～中性子星の観測と理論～　研究活性化ワークショップ 2025

On the proton effective mass in dilute nuclear matter

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H. Tajima, H. Moriya, W. Horiuchi, E. Nakano, and K. Iida, PLB **851**, 138567 (2024).
田島裕之,「ポーラロン描像から探る核物質の世界」, 日本物理学会誌 (掲載受理)

Outline

- **Proton effective mass in neutron-rich matter**
- **Short-range correlations**
- **Numerical results**
- **Summary**

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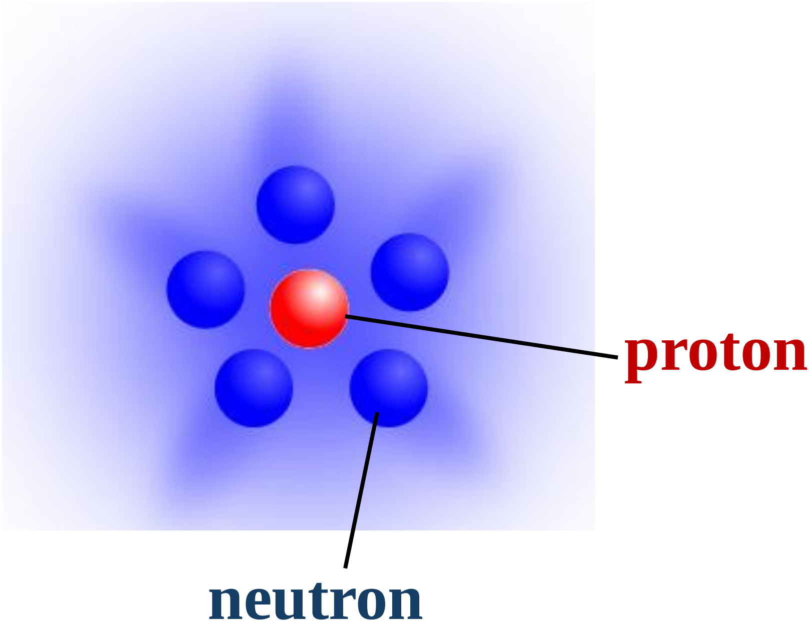
In this talk…

We are interested in impurity-like protons in dilute nuclear matter relevant to at the surface of neutron-rich nuclei and astrophysical environment.



H. Tajima, et al., PLB **851**, 138567 (2024).

Effective mass m^* in the kinetic energy

$$\varepsilon_{\boldsymbol{k}} = \frac{k^2}{2m^*}$$

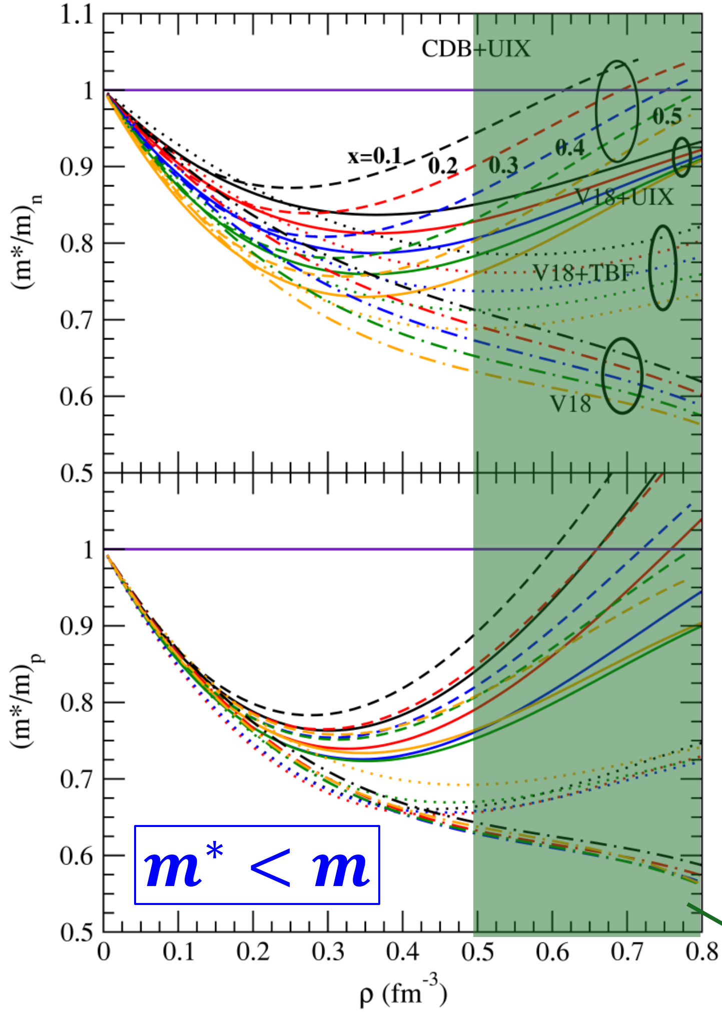
Question:
Is the nucleon effective mass in medium
larger or **smaller**
compared to bare one in vacuum?

Recent theoretical studies of nucleon effective mass

Bruckner Hartree-Fock result

Neutron effective mass

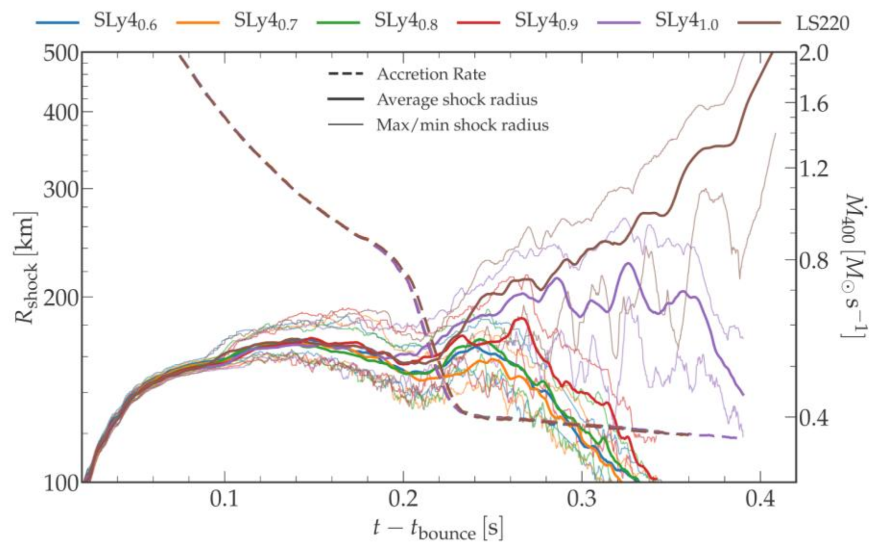
Proton effective mass



Quark DOF
becomes important

M. Baldo et al., PRC **89**,048801 (2014)

Supernova simulation with different *m**



Astrophysics community seems to accept **smaller** effective mass and examines the case with only $m^* < m$

A. S. Schneider, et el, PRC **100**, 055802 (2019)

Larger m^* ➡ Good for supernova

On the effective mass of proton in neutron star matter

G. Baym, H. A. Bethe, and C. J. Pethick, *Neutron star matter*, Nucl. Phys. A **175**, 225 (1971).

$$\frac{m_{\text{bare}}}{m_p^*} = Z\left[1 + \text{Re}\frac{\partial^2 \Sigma_{\text{p}}(\boldsymbol{k},\omega)}{\partial k^2}\right]^{-1}$$

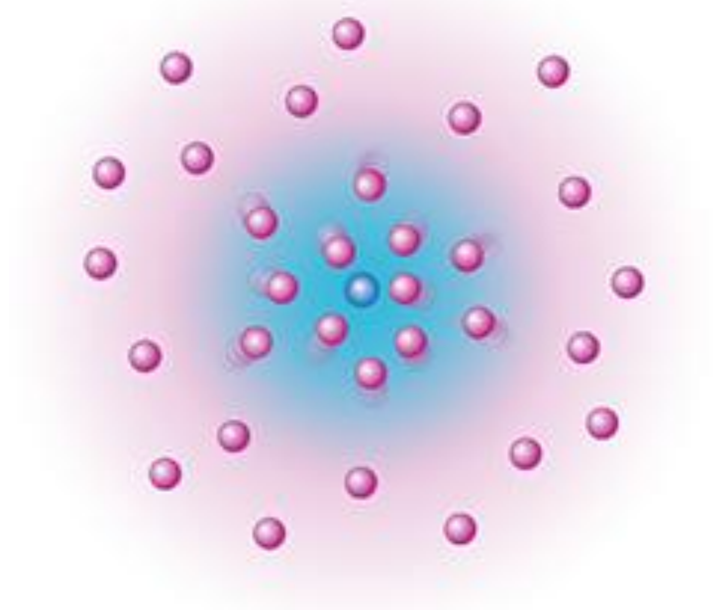
Z: quasiparticle residue (w.f. renormalization) Σ_{p}: proton self-energy

$m_{\text{p}}^*(k)$ is the effective mass of a single proton in pure neutron matter. Not having calculations of $m_{\text{p}}^*(k)$ we take it to be equal to m_{n}. One expects however that because of the strong proton-neutron attraction a single proton in a pure neutron gas will carry a considerable dressing cloud of neutrons with it, which will lead to a significant enhancement of the proton effective mass. This should be contrasted with symmetric nuclear matter where empirically $m_{\text{p}}^*/m_{\text{n}}$ on the Fermi surface is close to unity (see

Larger effective mass $\frac{m_{\text{bare}}}{m_p^*} < 1$

Protons in neutron matter $\simeq$ Fermi polaron

Impurity in Fermi sea
-Fermi polaron-



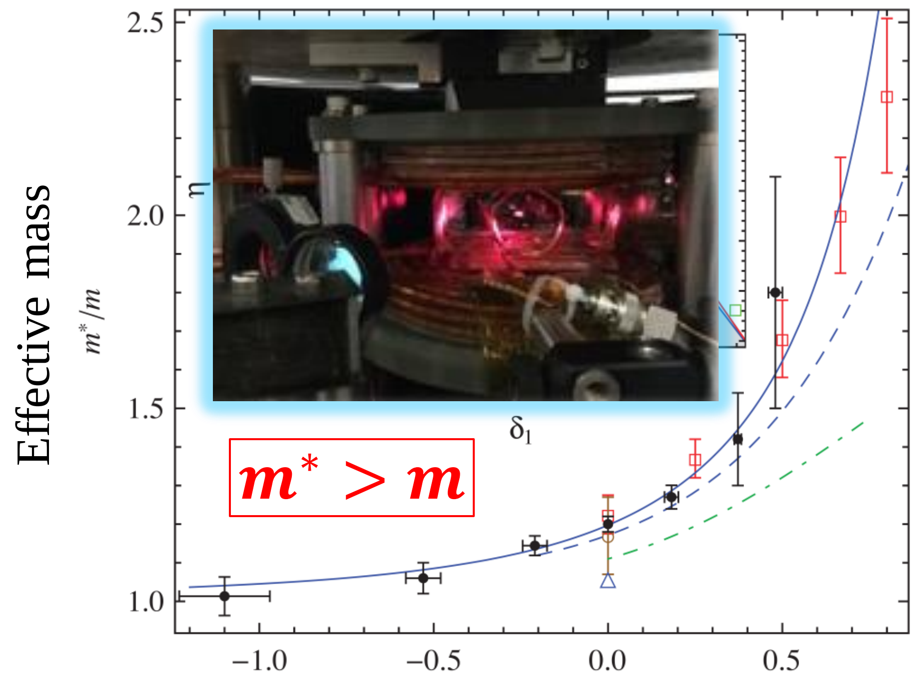
F. Chevy, Physics **9**, 86 (2016).

Impurity (~proton)

Medium (~neutron)

Resonant interaction (np interaction)

Effective mass of Fermi polaron
in ultracold atom experiment



Inverse scattering length $1/a\sqrt{2m\mu_\uparrow}$

Weak attraction

Strong attraction

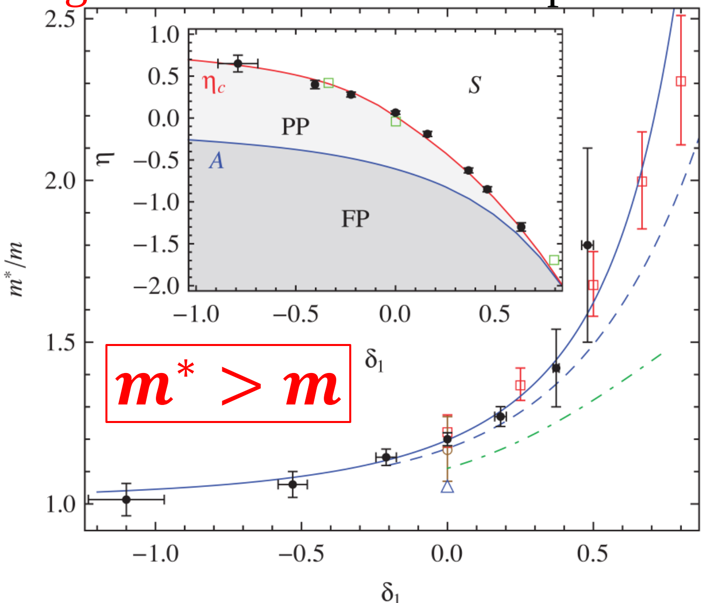
N. Navon et al., Science **328**, 729 (2010).

Photo: http://www.sci.osaka-cu.ac.jp/phys/laser/research_Li.html

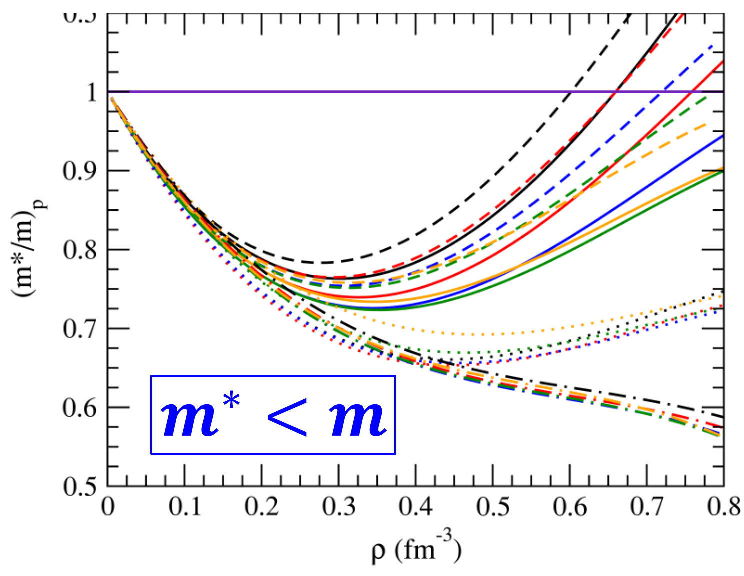
Why are there such differences?

- Can we bridge two cases with large and small effective masses of a proton in neutron-rich matter?
- When is the proton effective mass larger (smaller) than the bare mass?

Larger mass in cold-atom experiment



Smaller mass in nuclear theories



Why are there such differences?

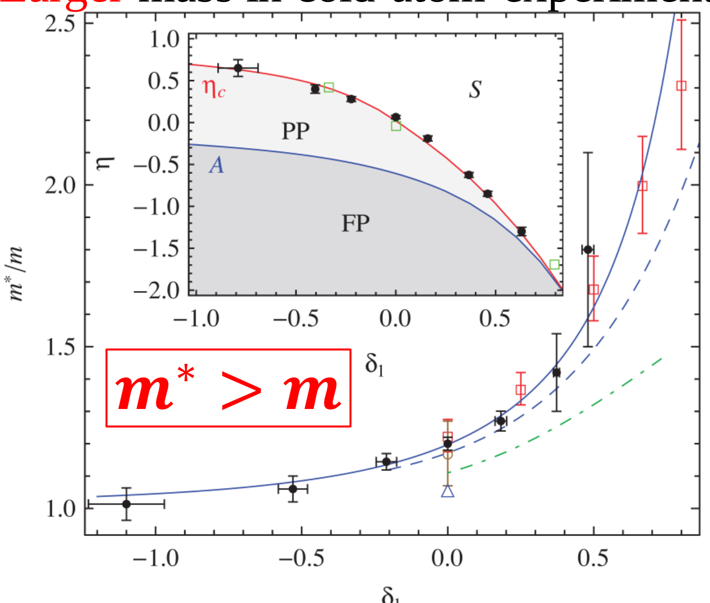
- Can we bridge two cases with large and small effective masses of a proton in neutron-rich matter?
- When is the proton effective mass larger (smaller) than the bare mass?

"My naïve speculation"

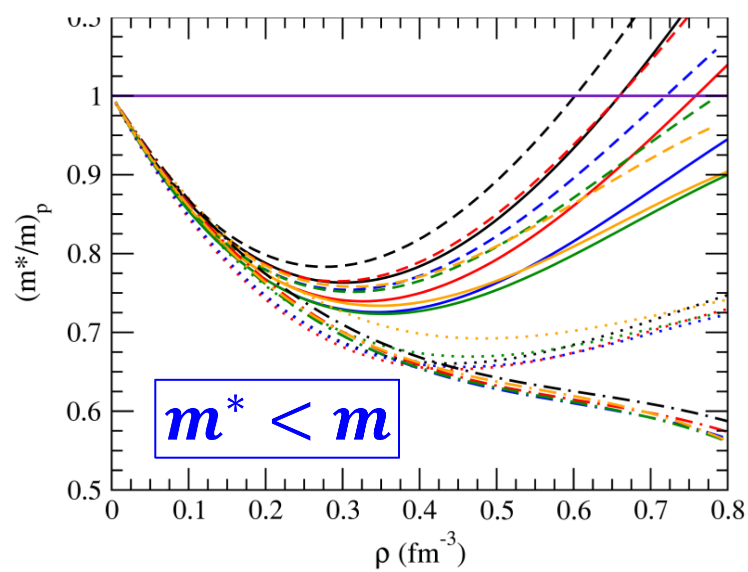
Short-range correlations in dilute regime

Long-range correlations around saturation density

Larger mass in cold-atom experiment



Smaller mass in nuclear theories



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- **Short-range correlations**
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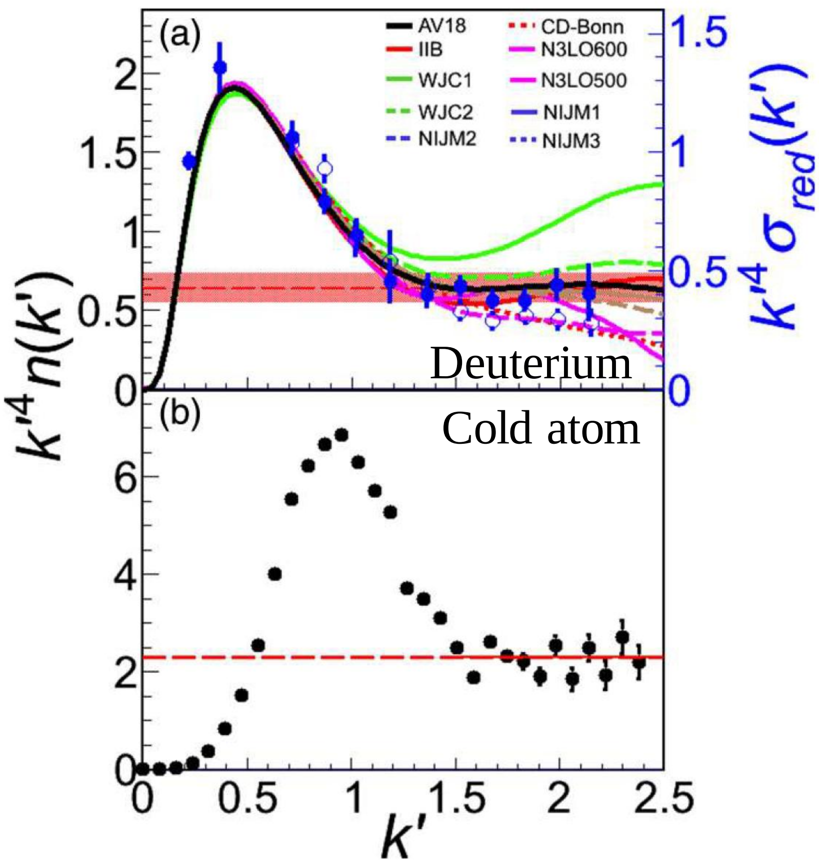
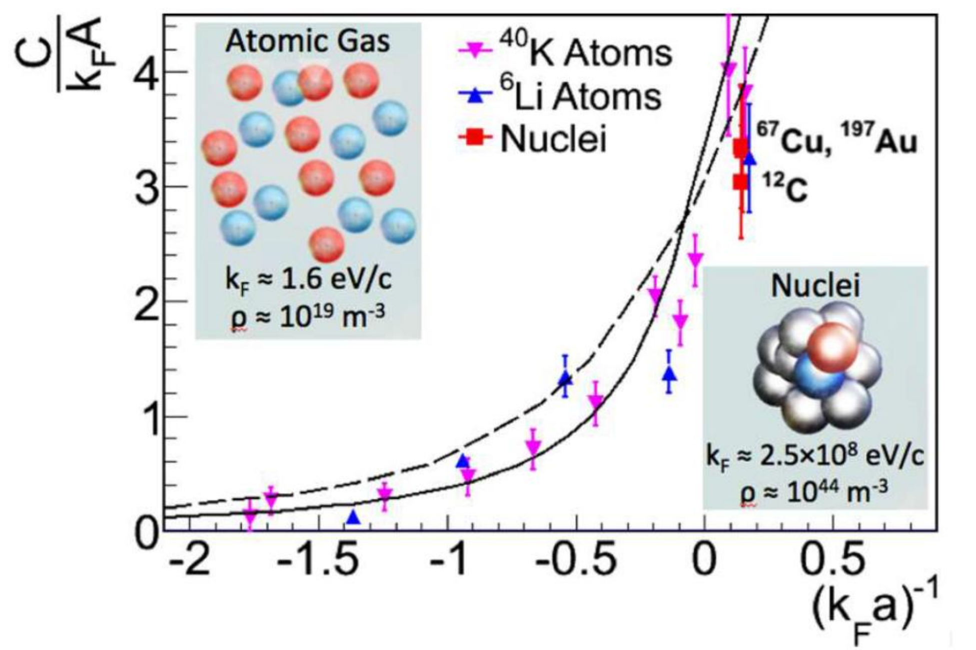
Short-range correlations

Power law in the momentum distribution



Tan's contact (k^4 coefficient)

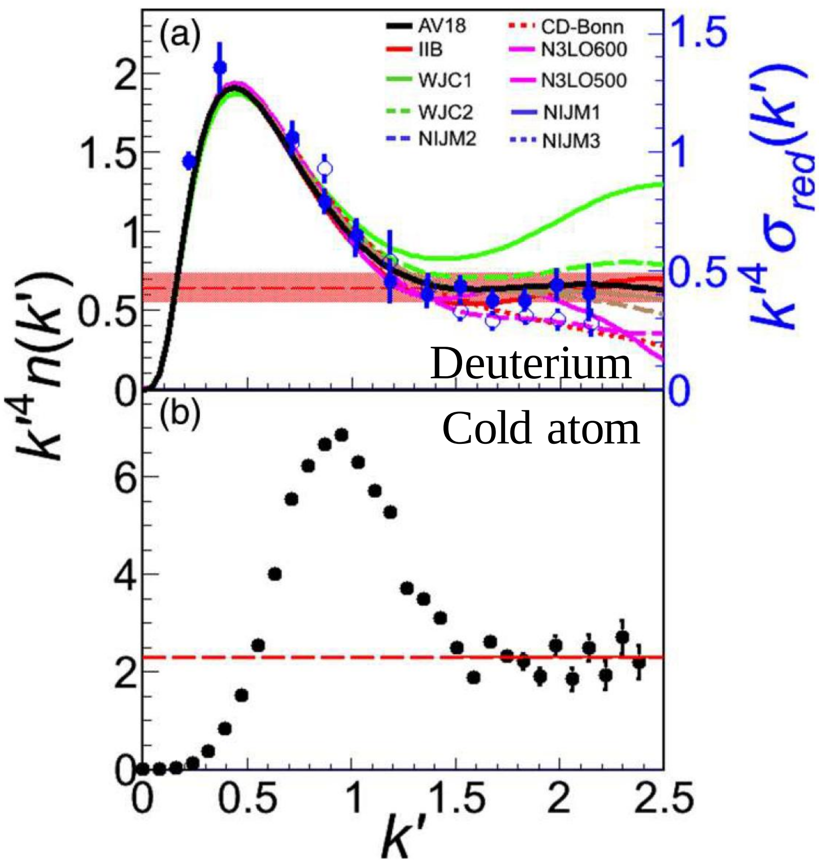


O. Hen, et al., PRC **92**,045205 (2015)

Short-range correlations compared to what?

$k \gg k_F$:Nucleon Fermi momentum

Power law in the momentum distribution



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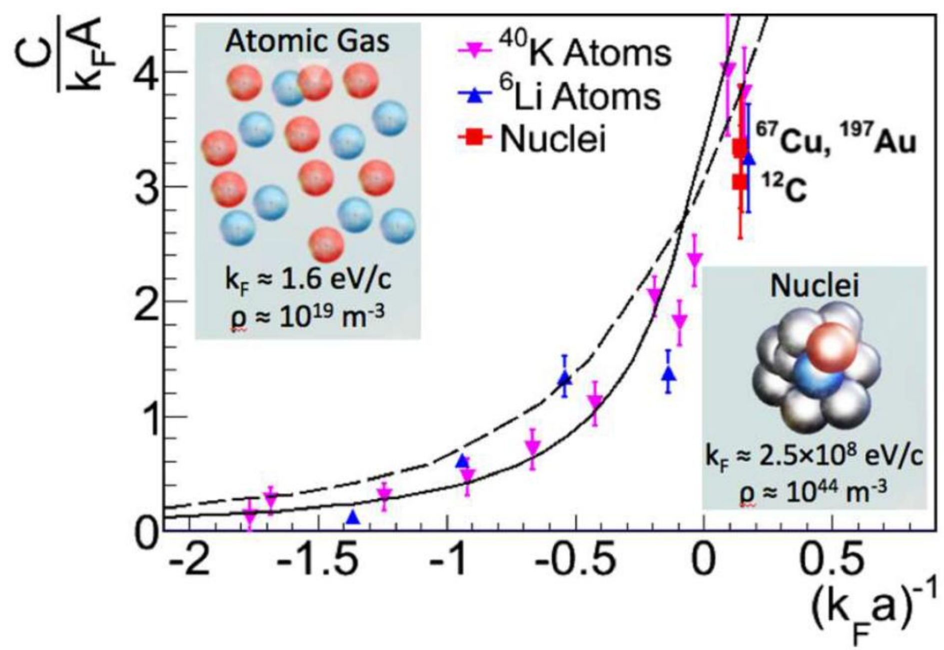
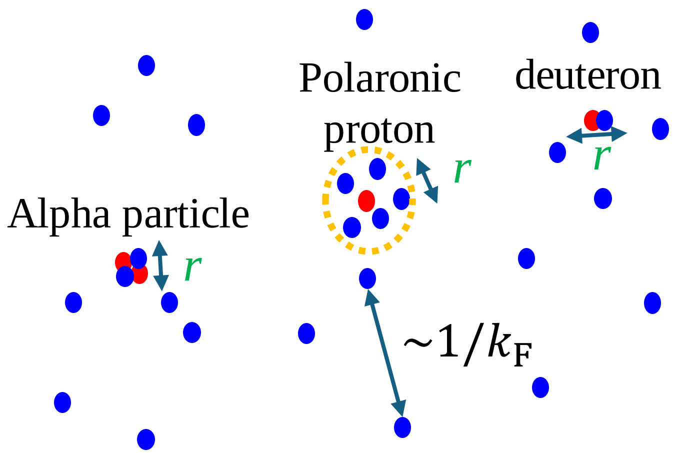


O. Hen, et al., PRC **92**,045205 (2015)

From short-range to long-range = From dilute to dense

Comparing the interaction range r with the interparticle distance $\sim 1/k_\mathrm{F}$

Dilute asymmetric nuclear matter (or surface region of nuclei)

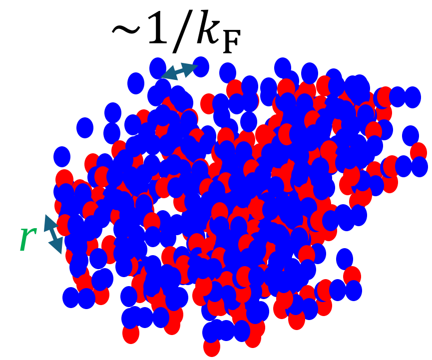


r ~ cluster or polaron size

$r \lesssim 1/k_\mathrm{F}$

"Short-range" correlations

Dense asymmetric nuclear matter (or central region of nuclei)



Interaction range ≳ Interparticle distance

$r \gtrsim 1/k_\mathrm{F}$

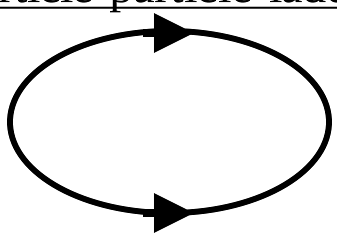
"Long-range" correlations
(or exchange term)

Diagrammatic representation of short-range and long-range correlations

- **Short-range correlations**

= Large momentum compared to k_{F}

Particle-particle ladder (ultraviolet divergence for contact interaction)

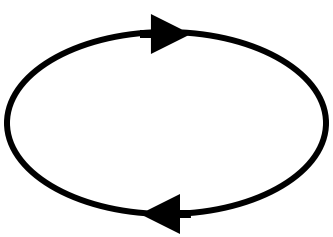


$$\sim \sum \frac{1 - f(\xi_{k+q}) - f(\xi_k)}{E - \xi_{k+q} - \xi_k} \;\xrightarrow{f(\xi_p) \to 0}\; \sim \Lambda_{\mathrm{cutoff}} \simeq \frac{1}{r_{\mathrm{eff}}}$$

- **Long-range correlations**

= Small momentum compared to k_{F}

Particle-hole bubble (important for long-range interaction (e.g., Coulomb))



$$\sim \sum \frac{f(\xi_{k+q}) - f(\xi_k)}{E + \xi_k - \xi_{k+q}} \;\xrightarrow{q \to 0}\; \sim \text{constant}$$

$f(\xi_p)$: Fermi distribution function

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Interaction potential

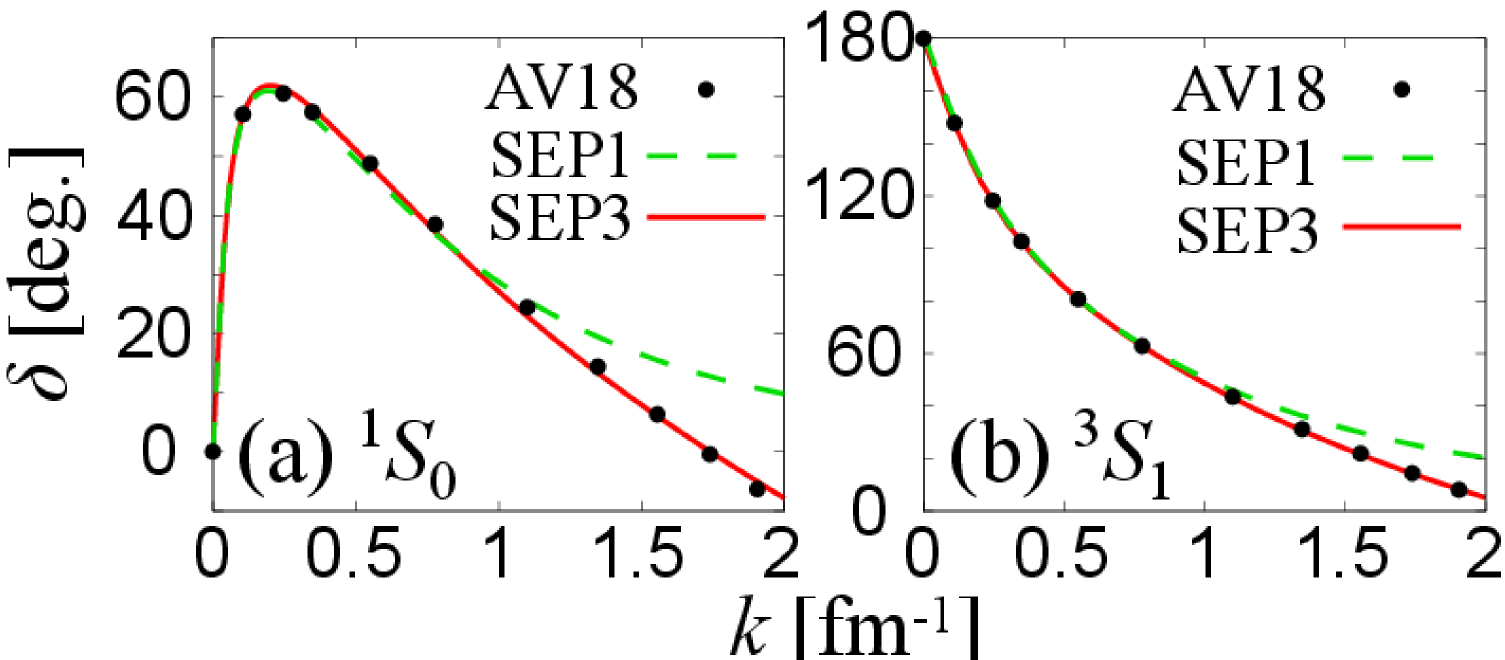
- Separable multi-rank potential (SEP n_{rank})

$$V(\boldsymbol{k},\boldsymbol{k}') = \sum_{n=1}^{n_{\mathrm{rank}}} \eta_n \gamma_n(\boldsymbol{k})\gamma_n(\boldsymbol{k}') = \boldsymbol{\gamma}^t(\boldsymbol{k})\hat{\eta}\boldsymbol{\gamma}(\boldsymbol{k}')$$

$$\gamma_n(\boldsymbol{k}) = \frac{u_n}{k^2+\Lambda_n^2} \text{ : form factor} \qquad \boldsymbol{\gamma}(\boldsymbol{k}) = \begin{pmatrix} \gamma_1(\boldsymbol{k}) \\ \gamma_2(\boldsymbol{k}) \\ \vdots \end{pmatrix} \qquad \hat{\eta} = \mathrm{diag}(\eta_1, \eta_2, \dots) \quad \eta_n = \pm 1$$

- s-wave phase shift

AV18: PRC, **51** 38 (1995).

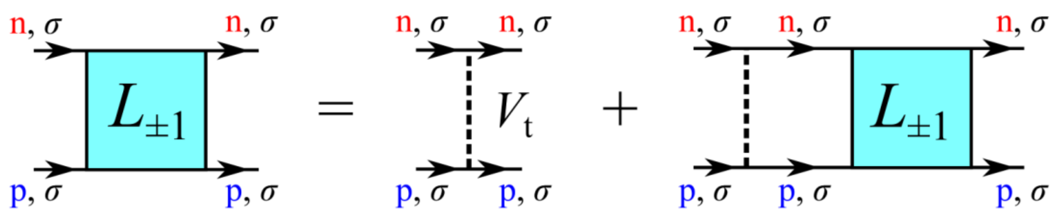


Spin-triplet neutron-proton correlations

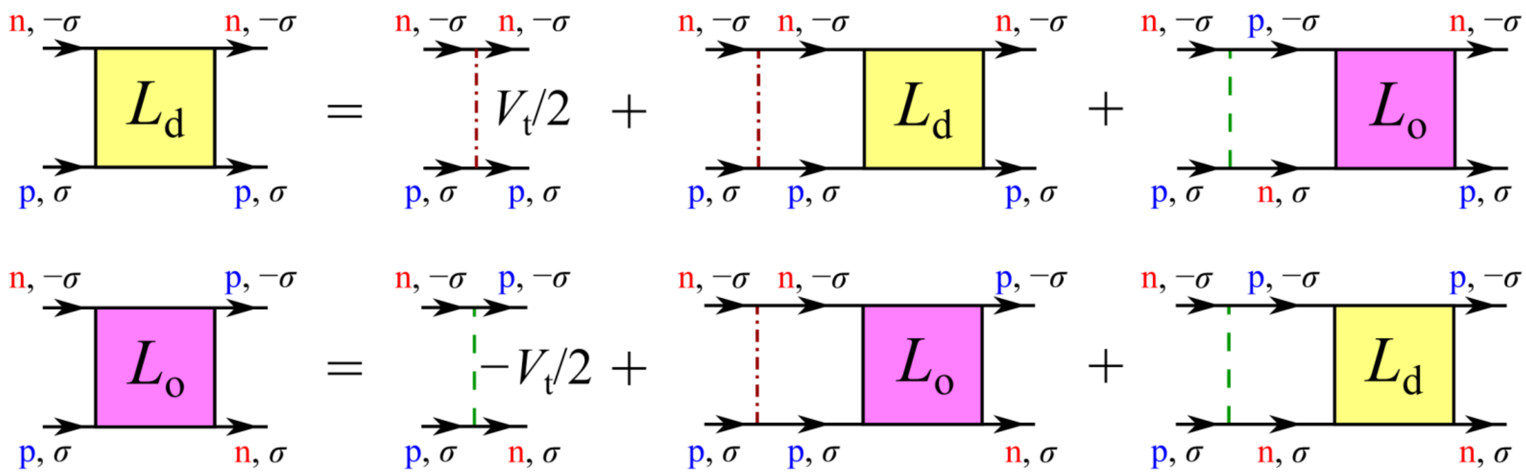
Particle-particle Ladder for short-range correlations

$$\Gamma_{\sigma\sigma'}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q},i\nu_\ell) = \left[L_1(\boldsymbol{q},i\nu_\ell)\delta_{\sigma,\sigma'} + L_0(\boldsymbol{q},i\nu_\ell)\left(1-\delta_{\sigma,\sigma'}\right)\right]\gamma_k\gamma_{k'}$$

(a) $S_z = \pm 1$ $\quad L_{\pm 1}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q},i\nu_\ell) = L_1(\boldsymbol{q},i\nu_\ell)\gamma_k\gamma_{k'}$



(b) $S_z = 0$ $\quad L_{\rm d}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q},i\nu_\ell) = -L_{\rm o}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q},i\nu_\ell) \equiv L_0(\boldsymbol{q},i\nu_\ell)\gamma_k\gamma_{k'}$



Self-energy of protonic polaron

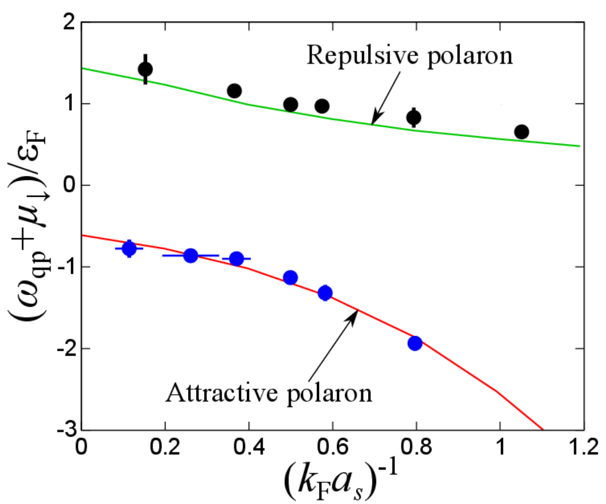
Self-energy within the particle-particle ladders (short-range correlations)

$$\Sigma_{p\sigma}(\boldsymbol{k}, i\omega_n) = T\sum_{\boldsymbol{q}}\sum_{\sigma'}\sum_{i\nu_\ell}\Gamma_{\sigma\sigma'}(\boldsymbol{q}/2-\boldsymbol{k}, \boldsymbol{q}/2-\boldsymbol{k}; \boldsymbol{q}, i\nu_\ell)G_{\rm n}(\boldsymbol{q}-\boldsymbol{k}, i\nu_\ell - i\omega_n)$$

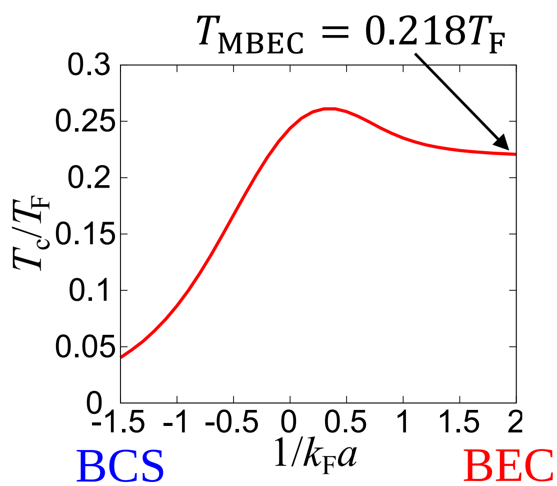
$G_{\rm n}$: neutron propagator

Reproducing cold atom experiments quantitatively

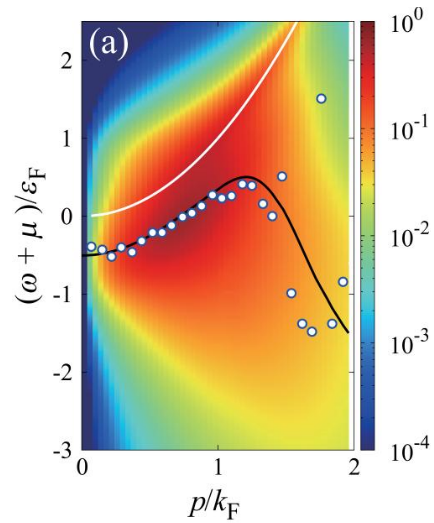
Atomic Fermi polaron energy



Critical temperature



Photoemission spectra

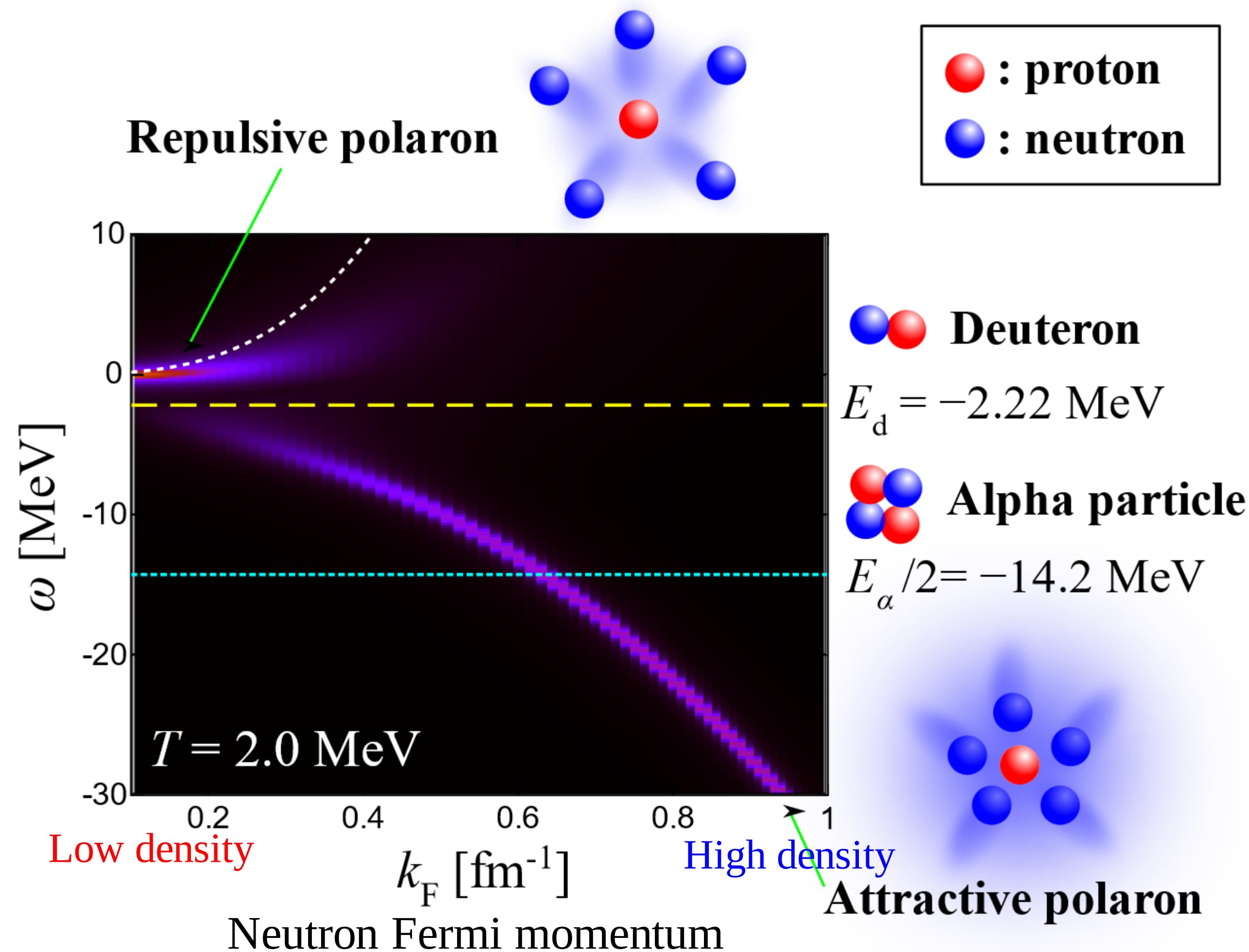


HT and S. Uchino, NJP, **20**, 073048 (2018).

M. Ota, *et al.*, PRA **95**,053623 (2017).

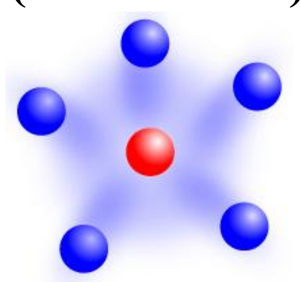
From clusters to polaronic protons

Proton spectral weight: $A_{p\sigma}(\boldsymbol{k}=\boldsymbol{0},\omega) = -\frac{1}{\pi}\mathrm{Im}G_{p\sigma}(\boldsymbol{k}=0,\omega)$



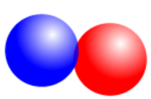
Proton effective mass

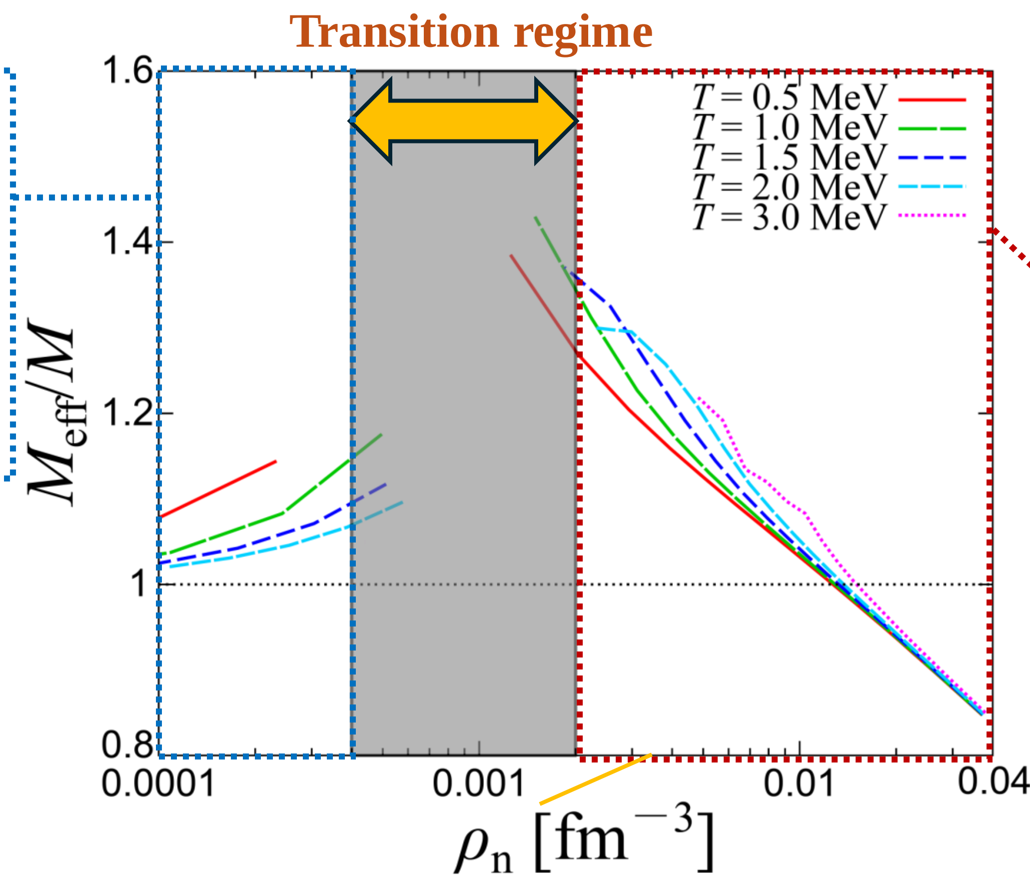
Repulsive polaron
(Excited state)



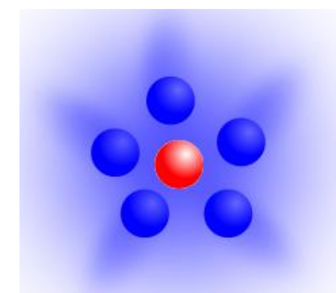
Higher energy

Deuteron clusters
(Ground state)





Attractive polaron
(ground state)

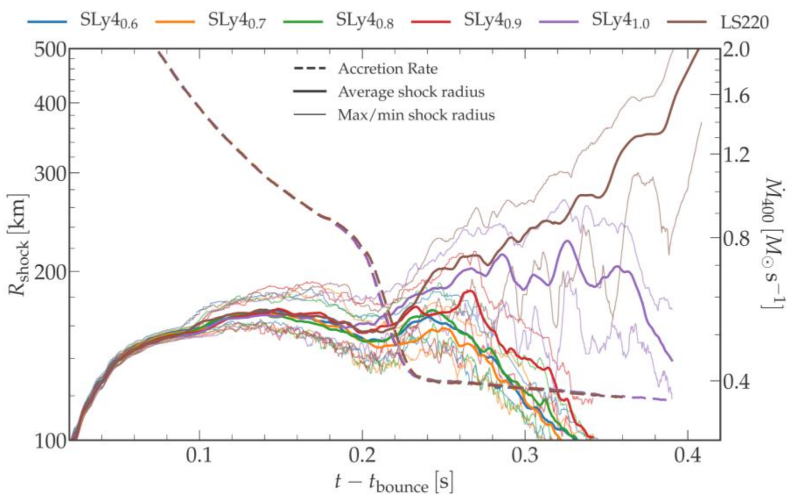


$M_{\rm eff} > M$ at low density
Consistent with a conjecture in
G. Baym, H. A. Bethe, and C. J. Pethick, Nucl. Phys. A **175**, 225 (1971).

$M_{\rm eff} < M$ at high density
Consistent with a conventional nuclear theories w/o SRC
M. Baldo et al., PRC **89**,048801 (2014).

Larger effective mass…, so what?

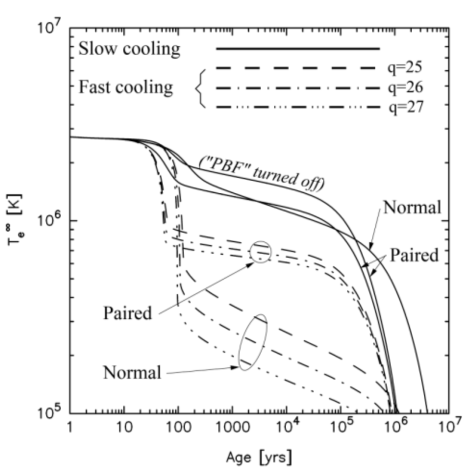
Beneficial for SN explosion



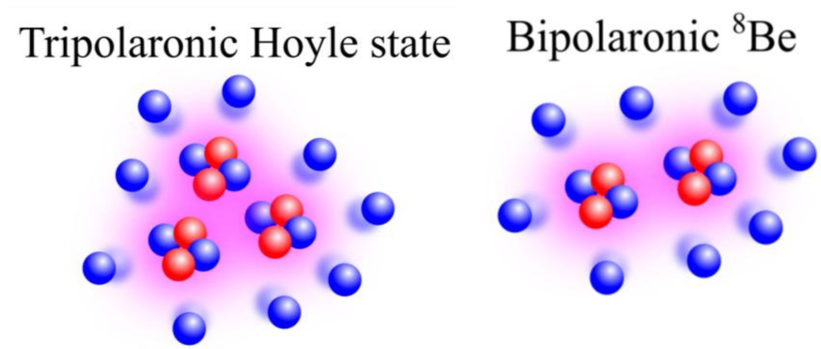
A. S. Schneider, et el, PRC **100**, 055802 (2019).

New insights on Cooling Curve

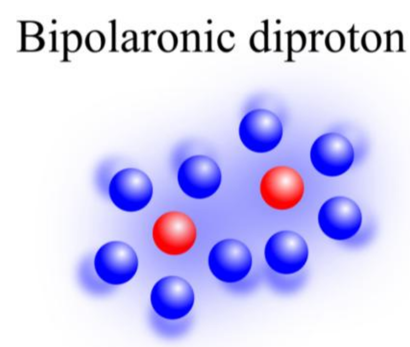
D. Page, Fifty Years of Nuclear BCS 324-447 (2013).



Unbound systems can be bound



PRC **104**, 065801 (2021).



PLB **851**, 138567 (2024).

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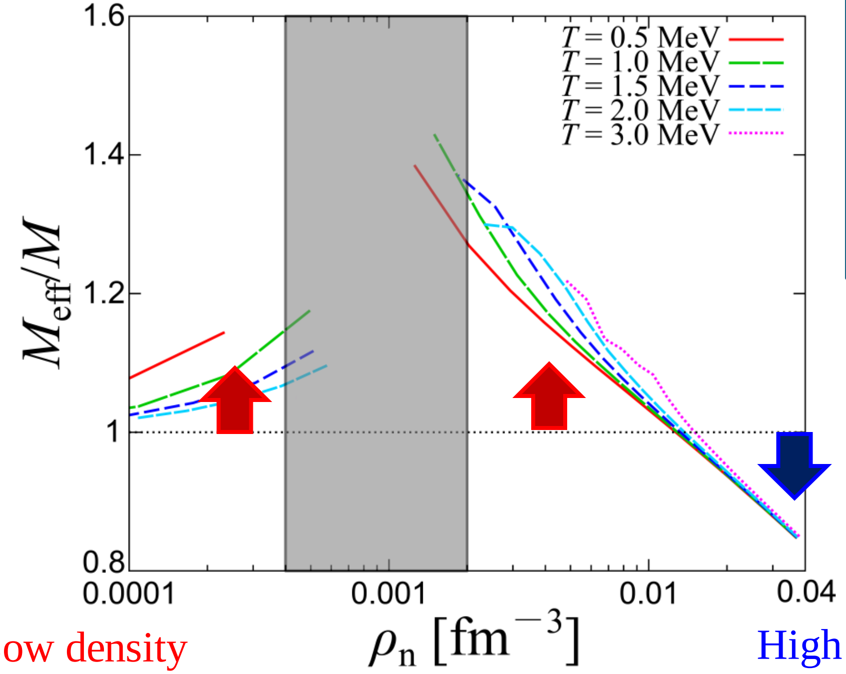
Summary

- We showed the enhanced effective mass M_{eff} of protons in dilute asymmetric nuclear matter.
- While the short-range correlations enhance M_{eff} being consistent with the Baym-Bethe-Pethick conjecture and the cold-atom experiment, the long-range correlations suppress M_{eff} at high density.
- Larger M_{eff} may give significant impact on astrophysical phenomena (e.g., supernova, cooling, clustering, etc…).

$M_{\mathrm{eff}} > M$ at low density
Due to the short-range correlations
Consistent with a conjecture in G. Baym, H. A. Bethe, and C. J. Pethick, Nucl. Phys. A **175**, 225 (1971).

H. Tajima, et al., PLB **851**, 138567 (2024).



Low density

High density

Other remarks
- Polaron spectral peak = Symmetry energy
- Peak width = Transport coefficient

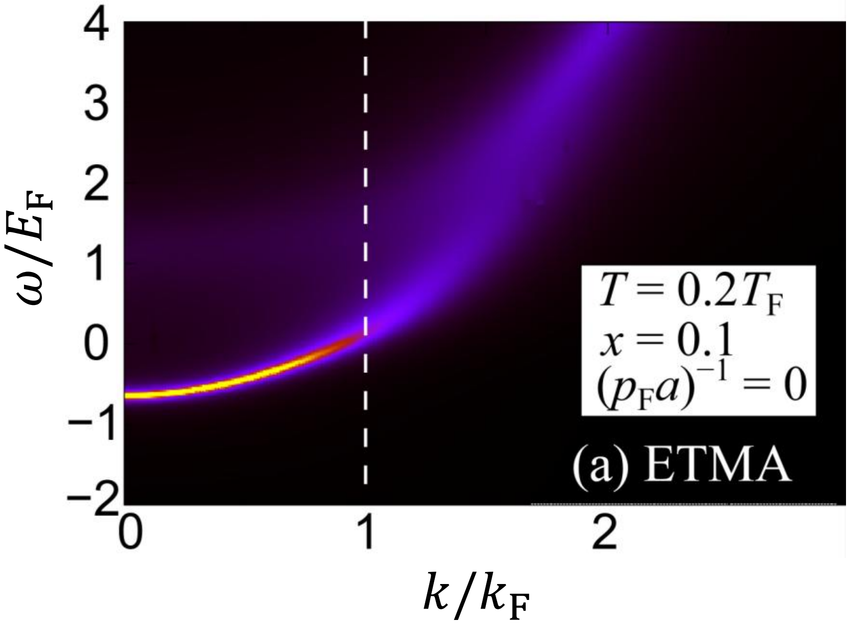
$M_{\mathrm{eff}} < M$ at high density due to the long-range correlations

Thank you for your attention!

Backup slides

Definition of the polaron effective mass

Fermi polaron spectra



Y. Sekino, HT, and S. Uchino,
Phys. Rev. Research **2**, 023152 (2020).

k_F: Fermi momentum of background Fermi sea

Protonic polaron propagator

$$G_i(\boldsymbol{k},\omega) = \frac{1}{\omega + i\delta - \varepsilon_{\boldsymbol{k},i} - \Sigma_i(\boldsymbol{k},\omega)}$$

$$\simeq \frac{Z}{\omega + i\delta - \frac{k^2}{2M^*} - E_P + i\Gamma/2}$$

Expanding self-energy at $k = 0$*

$$E_P = \mathrm{Re}\Sigma_i(\mathbf{0}, E_P),$$

$$Z = \left[1 - \mathrm{Re}\left(\frac{\partial \Sigma_i(\mathbf{0},\omega)}{\partial\omega}\right)_{\omega=E_P}\right]^{-1},$$

$$\frac{M}{M^*} = Z\left[1 + M\mathrm{Re}\left(\frac{\partial^2\Sigma_i(\boldsymbol{k},E_P)}{\partial k^2}\right)_{\boldsymbol{k}=\mathbf{0}}\right]$$

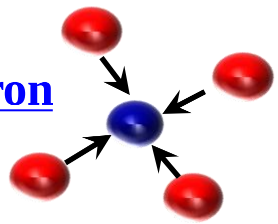
$$\Gamma = -2Z\mathrm{Im}\Sigma_i(\mathbf{0},E_P).$$

*Here we consider dilute limit of proton at $T \neq 0$ where $k_{Fp} \to 0$

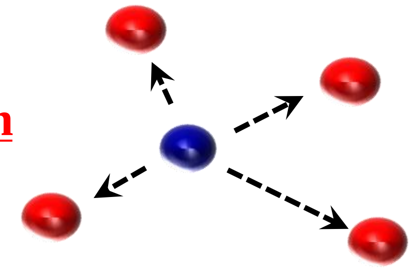
Fermi polarons in ultracold atoms

Attractive and repulsive Fermi polarons have been observed in ^{6}Li population-imbalanced Fermi gases with a positive scattering length *a*.

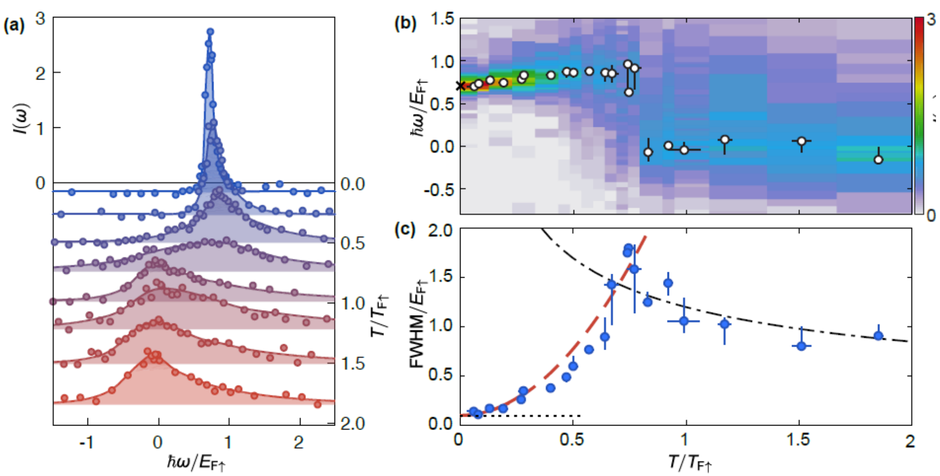
Attractive polaron



Repulsive polaron (excited branch)



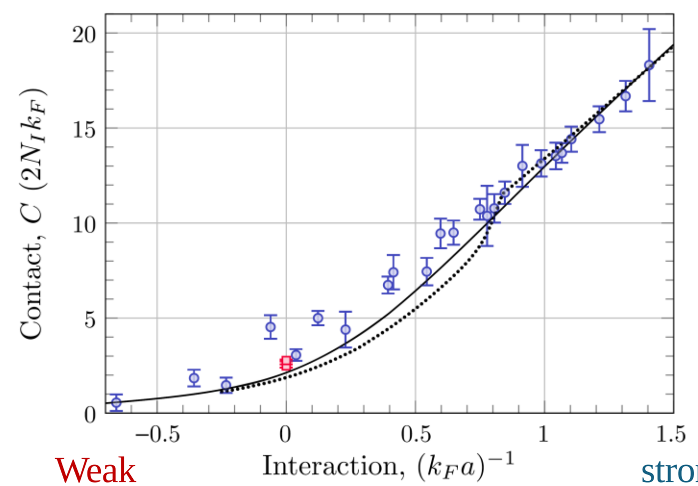
Thermal evolution of Fermi polaron spectra



Temperature

Z. Yan, et al., PRL **122**, 093401 (2019).

Interaction dependence of polaron contact



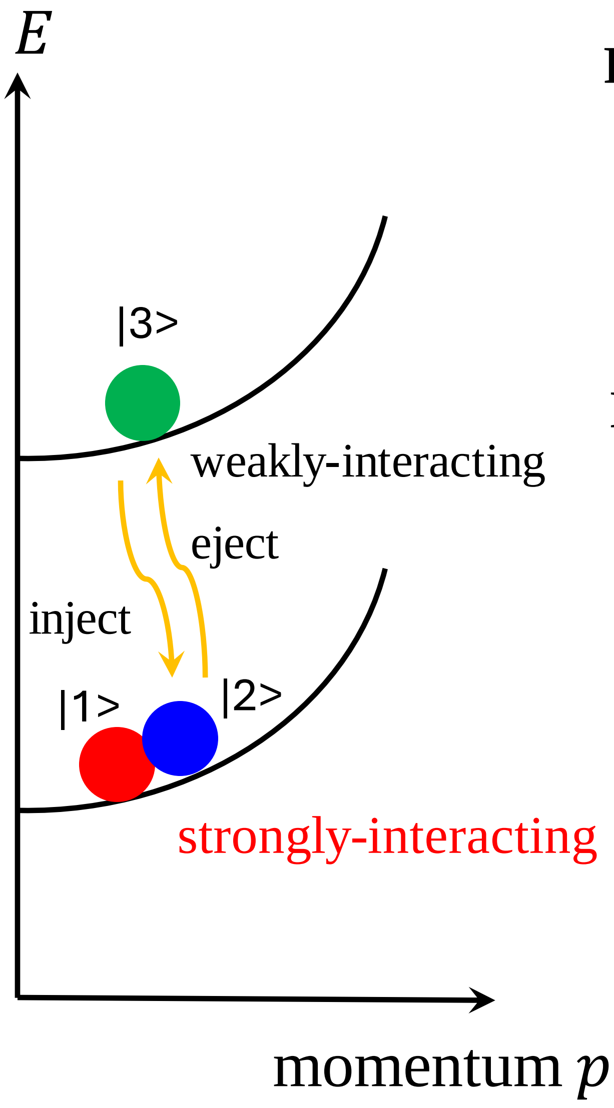
Weak coupling

interaction

strong coupling

G. Ness, et al., PRX **10**, 041019 (2020).

Radio-frequency (RF) spectroscopy in cold atoms



Polaron spectral function

$$A_{\mathrm{i}}(\boldsymbol{k},\omega) = -\frac{1}{\pi}\mathrm{Im}G_{\mathrm{i}}(\boldsymbol{k}, i\omega_n \to \omega + i\delta)$$

Injection RF spectroscopy

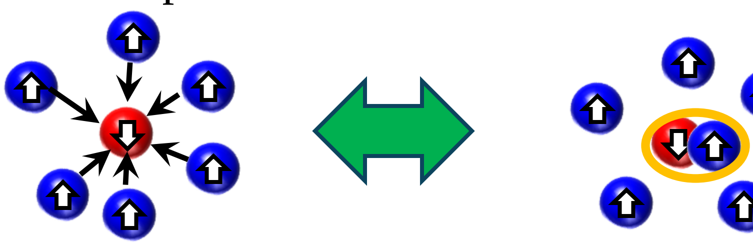
$$I_{\mathrm{in}}(\omega) = 2\pi\Omega_{\mathrm{R}}^2 \sum_{\boldsymbol{k}} f(\varepsilon_{\boldsymbol{k},\mathrm{ref.}}) A_{\mathrm{i}}(\boldsymbol{k}, \varepsilon_{\boldsymbol{k},\mathrm{i}} + \omega)$$

Ejection RF spectroscopy

$$I_{\mathrm{ej}}(\omega) = 2\pi\Omega_{\mathrm{R}}^2 \sum_{\boldsymbol{k}} f(\varepsilon_{\boldsymbol{k},\mathrm{i}} - \omega) A_{\mathrm{i}}(\boldsymbol{k}, \varepsilon_{\boldsymbol{k},\mathrm{i}} - \omega)$$

Polaron-molecule transition

Attractive polaron In-medium molecule

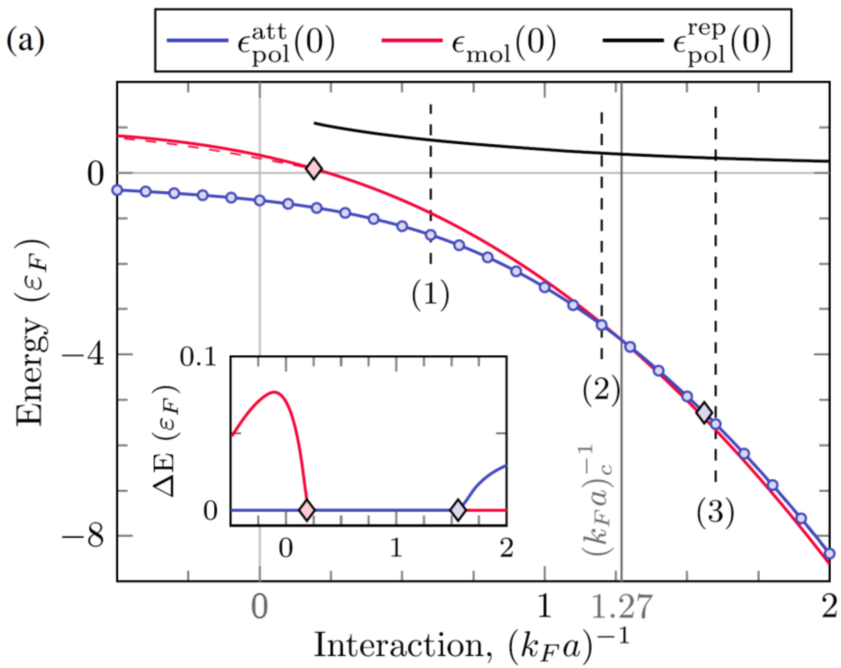


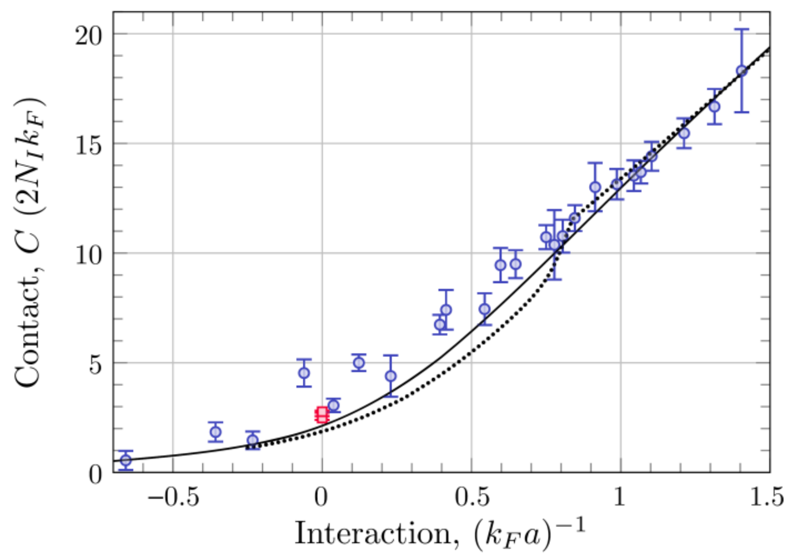
First-order transition within the single polaron ansatz at $T = 0$

Tan's contact $C = -4\pi m \dfrac{dE}{da^{-1}}$

Discontinuous change at the transition

M. Punk, et al., PRA **80**, 053605 (2009).

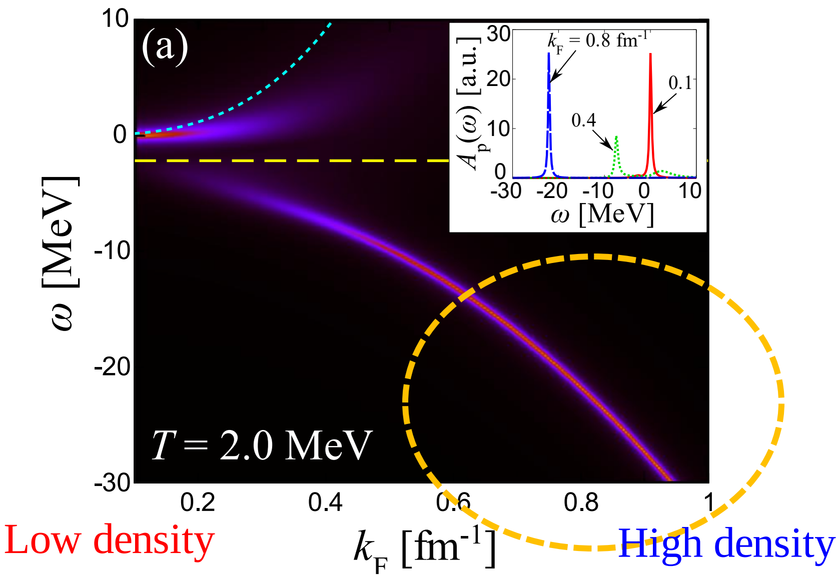




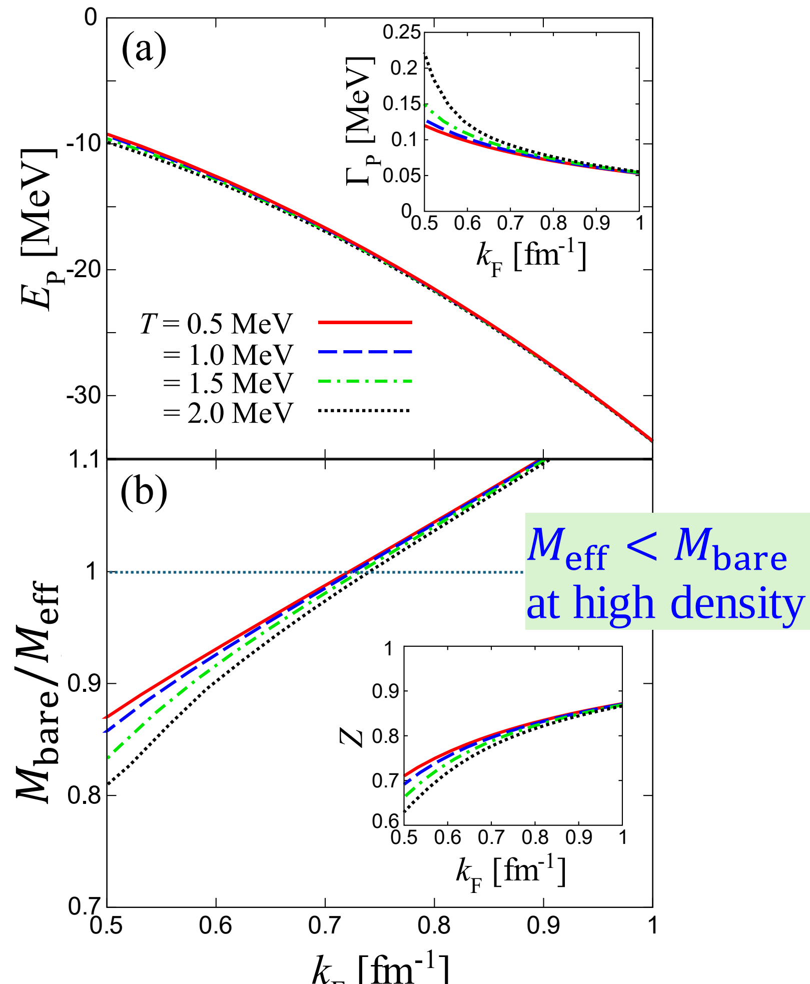
G. Ness, et al., PRX **10**, 041019 (2020).

Smooth due to the finite T and finite momentum

Proton effective mass



$$G_{p\sigma}(\boldsymbol{k},\omega) \simeq \frac{Z}{\omega_+ - \frac{k^2}{2M_{\rm eff}} - E_P + i\Gamma_P/2}$$



$M_{\rm eff} > M_{\rm bare}$ at low density

Consistent with a conjecture in

G. Baym, H. A. Bethe, and C. J. Pethick,

Nucl. Phys. A **175**, 225 (1971).

$m_p^*(k)$ is the effective mass of a single proton in pure neutron matter. Not having calculations of $m_p^*(k)$ we take it to be equal to m_n. One expects however that because of the strong proton-neutron attraction a single proton in a pure neutron gas will carry a considerable dressing cloud of neutrons with it, which will lead to a significant enhancement of the proton effective mass. This should be contrasted with symmetric nuclear matter where empirically m_p^*/m_n on the Fermi surface is close to unity (see

Polaron equation of state

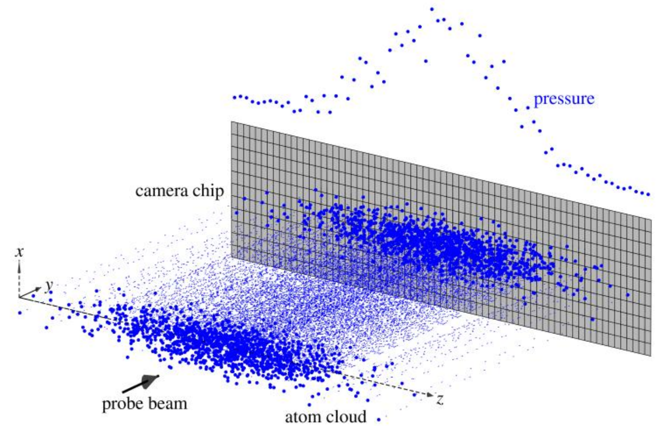
Ground-state energy

$$E = E_{\mathrm{FG}}\left[1 + \frac{5}{3}\frac{E_{\mathrm{P}}}{E_{\mathrm{F}}}\left(\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{m}}}\right) + \frac{M}{M^*}\left(\frac{\rho_{\mathrm{i}}}{\rho_{\mathrm{m}}}\right)^{\frac{5}{3}} + \cdots\right]$$

$\rho_{\mathrm{i(m)}}$: impurity (majority) density

E_{FG} : majority ground-state energy w/o impurities

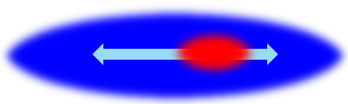
E_{F} : majority Fermi energy w/o impurities



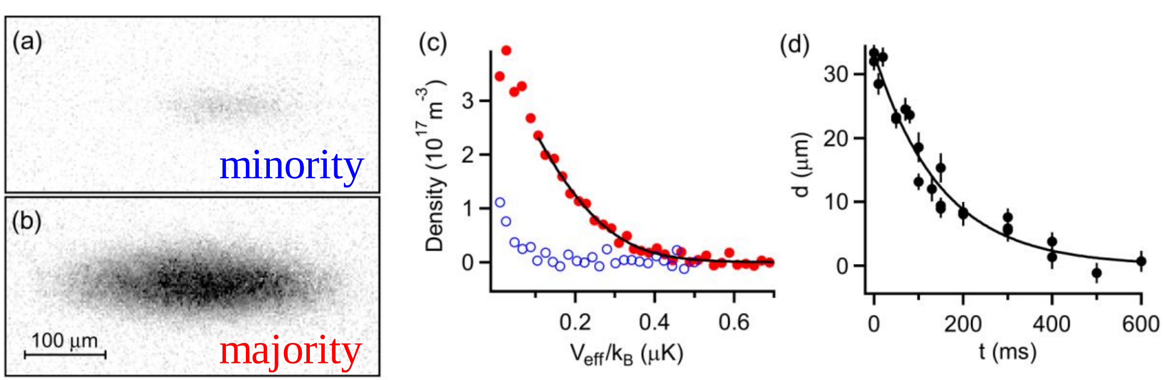
S. Nascimbène *et al* 2010 *New J. Phys.* **12** 103026

Spin-dipole frequency

Dipole oscillation of polaron cloud



$$\omega^* = \omega\sqrt{\left(1 - \frac{E_{\mathrm{P}}}{E_{\mathrm{F}}}\right)\frac{M}{M^*}}$$



A. Sommer *et al* 2011 *New J. Phys.* **13** 055009

At unitarity limit

Polaron energy

$E_{\mathrm{P}} \simeq -0.6E_{\mathrm{F}}$

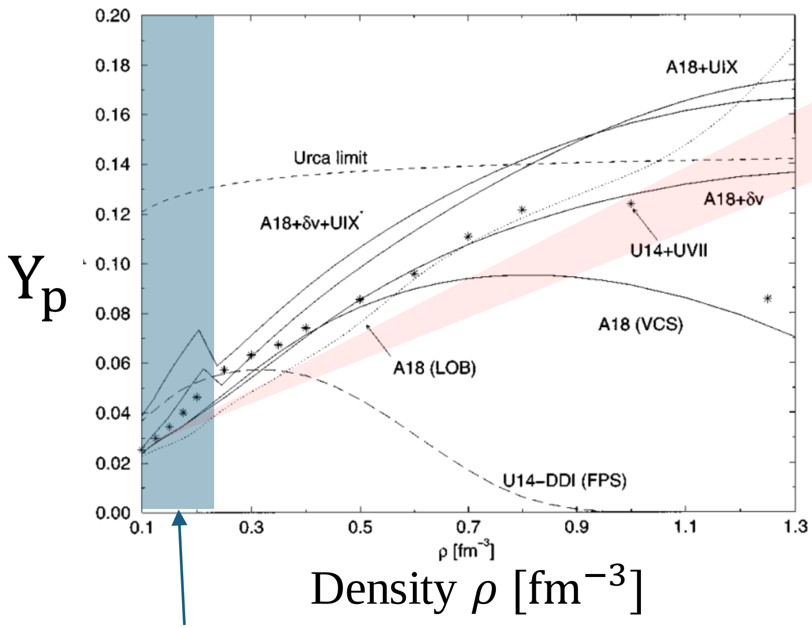
Effective mass

$M^* \simeq 1.17M$

Protons in neutron star matter

Protons may exist due to the beta equilibrium, but its fraction is smaller than neutron's one. Moreover, protons have a strong isoscalar (spin-triplet) interaction with a neutron.

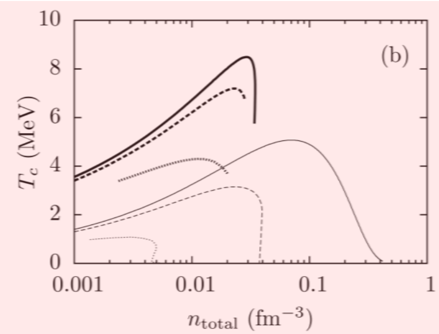
Proton fraction: $Y_\mathrm{p} = \dfrac{\rho_\mathrm{p}}{\rho_\mathrm{n} + \rho_\mathrm{p}}$



Typically $Y_\mathrm{p} \simeq 0.01 \sim 0.1$

➡ Impurity-like

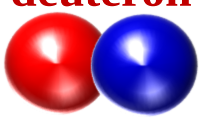
APR, PRC, **58**, 1804 (1998)

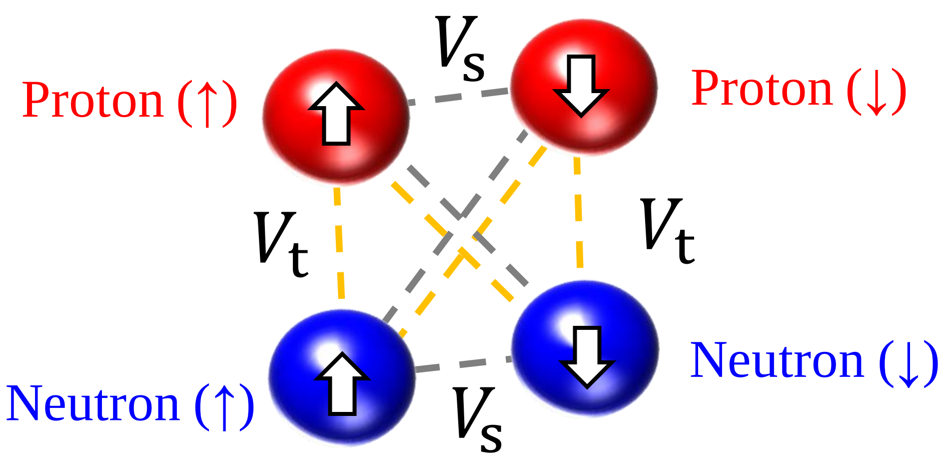


Protons remain beyond alpha Mott density $n_\mathrm{Mott} \simeq O(10^{-2})\ \mathrm{fm}^{-3}$

T. Sogo, G. Röpke, and P. Schuck, PRC **82**, 034322 (2010).

Strong spin-triplet interaction V_t ➡ deuteron



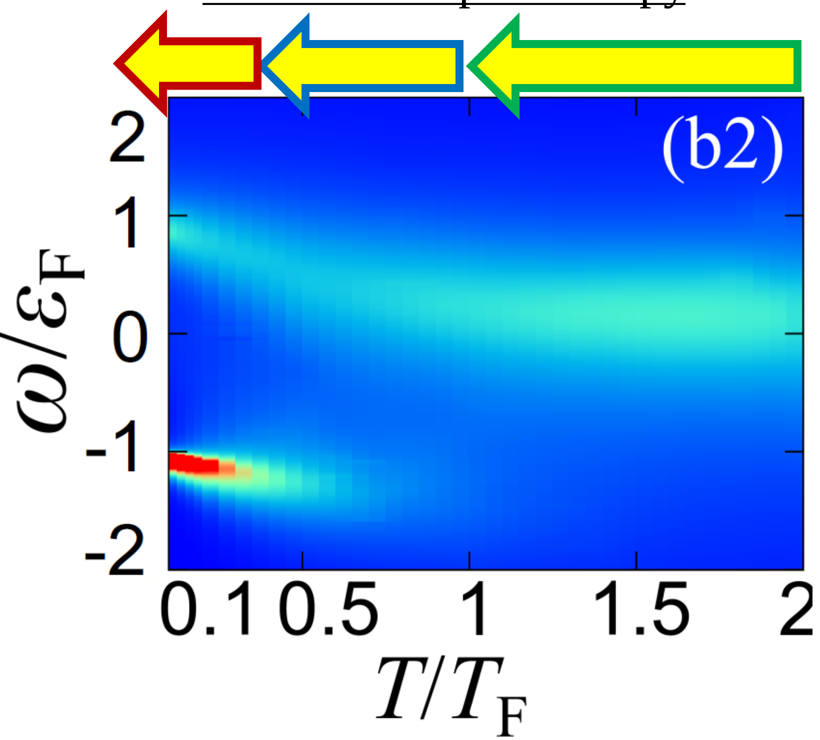


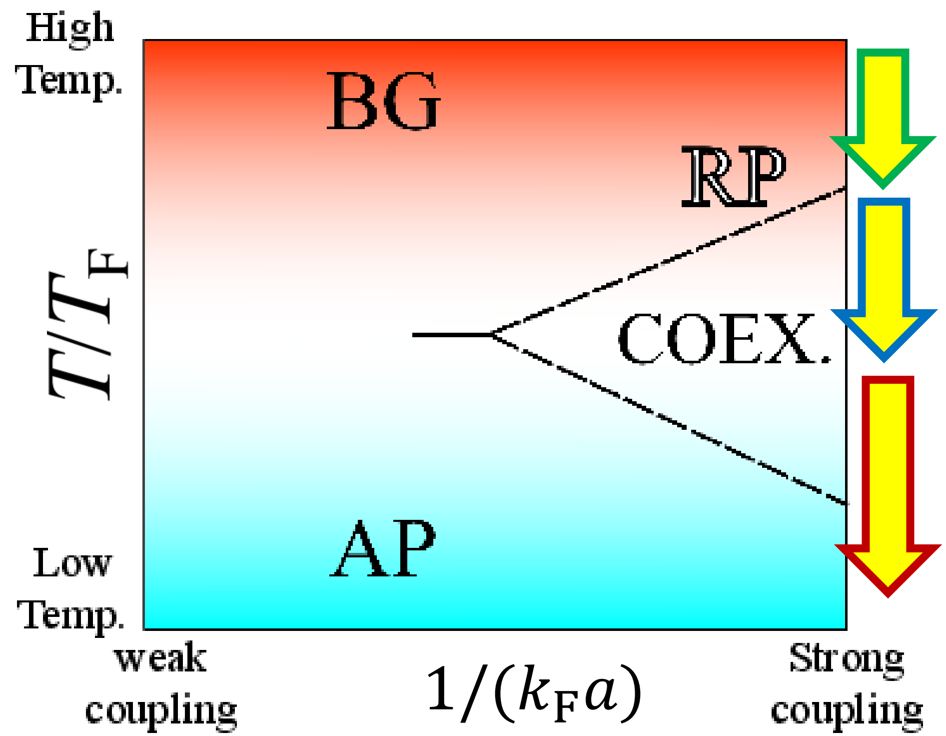
V_s: spin-singlet interaction

Thermal evolution of Fermi polarons

HT and S. Uchino, Phys. Rev. A **99**, 063606 (2019).

Reverse RF spectroscopy



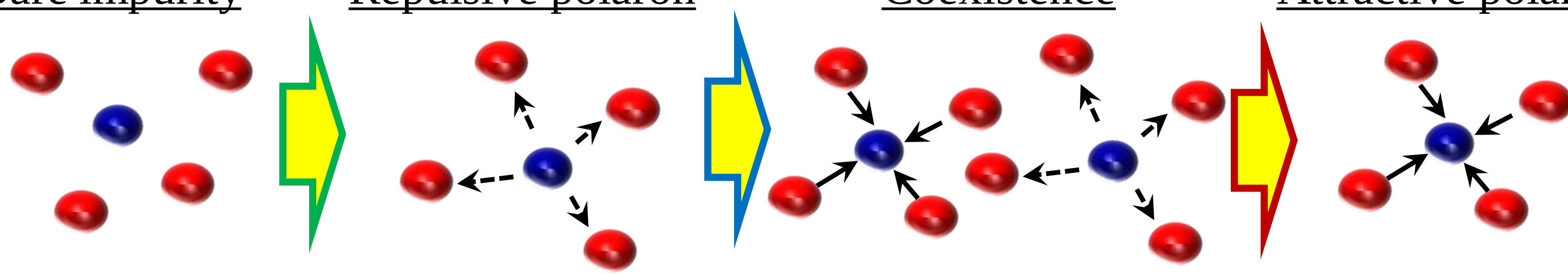


Bare impurity

Repulsive polaron

Coexistence

Attractive polaron



Thermal evolution of protonic polaron

Proton spectral weight in neutron matter