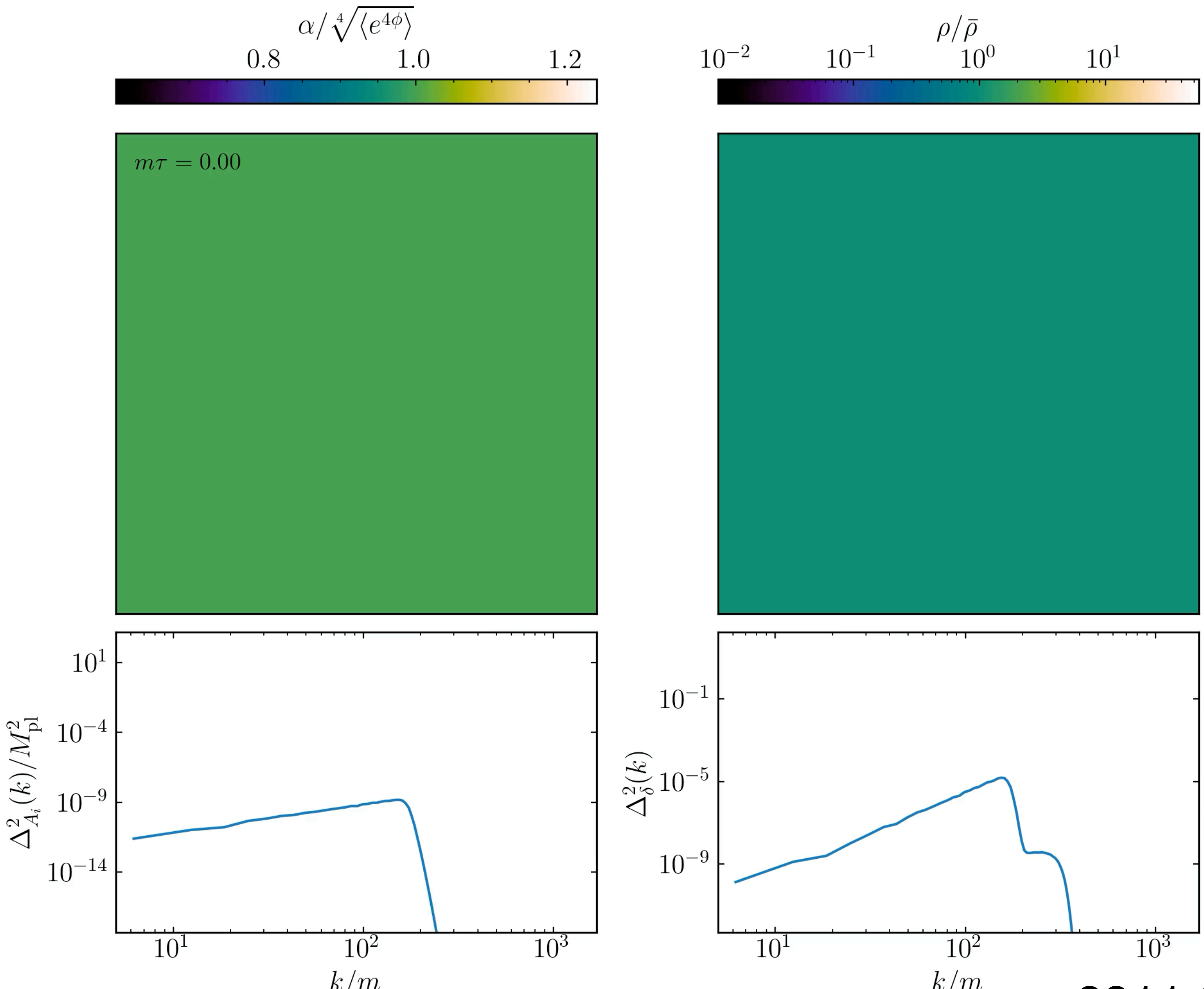
$$\frac{\alpha_g}{f} = 65\, m_{\rm pl}^{-1}$$



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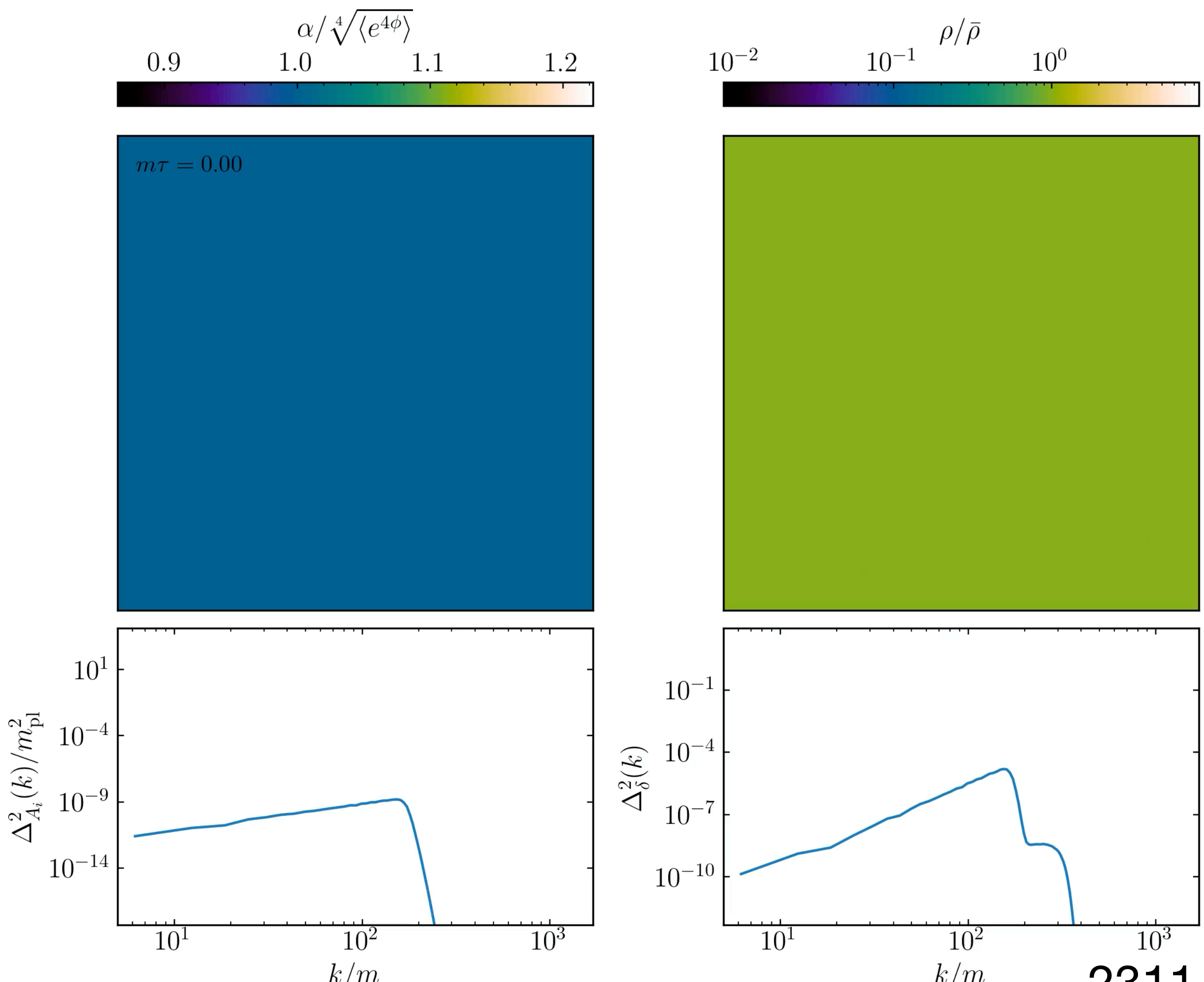
$$\frac{\alpha_g}{f} = 70\, m_{\rm pl}^{-1}$$



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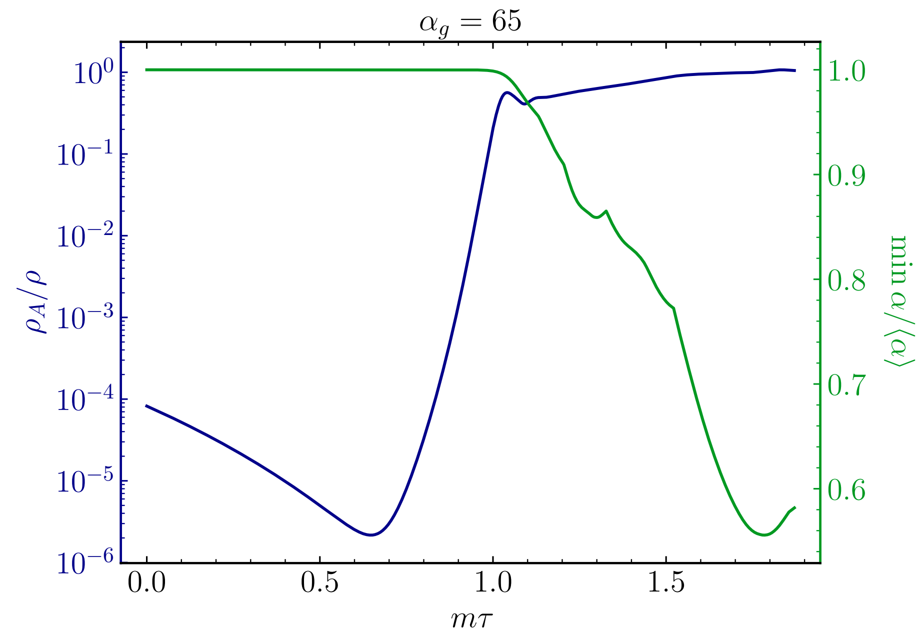
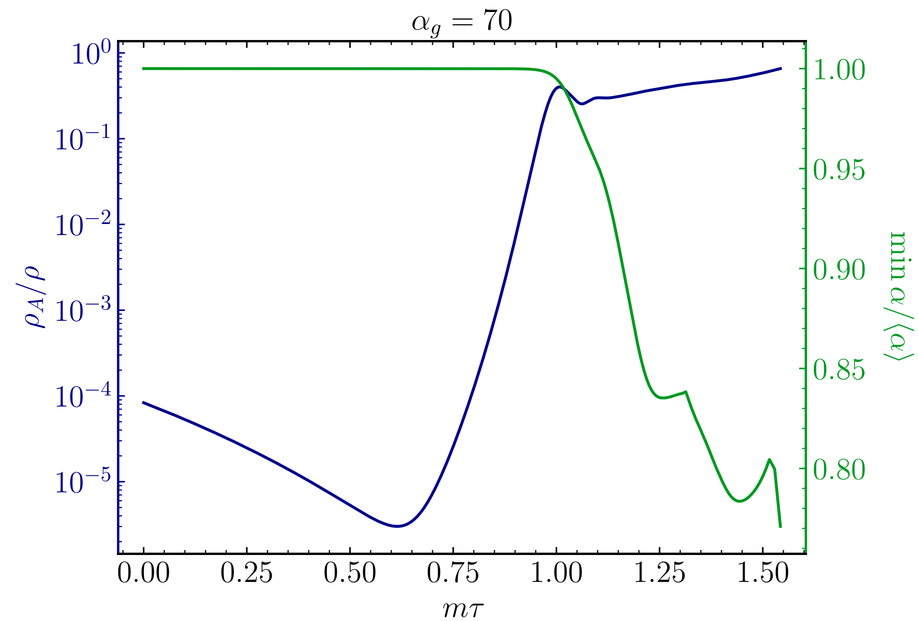
But … there are no PBH

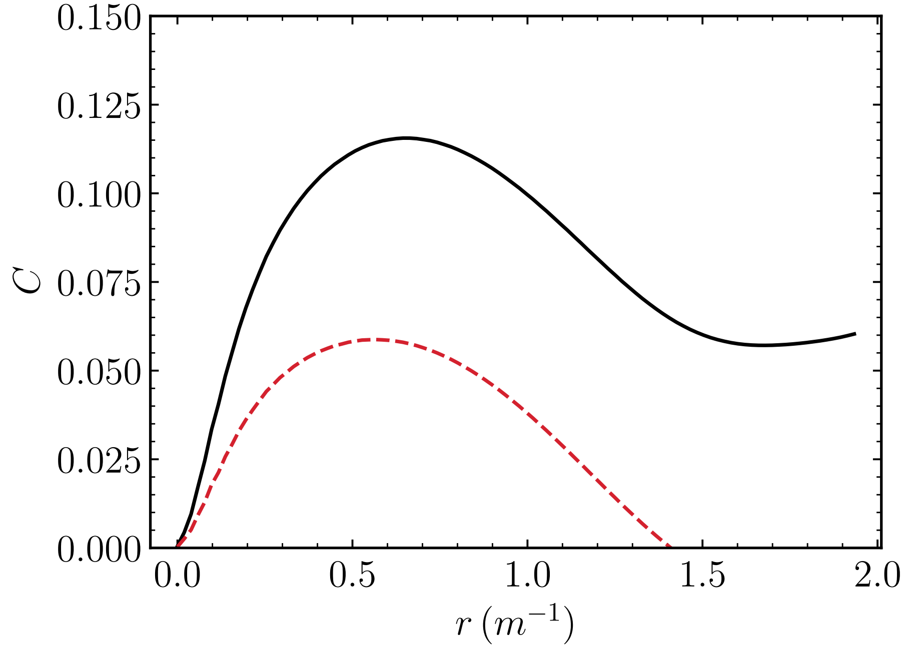


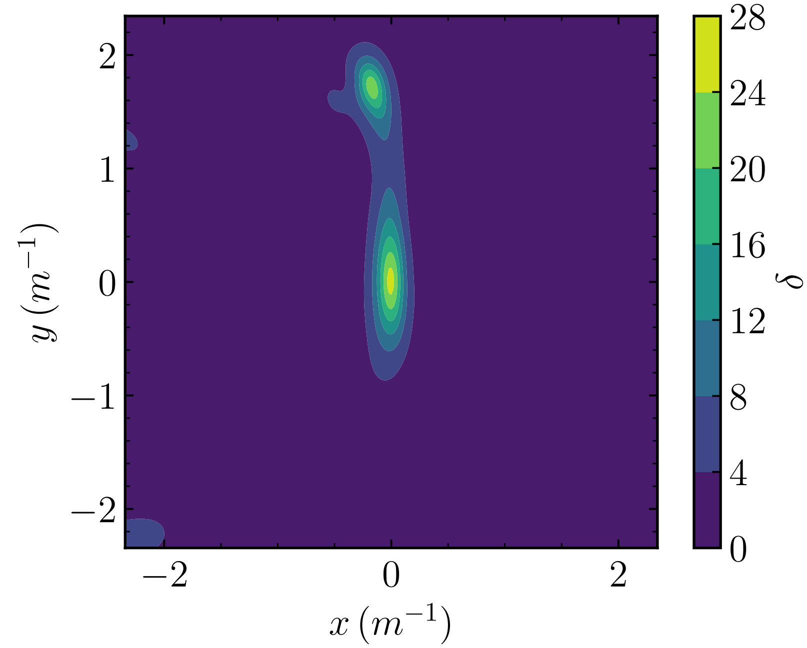


The minimum value of the lapse throughout the grid doesn’t approach zero

2311.01504

What does it look like?

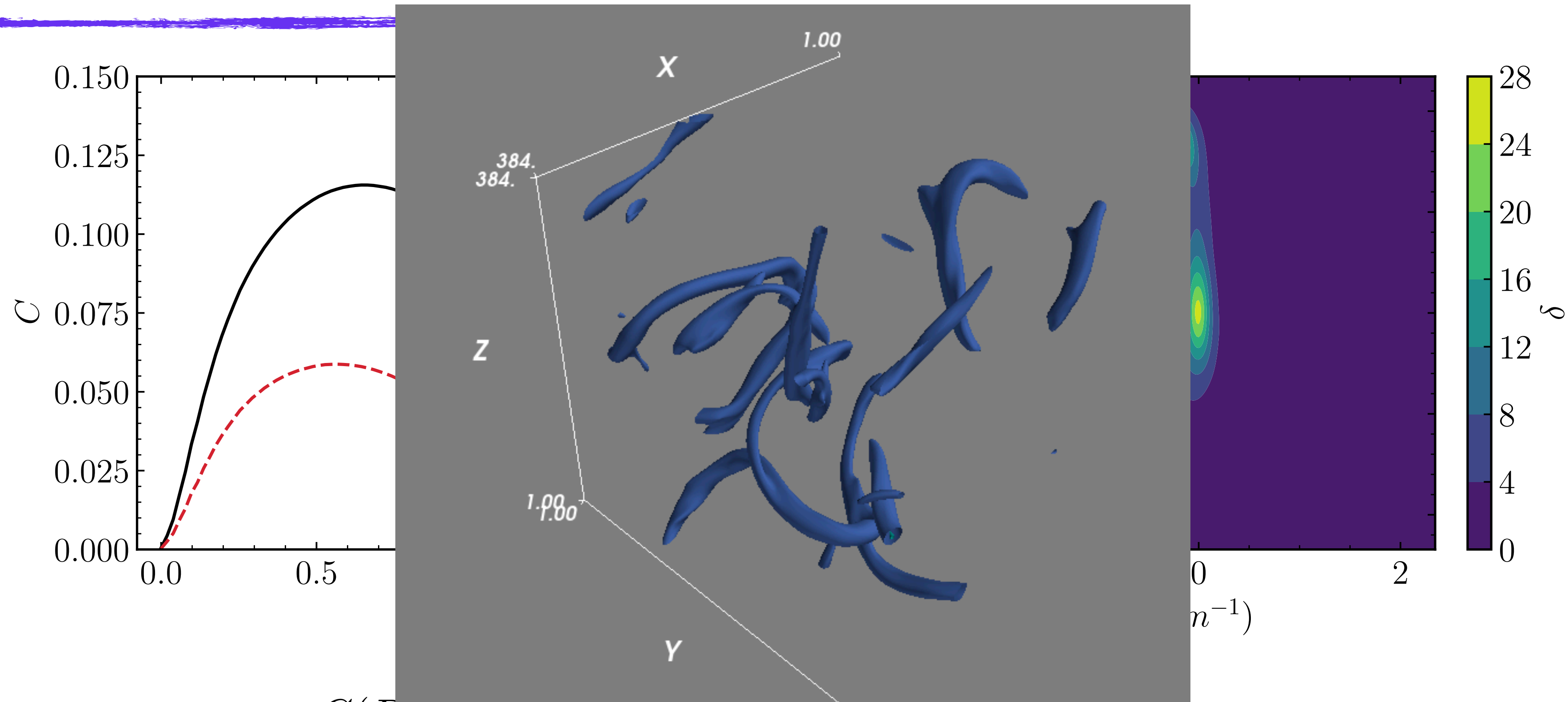




$$C(R) = \frac{G\delta M}{R}$$

2311.01504

What does it look like?



$$C(R) = \frac{\quad}{R}$$

2311.01504

Lesson #2:
Large density contrasts, near the horizon, do not necessarily collapse to Black Holes

Lesson #2:
Large density contrasts, near the horizon, do not necessarily collapse to Black Holes

BUT WHY NOT!?!

Not spherical enough? - à la Albert Escrivà and Chulmoon Yoo

Gauge fields are problematic?

Overdensities aren't big enough, fast enough?

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Gauge fields are

Simulations of Ellipsoidal Primordial Black Hole Formation

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and Chulmoon Yoo

Primordial black hole formation with full numerical relativity

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Received November 2, 2021
Revised February 10, 2022
Accepted February 19, 2022
Published March 14, 2022

JCAP03(2022)

Gauge fields are problematic?

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BU WHY NOT!?!

Primordial black hole formation during slow-reheating: A review

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(Dated: February 7, 2024)

In this paper we review the possible mechanisms for the production of primordial black holes (PBHs) during a slow-reheating period in which the energy transfer of the inflaton field to standard model particles becomes effective at slow temperatures, offering a comprehensive examination of the theoretical foundations and conditions required for each of formation channel. In particular, we focus on post-inflationary scenarios where there are no self-resonances and the reheating epoch can be described by the inflaton evolving in a quadratic-like potential. In the hydrodynamical

scrivà

Gauge fields are problematic?

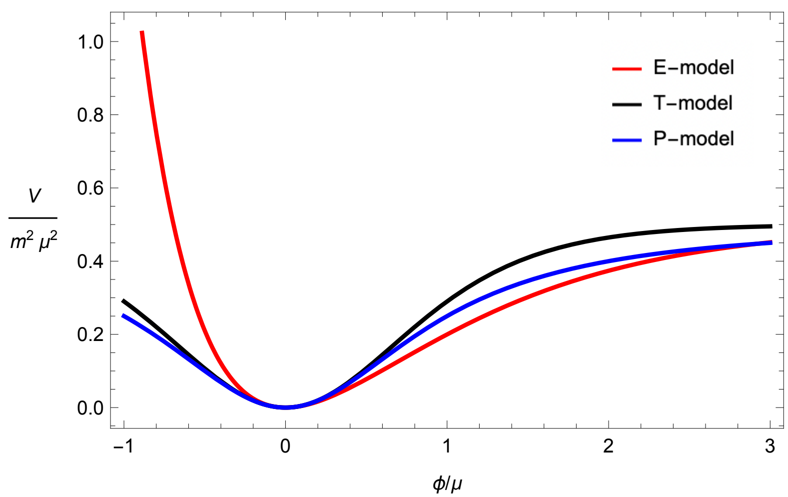
Overdensities aren't big enough, fast enough?

So what’s next?

Continuing on with Preheating

Alpha-attractors are an intriguing possibility:

$$\mathcal{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{e^{2\phi/\mu}}{2}\left(\partial\chi\right)^2 - V(\phi)$$



E-model: $V = \frac{m^2\mu^2}{2}\left(1 - e^{-\frac{\phi}{\mu}}\right)^2$

T-model: $V = \frac{m^2\mu^2}{2}\tanh^2\left(\frac{\phi}{\mu}\right)$

P-model: $V = \frac{m^2\mu^2}{2}\frac{\phi^2}{\phi^2+\mu^2}$

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Next Steps

Take, e.g. the E-model:

$$\mathcal{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$

Next Steps

Take, e.g. the E-model:

$$\mathcal{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$

One of the parameters (m) is directly set by CMB

Next Steps

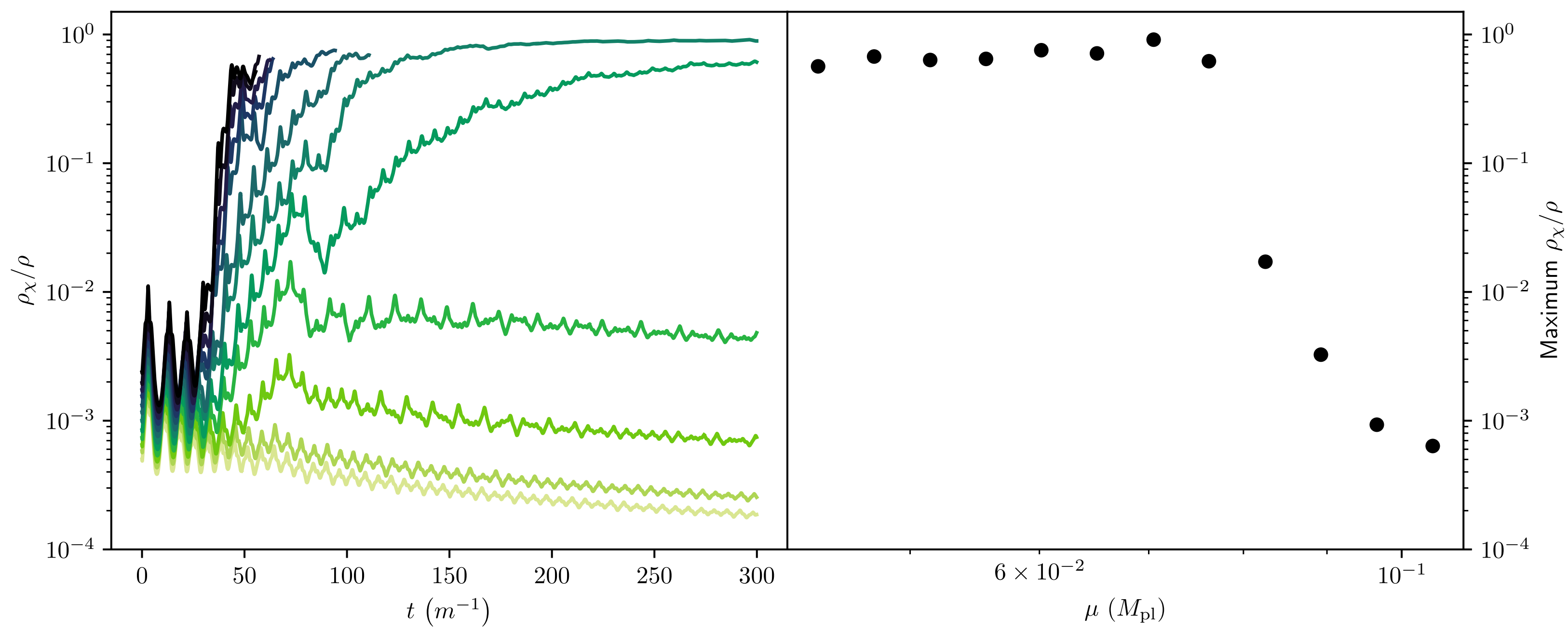
Take, e.g. the E-model:

$$\mathcal{L} = -\frac{1}{2}\left(\partial\phi\right)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$

Which leaves only *one* parameter that appears in both the potential and the kinetic coupling

Indeed, the preheating is violent

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{e^{2\phi/\mu}}{2}(\partial\chi)^2 - \frac{m^2\mu^2}{2}\left(1 - e^{-\phi/\mu}\right)^2$$



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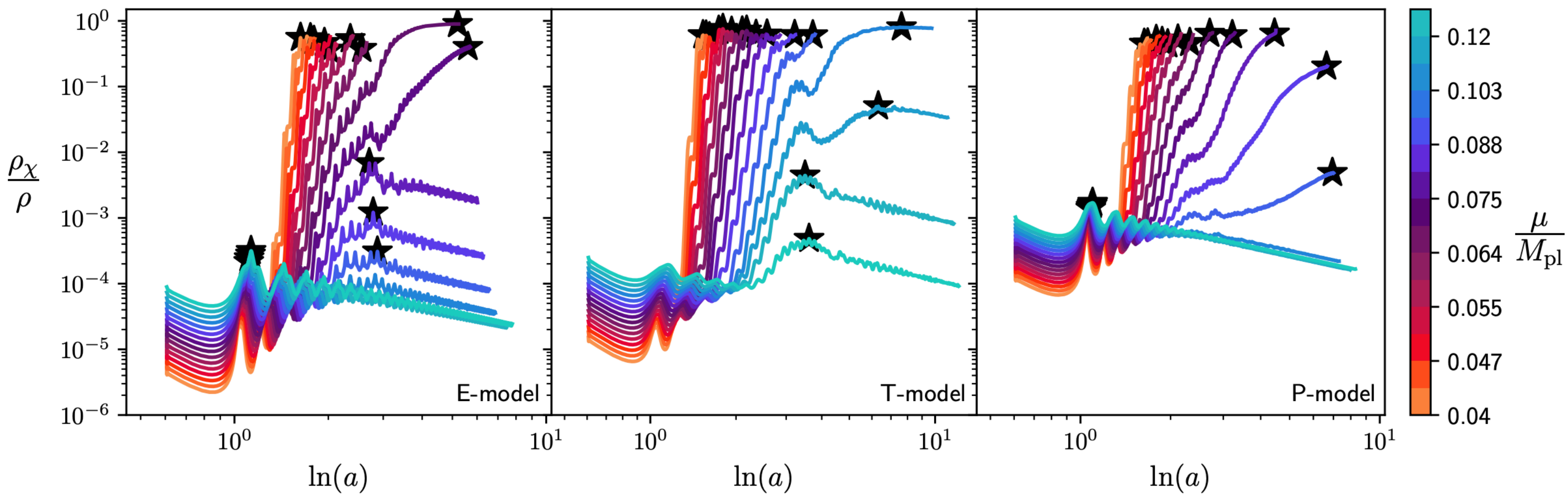
Which is only weakly model dependent

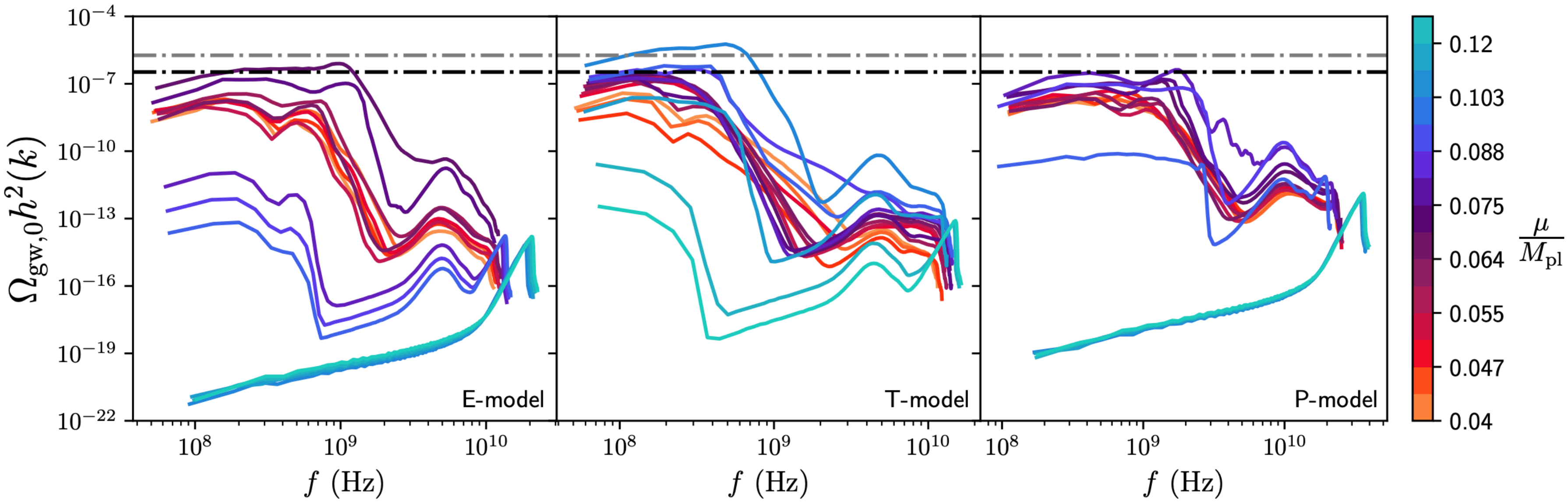
$$V = \frac{m^2\mu^2}{2}\left(1 - e^{-\frac{\phi}{\mu}}\right)^2 \qquad V = \frac{m^2\mu^2}{2}\tanh^2\left(\frac{\phi}{\mu}\right) \qquad V = \frac{m^2\mu^2}{2}\frac{\phi^2}{\phi^2+\mu^2}$$



2402.16152

They have lots of Gravitational Waves!

$$V = \frac{m^2\mu^2}{2}\left(1 - e^{-\frac{\phi}{\mu}}\right)^2 \qquad V = \frac{m^2\mu^2}{2}\tanh^2\left(\frac{\phi}{\mu}\right) \qquad V = \frac{m^2\mu^2}{2}\frac{\phi^2}{\phi^2 + \mu^2}$$



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Moving this to full Numerical Relativity

$$\Box\phi = -\frac{\partial \mathcal{L}}{\partial \phi}$$

$$\Pi = \frac{1}{\alpha}\left(\partial_t \varphi - \beta^i \partial_i \varphi\right)$$

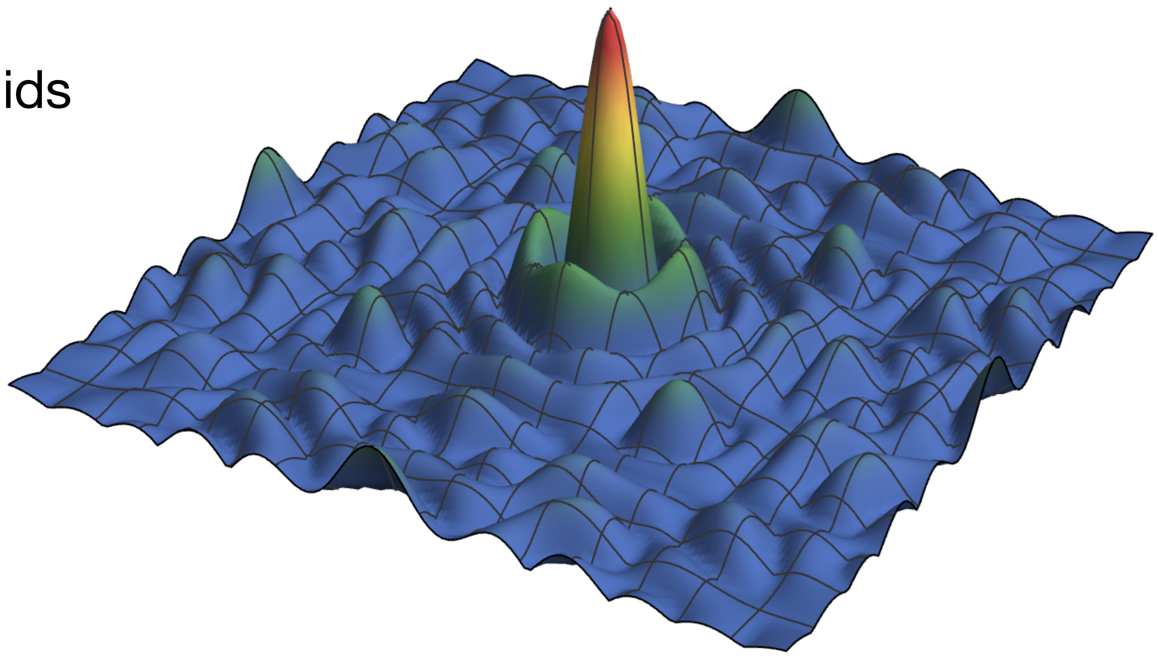
$$\partial_0 \Pi = \beta^i \partial_i \Pi - \alpha g^{ij} \partial_j \partial_i \varphi + \alpha g^{ij} \partial_k \varphi \Gamma^k_{ij} - g^{ij} \partial_j \varphi \partial_i \alpha + \alpha K \Pi$$

$$\begin{aligned}
\partial_0 \Pi =& \beta^i \partial_i \Pi - \alpha g^{ij} \partial_j \partial_i \varphi + \alpha g^{ij} \partial_k \varphi \Gamma^k_{ij} - g^{ij} \partial_j \varphi \partial_i \alpha + \alpha K \Pi \\
& - \frac{1}{2}\frac{\partial W}{\partial \varphi}\left[- \Pi^2 - 2\alpha^{-1}\beta^i \Pi \partial_i \chi + \left(\gamma^{ij} - \alpha^{-2}\beta^i\beta^j\right)\partial_i \chi \partial_j \chi\right] - \frac{\partial V}{\partial \varphi} \\
\partial_0 \Theta =& \beta^i \partial_i \Theta - \alpha g^{ij} \partial_j \partial_i \chi + \alpha g^{ij} \partial_k \chi \Gamma^k_{ij} - g^{ij} \partial_j \chi \partial_i \alpha + \alpha K \Theta \\
& + \frac{1}{W(\varphi)}\left(g^{ij}\partial_j \chi \partial_i W + \alpha^{-1}\Theta \partial_0 W - \alpha^{-1}\beta^i \Theta \partial_i W\right)
\end{aligned}$$

Eve Currens ’25
(Kenyon)

Next Steps

- Replace fields with fluids



Starting from a (violent) cosmological process does not (yet) seem to produce black holes

Next Steps

- Let's forget about *physical mechanisms* and focus on when *cosmological messiness* ruins everything!

$$T^{\mu\nu} = \left(\rho_0 + \frac{p}{c^2}\right) U^\mu U^\nu + p g^{\mu\nu}$$

$$E_* \equiv \sqrt{\gamma}\frac{\rho_0}{\alpha}\left[\frac{4}{3}W^2 - \frac{1}{3}\right] \qquad P_*^j \equiv \sqrt{\gamma}\frac{4}{3}W^2\rho_0 v^j$$

Which leads to a dynamical System

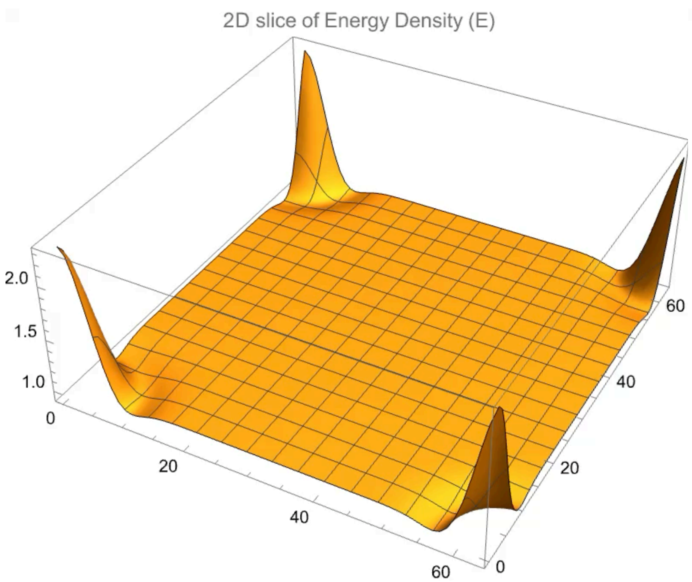
$$\nabla_\beta \left(n_\alpha T^{\alpha\beta}\right) = \frac{1}{\sqrt{-g}}\partial_\beta \left(\sqrt{-g} n_\alpha T^{\alpha\beta}\right)$$

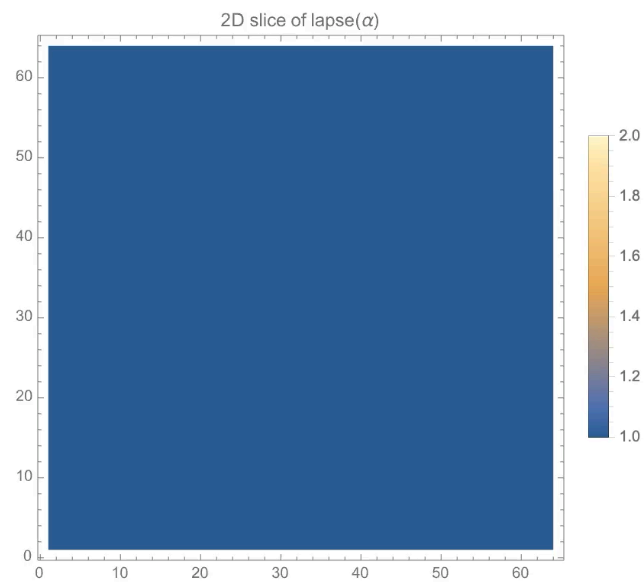
$$\partial_0 E = -\alpha(\partial_i P^i) + \beta^i(\partial_i E) + E(\partial_i \beta^i)$$
$$- 2P^i(\partial_i \alpha) + P^i \beta^j K_{ij} + K_{ij} S^{ij}_{code} - P^j \beta^i K_{ij}$$

$$\partial_t \left(\sqrt{-g} T^0{}_j\right) + \partial_i \left(\sqrt{-g} T^i{}_j\right) = \frac{1}{2}\sqrt{-g} T^{\alpha\beta} \partial_j g_{\alpha\beta}$$

$$\partial_0 P_j = -\gamma_{kj} \partial_i S^{ik}_{\mathrm{code}} - S^{ik}_{\mathrm{code}} \partial_i \gamma_{kj} + P_j \partial_i \beta^i - \beta^i \partial_i P_j$$
$$- E(\partial_j \alpha) + \frac{P^i}{2} \beta^k (\partial_j \gamma_{ik}) + P_k \partial_j \beta^k$$
$$+ \frac{1}{2} S^{ik}_{\mathrm{code}} (\partial_j \gamma_{ik}) - \frac{1}{2} P^k \beta^i (\partial_j \gamma_{ik}$$

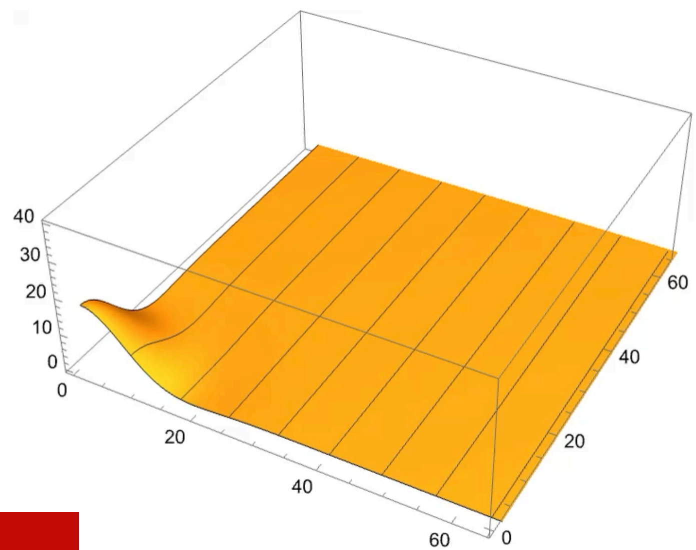
What that means…





An overdense sub-horizon region

For higher super-horizon



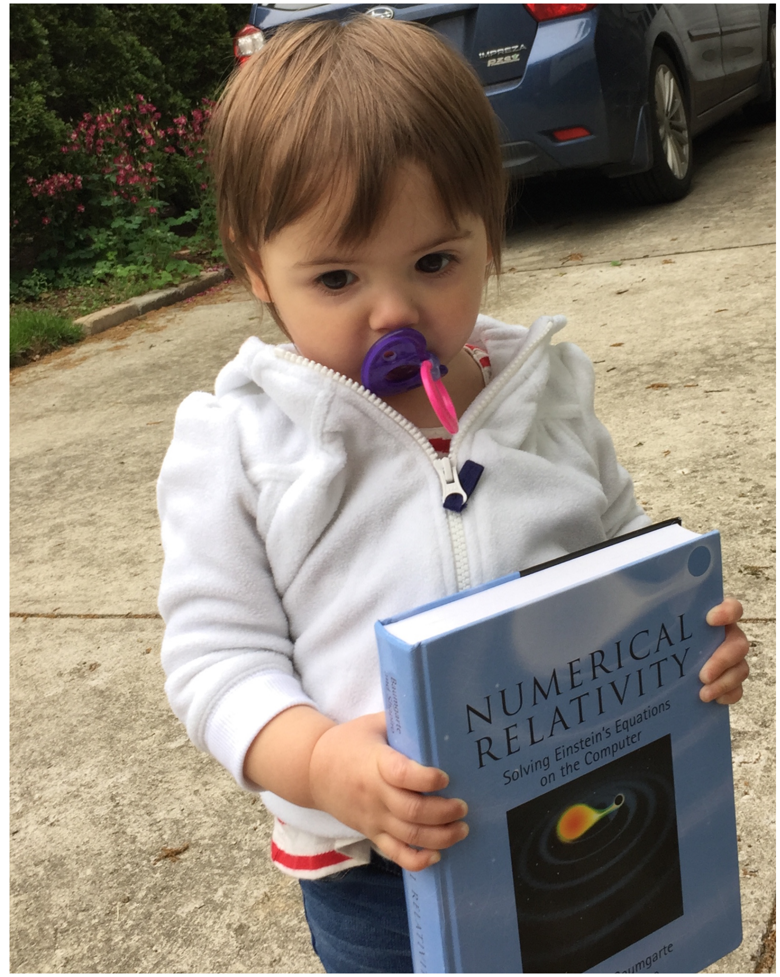
Code development is ongoing

Mary Gerhardinger '22 (Penn)

Amanda Miller '25 (Kenyon)

My Thoughts

- Gravity seems to be important when studying black holes
- We need to *do* the problem before we write down the answer
- **Black hole formation is not a direct consequence of the failure of perturbation theory**



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