Fluctuation effects and peaked speed of sound in the hadron-quark crossover

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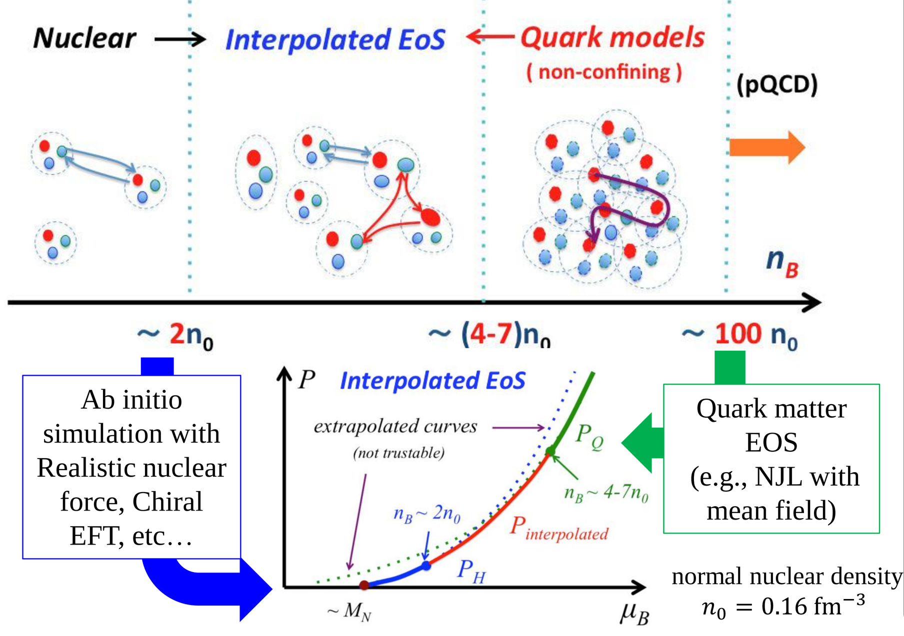
—The purpose of this poster—

Elucidating a microscopic mechanism of the hadron-quark crossover in massive neutron star is one of the most important subjects in nuclear astrophysics. In analogy with the BEC-BCS crossover well-established in condensed-matter physics, we discuss the role of *tripling* fluctuations (three-quark correlations) in the hadron-quark crossover. Considering tripling fluctuations within the phase-shift representation of three-body propagators, we investigate their impact on equation of state as well as momentum distributions of quarks and baryon-like clusters.

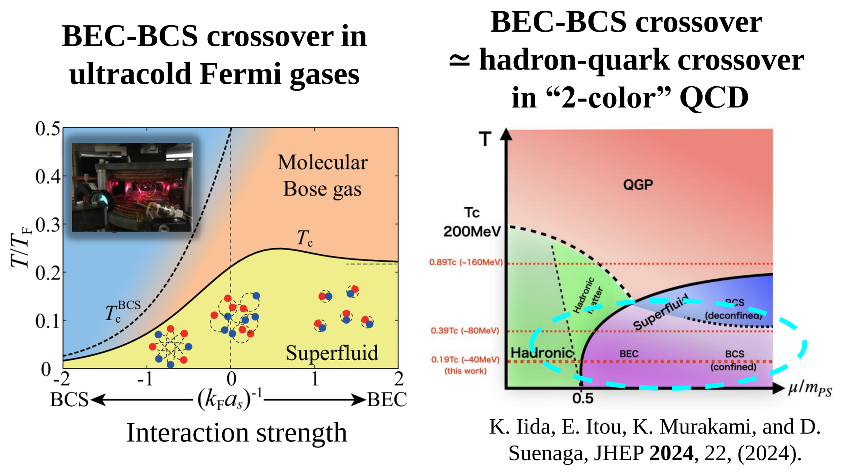
Introduction

From nuclear matter to quark matter in massive stars: Is it crossover?

G. Baym, *et al.*, Rep. Prog. Phys. **81**, 056902 (2018).



Can we understand the hadron-quark crossover in 3-color QCD in analogy with the BEC-BCS crossover?



K. Iida, E. Itou, K. Murakami, and D. Suenaga, JHEP **2024**, 22, (2024).

Review: Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

Photo: http://www.scl.osaka-cu.ac.jp/phys/laser/research_Li.html

Recent nuclear theories in terms of "BEC-BCS crossover theory"

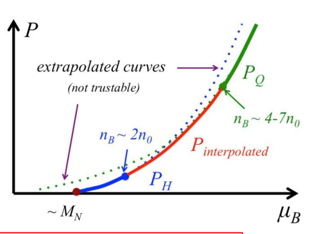
Theoretical approaches to nuclear equation of state (EOS)

Conventional nuclear EOS = Effective theory of nucleons (Nucleons + nuclear force)

Nuclear Hamiltonian
$H = H_0 + V_{NN} + V_{NNN} + \cdots$

Nuclear force

Interpolated EOS based on the hadron-quark crossover



Interpolate!

$$P(\rho) = P_H(\rho) f_-(\rho) + P_Q(\rho) f_+(\rho),$$
$$f_\pm(\rho) = \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho-\bar\rho}{\Gamma}\right)\right).$$

G. Baym, *et al.*, Rep. Prog. Phys. **81**, 056902 (2018).

In the sense of BEC-BCS crossover…

Effective theory of dimers
➡ never describe the crossover at $T \neq 0$

Molecular BEC EOS
$$E = \frac{N}{2}E_b + N\frac{\pi\hbar^2 a_{dd}}{2m} n \left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na_{dd}^3} + \cdots\right)$$

Dimer-dimer interaction

a_{dd}: dimer-dimer scattering length

N. Navon et al., Science **328**, 729 (2010).

Phenomenologically interpolating BCS and BEC EOS
➡ require microscopic mechanism

Molecular BEC EOS
$$E = \frac{N}{2}E_b + N\frac{\pi\hbar^2 a_{dd}}{2m} n \left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na_{dd}^3} + \cdots\right)$$

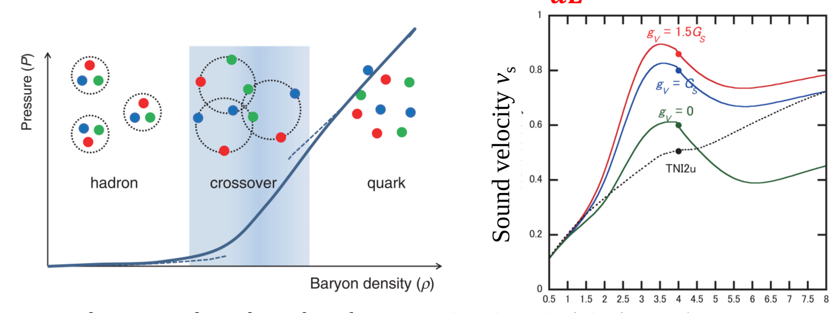
Interpolate!

Fermi gas EOS at weak coupling
$$E = \frac{3}{5}N\varepsilon_F\left(1 + \frac{10}{9\pi}k_F a + 0.18(2)(k_F a)^2 + 0.03(2)(k_F a)^3 + \cdots\right)$$

Hints from the interpolation model and the quarkyonic-matter model

1. Peaked speed of sound

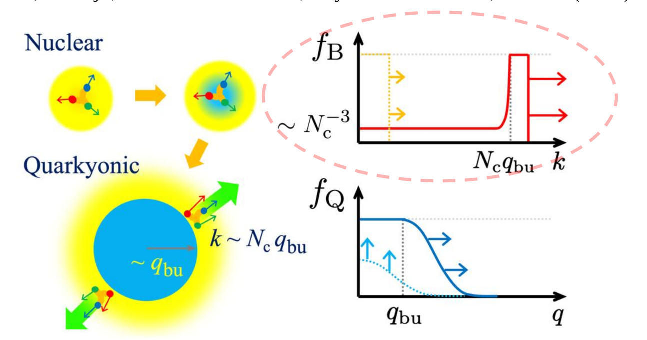
Rapid increase of $P(\rho)$ ➡ $c_s^2 = \frac{dP}{dE}$ exhibits a peak



K. Masuda, T. Hatsuda, and T. Takatsuka, PTEP **2013**, 073D01 (2013).

2. Exotic baryon-momentum distribution

Explicit duality model:
Y. Fujimoto, T. Kojo, and L. D. McLerran, Phys. Rev. Lett. **132**, 112701 (2024).



Toward the many-body theory of hadron-quark crossover: Lesson from the BEC-BCS crossover physics

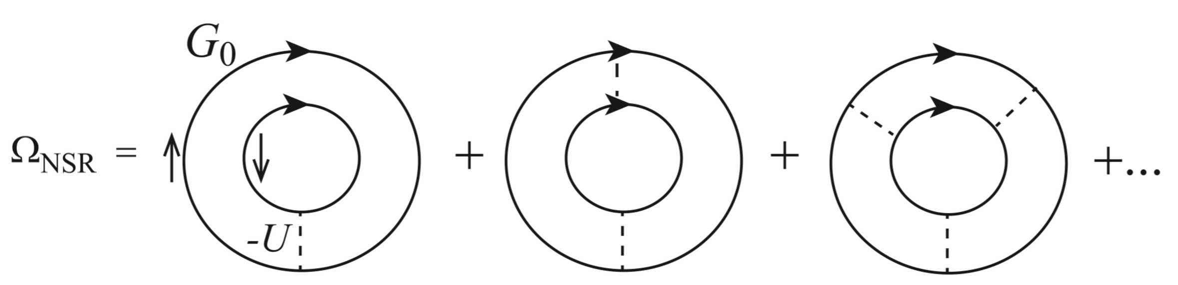
In the case of the BEC-BCS crossover, the **mean-field theory** (BCS-Eagles-Leggett theory) is "qualitatively" valid at $T = 0$.

Mean field = Superfluid/superconducting order parameter Δ

Y. Ohashi, HT, and P. van Wyk, Prog. Part. Nucl. Phys. **111**, 103739 (2020).

In the case of the hadron-quark crossover, the mean-field theory is **INVALID*** and we need to consider **fluctuations**

Nozieres-Schmitt-Rink (NSR) approach to pairing fluctuations



P. Nozières, and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).

➡ **Theory for "tripling" fluctuations is needed**

*Exception at weak coupling: S. Akagami, HT, and K. Iida, Phys. Rev. A **104**, L041302 (2021).

Toy model for the demonstration of tripling fluctuations

Total Hamiltonian: $\hat H = \hat H_0 + \hat V_3$

Kinetic Hamiltonian of 1D three-color fermions

$$\hat H_0 = \sum_{a=r,g,b} \psi_a^\dagger \left(-\frac{\partial_x^2}{2m} - \mu\right)\psi_a$$

μ: chemical potential m: mass
a = r, g, b: color $\psi_a^\dagger, \psi_a$: fermionic field operator

Short-range three-body interaction
(involving quantum anomaly with asymptotic freedom)

$$\hat V_3 = V(\psi_r^\dagger \psi_r)(\psi_g^\dagger \psi_g)(\psi_b^\dagger \psi_b)$$

J. Drut, et al., PRL **120**, 243002 (2018).

Three-body (baryon) binding energy

$$\mathcal{B} = \frac{\Lambda^2}{m}\exp\left(\frac{2\sqrt{3}\pi}{mV}\right)$$

Clustering fluctuation theory

***N*-body clustering fluctuations on thermodynamic potential:**

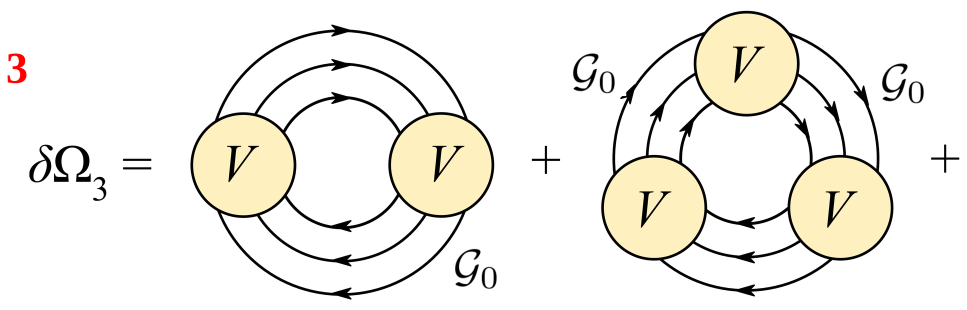
$$\delta\Omega_N = T\sum_{\boldsymbol K}\int_{-\infty}^{\infty}\frac{d\omega}{\pi}\ln\left(1 + e^{-\omega/T}\right)\partial_\omega\varphi$$

(N = 2 reproduces NSR theory)

***N*-body propagator and phase shift:** $\mathcal{G}/\mathcal{G}_0 = |\mathcal{G}/\mathcal{G}_0|e^{i\varphi}$

Tripling fluctuations: N = 3

V: short-range interaction responsible for N-body cluster formation



Derivative of the phase shift

Bound state v.s. **Scattering state**

$$\partial_\omega\varphi(\omega) = \pi\delta(\omega + \mathcal{B}) - \Theta(\omega) f_{\mathrm{scatt.}}(\omega)$$

$\mathcal{B}$: Cluster binding energy

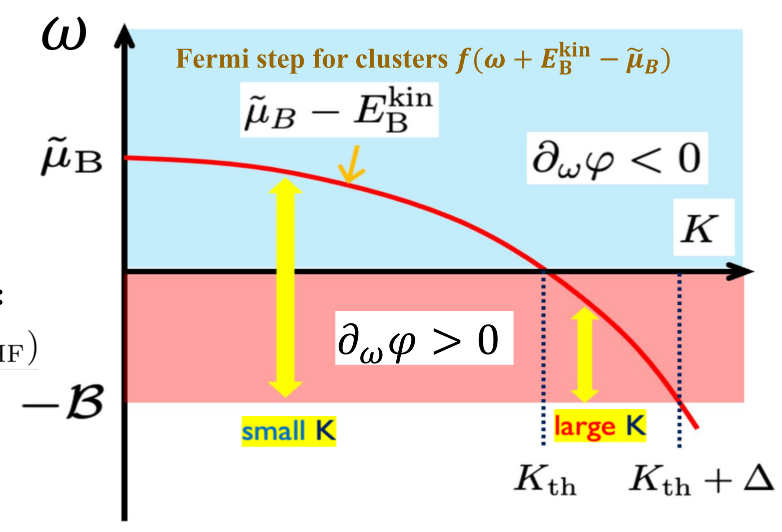
Exact constraint of φ
$\varphi(\omega = -\infty) = \varphi(\omega = \infty) = 0$
The phase shift induced by the bound state should be reduced back to zero by the scattering state

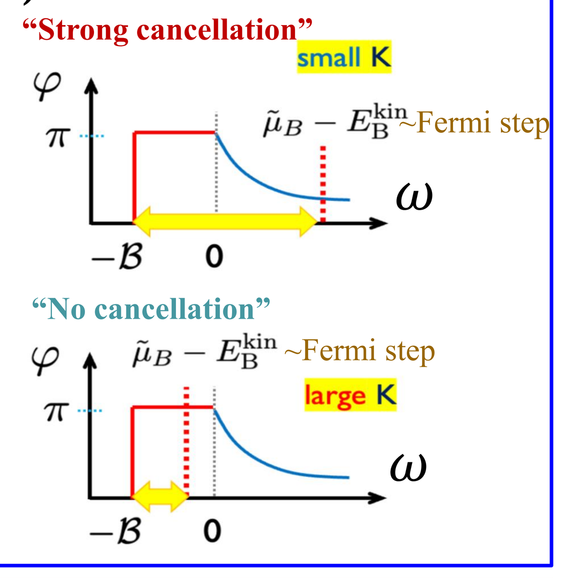
$K - \omega$ plane for positive (bound state) and negative (scattering state) contributions of fluctuations

K: three-body momentum
ω: three-body frequency

Baryon kinetic energy:
$E_B^{\mathrm{kin}}(K) = K^2/2M_B$

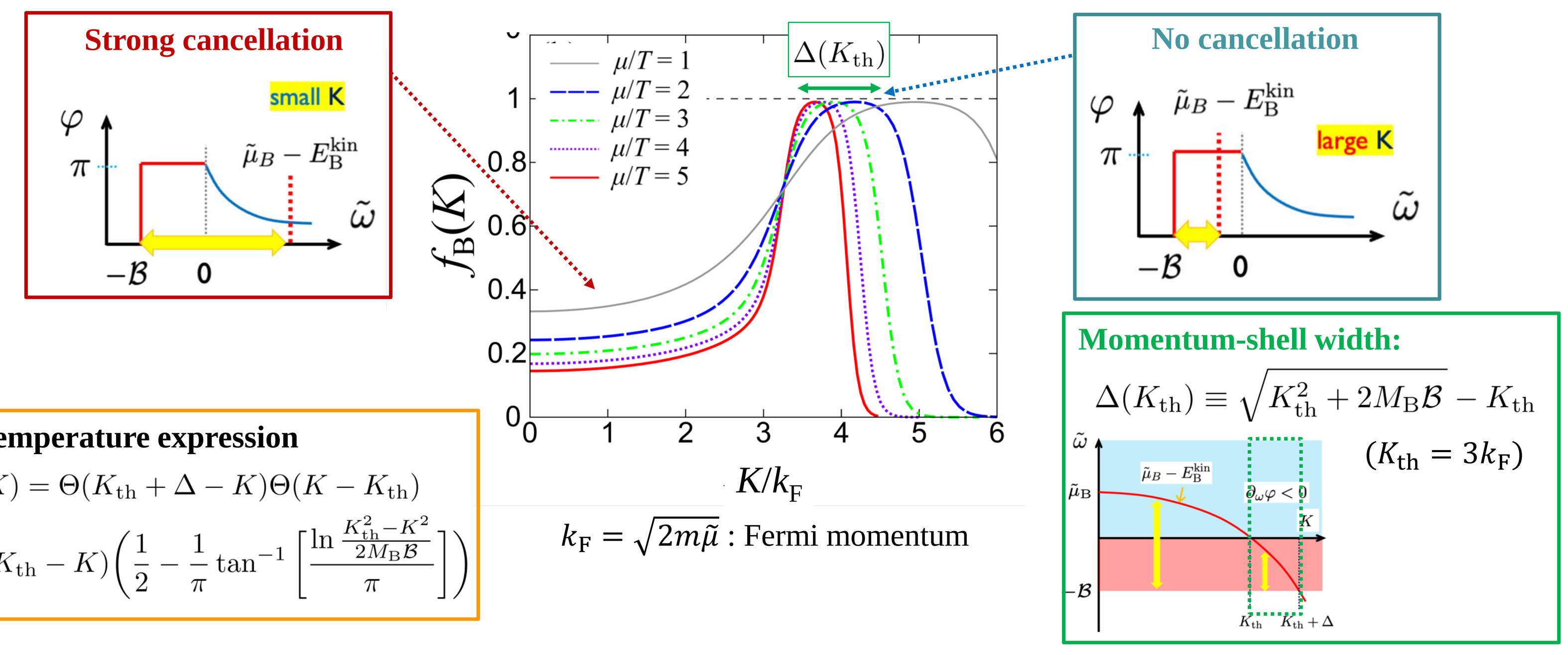
Baryon chemical potential:
$\tilde\mu_B = 3\tilde\mu \equiv 3(\mu - \Sigma_{\mathrm{HF}})$





Result 1: Exotic baryon momentum distribution

Tripling fluctuation theory can reproduce the baryon momentum distribution in quarkyonic matter model



Zero-temperature expression

$$f_B^{T=0}(K) = \Theta(K_{\mathrm{th}} + \Delta - K)\Theta(K - K_{\mathrm{th}}) + \Theta(K_{\mathrm{th}} - K)\left(\frac{1}{2} - \frac{1}{\pi}\tan^{-1}\left[\frac{\ln\frac{K_{\mathrm{th}}^2 - K^2}{2M_B\mathcal{B}}}{\pi}\right]\right)$$

$k_F = \sqrt{2m\tilde\mu}$: Fermi momentum

Momentum-shell width:

$$\Delta(K_{\mathrm{th}}) \equiv \sqrt{K_{\mathrm{th}}^2 + 2M_B\mathcal{B}} - K_{\mathrm{th}}$$

$(K_{\mathrm{th}} = 3k_F)$

Result 2: Peaked speed of sound

Peaked speed of sound is induced by suppressed baryon momentum distributions at low momenta

$$c_s^2 = \frac{n}{m}\left(\frac{\partial n}{\partial\mu}\right)^{-1}$$

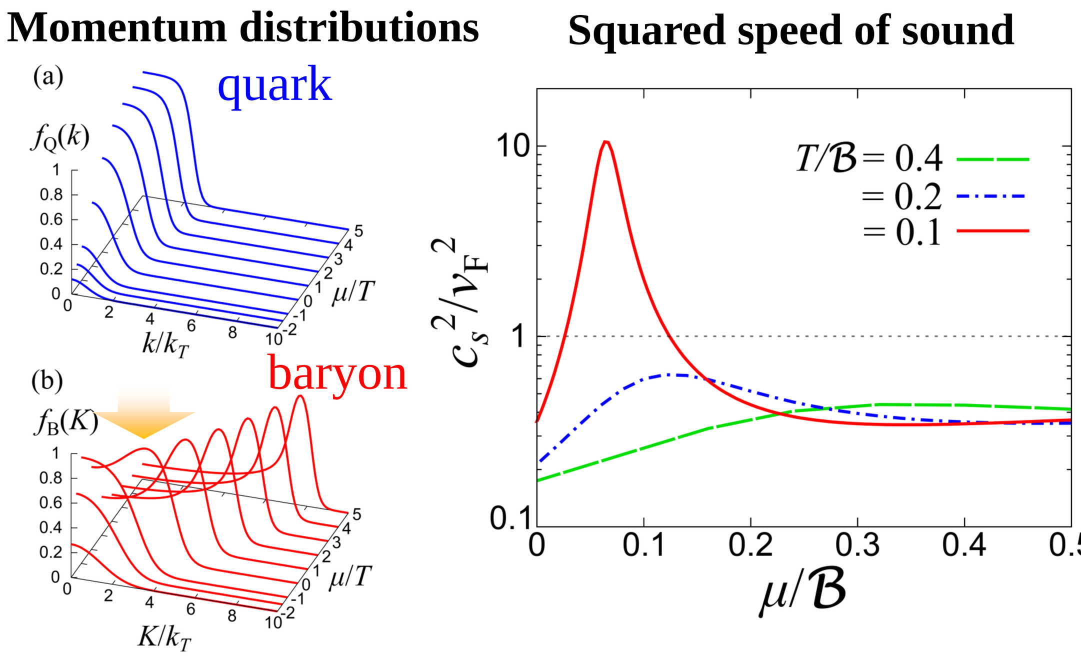
Density susceptibility:

$$\frac{\partial n}{\partial\mu} = \frac{\partial n_Q}{\partial\mu} + \frac{\partial n_B}{\partial\mu}$$

$$\frac{\partial n_Q}{\partial\mu} = 3\sum_k \frac{\partial f_Q(k)}{\partial\mu} > 0$$

$$\frac{\partial n_B}{\partial\mu} = 3\sum_K \frac{\partial f_B(K)}{\partial\mu} < 0$$

Momentum distributions **Squared speed of sound**



Summary

We have explored a possible microscopic mechanism of the hadron-quark crossover in analogy with the condensed-matter theory of the BEC-BCS crossover well established in cold atomic physics. The tripling fluctuation theory for baryon-cluster formation can explain both peaked speed of sound and exotic baryon momentum distributions expected in the crossover regime.

Future perspective: Application to more realistic systems relevant to neutron star matter