Entanglement suppression and hadron scatterings

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  S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
  I. Low, T. Mehen, PRD104, 074014 (2021)

  - Entanglement power
  - Entanglement suppression for NN scattering

- **SU(3) baryon decuplet scattering**

  T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]

  - Spin 3/2 scattering
  - Emergent symmetries
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- **Summary**

Introduction to entanglement suppression

Entanglement power (EP) of S-matrix $E(\hat{S})$

- ability of generating entanglement

$$|\text{out}\rangle = \hat{S}|\text{in}\rangle$$

entangle?

Entanglement suppression

S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$E(\hat{S}) = 0$ **—> emergent symmetries of** NN **scattering**

Scatterings of octet baryons, pd**,** nd**,** dd**, heavy mesons, …**

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023);
T. Kirchner, W. Elkamhawy, H.-W. Hammer, Few-Body Syst. 65, 29 (2024);
T.R. Hu, S. Chen, F.K. Guo, PRD110, 014001 (2024); …

Symmetries of NN scattering

Fundamental symmetries <— QCD

- **spin symmertry** $\mathrm{SU}(2)_{\mathrm{spin}}$ $\{|\uparrow\rangle, |\downarrow\rangle\}$
- **isospin symmetry** $\mathrm{SU}(2)_{\mathrm{isospin}}$ $\{|p\rangle, |n\rangle\}$

Low-energy (s-wave) NN **scattering under** $\mathrm{SU}(2)_{\mathrm{spin}} \times \mathrm{SU}(2)_{\mathrm{isospin}}$

—> two components, $^1\mathrm{S}_0$ **and** $^3\mathrm{S}_1$

$^1\mathrm{S}_0$ **and** $^3\mathrm{S}_1$ **are independent, no constraint on their strength**

Emergent symmetries <— entanglement suppression?

- **spin-flavor symmetry** $\mathrm{SU}(4)$ $|1/a_0(^1\mathrm{S}_0)| \sim |1/a_0(^3\mathrm{S}_1)|$

  E. Wigner, PR51, 106 (1937), …, D.B. Kaplan, M.J. Savage, PLB365, 244 (1996), …

- **nonrelativistic conformal symmetry** $|a_0| \gg 1/m_\pi$

  T. Mehen, I.W. Stewart, M.B. Wise, PLB474, 145 (2000)

Entanglement measures

Entanglement measure for bipartite state $|\psi\rangle \in \mathscr{H}_A \otimes \mathscr{H}_B$

- von Neumann entropy

$$\mathscr{E}_{\rm vN}(|\psi\rangle) = -\,\mathrm{Tr}_A[\rho_A \ln \rho_A], \quad \rho_A = \mathrm{Tr}_B |\psi\rangle\langle\psi|$$

- linear entropy (expansion $\ln \rho_A \sim (\rho_A - 1)$)

$$\mathscr{E}(|\psi\rangle) = 1 - \mathrm{Tr}_A[\rho_A^2]$$

- product state: minimum $\mathscr{E}(|\psi\rangle) = 0$

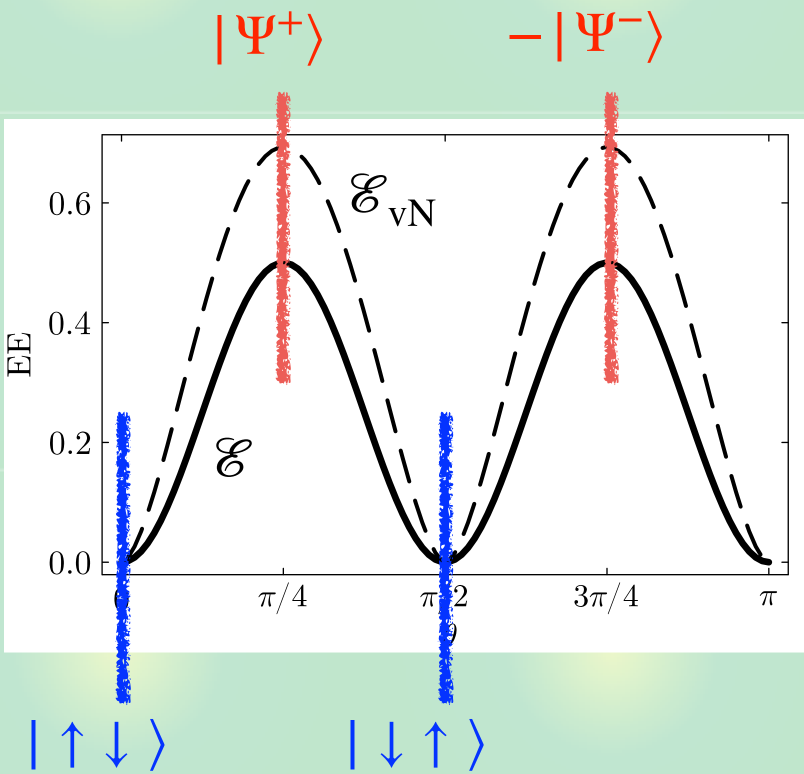
$$|\psi\rangle = |\uparrow_A\rangle \otimes |\downarrow_B\rangle = |\uparrow\downarrow\rangle$$

- Bell state: maximum $\mathscr{E}(|\Psi^\pm\rangle) = 1/2$

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle \pm \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle$$

Example:

$$|\psi\rangle = \cos\theta|\uparrow\downarrow\rangle + \sin\theta|\downarrow\uparrow\rangle$$



Entanglement power

Entanglement power (EP) of operator U acting on $|\psi\rangle$

P. Zanardi, C. Zalka, L. Faoro, PRA62, 030301 (2000)

- average over entanglement measure of $U|\psi_A\rangle \otimes |\psi_B\rangle$

$\mathscr{E}(|\psi_A\rangle \otimes |\psi_B\rangle) = 0$

$$E(U) = \overline{\mathscr{E}(U|\psi_A\rangle \otimes |\psi_B\rangle)}$$

$$= \int d\omega_A d\omega_B \; \mathscr{E}(U|\psi_A(\omega_A)\rangle \otimes |\psi_B(\omega_B)\rangle)$$

—> ability of U to generate entanglement

- **spin 1/2 (qubit):** rays in $\mathbb{C}^2 \simeq \mathbb{CP}^1$ **—> Fubini-Study measure**

$$|\psi(\omega_2)\rangle = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 e^{i\nu_1} \end{pmatrix}, \quad d\omega_2 = \frac{1}{\pi} d\theta_1 \cos\theta_1 \sin\theta_1 d\nu_1 \qquad \begin{matrix} \theta_1 \in [0,\pi/2) \\ \nu_1 \in [0,2\pi) \end{matrix}$$

(equivalent to Bloch sphere with $\theta_1 = \theta/2,\ \nu_1 = \phi,\ d\omega_2 = d\Omega/4\pi$)

S-matrix for *NN* scattering

S-matrix for *pn* scattering

$$\hat{S} = \underbrace{\exp\{2i\delta_0\}}_{{}^1\mathrm{S}_0}\frac{1-\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4} + \underbrace{\exp\{2i\delta_1\}}_{{}^3\mathrm{S}_1}\frac{3+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4}$$

(isospin is automatically determined by Pauli principle)

Entanglement power of *NN* scattering

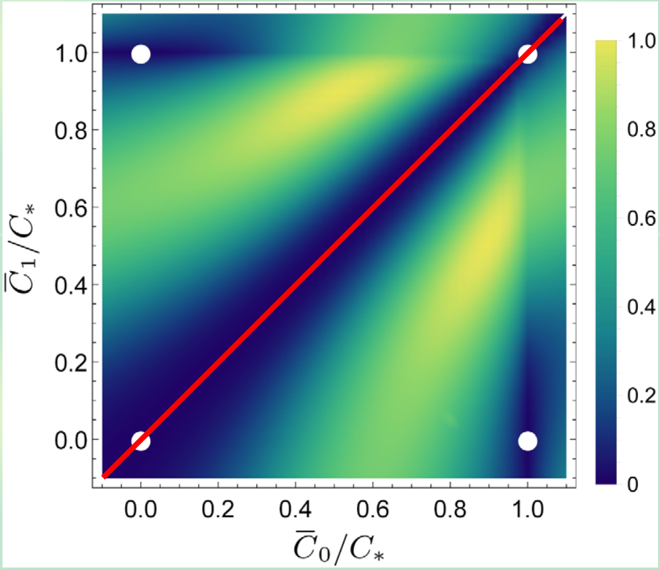
S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019)

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0 - \delta_1)] \geq 0$$

$E(\hat{S}) = 0$ **is achieved if**

$$\begin{cases} \delta_0 = \delta_1 & \text{SU(4) symmetry} \\ (\delta_0, \delta_1) = (0,0), (0,\pi/2), (\pi/2,0), (\pi/2,\pi/2) & \end{cases}$$

NR conformal symmetry



Interpretation by quantum information

S-matrix with $E(\hat{S}) = 0$: product state —> product state

I. Low, T. Mehen, PRD104, 074014 (2021)

- **Identity gate:** $|\delta_0 - \delta_1| = 0$

$$\hat{S} \propto \frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} + \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = 1, \quad 1\,|\psi_A\rangle \otimes |\psi_B\rangle = |\psi_A\rangle \otimes |\psi_B\rangle$$

- **SWAP gate:** $|\delta_0 - \delta_1| = \pi/2$ **(spin exchange operator P_s)**

$$\hat{S} \propto -\frac{1 - \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} + \frac{3 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{4} = \frac{1 + \boldsymbol{\sigma} \cdot \boldsymbol{\sigma}}{2} = \mathrm{SWAP}$$

$$\mathrm{SWAP}\,|\psi_A\rangle \otimes |\psi_B\rangle = |\psi_B\rangle \otimes |\psi_A\rangle$$

Identity and SWAP are the only minimal entanglers

<— parametrization of $\mathrm{SU}(4)/(\mathrm{SU}(2) \times \mathrm{SU}(2))$

Summary of overview part

- **Entanglement power** of S-matrix ~ ability of generating entanglement

$$E(\hat{S}) = \overline{\mathcal{E}(\hat{S}\,|\psi_A\rangle \otimes |\psi_B\rangle)}$$

- Entanglement suppression: **requireing** $E(\hat{S}) = 0$ for NN scattering, the following symmetries emerge

  - $\mathrm{SU}(4)$ spin-flavor symmetry ($\hat{S} \propto 1$)

  - NR conformal symmetry ($\hat{S} \propto \mathrm{SWAP}$)

S. Beane, D.B. Kaplan, N. Klco, M.J. Savage, PRL122, 102001 (2019);
I. Low, T. Mehen, PRD104, 074014 (2021)

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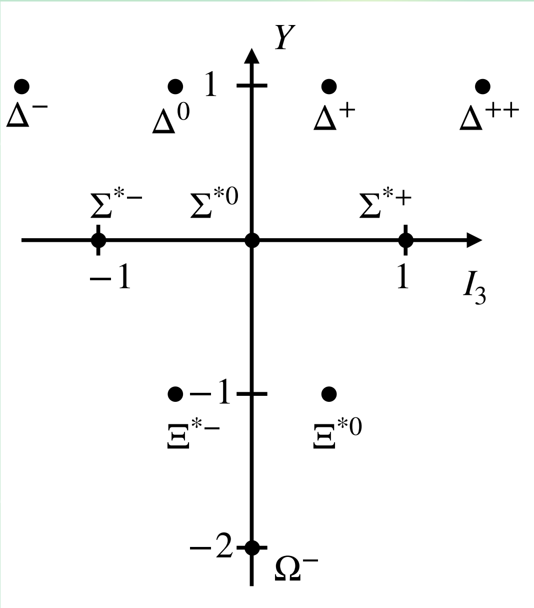
SU(3) baryon decuplet scattering

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]

- Spin 3/2 scattering
- Emergent symmetries
- Sub-unitary S-matrix

Summary

Motivation



Decuplet baryons

- spin 3/2
- Octet + decuplet: ground state of SU(6)

Possibly large scattering length

- $\Delta\Delta$ dibaryon, $d^*(2380)$?

F. Dyson, N.H. Xuong, PRL13, 815 (1964);
P. Adlarson *et al.*, (WASA-at-COSY), PRL102, 052301 (2009)

- lattice QCD for $\Omega\Omega$ scattering: $a_0(^1\mathrm{S}_0) = 4.6\ \mathrm{fm}$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Quantum information viewpoint

- scattering of spin 3/2 particles = two-qudit quantum gate

Spin 3/2 state

Spin 3/2 (qudit): rays in $\mathbb{C}^4 \simeq \mathbb{CP}^3$

$$|\psi_{\mathbf{4}}\rangle = \begin{pmatrix} \cos\theta_1 \sin\theta_2 \sin\theta_3 \\ \sin\theta_1 \sin\theta_2 \sin\theta_3 e^{i\nu_1} \\ \cos\theta_2 \sin\theta_3 e^{i\nu_2} \\ \cos\theta_3 e^{i\nu_3} \end{pmatrix}, \quad d\omega_{\mathbf{4}} = \frac{3!}{\pi^3} \prod_{i=1}^{3} d\theta_i d\nu_i \cos\theta_i \sin^{2i-1}\theta_i,$$

$$\theta_i \in [0,\pi/2), \quad \nu_i \in [0,2\pi)$$

Spin decomposition (2 symmetric, 2 anti-symmetric)

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\mathrm{A}} \oplus \underbrace{1 \oplus 3}_{\mathrm{S}}$$

S-matrix: 4 components, 4 phase shifts

$$\hat{S} = \mathscr{J}_0 \exp\{2i\delta_0\} + \mathscr{J}_1 \exp\{2i\delta_1\} + \mathscr{J}_2 \exp\{2i\delta_2\} + \mathscr{J}_3 \exp\{2i\delta_3\}$$

$$\mathscr{J}_0 = \frac{33}{128} + \frac{31}{96}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right) - \frac{5}{72}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right)^2 - \frac{1}{18}\left(\boldsymbol{t}_{3/2} \cdot \boldsymbol{t}_{3/2}\right)^3, \cdots$$

Entanglement power

Entanglement power (~3 days on mathematica)

$$
\begin{aligned}
E(\hat{S}) = \frac{1}{200000}\Big\{ & 77482 - 2100\cos\left[2\left(\delta_0+\delta_1-\delta_2-\delta_3\right)\right] \\
& -2100\cos\left[2\left(\delta_0-\delta_1+\delta_2-\delta_3\right)\right] - 2100\cos\left[2\left(\delta_0-\delta_1-\delta_2+\delta_3\right)\right] \\
& -1200\cos\left[2\left(\delta_0-2\delta_1+\delta_2\right)\right] - 4200\cos\left[2\left(\delta_0+\delta_2-2\delta_3\right)\right] \\
& -8400\cos\left[2\left(\delta_1-2\delta_2+\delta_3\right)\right] \\
& -375\cos\left[4\left(\delta_0-\delta_1\right)\right] - 10800\cos\left[2\left(\delta_0-\delta_2\right)\right] - 625\cos\left[4\left(\delta_0-\delta_2\right)\right] \\
& -875\cos\left[4\left(\delta_0-\delta_3\right)\right] - 2175\cos\left[4\left(\delta_1-\delta_2\right)\right] - 26376\cos\left[2\left(\delta_1-\delta_3\right)\right] \\
& -5481\cos\left[2\left(\delta_1-\delta_3\right)\right] - 10675\cos\left[2\left(\delta_2-\delta_3\right)\right]\Big\}
\end{aligned}
$$

$E(\hat{S}) = 0$ **is achieved if (all cos = +1)**

$$
\delta_0 = \delta_2 = \delta_{\rm even}, \quad \delta_1 = \delta_3 = \delta_{\rm odd}, \quad \begin{cases} |\delta_{\rm even} - \delta_{\rm odd}| = 0 \\ |\delta_{\rm even} - \delta_{\rm odd}| = \pi/2 \end{cases}
$$

1 and SWAP?

SWAP operators

SWAP operator for general spin scattering

$$\mathrm{SWAP} = \sum_i \mathcal{S}_i - \sum_i \mathcal{A}_i$$

symmetric anti-symmetric

—> one can show that $\mathrm{SWAP}\,|\psi_A\rangle \otimes |\psi_B\rangle = |\psi_B\rangle \otimes |\psi_A\rangle$

- spin 1/2 scattering

$$\mathrm{SWAP} = \frac{3 + \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4} - \frac{1 - \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}}{4} = \mathcal{J}_1 - \mathcal{J}_0$$

symmetric anti-symmetric

- spin 3/2 scattering

$$\mathrm{SWAP} = \mathcal{J}_1 + \mathcal{J}_3 - \mathcal{J}_0 - \mathcal{J}_2$$

symmetric anti-symmetric

Minimal entanglers

S-matrix with $E(\hat{S}) = 0$

- **Identity gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = 0$

  $$\hat{S} \propto \mathcal{J}_0 + \mathcal{J}_1 + \mathcal{J}_2 + \mathcal{J}_3 = 1$$

- **SWAP gate:** $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2$

  $$\hat{S} \propto -\mathcal{J}_0 + \mathcal{J}_1 - \mathcal{J}_2 + \mathcal{J}_3$$

Identity and SWAP are the only minimal entanglers, also for spin 3/2 scattering

Shown to hold for any bipartite system with same dimension

E. Alfsen, F. Shultz, JMP51, 052201 (2010);…

Decuplet-decuplet scattering

Spin and flavor decomposition

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\mathrm{A}} \oplus \underbrace{1 \oplus 3}_{\mathrm{S}}$$

$$\mathbf{10} \otimes \mathbf{10} = \underbrace{\mathbf{27} \oplus \mathbf{28}}_{\mathrm{S}} \oplus \underbrace{\overline{\mathbf{10}} \oplus \mathbf{35}}_{\mathrm{A}}$$

S-matrix in s-wave: spin⊗flavor should be A

$$\hat{S} = \mathcal{J}_0 \otimes \left(\mathcal{F}_{\mathbf{27}}\, e^{2i\delta_{0,\mathbf{27}}} + \mathcal{F}_{\mathbf{28}}\, e^{2i\delta_{0,\mathbf{28}}}\right) + \mathcal{J}_1 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}}\, e^{2i\delta_{1,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}}\, e^{2i\delta_{1,\mathbf{35}}}\right)$$

$$+ \mathcal{J}_2 \otimes \left(\mathcal{F}_{\mathbf{27}}\, e^{2i\delta_{2,\mathbf{27}}} + \mathcal{F}_{\mathbf{28}}\, e^{2i\delta_{2,\mathbf{28}}}\right) + \mathcal{J}_3 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}}\, e^{2i\delta_{3,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}}\, e^{2i\delta_{3,\mathbf{35}}}\right)$$

$E(\hat{S}) = 0$ is achieved if

$$\delta_{0,F} = \delta_{2,F} = \delta_{\mathrm{even}}, \quad \delta_{1,F} = \delta_{3,F} = \delta_{\mathrm{odd}}, \quad \begin{cases} |\delta_{\mathrm{even}} - \delta_{\mathrm{odd}}| = 0 \\ |\delta_{\mathrm{even}} - \delta_{\mathrm{odd}}| = \pi/2 \end{cases}$$

Emergent symmetries

Emergent symmetry <— effective Lagrangian

Q. Liu, I. Low, T. Mehen, PRC107, 025204 (2023)

- spin $i = 1,2,3,4$, flavor $T = (\Delta^{++}, \ldots, \Omega^{-})$

Effective Lagrangian for $|\delta_{\rm even} - \delta_{\rm odd}| = 0$

$$\mathscr{L} = c_1 (\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{i})^2$$

- rotation of 40 component spin-flavor vector
  —> $\mathrm{SU}(40)$ spin flavor symmetry

Effective Lagrangian for $|\delta_{\rm even} - \delta_{\rm odd}| = \pi/2$

$$\mathscr{L} = -\frac{2\pi}{\mu\Lambda}\left[-\frac{1}{4}\left(\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{i}\right)^2 \pm \frac{1}{4}\left(\boldsymbol{T}^{\dagger i} \cdot \boldsymbol{T}^{j}\right)\left(\boldsymbol{T}^{\dagger j} \cdot \boldsymbol{T}^{i}\right)\right]$$

- independent rotation of 4 spin and 10 flavor
  —> $\mathrm{SU}(4)_{\rm spin} \times \mathrm{SU}(10)_{\rm flavor}$ + NR conformal symmetry

Idntical particles and sub-unitary S-matrix

Identical particles: symmetric spin states are forbidden

- **nn scattering**

$$\hat{S}_{nn} = \mathcal{J}_0 \exp\{2i\delta_0\}$$

- **$\Omega\Omega$ scattering**

$$\hat{S}_{\Omega\Omega} = \mathcal{J}_0 \exp\{2i\delta_{0,\mathbf{28}}\} + \mathcal{J}_2 \exp\{2i\delta_{2,\mathbf{28}}\}$$

Naive calculation: larger than Bell state?

$$E(\hat{S}_{nn}) = \frac{23}{24}$$

S-matrix is not unitary in full space: $\hat{S}_{nn}\hat{S}^{\dagger}_{nn} = \mathcal{J}_0 \neq 1$

- **density matrix is not properly normalized**

$$\rho = |\psi_{\rm out}\rangle\langle\psi_{\rm out}| = \hat{S}_{nn}|\psi_{\rm in}\rangle\langle\psi_{\rm in}|\hat{S}^{\dagger}_{nn}, \quad {\rm Tr}[\rho] = \langle\psi_{\rm in}|\mathcal{J}_0|\psi_{\rm in}\rangle < 1$$

Normalized entanglement power

Normalized density matrix

M.A. Nielsen, I.L. Chuang, *Quantum Computation and Quantum Information*, Cambridge

$$\tilde{\rho} = \frac{|\psi_{\rm out}\rangle\langle\psi_{\rm out}|}{\langle\psi_{\rm out}|\psi_{\rm out}\rangle}$$

k-weighted normalized entanglement power

$$E_k(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \langle\psi_{\rm in}|\mathscr{P}_A|\psi_{\rm in}\rangle^k \ {\rm Tr}_A\left[\tilde{\rho}_A{}^2\right]}{\int d\omega_A d\omega_B \langle\psi_{\rm in}|\mathscr{P}_A|\psi_{\rm in}\rangle^k}$$

For nn scattering, result does not depend on k

$$E_k(\hat{S}_{nn}) = \frac{1}{2}$$

$k = 2$ is practically convenient

$$E_2(\hat{S}) = 1 - \frac{\int d\omega_A d\omega_B \,{\rm Tr}_A\left[\rho_A{}^2\right]}{\int d\omega_A d\omega_B \langle\psi_{\rm in}|\mathscr{P}_A|\psi_{\rm in}\rangle^2}$$

ΩΩ scattering

Entanglement power of ΩΩ S-matrix

$$E_2(\hat{S}_{\Omega\Omega}) = \frac{1}{48}\left\{25 - \cos\left[4\left(\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}\right)\right]\right\}$$

$E_2(\hat{S}) = 0$ **is achieved if**

$$\begin{cases} |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0 \\ |\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2 \end{cases}$$

Lattice QCD shows $a_0(^1S_0) = 4.6\ \mathrm{fm} \gg 1/m_\pi$ **—>** $\delta_{0,\mathbf{28}} \sim \pi/2$

S. Gongyo, *et al.*, (HAL QCD), PRL120, 212001 (2018)

Entanglement suppression suggests either

- **spin 2 channel is near the unitary limit (**$|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = 0$**), or**
- **spin 2 channel is almost noninteracting (**$|\delta_{0,\mathbf{28}} - \delta_{2,\mathbf{28}}| = \pi/2$**)**

Summary

- **Entanglement suppression for baryon decuplet**
- $E(\hat{S}) = 0$ **is achieved only by** $\hat{S} \propto 1$ **or** $\hat{S} \propto \mathrm{SWAP}$ **for spin 3/2 scattering**
- **Largest emergent symmetries**
  - $\mathrm{SU}(40)$ **spin-flavor symmetry (**$\hat{S} \propto 1$**)**
  - $\mathrm{SU}(4)_{\mathrm{spin}} \times \mathrm{SU}(10)_{\mathrm{flavor}}$ **+ NR conformal (**$\hat{S} \propto \mathrm{SWAP}$**)**
- **Sub-unitary S-matrix**
  - **final state should be normalized**
  - $\Omega\Omega$ **scattering: implication to spin 2 channel**

T.R. Hu, K. Sone, F.K. Guo, T. Hyodo, I. Low, arXiv:2506.08960 [hep-ph]