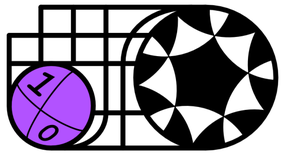
Equation of state of QCD(-like) theory using Lattice Monte Carlo simulations

Etsuko Itou (YITP, Kyoto U./ RIKEN iTHEMS)



From Quarks to Neutron Stars: Insights from kHz gravitational waves, Koshiba Hall, University of Tokyo. 2025/04/23

Introduction: from quarks…

QCD Lagrangian

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi$$

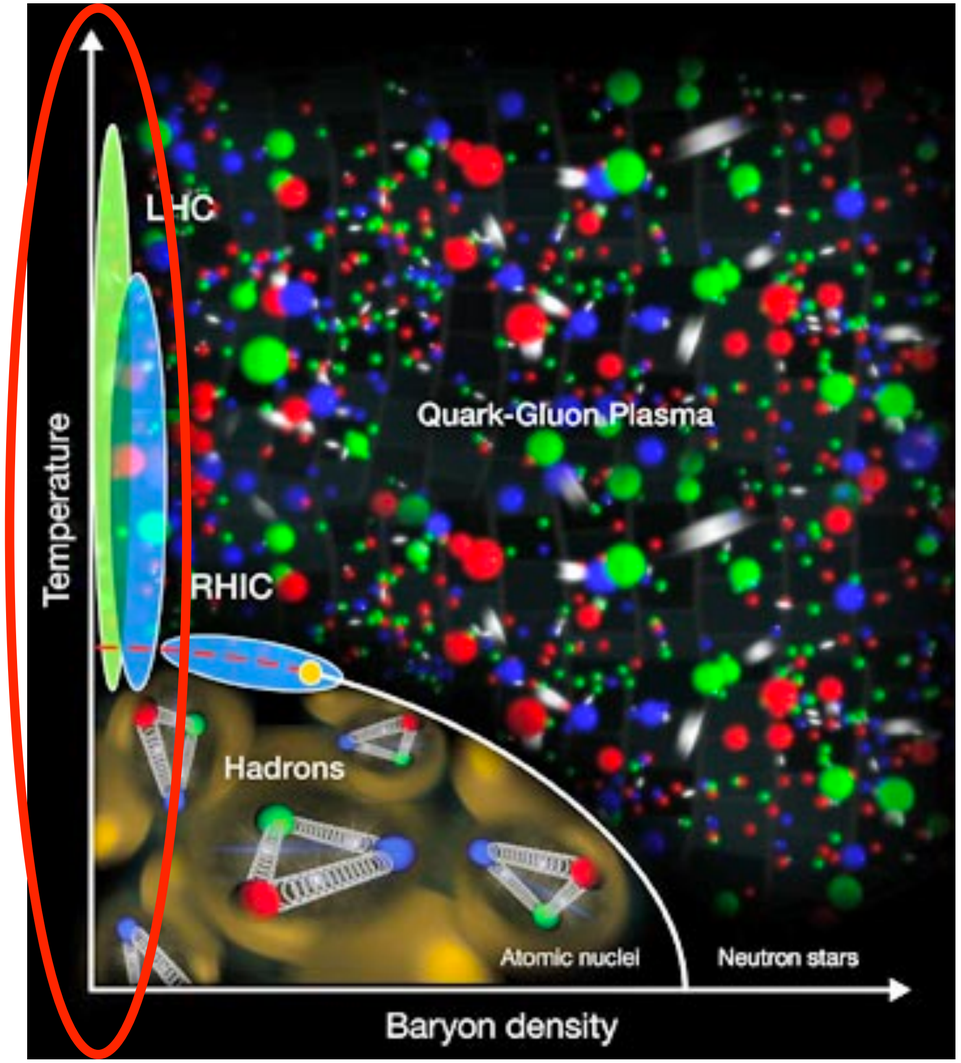
- Fundamental theory for protons and neutrons is given by QCD.
  non-abelian gauge theory (SU(3) gauge), strong interaction

- Lattice gauge theory is only known nonperturbative and gauge invariant regularization method

- Lattice MC from the QCD action shows the precise agreements with experimental data

- Lattice MC simulation is now recognized as an ab initio approach to general gauge theories (gauge principle / exact algorithm)
  It serves as a numerical experiment for gauge theories

EoS and sound velocity at zero μ



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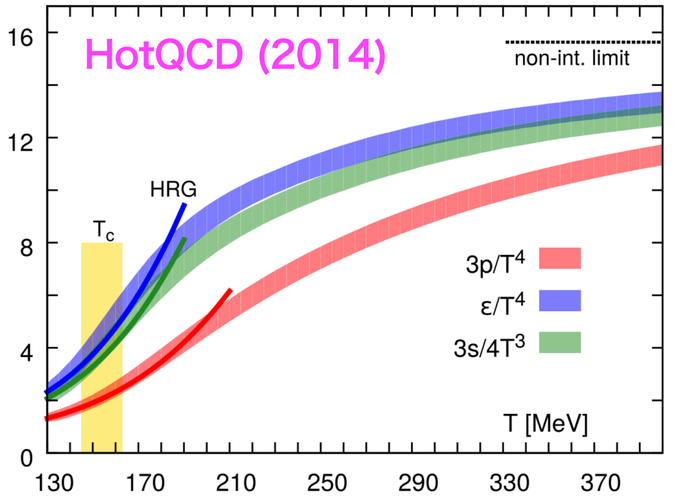
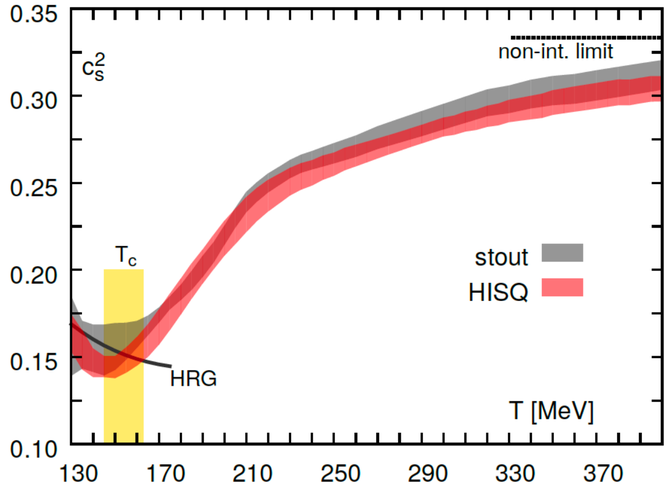
Finite Temperature transition (Nf=2+1 QCD)

EoS (p and ε)



Sound velocity $c_s^2/c^2 = \partial p/\partial \epsilon$



cf.) $c_s^2/c^2 = 1/3$: relativistic free theory where $e = 3p$

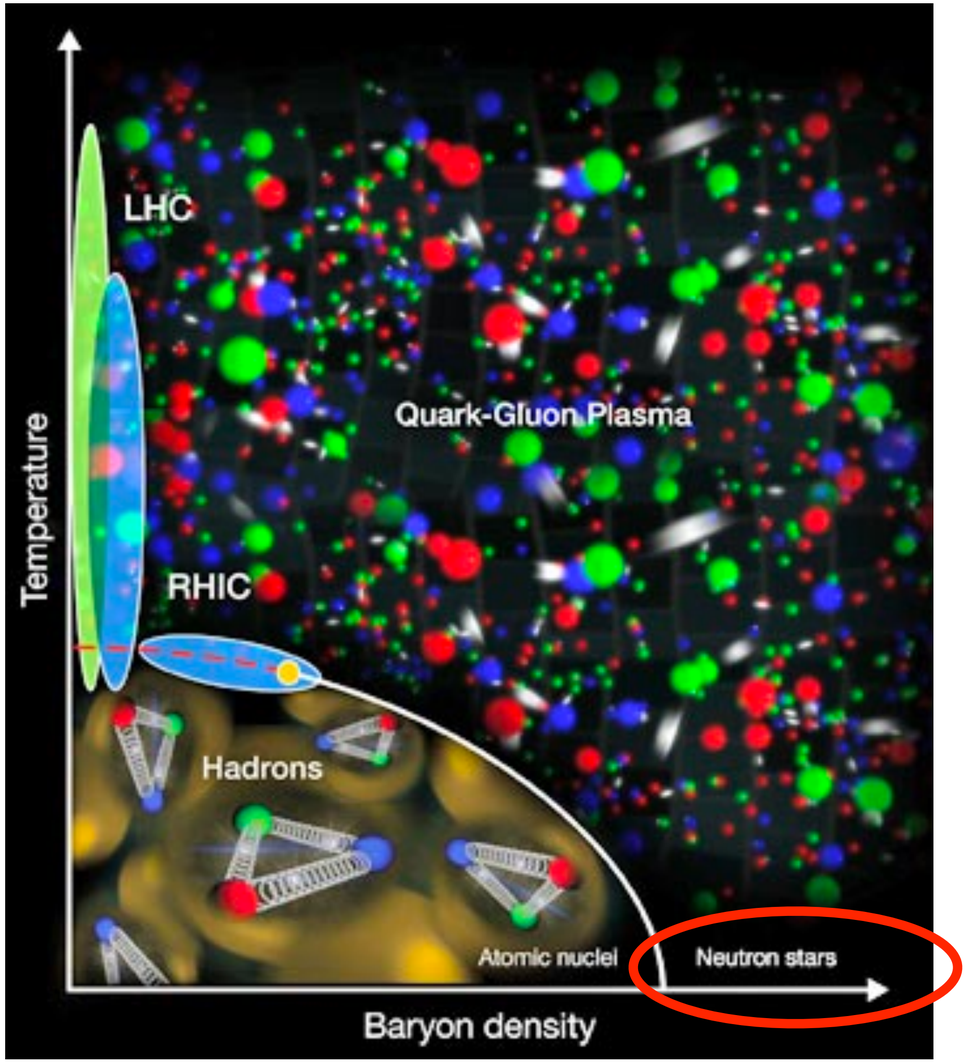
($c_s^2/c^2 \leq 1/3$: conformal bound)

It gives the input for $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$

To Neutron star (non-zero μ QCD)

expected QCD phase diagram

T



$\propto \mu$ ©BNL/RHIC

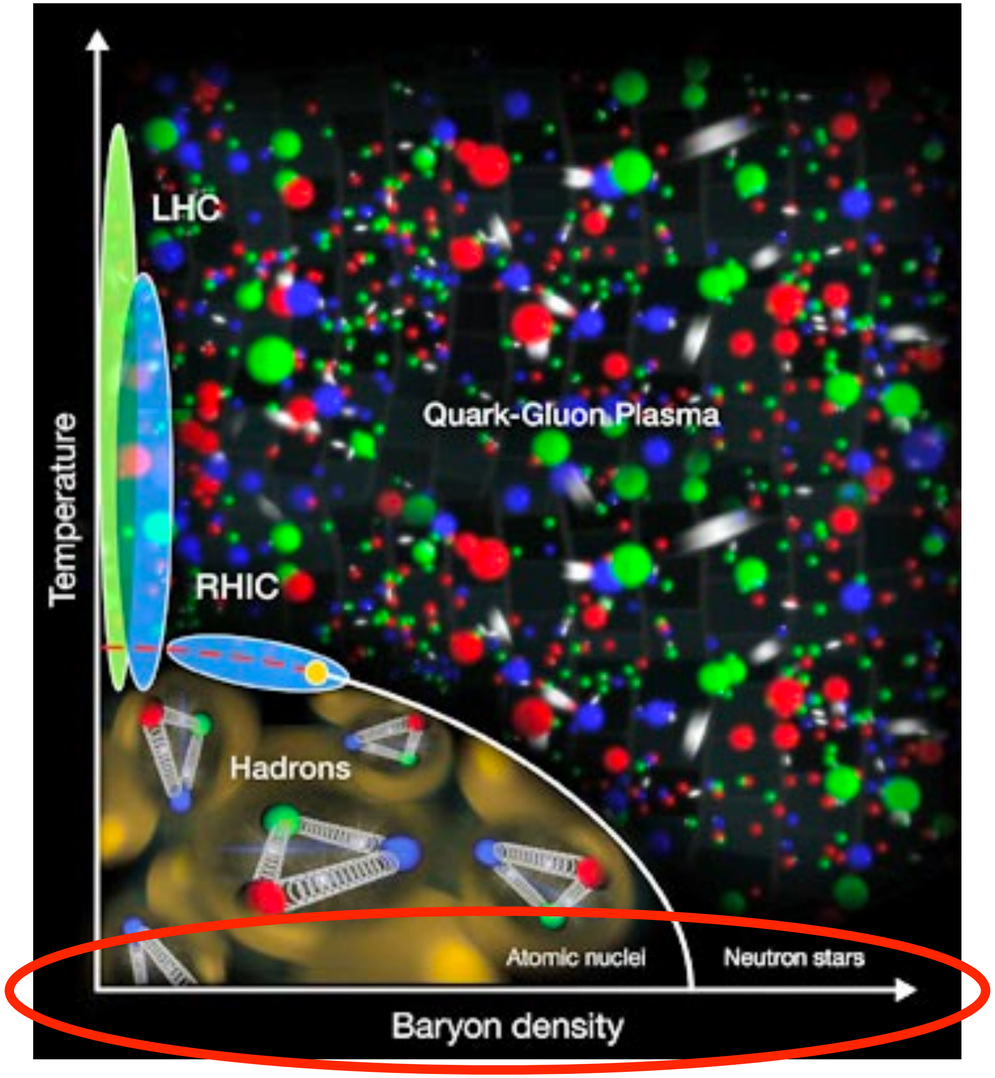
Neutron star ~ Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- μ : quark chemical potential
  - ~ density of matter
  - ~ baryon chemical potential
  - $(\mu_B = N_c\mu)$

Sound velocity in finite density regime?

EoS and sound velocity at low-T and high-μ



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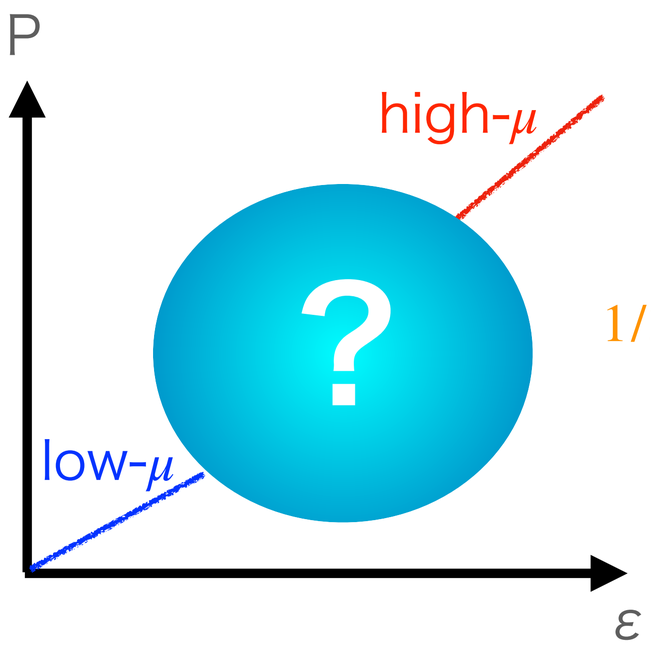
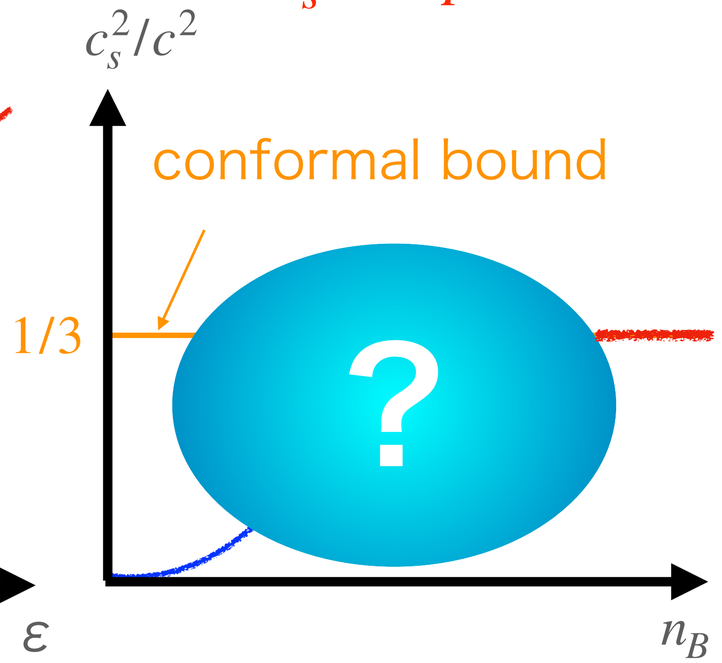
EoS

$p(\mu)$ vs $\varepsilon(\mu)$



Sound velocity

$c_s^2 = \partial p / \partial \epsilon$



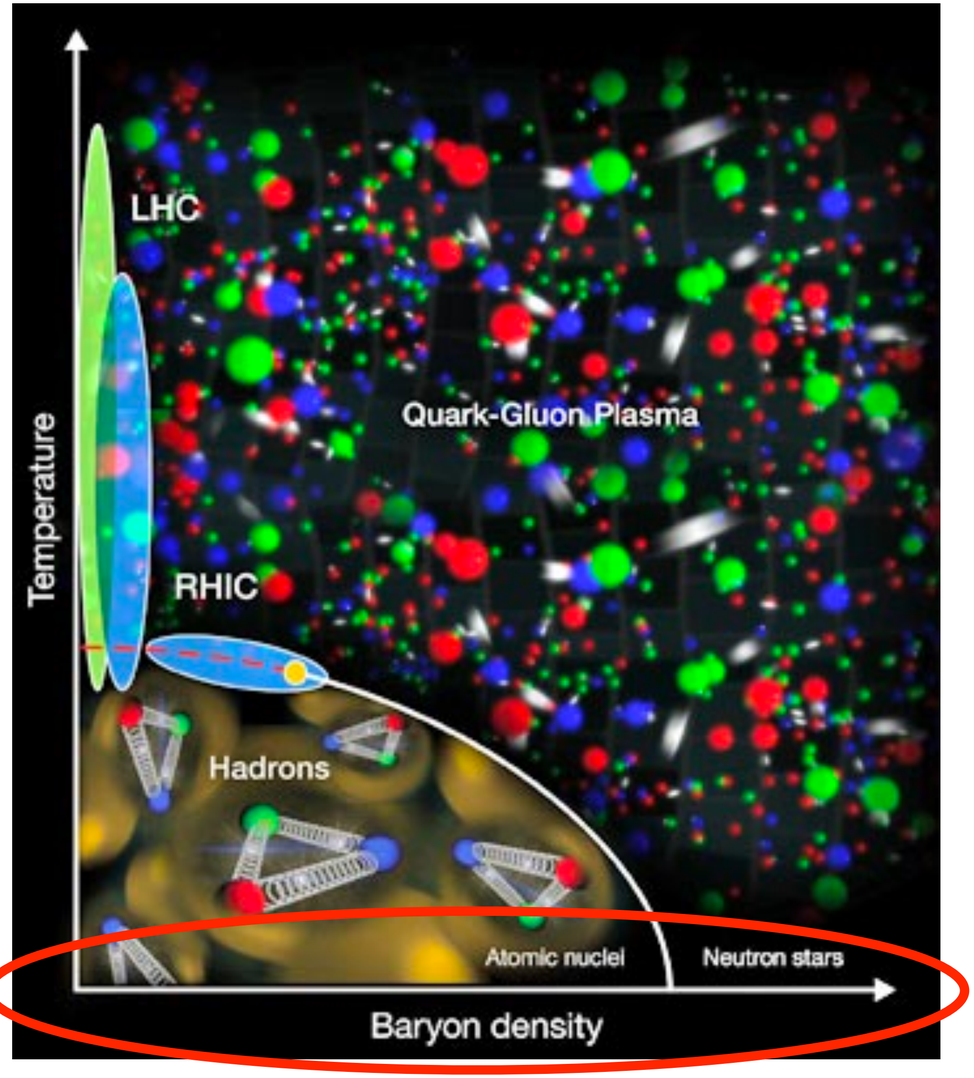
low -μ ($n_B \lesssim 2n_0$): Hadronic matter

high-μ ($5n_0 < n_B$): Quark matter

-> pQCD ($50n_0 < n_B$)

Non-zero μ QCD is impossible in MC

expected QCD phase diagram



T

$\propto \mu$

©BNL/RHIC

Neutron star ~ Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}(i\gamma_\mu D_\mu + m)\psi + \mu\bar{\psi}\gamma_0\psi$$

- In $\mu \neq 0$ regime, MC simulation suffers from the sign problem

K.Nagata, Finite-density lattice QCD and sign problem: Current status and open problems
Prog.Part.Nucl.Phys. 127 (2022) 103991

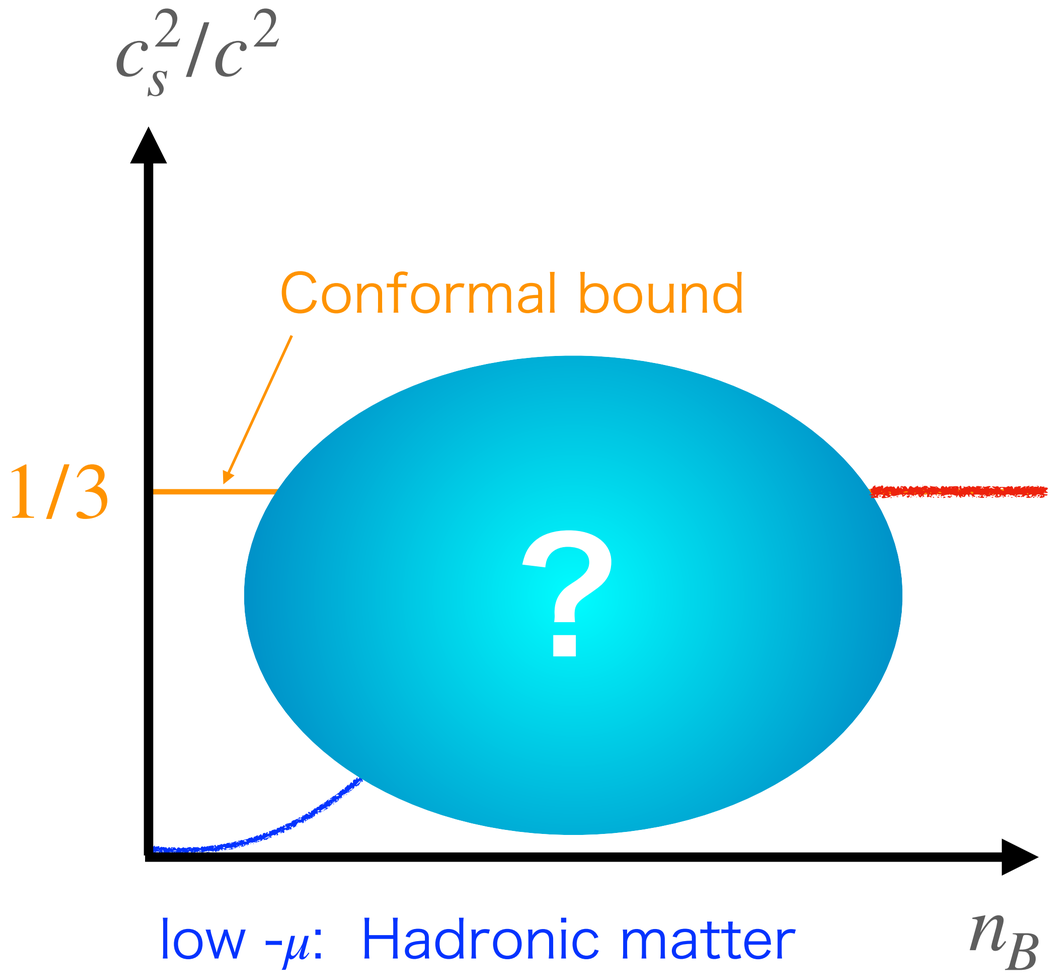
- Sign problem is NP-hard

Troyer and Weise, 2005

Need to change the theory
or the algorithm (quantum computation)

Prediction by phenomenology and effective models

Sound velocity has a peak?



low -μ: Hadronic matter

high-μ: Quark matter ~ pQCD

- Quark-hadron crossover picture consistent with observed neutron stars (M-R) suggests

  c_s^2 peaks at $n_B = 1 - 10 n_0$

  Masuda,Hatsuda,Takatsuka (2013)
  Baym, Hatsuda, Kojo(2018)

- Quarkyonic matter model

  c_s^2 peaks at $n_B = 1 - 5 n_0$

  McLerran and Reddy (2019)

- Microscopic interpretation on the origin of the peak = quark saturation

  (work for any # of color)

  Kojo (2021), Kojo and Suenaga (2022)

➡ Lattice study on 2color dense QCD
the sign problem is absent!!

2color QCD ≈ 3color QCD

Finite density QCD

$$\mathscr{L} = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \bar{\psi}^i(i\gamma_\mu D_\mu + m)\psi^i + \mu\bar{\psi}^i\gamma_0\psi^i$$

For gluons

3color QCD: a=1 - 8, 2color QCD: a=1 - 3

For quarks

3color QCD: i=1 - 3, 2color QCD: i=1 or 2

- Reduced model the color d.o.f.
  SU(2) gauge theory + dynamical 2 color quarks
- Same nonperturbative properties with QCD at $\mu = 0$
  - confinement
  - chiral sym. breaking
  - topological configuration
- Lattice MC for 2color QCD in finite-density regimes gives a hint for dense-QCD

2color QCD phase diagram

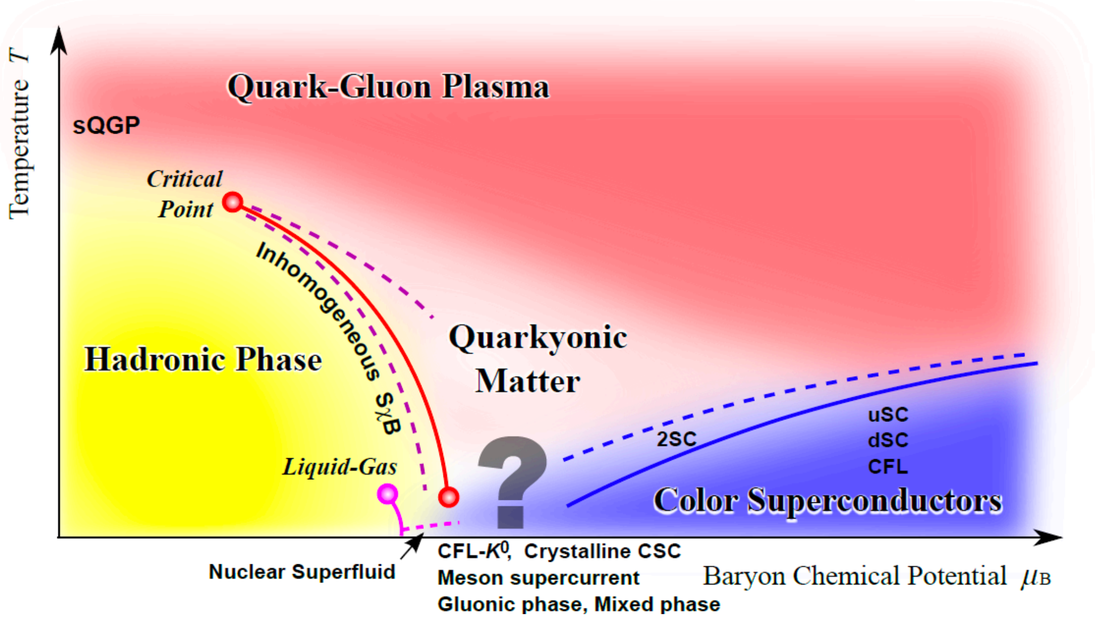
(1) K.Iida, EI, K.Murakami, D.Suenage arXiv: 2405.20566 [hep-lat]
(2) K.Iida, K.Ishiguro , EI, arXiv: 2111.13067
(3) K.Iida, EI, T.-G. Lee: PTEP2021(2021) 1, 013B0
(4) K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
(5) T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Phase diagram in T-μ plane

3 color QCD

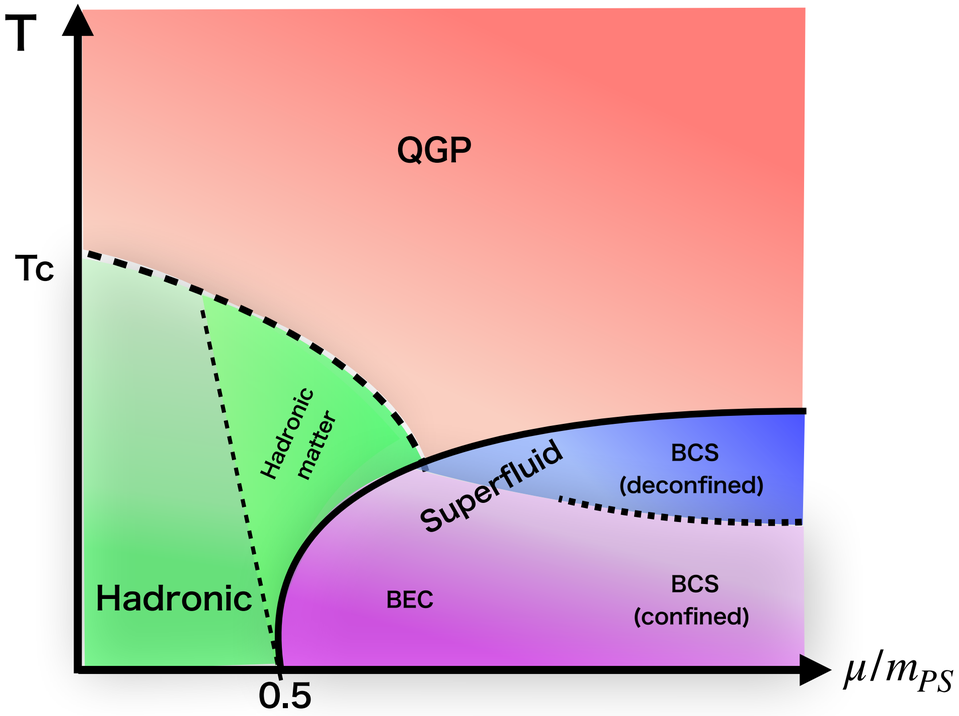
expected phase diagram

Fukushima-Hatsuda (2010)

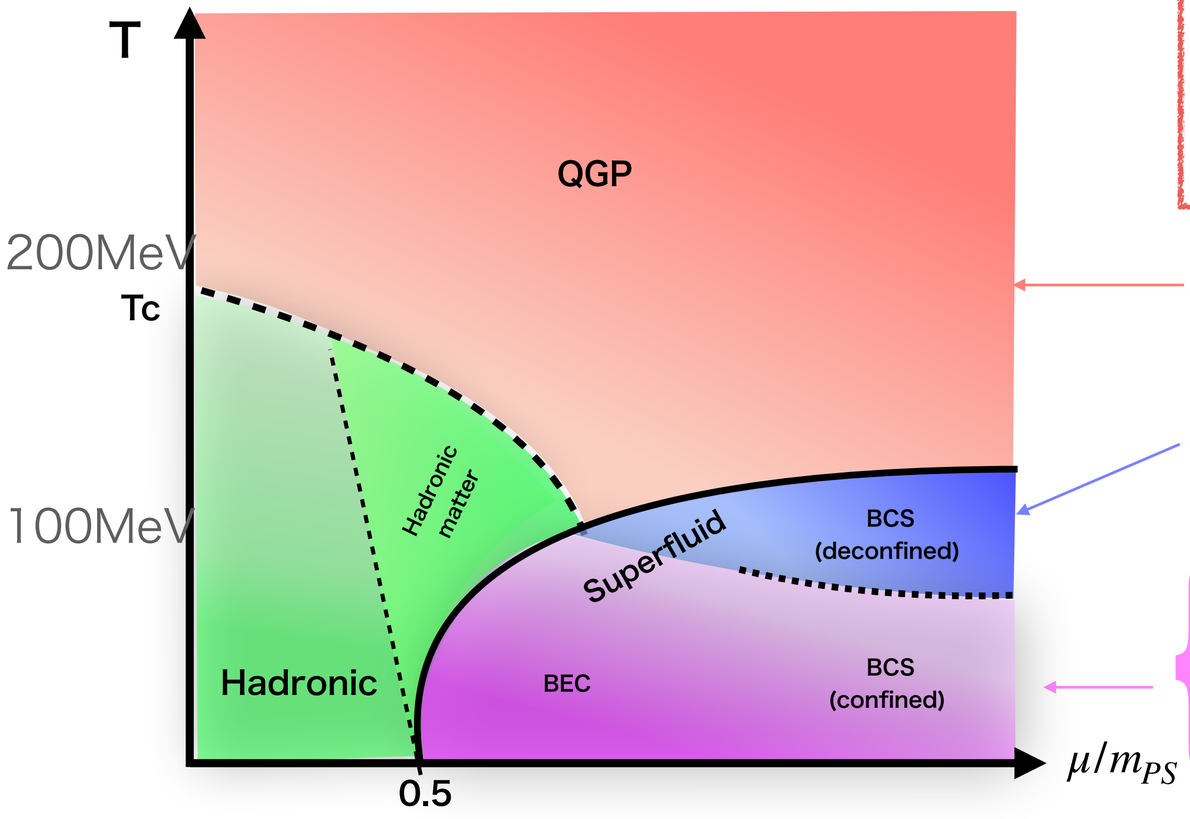


2 color QCD

numerically determined phase diagram



Current status on 2color QCD phase diagram



At least 4 independent groups are studying the phase diagram

(1) S. Hands group : Wilson-Plaquette gauge + Wilson fermion
(2) Russian group : tree level improved Symanzik gauge + rooted staggered fermion
(3) Our group : Iwasaki gauge + Wilson fermion, Tc=200 MeV to fix the scale
(4) von Smekal group: Wilson/Improved gauge + rooted staggered fermion

T=158 MeV (**deconfined**, hadron -> QGP phase transition occurs)
T=130 MeV (**deconfined? QGP phase?** , 2019)

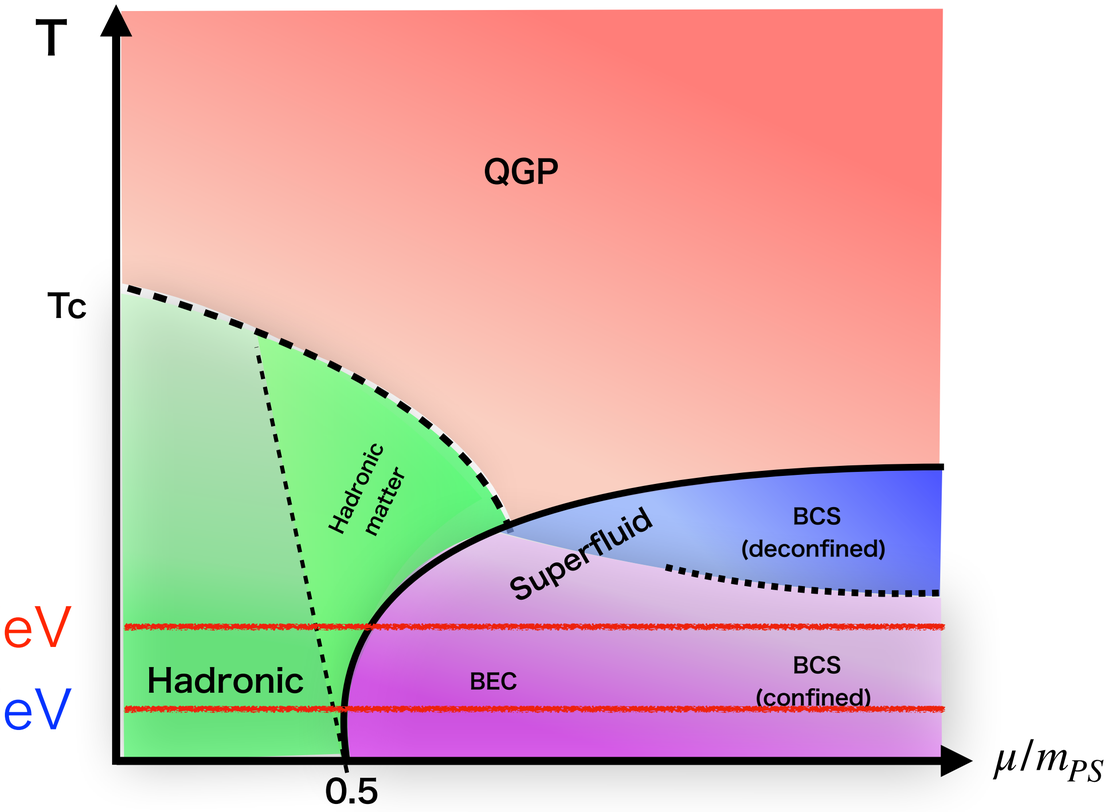
T=140 MeV (**deconfined** in high mu, <qq> is not zero, 2017, 2018, 2020)
T= 93 MeV (**deconfined** in high mu ?, also <qq> is not zero?, 2017)

T=87 MeV (**confined** in 2019)
T=79 MeV (**confined** even in high mu.2019)
T=55 MeV (**confined** in high mu, 2016)
T=47 MeV (**deconfined** coarse lattice in 2012, but **confined** in 2019)
T=45 MeV (**confined** in 2019)
T=40 MeV (**confined, 2024**)

- Even $T \approx 100\mathrm{MeV}$ and $\mu/m_{PS} = 0.5$, superfluid phase emerges
- T_d (confine/deconfine)$\leq T_{SF}$ (superfluid/QGP) : constraint from 't Hooft anomaly matching

T.Furusawa, Y.Tanizaki, EI: PRResearch 2(2020)033253

Current status on 2color QCD phase diagram



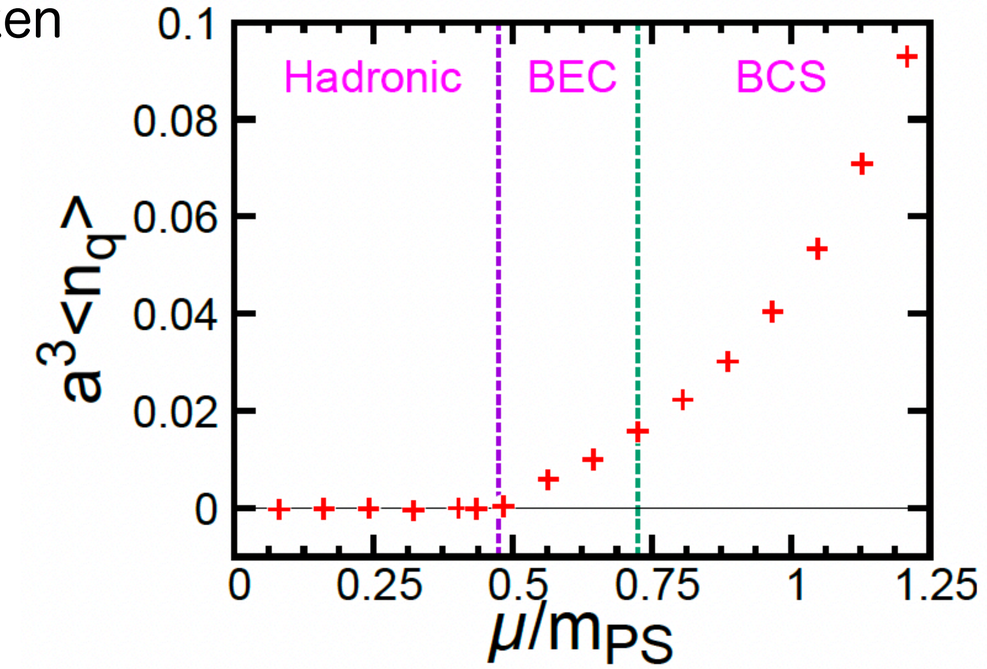
80MeV

40MeV

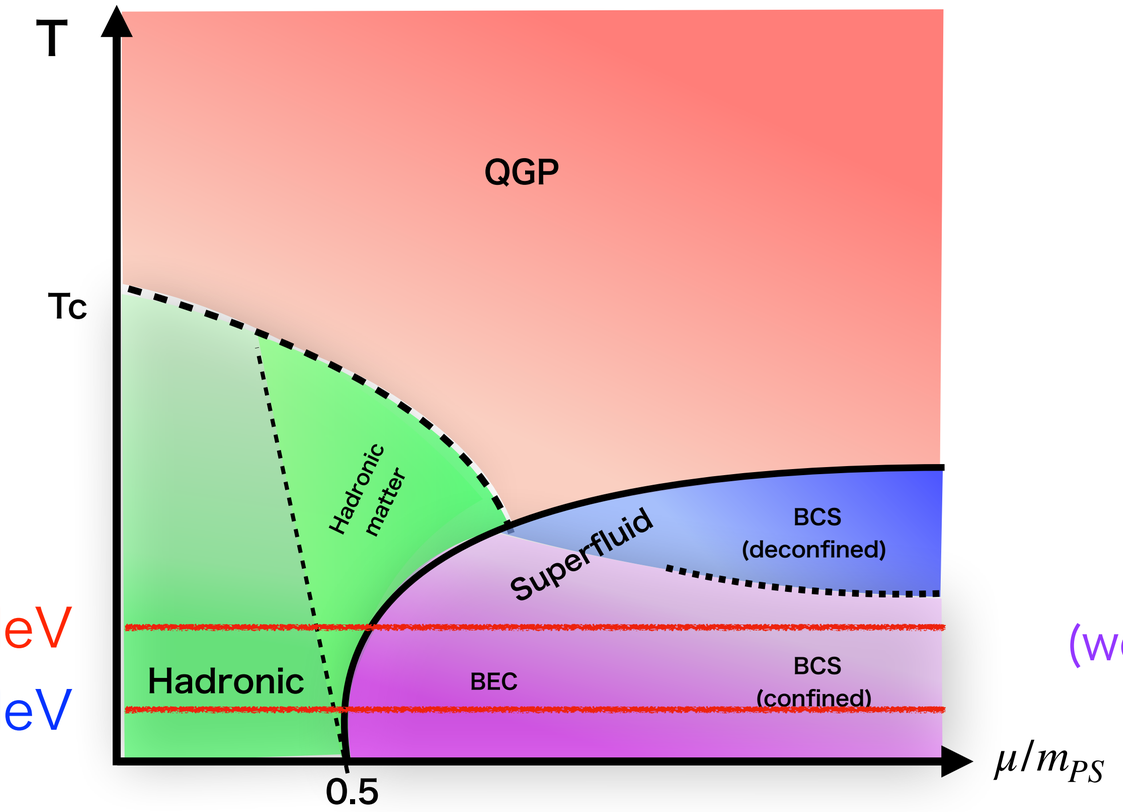
K.Iida, EI, T.-G. Lee: JHEP2001(2020)181
K.Iida, EI, K.Murakami, D.Suenage
arXiv: 2405.20566 [hep-lat]

- We investigated T=80MeV and 40MeV
- Hadronic / Superfluidity phase transition around $\mu = m_{PS}/2$

  Quark number density becomes non-zero because of pair-creation of lightest hadrons and the baryon symmetry is spontaneously broken



Current status on 2color QCD phase diagram



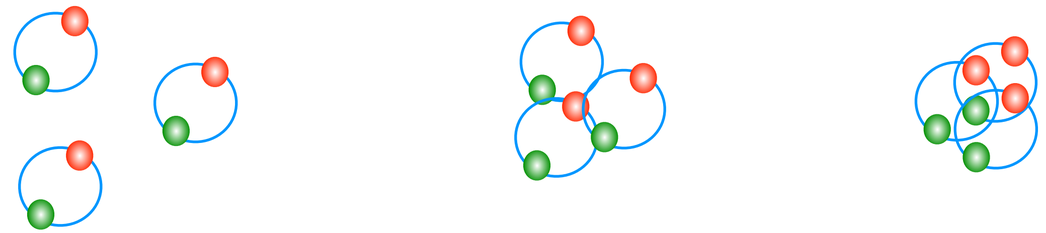
80MeV

40MeV

- We investigated T=80MeV and 40MeV
- Hadronic / Superfluidity phase transition
- BEC/BCS crossover in SF phase

BEC phase
strong coupled
(well-described by ChPT)

BCS phase
weakly coupled



Δ^{-1} Δ: gap energy

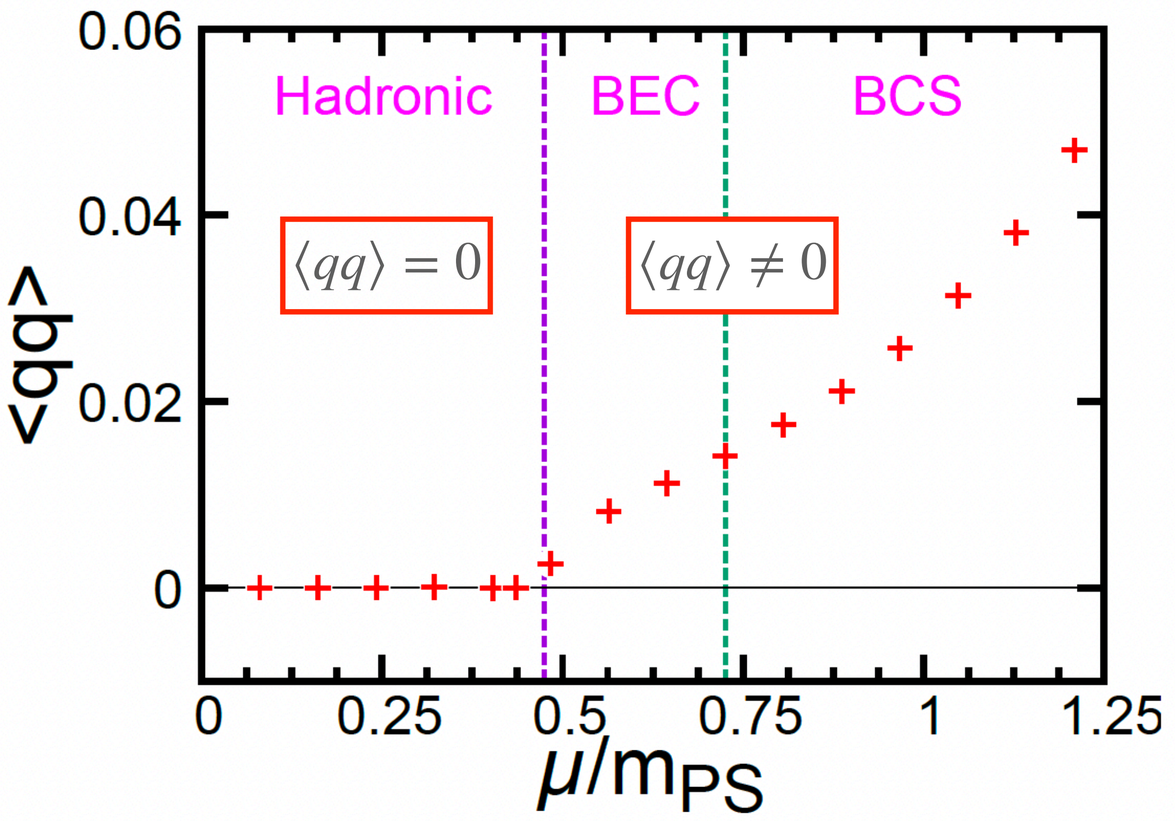
Distance between quarks $\gg \Delta^{-1}$

Distance between quarks $\ll \Delta^{-1}$
Quarks behave free particles

density

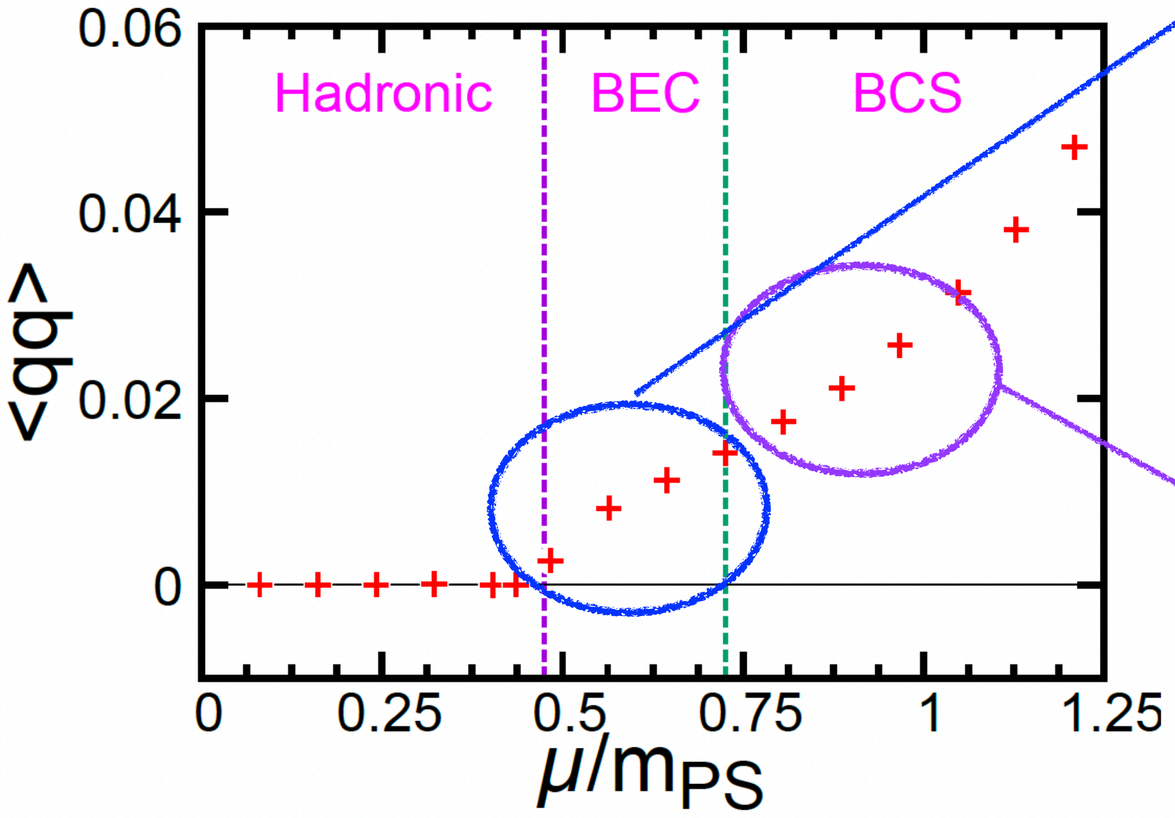
Order parameter of Superfluidity : $\langle qq \rangle$

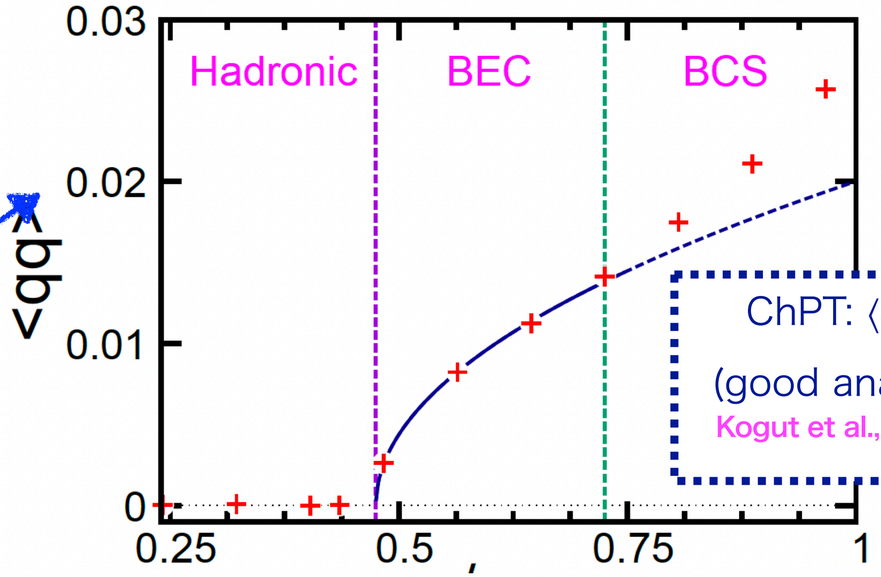
Diquark condensate



Order parameter of Superfluidity : $\langle qq \rangle$

Diquark condensate

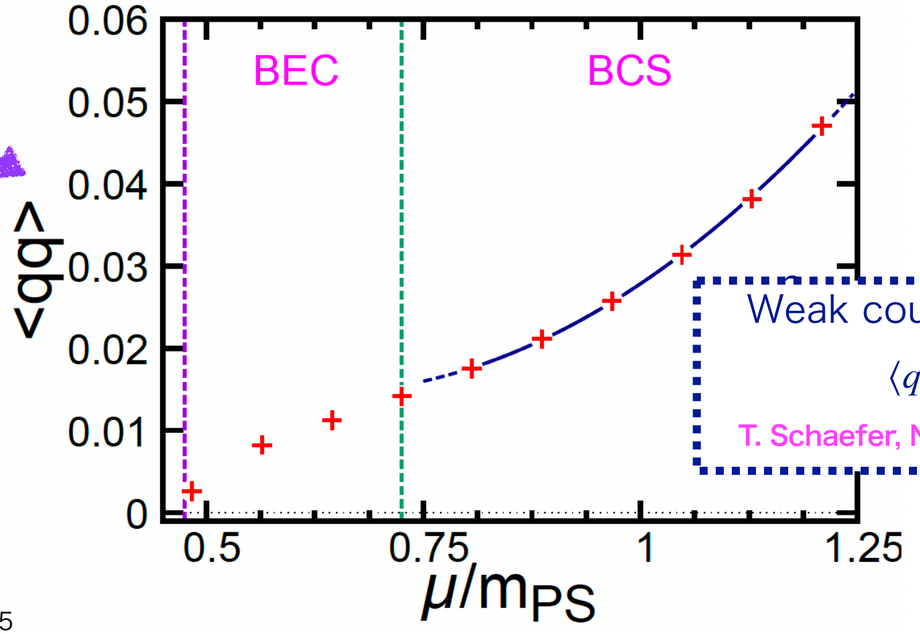




ChPT: $\langle qq \rangle \propto (\mu - \mu_c)^{1/2}$

(good analysis for $\mu \approx \mu_c$)

Kogut et al., NPB 582 (2000) 477

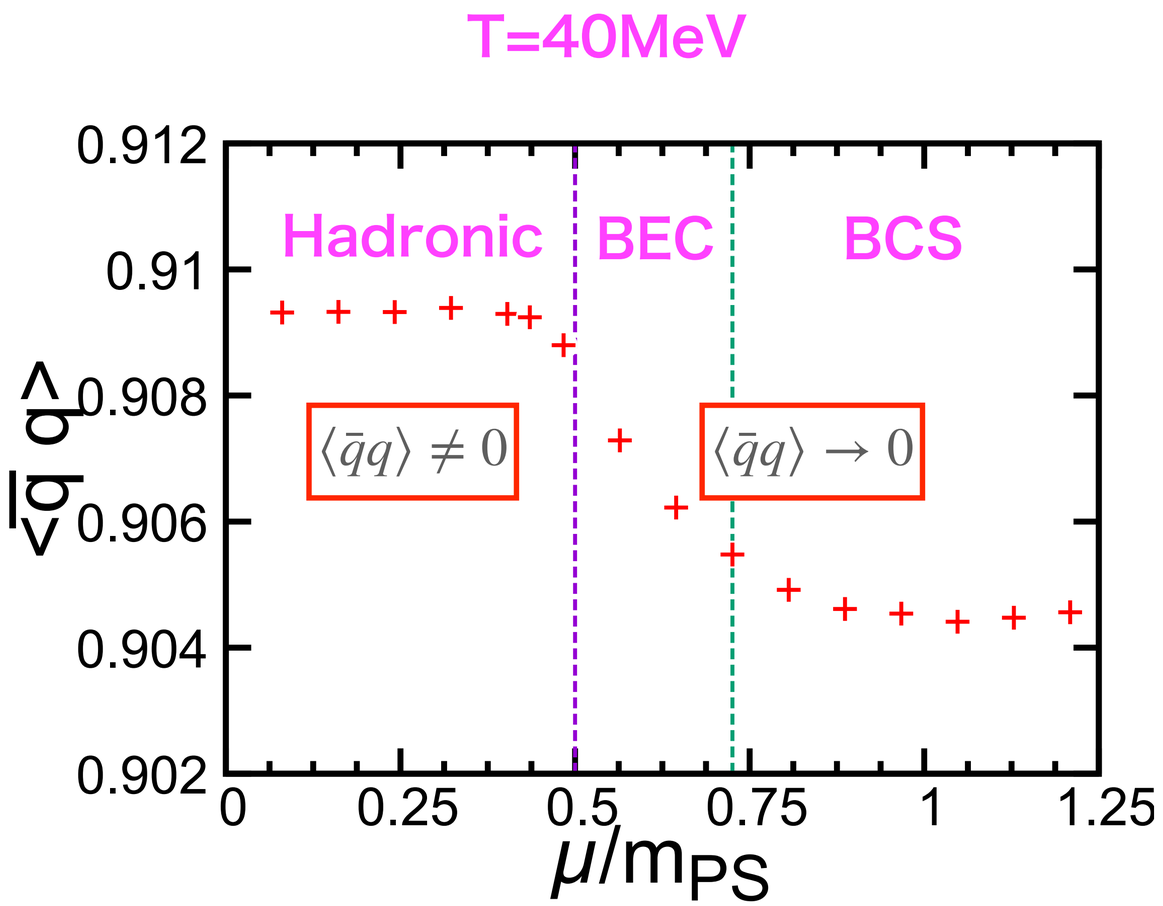


Weak coupling analysis

$\langle qq \rangle \propto \mu^2$

T. Schaefer, NPB 575 (2000) 269

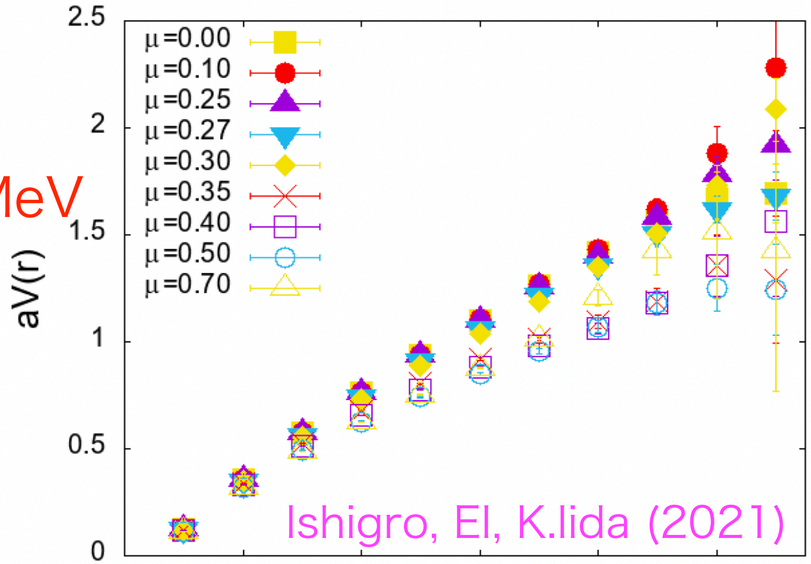
Chiral condensate : $\langle \bar{q}q \rangle$

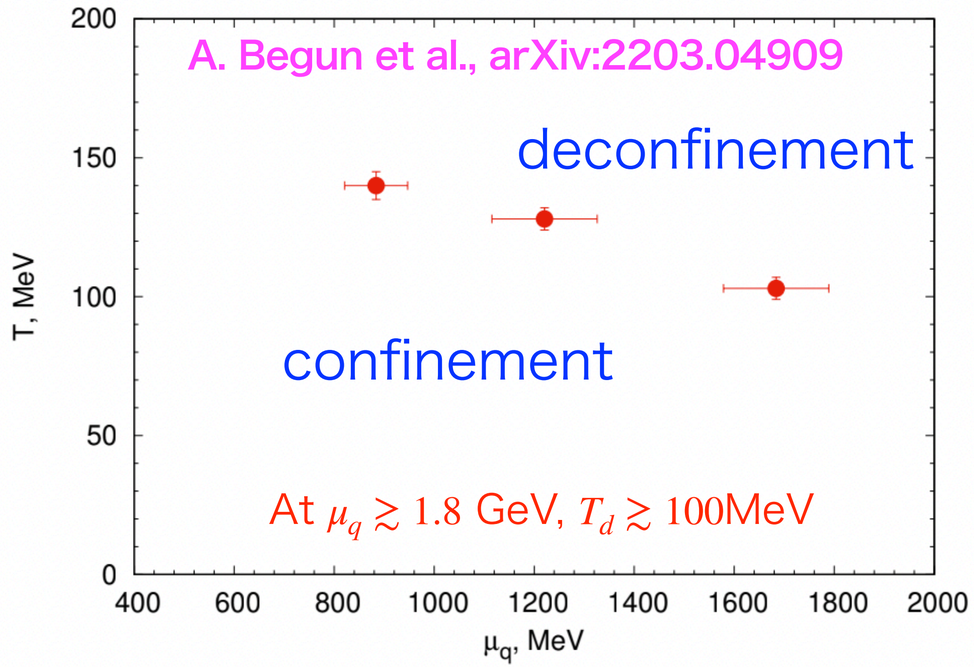


- Chiral symmetry is getting restored in Superfluid phase
- Our simulation uses the Wilson fermion, so does not show $\langle \bar{q}q \rangle = 0$ because of additive renormalization. But Russian group using the staggered fermion gives $\langle \bar{q}q \rangle = 0$

Confinement remains even in high density

T=40MeV



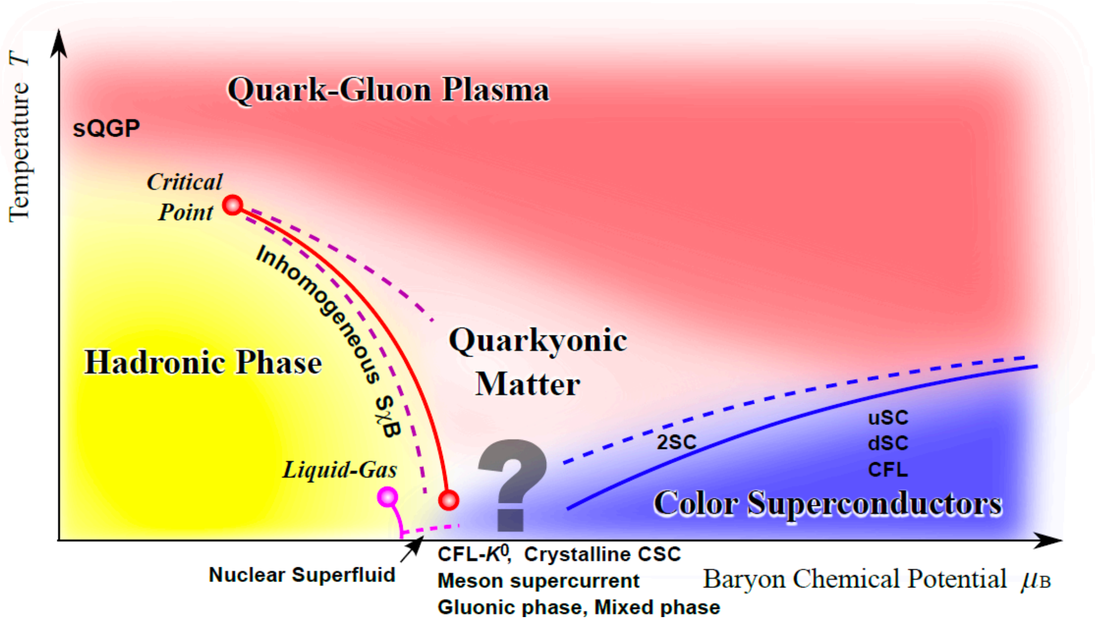


- $q\bar{q}$ potential at T=40MeV also show a linear potential (=confinement)
- Other 2 groups also show: T~90-100MeV is the critical T for deconfinement
- In 2color QCD, the confinement occurs even at high-density. Hadronic superfluidity
- cf.) In real dense QCD, it is expected that quark dof would be relevant Condensate has color charge: $\langle (qq)^a \rangle$

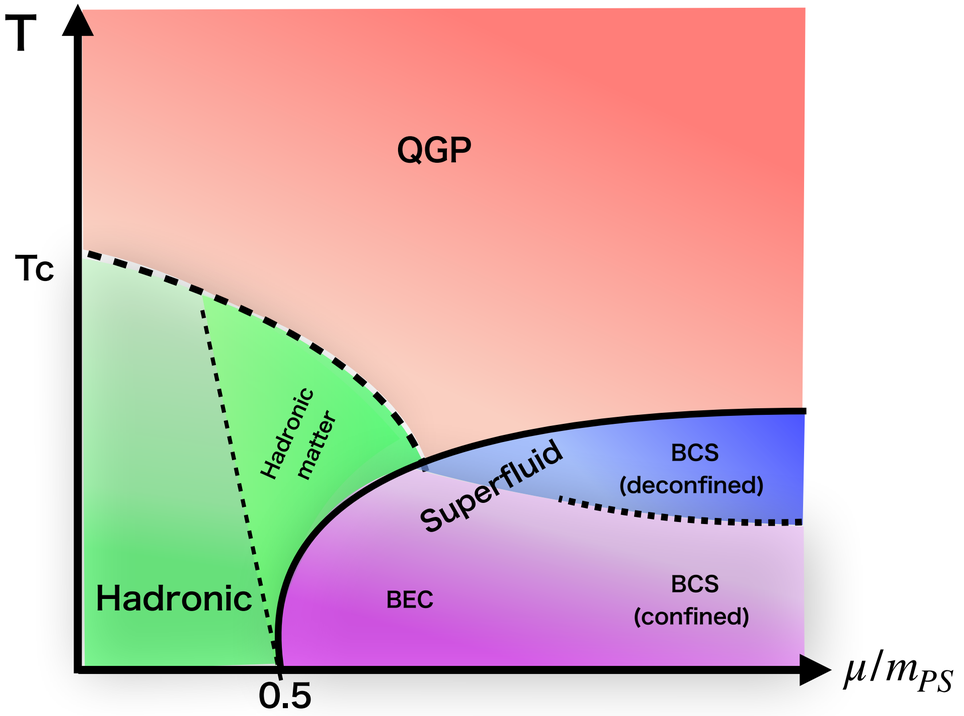
Phase diagram in T-μ plane

3 color (QCD)
expected phase diagram

Fukushima-Hatsuda (2010)



2 color (QCD)
numerically determined phase diagram

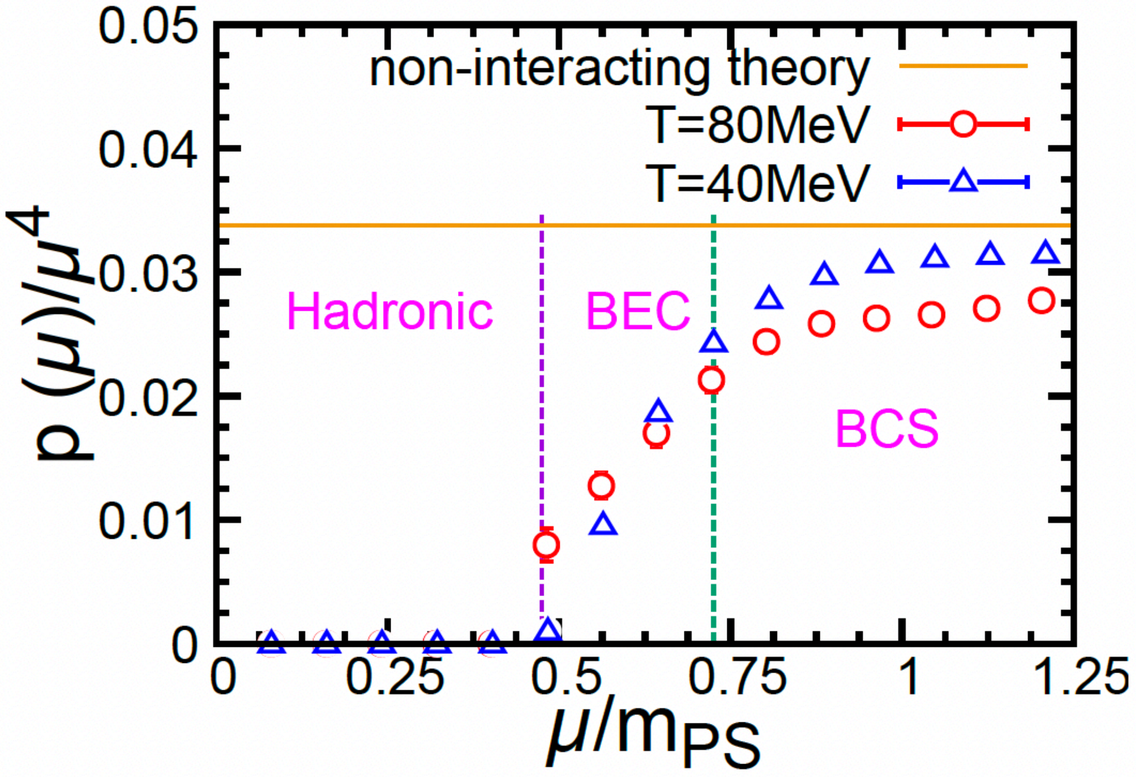


2color QCD phase diagram has been determined by independent works!

Equation of state

K.Iida and EI, PTEP 2022 (2022) 11, 111B01
K.Iida, EI, K.Murakami, D.Suenaga, e-Print: 2405.20566

T dependence of EoS

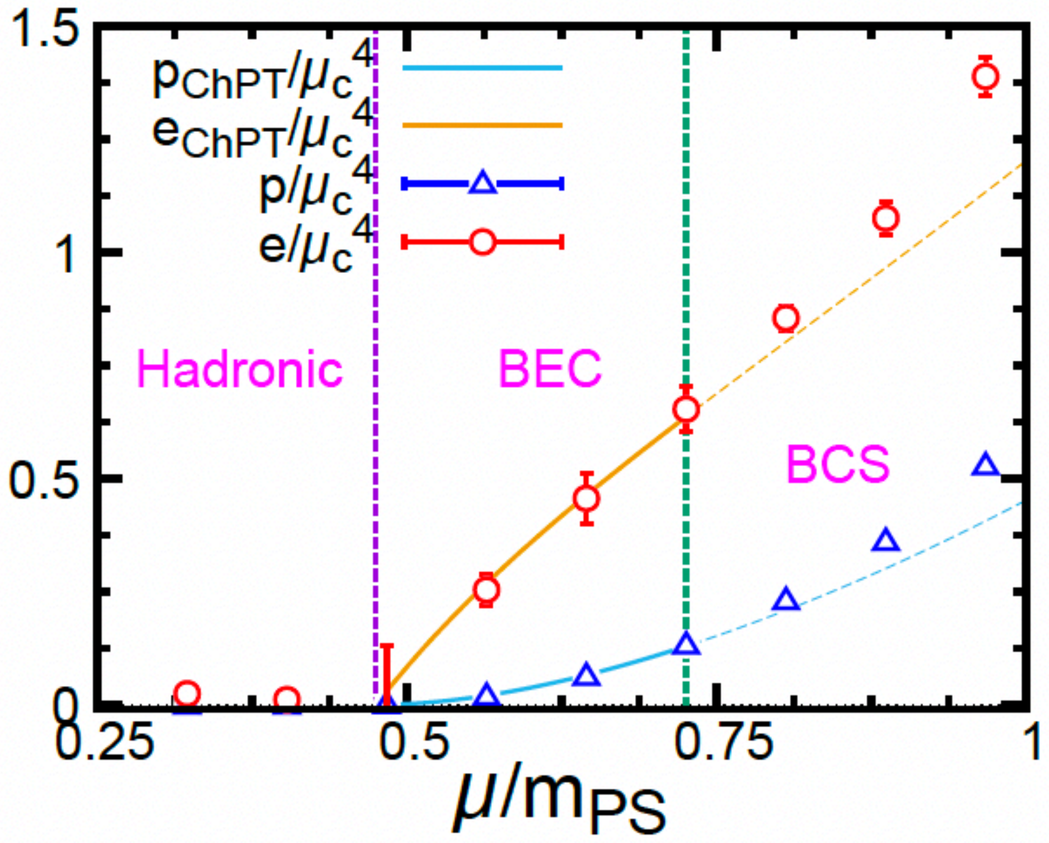


- p increases more rapidly near the critical point at lower-T
- In high-μ, the data approaches the Stefan-Boltzmann limit (=non-interacting theory)

$$p_{SB}/\mu^4 = N_c N_f/(12\pi^2) \approx 0.03$$

- Our largest data of p at T=40MeV reaches at 93% of p_{SB}

EoS and consistency with ChPT result in BEC



- ChPT prediction (valid for near μ_c)

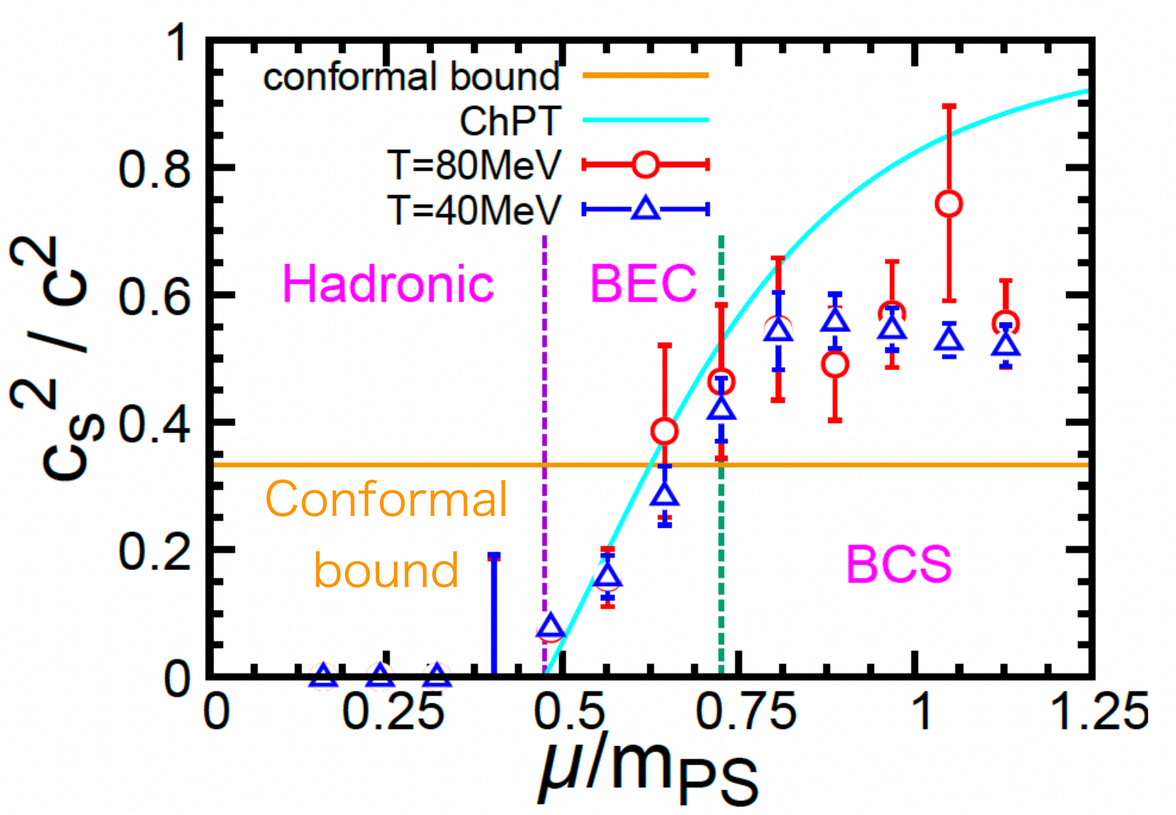
$$p_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)^2$$

$$e_{\mathrm{ChPT}} = 4N_f F^2 \mu^2 \left(1 - \frac{\mu_c^2}{\mu^2}\right)\left(1 + 3\frac{\mu_c^2}{\mu^2}\right)$$

- We obtain the pion decay constant(F)
  from fit of p : F=51.1(5) MeV
  from fit of e : F=56.7(7) MeV
  cf.) F=60.8(1.6) by fitting of $\langle n_q \rangle$ at 140MeV
  (different mass, staggered fermion)

N. Astrakhantsev et al. (2020)

Square of sound velocity ($c_s^2/c^2 = \Delta p/\Delta e$)



- T-dependence of the sound velocity is negligible!
- In BEC phase, result is consistent with ChPT

  Chiral Perturbation Theory (ChPT)

  $c_s^2/c^2 = \dfrac{1-\mu_c^4/\mu^4}{1+3\mu_c^4/\mu^4}$: no free parameter!

  Son and Stephanov (2001) : 3color QCD with isospin μ

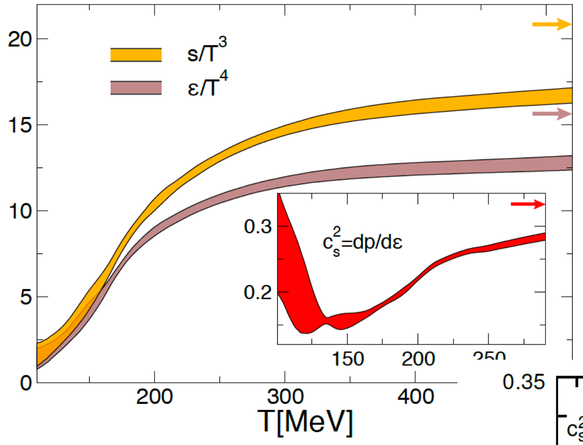
  Hands, Kim, Skullerud (2006) : 2color QCD with real μ

- c_s^2/c^2 exceeds the conformal bound

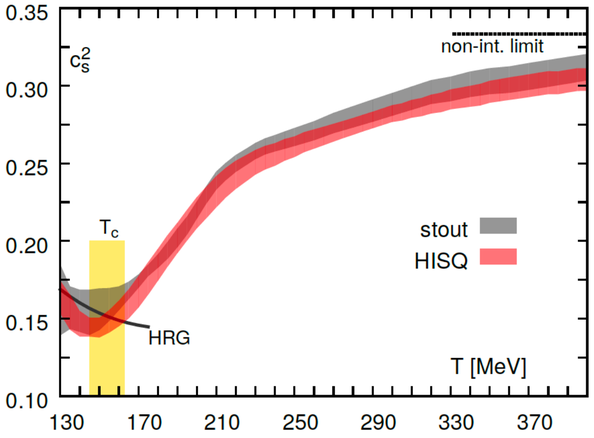
Sound velocity and phase transition

Finite Temperature transition (Nf=2+1 QCD)

Borsanyi et al. (2013)



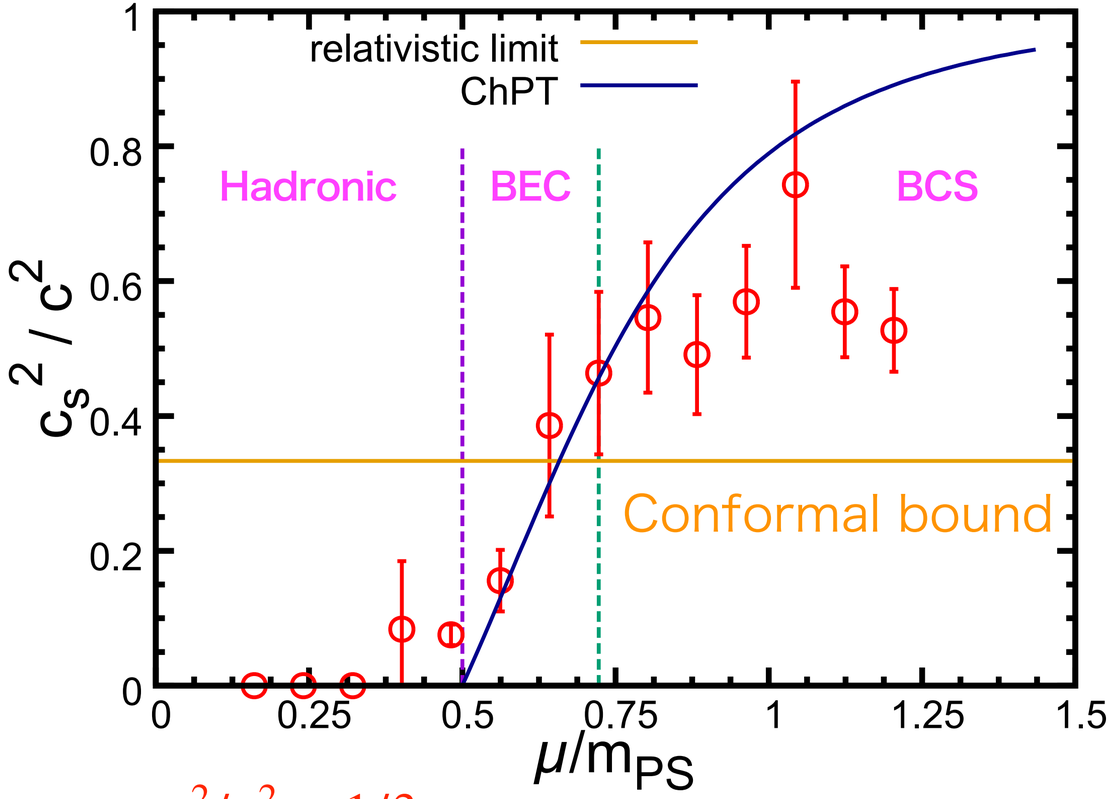
HotQCD (2014)



- Minimum around Tc
- Monotonically increases to $c_s^2/c^2 = 1/3$

Finite Density transition (Nf=2 2color QCD)

Iida and El arXiv: 2207.01253



- $c_s^2/c^2 > 1/3$
- previously unknown from any lattice calculations for QCD-like theories

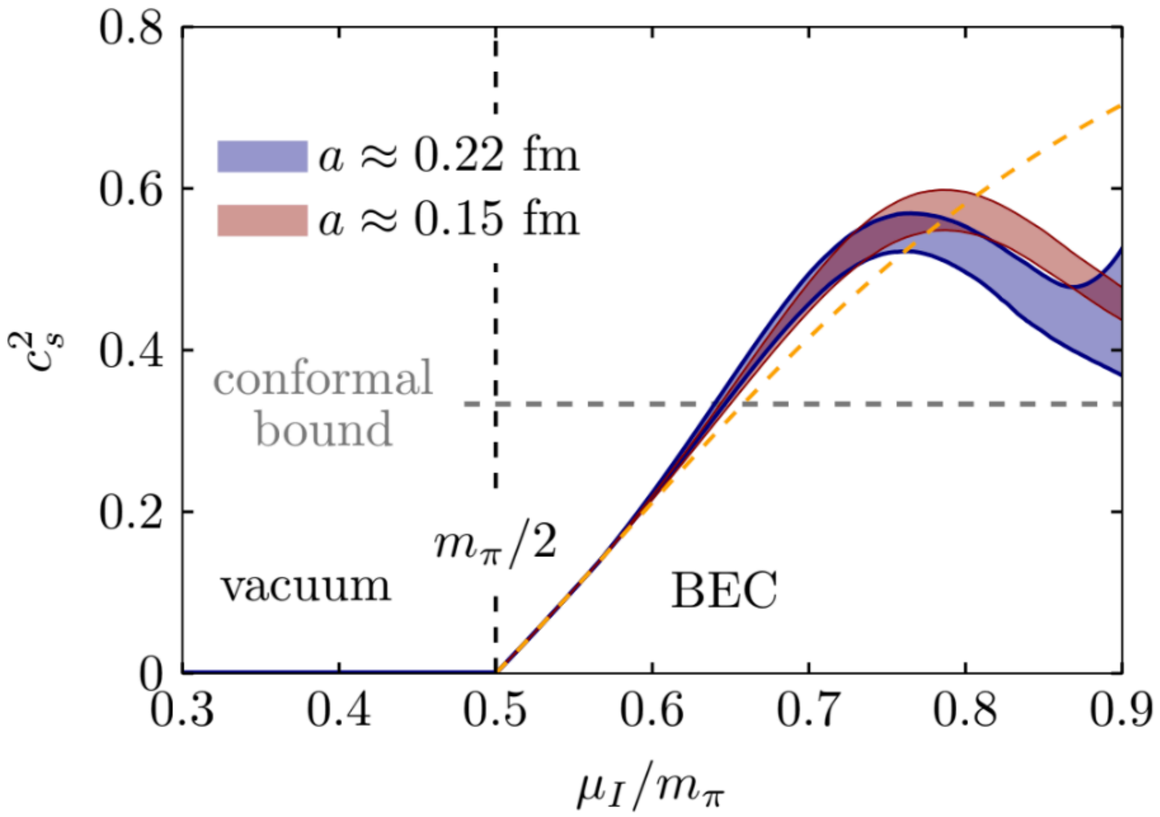
Lattice MC for 3 color QCD with isospin chemical potential

3 color QCD w/ Isospin-$\mu_I \approx$ 2color QCD w/ real μ

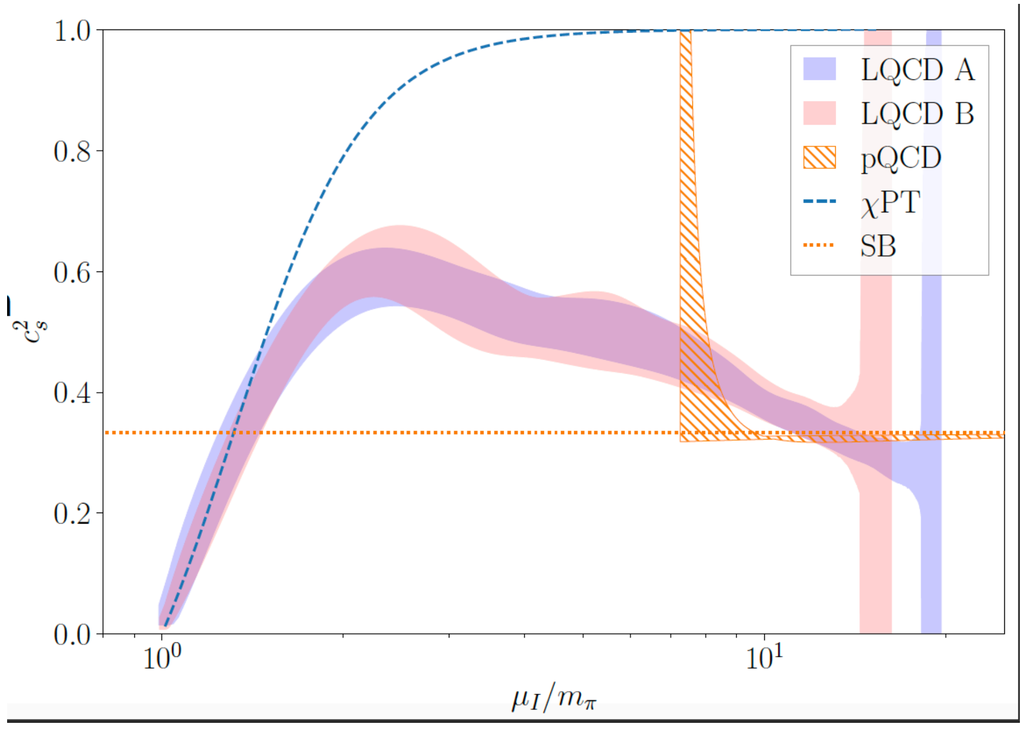
B. B. Brandt, F. Cuteri , G. Endrodi, arXiv: 2212.14016

R. Abbott et al. arXiv:2307.15014

Result with spline interpolation



New algorithm for n-point fn. calc.



Summary and future work

- Lattice numerical simulation for QCD-like theory w/o the sign problem has been ongoing
  2color finite-density QCD and 3color w/ isospin chemical potential are doable using exact algorithm

- Sound velocity exceeds the conformal bound in finite-μ QCD-like theory

  (All Lattice Monte Carlo results have satisfied $c_s^2/c^2 \leq 1/3$ for 40years!)

- Large sound velocity suggests stiffer-than-conventional picture of QCD matter

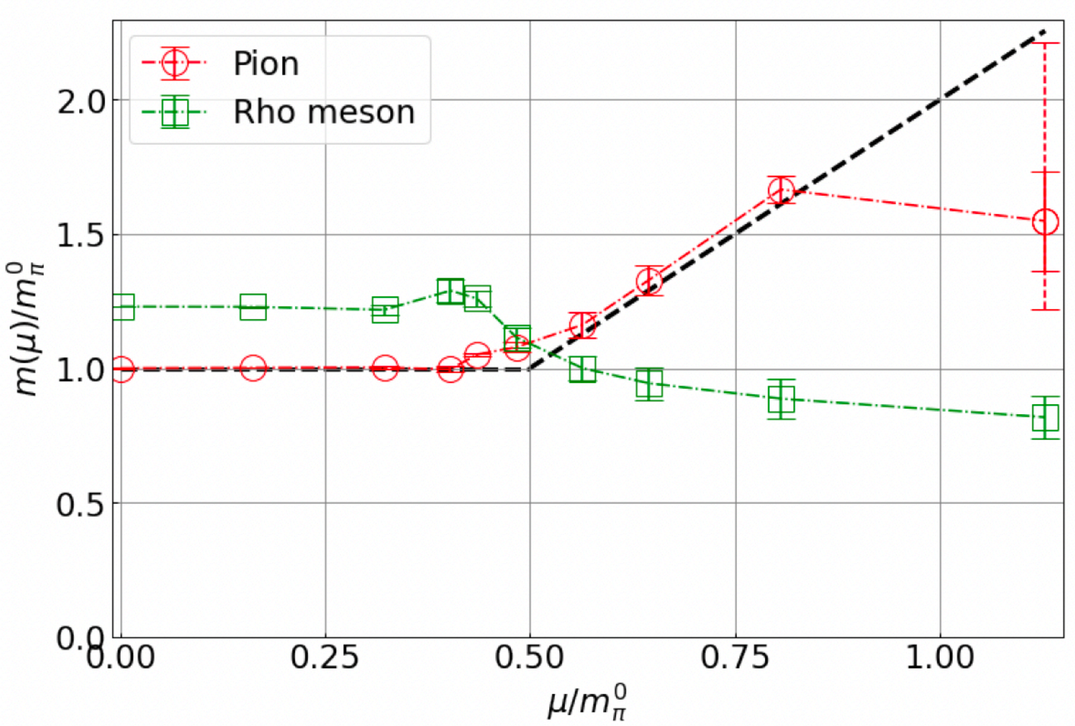
- Find a mechanism of a peak structure
  - quark saturation?(Kojo,Suenaga), negative trace anomaly?
    (McLerran, Fukushima, Fujimoto et al.), others?
  - Effective model analyses combined with the lattice results are also ongoing

- Ongoing Lattice studies:
  => mass spectrum in superfluid phase
  => extended HAL QCD method in finite density
  => Find an explicit evidence of superfluidity (Fermi surface...)

cf.) D.Suenaga, Y.Fujimoto
Minato and Fukushima...

Other interesting phenomena in superfluid phase

Mass spectrum



K.Murakami, D.Suenaga, K.Iida, EI,
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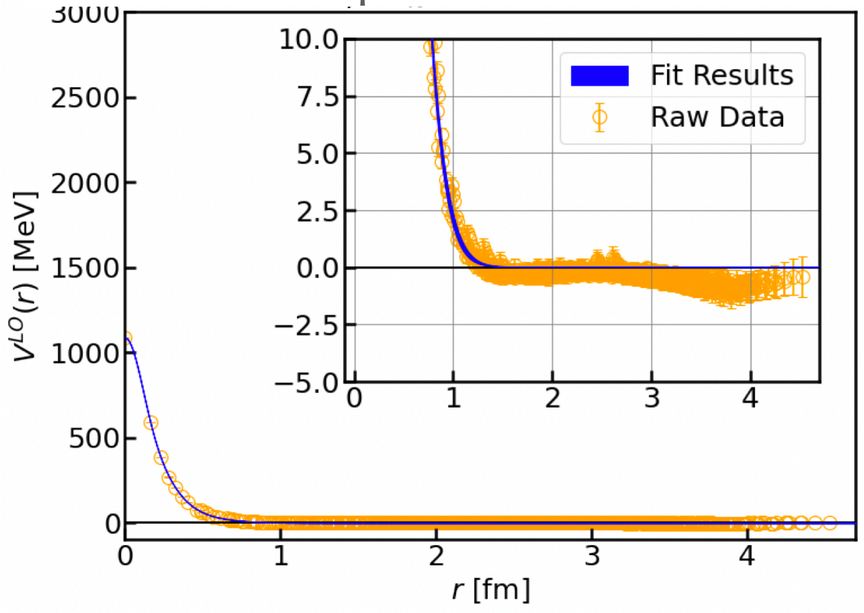
- It is observed that the order of hadron spectra are changed in superfluid phase
- rho meson becomes lighter than pion
- Such a changing is also predicted in 3color QCD Hatsuda-Lee(1992)

Hadron potential by HAL QCD method

- In hadronic phase, pion and diquark potential are equivalent because of extended flavor symmetry.

- Pion potential for 2color and 3color QCD are qualitatively same

Diquark-diquark potential
in hadronic phase of 2color QCD



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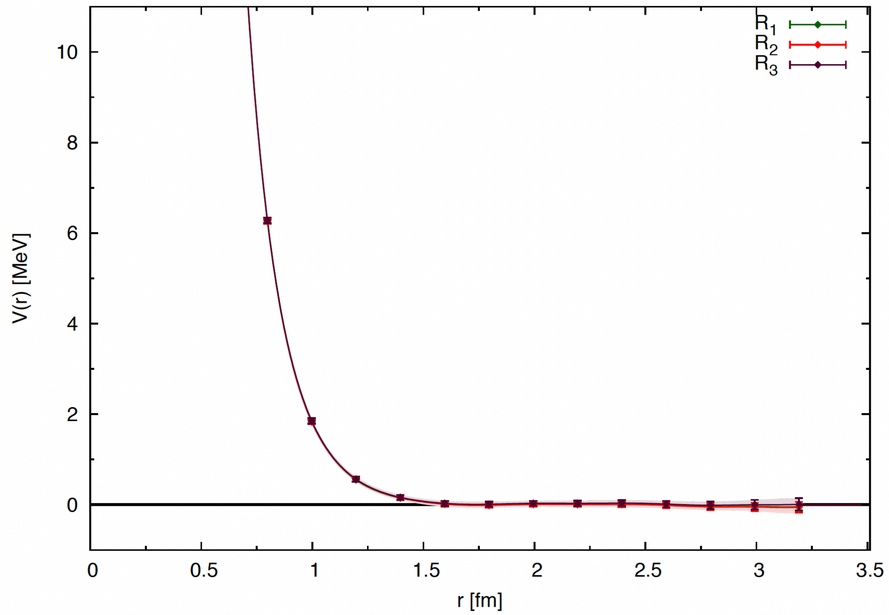
I=2 $\pi\pi$ potential of 3color QCD



T.Kurth et al.(HAL QCD coll.), JHEP12(2013)015