Oscillation and deformation on neutron stars leading to GW detection

Yasufumi Kojima

From Quarks to Neutron Stars:
Insights from kHz gravitational waves

23 - 24 April 2025 @ Tokyo

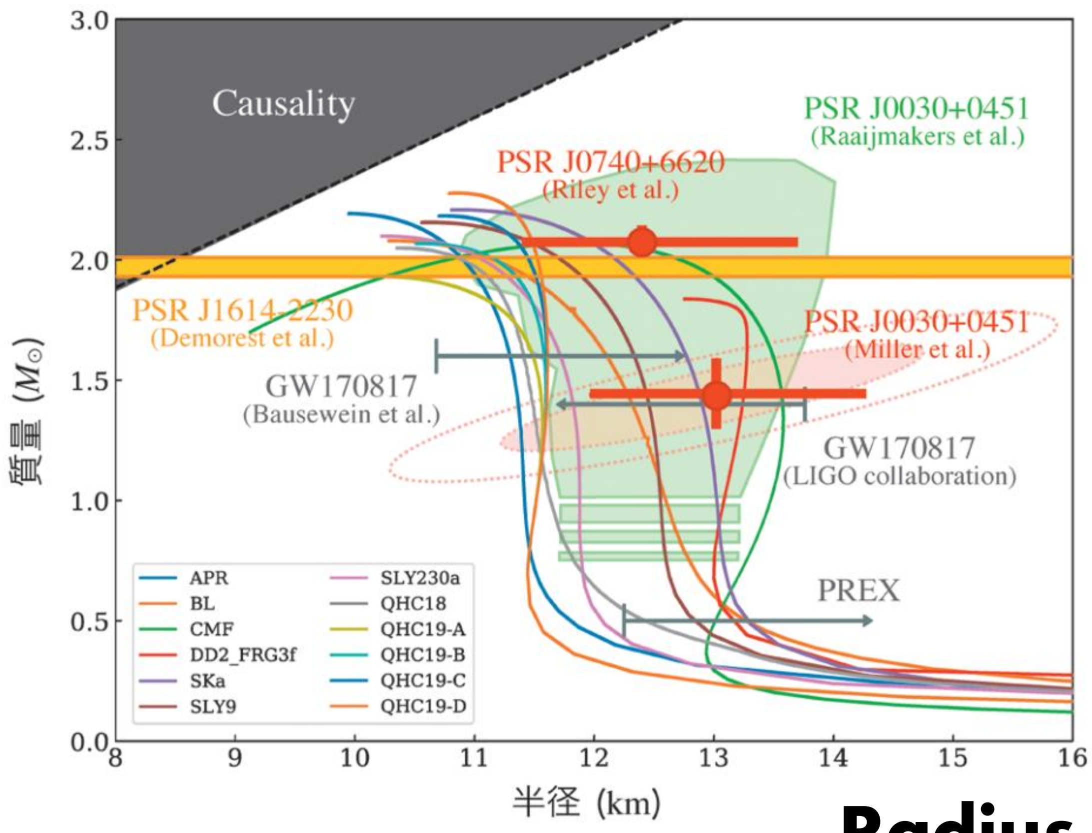
Constraints to EOS

Remarkable development

- Maximum mass
- GR red shift of line
- GR light bending(NICER)
- GW (tidal deformability)

Further constrains by GW in kHz band ?

Mass **Mass-radius diagram of NS**



Radius

Enoto & Yasutake (2021) Butsuri in Japanese

General consideration

For $f \sim$ kHz GW from NS

$f^{-1} \sim$ ms   dynamical timescale (<1ms) of NS

Dynamical aspect of NS   ⇔ Static star with M and R

However, amplitude h is small (rare chance) in high f because $\Delta F \propto (f h)^2$

kHz GW is challenging issue, important information if detected

Potential source <u>except for binary NS</u> in next GW astronomy

- Bursts

Oscillations in PNS / Glitches / Magnetar flares

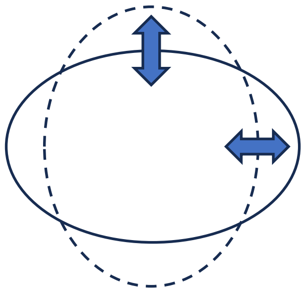
- CGW

Deformation of (rapidly) rotating NS

f-mode oscillation of NS

- Almost volume-preserving

  Global oscillation of entire star



- Oscillation frequency

  $f \propto (GM/R^3)^{1/2} \propto \bar{\rho}^{1/2}$~1.5 – 3 kHz

- Damped by GW

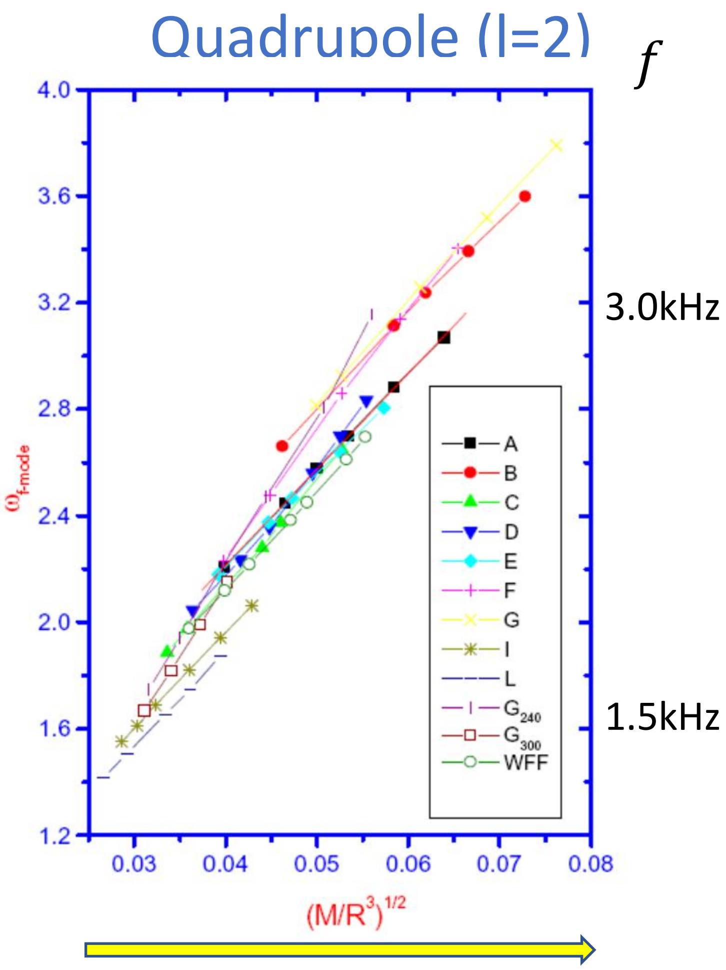
  $\tau_{GW} \sim c^5 R^4/(GM)^3 \sim 0.1 - 0.5(< 1)sec$

➔Efficient source of GW

However, how is it excited?

SN Glitch Flare

Andersson & Kokkotas (1998)



Relativistic, Massive, Compact

Various oscillations of PNS

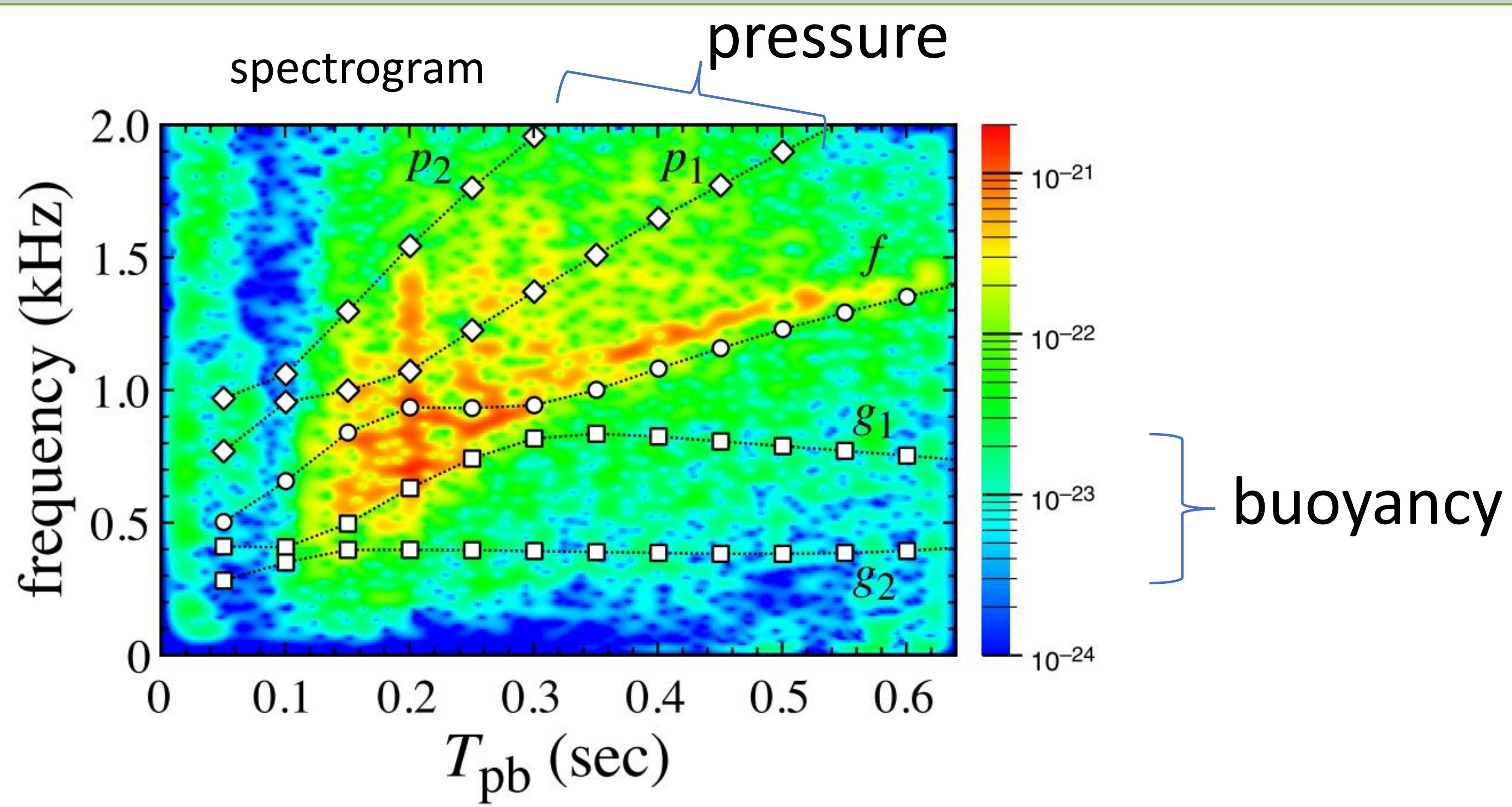


Figure 5. Comparison between the gravitational wave signals obtained from the numerical simulation (background contour) and several eigenfrequencies for the PNS with $\rho_s = 10^{11}\ \mathrm{g\,cm^{-3}}$, where the circles, the diamonds, and the squares denote the f-, p_i-, and g_i-modes for $i = 1$ or 2. The source distance is assumed to be $D = 10$ kpc.

Sotani & Takiwaki (22)

Study including many physics

Oscillations are identified in numerical simulation models

Sotani +(16,…) ; Muller+(13,..); Powell+(23,24,..);Morozova+(18,...) ;

Torres-Forne +(18,…); Cusinato+(25);

(1D) 2D/3D, GR/Newton, neutrino transport, Stellar models,

+ Rotation + magnetic field, ….

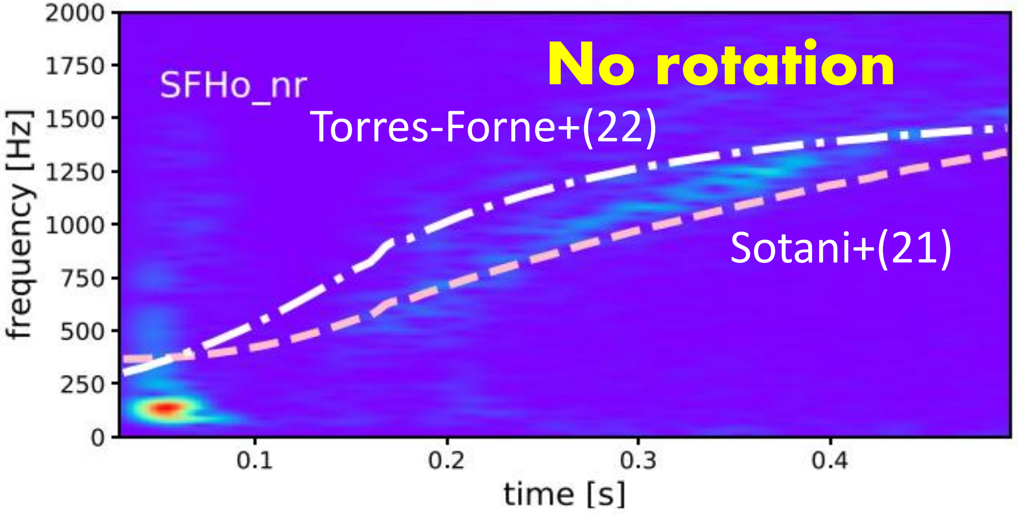
Typical values

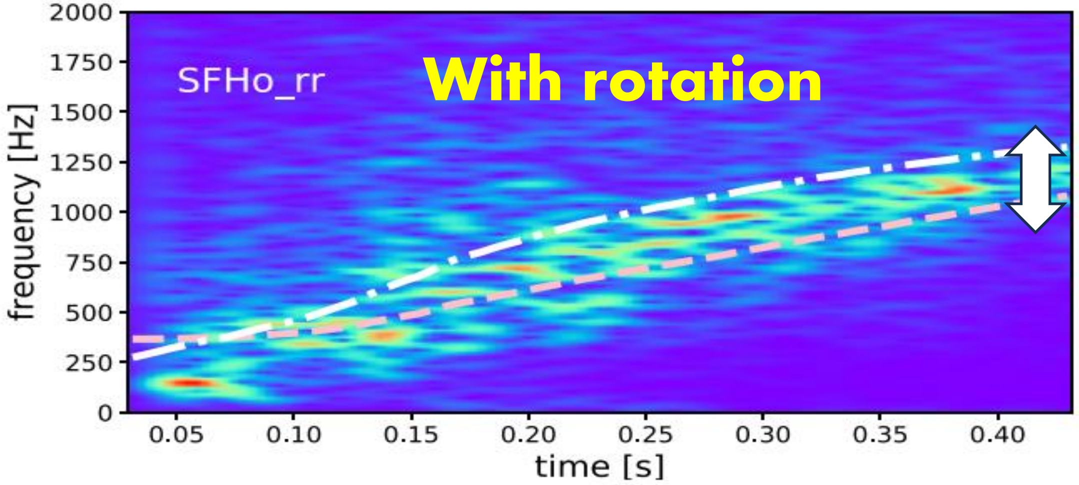
$E_{GW} \sim 10^{46} - 10^{47} \ll E_{ex} \sim 10^{51} erg$

$h_{GW} \sim 10^{-22}$ (d~10 kpc)

spectrogram

Powell & Muller (24)
15M_sun





Broad width by mode splitting

Nonrotating -> rigid rotation with Ω

$$f_0 \quad \rightarrow f \approx f_0[1 + m(\Omega/\Omega_K) \times c]$$

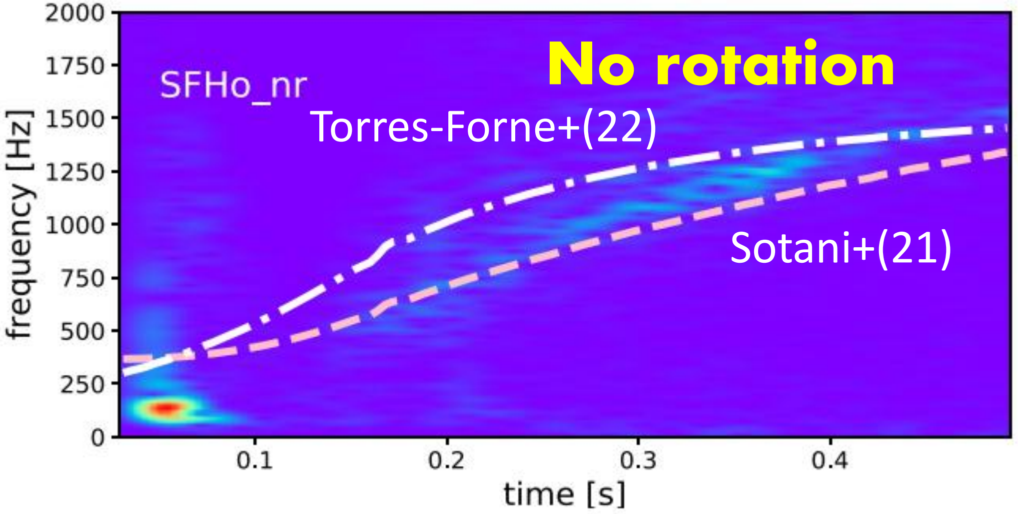
$$m = -l, \ldots + l \quad (2l+1)$$

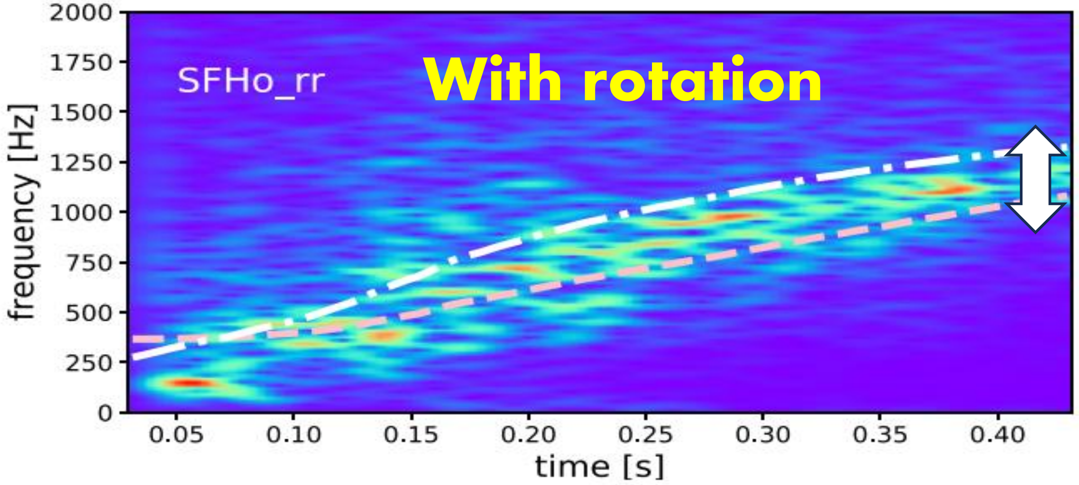
$c \approx 0.4 - 0.7$ (e.g. YK(93), Doneva+(13))

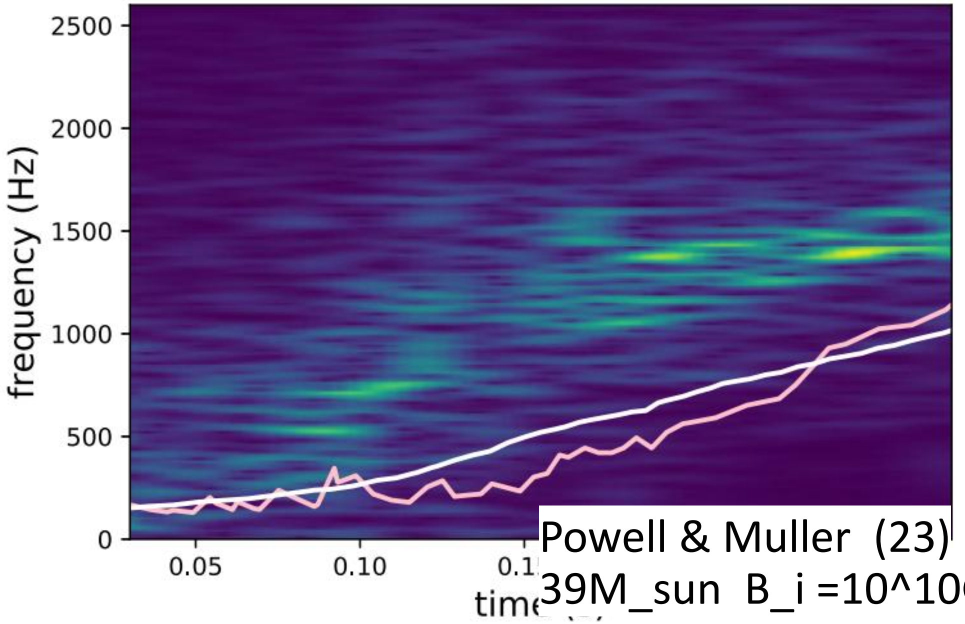
$\Omega/\Omega_K \sim 0.05 \quad T \sim 10^{50} < E_{ex}\, ; T/W < 0.01$

spectrogram

Powell & Muller (24)
15M_sun







Powell & Muller (23)
39M_sun B_i =10^10G

With magnetic field

Complicated behavior

Spin down by mag. Torque

Initial ang. Mom.

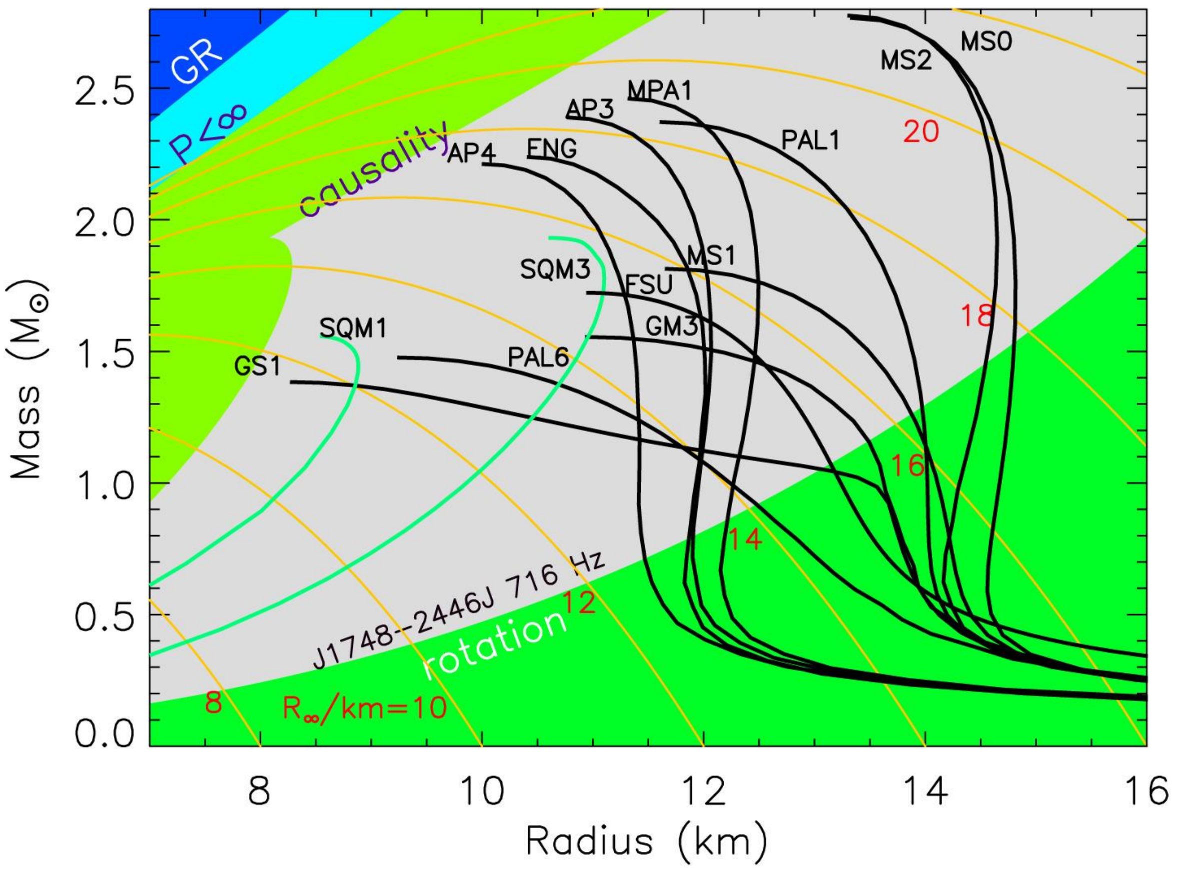
$$J_0 = 10^{48}\ gcm^2\ s^{-1} \rightarrow\ 4-5\times 10^{47}$$

Extreme cases are interesting, but need more studies

M-R of NS

Strange Quark Model

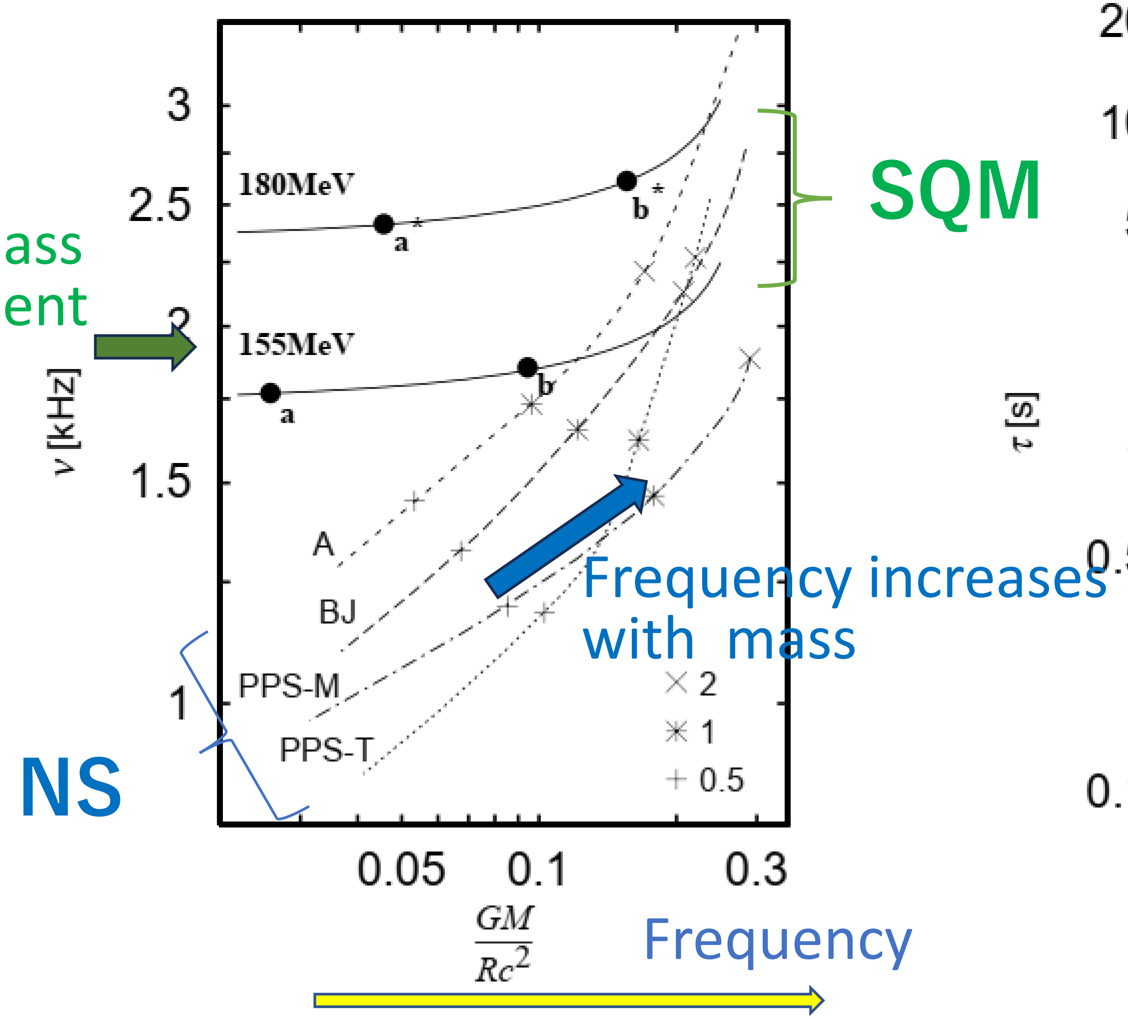
RXJ1856-3735
-> XDIM



Lattimer& Prakash (2016) Physics Reports

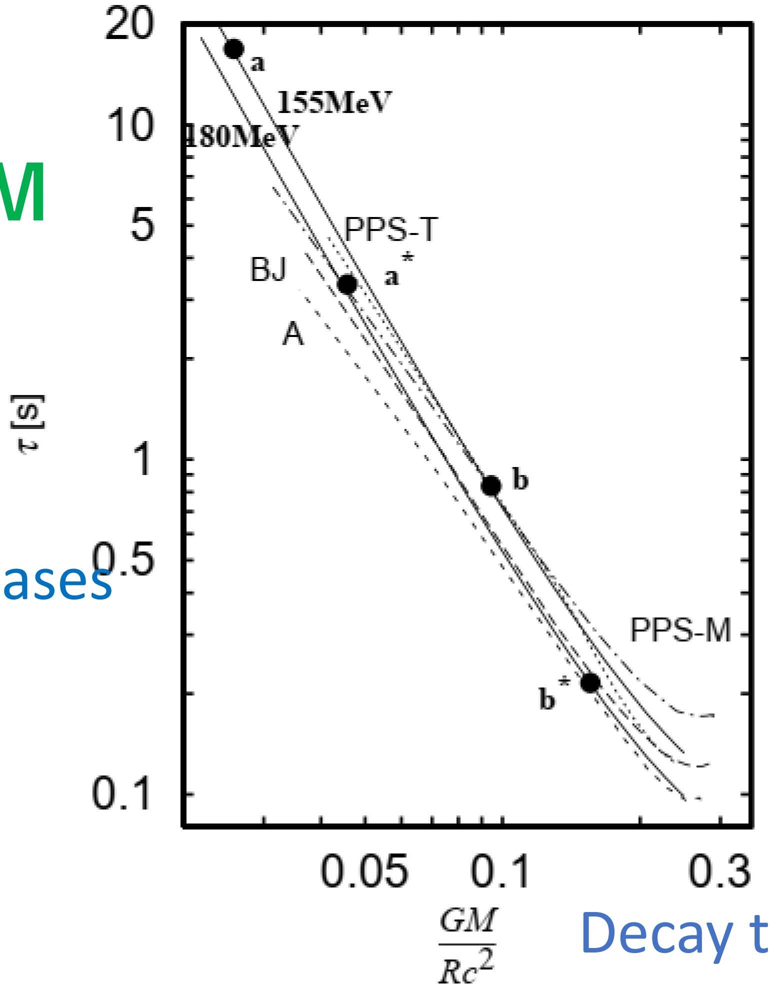
Gravitational bound object
or
Quark -confinement

Almost mass independent



Frequency

Relativistic, Massive, Compact



Decay time

YK& Sakata (2002)PTP

Different compact object detected by GW

✓Direct evidence: Oscillation frequency is not explained by any NS models

unclear excitation mechanism with large amplitude

✓Indirect evidence: Freq. distribution according to stellar mass because f depends on mass

Scenario in Wilson & Ho(2024)PRD “f-mode oscillation induced by glitch $E_{osc} = E_{glit} = I\Omega\Delta\Omega{\sim}10^{40} erg$ is damped with $\tau_{GW}{\sim}0.1 sec$ “

$h_0{\sim}10^{-23}$ @d=1kpc; SNR$\sim h_0\sqrt{\tau_{GW}/2S_h}$

Synthesized population from known pulsar and glitches catalogues

Gravitational waves from glitch-induced f -mode oscillations in quark and neutron stars

Wilson & Ho(2024)PRD: Synthesized population for NS (#10k) and QS(#10k) from known pulsars and glitches catalogues

QS

NS

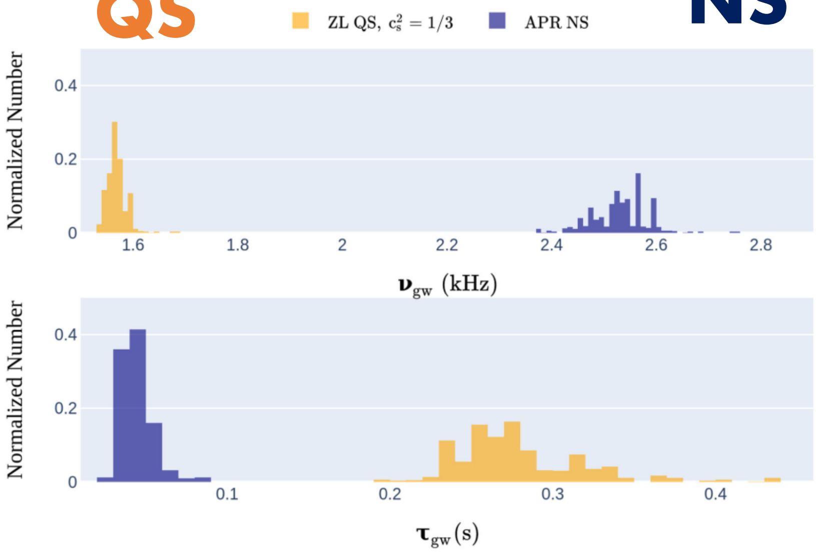


FIG. 5. Normalized distributions of GW frequency ν_{gw} and damping time τ_{gw} using the ZL QS $c_s^2 = 1/3$ EOS and APR NS EOS and Population 1 (known pulsars and glitches and assuming a Gaussian mass distribution).

Different distribution scenario

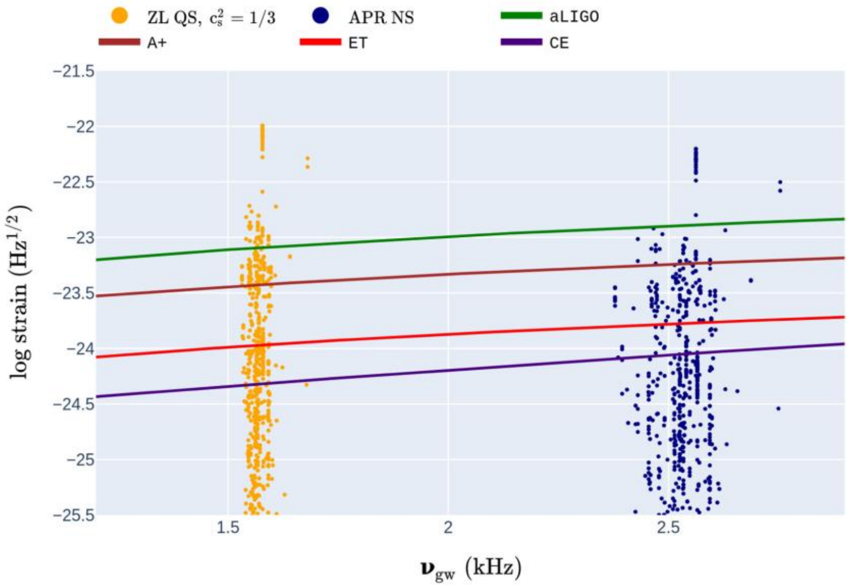


FIG. 6. GW strain ($h_0\sqrt{\tau_{gw}}$) from glitch-induced f modes, calculated using the ZL QS $c_s^2 = 1/3$ EOS and APR NS EOS and Population 1 (known pulsars and glitches and assuming a Gaussian mass distribution). Solid lines indicate current and future GW detector sensitivities.

Gravitational waves from glitch-induced f -mode oscillations in quark and neutron stars

Frequency hardly depends on mass in QS models

Small variation

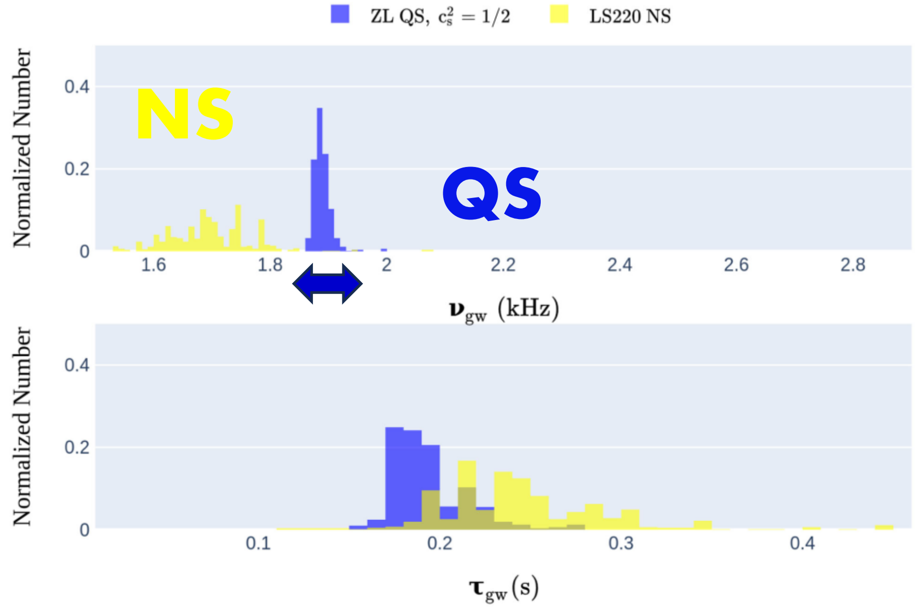


FIG. 8. Normalized distributions of GW frequency $\nu_{\rm gw}$ and damping time $\tau_{\rm gw}$ using the ZL QS $c_s^2 = 1/2$ EOS and LS220 NS EOS and Population 1 (known pulsars and glitches and assuming a Gaussian mass distribution).

II Overlapping case

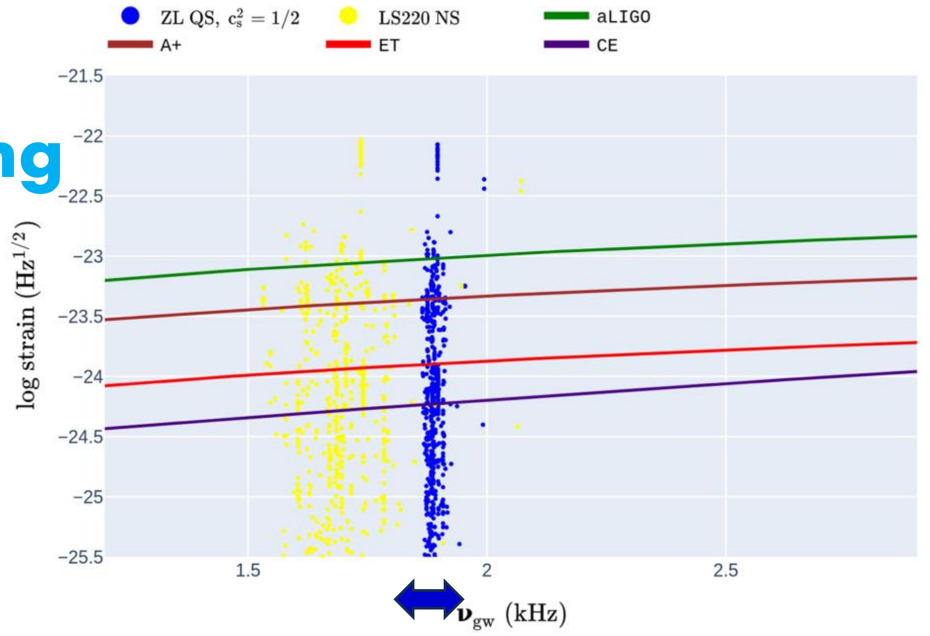


FIG. 9. GW strain ($h_0\sqrt{\tau_{\rm gw}}$) from glitch-induced f modes, calculated using the ZL QS $c_s^2 = 1/2$ EOS and LS220 NS EOS and Population 1 (known pulsars and glitches and assuming a Gaussian mass distribution). Solid lines indicate current and future GW detector sensitivities.

Different compact object detected by GW

✓Direct evidence: Oscillation frequency is not explained by any NS models

unclear excitation mechanism with large amplitude

✓Indirect evidence: Freq. distribution according to stellar mass because f depends on mass

Scenario in Wilson & Ho(2024)PRD "f-mode oscillation induced by glitch $E_{osc} = E_{glit} = I\Omega\Delta\Omega \sim 10^{40} erg$ is damped with $\tau_{GW} \sim 0.1 sec$ "

$h_0 \sim 10^{-23}$ @d=1kpc; SNR$\sim h_0\sqrt{\tau_{GW}/2S_h}$

Synthesized population from known pulsar and glitches catalogues

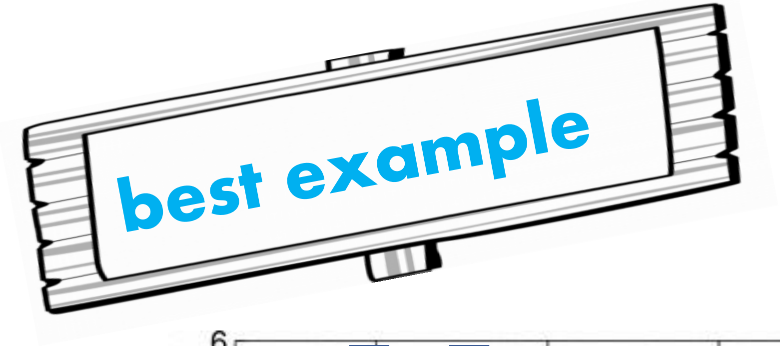
Bimodal distribution -> Another type of compact star

However, optimistic estimate

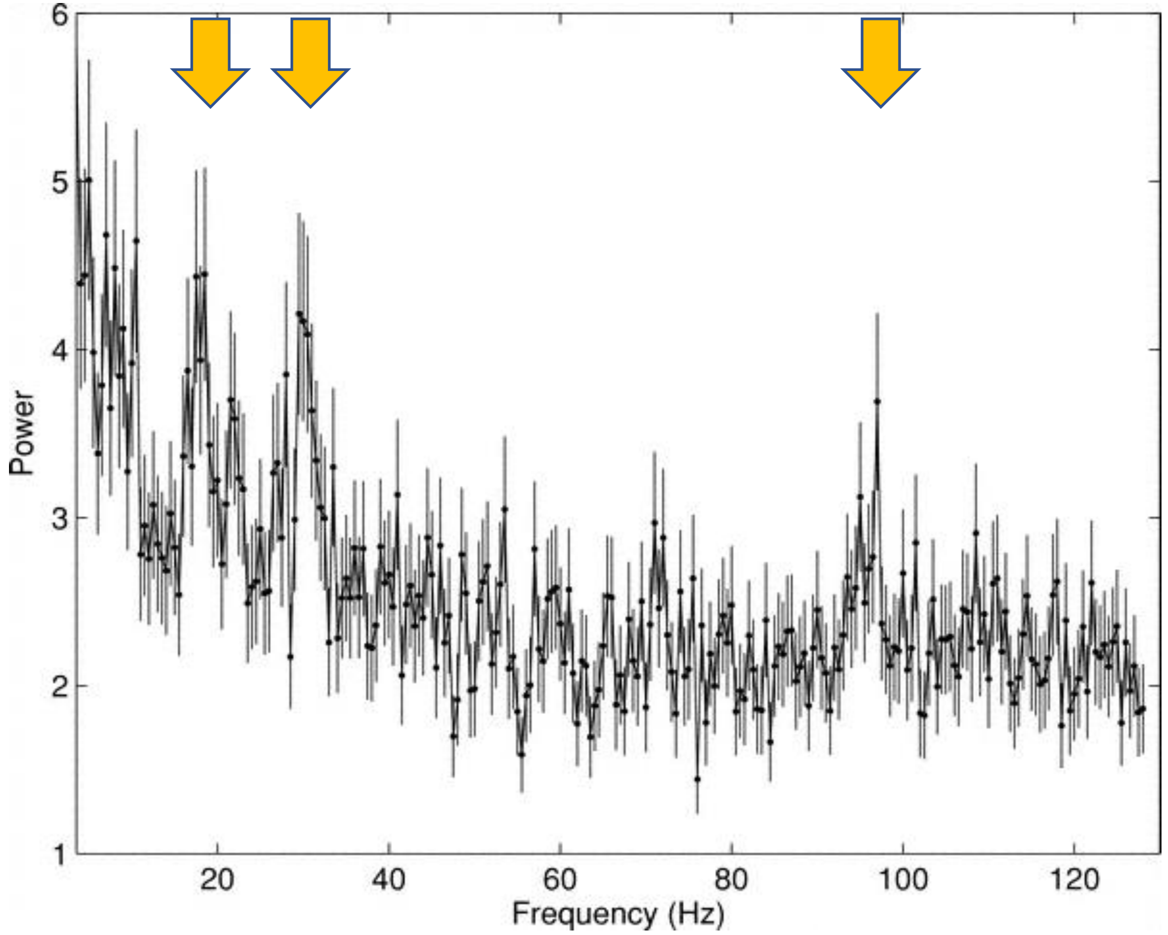
Magnetar flare

Energetic event $E_{EM} \sim 10^{46} erg$ e.g., Giant flare in 2004 SGR1806-20
Magnetic energy $E_{gr} \approx 10^{53}, E_{mag} \approx 10^{48}, E_{rot} \approx 10^{45}, E_{th} \approx 10^{43} erg, \ldots$

✓ QPO's observed in 20-100 Hz kHz (unclear ?)

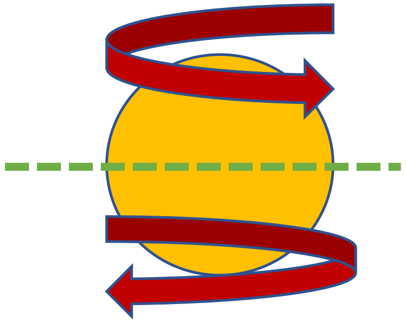


SGR1806-20 Israel+ 2005



Peaks at 20, 30, 95Hz during 200-300s

Torsional shear oscillation in crust



$$C_s \approx (\mu/\rho)^{1/2} \approx 10^8 cm/s$$
$$f = T^{-1} \approx [(l(l+1)-2)^{1/2}C]/2\pi R$$
$$\approx a \times 10Hz$$

Exploring flare

Torsional shear oscillation in solid crust ($\Delta r/R < 0.1$)

<-> Constrain to NS model <-> K,L parameters of nuclear matter

($\Delta r/R$ decreases with increase of M/R , L) (Rev. by Sotani 24)

Other modes e.g., f-modes excited?

Energy of f-mode oscillation is $\sim 10^{-6} E_{mag} \sim 10^{40}$ erg , when flare is induced by release of internal magnetic energy (Levin+ 11)

Different origin? Another scenario?

➔ Solved only by observation (GW observation is important at the flare)

Ref $\Delta E \leq 7.7 \times 10^{46} erg$; $h_{rms} \leq 4.5 \times 10^{-22} Hz^{-1/2}$, Giant Flare 04

Non-axisymmetric deformation of rotating NS

GW source

$$h = (16G\pi^2/c^4) I_{zz} \varepsilon \, {f_{rot}}^2 \; d^{-1}$$

Spin-down limit

$$\dot{E}_{GW} \; (\propto h^2 \,, \; \varepsilon^2) \; \leq I\Omega\dot{\Omega}$$

Upper limit by GW (no signal)

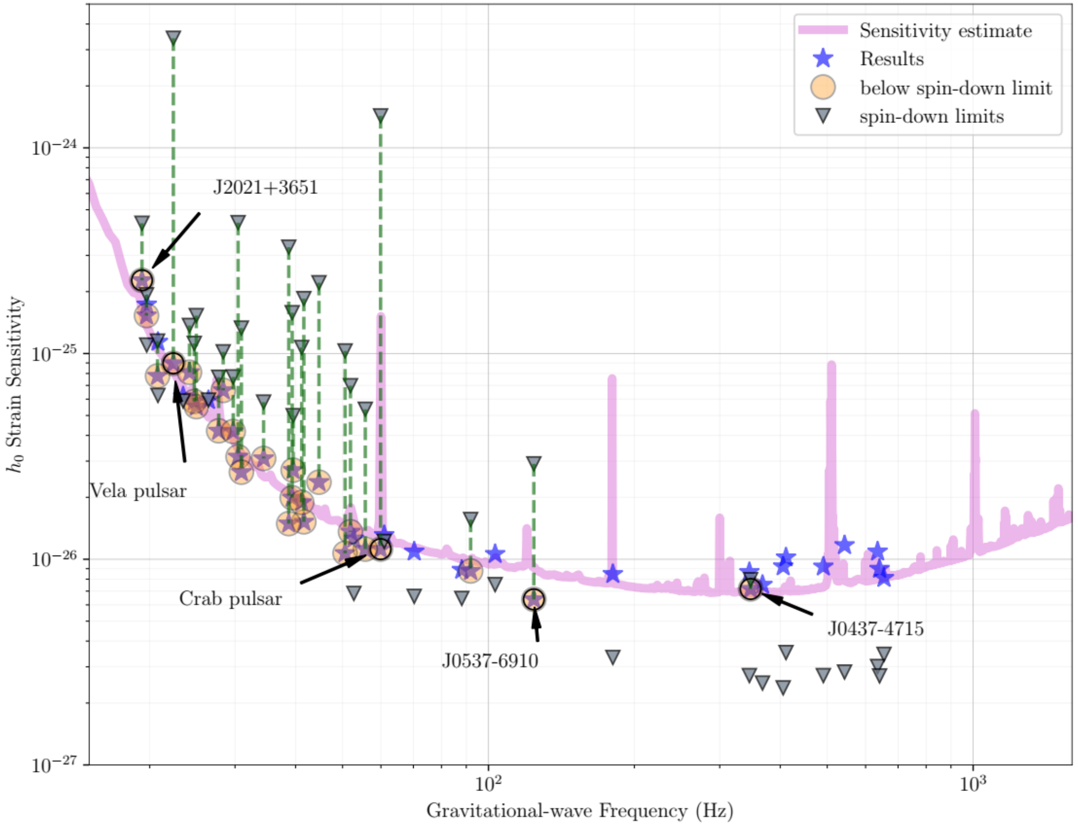
Current upper limit by GW

$$h = 8\times10^{-26}(\varepsilon/10^{-6})(f/kHz)^2(d/10kpc)^{-1}$$

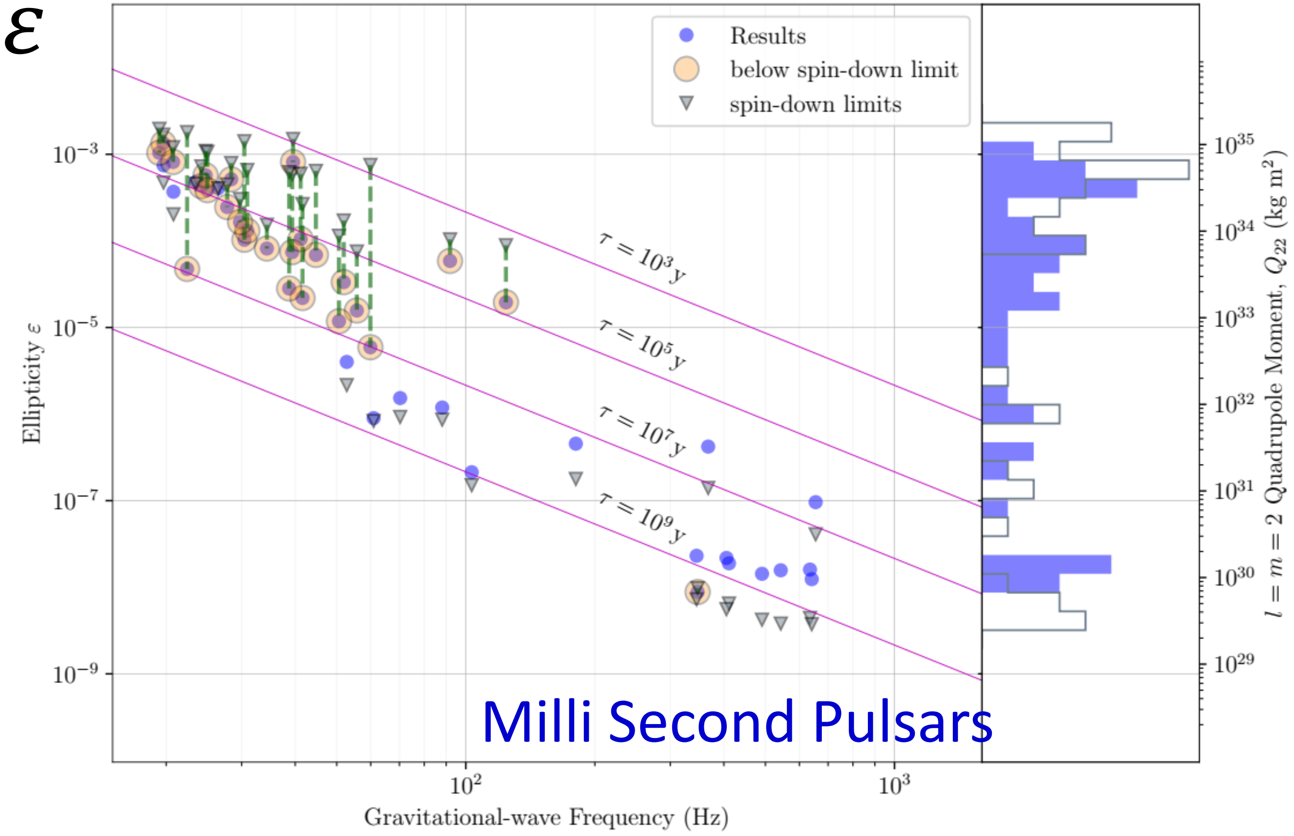
$$h_{sd} = 3\times10^{-27}((\dot{f}/f)_{-15})^{1/2}(d/10kpc)^{-1}$$

Many pulsars below spin-down limit

h



ε



Milli Second Pulsars

LVK O4a 2501.01495

Small spin-down limit

Non-axisymmetric deformation of rotating NS

GW source
$$h = (16G\pi^2/c^4) I_{zz} \varepsilon \, f_{rot}{}^2 \; d^{-1}$$

Spin-down limit
$$\dot{E}_{GW} \, (\propto h^2 \, , \, \varepsilon^2) \; \leq I\Omega\dot{\Omega}$$

Upper limit by GW (no signal)

Theoretical models to determine ε

Non-isotropic stress is necessary

- Magnetic deformation/ thermal mountain
- Deformation is estimated by energy ratio to gravity

$$\varepsilon \sim E_{mag}/E_{gr} \; \sim 10^{-6} \; \times \bar{B}_{15}{}^2$$
$$\sim 10^{-5} \times (H_{cr,16} \, B_{s15} \,)$$
$$\varepsilon \approx 10^{-7} \, B_{s,14}^2$$

Magnetic deformation

Type-II superconductors

Sustained by solid crust

- Realistic formation (recent theoretical model)

Deformation of magnetars
NS with intense mag. $(> 10^{14}\ G)$

Precession observed in seven magnetars (Makishima + 2014,…)

4U 0142 + 61,1E 1841-045, 1E 1547-5408,SGR 1900 + 14,SGR 1806 −20,SGR 0501+4516,RXJ 1708-4009

Universal

Prolate shape $\varepsilon = (I_1 - I_3)/I_3 \sim 10^{-4}$

⇔ toroidal field $(\sim 10^{16}\ G)$

or unknown mechanism to produce the shape

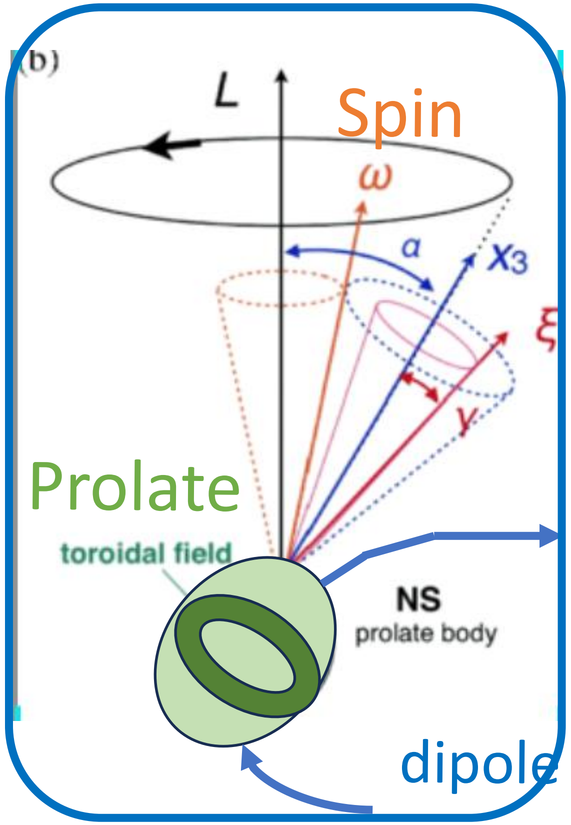


Table 3. Summary of PPM measurements and estimates of B_t.

Source	Year	P_0 (s)	T_{ppm} (ks)	P_0/T_{ppm} (10^{-4})	B_t' [a] (10^{16} G)	B_d [b] (10^{14} G)	B_t'/B_d	τ_c [c] (kyr)	Ref. [d]
4U 0142+61	2009	8.689	55 ± 4	1.6	1.3	1.3	100	68	[1]
SGR 0501+4516	2008	5.762	$16.4^{+0.6}_{-0.5}$	3.5	1.9	1.9	100	15	[2]
RXJ 1708−4009	2010	11.006	46.5 ± 3.1	2.4	1.5	4.7	32	9.0	[2]
1E 1841−045	2006	11.783	$23.4^{+0.5}_{-0.7}$	5.0	2.2	7.0	31	4.6	[2]
SGR 1900+14	2016	5.227	40.5 ± 0.8	1.3	1.1	7.0	16	0.90	[1]
1E 1547.0−5408	2016	2.087	36.0 ± 2.3	0.58	0.76	3.2	24	0.69	[1]
SGR 1806−20	1993	7.469	16.435 ± 0.024	4.5	2.1	20	11	0.24	[1]

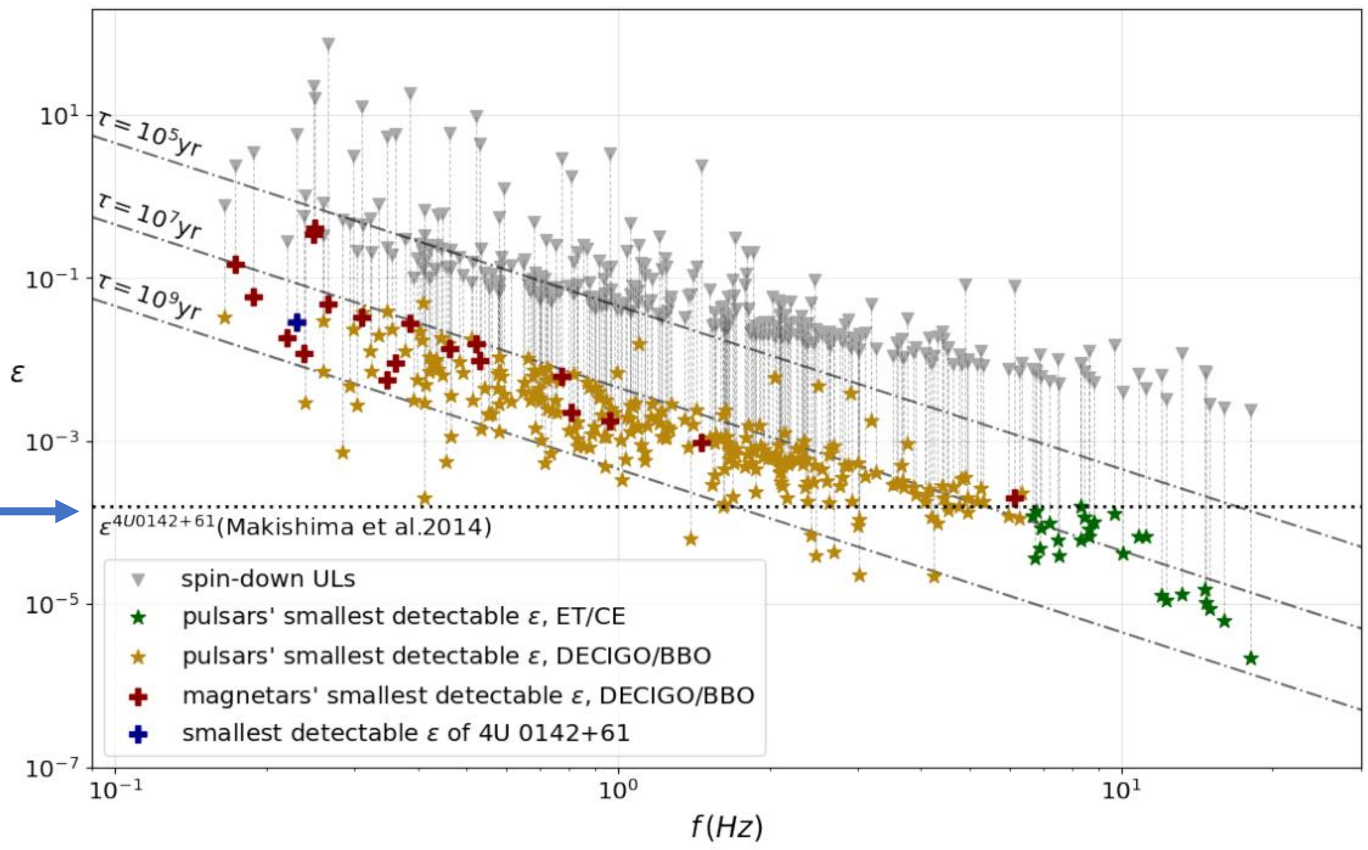
Makishima + (2024)

Detection in space

DECIGO, BBO

Figure 1. There are 273+27 (see previous table) coloured markers (stars and crosses), corresponding to sources from the considered samples, detectable by various detectors. The grey triangles show the spin-down ellipticity for every source. The horizontal dotted line is the ellipticity inferred by Makishima et al. (2014) for the magnetar 4U 0142+61. The dash-dotted lines indicate characteristic ages assuming only gravitational wave braking.

$\varepsilon \sim 10^{-4}$



GWs from magnetars with a big deformation are expected, but the frequency is low

Pagliaro+ 2503.17087

Evolution of newly borne NS

Hypothetical milli sec. NS

$$f_{spin} \sim kHz \quad \& \quad \varepsilon \sim 10^{-4}$$

EM

GW

Spin evolution

Angular momentum loss by GW and magnetic dipole

$$I\Omega\dot{\Omega} = -\beta\Omega^4 - \gamma\Omega^6$$

Magnetic dipole $\beta = \frac{1}{6c^3}B^2\ R^6$; Quadrupole GW radiation $\gamma = \frac{32G}{6c^5}\varepsilon^2\ I^2$

Parameters B , ε and initial value $\Omega_0 = 2\pi f_0$ (max. rotation)

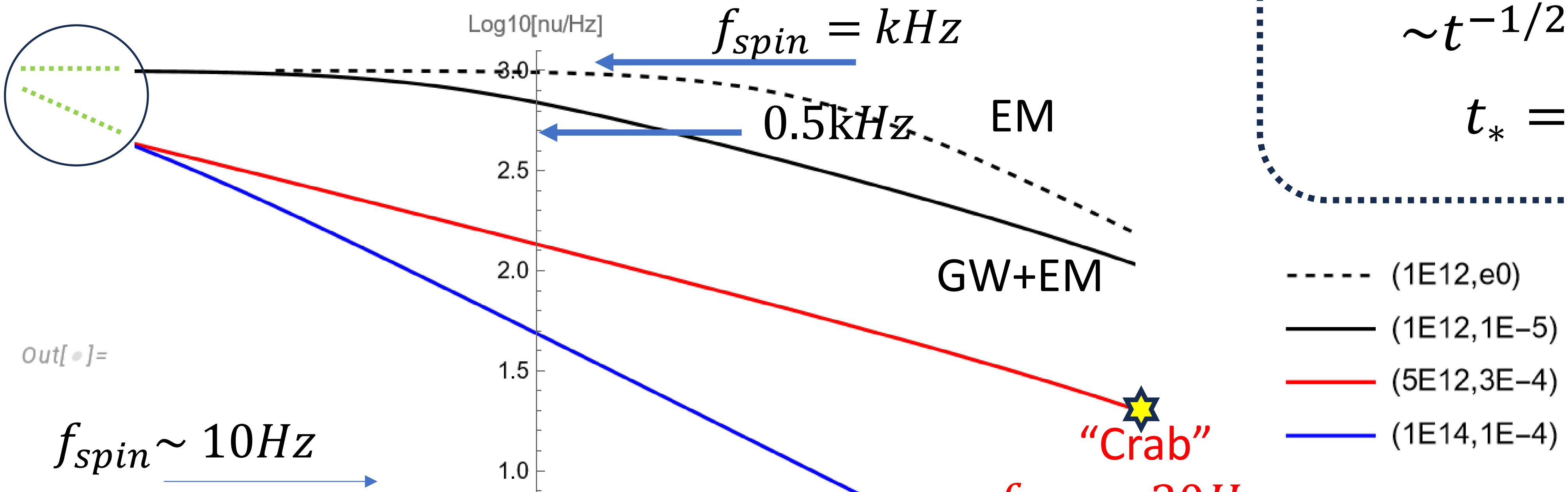
Classical paper;

Model of Crab PSR(born in 1054, discovered in 1968, Gunn & Ostriker 1969)

$B = 5 \times 10^{12} G$ and $\varepsilon = 4 \times 10^{-4}$

Large number ε might inspire resonance-type experiment in the past, but was not correct; current upper limit is $\underline{\varepsilon\ < 6 \times 10^{-6}}$ O4 (2025)

Secular evolution



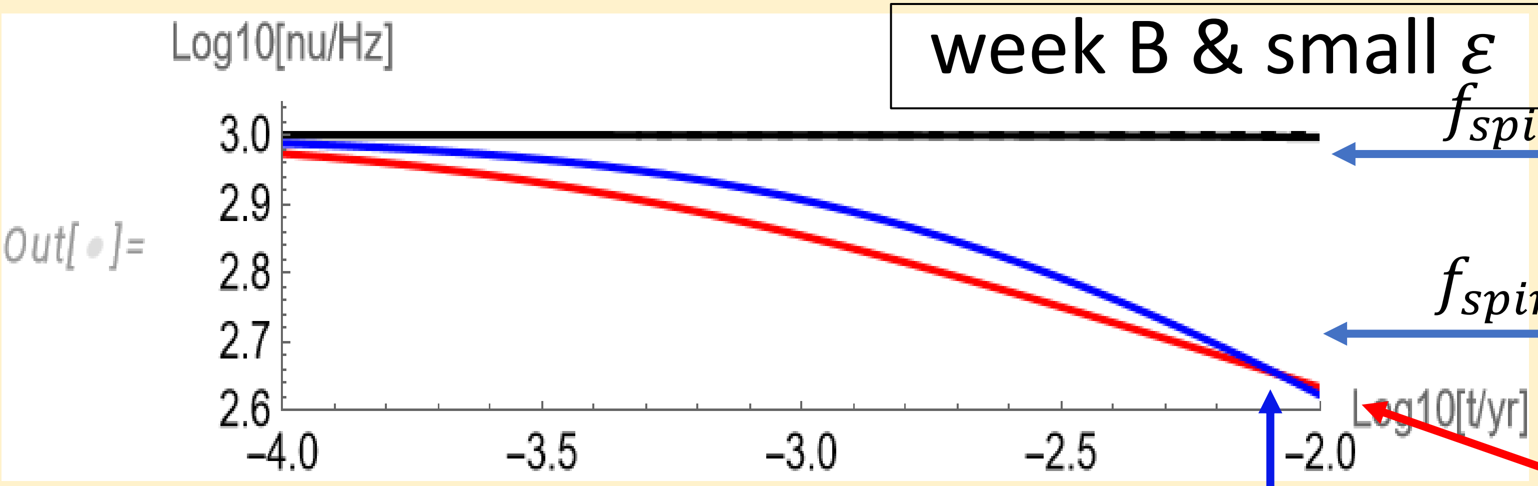
EM only

$$f/f_0 = (1 + t/t_*)^{-1/2}$$
$$\sim t^{-1/2} \ (t \gg t_*)$$
$$t_* = 24 B_{12}{}^{-2} \ yr$$

Magnetar with $B_d = 10^{14}$ G

Magnetic torque dominates

Early phase (< a few days)



week B & small ε

$f_{spin} = 1kHz$

$f_{spin} = 500Hz$

$kHz\ GW$

1 day

1 week

$B = 10^{14}$

$\varepsilon = 3 \times 10^{-4}$

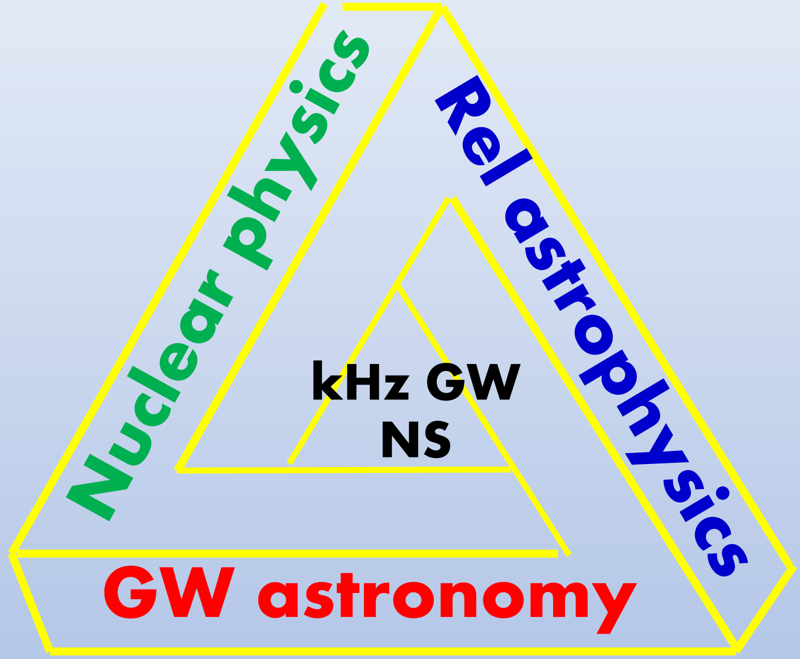
Within three days, frequency decreases to half
either $B > 9 \times 10^{13} G$ or $\varepsilon > 4 \times 10^{-4}$
$h_0 \sqrt{\tau} \sim 10^{-22} Hz^{-1/2}$ d=10Mpc
Or no signal for magnetar with small ε

Origin of B and ε

When, How much?

Summary

Oscillations associated with CCSN, glitches, magnetar flares and rotating NS with non-axisymmetric deformation are discussed



Many unsolved issues

kHz GW is challenging issue, and provides valuable information