Probing the Dense Matter Equation of State via GW Asteroseismology

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Tokyo, Apr 23, 2025

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NSF
INSTITUTE for NUCLEAR THEORY

Oscillations of NS

Fluid perturbations

- Radial oscillation (l=0): $\xi^r = W_n^r(r) e^{\mathbf{i}\omega t}$
  don't couple to gravitational waves

- Non-radial oscillation (l>=2):

$$\xi_{even}^{\theta,\phi} = \partial_{\theta,\phi}\left(V_{n,l,m}(r) Y_m^l(\theta,\phi) e^{\mathbf{i}\omega t}\right)$$

$$\xi_{odd}^{\theta,\phi} = \hat{r} \times \partial_{\theta,\phi}\left(V_{n,l,m}(r) Y_m^l(\theta,\phi) e^{\mathbf{i}\omega t}\right)$$

f-mode (fundamental n=0) (even),
p-modes (pressure n=1,2,…) (even)
g-modes (gravity n=-1,-2,…) (even)
r-modes (rotation m=+-1,+-2,…) (odd)

$$\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

	ν (kHz)	τ (s)
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

- **Spacetime perturbations**:
  Family I w-modes (even)
  Family II w-modes (odd)
  important for BBH ring-down

even-parity (polar mode)

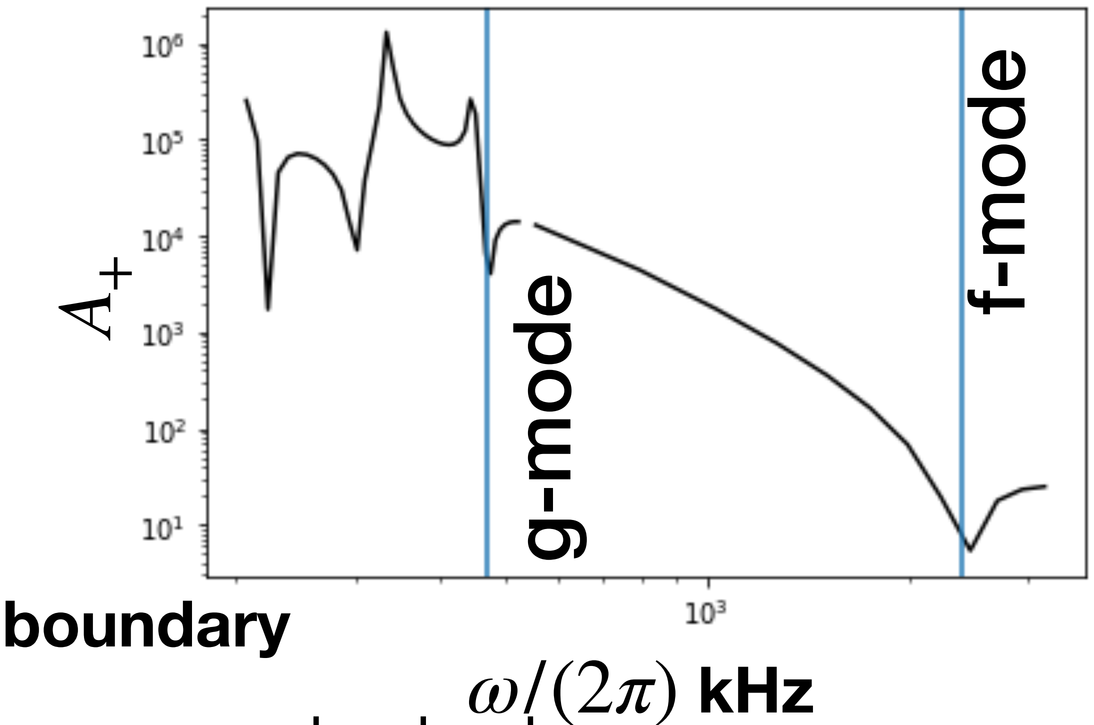
$$h_{\mu\nu}^{even} = \begin{pmatrix} H_0 e^{\nu} & H_1 & 0 & 0 \\ H_1 & H_0 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta K \end{pmatrix} P_l(\cos\theta)$$

odd-parity (axial modes)

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & 0 & H_0' \\ 0 & 0 & 0 & H_1' \\ 0 & 0 & 0 & 0 \\ H_0' & H_1' & 0 & 0 \end{pmatrix} \sin\theta \partial_\theta P_l(\cos\theta)$$

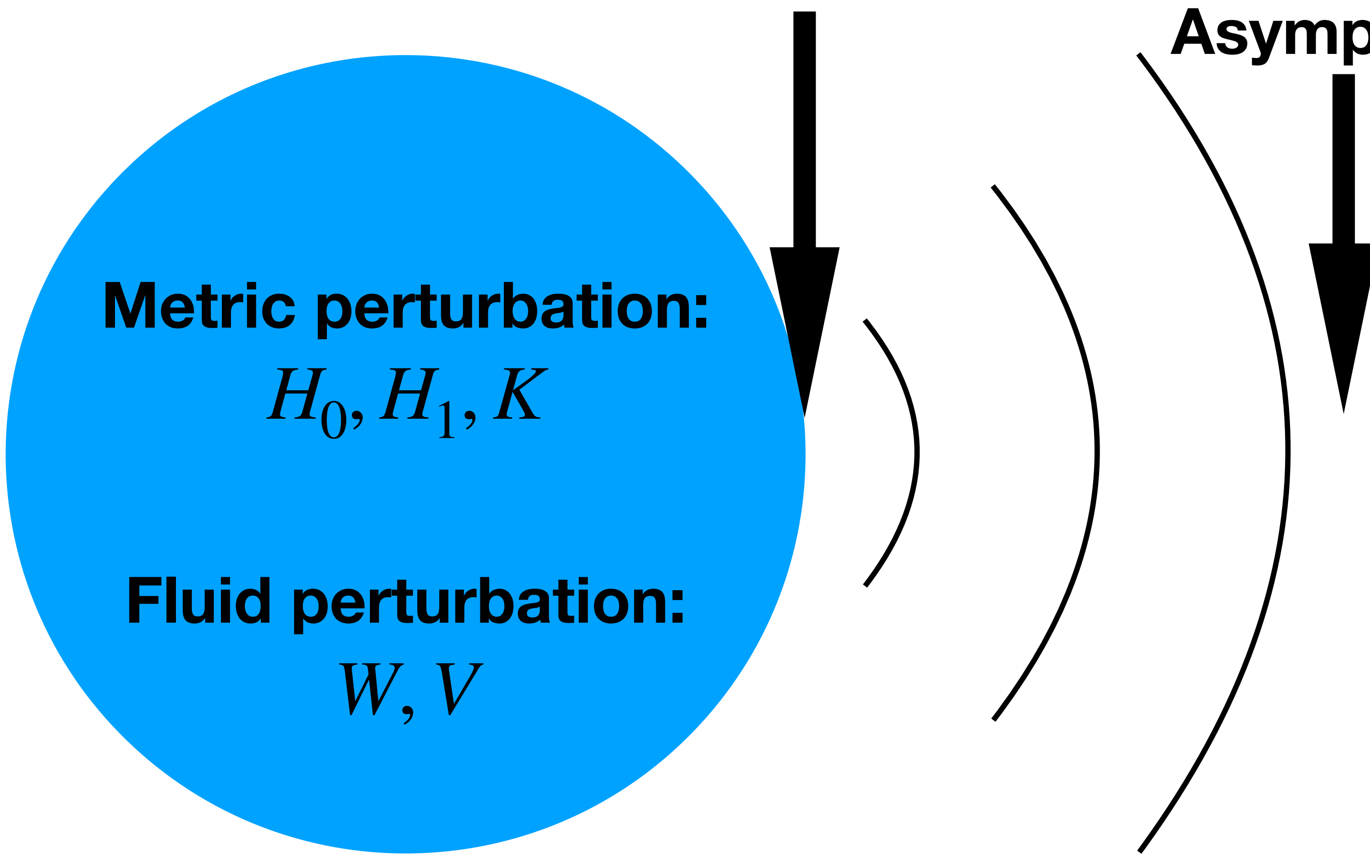
- Solid crust modes (elasticity), see Sotani's

Perturbation ODEs



Surface boundary

Asymptotic boundary



$$Z_\pm \propto \frac{A_\pm}{r} e^{\pm \frac{i\omega r}{c}}$$

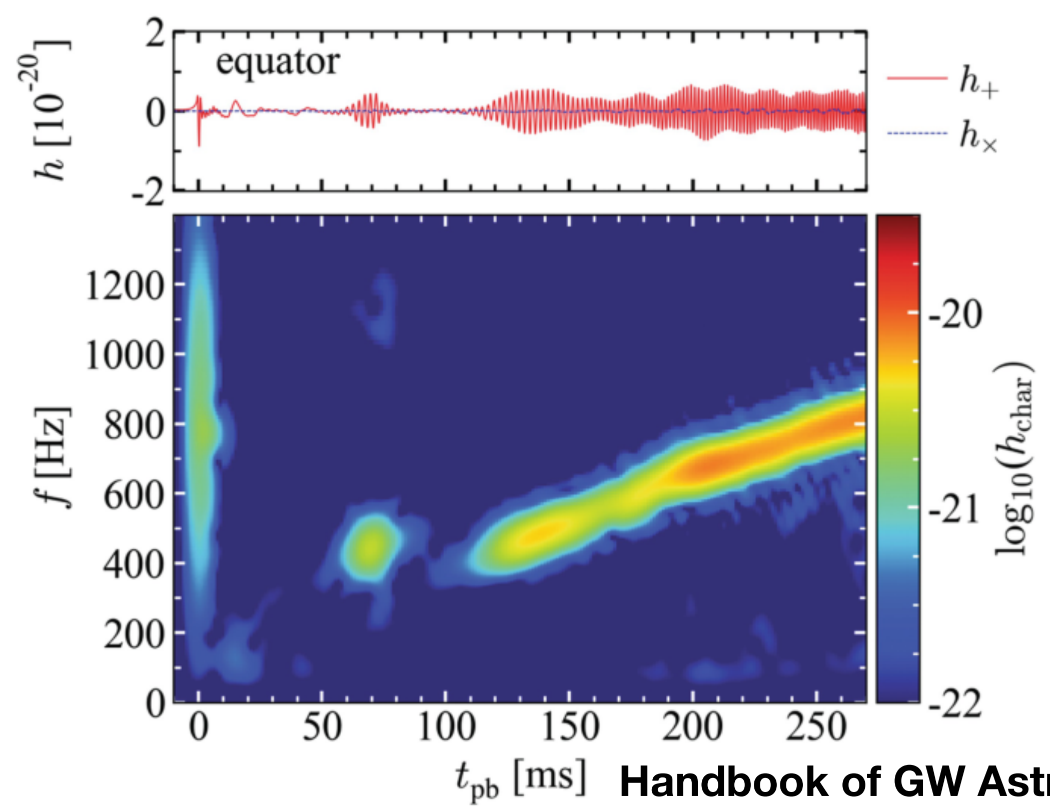
Observer:
LIGO, Virgo, KAGRA
GW strain h

- Thorne, Kip S. 1967: four 1st ODEs

Metric perturbation:

$$\underbrace{H_0, H_1, K}_{Z}$$

- Zerilli's Equation: one 2nd ODEs



Handbook of GW Astro. 2020

Observation of Oscillations of NS

- Direct observation:

matter motion { 1. BNS merger remnant
2. Core-collapse SNe
3. Star quake (glitches)
4. NS close encounter }

↓

spacetime variation

↓

gravitational wave radiation

- Indirect observation:

Binary NS inspiring

↓

Orbital angular momentum transfer

↓

gravitational wave form information

- Instrument:
  Comic Explorer (US)
  Einstein Telescope (Europe)

Gravitational radiation of NS oscillation

- The amplitude of observed oscillations is

$$h(t) = h_0 e^{-t/\tau} \cos \omega t \qquad A(r) e^{i\omega t} \qquad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

- The observed GW energy flux is

$$F(t) = \frac{c^3 \omega^2 h_0^2}{16\pi G} e^{-2t/\tau} = 3.17 e^{-2t/\tau} \left(\frac{\nu}{\mathbf{kHz}}\right)^2 \left(\frac{h_0}{10^{-22}}\right)^2 \ \mathbf{ergs\ cm^{-2} s^{-1}}$$

- The total GW energy is

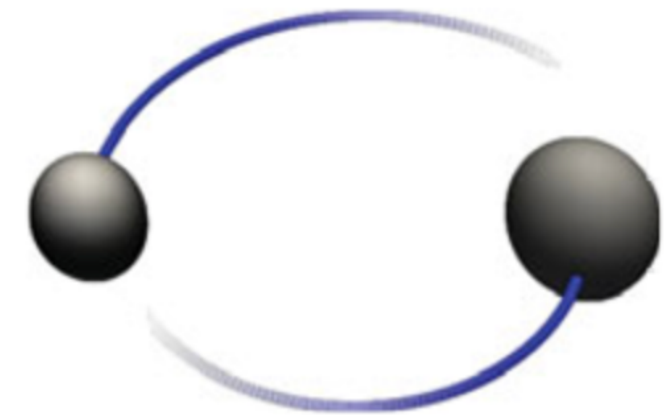
$$E = \frac{c^3 \omega^2 h_0^2 \tau D^2}{8G} = 4.27 \times 10^{49} \left(\frac{\nu}{\mathbf{kHz}}\right)^2 \left(\frac{h_0}{10^{-23}}\right)^2 \left(\frac{\tau}{0.1\ \mathbf{s}}\right) \left(\frac{D}{\mathbf{15\ Mpc}}\right)^2 \mathbf{ergs}$$

- supernovae remnant: 10^44-10^47 ergs

  merger remnant: 10^51-10^52 ergs

	aLIGO	3G
supernovae remnant	$D < 20$ **kpc**	$D < 200$ **kpc** **A few per century**
merger remnant	$D \lesssim 20 - 45$ **Mpc**	$D \lesssim 200 - 450$ **Mpc** **0.06-4 per year**

Dynamic Tidal deformability from NS merger



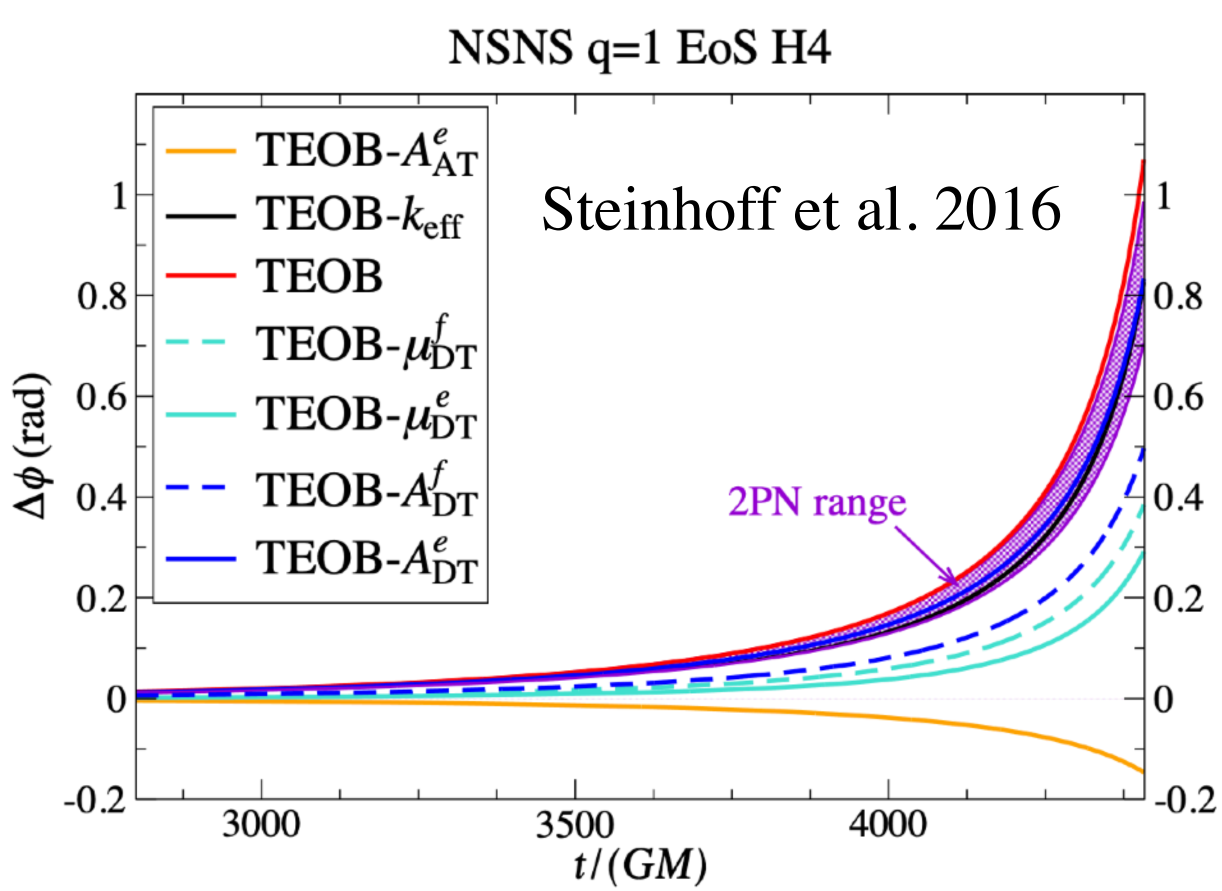
- Classical dynamic tidal Lagrangian:

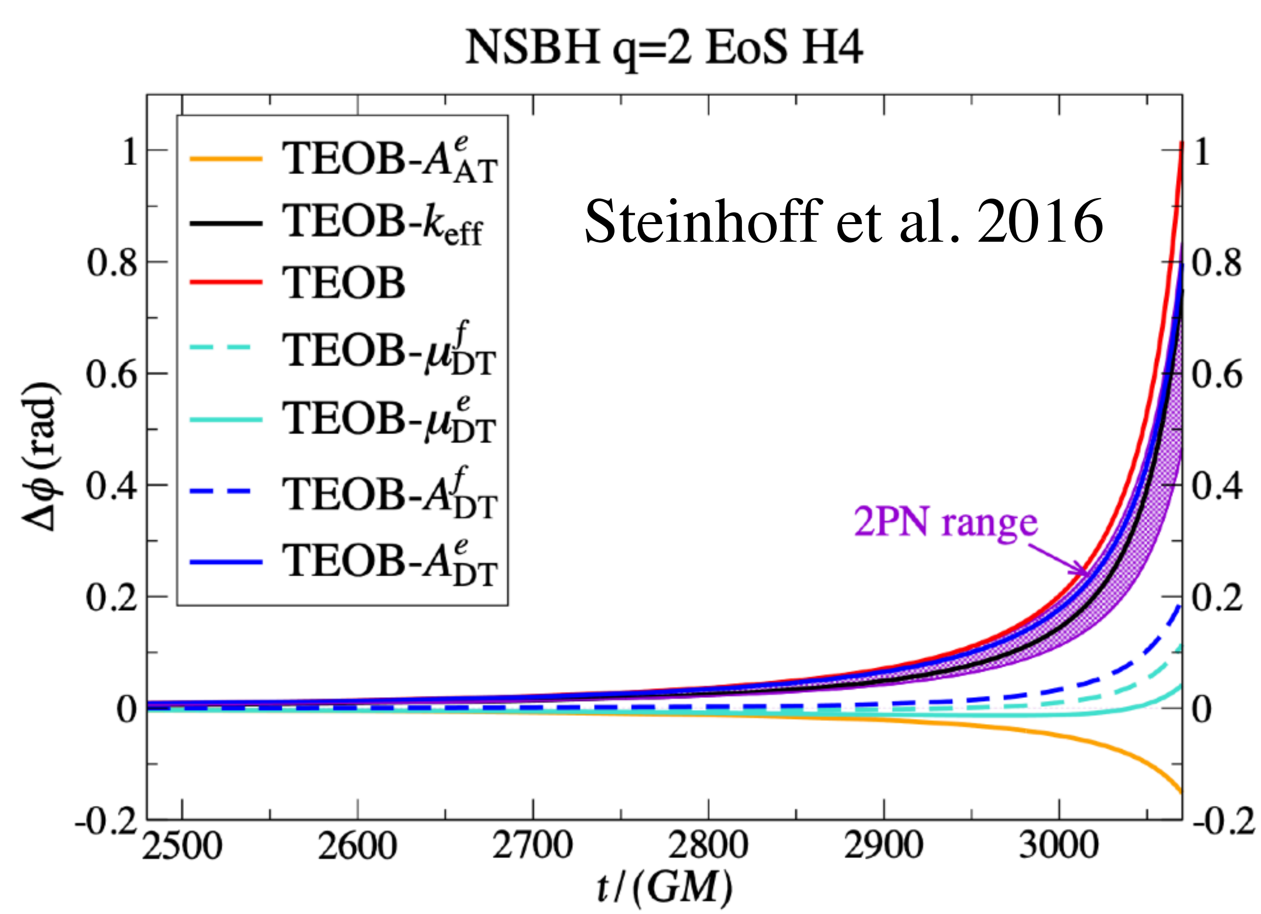
$$\mathcal{L}_{DT} = \frac{1}{4\lambda\omega_f^2}\left[\dot{Q}^{ij}\dot{Q}^{ij} - \omega_f^2 Q^{ij}Q^{ij}\right] - \frac{1}{2}\varepsilon_{ij}Q^{ij}$$

where $E_{ij} = \partial_i\partial_j\Phi$ is the tidal field,

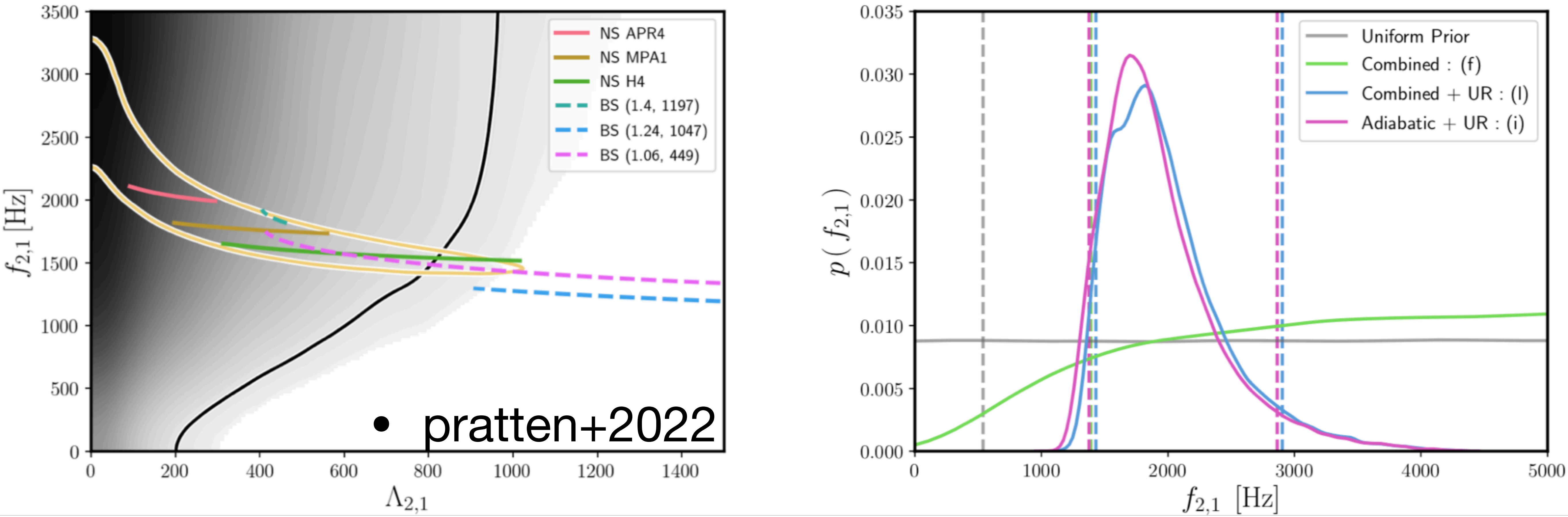
and Q_{ij} is the quadrupole moments: $Q^{ij}(t) = \int \rho(\mathbf{r})\left[x^i\,\xi_f^j(\mathbf{r}) + x^j\,\xi_f^i(\mathbf{r})\right]\,d^3r$

- Replacing the static (adiabatic) tidal assumption $-\lambda\varepsilon_{ij}(t) = Q_{ij}(t)$ with the EOM of $\mathcal{L}_{DT}$.





Dynamical tidal effect of GW170817



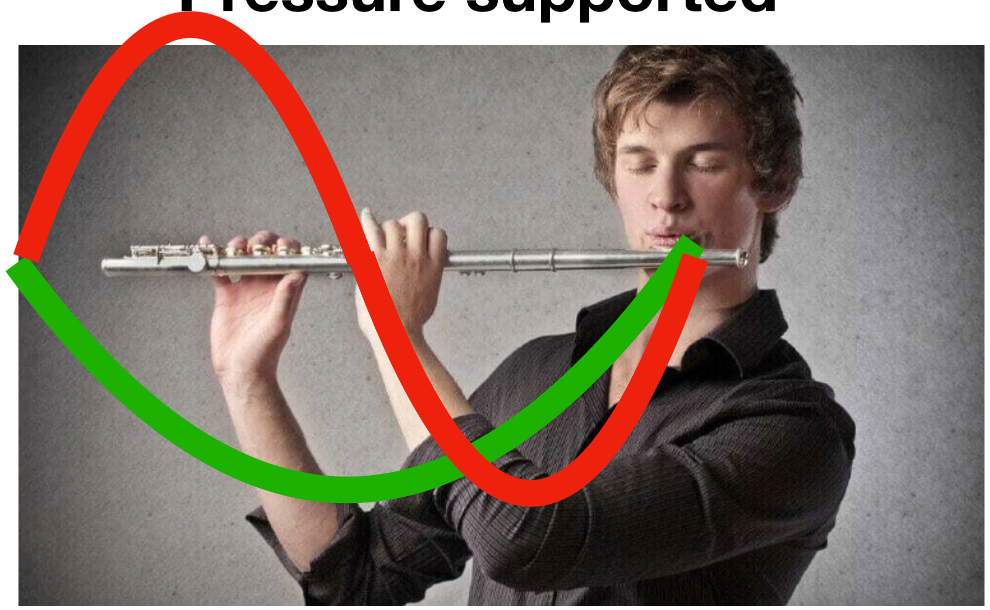
- pratten+2022

- 90% credible interval of f-mode frequency for GW170817:
  1.43 kHz ~ 2.90 kHz for the more massive star
  1.48 kHz ~ 3.18 kHz for the less massive star

Oscillation modes

$$A(r)e^{i\omega t} \qquad \omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

Pressure supported



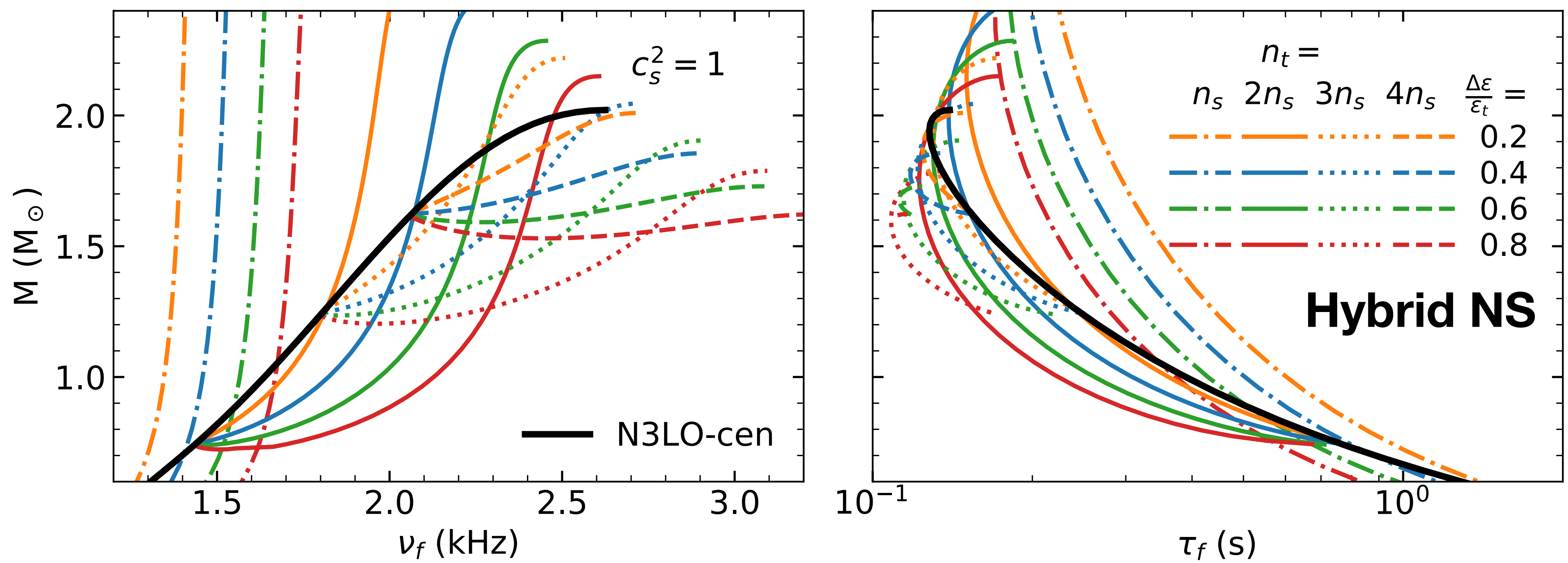
Standing sound wave of order n:

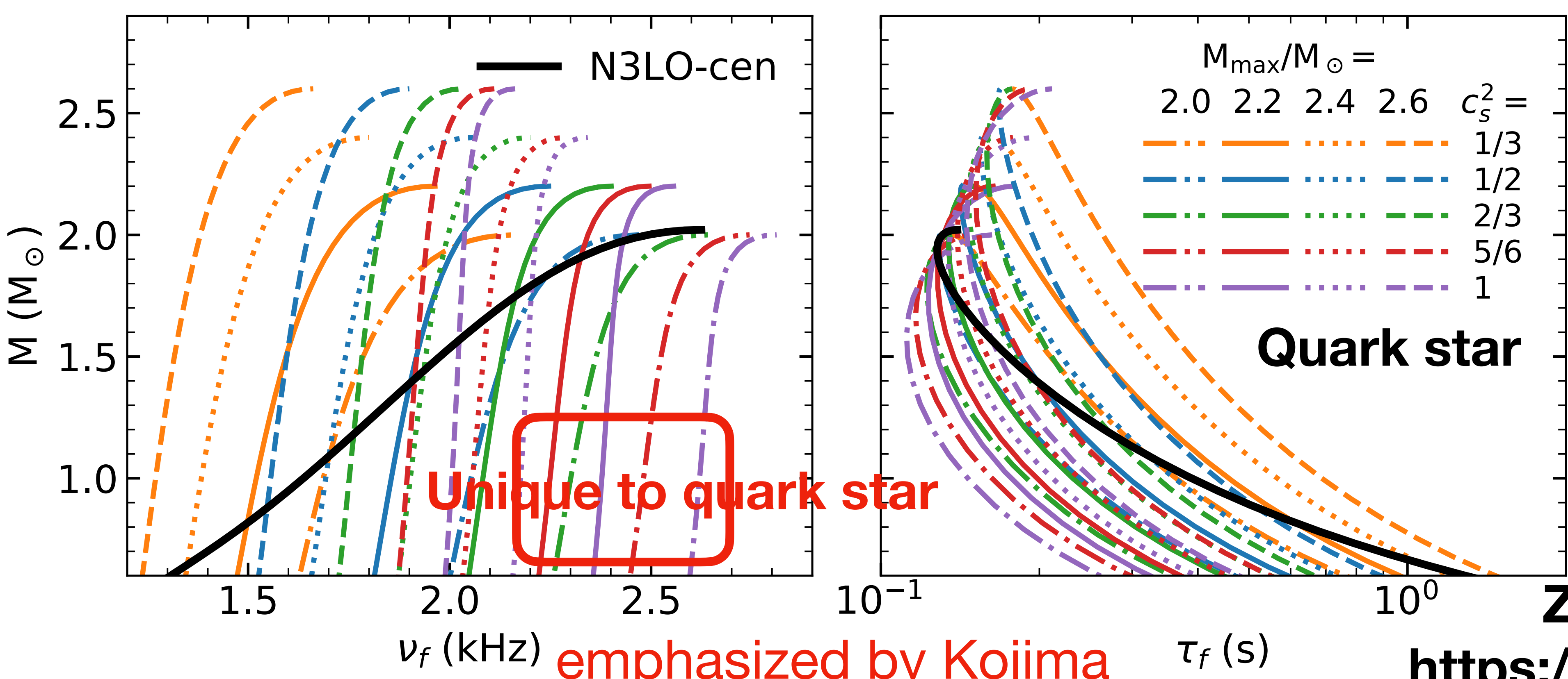
$$\omega^2 \approx \frac{dp}{d\varepsilon}k^2 \qquad k = \frac{\sqrt{l(l+1)}n}{2\pi R}$$

n=0 f-mode (fundamental)

n=1 p-mode (pressure)

f-mode with Hybrid and Quark EOS





Frequency:

$\nu_f \in (1.3 - 2.8)$ kHz

Damping time:

$\tau \in (0.1, 1)$ s

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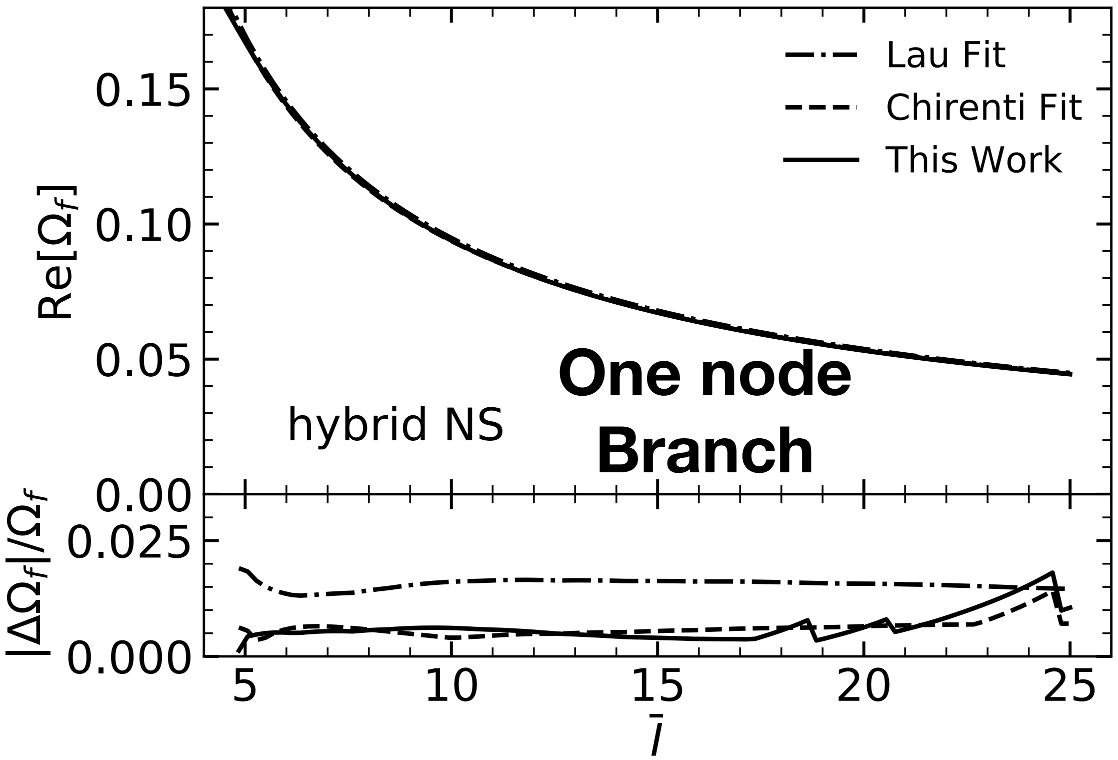
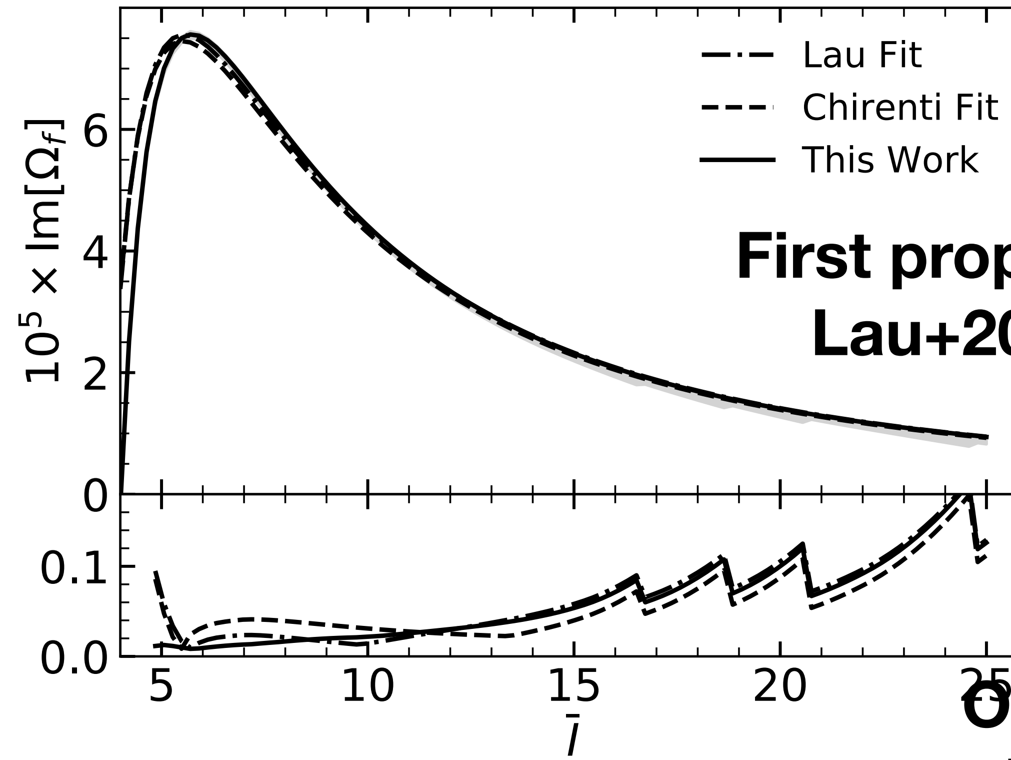
https://arxiv.org/abs/2204.03037

f-mode universal relations



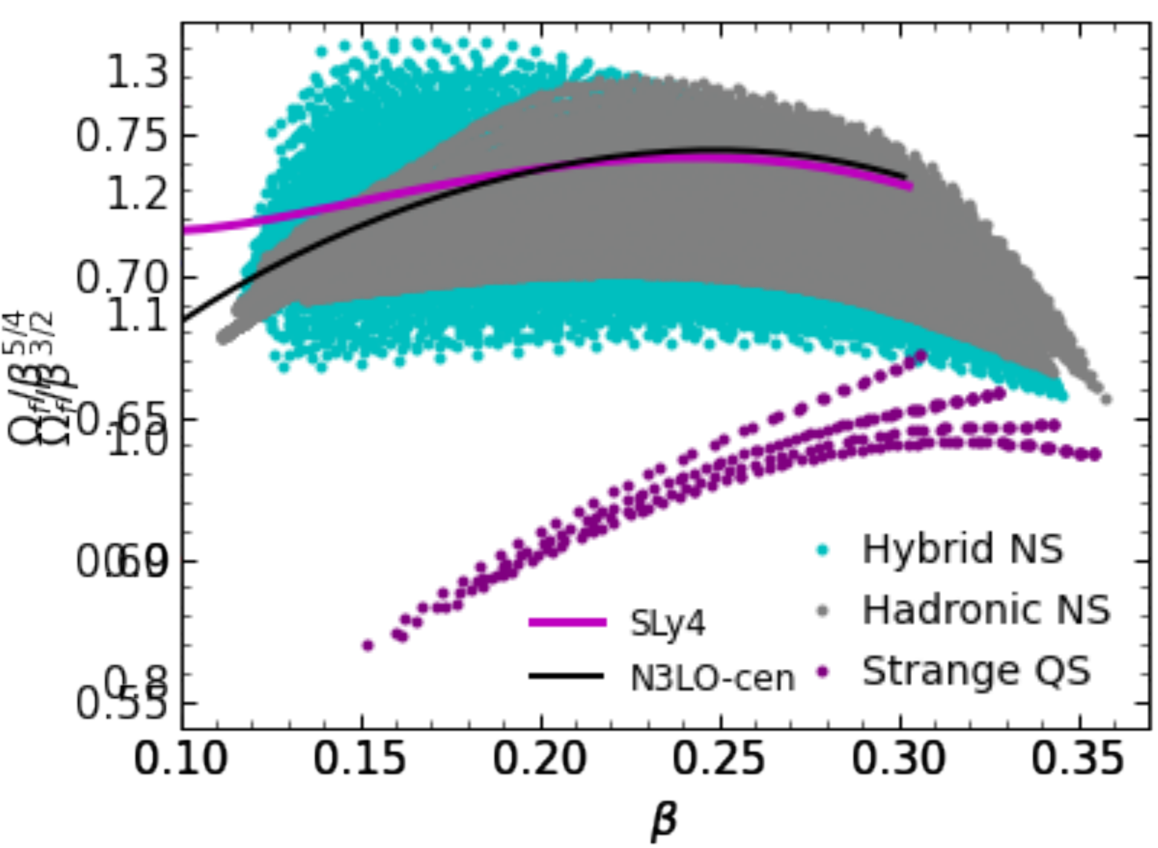


First proposed Lau+2010

One-node Branch

0.1% deviation except for one-node branches

1% deviation except for one-node branches



$$\Omega_f = GM\omega_f/c^3 \qquad (\propto \beta^{3/2} \text{ in Newtonian})$$

1. $\Omega_f - \Lambda$ **is slightly weaker than** $\Omega_f - \bar{I}$
2. Ω_f **is close related to compactness** β

$$\Omega_f = (0.887 \pm 0.061)\ \beta^{3/2}$$ **Quark star**

$$\Omega_f = (0.714 \pm 0.056)\ \beta^{5/4}$$ **Hadronic & hybrid NS**

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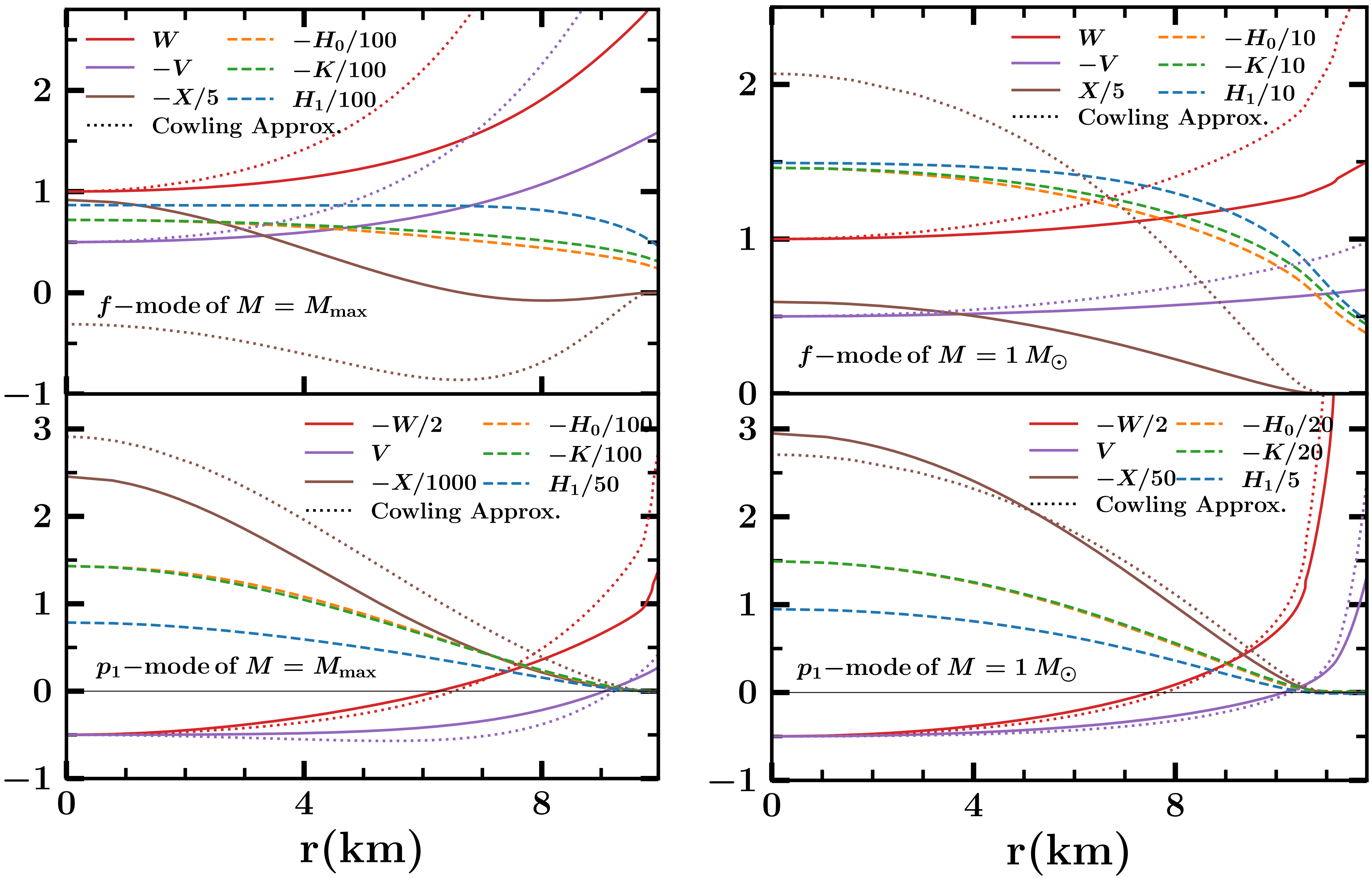
BACK

f-mode vs p-mode oscillations



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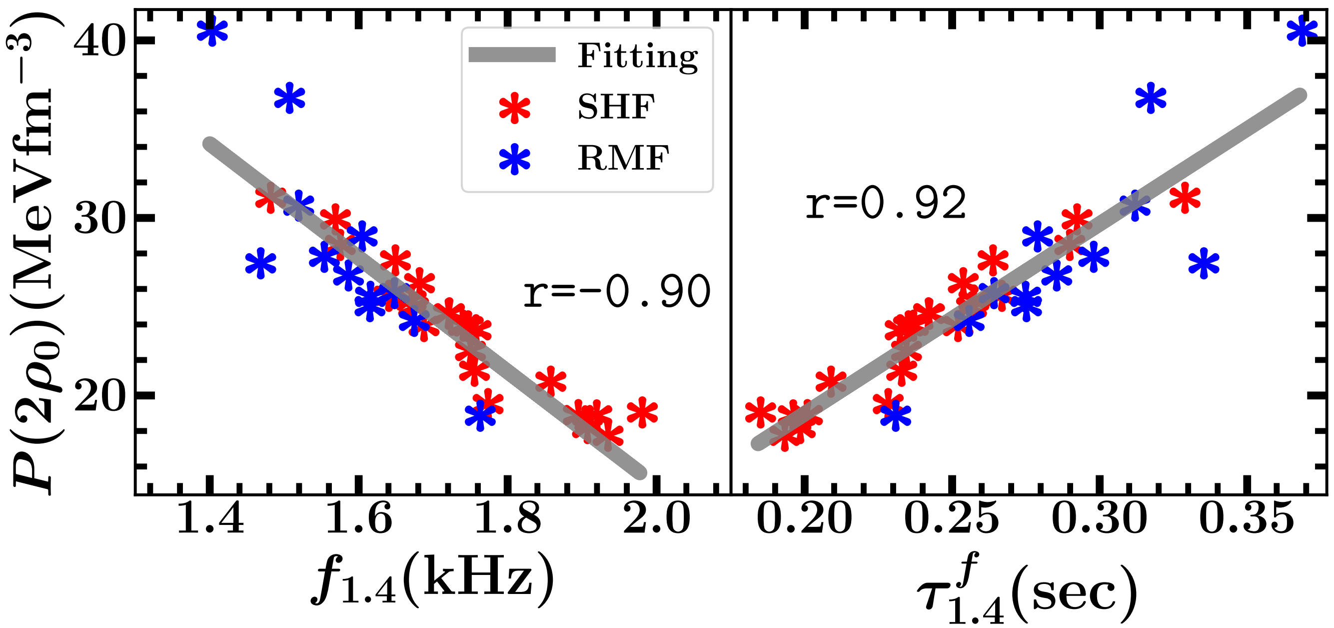
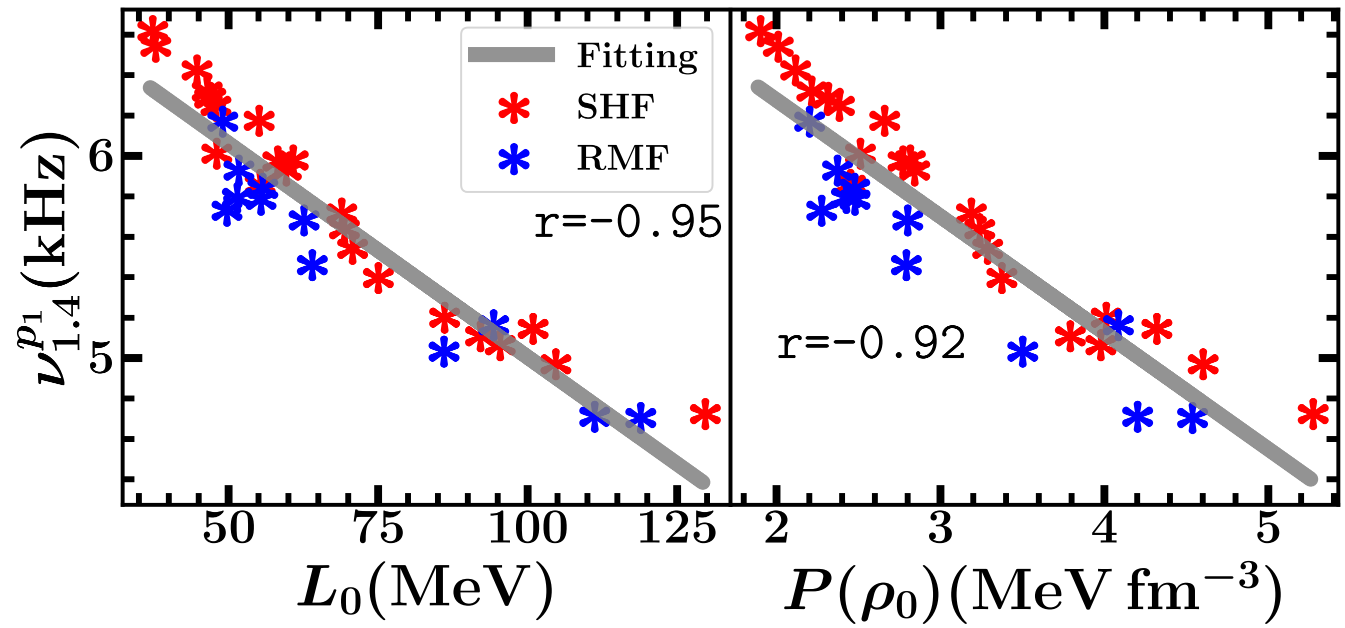
p-modes with SHF and RMF EOSs





$\nu^{p_1}_{1.4}$ is sensitive to EOS around saturation

$f_{1.4}$ and $\tau^f_{1.4}$ are sensitive to EOS around twice saturation density

Higher order p-mode is sensitive to EOS at lower density

	$P(\rho_0)$	$P(1.5\rho_0)$	$P(2\rho_0)$	$P(2.5\rho_0)$	K_0	Q_0	L_0	$K_{sym,0}$	$R_{1.4}$	$\Lambda_{1.4}$
$\nu^{p_1}_{1.4}$	-0.92	-0.85	-0.61	-0.42	-0.48	-0.16	-0.95	-0.57	-0.82	-0.76
$\tau^{p_1}_{1.4}$	-0.31	0.09	0.56	0.72	0.24	0.73	-0.25	0.19	0.33	0.41
$\tau^f_{1.4}$	0.67	0.87	0.92	0.84	0.64	0.53	0.73	0.76	0.97	1.0
$f_{1.2}$	-0.72	-0.89	-0.88	-0.76	-0.62	-0.45	-0.77	-0.77	-0.98	-0.96
$f_{1.4}$	-0.63	-0.85	-0.9	-0.81	-0.62	-0.52	-0.69	-0.75	-0.97	-0.97
$f_{1.6}$	-0.54	-0.8	-0.9	-0.84	-0.61	-0.58	-0.61	-0.72	-0.94	-0.96
$f_{1.8}$	-0.45	-0.73	-0.9	-0.87	-0.6	-0.64	-0.52	-0.67	-0.91	-0.93

1.00 0.75 0.50 0.25 0.00 -0.25 -0.50 -0.75 -1.00

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Oscillation modes

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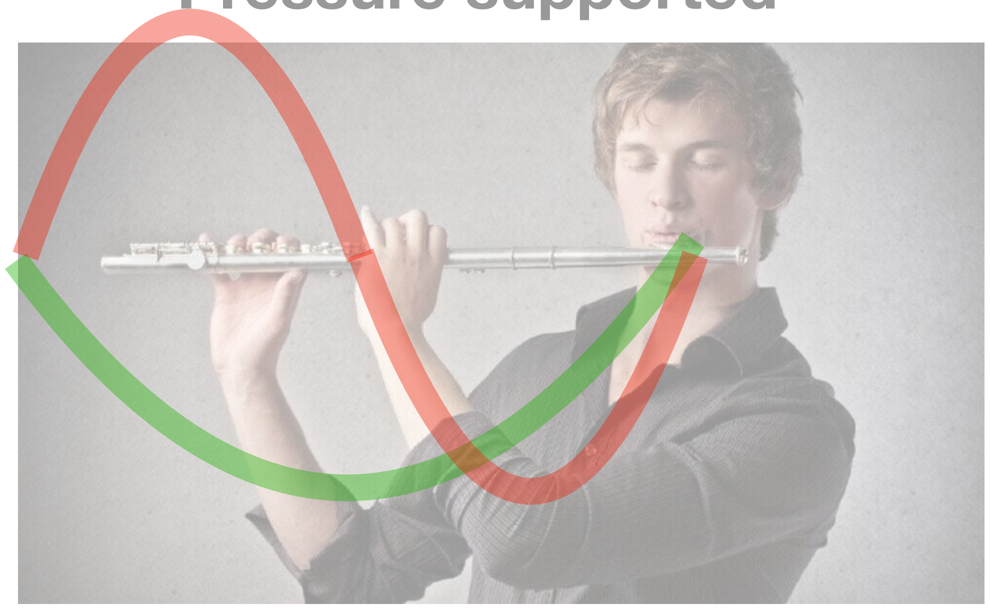
Pressure supported



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n=0 f-mode (fundamental)

n=1 p-mode (pressure)

Gravity & interface

Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

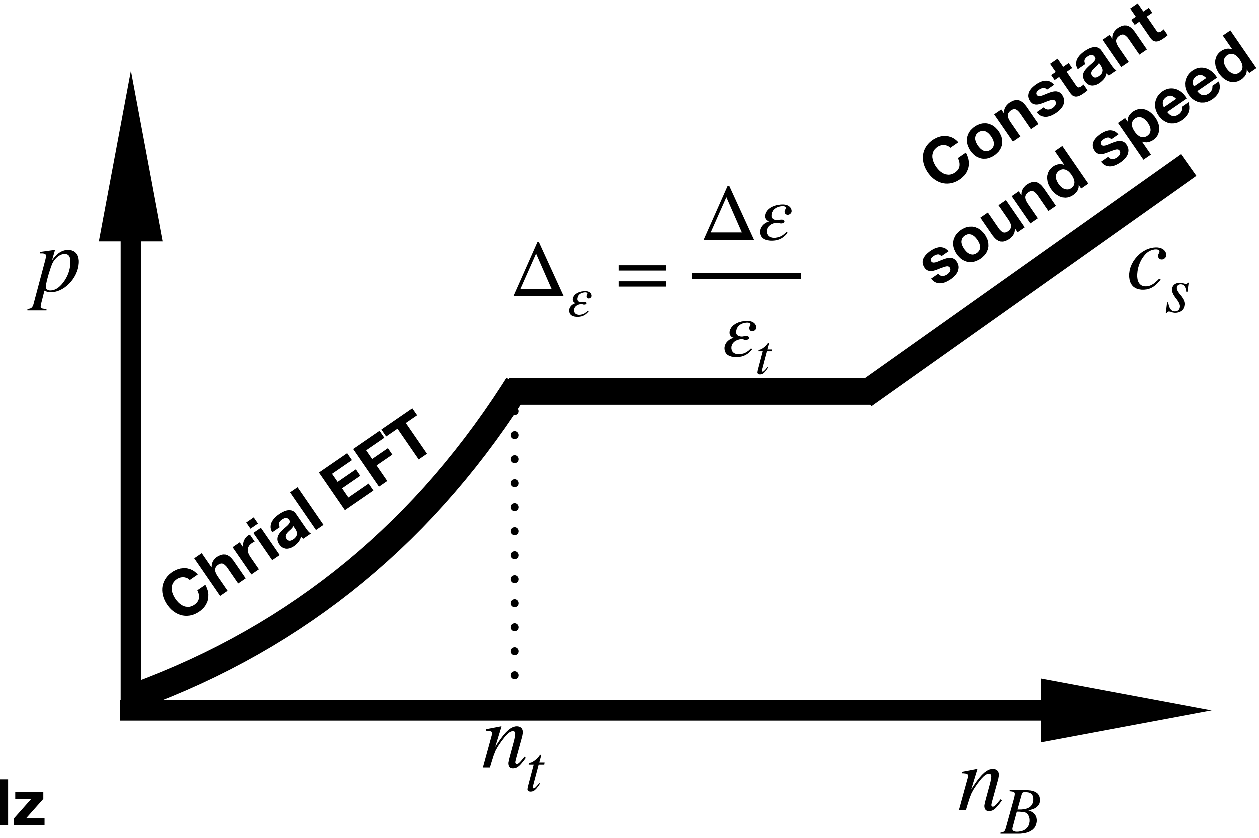
Discontinuity g-mode (interface gravity mode)

Discontinuity g-mode

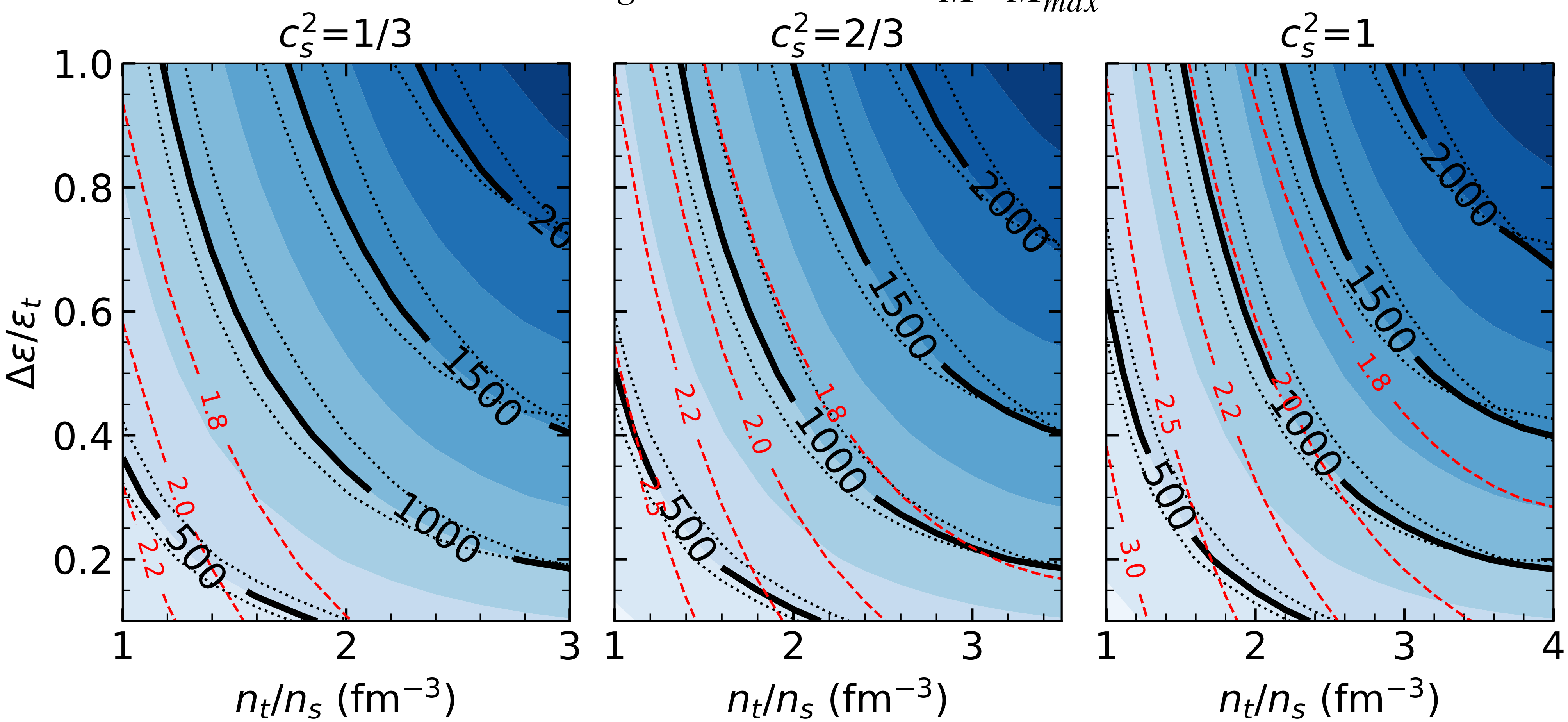
First order transition

$$\Omega_g^2 \approx \frac{\beta^3 (M_t/M)(R/R_t)^3 (\Delta\varepsilon/\varepsilon_t) D \tanh[D]}{1 + \Delta\varepsilon/\varepsilon_t + \tanh[D]/\tanh[D(R/R_t - 1)]}$$

Ω_g is sensitive to structure factors, $\frac{R_t}{R}, \frac{M_t}{M}, \frac{\Delta\varepsilon}{\varepsilon}$



Contour of $\nu_g(n_t, \Delta_\varepsilon, c_s^2)|_{M=M_{max}}$ **Hz**



Dotted: Chrial EFT Uncertanty 5%

Dashed: maximum mass

Frequency: $\nu_g < 0.8\ (1.5)$ kHz for $c_s^2 = c^2/3\ (c^2)$

Damping time: $\tau > 100\ (10000)$ s

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Oscillation modes

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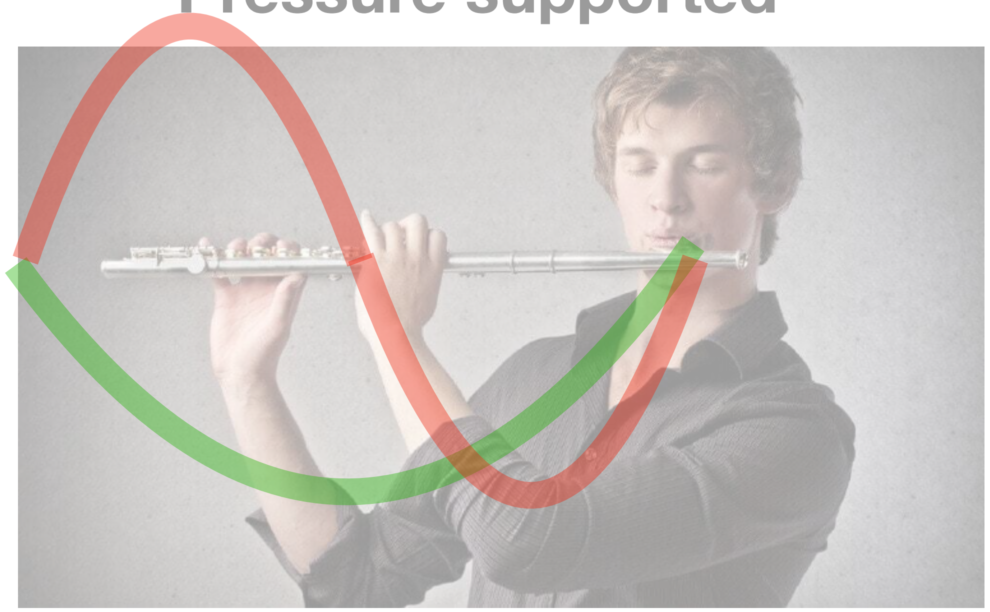
Pressure supported



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n=0 f-mode (fundamental)

n=1 p-mode (pressure)

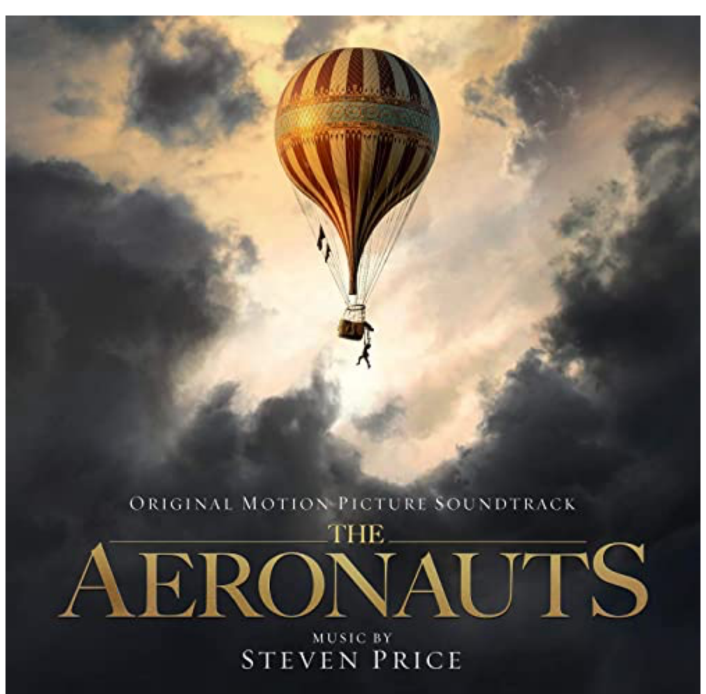
Gravity & interface

Stratified fluid in uniform gravity g:

$$\omega^2 = \frac{(\varepsilon_+ - \varepsilon_-)gk}{\varepsilon_+/\tanh(kd_+) + \varepsilon_-/\tanh(kd_-)}$$

Discontinuity g-mode (interface gravity mode)

Gravity & x gradient



Buoyancy oscillation in uniform gravity g: $\{x\} = \{\frac{n_p}{n_B}, \frac{n_q}{n_B}, S, \ldots\}$

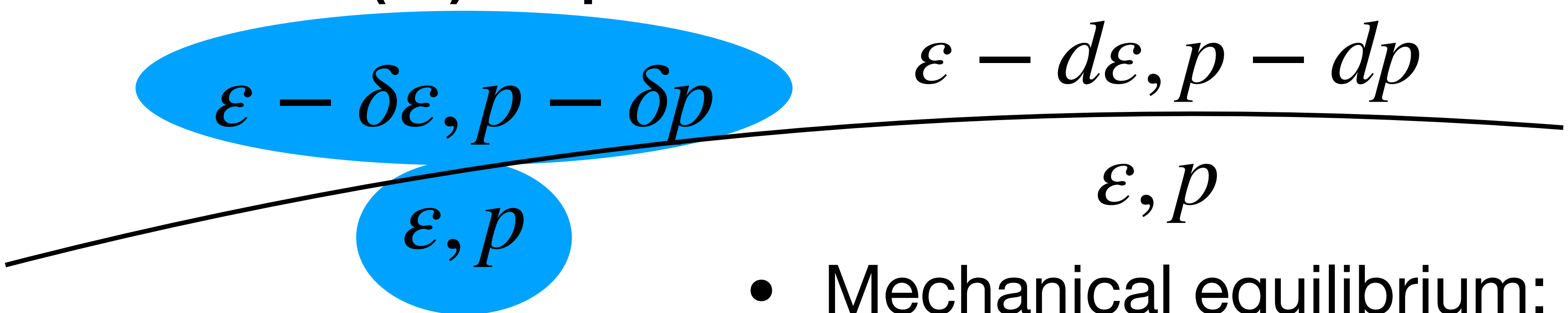
$$\omega^2 \approx \mathcal{N}_{BV}^2 = -\frac{g}{\varepsilon}\frac{d\varepsilon - \delta\varepsilon}{dr} = -\frac{g}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial x}\right)_p \frac{dx}{dr} = g^2\left(\frac{1}{c_{eq}^2} - \frac{1}{c_{ad}^2}\right)$$

Chemical g-mode (gravity with composition gradient)

g-mode

Adiabatic (de)compression:

Ambient environment:

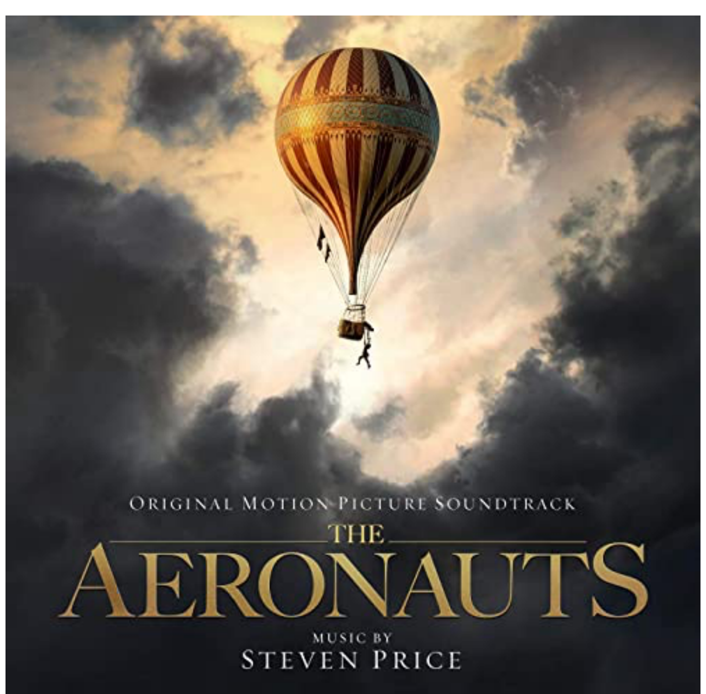


Gravity field $\vec{g}$

- Mechanical equilibrium: $p - \delta p = p - dp$
- Gravity as recovering: $\varepsilon - \delta\varepsilon > \varepsilon - d\varepsilon$
- Non-convecting(stable): $c_{ad}^2 = \frac{\delta p}{\delta \varepsilon} > \frac{dp}{d\varepsilon} = c_{eq}^2$

$$c_{eq}^2 = \frac{dp}{dr}\left[\frac{d\varepsilon}{dr}\right]^{-1} \qquad c_{ad}^2 = \left[\frac{\partial p}{\partial n}\right]_{\{x\}}\left[\frac{\partial \varepsilon}{\partial n}\right]_{\{x\}}^{-1}$$

Gravity & x gradient



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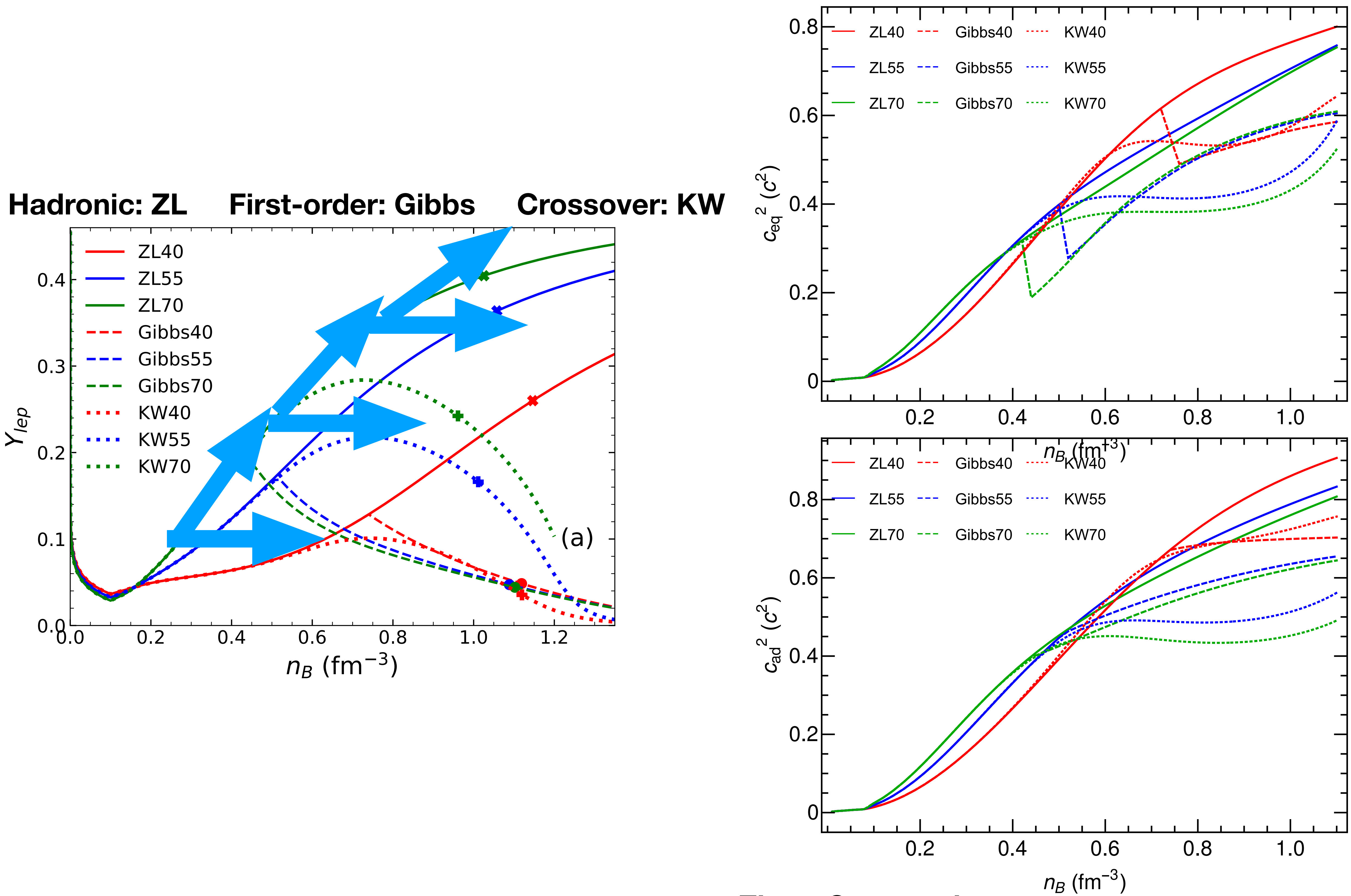
Chemical g-mode (gravity with composition gradient)

Compositional g-mode universal relation

Hadronic: ZL First-order: Gibbs Crossover: KW



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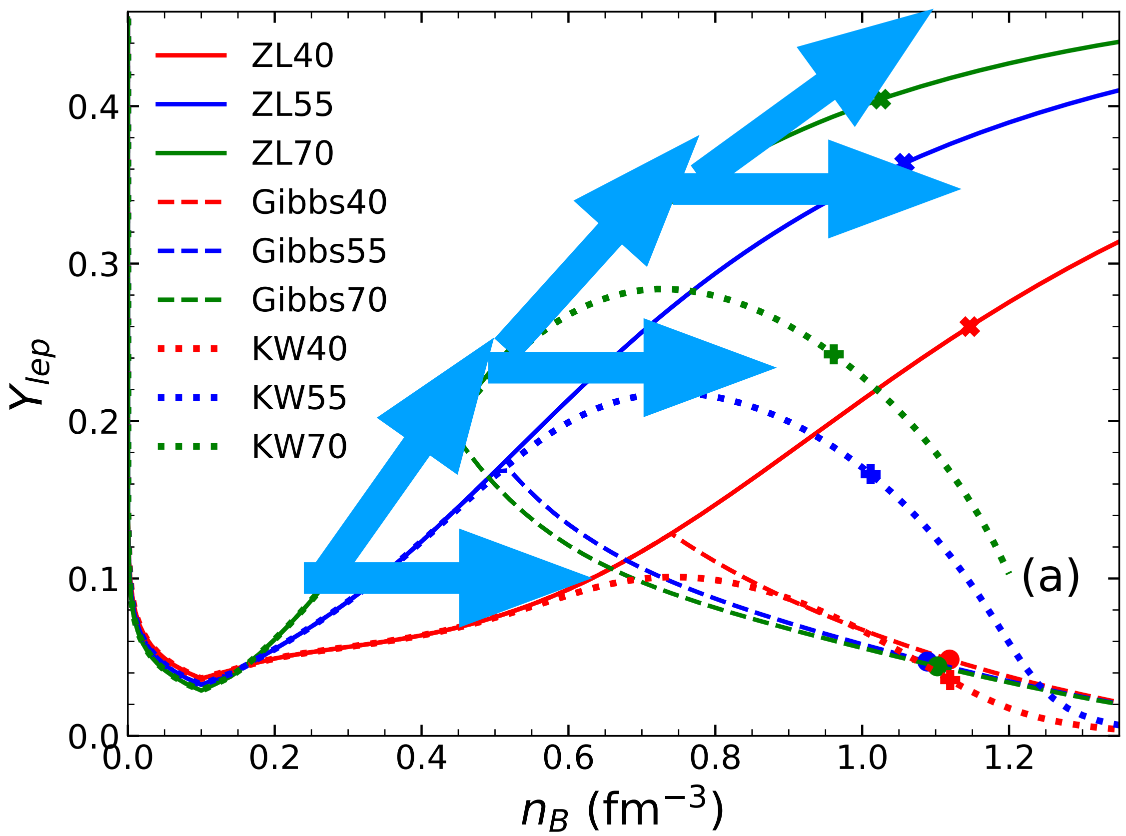
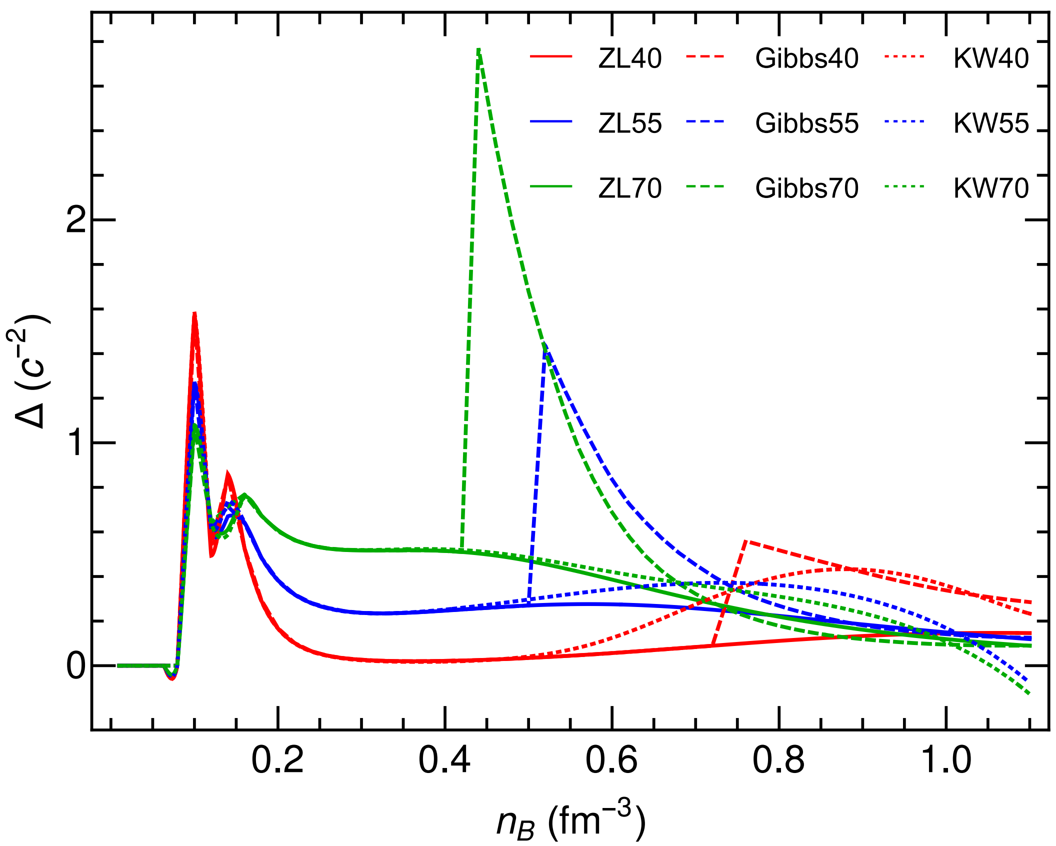
Compositional g-mode universal relation

$$\mathcal{N}_{BV}^2 = g^2 e^{\nu-\lambda}\Delta(c^{-2}) \qquad \Delta(c^{-2}) = \frac{1}{c_{ad}^2} - \frac{1}{c_{eq}^2}$$

Hadronic: ZL First-order: Gibbs Crossover: KW





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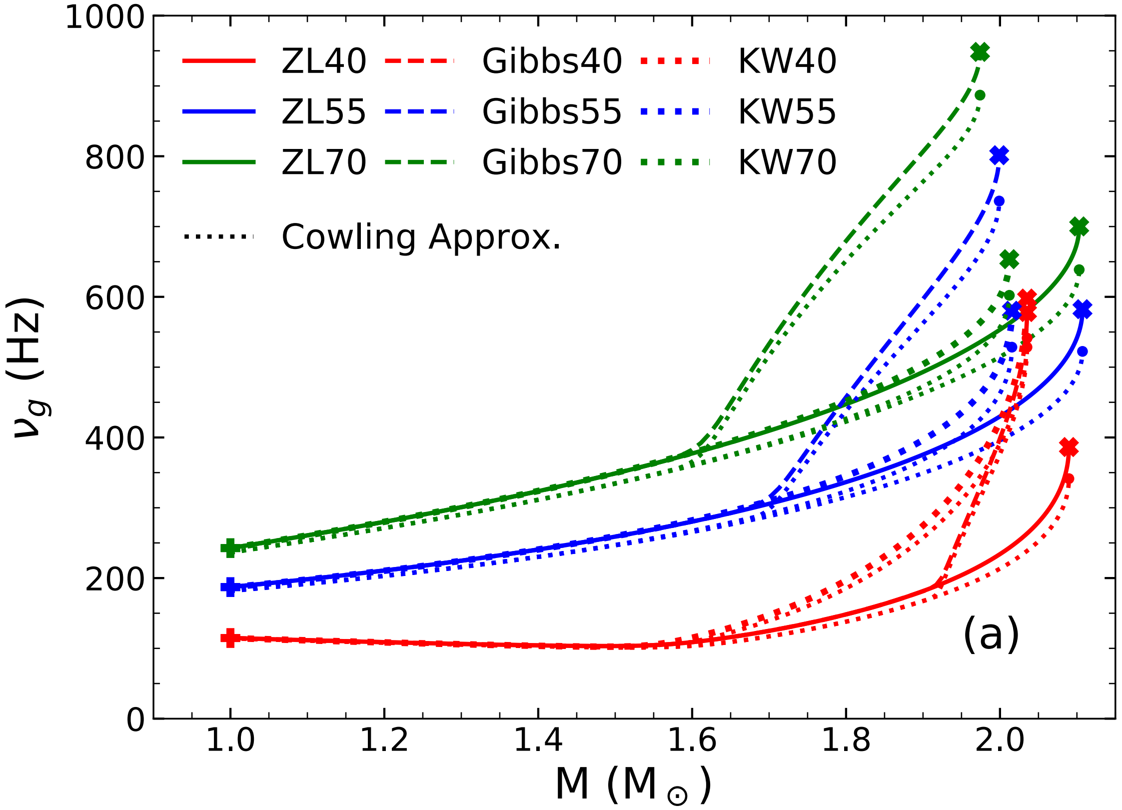
Compositional g-mode universal relation

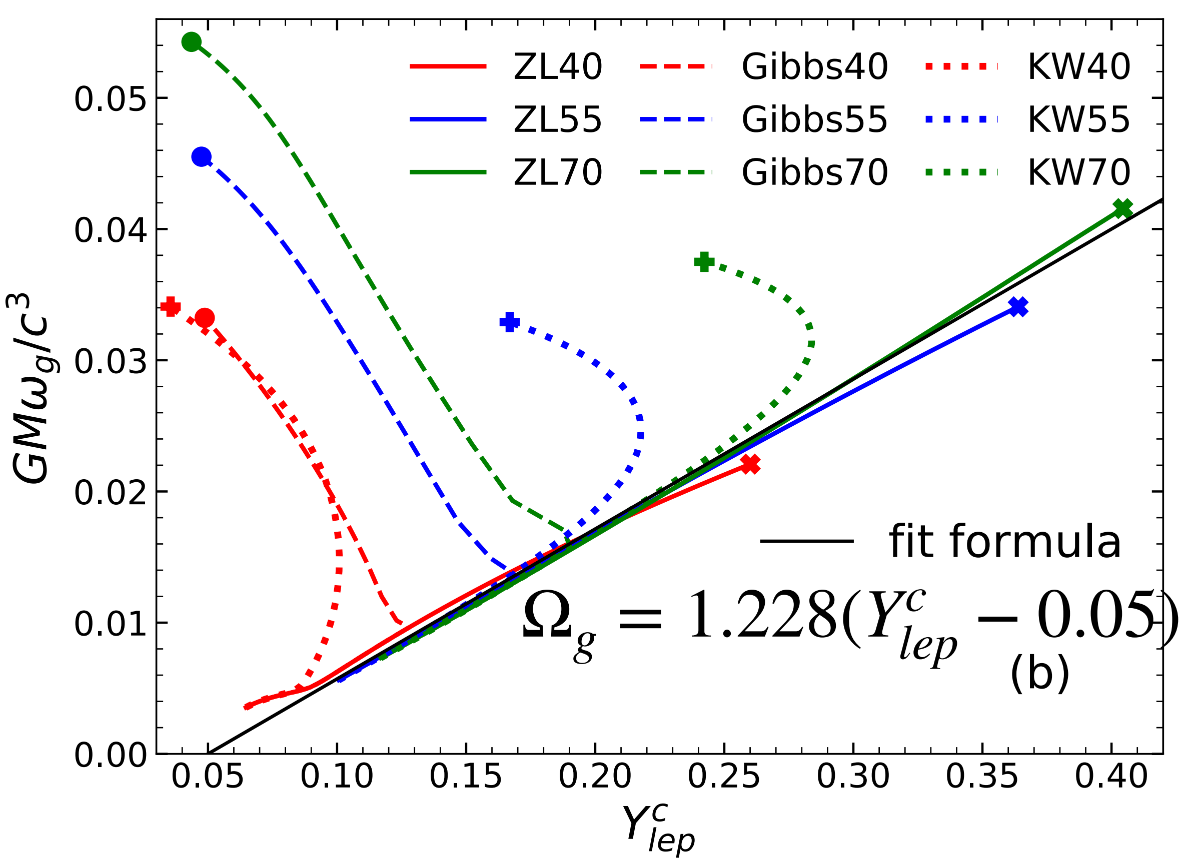
Hadronic: ZL   First-order: Gibbs   Crossover: KW

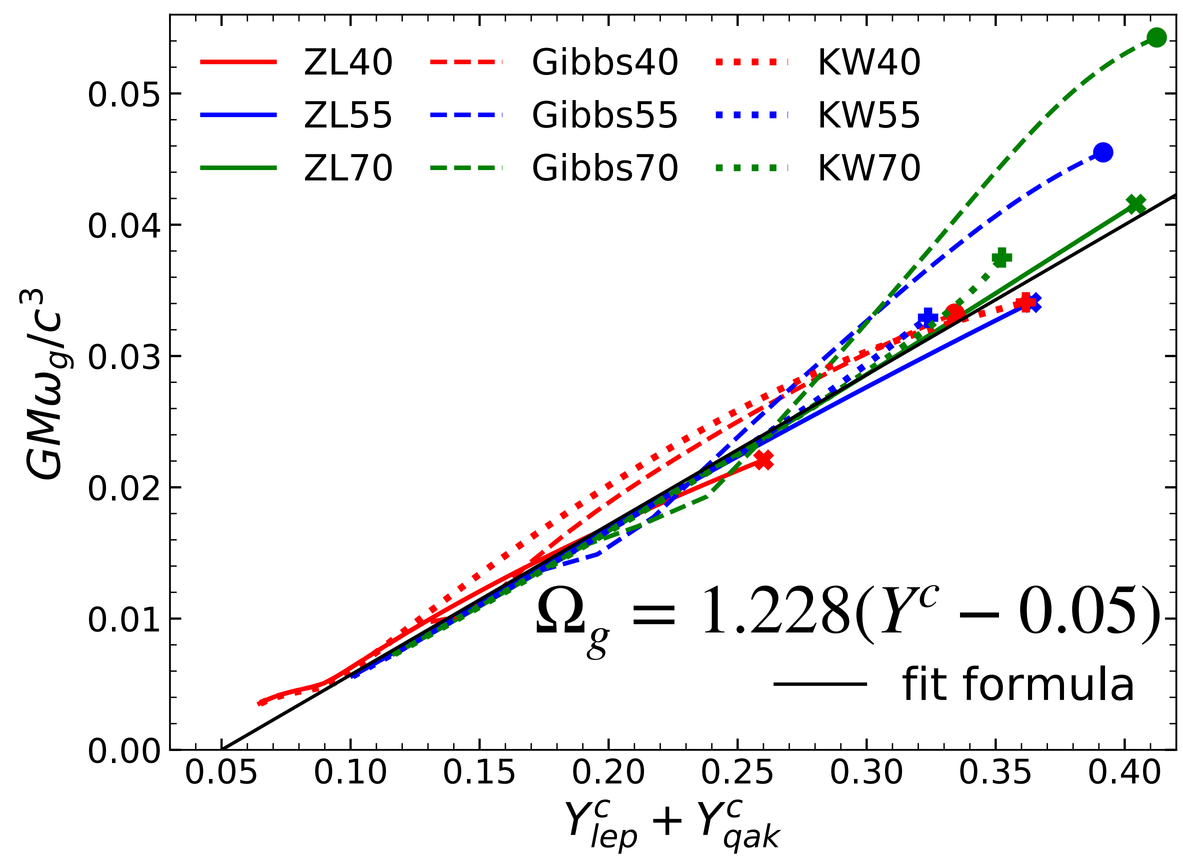


$$\mathcal{N}_{BV}^2 = g^2 e^{\nu-\lambda} \Delta(c^{-2}) \qquad \Delta(c^{-2}) = \frac{1}{c_{ad}^2} - \frac{1}{c_{eq}^2}$$

is sensitive to:

1. **Symmetry energy S(n)**
2. **Number of particle species**
3. **Proto-NS neutrino emission**
4. **Bulk viscosity**





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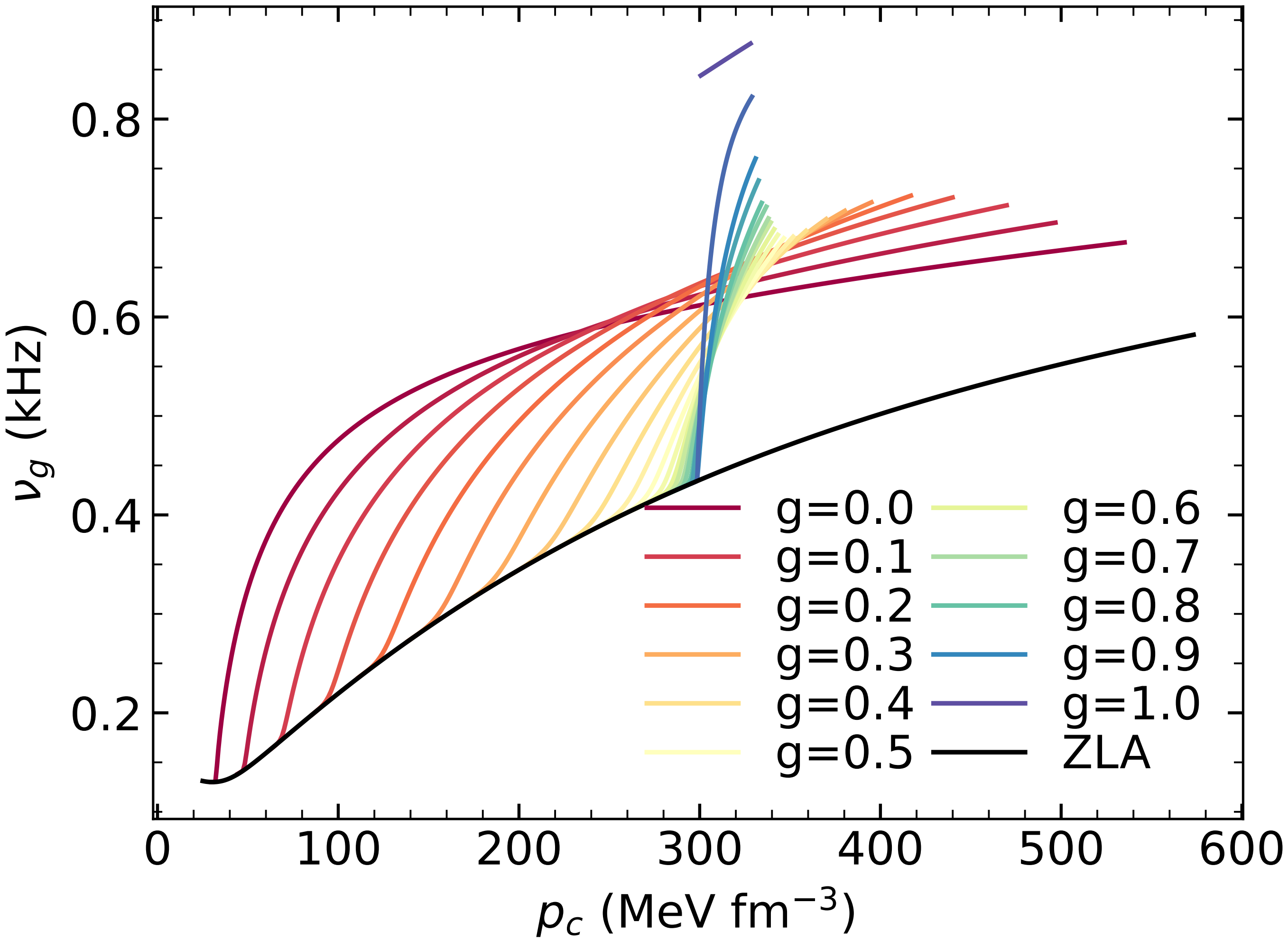
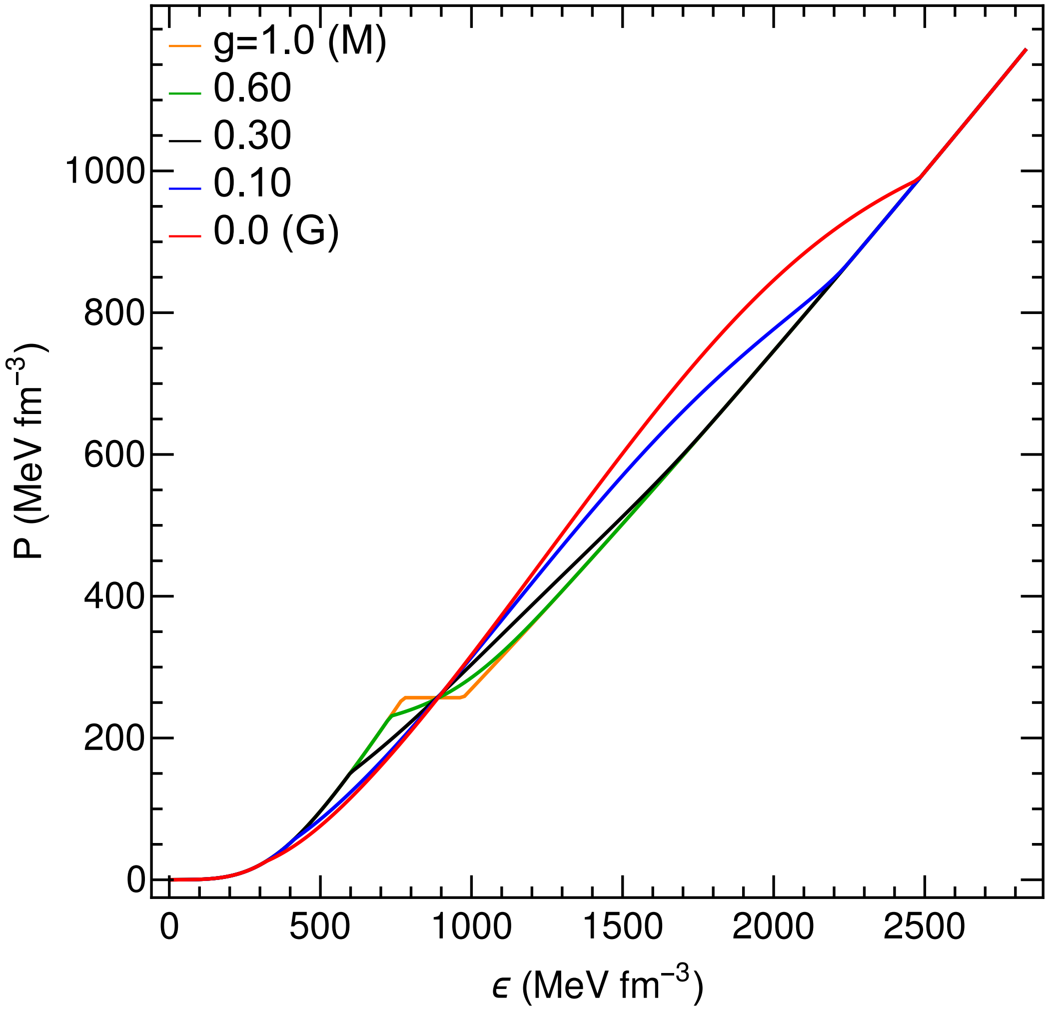
Discontinuity g-mode as special Compositional g-mode

- Quark-hadron surface tension increases with g: g=0 (Gibbs transition with continuous transition) g=1 (Maxwell transition with density discontinuity)





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Between Maxwell & Gibbs

Extend to finite temperature

- Relativistic Fermi integrals, JEL polynomials.
- Introduce anti-particles as,

$$\mu_{e^-} = -\mu_{e^+}$$

$$\mu_{\mu^-} = -\mu_{\mu^+}$$

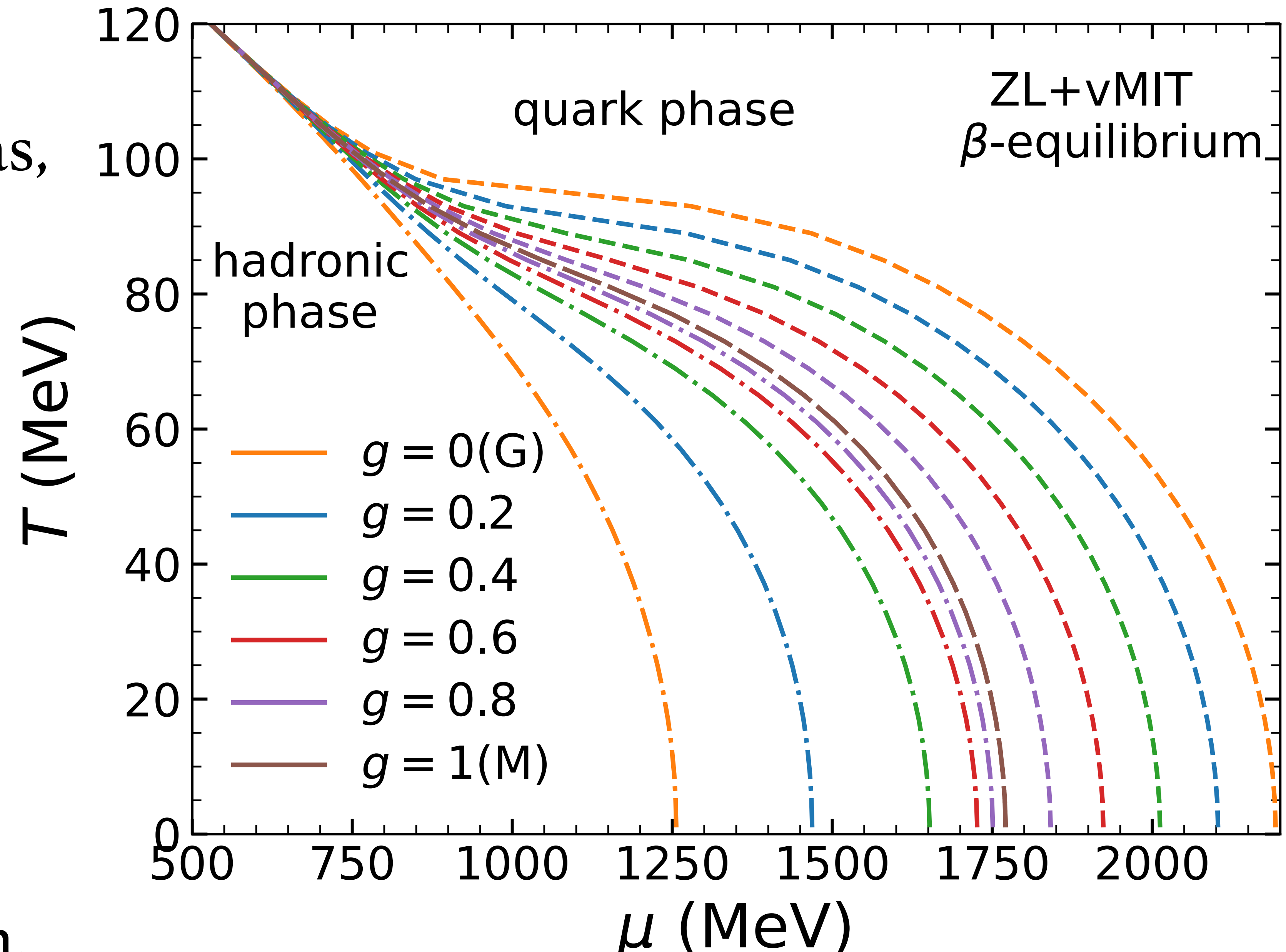
$$\mu_{u^-} = -\mu_{u^+}$$

$$\mu_{d^-} = -\mu_{d^+}$$

$$\mu_{s^-} = -\mu_{s^+}$$

- Add photon contribution,

$$\varepsilon_{photon} \propto T^4$$



Preliminary: arXiv: 25xx.xxxxx

Between Maxwell & Gibbs

Extend to off-β-equilibrium:

- Ignore β-equilibrium condition,

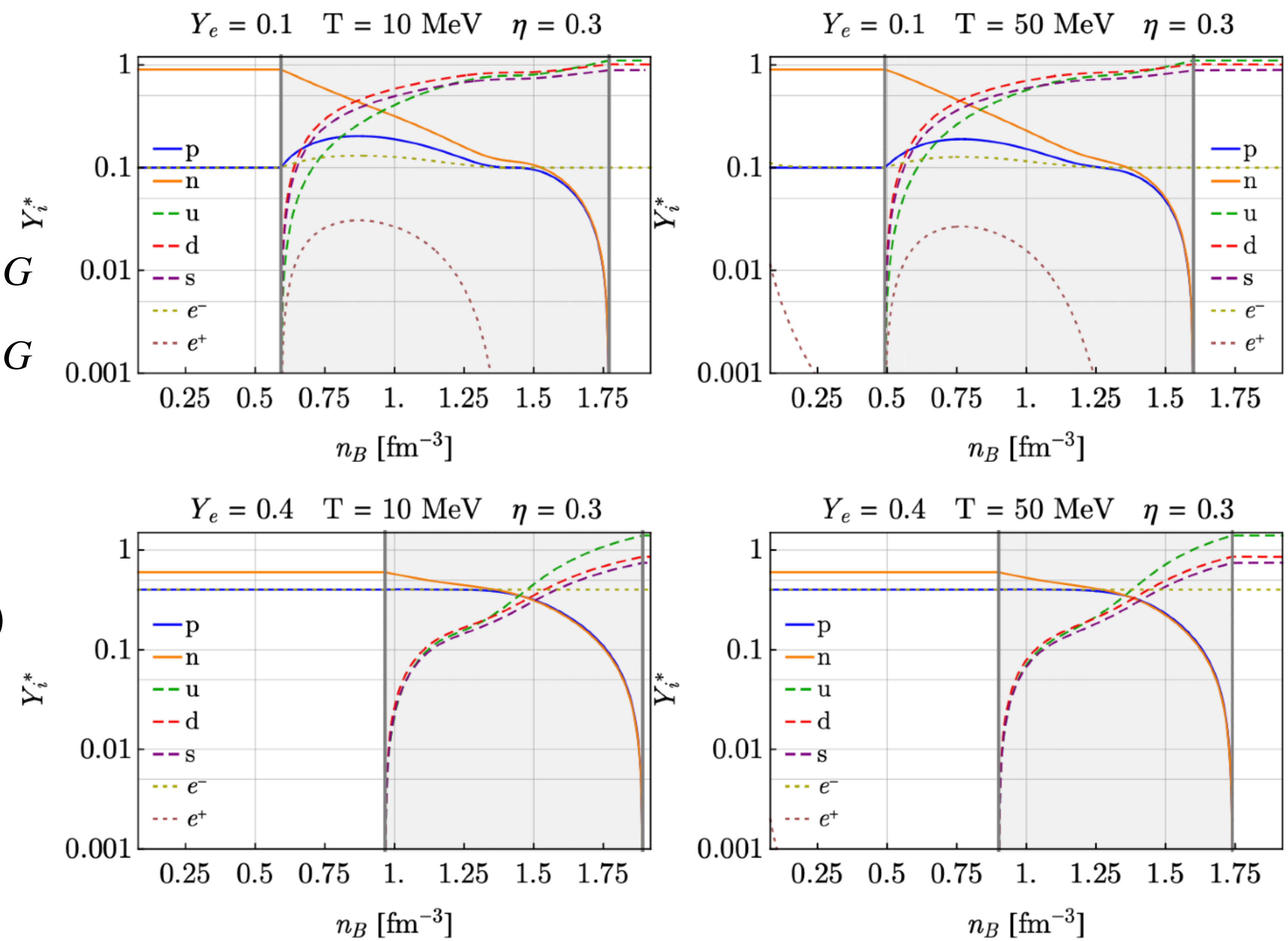
$$\mu_d = \mu_u + g\mu_{e,Q} + (1-g)\mu_{e,G}$$
$$\mu_n = \mu_p + g\mu_{e,N} + (1-g)\mu_{e,G}$$

- And replace it with,

$$n_e = n_B Y_e = (1-g)n_{e,G} + g(f n_{e,N} + (1-f)n_{e,Q})$$

- The final EOS is,

$$\varepsilon = \varepsilon(n_B, Y_e, T)$$
$$p = p(n_B, Y_e, T)$$



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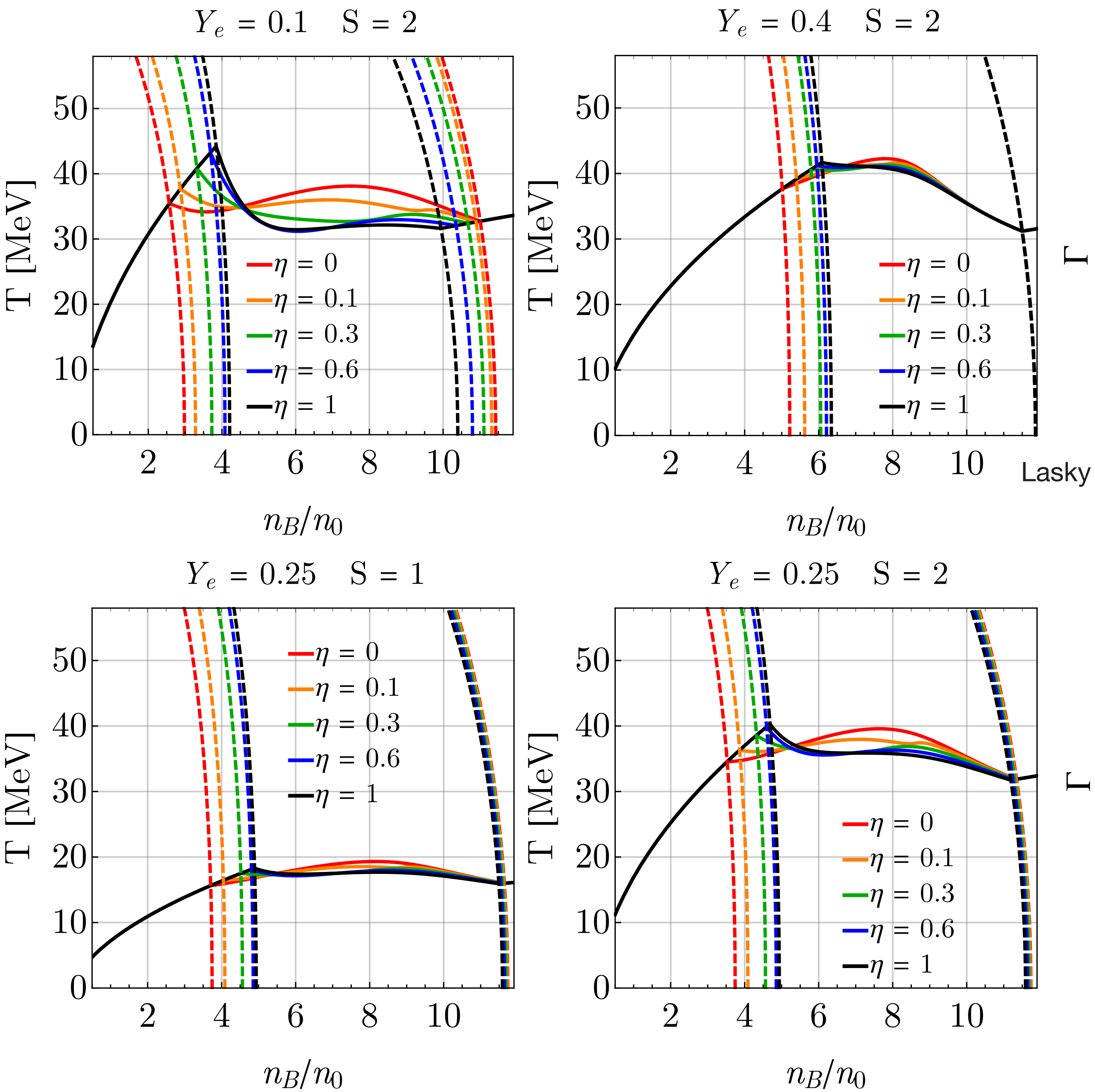
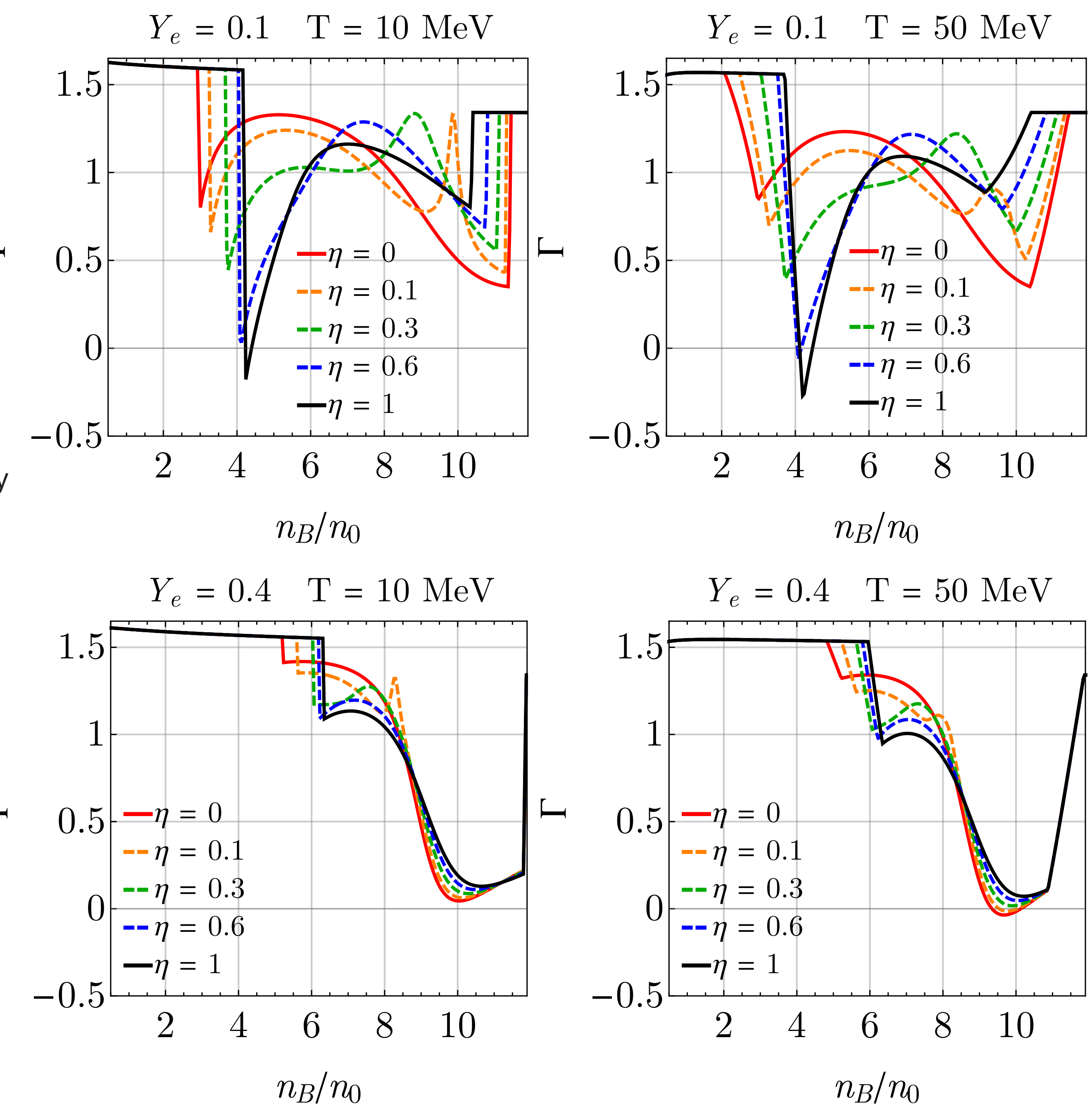
Between Maxwell & Gibbs

Extend to off-β-equilibrium:





Comparable to Kotake's work

Related to Lasky's comments

Preliminary: arXiv: 25xx.xxxxx

Dissipation from Bulk viscosity

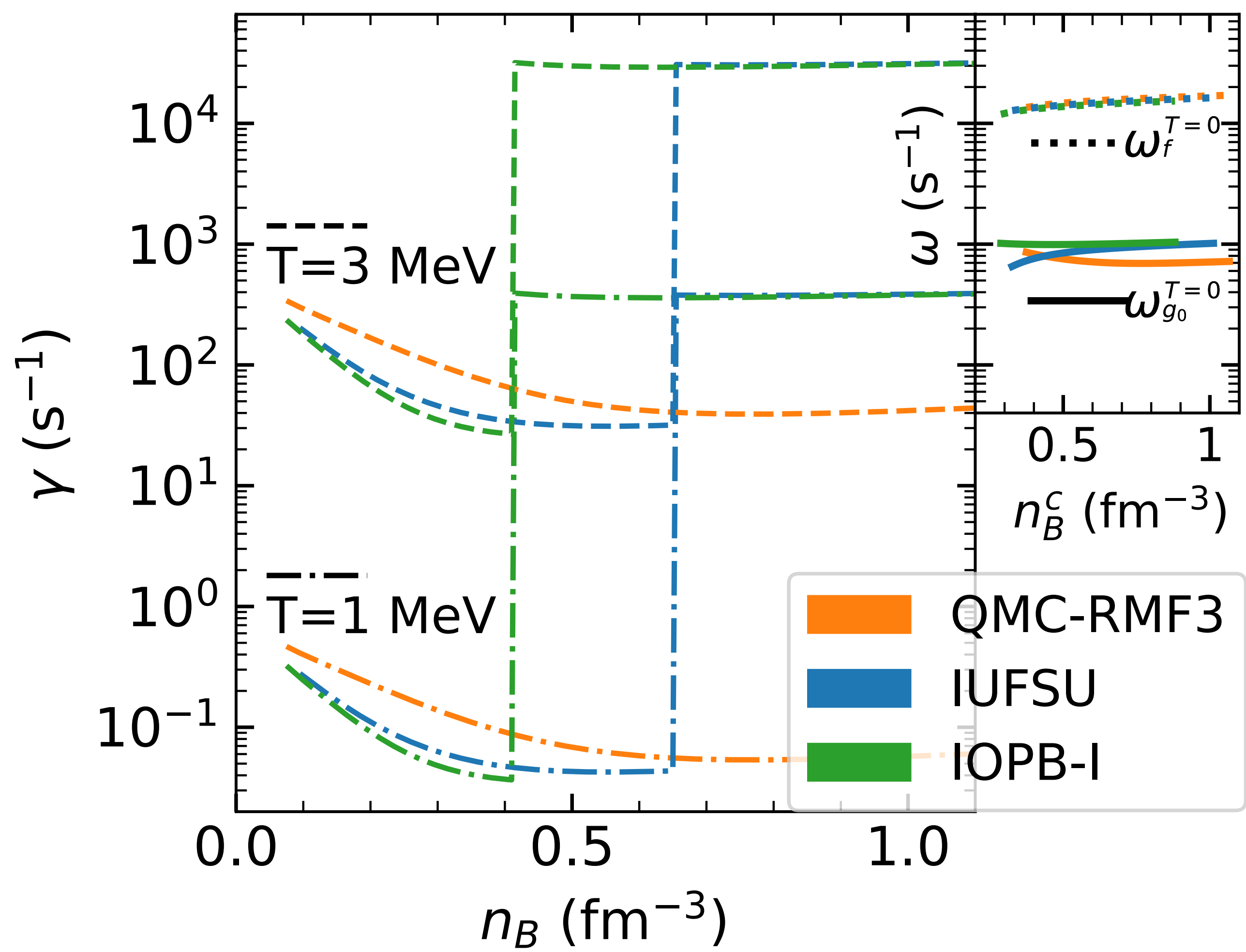
$$\frac{d\varepsilon}{dt} = -\zeta(\nabla \cdot \mathbf{v})^2$$
$$\mathbf{v} = \partial_t \xi$$

- Nucleon weak β-equilibrium at low neutrino opacity,
  $\bar{\nu} + p + e^- \leftarrow n \quad (+N)$
  $(N+)\ p + e^- \rightarrow n + \nu$
  leads to $\delta_\mu = \mu_n - \mu_p - \mu_e = 0$, and $x = \frac{n_p}{n_B} = x_{eq}$.

Zhao, Rau, Haber, Harris, Constantinou, Han 2025

- off β-equilibrium, when $x_\delta = x - x_{eq}$,
  $\partial_t x_\delta = \frac{\lambda \delta_\mu}{n_B}$,
  $$\lambda(T, n_B) = \left(\frac{\partial(\Gamma_{n\rightarrow p} - \Gamma_{p\rightarrow n})}{\partial \delta_\mu} \right)_{\delta_\mu = 0}$$

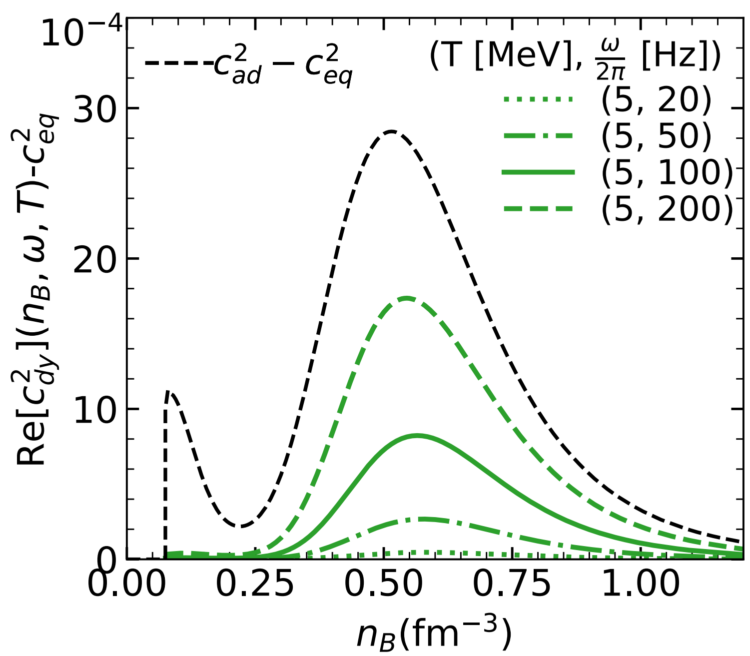
  which can be linearized to:
  $\partial_t x_\delta = -\gamma x_\delta, \gamma = -\frac{\lambda}{n_B}\frac{\partial \delta_\mu}{\partial x_\delta}|_{n_B,S}$
  γ represent impact of bulk viscosity.

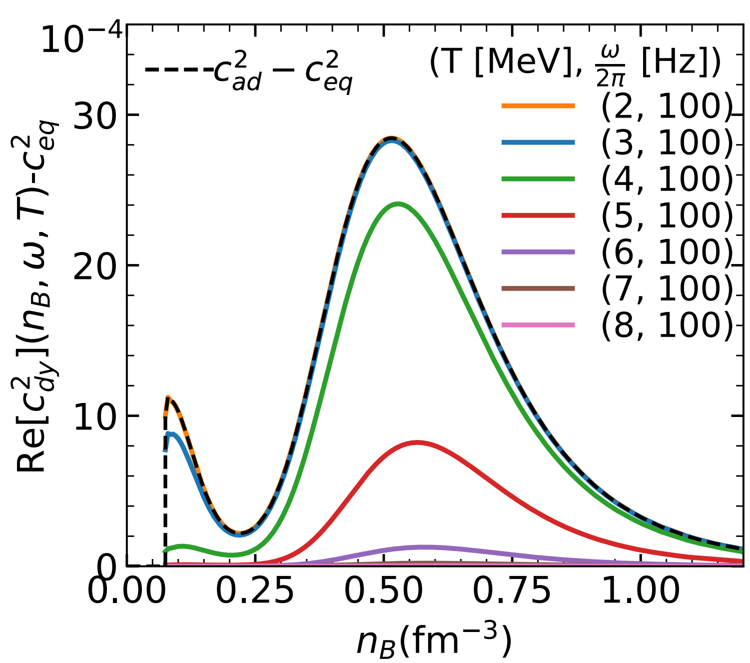


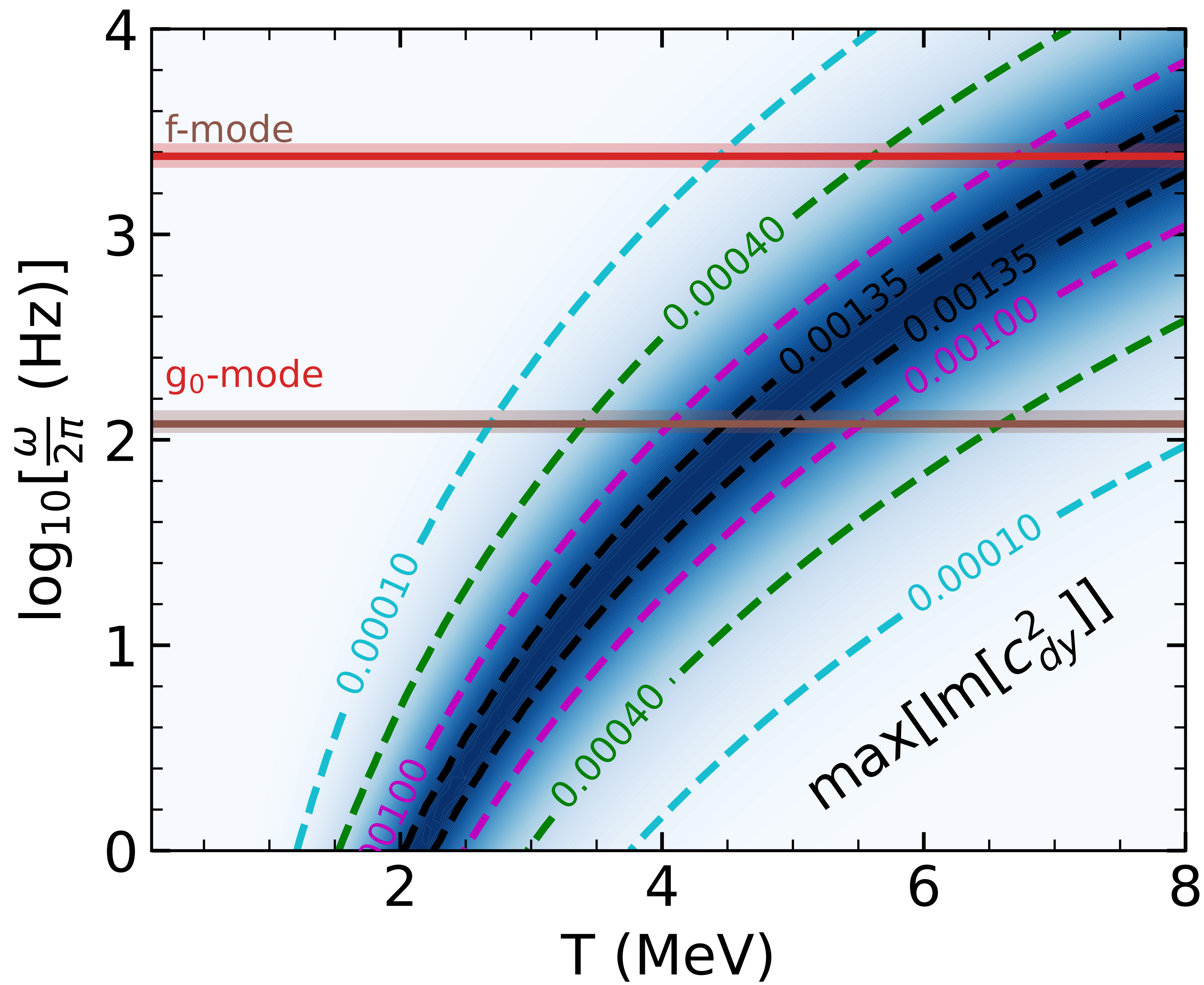
Dynamic sound speed

$$c_{dy}^2 = c_{eq}^2 + \frac{c_{ad}^2 - c_{eq}^2}{1 - i\frac{\gamma}{\omega}}$$

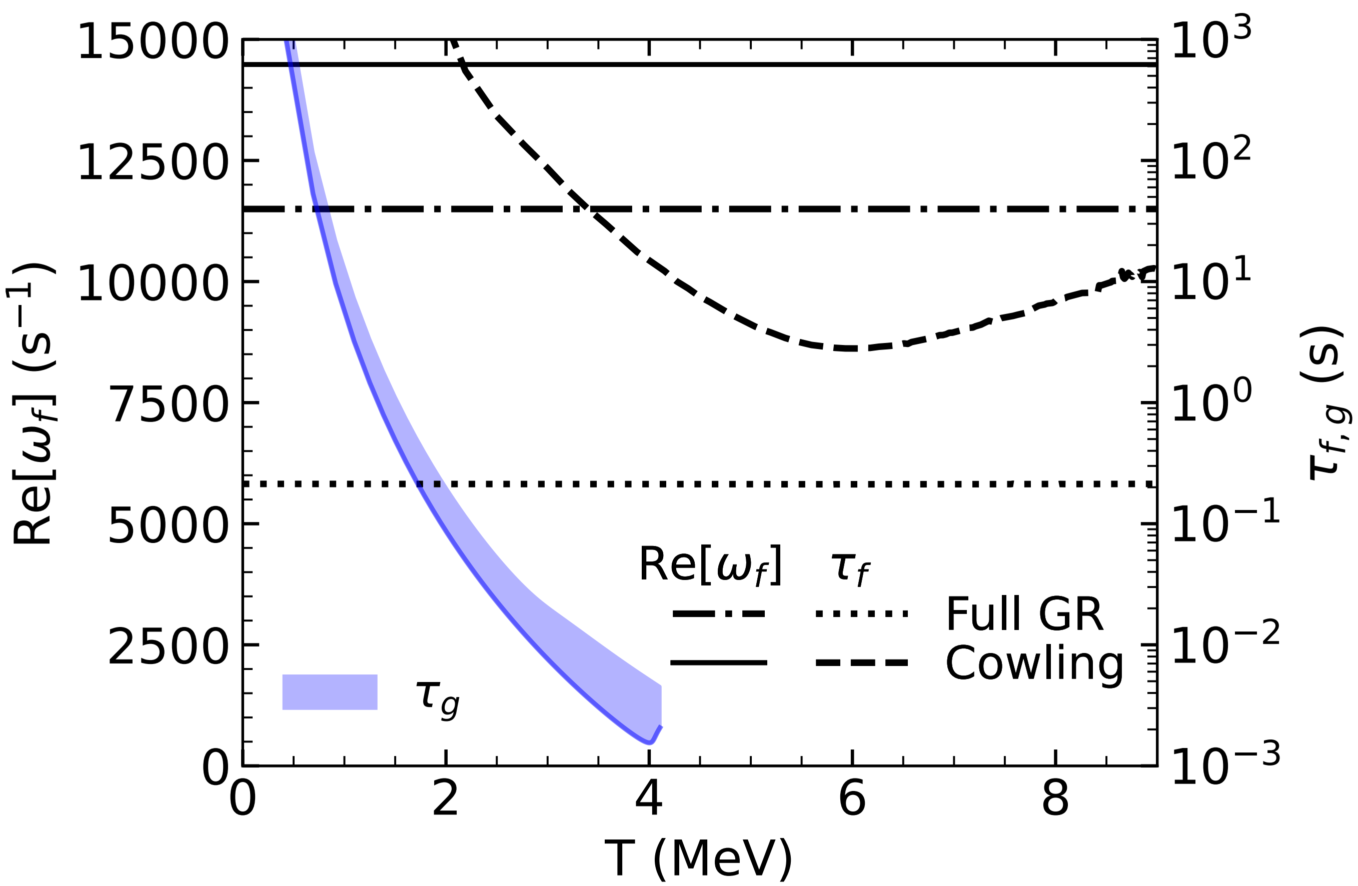
$$\mathrm{Re}\left[c_{dy}^2\right] = c_{eq}^2 + \left(c_{ad}^2 - c_{eq}^2\right)\frac{\omega^2}{\omega^2+\gamma^2} \qquad \mathrm{Im}\left[c_{dy}^2\right] = \left(c_{ad}^2 - c_{eq}^2\right)\frac{\omega\gamma}{\omega^2+\gamma^2} = \frac{\omega\zeta}{\varepsilon+p}$$

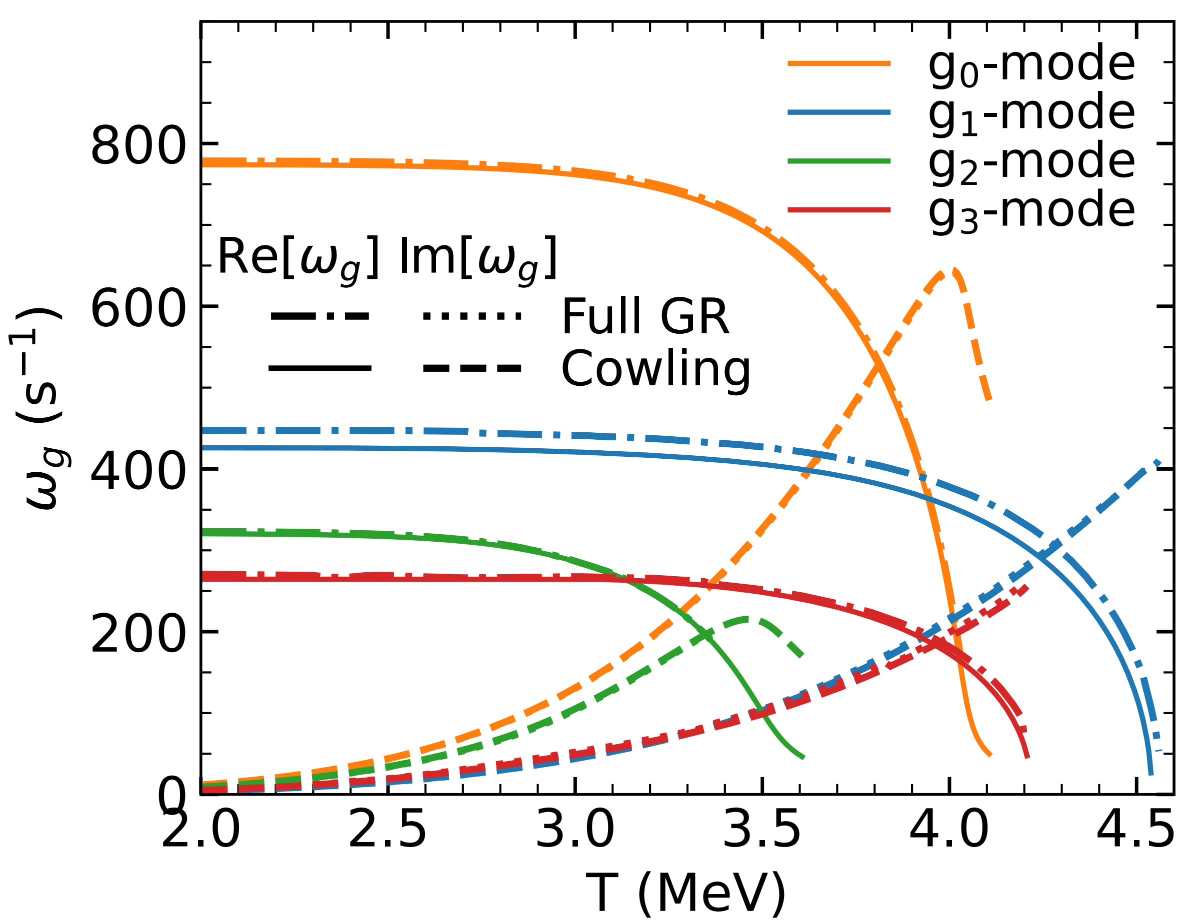






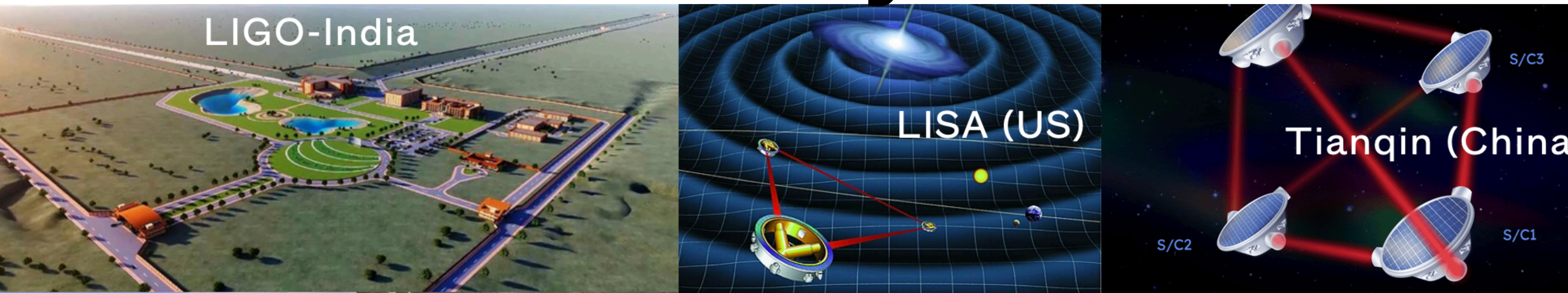
Impact of bulk viscosity





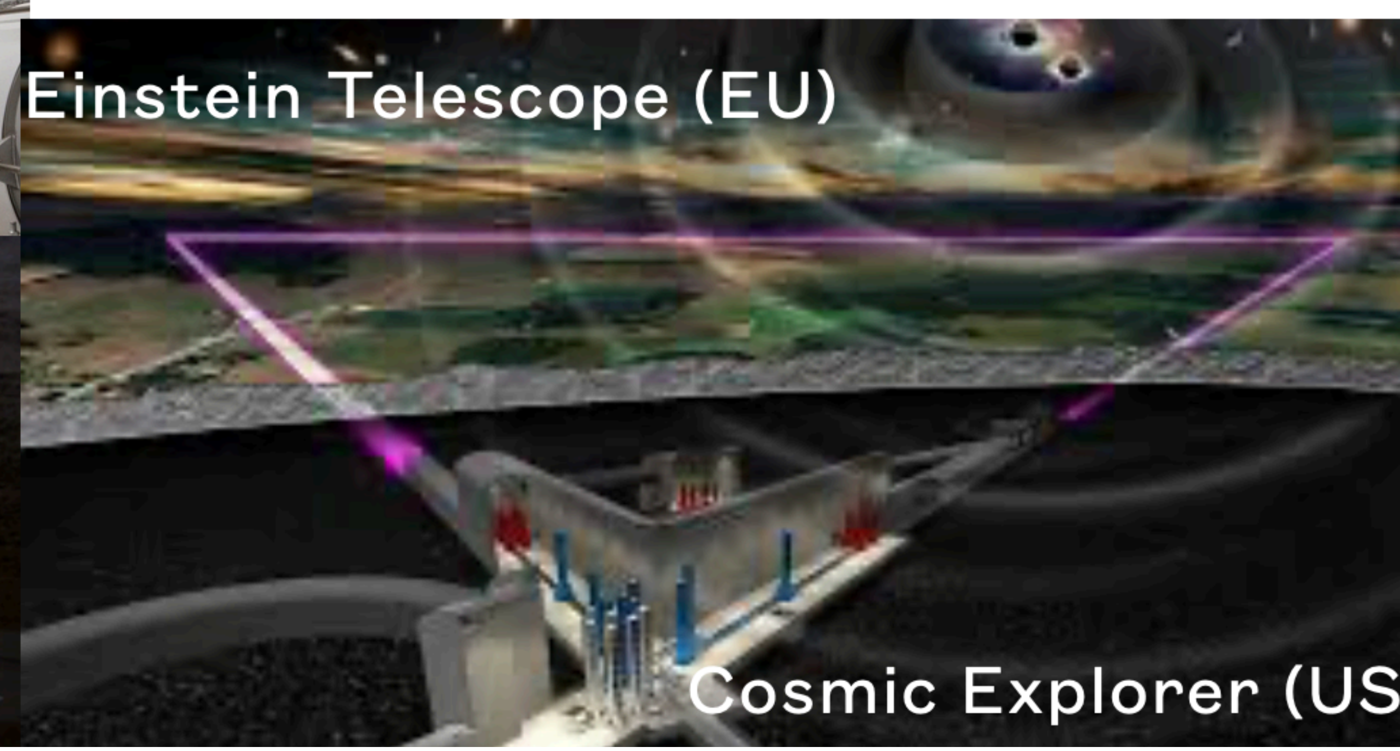
- No impact to f-mode. f-mode is semi divergence-free.
- g-mode gets damped at larger T. Avoided crossing between g-modes.

Thank you!



Gravitational wave observatory:

Binary inspiraling, Remnant ringing down, Non-radial Oscillation modes, ...



Tianqi Zhao Apr 23, 2025

Reference

- Universal relations for neutron star -mode and -mode oscillations
  T Zhao, JM Lattimer Physical Review D 106 (12), 123002 [2022]
- Impact of the equation of state on - and - mode oscillations of neutron stars
  A Kunjipurayil, T Zhao, B Kumar, BK Agrawal, M Prakash
  Physical Review D 106 (6), 063005 [2022]
- Quasinormal g-modes of neutron stars with quarks
  T Zhao, C Constantinou, P Jaikumar, M Prakash
  Physical Review D 105 (10), 103025 [2022]
- Framework for phase transitions between the Maxwell and Gibbs constructions
  C Constantinou, T Zhao, S Han, M Prakash
  Physical Review D 107 (7), 074013 [2023]
- Suppression of composition g-modes in chemically-equilibrating warm neutron stars. T Zhao, P Rau, A Haber, S Harris, C Constantinou, S Han [2025]
  arXiv:2504.12230
- Phase transitions between the Maxwell and Gibbs constructions at finite temperature
  C Constantinou, M Guerrini, T Zhao, S Han, M Prakash [Preliminary: arXiv: 25xx.xxxxx]

BACK

BACK UP SLIDES

ODEs of Non-radial Adiabatic Oscillation

Eigen value problem of even quasi-normal modes

- Linearized Full GR:
  4 1st ODEs (inside) Thorne, Kip S. 1967
  1 2nd ODEs (outside)
  Zerilli's Eq Fackerell, Edward D. 1971
  Lee Lindblom and Steven L. Detweiler 1983

(fluid)
Drop spacetime perturbation
Zhao, Constantinou, Jaikumar, Prakash 2022
https://arxiv.org/abs/2202.01403

(Inverse Cowling)

- Relativistic Cowling approximation:
  2 1st-order ODEs
  or 1 2nd-order ODE
  P. N. McDermott et. al. 1983

Take Newtonian limit for static gravity and perturbation

Take Newtonian limit for static gravity

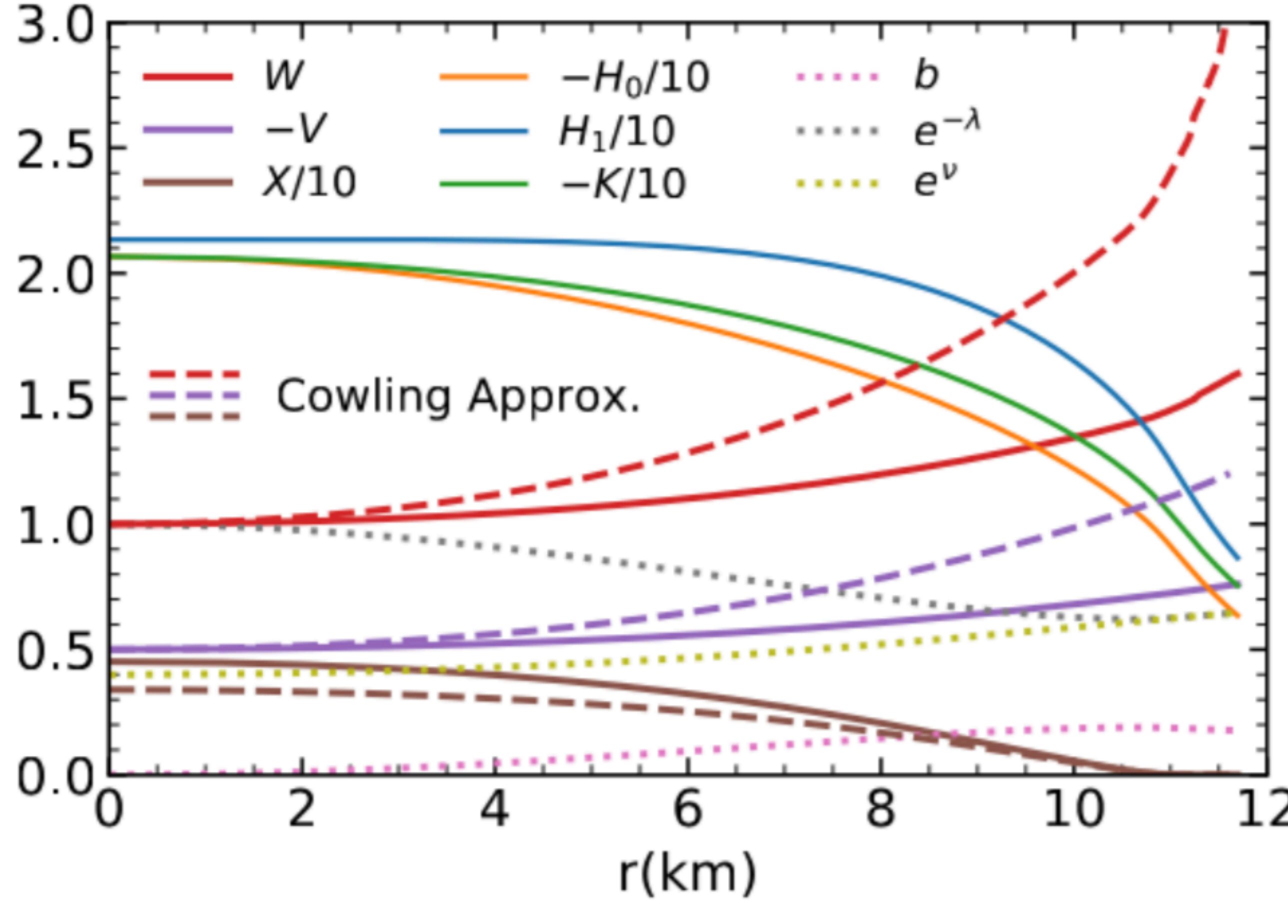
- Newtonian: Cox, John P. 1980
  2 1st ODEs + 1 2nd ODE
  Analytical for some modes,
  e.g. f-mode and interface g-mode
  in stellar asteroseismology.

Drop gravity perturbation

(Inverse Cowling)

- Newtonian Cowling approximation:
  2 ODEs
  Cowling, Thomas G 1941

Outside metric perturbation (f-mode as example)

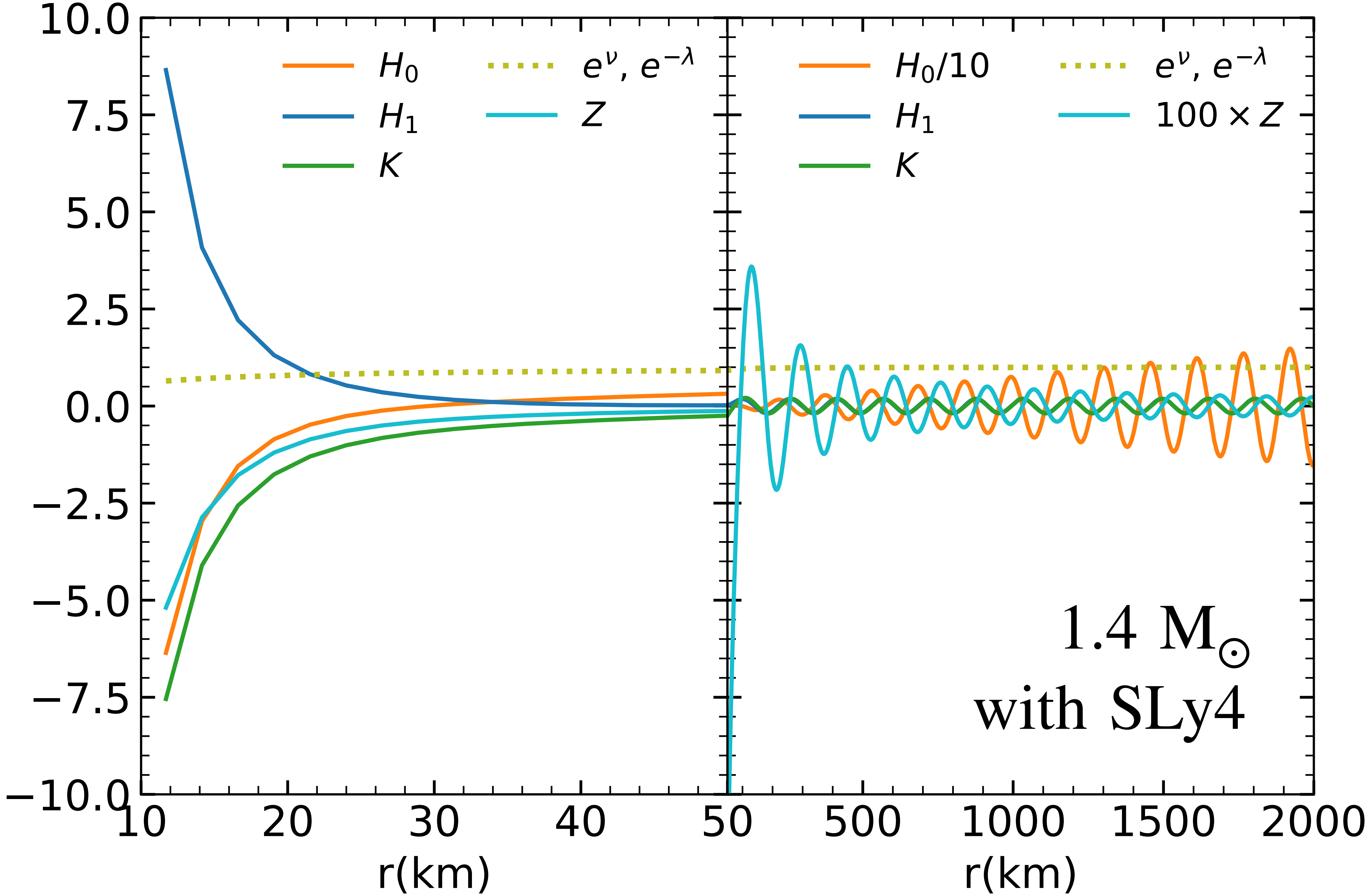


$1.4\ M_\odot$ with SLy4

FIG. 14. Metric perturbation amplitudes, fluid perturbation amplitudes for non-radial oscillations with $\ell = 2$ with (dashed curves) and without (solid curves) the Cowling approximation, and static metric functions (dotted curves) inside a $1.4M_\odot$ NS computed with the Sly4 EOS [85]. H_0, H_1 and K are in units of $\varepsilon_s = 152.26$ MeV fm^{-3}, X is in units of ε_s^2, and W, V, ν and λ are dimensionless. Only real parts of the perturbation amplitudes are plotted.

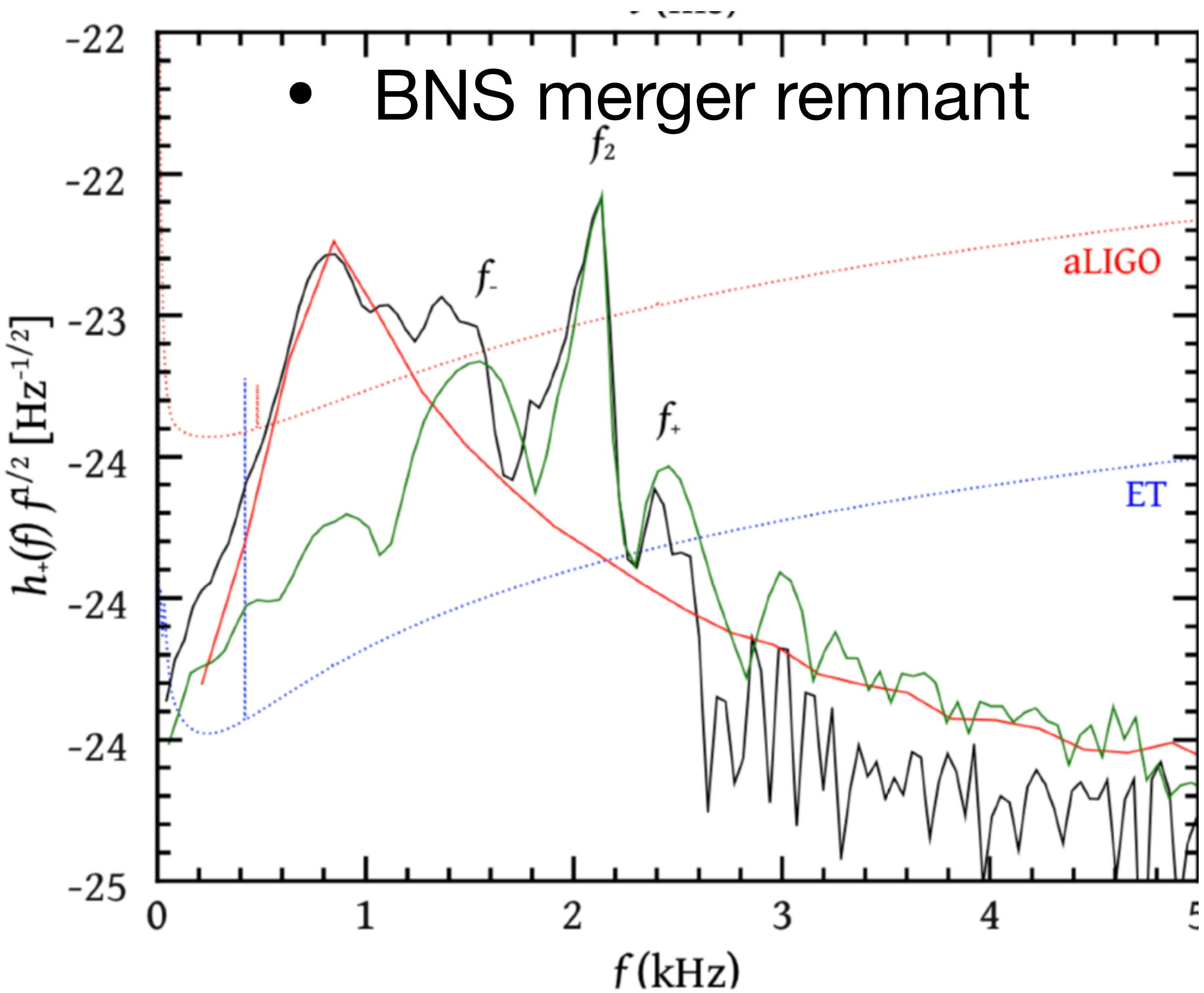
Zhao & Lattimer 2022
https://arxiv.org/abs/2204.03037

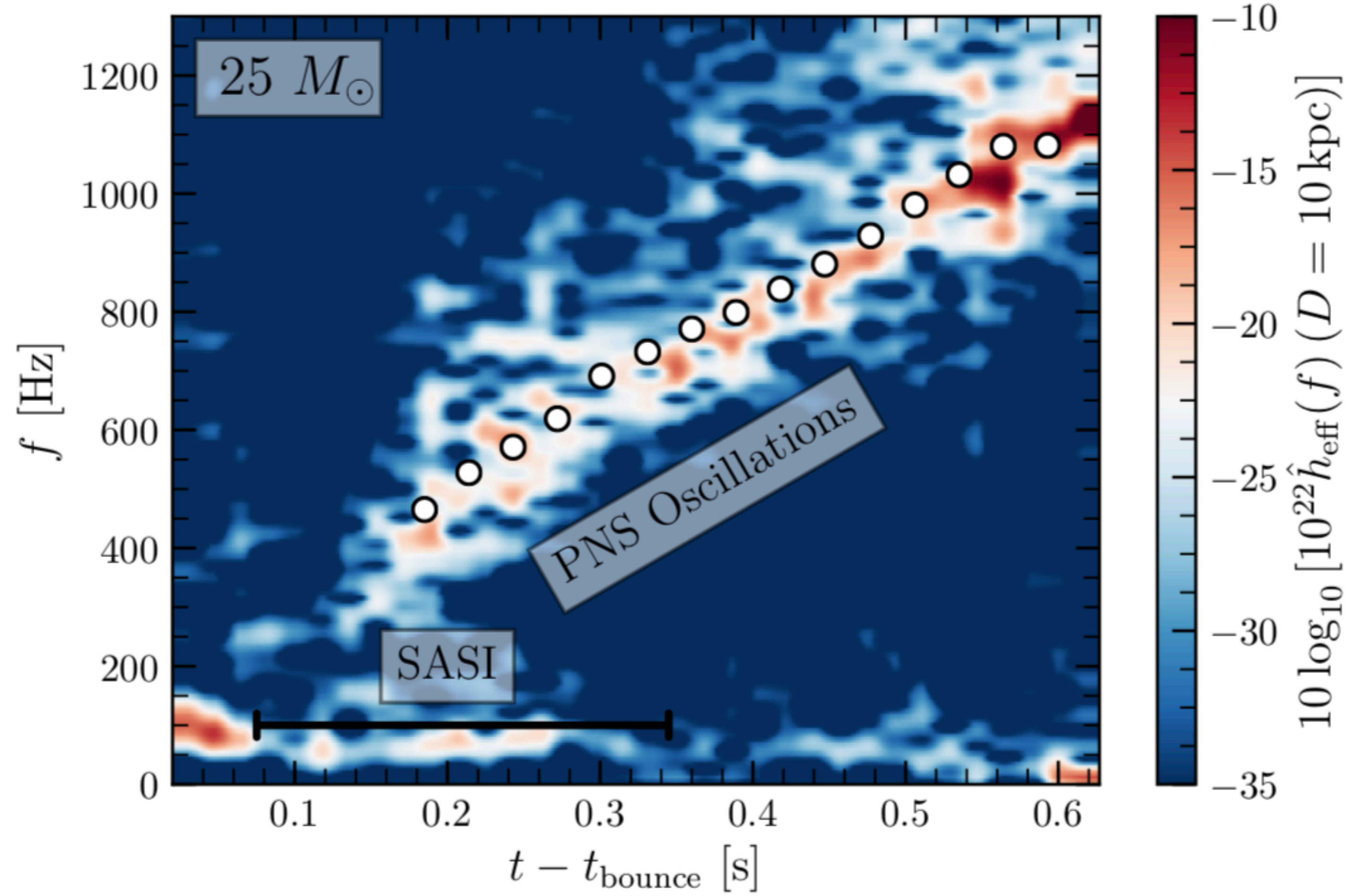
Outside metric perturbation (f-mode as example)



Oscillations of NS in simulation

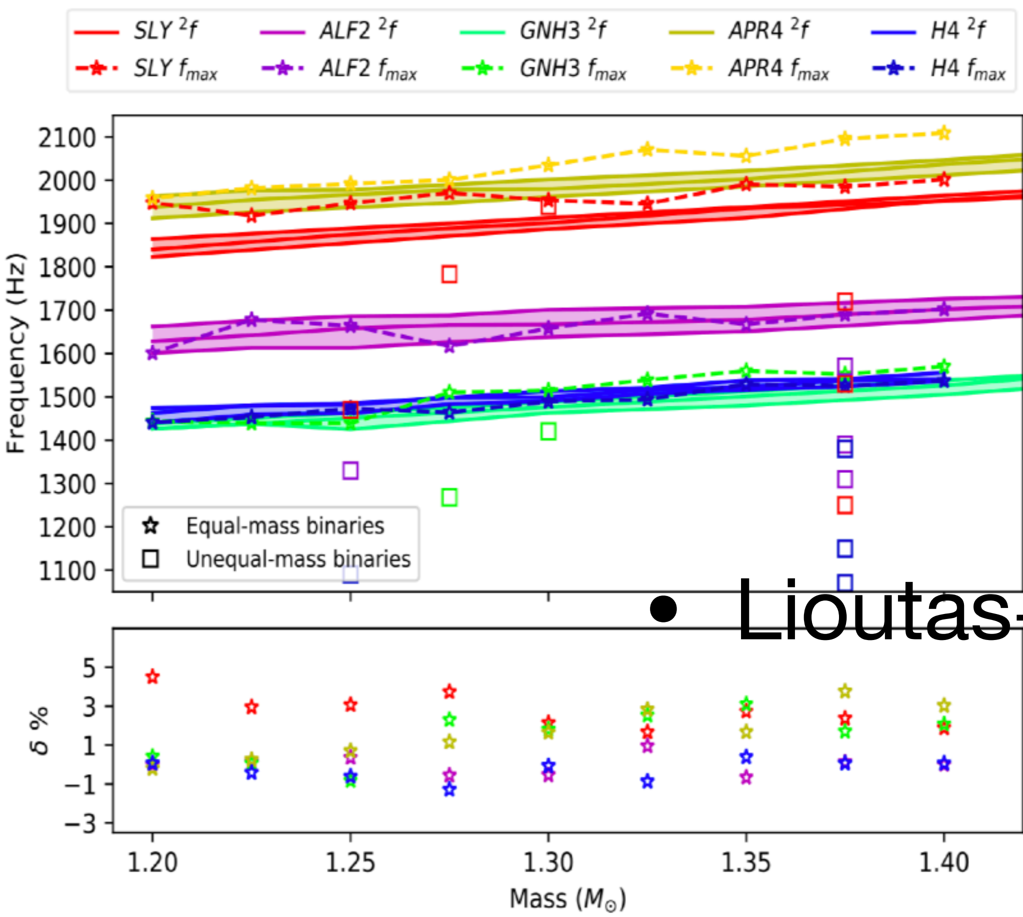
- Core-collapse SNe
- BNS merger remnant



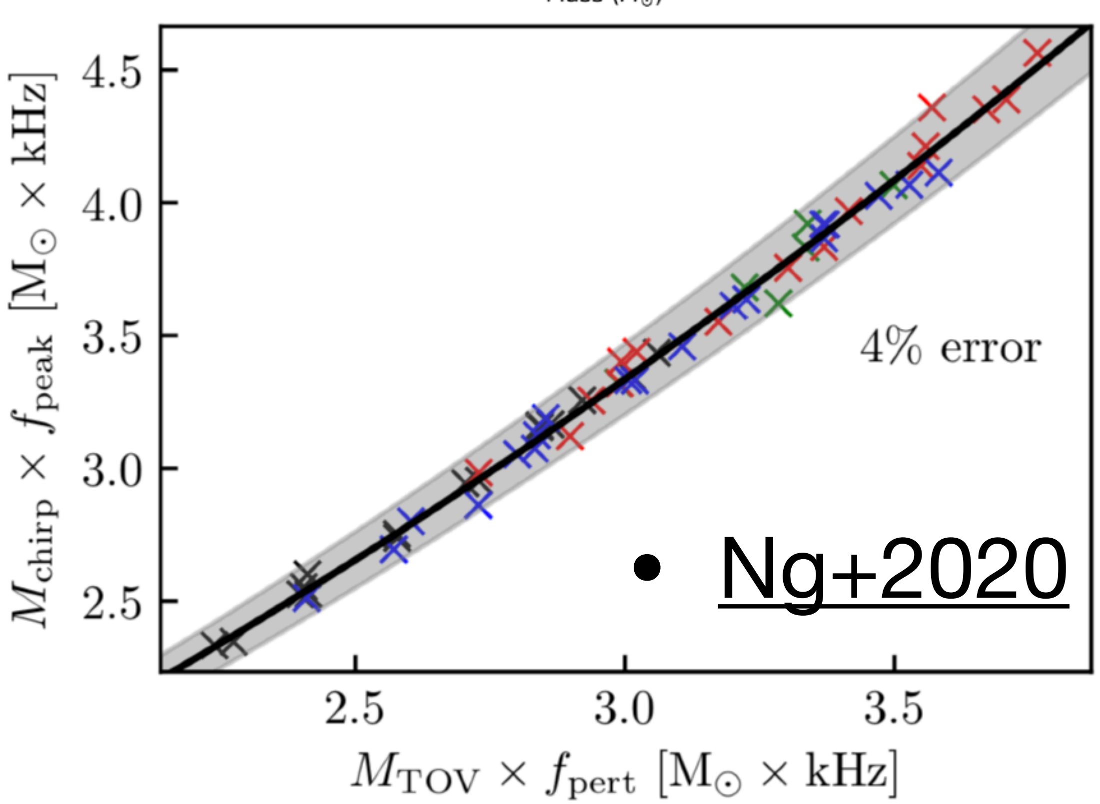


- Stergioulas+2011
- Radice+2019

Isolated oscillation VS merger remnant



- Lioutas+2021



- Ng+2020

- Strong correlation with the isolated NS f-mode frequency and the peak frequency in post merger.

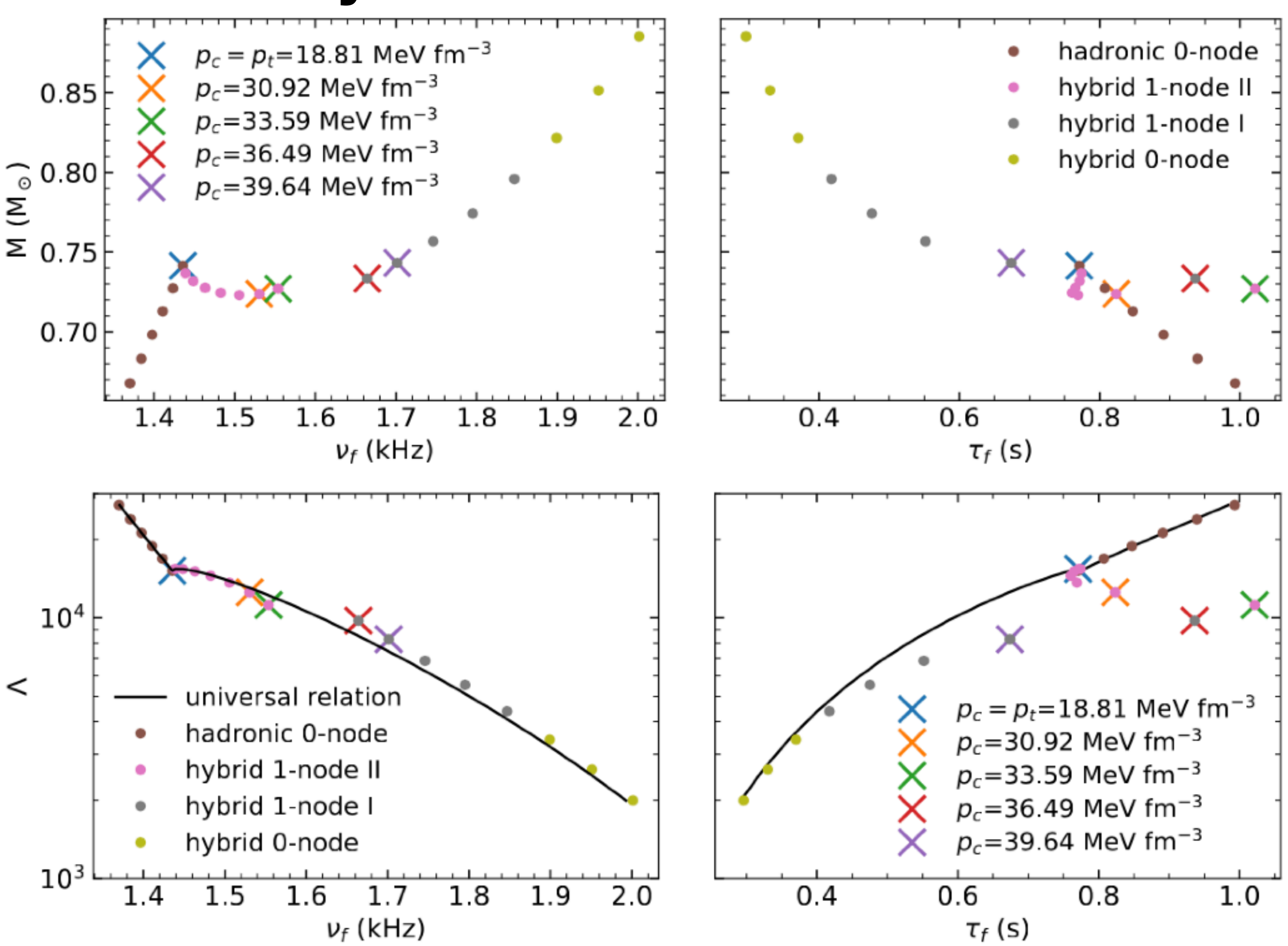
- case of equal-mass mergers, the peak frequency in supramassive NSs is almost equal to that of the non-rotating f-mode frequency of isolated NSs with the same mass as each of the merging components

One node branch

- Lowest order pressure mode have zero node which is named as f-mode (fundamental).
- However, in case of hybrid NS lowest order pressure mode sometimes have one node due to strong density discontinuity.
- Stars with radial nodes in V only we refer to as 1-node I.
- hybrid stars have a radial node (zero) in the fluid and metric perturbation amplitudes X, W, H_0, H_1, K (but not V, which, however discontinuously changes sign) at a radius slightly larger than the phase transition radius R_t. We will call this type of behavior 1-node II

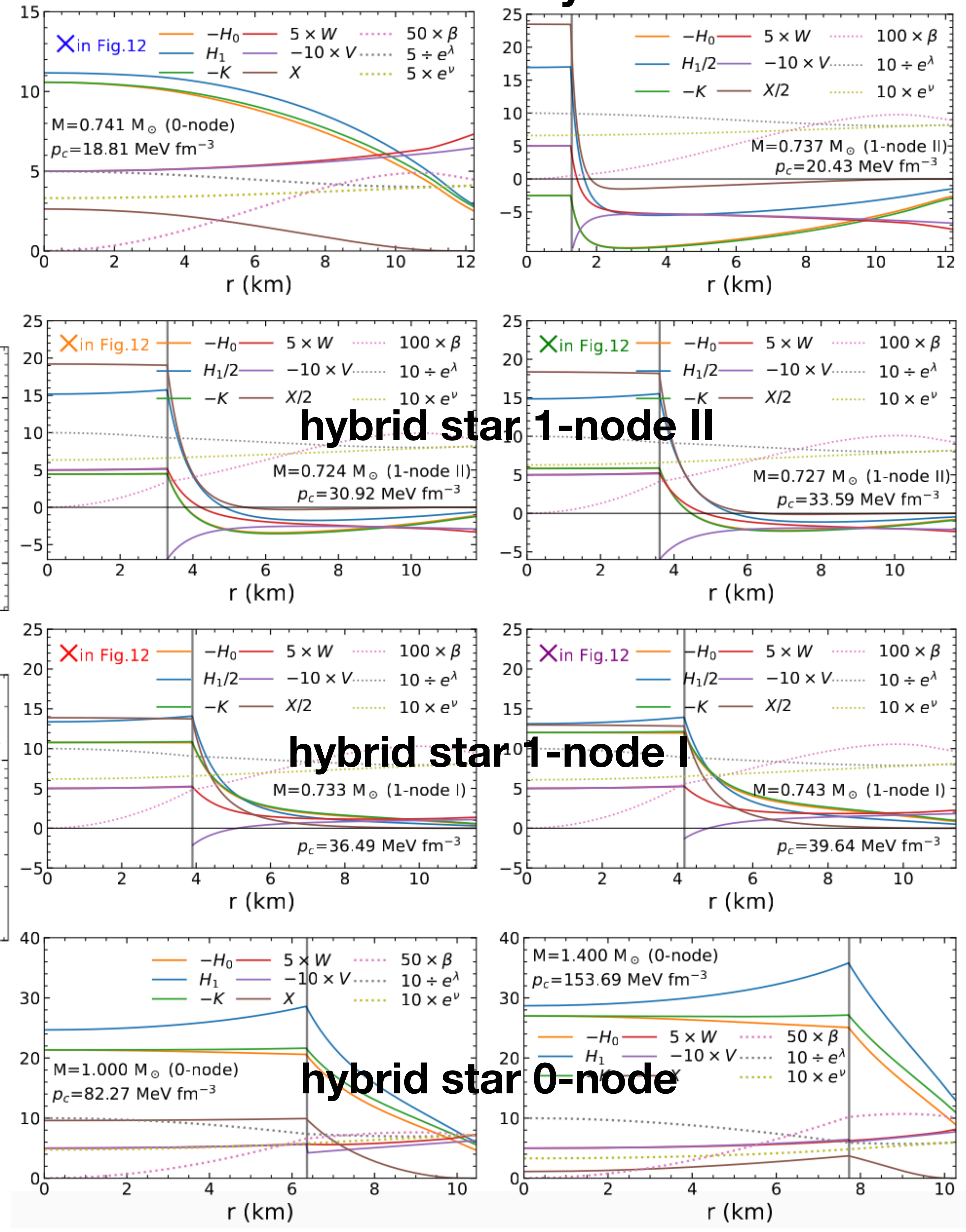
One node branch

hybrid star with 1-node deviates away from f-I-love-Q relation



Zhao & Lattimer 2022

hadronic star 0-node **hybrid star 1-node II**

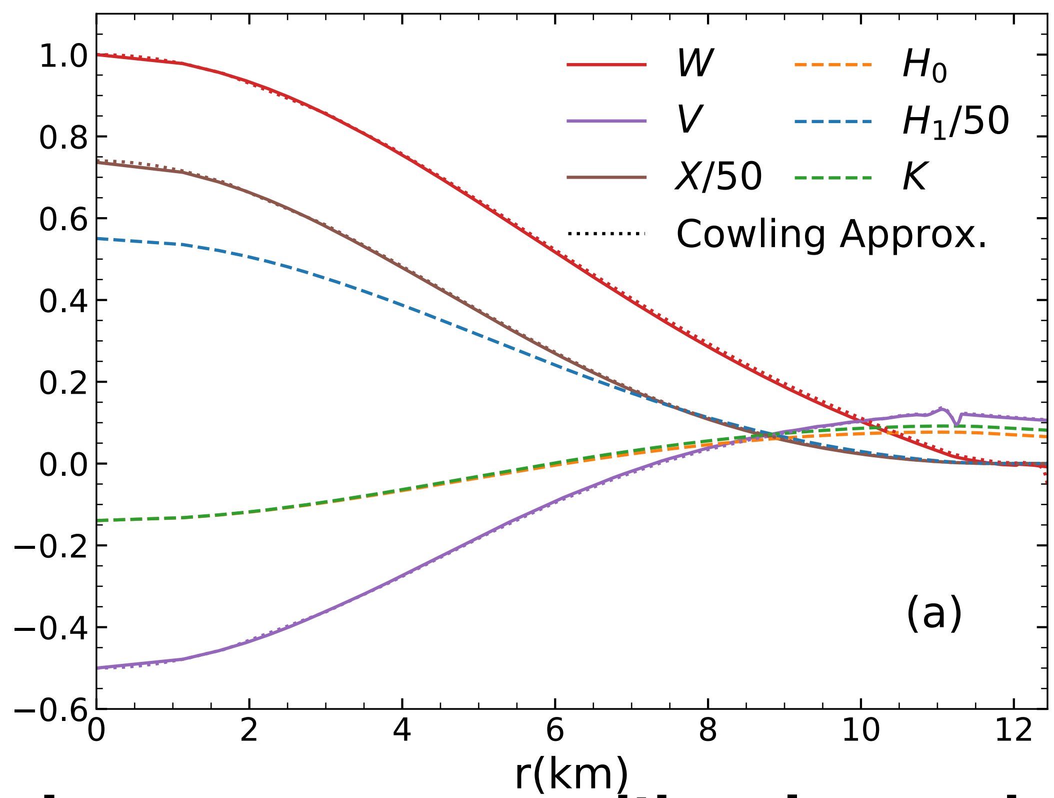


hybrid star 1-node II

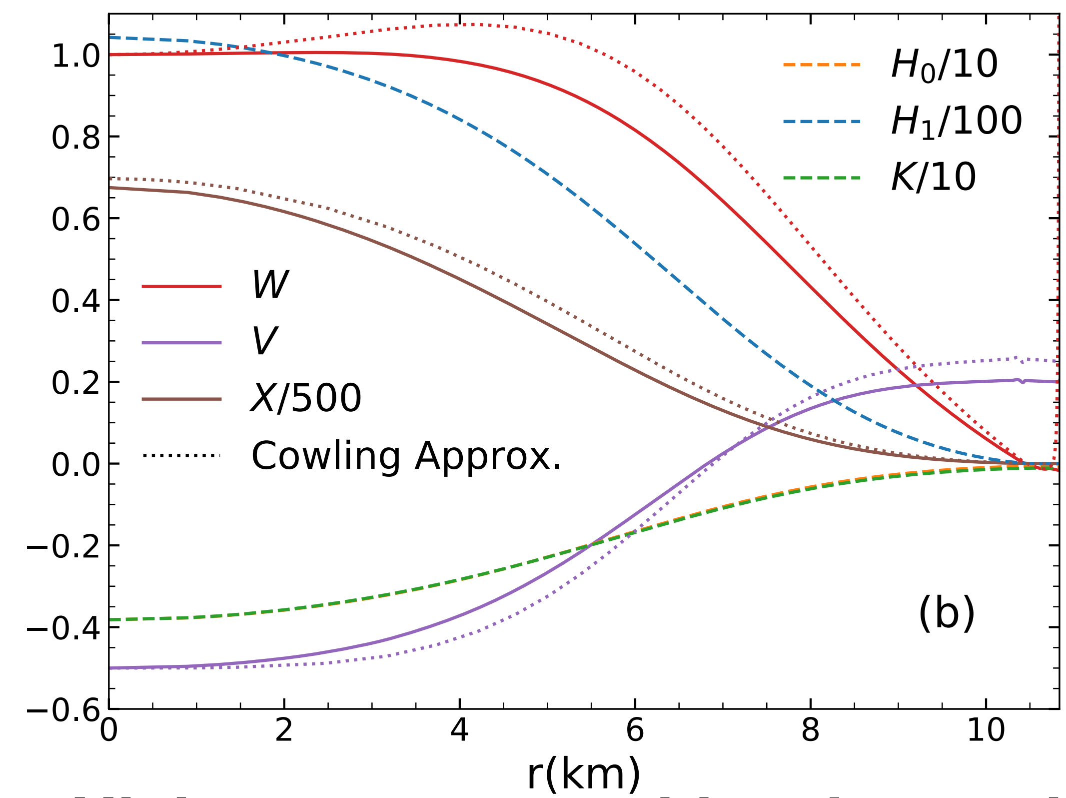
hybrid star 1-node I

hybrid star 0-node

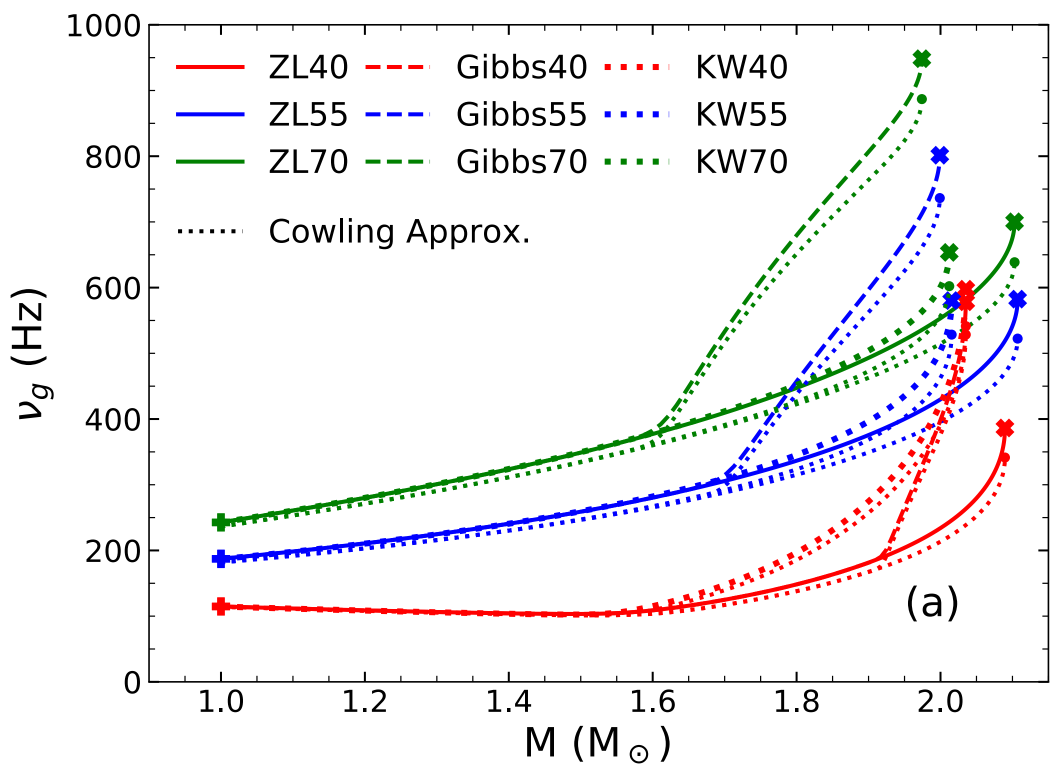
Cowling Approximation in Compositional g-modes



Low mass compositional g-mode



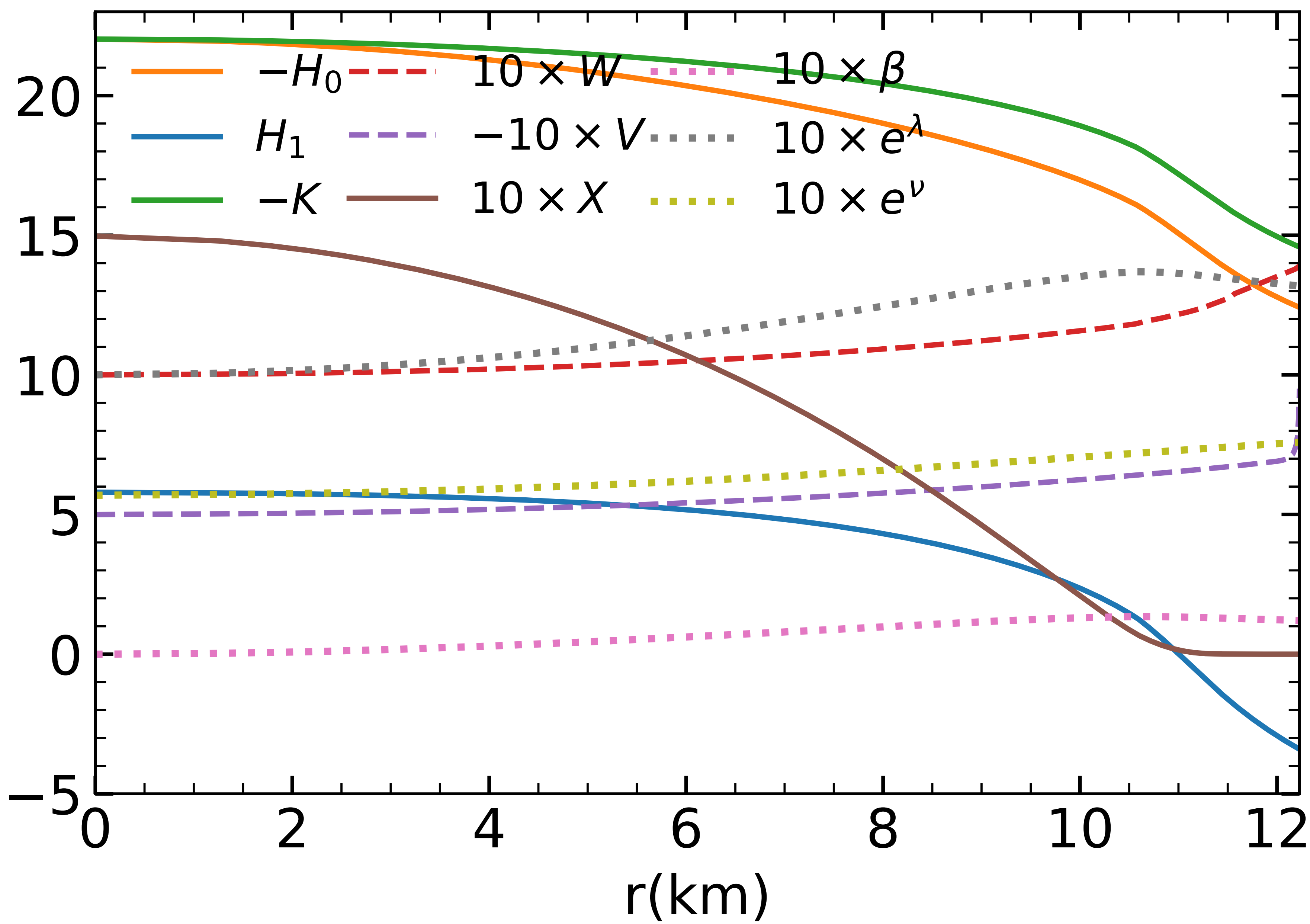
High mass compositional g-mode



Cowling approximation: up to 10% deviation from the linearized full GR

Zhao, Constantinou, Jaikumar and Prakash 2022
https://arxiv.org/abs/2202.01403

Compositional g-mode of hadronic NS



Discontinuity g-mode of hadronic NS