Asymptotic states of fast neutrino-flavor conversions in the three-flavor framework

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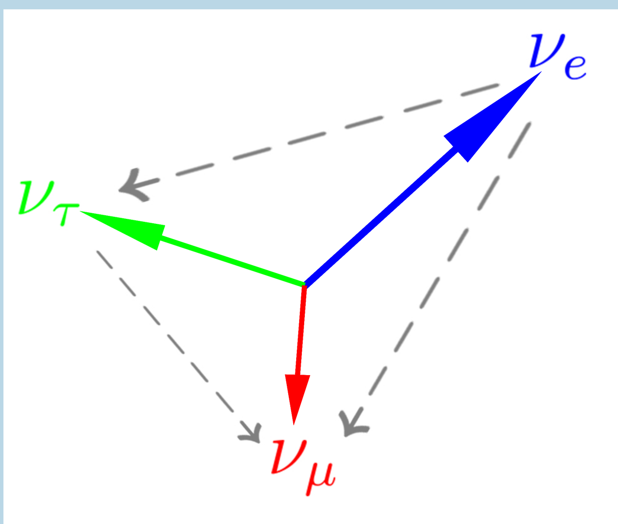
Background & Motivation

Neutrino flavor conversions are expected to play a vital role in **core-collapse supernovae** and **neutron star mergers**, impacting explosion dynamics, nucleosynthesis, and the emitted neutrino signal.

Accurately modeling these effects requires solving the **quantum kinetic equations (QKEs)**, which are **computationally demanding** due to their high dimensionality and nonlinear flavor evolution.

To address this challenge, we propose a **computationally inexpensive method** that predicts the **late-time asymptotic states** of **fast flavor conversions** within the **three-flavor framework**.

In contrast to simpler two-flavor systems, three-flavor dynamics involve **interacting coherence sectors** (ν_e–ν_μ, ν_e–ν_τ, ν_μ–ν_τ), which exhibit **nonlinear competition**. Our method captures this interplay without solving full QKEs.



Fast Flavor Conversions

In dense astrophysical environments, neutrinos undergo **collective flavor conversions** driven by nonlinear forward scattering. These are governed by the **quantum kinetic equation (QKE)**:

$$(\partial_t + \boldsymbol{v}\cdot\nabla) f = -i[H, f],$$

where f is the neutrino density matrix and H includes self-interaction potentials that depend on the angular and spectral distributions of surrounding neutrinos.

As the system evolves, flavor distributions tend toward a **quasi-steady (asymptotic)** state. This motivates a **relaxation-type approximation**, modeled after the **Bhatnagar–Gross–Krook (BGK)** form [1]:

$$(\partial_t + \boldsymbol{v}\cdot\nabla) f = -\frac{1}{\tau}(f - f^a), \tag{2}$$

where τ is an effective relaxation timescale and f^a is the asymptotic flavor distribution.

In the **three-flavor framework**, flavor evolution involves multiple coherence sectors that are **nonlinearly coupled**. It makes direct estimation of τ and f^a challenging, as instabilities can grow at different rates in different sectors.

Our method addresses this by modeling each sector's relaxation independently, while preserving their interaction through coupled evolution.

Subgrid BGK Scheme

To approximate the **asymptotic state** of fast flavor conversions in the three-flavor system, we extend the BGK formalism by introducing **sector-wise competition**.

The time evolution of each flavor's distribution function is governed by:

$$\begin{aligned}
\frac{df_e}{dt} &= -\frac{f_e - f_e^{a,e\mu}}{\tau_{e\mu}} - \frac{f_e - f_e^{a,e\tau}}{\tau_{e\tau}},\\
\frac{df_\mu}{dt} &= -\frac{f_\mu - f_\mu^{a,e\mu}}{\tau_{e\mu}} - \frac{f_\mu - f_\mu^{a,\mu\tau}}{\tau_{\mu\tau}},\\
\frac{df_\tau}{dt} &= -\frac{f_\tau - f_\tau^{a,e\tau}}{\tau_{e\tau}} - \frac{f_\tau - f_\tau^{a,\mu\tau}}{\tau_{\mu\tau}}.
\end{aligned} \tag{3}$$

Each term models relaxation toward a sector-wise reference state $f_i^{a,ij}$, weighted by a timescale τ_{ij} based on the strength of instability between sectors.

Key features:

- Preserves **nonlinear coupling** via simultaneous sector evolution
- Reaches asymptotic state **without solving full QKE**
- **Lightweight** and compatible with classical transport codes

This formulation captures late-time flavor dynamics even when sectoral instabilities develop at different rates.

Asymptotic Estimation

We estimate the asymptotic state $f_i^{a,ij}$ and relaxation rate τ_{ij}^{-1} using the flavor potential:

$$G_{ij}^{\nu} \equiv \sqrt{2} G_F \left((\bar{f}_i - \bar{f}_j) - (f_i - f_j) \right).$$

The angular domain is split into two regions:

$$A = \left| \int_{G_{ij}^\nu < 0} d\Omega\, G_{ij}^\nu \right|, \quad B = \int_{G_{ij}^\nu > 0} d\Omega\, G_{ij}^\nu.$$

The relaxation rate is:

$$\tau_{ij}^{-1} = \frac{2\pi}{\sqrt{AB}}.$$

The asymptotic state is computed using a survival probability [2]:

$$\begin{aligned}
f_i^{a,ij} &= P_{ij} f_i + (1 - P_{ij}) f_j,\\
f_j^{a,ij} &= (1 - P_{ij}) f_i + P_{ij} f_j,
\end{aligned}$$

with (for $B \geq A$)

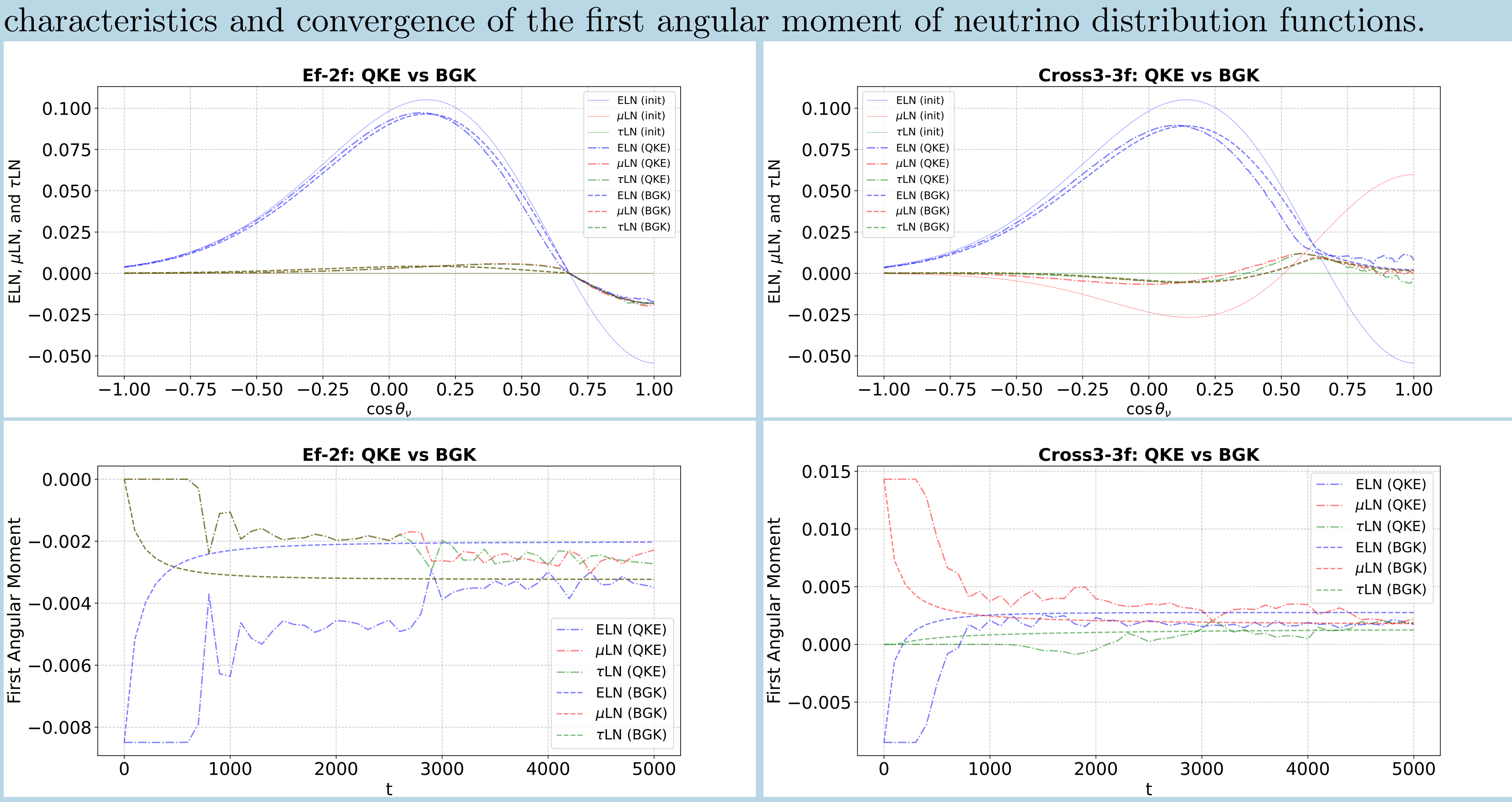
$$P_{ij} = \begin{cases} 1 - \frac{A}{2B} & \text{if } G_{ij}^\nu > 0,\\ \frac{1}{2} & \text{if } G_{ij}^\nu < 0, \end{cases}$$

and (for $B < A$)

$$P_{ij} = \begin{cases} 1 - \frac{B}{2A} & \text{if } G_{ij}^\nu < 0,\\ \frac{1}{2} & \text{if } G_{ij}^\nu > 0. \end{cases}$$

Validation Against QKE Simulations

We validate the BGK approximation by comparing it to full QKE simulations in various settings. We compare (1) the predicted asymptotic state of neutrino distribution functions and (2) temporal characteristics and convergence of the first angular moment of neutrino distribution functions.



Key observations:

- BGK tracks the asymptotic state and coherence decay well.
- Relaxation timescales and flavor mixing trends match QKE.

Implementation

Our BGK-based scheme is lightweight and easy to integrate into classical neutrino transport frameworks, without requiring full QKE solutions.

The method can be implemented as a subgrid module:

- **Step 1:** Compute one timestep using classical neutrino transport
- **Step 2:** If a flavor instability is detected, solve Eq. (3) until convergence to estimate f^a and τ
- **Step 3:** Replace the classical update with the relaxation step from Eq. (2)

This structure makes the BGK scheme well-suited for both moment-based and momentum-space discretized transport codes in core-collapse supernova and neutron star merger simulations.

References

[1] Hiroki Nagakura, Lucas Johns, and Masamichi Zaizen. *Phys. Rev. D* **109**, 083013 (2023).

[2] Masamichi Zaizen and Hiroki Nagakura. *Phys. Rev. D* **107**, 103022 (2023).