Check of Treatment for Gravitational Wave Signals From a Core-Collapse Supernova in Empirical Mode Decomposition

"From Quarks to Neutron Stars: Insights from kHz gravitational waves" @ U. Tokyo (Apr. 23 - 24, 2025)

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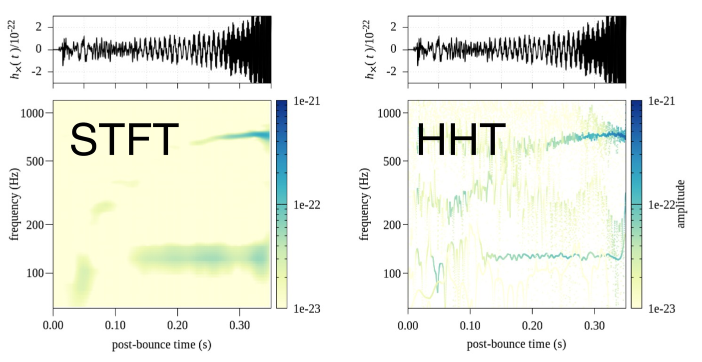
1. Introduction & Aim

- **Hilbert-Huang Transform (HHT)** [1] is one of the **time-frequency** analysis methods with the **high resolution**.
- HHT was applied to Gravitational Wave (GW) data analysis first by Camp *et al.* [2], Stroeer *et al.* [3], and Japanese group follows them.
  - Binary neutron star merger [4], [9]
  - Quasinormal modes in ringdown [5]
  - Core-collapse supernova (CCSN) — SASI [7]

(Modified by YH. Original figure is from [7])

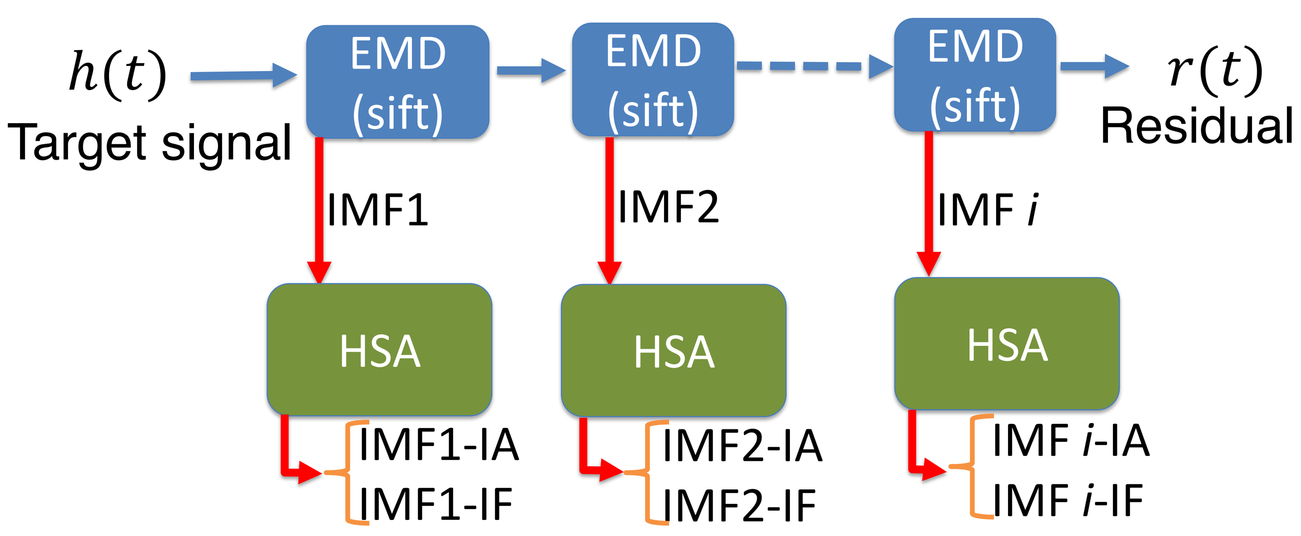


- In this work, check about effect of range of target signals from a CCSN in Empirical Mode Decomposition (EMD). We set 2 type, GW signal + extend range (long) and GW signal only range (short).

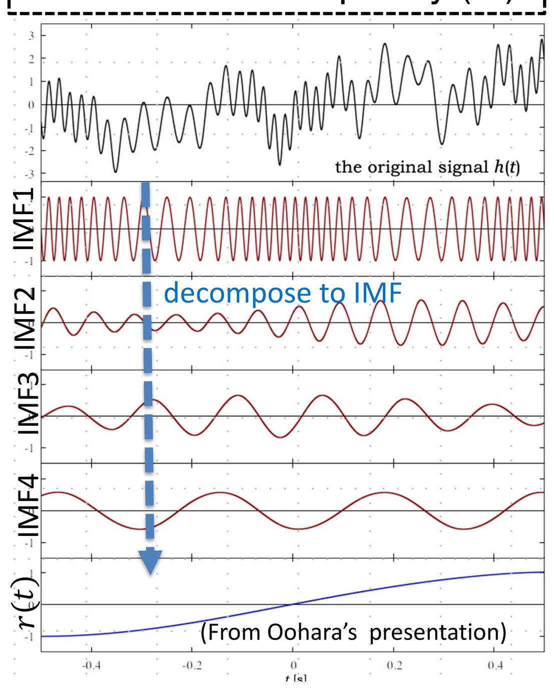
2. Hilbert-Huang Transform (HHT)

◆ Overview of the HHT

The HHT consists of two parts
✓ **Empirical Mode Decomposition (EMD)**
✓ **Hilbert Spectral Analysis (HSA)**



Intrinsic Mode Function (IMF)
Instantaneous Amplitude (IA)
Instantaneous Frequency (IF)



Empirical Mode Decomposition (EMD)

✓EMD sifts original signal, like a high pass filter.

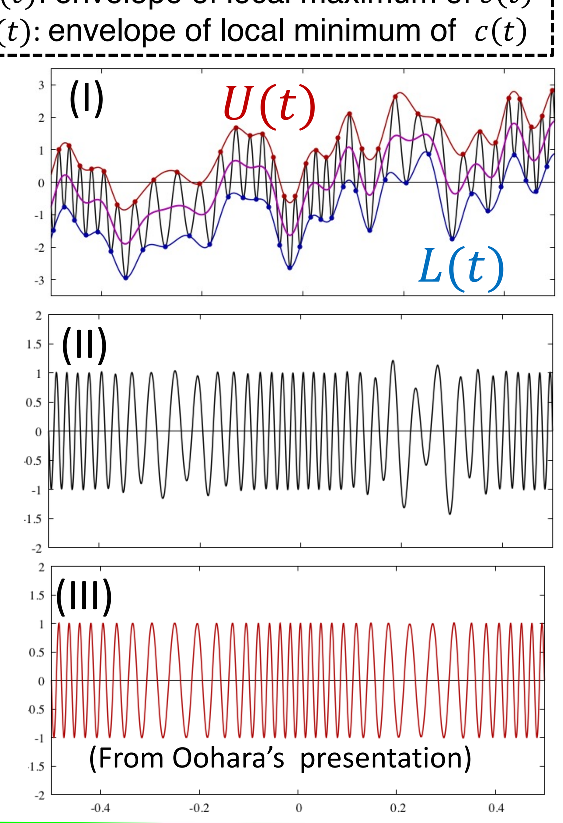
$U(t)$: envelope of local maximum of $c(t)$
$L(t)$: envelope of local minimum of $c(t)$

- set $h_0(t) \leftarrow h(t)$

1. $c(t) \leftarrow h_0(t)$
2. $m(t) \leftarrow (U(t) + L(t))/2$ …(I)
3. $c_{\text{new}}(t) \leftarrow c(t) - m(t)$ …(II)

if a stoppage criterion is satisfied *then*
 get IMF $= c(t)$, …(III)
 $h_0(t) \leftarrow h_0(t) - c(t)$,
 goto 1. ➔ Search next IMF
else
 $c(t) \leftarrow c_{\text{new}}(t)$,
 goto 2.
end if

"EMD stoppage criterion"
$$\frac{\sum_i |m(t_i)|^2}{\sum_i |c(t_i)|^2} < \varepsilon$$
(In this poster, ε is fixed to be 10^{-1})



Hilbert Spectral Analysis (HSA)

✓ Get IA, IF of each IMF ($x(t)$).

$$\left(\mathcal{H}[x(t)] = \frac{1}{\pi} PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau\right)$$
(PV: Principal value)

$$z(t) = x(t) + i\mathcal{H}[x(t)] = a(t)e^{i\theta(t)}$$

$a(t) = \sqrt{x^2(t) + \mathcal{H}[x(t)]^2}$: **Instantaneous Amplitude (IA)**

$f(t) = \frac{1}{2\pi}\frac{d\theta}{dt}$: **Instantaneous Frequency (IF)**

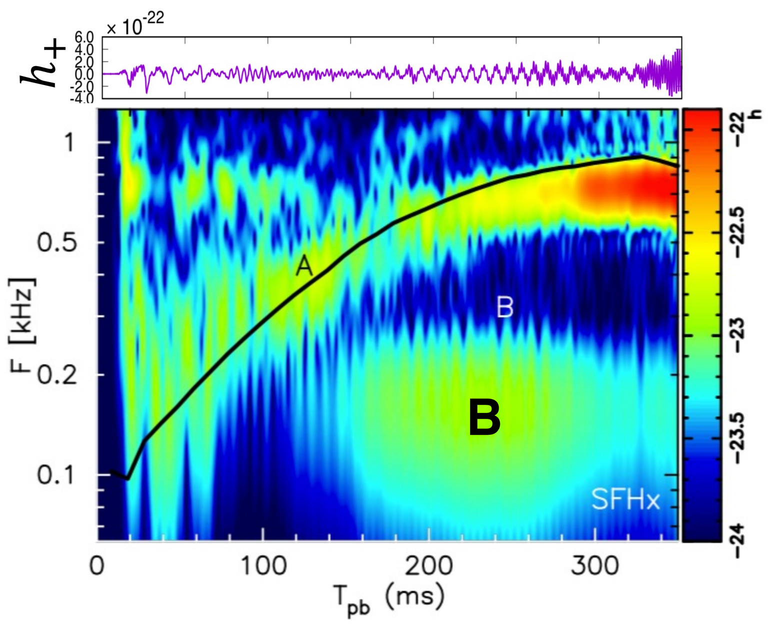
3. Ensemble Empirical Mode Decomposition (EEMD)

Add a white Gaussian noise with a standard deviation σ_e to the original data $h(t)$.
($\sigma_e = \sigma_h \times \sigma_{\text{eemd}}$ with a pre-determined $\boldsymbol{\sigma_{\text{eemd}}}$)($\sigma_h$:standard deviation of $h(t)$)

(i) Decompose the data with the white noise into IMFs.
(ii) Repeat steps (i) and (ii) many times but with different white Gaussian noise series at each time.
(iii) Obtain the **ensemble means** for the series of the **obtained IMFs**. The number of ensemble trials, $\boldsymbol{N_{\text{eemd}}}$, has to be large enough.

(E)EMD parameters
ε : stoppage criterion
N_{eemd} : ensemble number
σ_{eemd} : injection noise parameter

4. Target Singal & Set Up

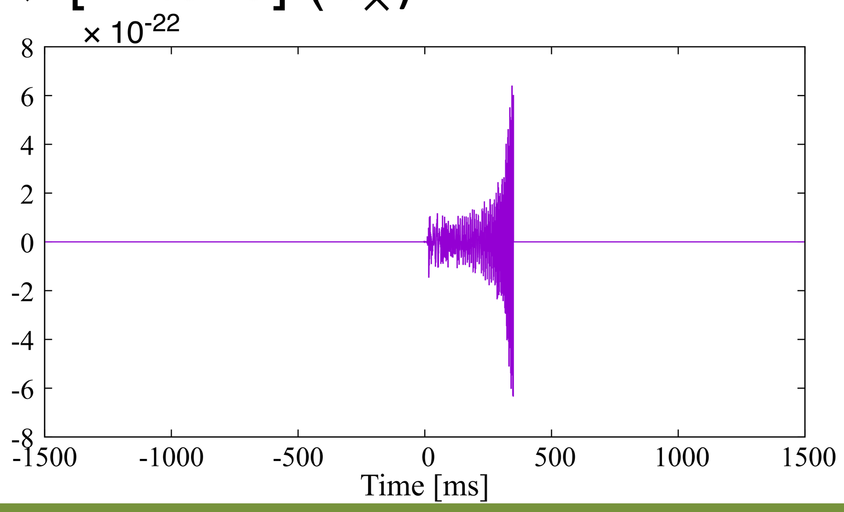


Kuroda *et al.* [6]: SFHx

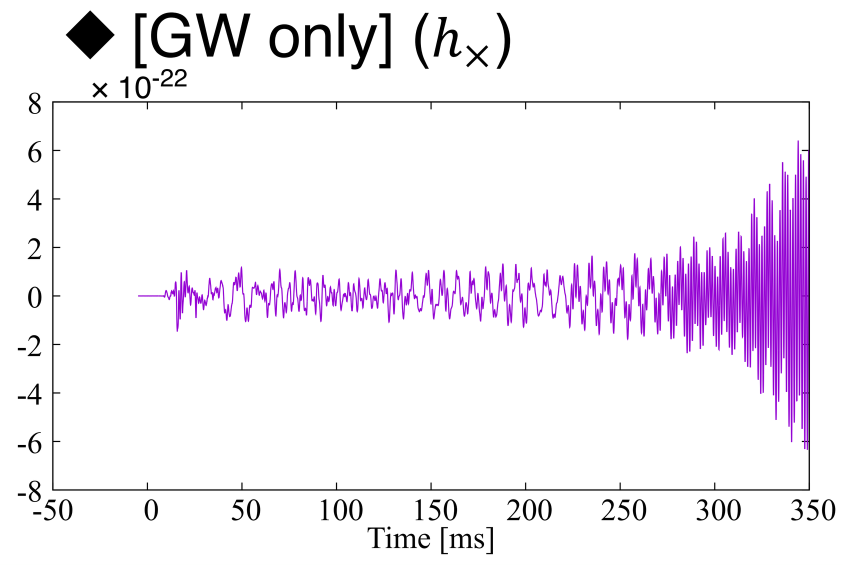
- 3-D core collapse supernova simulation of a no-rotating $15M_\odot$ star.
- Standing accretion shock instability (SASI) is active in this case (B part in the left figure).

(Modified by YH. Original fig. is from [6])

◆ [Extend] ($h_\times$)



◆ [GW only] ($h_\times$)



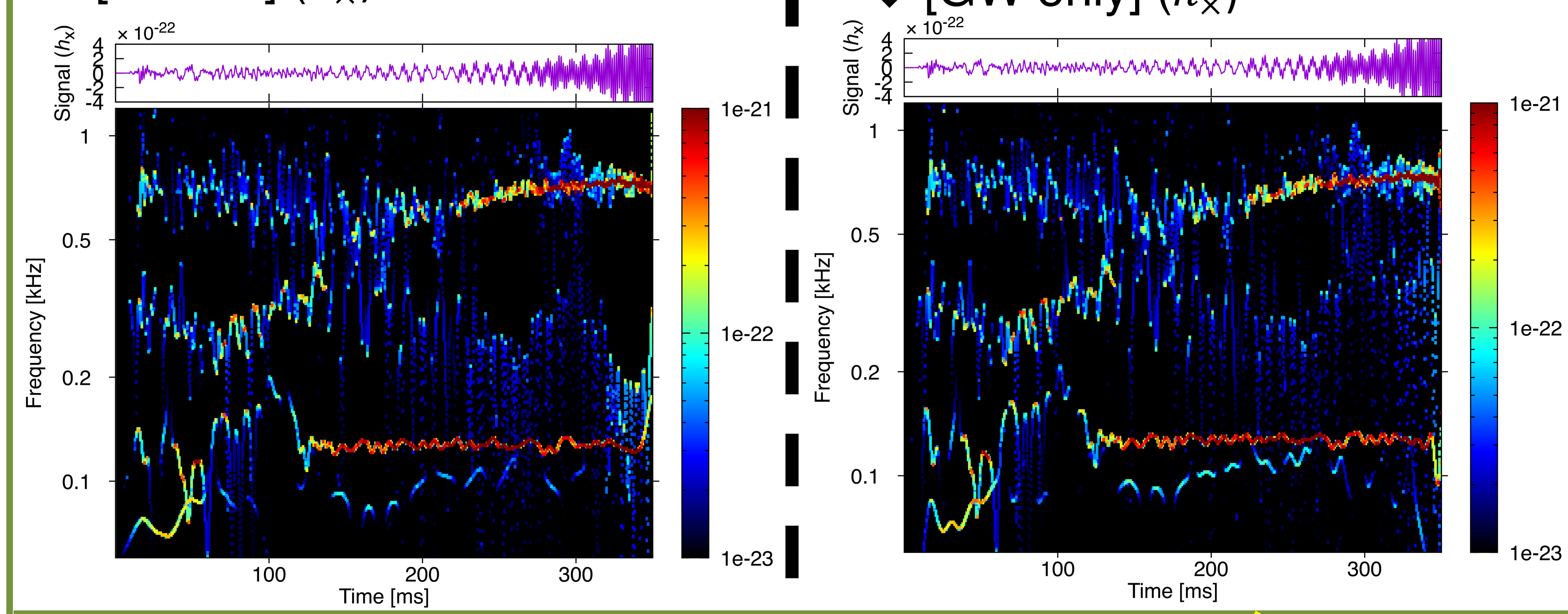
5. Results

5.1 CASE1

Preliminary

EEMD parameters: $\varepsilon = 0.1, N_{\text{eemd}} = 10^5, \sigma_{\text{eemd}} = \mathbf{10}$

◆ [Extend] ($h_\times$) ◆ [GW only] ($h_\times$)

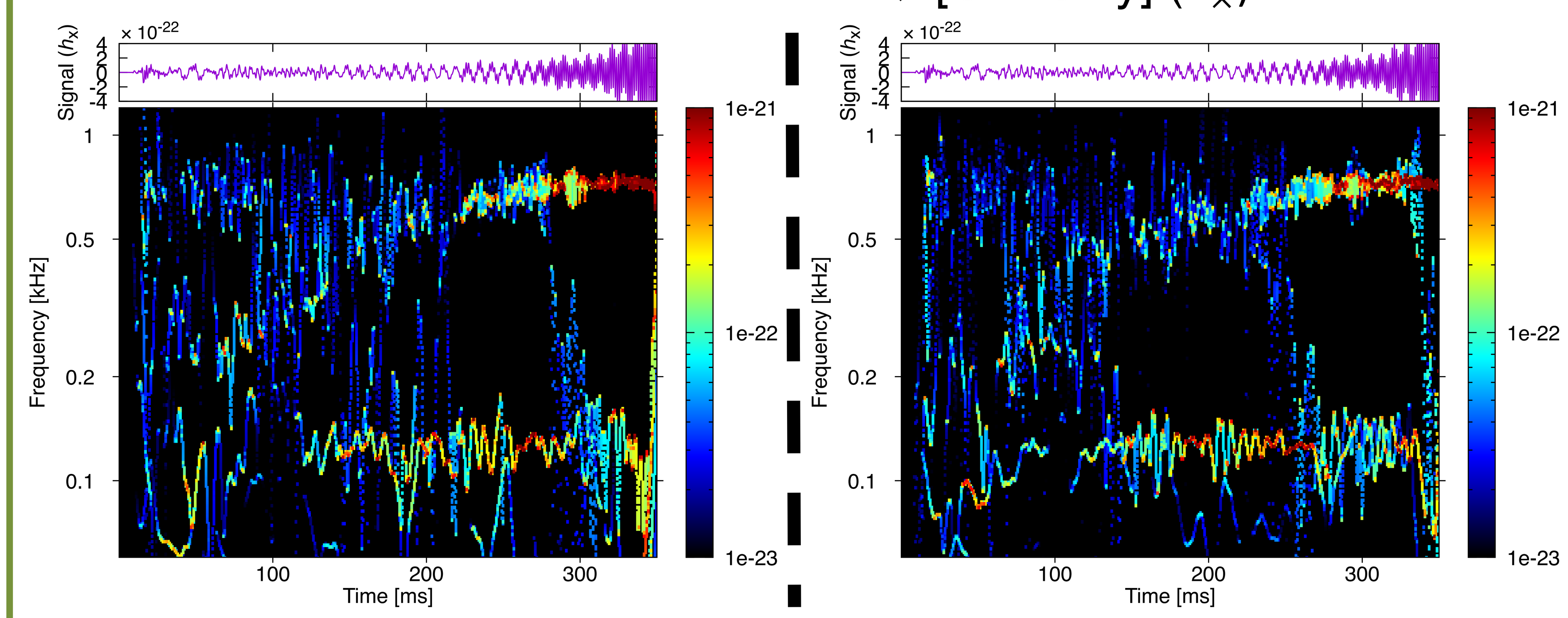


5.2. CASE2

Preliminary

EEMD parameters: $\varepsilon = 0.1, N_{\text{eemd}} = 10^5, \sigma_{\text{eemd}} = \mathbf{0.1}$

◆ [Extend] ($h_\times$) ◆ [GW only] ($h_\times$)



6. Summary

In large noise ($\sigma_{\text{eemd}} = 10$) case, time-frequency maps (T-F maps) are not much difference "Extend" and "GW Only". But T-F maps in small noise ($\sigma_{\text{eemd}} = 0.1$) case, they seem to be difference, specially SASI part.
→When EEMD parameter search, may need to care about target signal range in EMD.

Acknowledgements

YH used HHT code of `KAGALI` (KAGRA Algorithmic Library).

Reference

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