Strangess is the key： from $\bar{K}N$ to $\bar{D}_s DK$

Li-Sheng Geng（耿立升）@ Beihang U.

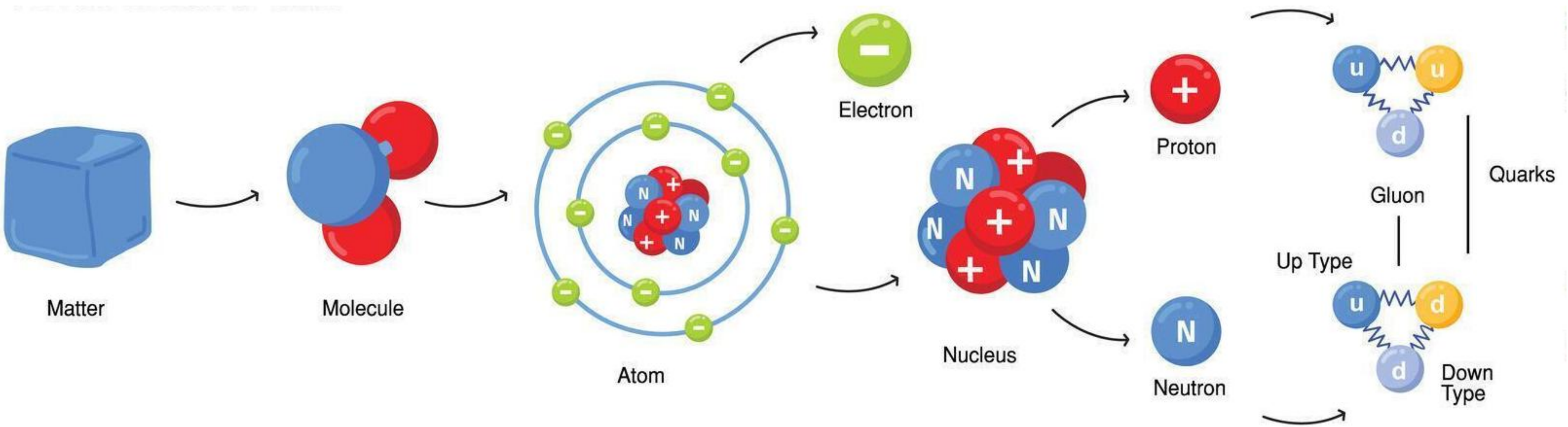
Two-pole structures as a universal phenomenon dictated by coupled-channel chiral dynamics
Jia-Ming Xie, Jun-Xu Lu, LSG*, Bing-Song Zou, Phys.Rev.D. 108 (2023) L111502

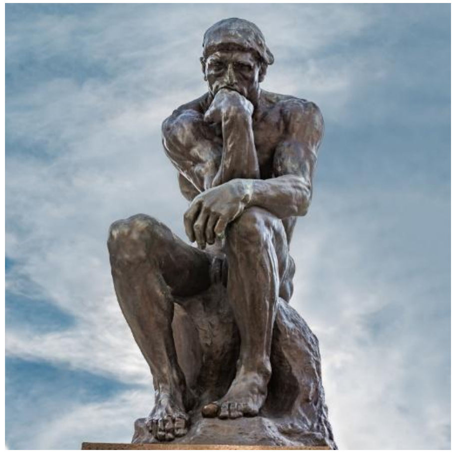
Implication of the Existence of JPC=0-- $\bar{D}$sDK Bound State on the Nature of Ds0*(2317), and a New Configuration of Exotic State
Tian-Wei Wu, Ming-Zhu Liu*, LSG*, Phys.Rev.Lett. 135 (2025) 03190202

Contents

- **Motivation: exotic hadrons as hadronic molecules**
- $\Lambda(1405)$ **as a** $\bar{K}N$ **bound state and its two pole structure**
- $D_{s0}^{*}(2317)$ **as a** DK **molecule and the existence of a unique** $\bar{D}_s DK$ **state**
- **Summary and outlook**

One central question in physics





The Thinker by Auguste Rodin

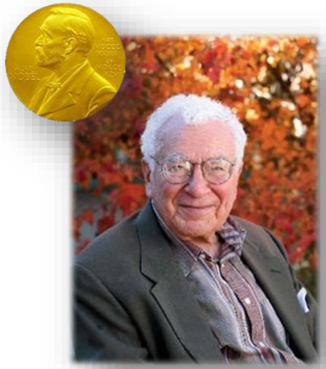
- The most **efficient building blocks** of NATURE at different scales
- How they interact with one another — **interaction**

Hadrons in the Constituent Quark Model

- 1964, Gell-Mann and Zweig proposed the legendary **constituent Quark Model** and successfully classified all **strongly** interacting particles into mesons and baryons (**hadrons**)

baryons

mesons



A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN

California Institute of Technology, Pasadena, California

4616 citations Received 4 January 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

CERN LIBRARIES, GENEVA

G. Zweig *)

CERN - Geneva

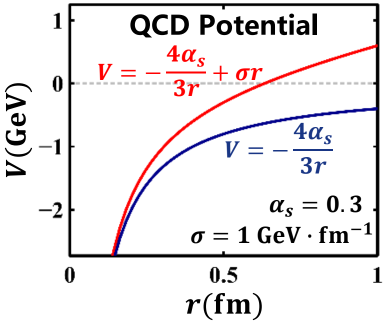
929 citations

- In the QM, the building blocks are **constituent quarks**, which are bound into hadrons by **quark-(anti)quark potentials**. QM proved highly successful up to around 2003 with few exceptions, such as $\Lambda(1405)$

➢ **Cornell model**

$$V_0(r) = -\frac{4\alpha_s}{3r} + \sigma r$$

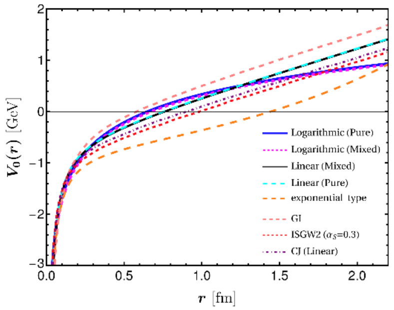
E. Eichten, et al.
Phys.Rev.Lett. 34 (1975) 369-372
1366 citations



➢ **Goldfrey-Isgur model**

$$V = V^{\text{conf}} + V^{\text{hyp}} + V^{\text{so}} + V_A$$

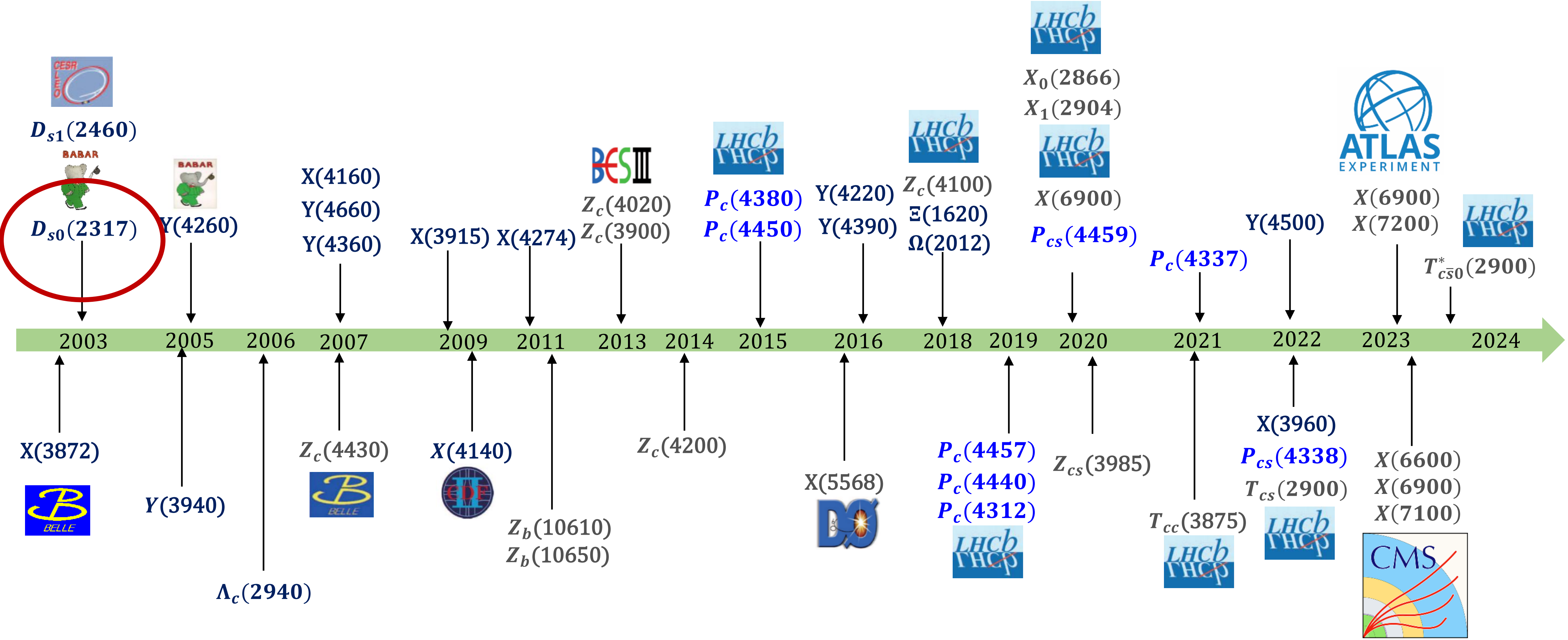
S. Godfrey, et al.
Phys.Rev.D 32 (1985) 189-231
3422 citations



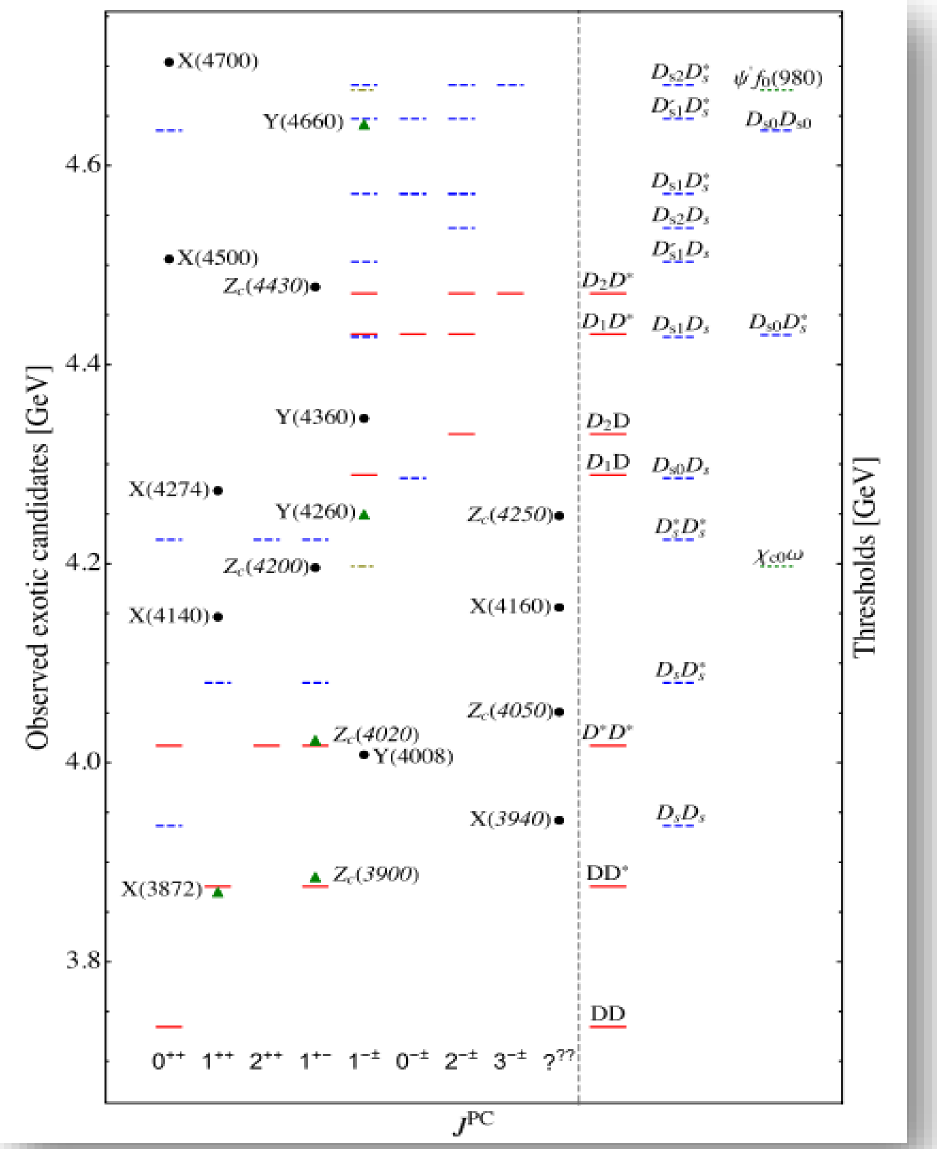
B. Pandya, et al. Phys.Rev.D 110 (2024) 094021

Exotic hadrons discovered since 2003 challenged the CQM

Exotic mesons and baryons **Tetraquarks** **Pentaquarks**



Most of them close to thresholds— hadronic molecules



Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner, Qian Wang, Qiang Zhao, Bing-Song Zou. Rev.Mod.Phys. 90 (2018) 015004

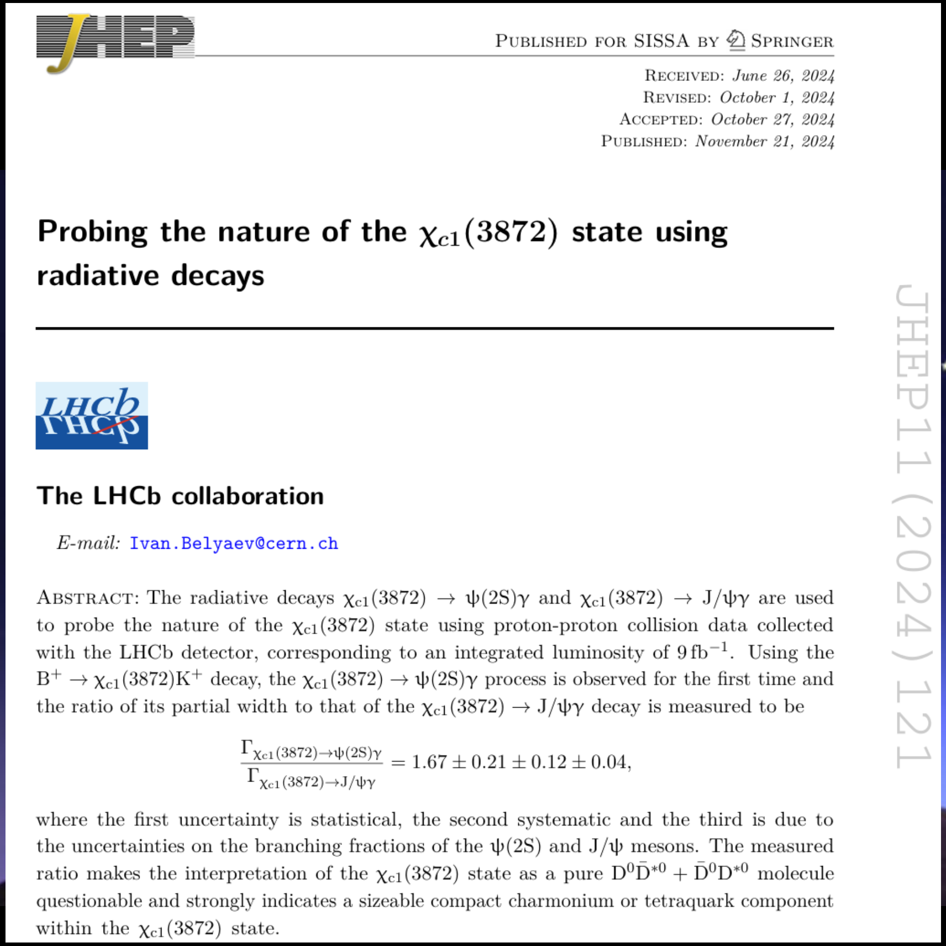
Richard F. Lebed, Ryan E. Mitchell, Eric S. Swanson, Prog.Part.Nucl.Phys. 93 (2017) 143

Atsushi Hosaka, Toru Iijima, Kenkichi Miyabayashi, Yoshihide Sakai , Shigehiro Yasui, PTEP 2016 (2016) 062C01

Hua-Xing Chen, Wei Chen, Xiang Liu Shi-Lin Zhu, Phys. Rept.639 (2016) 1

……

How to check the molecular picture?



JHEP

Published for SISSA by Springer

Received: June 26, 2024
Revised: October 1, 2024
Accepted: October 27, 2024
Published: November 21, 2024

Probing the nature of the $\chi_{c1}(3872)$ state using radiative decays

LHCb

The LHCb collaboration

E-mail: Ivan.Belyaev@cern.ch

Abstract: The radiative decays $\chi_{c1}(3872) \to \psi(2S)\gamma$ and $\chi_{c1}(3872) \to J/\psi\gamma$ are used to probe the nature of the $\chi_{c1}(3872)$ state using proton-proton collision data collected with the LHCb detector, corresponding to an integrated luminosity of $9\,\mathrm{fb}^{-1}$. Using the $B^+ \to \chi_{c1}(3872)K^+$ decay, the $\chi_{c1}(3872) \to \psi(2S)\gamma$ process is observed for the first time and the ratio of its partial width to that of the $\chi_{c1}(3872) \to J/\psi\gamma$ decay is measured to be

$$\frac{\Gamma_{\chi_{c1}(3872)\to\psi(2S)\gamma}}{\Gamma_{\chi_{c1}(3872)\to J/\psi\gamma}} = 1.67 \pm 0.21 \pm 0.12 \pm 0.04,$$

where the first uncertainty is statistical, the second systematic and the third is due to the uncertainties on the branching fractions of the $\psi(2S)$ and J/ψ mesons. The measured ratio makes the interpretation of the $\chi_{c1}(3872)$ state as a pure $D^0\bar{D}^{*0} + \bar{D}^0D^{*0}$ molecule questionable and strongly indicates a sizeable compact charmonium or tetraquark component within the $\chi_{c1}(3872)$ state.

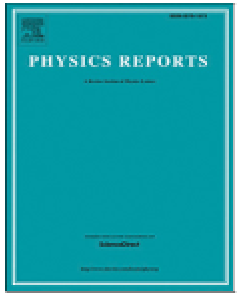
JHEP11(2024)121

Physics Reports 1108 (2025) 1–108

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep



Three ways to decipher the nature of exotic hadrons: Multiplets, three-body hadronic molecules, and correlation functions

Ming-Zhu Liu [a,b], Ya-Wen Pan [c], Zhi-Wei Liu [c], Tian-Wei Wu [d], Jun-Xu Lu [c], Li-Sheng Geng [c,e,f,g,h,*]

[a] *Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, China*
[b] *School of Nuclear Science and Technology, Lanzhou University, Lanzhou 730000, China*
[c] *School of Physics, Beihang University, Beijing 102206, China*
[d] *School of Science, Shenzhen Campus of Sun Yat-sen University, Shenzhen 518107, China*
[e] *Centrale Pekin, Beihang University, Beijing 100191, China*
[f] *Peng Huanwu Collaborative Center for Research and Education, Beihang University, Beijing 100191, China*
[g] *Beijing Key Laboratory of Advanced Nuclear Materials and Physics, Beihang University, Beijing 102206, China*
[h] *Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, China*

Contents

- Motivation: exotic hadrons as hadronic molecules
- $\Lambda(1405)$ as a $\bar{K}N$ bound state and its two pole structure
- $D_{s0}^{*}(2317)$ as a DK molecule and the existence of a unique $\bar{D}_s DK$ state
- Summary and outlook

$\Lambda(1405)$: why is it special

In the constituent quark model, $\Lambda(1405)$ is the first P-wave orbital excitation of the ground-state baryon $\Lambda(1115)$

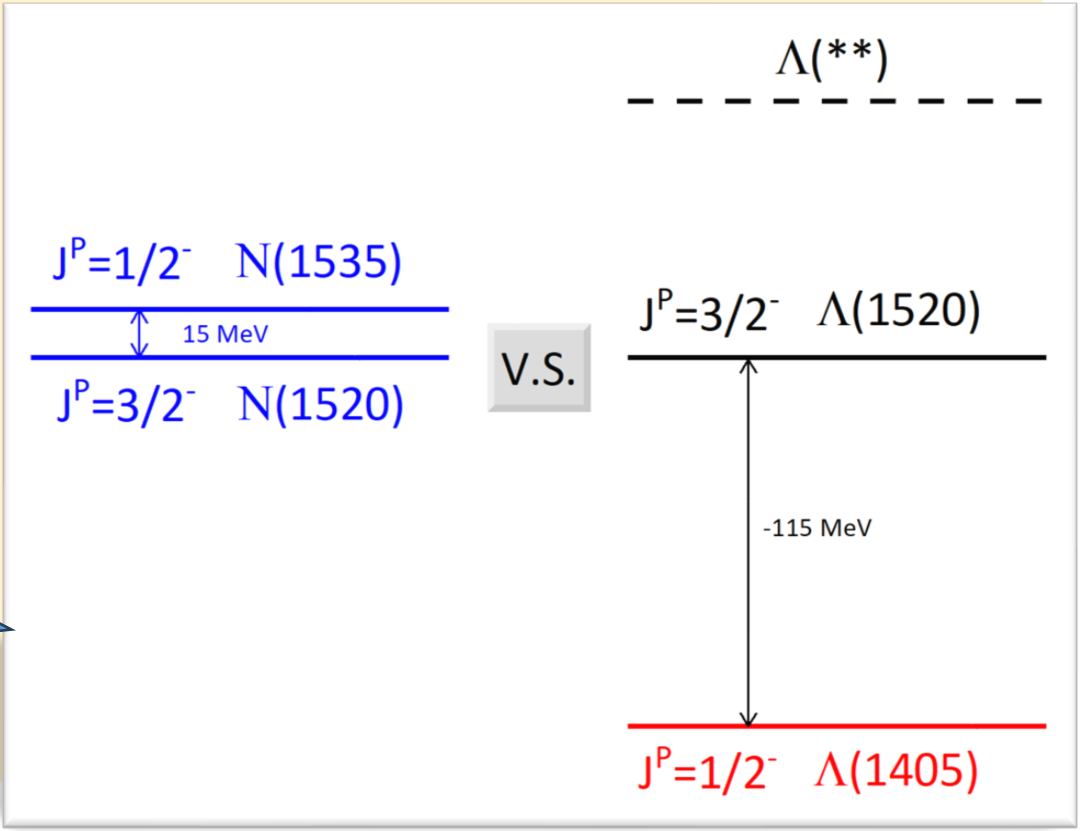
PDG

$I(J^P) = 0(1/2^-), S = -1$

$M = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma = 50.5 \pm 2.0$ MeV

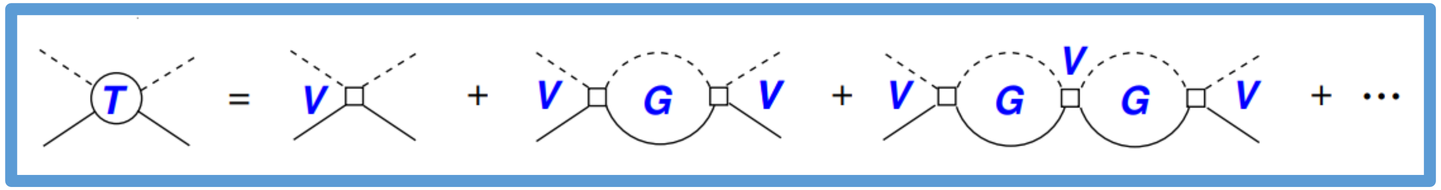
Two puzzles

Low mass vs. N*(1535)
large spin splitting vs. $\Lambda(1520)$



$\Lambda(1405)$ as a dynamically generated state

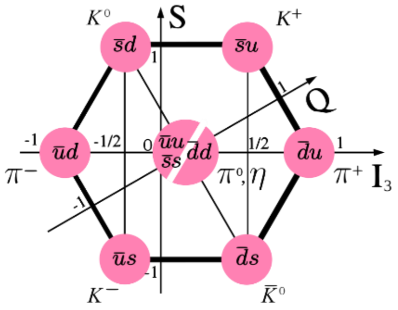
- **Modern picture for $\Lambda(1405)$: a $\bar{K}N$ bound state dynamically generated by coupled-channel chiral dynamics implemented in the so-called chiral unitary approaches**

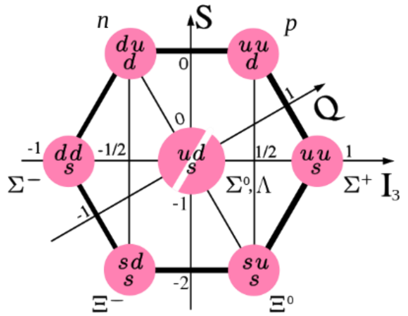


Bethe-Salpeter Equation

- **Weinberg-Tomozawa (WT) potential**

$$V_{ij} = -\frac{C_{ij}}{4f^2}(E_i + E_j)$$



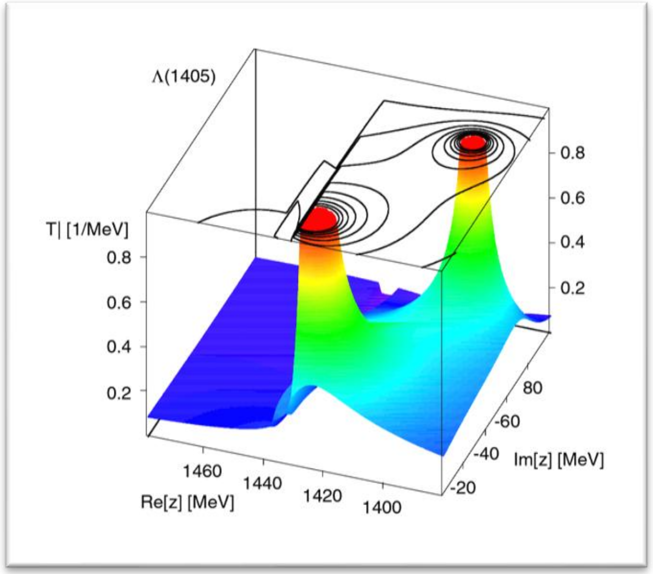


- LO&NLO, Kaiser, Siegel, Weise, NPA594, 325(1995), **782** citations
- LO, Oset and Ramos, NPA635, 99(1998), **923** citations
- NLO, Oller and Meissner, PLB500, 263(2001), **996** citations

as of 2025.09.28

Unexpected two-pole structure!

- **Two poles:** $W_H = 1424.3 - 17.1i$, $W_L = 1389.1 - 64.1$



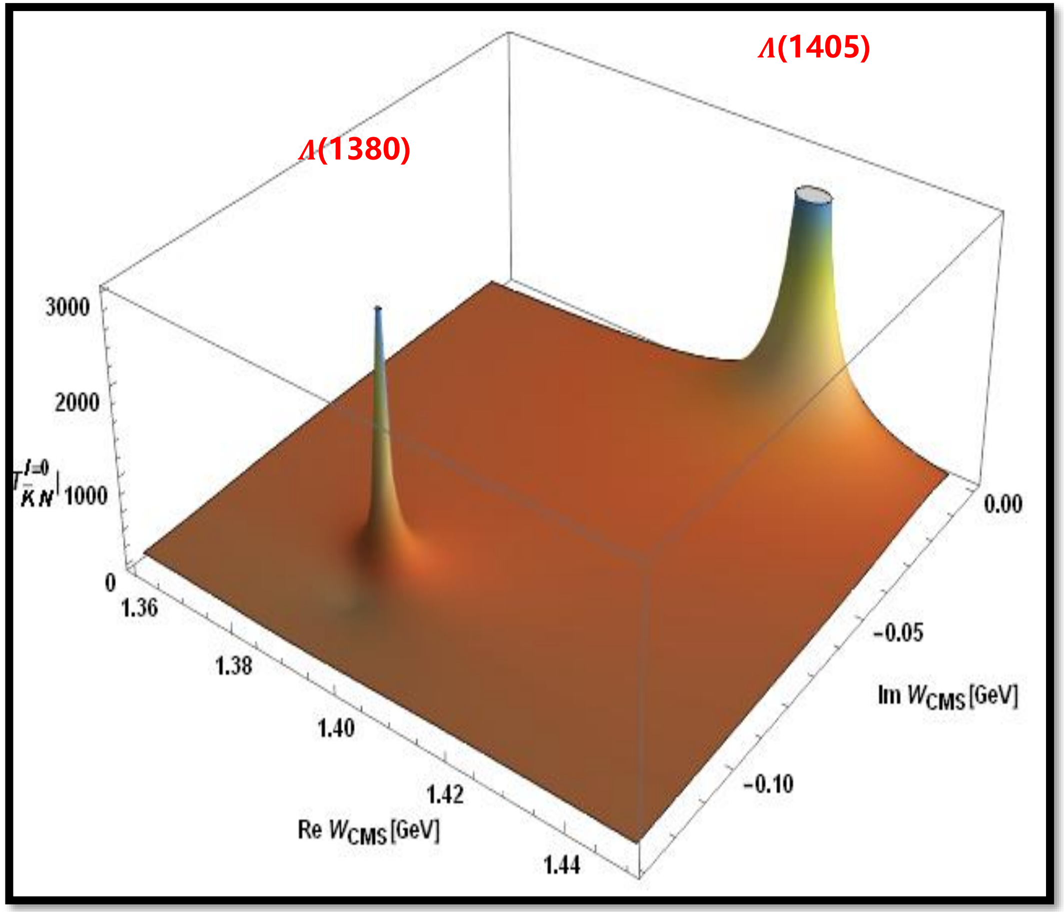
	$\Lambda(1405)$	
z_R (MeV)	$1390 - 66i$	$1426 - 16i$
$\lvert g_i \rvert (\pi\Sigma)$	2.9	1.5
$\lvert g_i \rvert (\bar{K}N)$	2.1	2.7
$\lvert g_i \rvert (\eta\Lambda)$	0.77	1.4
$\lvert g_i \rvert (K\Xi)$	0.61	0.35

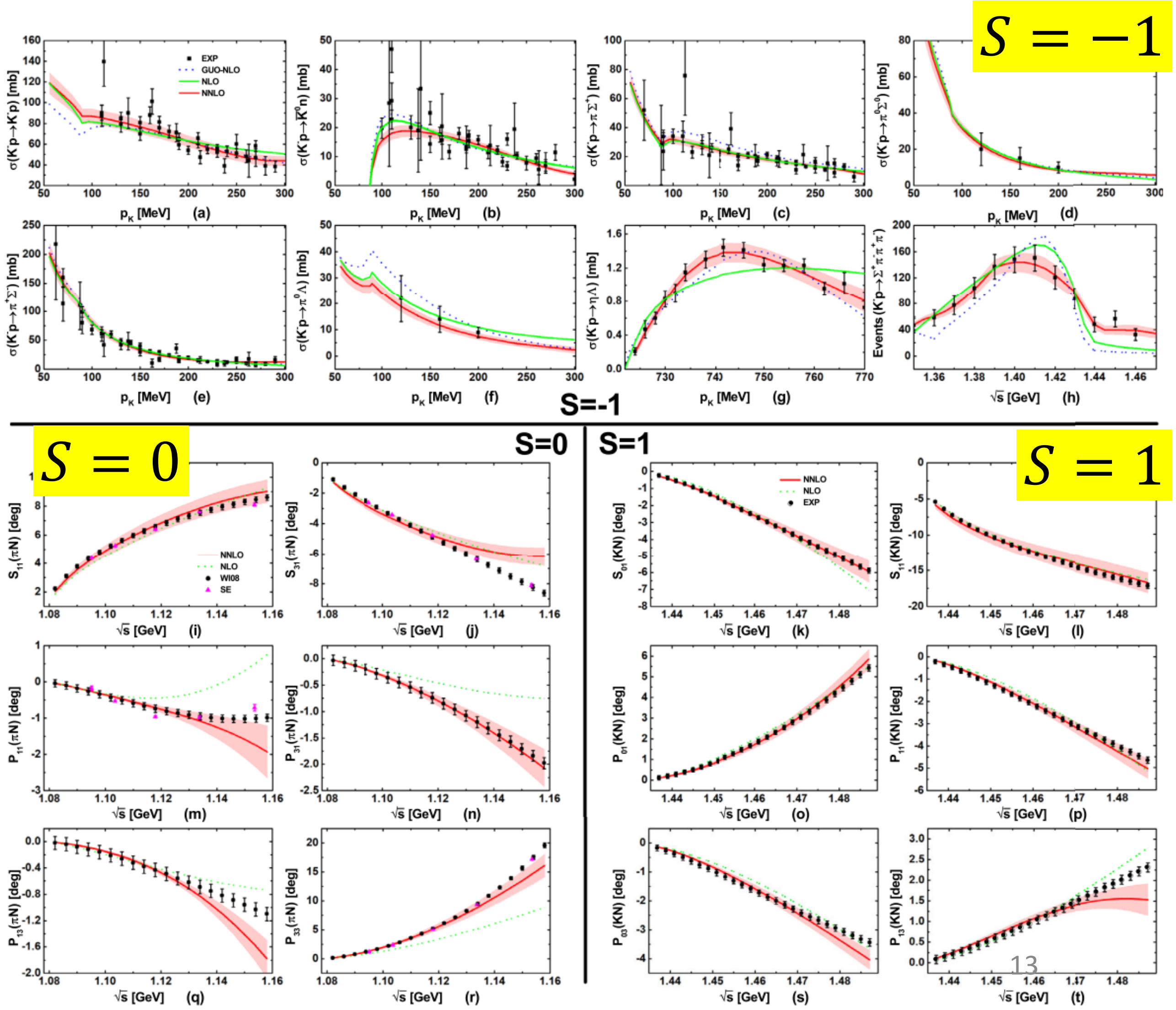
Isopin 0, four coupled channels: $\pi\Sigma(1330)$, $\bar{K}N(1433)$, $\eta\Lambda(1662)$, $K\Xi(1813)$ (renormalization scale $\mu = 630$ MeV, with four different $a_i(\mu)$)

Oller and Meissner, PLB500, 263(2001), **996** citations
Jido, Oller, Oset, Ramos, and Meissner, NPA725, 181 (2003), **772** citations
Hyodo and Jido, PPNP67, 55 (2012), **388** citations
Meissner, Symmetry 12 (2020) 981, **96** citations

The two-pole structure persists at N2LO

Meson-baryon scattering up to N2LO, Jun-Xu Lu, LSG*, M. Doering and M. Mai, PRL130, 071902(2023)

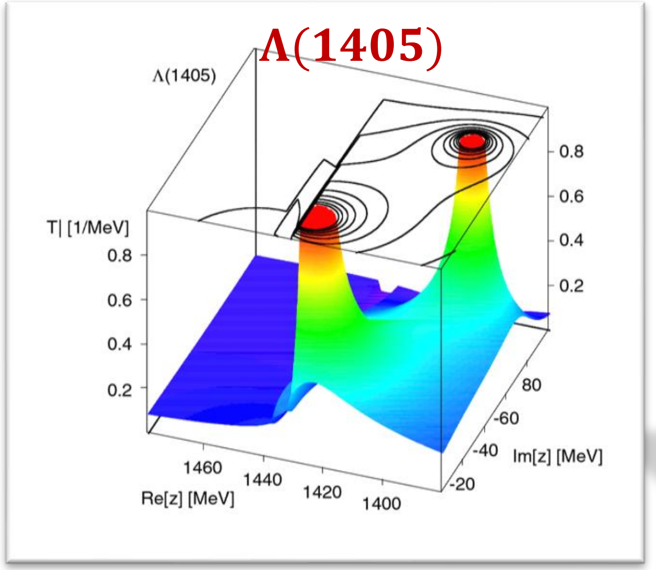




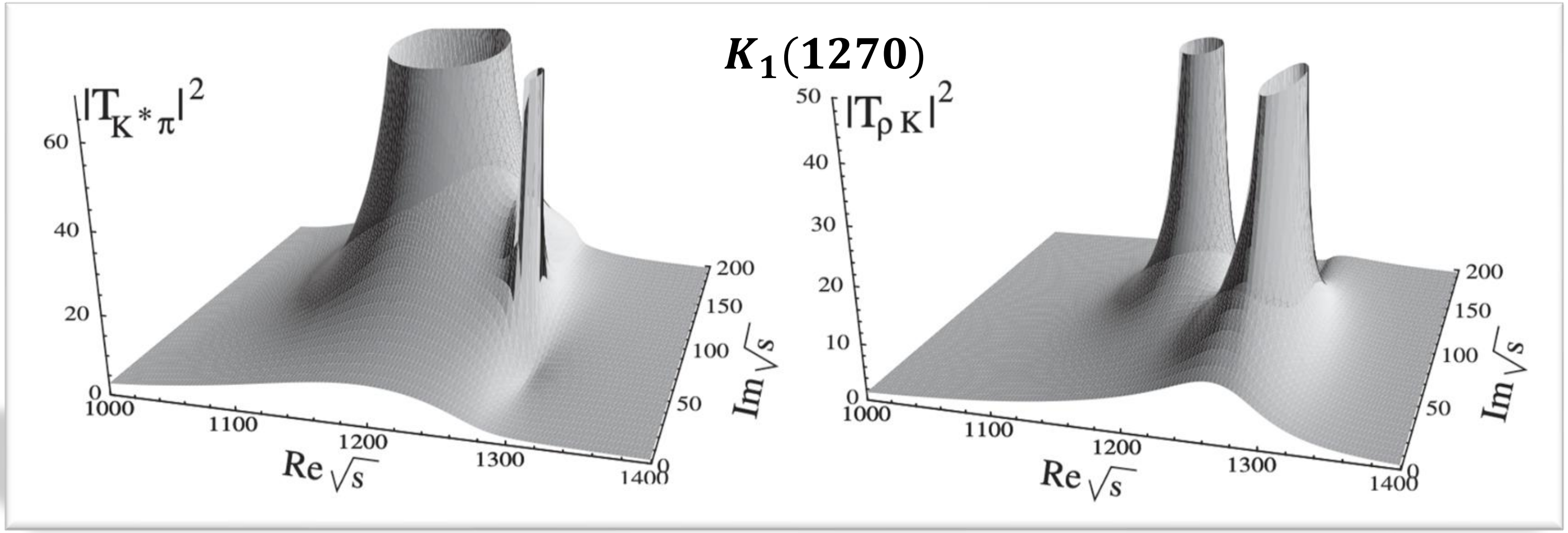
Two-pole structures are not simply two states!

① Two-pole structures refer to the fact that two dynamically generated states, **one resonant and one bound, are located close to each other** between two coupled channels and with a mass difference smaller than the sum of their widths.

② Two poles overlap, which creates the impression that **there is only one state in the invariant mass distribution** of their decay products.

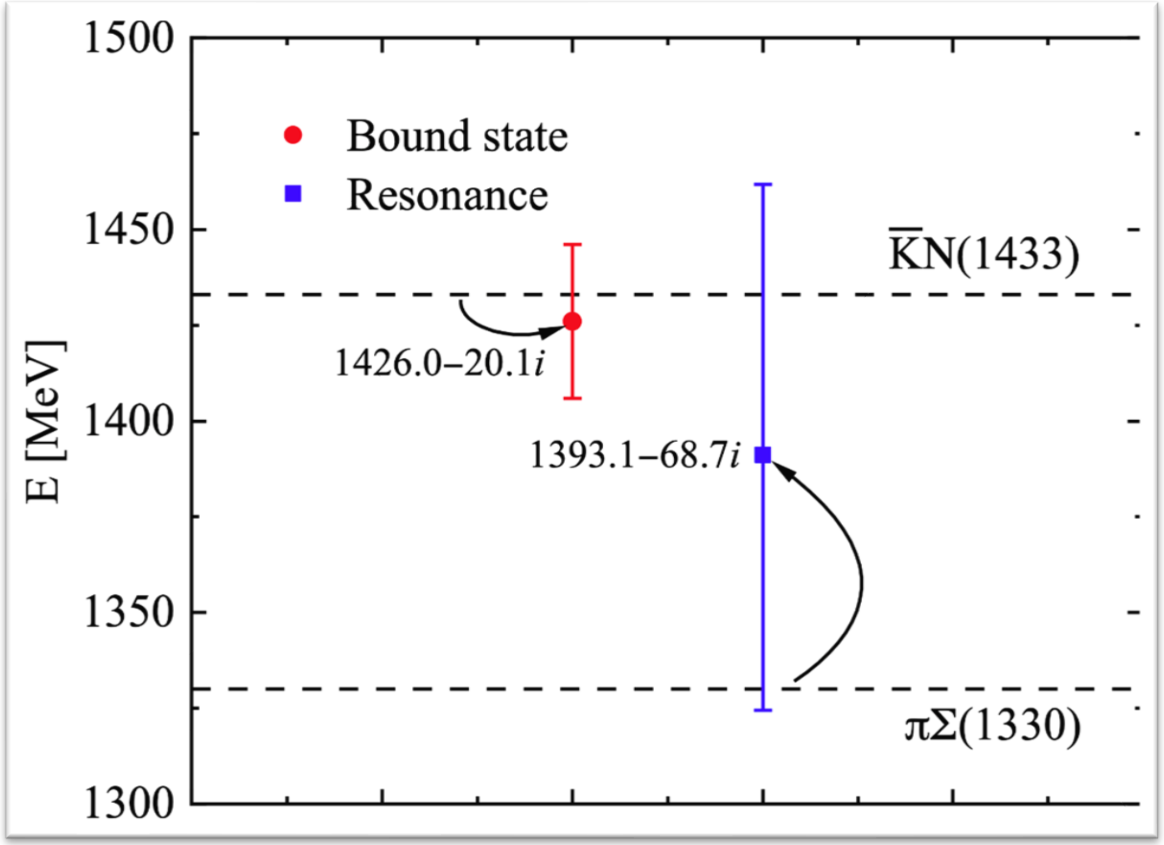


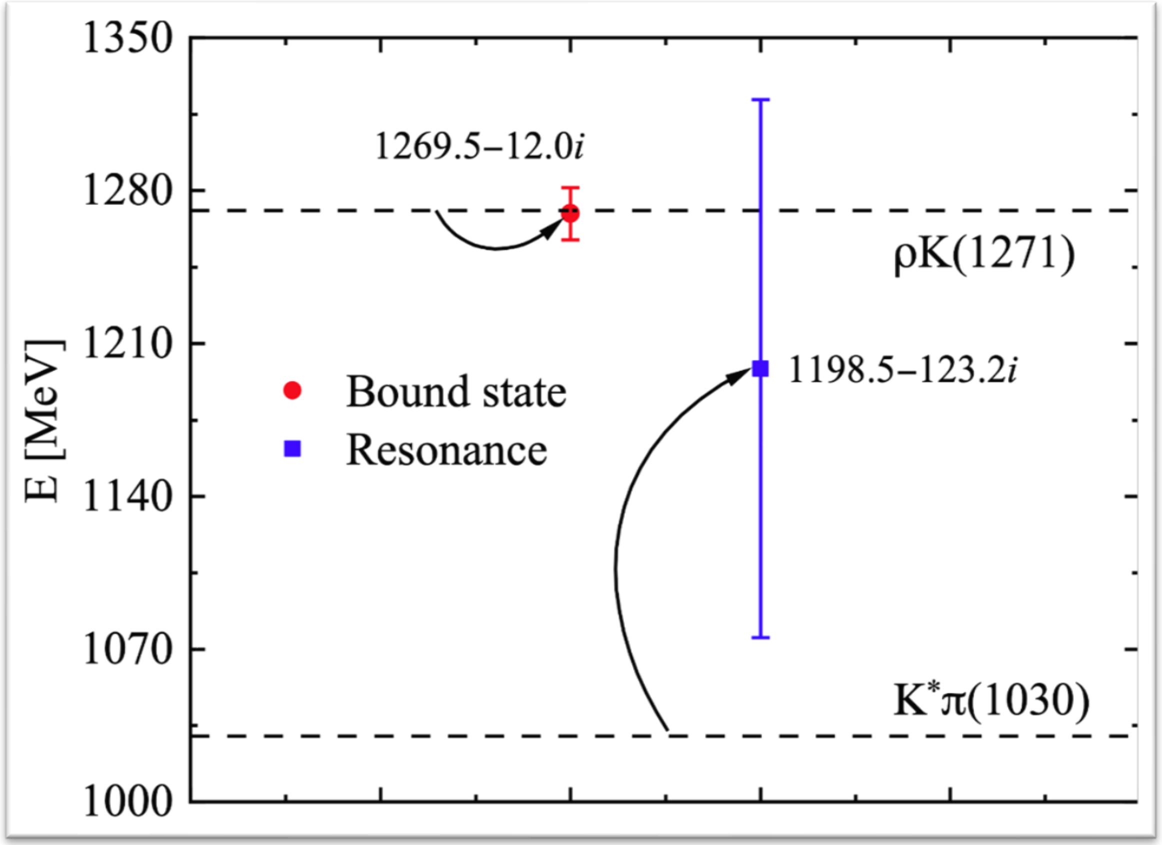
Hyodo and Jido, PPNP67, 55 (2012)



LSG, Oset, Roca, and Oller, PRD**75,** 014017 (2007)

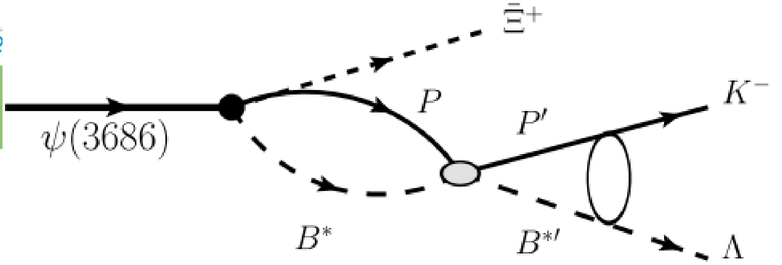
Two prominent examples: $\Lambda(1405)$ and $K_1(1270)$

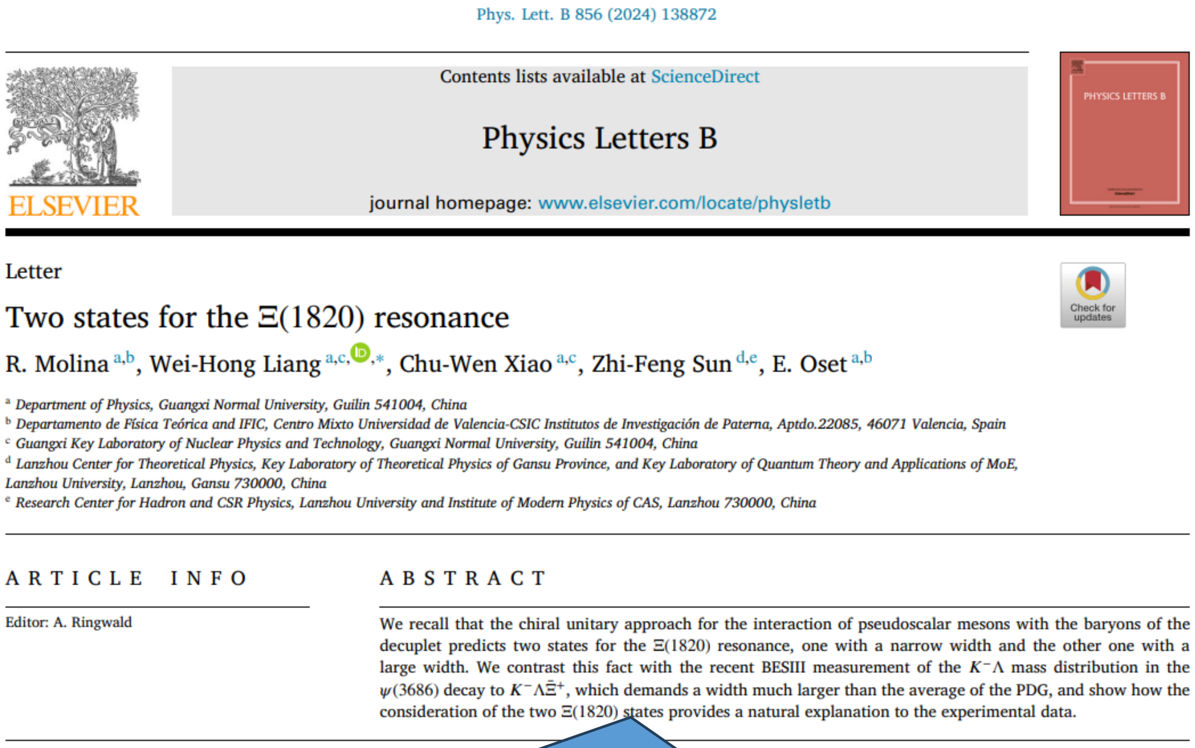




Experimental evidence?

Test the two-pole structure of the Ξ(1820) state
Wei-Hong Liang
Koshiba-Hall, Tokyo 11:05 - 11:30





Phys. Lett. B 856 (2024) 138872

Contents lists available at ScienceDirect

Physics Letters B

journal homepage: www.elsevier.com/locate/physletb

ELSEVIER

Letter

Two states for the Ξ(1820) resonance

R. Molina [a,b], Wei-Hong Liang [a,c,*], Chu-Wen Xiao [a,c], Zhi-Feng Sun [d,e], E. Oset [a,b]

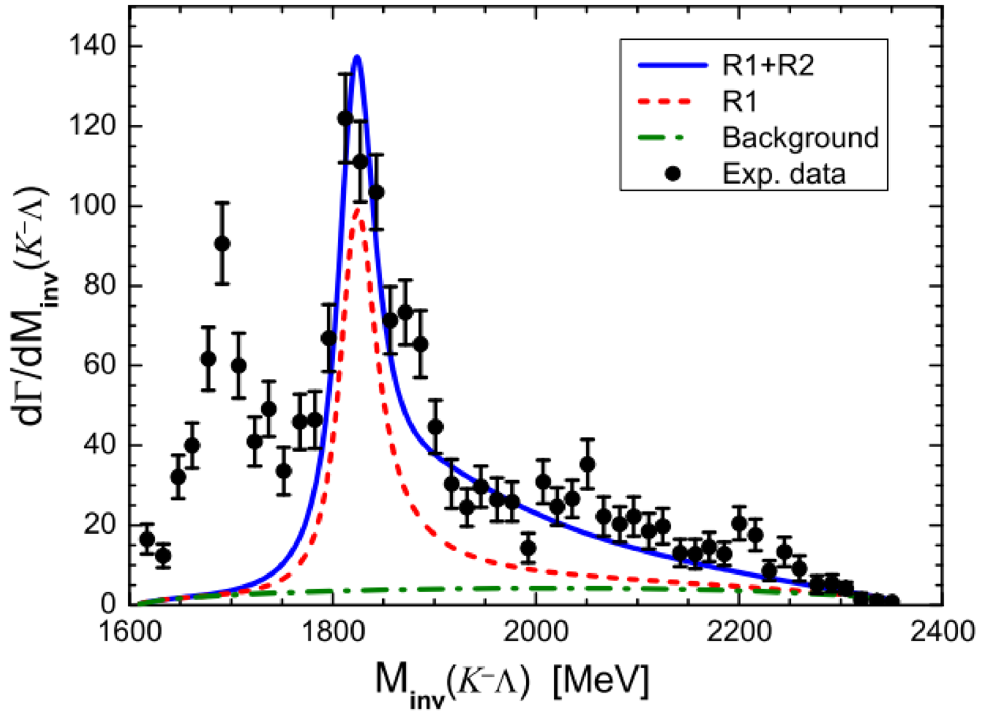
[a] Department of Physics, Guangxi Normal University, Guilin 541004, China
[b] Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC Institutos de Investigación de Paterna, Aptdo.22085, 46071 Valencia, Spain
[c] Guangxi Key Laboratory of Nuclear Physics and Technology, Guangxi Normal University, Guilin 541004, China
[d] Lanzhou Center for Theoretical Physics, Key Laboratory of Theoretical Physics of Gansu Province, and Key Laboratory of Quantum Theory and Applications of MoE, Lanzhou University, Lanzhou, Gansu 730000, China
[e] Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China

ARTICLE INFO

Editor: A. Ringwald

ABSTRACT

We recall that the chiral unitary approach for the interaction of pseudoscalar mesons with the baryons of the decuplet predicts two states for the Ξ(1820) resonance, one with a narrow width and the other one with a large width. We contrast this fact with the recent BESIII measurement of the $K^-\Lambda$ mass distribution in the $\psi(3686)$ decay to $K^-\Lambda\bar{\Xi}^+$, which demands a width much larger than the average of the PDG, and show how the consideration of the two Ξ(1820) states provides a natural explanation to the experimental data.



We contrast this fact with the recent BESIII measurement of the $K^-\Lambda$ mass distribution in the $\psi(3686)$ decay to $K^-\Lambda\Xi^+$, which demands a width much larger than the average of the PDG, and show how the consideration of the two Ξ(1820) states provides a natural explanation to the experimental data

More can be asked

1. Why are there two?
2. Why can they be mistaken for one state?
3. Why are they located between the two channels?
4. What kind of interactions can generate such two poles?
5. ……

One possible answer

—study the light-quark mass evolution of the two poles

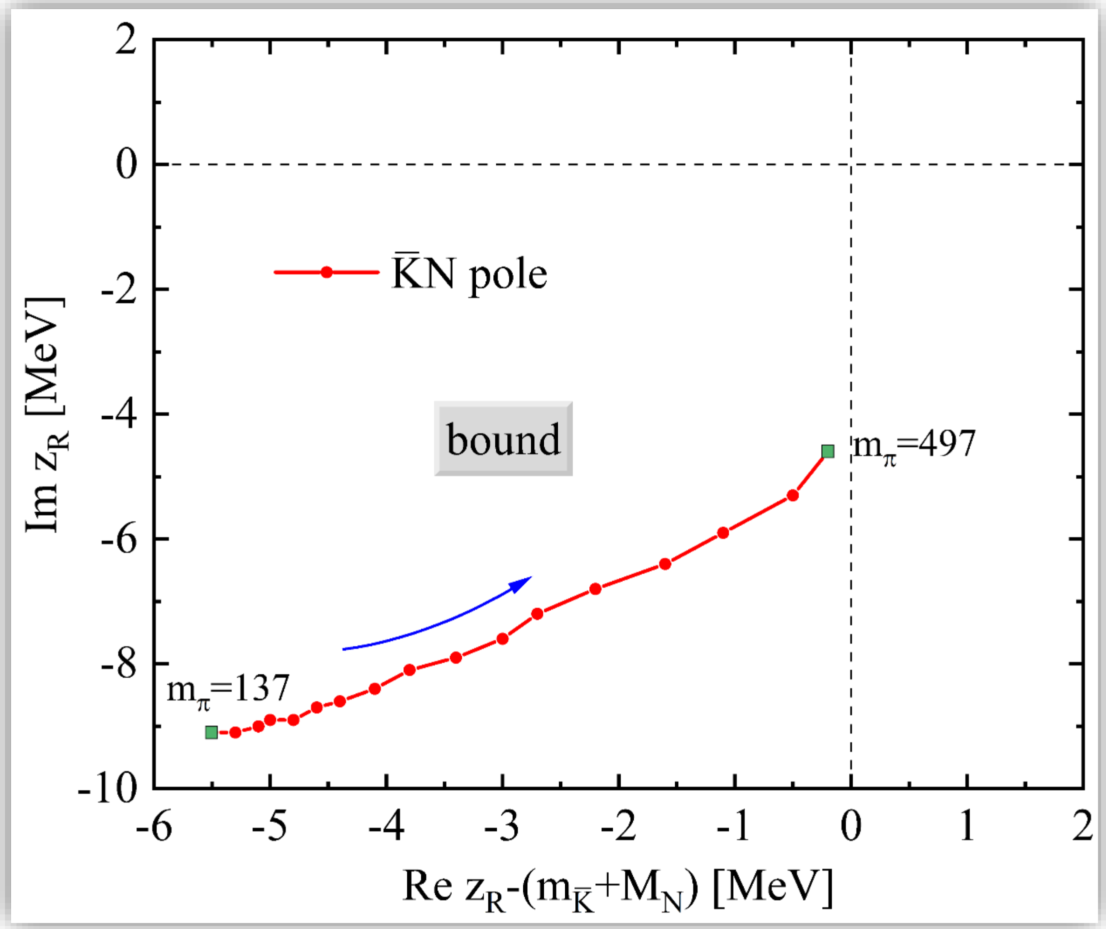
$$V_{ij} = -\frac{C_{ij}}{4f^2}(E_i + E_j)$$

Two-pole structures as a universal phenomenon dictated by coupled-channel chiral dynamics
Jia-Ming Xie, Jun-Xu Lu, LSG*, Bing-Song Zou,
Phys.RevD.108 (2023) L111502

Evolution of the higher pole with m_π: simple

- **As m_π increases**, both the real and the imaginary parts of the higher pole decrease, which indicates that **the effective $\bar{K}N$ attraction becomes weaker and the coupling to $\pi\Sigma$ decreases as well.**

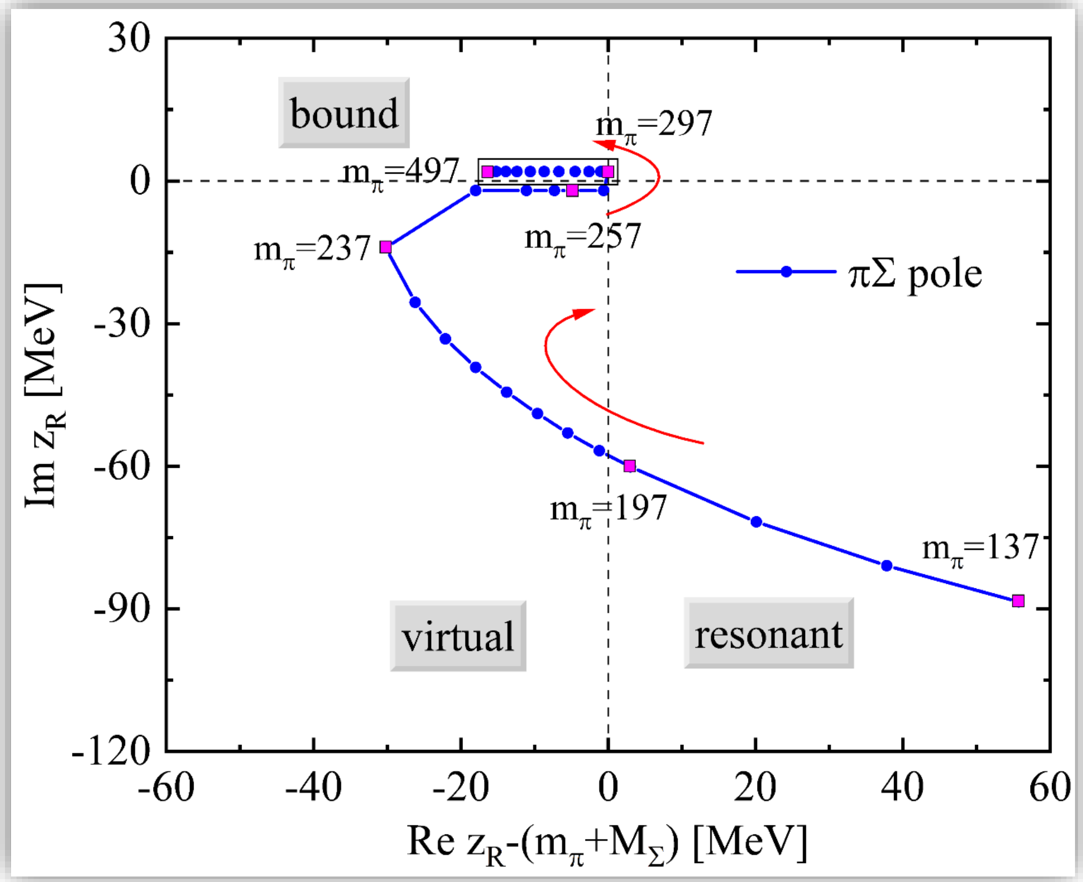
- Note that the two thresholds also increase as m_π increases.



Evolution of the lower pole with m_π: complicated

- ☐For $m_\pi \approx 200$ **MeV**, it becomes **a virtual resonance** from a resonant state.

- ☐For a pion mass of about **300 MeV**, it becomes **a bound state** and remains so up to the pion mass of **500 MeV.**



The evolution of the lower pole clearly demonstrates the chiral dynamics underlying the two-pole structure of $\Lambda(1405)$.

Latest lattice QCD study

$$\det[\widetilde{K}^{-1}(E_{\rm cm}) - B^{\boldsymbol{P}}(E_{\rm cm})] = 0 \qquad \frac{E_{\rm cm}}{m_\pi}\widetilde{K}_{ij} = A_{ij} + B_{ij}\Delta_{\pi\Sigma;}$$

Editors' Suggestion Open Access Acc

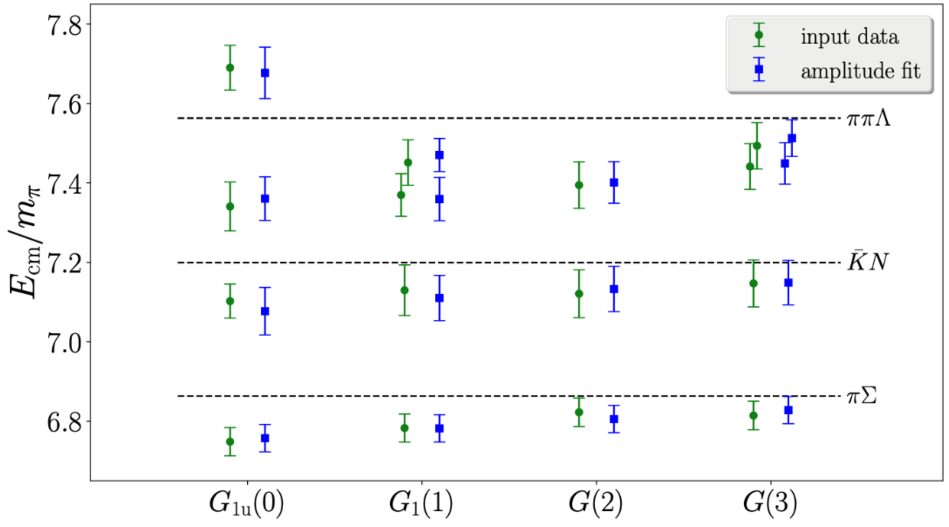
Two-Pole Nature of the $\Lambda(1405)$ Resonance from Lattice QCD

John Bulava, Bárbara Cid-Mora, Andrew D. Hanlon, Ben Hörz, Daniel Mohler, Colin Morningstar, Joseph Moscoso, Amy Nicholson, Fernando Romero-López, Sarah Skinner, and André Walker-Loud (Baryon Scattering (BaSc) Collaboration)

Phys. Rev. Lett. **132**, 051901 – Published 30 January 2024

This letter presents the first lattice QCD computation of the coupled channel $\pi\Sigma$–$\bar{K}N$ scattering amplitudes at energies near 1405 MeV. These amplitudes contain the resonance $\Lambda(1405)$ with strangeness $S=-1$ and isospin, spin, and parity quantum numbers $I(J^P) = 0(1/2^-)$. However, whether there is a single resonance or two nearby resonance poles in this region is controversial theoretically and experimentally. Using single-baryon and meson-baryon operators to extract the finite-volume stationary-state energies to obtain the scattering amplitudes at slightly unphysical quark masses corresponding to $m_\pi \approx 200$ MeV and $m_K \approx 487$ MeV, this study finds the amplitudes exhibit a virtual bound state below the $\pi\Sigma$ threshold in addition to the established resonance pole just below the $\bar{K}N$ threshold. Several parametrizations of the two-channel K-matrix are employed to fit the lattice QCD results, all of which support the two-pole picture suggested by $SU(3)$ chiral symmetry and unitarity.

$m_\pi \approx 200$ MeV,
a virtual bound state below $\pi\Sigma$ and a resonant (bound) state just below $\bar{K}N$, support the two-pole structure suggested



Two poles are found on the $(-,+)$ sheet, which is the one closest to physical scattering in the region between the two thresholds. Their locations are

$$\begin{aligned} E_1 &= 1392(9)(2)(16)\ \text{MeV}, \\ E_2 &= [1455(13)(2)(17) - i11.5(4.4)(4)(0.1)]\ \text{MeV}, \end{aligned} \qquad (6)$$

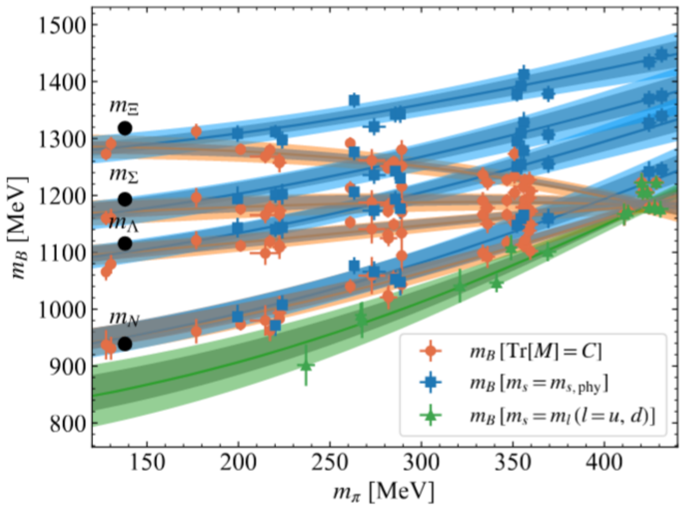
and their couplings

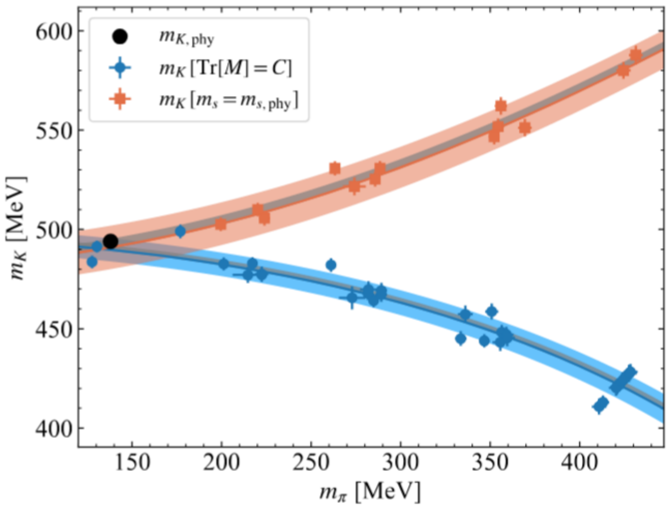
$$\left|\frac{c^{(1)}_{\pi\Sigma}}{c^{(1)}_{\bar{K}N}}\right| = 1.9(4)(6), \qquad \left|\frac{c^{(2)}_{\pi\Sigma}}{c^{(2)}_{\bar{K}N}}\right| = 0.53(9)(10). \qquad (7)$$

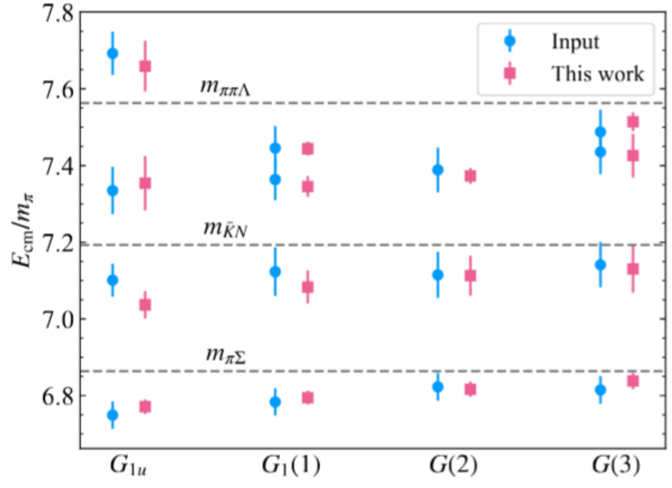
Can be described by UChPT and consistent with exp.

Zejian Zhuang, R. Molina∗, Jun-Xu Lu, and LSG, Sci.Bull. 70 (2025) 1953-1961

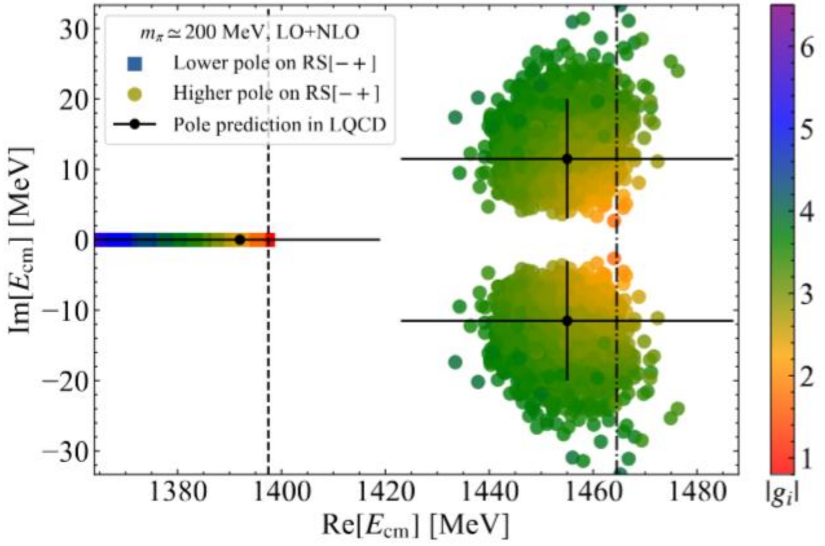
Fits

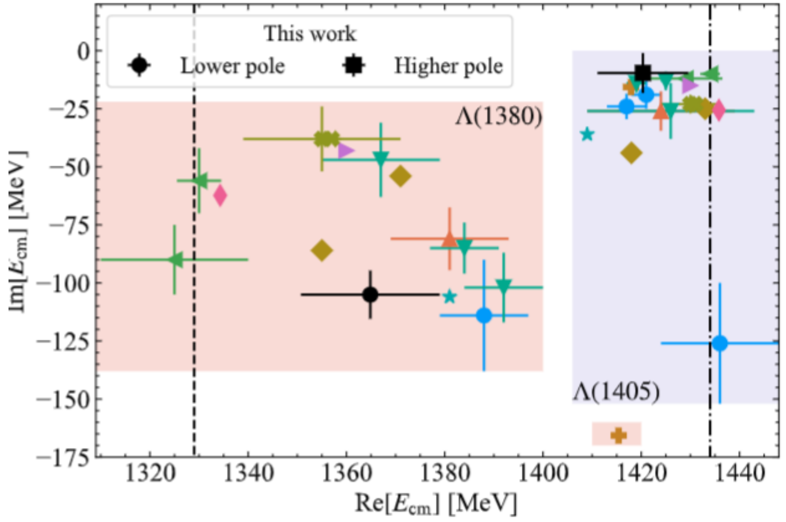


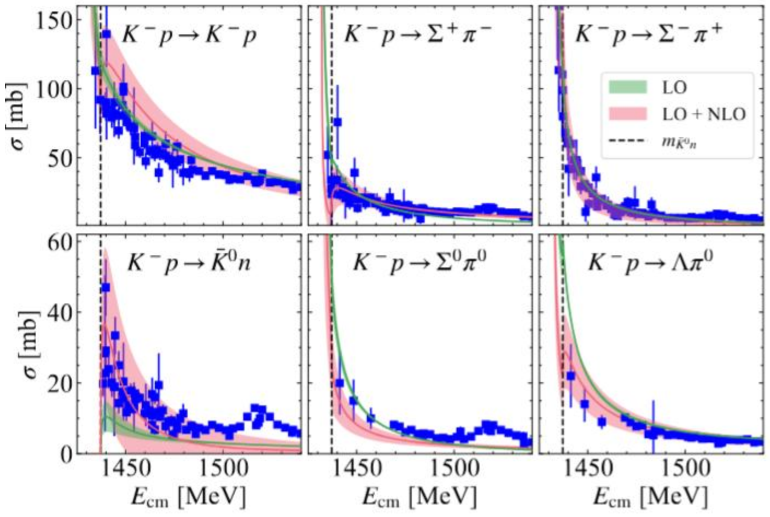




predictions

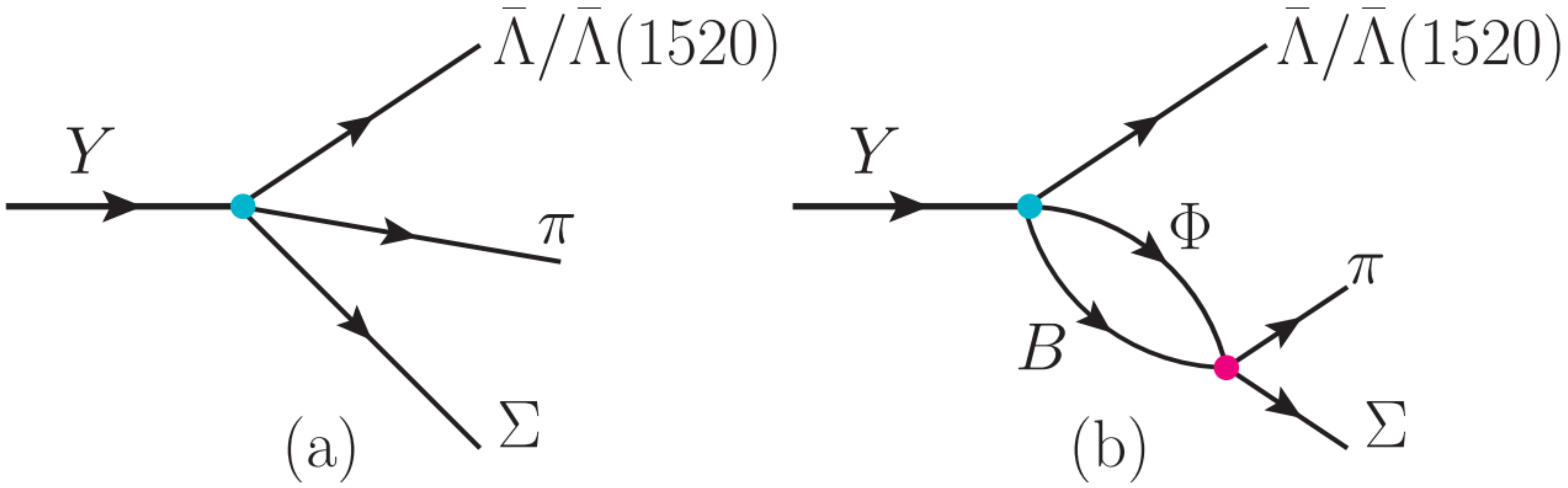






Identifying flavor-content of the two poles of Λ(1405)

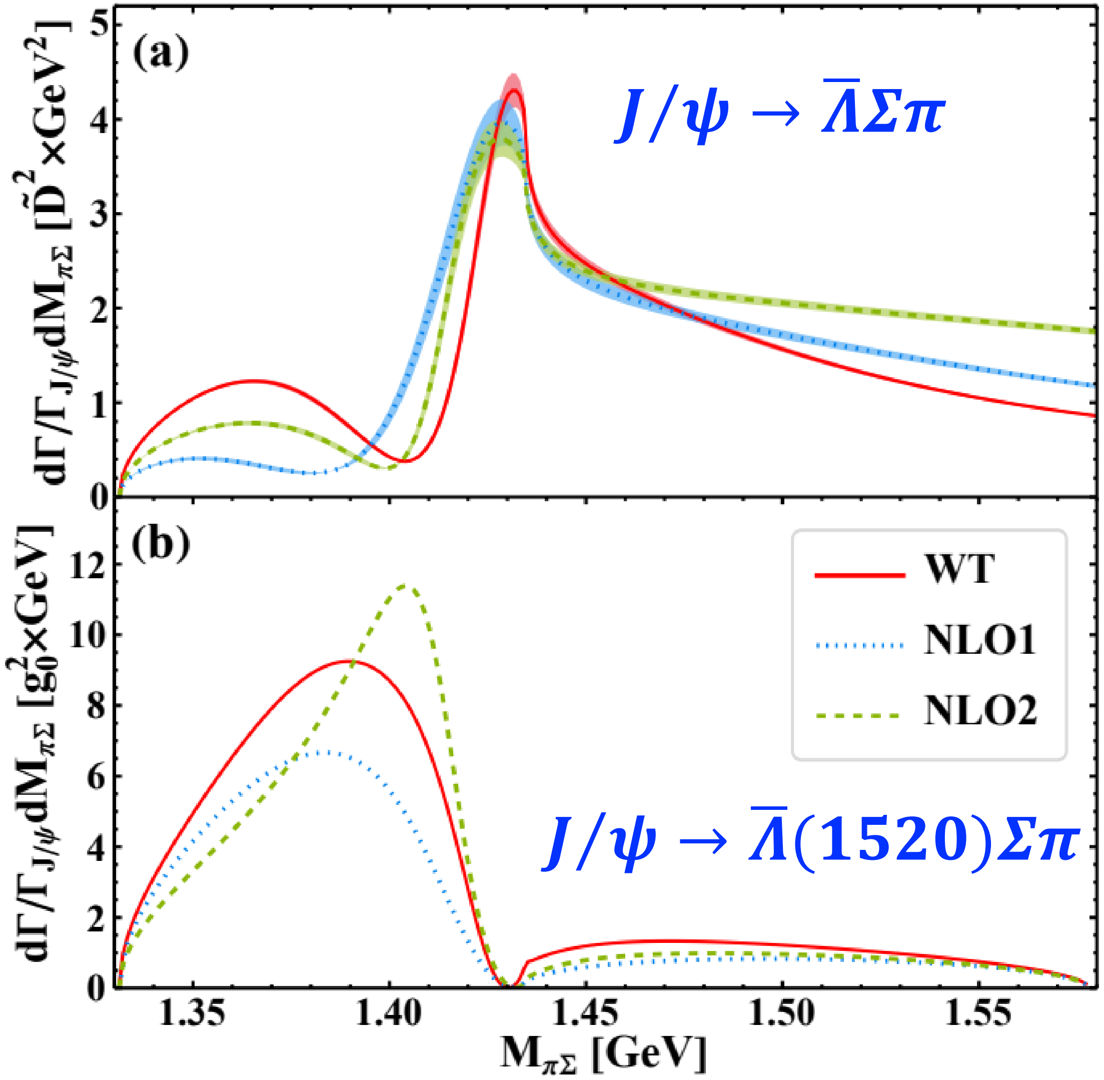
Ying-Bo He, Xiao-Hai Liu*, LSG*, Feng-Kun Guo*, Ju-Jun Xie*, 2407.13486



- Y is a charmonium or bottomonium, such as J/ψ, ψ_{2s}, χ_{c0}, but an SU(3) singlet
- $\bar{\Lambda}$ is an SU(3) octet, then the $\pi\Sigma$ pair must be an SU(3) octet—higher pole
- $\bar{\Lambda}$(1520) is an SU(3) singlet, then the $\pi\Sigma$ pair must be an SU(3) singlet—lower pole

Identifying flavor-content of the two poles of $\Lambda(1405)$

Ying-Bo He, Xiao-Hai Liu*, LSG*, Feng-Kun Guo*, Ju-Jun Xie*, 2407.13486



[NLO1] Y. Ikeda, T. Hyodo, and W. Weise, NPA881(2012)98

[NLO2] F.-K. Guo, Y. Kamiya, M. Mai, and U.-G. Meißner, PLB846(2023)13826

Contents

- □ **Motivation: exotic hadrons as hadronic molecules**
- □ $\Lambda(1405)$ **as a** $\bar{K}N$ **bound state and its two pole structure**
- □ $D_{s0}^{*}(2317)$ **as a** DK **molecule and the existence of a unique** $\bar{D}_s DK$ **state**
- □ **Summary and outlook**

What is special about $D_{s0}^*(2317)$ and $D_{s1}(2460)$

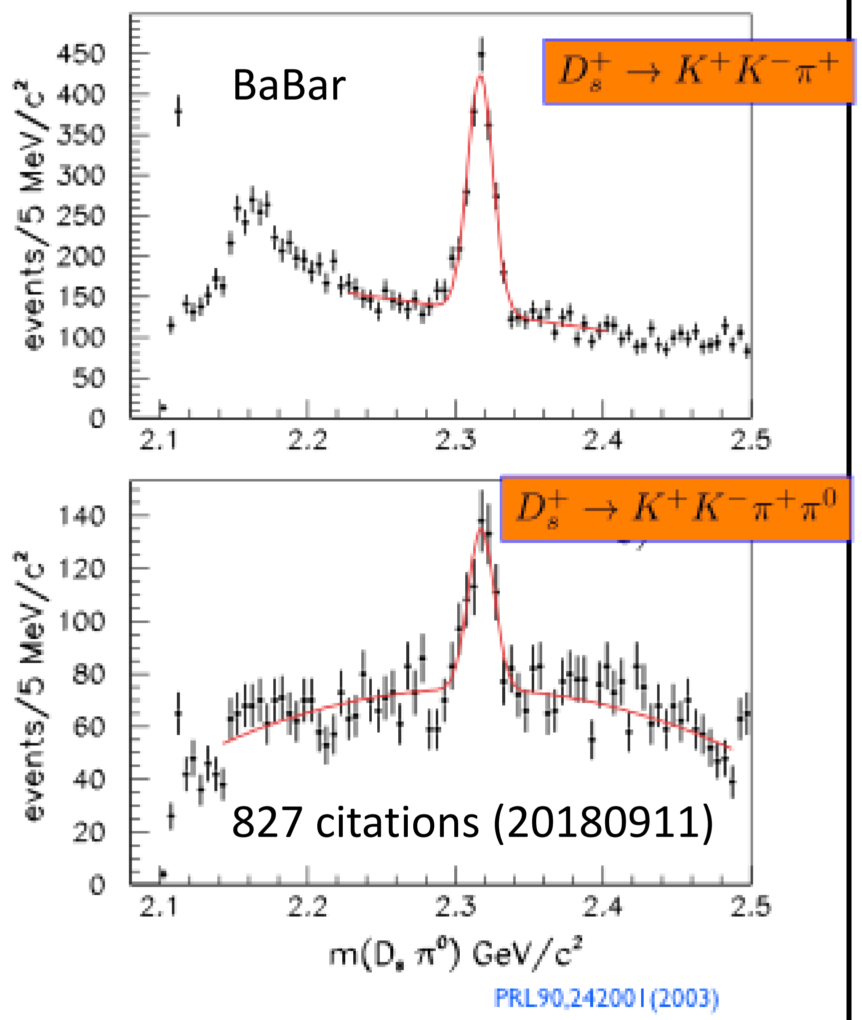
- **160/70 lower than the GI quark model predictions**—difficult to be understood as conventional csbar states.

- "Dynamically generated" from strong DK interaction
  - E. E. Kolomeitsev 2004,
  - F. K. Guo 2006,
  - D. Gamermann 2007
  - ...

$$m_{D_{s1}(2460)} - m_{D_{s0}^*(2317)} \approx m_{D^*} - m_D$$

Feng-Kun Guo, EPJ Web of Conferences **202**, 02001 (2019)

$\mathrm{M} = 2317.8 \pm 0.6$ and $\Gamma < 3.8$ MeV



Analogy between $\boldsymbol{KD}$ and $\bar{\bar{\boldsymbol{K}}}\boldsymbol{N}$

$\mathbf{D_{s0}^*(2317)}$

$\boldsymbol{\Lambda(1405)}$

- $\boldsymbol{KD}$ bound state
- **Dynamically generated**--Unitary heavy hadron chiral perturbation theory

- $\bar{\bar{\boldsymbol{K}}}\boldsymbol{N}$ bound state
- **Dynamically generated**--Unitary baryon chiral perturbation theory

The interaction between **a kaon and a heavy particle** seems to play an important role, described by the same WT at the leading order

UChPT in Bethe-Salpeter equation

Altenbuchinger, Geng, Weise, PRD89 (2014)014026

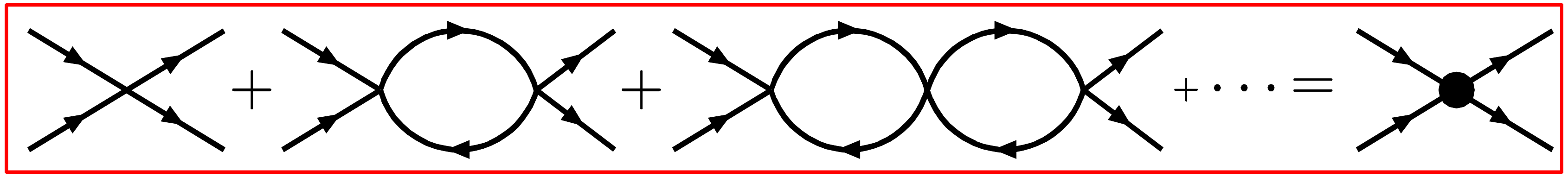
□ Model independent DK interaction from ChPT

$$\mathcal{V}_{\rm WT}(P(p_1)\phi(p_2) \to P(p_3)\phi(p_4)) = \frac{1}{4f_0^2}\mathcal{C}_{\rm LO}\,(s-u)$$

Weinberg-Tomazawa

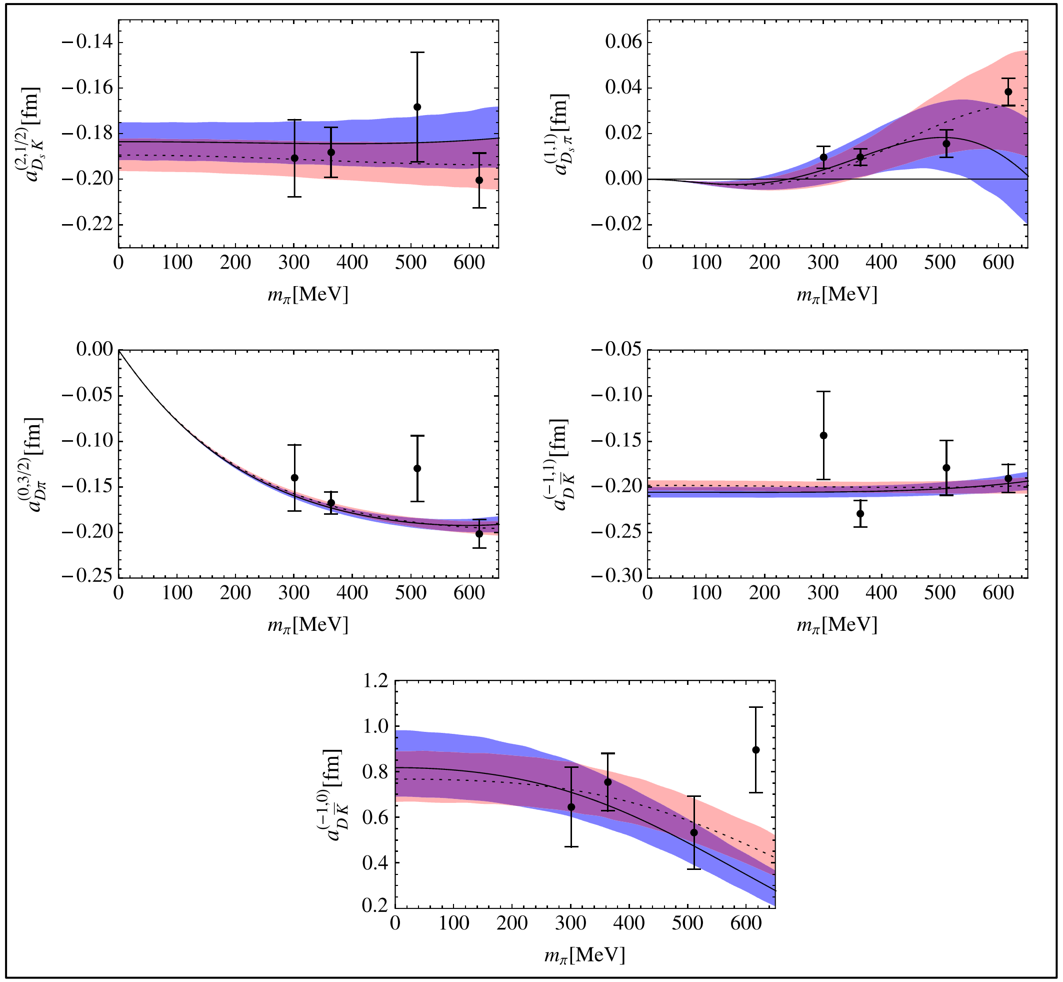
$$\begin{aligned}\mathcal{V}_{\rm NLO}(P(p_1)\phi(p_2) \to P(p_3)\phi(p_4)) = &-\frac{8}{f_0^2}\mathcal{C}_{24}\left(c_2\, p_2\cdot p_4 - \frac{c_4}{m_P^2}\left(p_1\cdot p_4\; p_2\cdot p_3 + p_1\cdot p_2\; p_3\cdot p_4\right)\right)\\ &-\frac{4}{f_0^2}\mathcal{C}_{35}\left(c_3\, p_2\cdot p_4 - \frac{c_5}{m_P^2}\left(p_1\cdot p_4\; p_2\cdot p_3 + p_1\cdot p_2\; p_3\cdot p_4\right)\right)\\ &-\frac{4}{f_0^2}\mathcal{C}_6\,\frac{c_6}{m_P^2}\left(p_1\cdot p_4\; p_2\cdot p_3 - p_1\cdot p_2\; p_3\cdot p_4\right)\\ &-\frac{8}{f_0^2}\mathcal{C}_0\, c_0 + \frac{4}{f_0^2}\mathcal{C}_1\, c_1\,, \qquad (11)\end{aligned}$$

□ Resumed in the Bethe-Salpeter equation (two-body elastic unitarity)

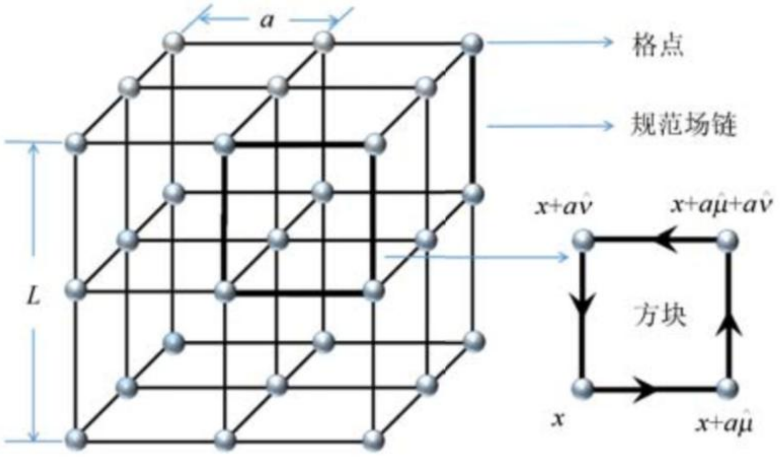


Fixing the LECs using latest LQCD* data

Altenbuchinger, LSG*, Weise, PRD89 (2014)014026



- NLO ChPT kernel: 5 LECs
- A quite good description of the 20 Lattice **scattering lengths of pseudoscalar mesons and D mesons （I=0 DK excluded）** can be achieved.



Ds0 and Ds1 dynamically generated

Altenbuchinger, LSG*, Weise, PRD89 (2014)014026

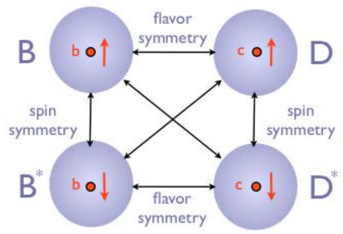
"Post-diction"

$D^*_{s0}(2317),\ D_{s1}(2460)$

□ Charm sector

TABLE V. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	2317 ± 10	2457 ± 17
(0, 1/2)	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$



□ Bottom Sector

TABLE VI. Pole positions $\sqrt{s} = M - i\frac{\Gamma}{2}$ (in units of MeV) of bottom mesons dynamically generated in the HQS UChPT.

(S, I)	$J^P = 0^+$	$J^P = 1^+$
(1, 0)	5726 ± 28	5778 ± 26
(0, 1/2)	$(5537 \pm 14) - i(118 \pm 22)$	$(5586 \pm 16) - i(124 \pm 25)$

Predicted Bs0 and Bs1 states

Altenbuchinger, LSG*, Weise, PRD89 (2014)014026

Physics Letters B 750 (2015) 17–21

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Predicting positive parity B_s mesons from lattice QCD

C.B. Lang [a], Daniel Mohler [b,*], Sasa Prelovsek [c,d], R.M. Woloshyn [e]

[a] Institute of Physics, University of Graz, A-8010 Graz, Austria
[b] Fermi National Accelerator Laboratory, Batavia, IL 60510-5011, USA
[c] Department of Physics, University of Ljubljana, 1000 Ljubljana, Slovenia
[d] Jozef Stefan Institute, 1000 Ljubljana, Slovenia
[e] TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

Table 5
Comparison of masses from this work to results from various model based calculations; all masses in MeV.

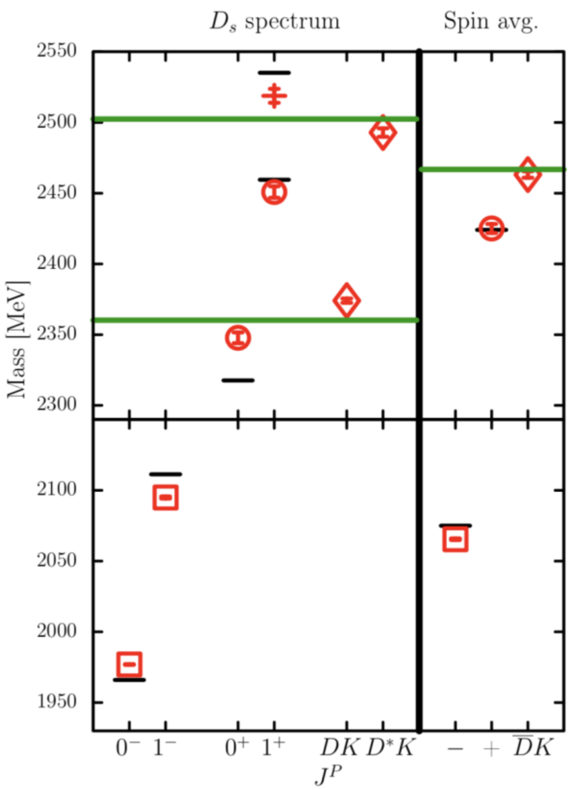
J^P	0^+	1^+
Covariant (U)ChPT [24]	5726(28)	5778(26)
NLO UHMChPT [19]	5696(20)(30)	5742(20)(30)
LO UChPT [17,18]	5725(39)	5778(7)
LO χ-SU(3) [16]	5643	5690
HQET + ChPT [20]	5706.6(1.2)	5765.6(1.2)
Bardeen, Eichten, Hill [15]	5718(35)	5765(35)
rel. quark model [5]	5804	5842
rel. quark model [22]	5833	5865
rel. quark model [23]	5830	5858
HPQCD [30]	5752(16)(5)(25)	5806(15)(5)(25)
this work	5713(11)(19)	5750(17)(19)

In agreement with lQCD

More support from following lQCD studies

- G.K.C. Cheung et al., arXiv:2008.06432[hep-lat].
- G. S. Bali et al., arXiv:1706.01247 [hep-lat].

C. B. Lang et al., arXiv:1403.8103 [hep-lat].
D. Mohler et al., arXiv:1308.3175 [hep-lat].

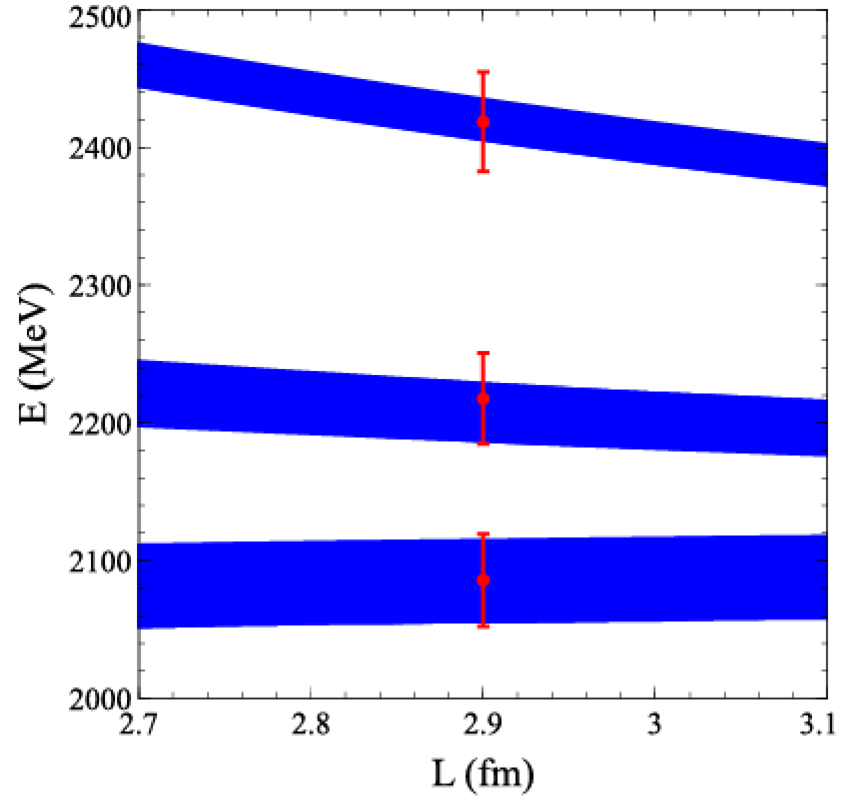


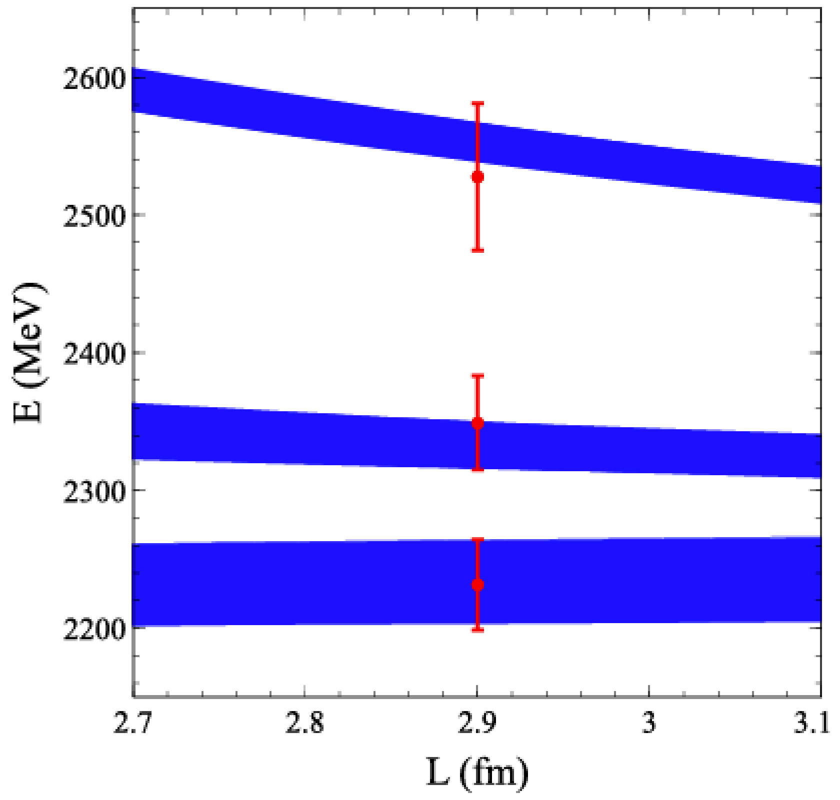
"DK components substantial"

FIG. 12. On the left, our final results for the lower lying D_s spectrum as detailed in Table VII. The short horizontal black lines indicate the corrected experimental values (see Section II) while the green horizontal lines give the positions of the DK and D^*K non-interacting thresholds. Our lattice results for the finite volume thresholds are labelled DK and D^*K, respectively. The errors indicated are statistical only. On the right, the negative parity spin-averaged $1S$ mass $m_- = \frac{1}{4}(m_{0^-} + 3m_{1^-})$ is shown and denoted $-$, while the same spin-average of the positive parity 0^+ and 1^+ states is labelled with $+$ and the weighted average of the threshold is labelled as $\overline{DK}$.

See as well Miguel Albaladejo et al. **arXiv:1805.07104**

Many re-analyses of the lattice finite volume data

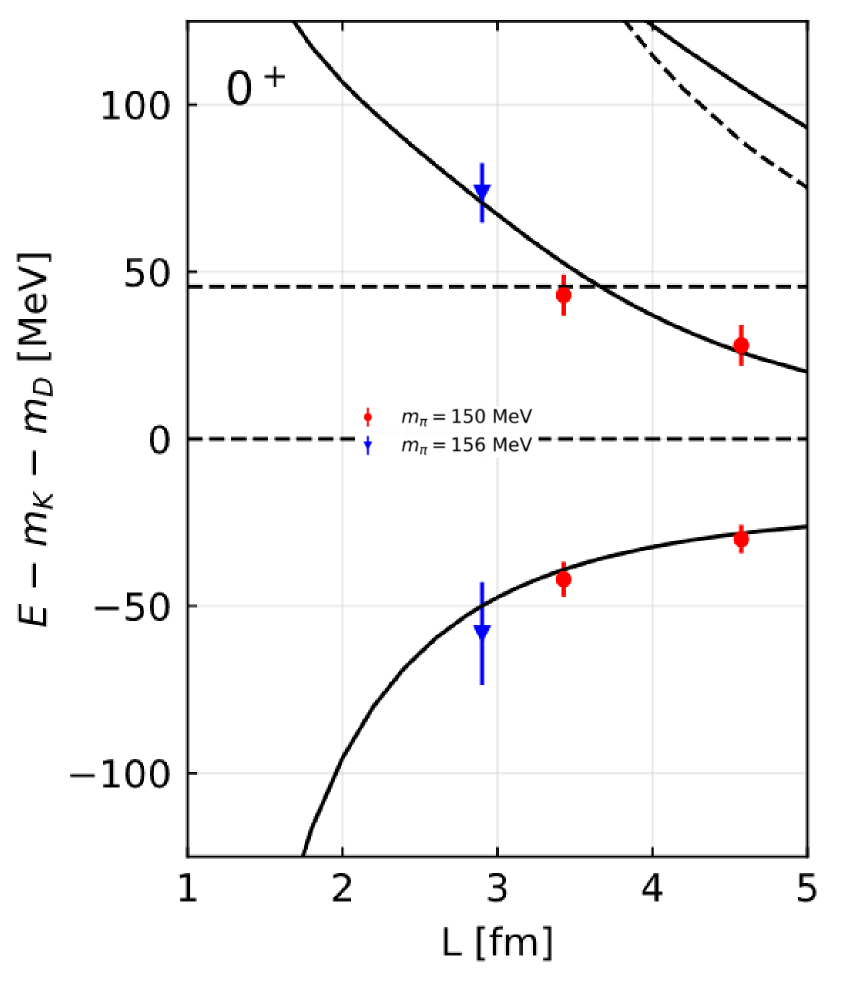




$$B(KD) = 38 \pm 18 \pm 9 \text{ MeV},$$
$$B(KD^*) = 44 \pm 22 \pm 26 \text{ MeV},$$
$$P(KD) = 72 \pm 13 \pm 5 \text{ \%},$$
$$P(KD^*) = 57 \pm 21 \pm 6 \text{ \%},$$
$$a(KD) = -1.3 \pm 0.5 \pm 0.1 \text{ fm},$$
$$a(KD^*) = -1.1 \pm 0.5 \pm 0.2 \text{ fm},$$
$$r_0(KD) = -0.1 \pm 0.3 \pm 0.1 \text{ fm},$$
$$r_0(KD^*) = -0.2 \pm 0.3 \pm 0.1 \text{ fm},$$

Torres, Oset, Prelovsekc, and Ramos, *JHEP* 05 (2015) 153

Support from LQCD+unquenched QM



***DK* about 70%**

Yang, Wang, Wu, Oka, Zhu,
Phys. Rev. Lett. 128, 112001 (2022)

TABLE II. The comparison of D_s pole masses (MeV) (Ours) with the experimental results. The script $P(c\bar{s})$ represents the content of the bare $c\bar{s}$ cores in the D_s states at $L = 4.57$ fm.

	$P(c\bar{s})(\%)$	Ours	Experimental results
$D_{s0}^*(2317)$	$32.0^{+5.2}_{-3.9}$	$2338.9^{+2.1}_{-2.7}$	2317.8 ± 0.5
$D_{s1}^*(2460)$	$52.4^{+5.1}_{-3.8}$	$2459.4^{+2.9}_{-3.0}$	2459.5 ± 0.6
$D_{s1}^*(2536)$	$98.2^{+0.1}_{-0.2}$	$2536.6^{+0.3}_{-0.5}$	2535.11 ± 0.06
$D_{s2}^*(2573)$	$95.9^{+1.0}_{-1.5}$	$2570.2^{+0.4}_{-0.8}$	2569.1 ± 0.8

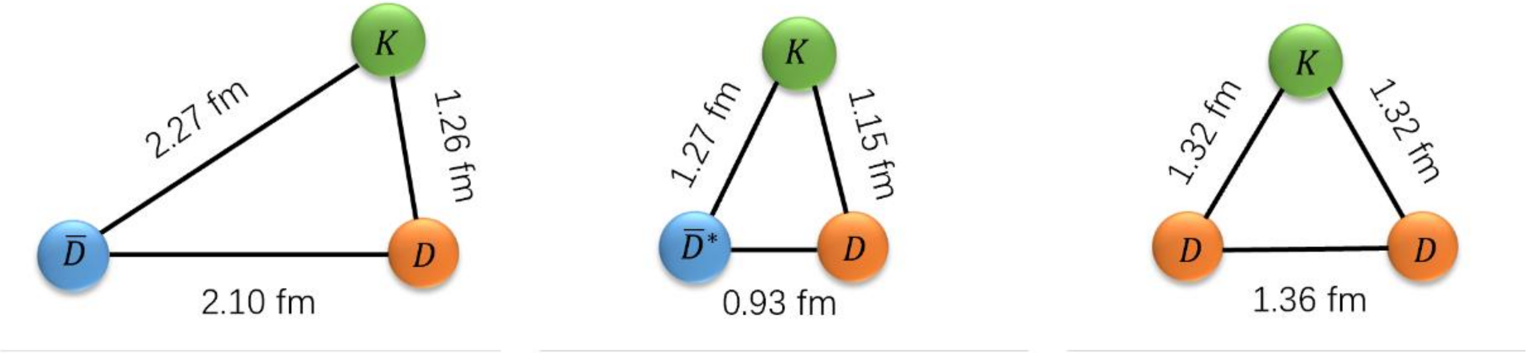
$$\boldsymbol{M(D_{s1}(2460)) - M(D_{s0}(2317)) = 120}$$ ‼ !

Further tests of the DK interaction

- **Experiments, theory, and lattice QCD all show that** DK or D^*K interaction is strong enough to form $\boldsymbol{D_{s0}^*(2317)}$ **and** $\boldsymbol{D_{s1}(2460}$
- **A natural question is: if we add one more** $\boldsymbol{D(\bar{D})}$ **or** $\boldsymbol{D^*(\overline{D^*})}$**, can they form molecules of three hadrons?**
- This seems to **be a rather straightforward and naive question**, but remains unexplored until quite recently

Three-body molecules built from the KD interaction

Tian-Wei Wu, Ming-Zhu Liu, and LSG*, Phys.Rev.D 103 (2021) L031501

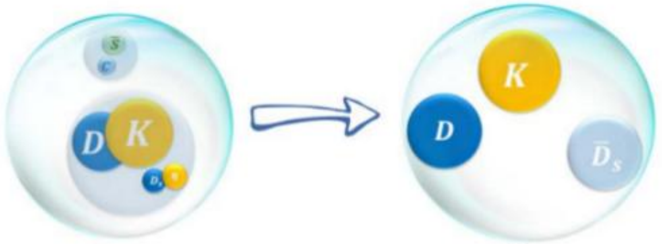


Kc(4180) K*(4307) R(4140)

	This work	Ref [28]	Ref [29]
Method	GEM(SE)	BOA(SE)	FCA(FE)
Interaction Models	χEFT+OBE	delocalized π bond	χEFT+OBE
$\frac{1}{2}(0^-)\ D\bar{D}K$	$4181.2^{+2.4}_{-1.4}(B_3 \simeq 48.9^{+1.4}_{-2.4})$	-	-
$\frac{1}{2}(1^-)\ D\bar{D}^*K$	$4294.1^{+6.6}_{-3.1}(B_3 \simeq 77.3^{+3.1}_{-6.6})$	$4317.92^{+6.13}_{-6.55}(B_3 \simeq 53.52^{+6.55}_{-6.13})$	$4307 \pm 2(B_3 \simeq 64 \pm 2)$

[28] L. Ma, Q. Wang, and U.-G. Meißner, Chin. Phys. C 43, 014102 (2019).
[29] X.-L. Ren, B.B. Malabarba, L.-S. Geng, K.P. Khemchandani, and A. Martínez Torres, Phys. Lett. B 785, 112 (2018).

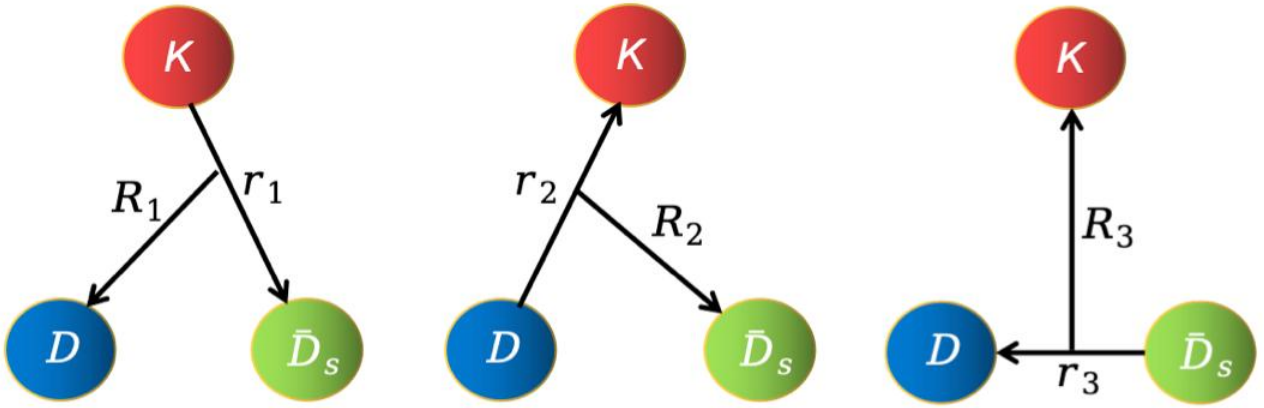
Why the $\bar{D}_s DK$ three-body system is different



$D_{s0}^*(2317)$ $\bar{D}_s DK$

- ☐ It is a three-body system involving C-parity interaction
- ☐ It does not mix with conventional hadrons— 0^{--} exotic quantum numbers
- ☐ The two-body system can not bind—suppressed by the OBE model or the OZI rule
- ☐ It can be produced in both e^+e^- and pp collisions—hidden charm/strange state easier to observe

Wave function of $\bar{D}_s D K$ with good C parity



$$\bar{D}_s D K$$

$$\Psi^C = \frac{1}{\sqrt{2}}(\Psi_{\bar{D}_s D K} + C\Psi'_{D_s \bar{D} \bar{K}}), \quad \langle \Psi^C | (H - E) | \Psi^C \rangle = 0$$

$$C = \pm 1$$

$$\Psi'_{D_s \bar{D} \bar{K}} = \hat{C}\Psi_{\bar{D}_s D K} = \sum_{c=1,3} \Phi(r'_c, R'_c).$$

$$H = T + V + V_C$$

Two-body interaction

C-parity interaction

Two-body interactions

- DK interaction parameterized by C_a-- in the contact-range EFT
- $DK - D_s\eta$ coupled channel interaction in matrix form

$$V_{DK-D_s\eta}^{J^P=0^+} = \begin{pmatrix} C_a & -\frac{\sqrt{3}}{2}C_a \\ -\frac{\sqrt{3}}{2}C_a & 0 \end{pmatrix}$$

- V_{DK} in a Gaussian form in coordinate space

$$V(r) = C_a \frac{e^{(r/R_c)2}}{\pi^{3/2}R_c^3},$$

- With SU(3) flavor symmetry and experimental fitting

$$C_a^{DK} : C_a^{\bar{D}_sK} : C_a^{\bar{D}_sD} \approx 1 : 0.5 : 0.1$$

DK interaction determined by fitting $D_{s0}^*(2317)$

- Considering $D_{s0}^*(2317)$ as a $DK - D_s\eta$ molecule + $c\bar{s}$ state

Momentum space

Couplings	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
$g_{D_{s0}^* DK}$ (GeV)	19.37	14.72	13.32	12.66	16.20	12.28	11.16	10.63
$g_{D_{s0}^* D_s \eta}$ (GeV)	13.23	9.54	8.40	7.86	10.42	7.70	6.89	6.50
$C_a(\mathrm{fm}^2)$	−5.78	−1.84	−1.03	−0.71	−6.96	−2.06	−1.12	−0.75
Compositeness	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
P_{DK}	0.92	0.90	0.89	0.88	0.65	0.63	0.62	0.62
$P_{D_s\eta}$	0.08	0.10	0.11	0.12	0.05	0.07	0.08	0.08

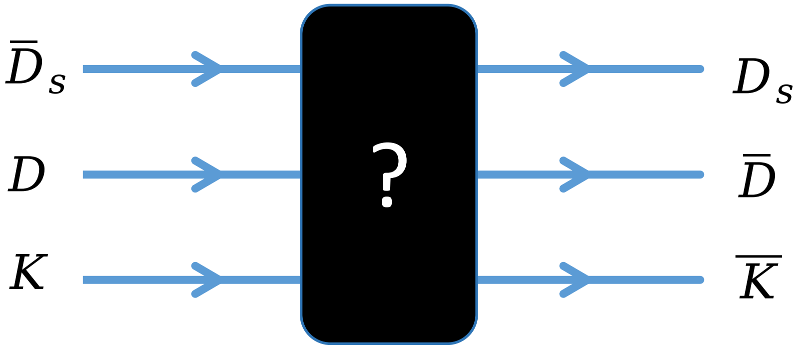
100% molecule (columns 2–5) 70 % molecule+30 % $c\bar{s}$ (columns 6–9)

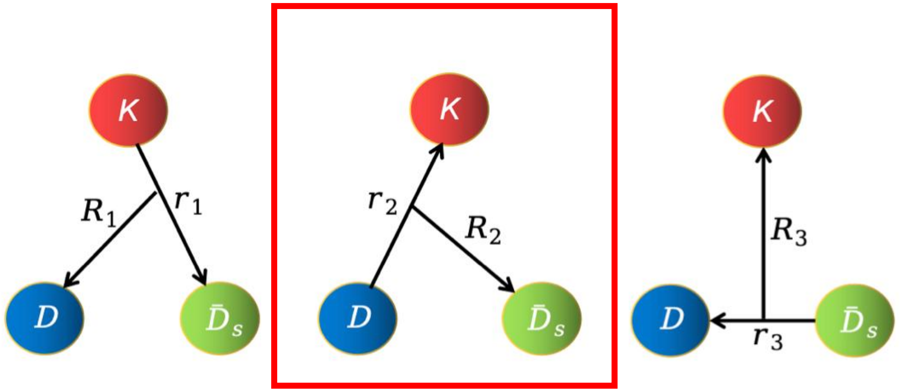
Coordinate space

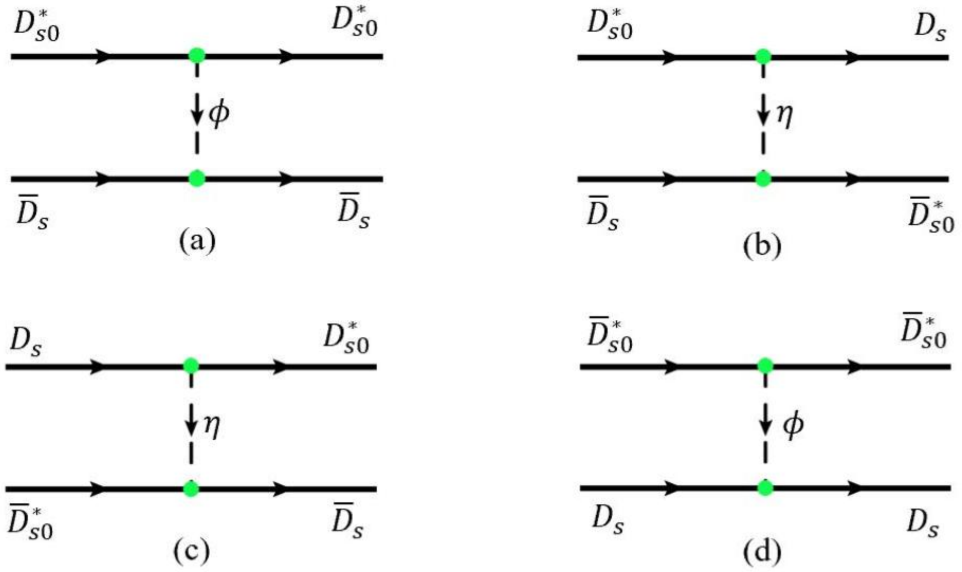
Components of $D_{s0}^*(2317)$	$M(DK - D_s\eta)$	$M(DK)$	$M(c\bar{s})$ [4]	$P(c\bar{s})$	$P(DK)$	$P(D_s\eta)$
70% molecule+30% $c\bar{s}$	2280	2349	2406	30%	60%	10%
100% molecule	2318	2358	2406	0%	90%	10%
50% molecule+50% $c\bar{s}$	2230	2336	2406	50%	42%	8%

$DK - D_s\eta > c\bar{s}$ $B_{DK} < 45$ MeV

C-parity dependent interaction $\bar{D}_s D_{s0}^* - D_s \bar{D}_{s0}^*$







η exchange potential

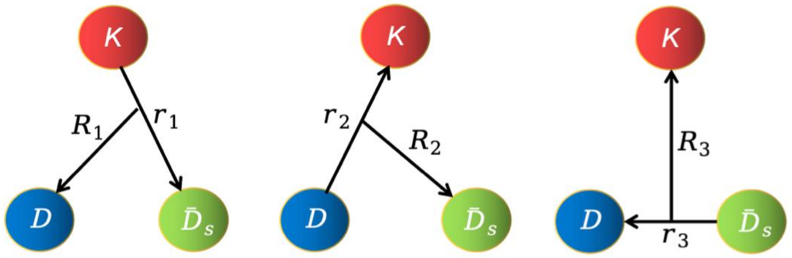
$$V^{C=\pm} = \mp\frac{2}{3}\frac{k^2}{f_\pi^2}q_0^2\left(\frac{e^{-mr}-e^{-\Lambda r}}{4\pi r} - \frac{\Lambda^2-m^2}{8\pi\Lambda}e^{-\Lambda r}\right)$$

$$q_0 = m_{D_{s0}^*} - m_{D_s}, k = 0.56,$$

$$\Lambda = \alpha\Lambda_{QCD} + m_{eff}, m_{eff} = \sqrt{m_\eta^2 - (m_{D_{s0}^*} - m_{D_s})^2}.$$

Binding energies, relative weights, and spatial structure

0^{--} $\bar{D}_s D K$ molecule: X(4310)



Scenarios	B.E.(0^{--})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha=1$	22^{+23}_{-14}	11^{+1}_{-1} %	78^{-1}_{+2} %	11^{+0}_{-1} %
$\alpha=2$	20^{+22}_{-13}	10^{+1}_{-1}%	80^{-1}_{+2} %	10^{+0}_{-1} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T\rangle$
$\alpha=1$	$1.6^{-0.4}_{+0.9}$	$1.2^{-0.3}_{+0.5}$	$1.4^{-0.3}_{+0.8}$	177^{+81}_{-74}
$\alpha=2$	$1.7^{-0.4}_{+1.4}$	$1.2^{-0.3}_{+0.7}$	$1.6^{-0.5}_{+1.3}$	169^{+82}_{-78}
Scenarios	$\langle V_{D_s\bar{K}}\rangle$	$\langle V_{DK}\rangle$	$\langle V_{D_s\bar{D}}\rangle$	$\langle V^{C=-}_{D_s\bar{D}^*_{s0}}\rangle$
$\alpha=1$	-40^{-25}_{+20}	-147^{-73}_{+62}	-14^{-7}_{+6}	2^{+0}_{-1}
$\alpha=2$	-37^{-25}_{+22}	-143^{-73}_{+64}	-13^{-6}_{+7}	3^{+2}_{-1}

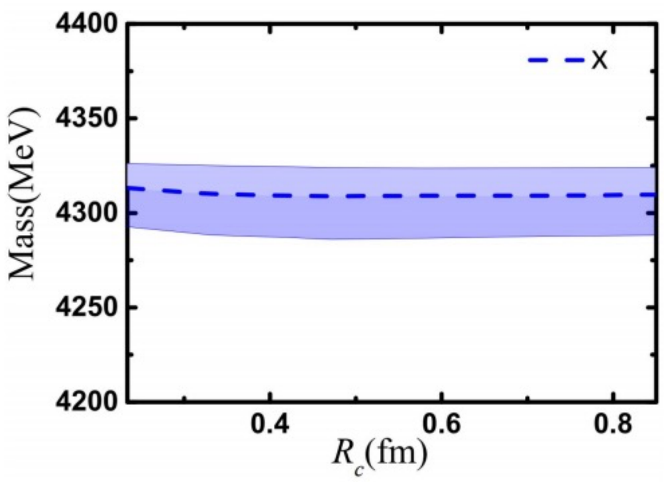
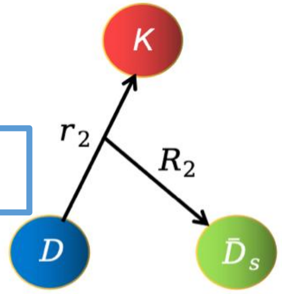


FIG. 5. Mass of X as a function of the cutoff R_c.

Dominant component

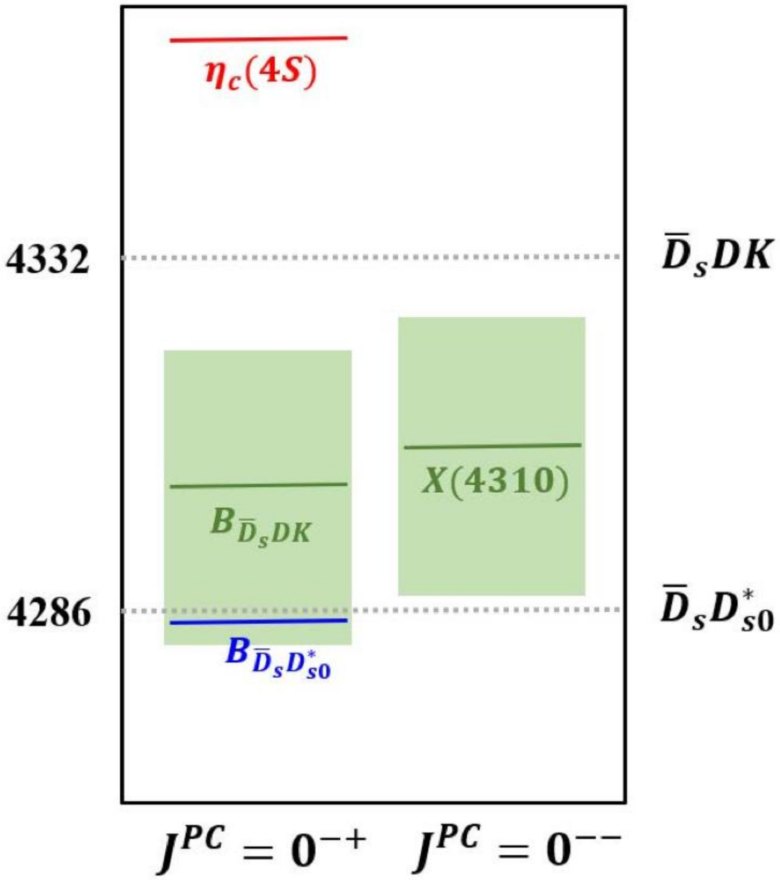


Cutoff $\Lambda = \alpha\Lambda_{QCD} + m_E$

Comparion with the 0^{-+} state

0^{-+} $\bar{D}_s D K$ molecule: 4304 MeV

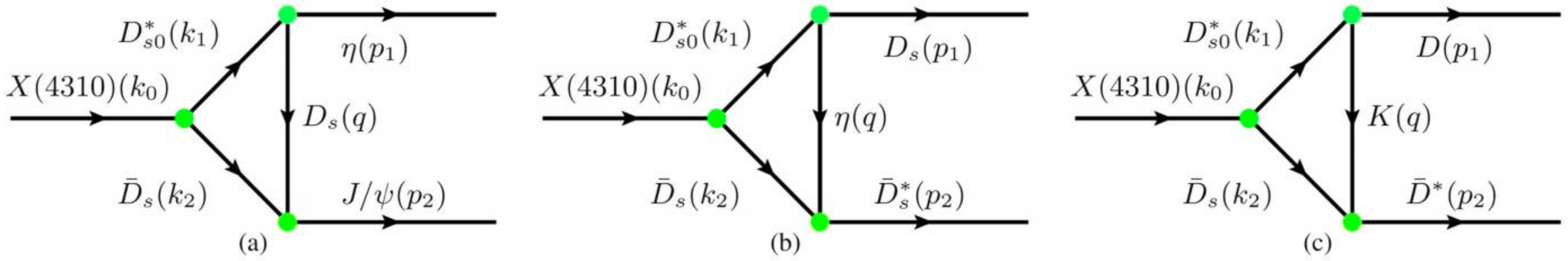
Sets	B.E.(0^{-+})	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha=1$	26^{+22}_{-16}	13^{+0}_{-1} %	76^{-0}_{+0} %	11^{-0}_{+1} %
$\alpha=2$	28^{+23}_{-17}	14^{+0}_{-1}%	74^{+1}_{+1} %	12^{-1}_{+0} %
Scenarios	$r_{\bar{D}_s K}$	r_{DK}	$r_{\bar{D}_s D}$	$\langle T\rangle$
$\alpha=1$	$1.5^{-0.3}_{+0.7}$	$1.1^{-0.2}_{+0.5}$	$1.3^{-0.3}_{+0.6}$	187^{+78}_{-73}
$\alpha=2$	$1.4^{-0.3}_{+0.6}$	$1.1^{-0.2}_{+0.4}$	$1.2^{-0.2}_{+0.5}$	195^{+77}_{-73}
Scenarios	$\langle V_{D_s\bar{K}}\rangle$	$\langle V_{DK}\rangle$	$\langle V_{D_s\bar{D}}\rangle$	$\langle V^{C=+}_{D_s\bar{D}^*_{s0}}\rangle$
$\alpha=1$	-44^{-24}_{+20}	-151^{-71}_{+61}	-16^{-6}_{+6}	-2^{-0}_{+1}
$\alpha=2$	-47^{-24}_{+21}	-154^{-70}_{+61}	-17^{-6}_{+6}	-4^{-1}_{+2}



- 0^{-+} $\bar{D}_s DK$: coupling with $c\bar{c}$ *and* $\bar{D}_s D^*_{s0}$
- 0^{--} $\bar{D}_s DK$: no coupling with $c\bar{c}$ *and* $\bar{D}_s D^*_{s0}$

Strong decays of the 0^{--} $\bar{D}_s D K$ molecule X(4310)

Triangle diagrams of the strong decays: three modes



$$i\mathcal{M}_a = g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}D_s\eta} g_{\psi\bar{D}_s D_s} \int \frac{d^4q}{(2\pi)^4}(k_2^\mu - q^\mu)\frac{1}{k_1^2 - m^2_{D^*_{s0}}}\frac{1}{k_2^2 - m^2_{\bar{D}_s}}\frac{1}{q^2 - m^2_{D_s}}\varepsilon_\mu(p_2)F(q^2),$$

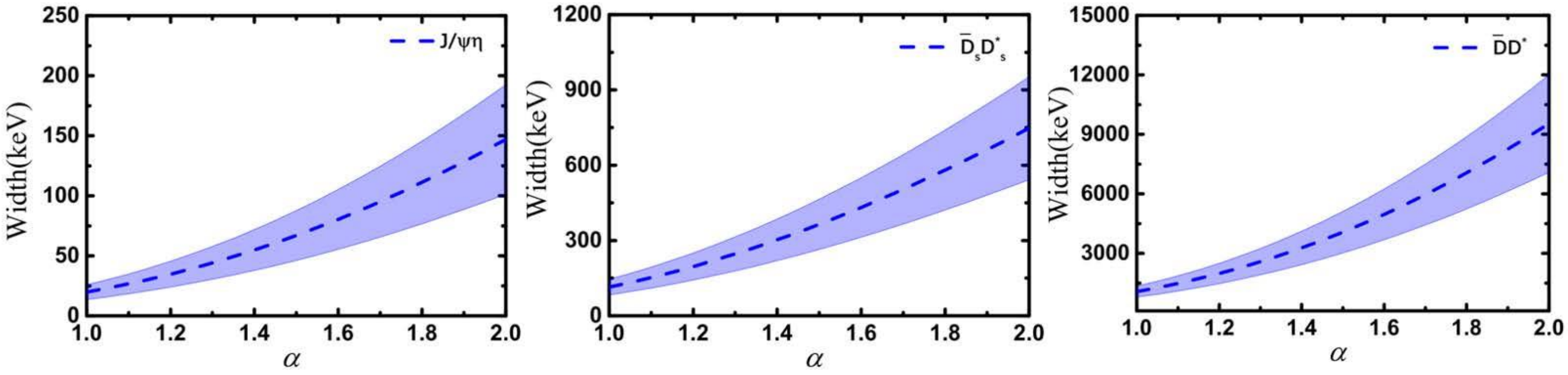
$$i\mathcal{M}_b = g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}D_s\eta} g_{\bar{D}_s D^*_s\eta} \int \frac{d^4q}{(2\pi)^4}q^\mu\frac{1}{k_1^2 - m^2_{D^*_{s0}}}\frac{1}{k_2^2 - m^2_{\bar{D}_s}}\frac{1}{q^2 - m^2_{\eta}}\varepsilon_\mu(p_2)F(q^2),$$

$$i\mathcal{M}_c = g_{XD^*_{s0}\bar{D}_s} g_{D^*_{s0}DK} g_{\bar{D}_s D^* K} \int \frac{d^4q}{(2\pi)^4}q^\mu\frac{1}{k_1^2 - m^2_{D^*_{s0}}}\frac{1}{k_2^2 - m^2_{\bar{D}_s}}\frac{1}{q^2 - m^2_{K}}\varepsilon_\mu(p_2)F(q^2),$$

$$\Gamma = \frac{1}{2J+1}\frac{1}{8\pi}\frac{|\vec{p}|}{M^2}|\bar{\mathcal{M}}|^2 \qquad F(q,\Lambda,m) = (\frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2})^2,$$

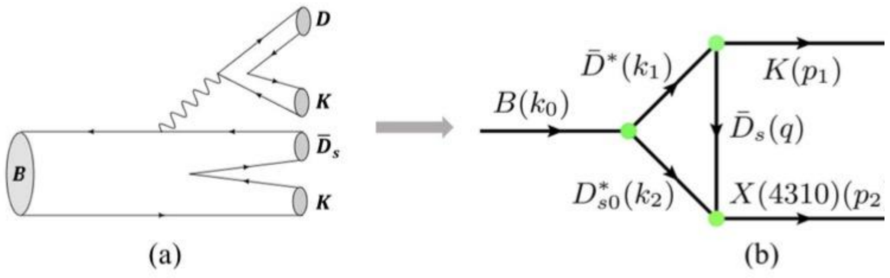
Partial decay widths

Dominant decay mode: $X \to \bar{D}^* D$

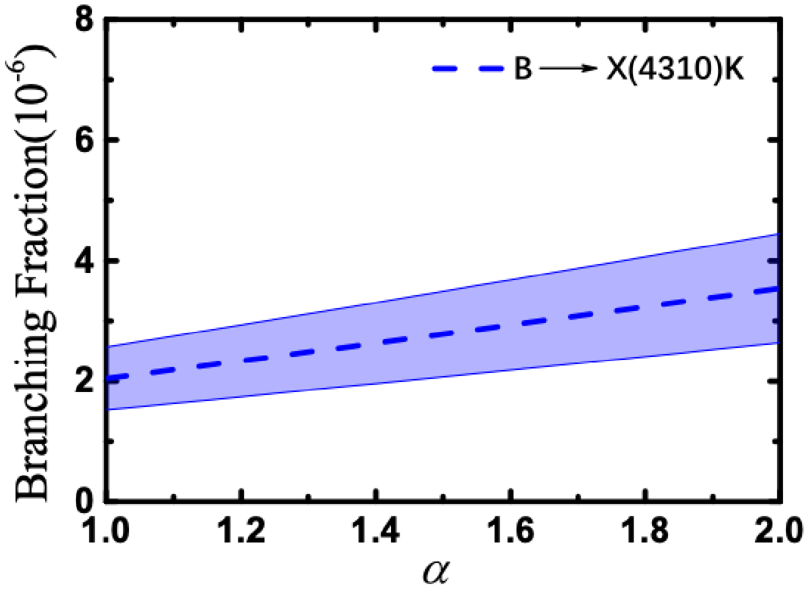


$X \to J/\psi\eta \sim 10^1$ $\quad X \to \bar{D}_s D_s^* \sim 10^2$ $\quad X \to \bar{D}^* D \sim 10^3$

Productions of the 0^{--} $\bar{D}_s DK$ molecule



0^{--} $\bar{D}_s DK$ molecule:



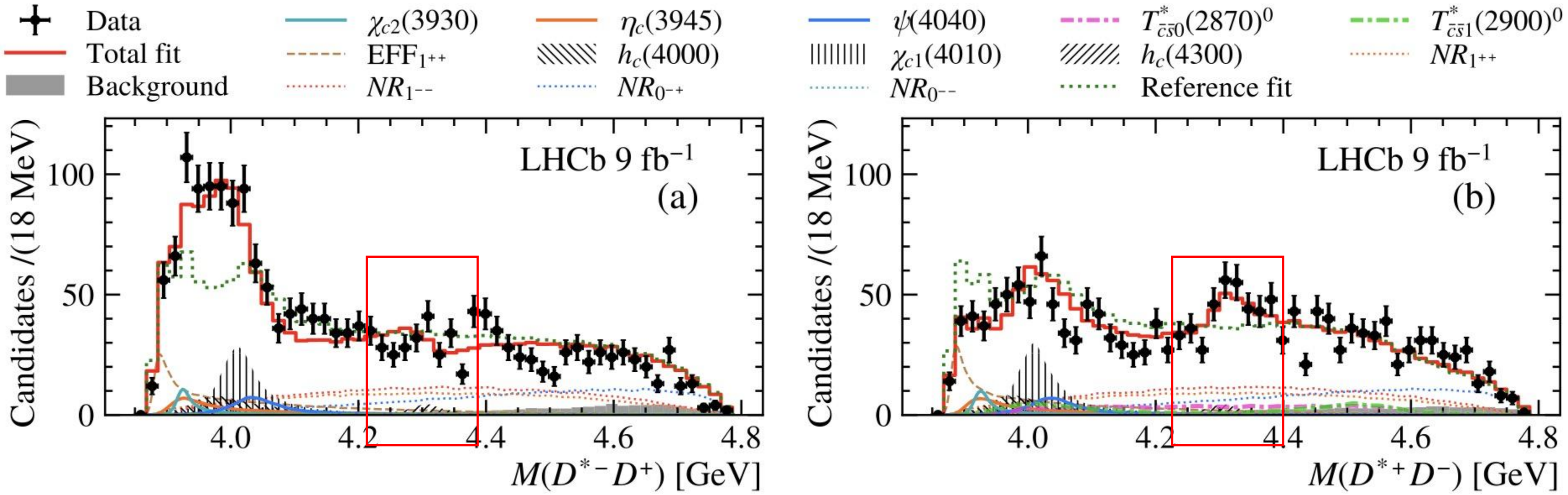
$$\mathcal{M} = g_{\bar{D}^*\bar{D}_s K} g_{D^*_{s0}\bar{D}_s X} \mathcal{A}(B \to D^*_{s0}\bar{D}^*)^\mu \frac{-g^{\mu\nu} + \frac{k_1^\mu k_1^\nu}{k_1^2}}{(k_1^2 - m^2_{\bar{D}^*})(k_2^2 - m^2_{D^*_{s0}})(q^2 - m^2_{\bar{D}_s})} p_1^\nu F(q^2),$$

$$\mathcal{A}(B \to D^*_{s0}\bar{D}^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 f_{D^*_{s0}} \{-q_1 \cdot \varepsilon(q_2)(m_{D^*} + m_B) A_1(q_1^2) + (k_0 + q_2) \cdot \varepsilon(q_2) q_1 \cdot (k_0 + q_2) \frac{A_2(q_1^2)}{m_{D^*} + m_B} + (k_0 + q_2) \cdot \varepsilon(q_2)[(m_{D^*} + m_B) A_1(q_1^2) - (m_B - m_{D^*}) A_2(q_1^2) - 2m_{D^*} A_0(q_1^2)]\},$$

Production rate: $B \to [X(4310) \to \bar{D}^* D]K \sim (3 \pm 1) \times 10^{-6}$

Possibility at the LHCb

LHCb, Phys. Rev. Lett. 133,131902(2024)



Component	$J^{P(C)}$	Fit fraction [%] $B^+ \to D^{*+}D^-K^+$	Fit fraction [%] $B^+ \to D^{*-}D^+K^+$	Branching fraction $[10^{-4}]$
$T^*_{\bar{c}\bar{s}0}(2870)^{0,\dagger}$	0^+	$6.5^{+0.9}_{-1.2}{}^{+1.3}_{-1.6}$	$\cdots$	$0.45^{+0.06}_{-0.08}{}^{+0.09}_{-0.10} \pm 0.04$
$T^*_{\bar{c}\bar{s}1}(2900)^{0\dagger}$	1^-	$5.5^{+1.1}_{-1.5}{}^{+2.4}_{-1.6}$	$\cdots$	$0.38^{+0.07}_{-0.10}{}^{+0.16}_{-0.11} \pm 0.03$
$\text{NR}_{1^{--}}(D^{*\mp}D^{\pm})$	1^{--}	$20.4^{+2.3}_{-0.6}{}^{+2.1}_{-2.6}$	$18.5^{+2.1}_{-0.5}{}^{+1.9}_{-2.3}$	$1.39^{+0.16}_{-0.04}{}^{+0.14}_{-0.17} \pm 0.12$
$\text{NR}_{0^{--}}(D^{*\mp}D^{\pm})$	0^{--}	$1.2^{+0.6}_{-0.1}{}^{+0.7}_{-0.6}$	$1.1^{+0.6}_{-0.1}{}^{+0.6}_{-0.5}$	$0.08^{+0.04}_{-0.01}{}^{+0.05}_{-0.04} \pm 0.01$
$\text{NR}_{1^{++}}(D^{*\mp}D^{\pm})$	1^{++}	$17.8^{+1.9}_{-1.4}{}^{+3.6}_{-2.6}$	$16.1^{+1.7}_{-1.3}{}^{+3.3}_{-2.3}$	$1.21^{+0.13}_{-0.10}{}^{+0.24}_{-0.17} \pm 0.11$
$\text{NR}_{0^{-+}}(D^{*\mp}D^{\pm})$	0^{-+}	$15.9^{+3.3}_{-1.2}{}^{+3.3}_{-3.3}$	$14.5^{+3.0}_{-1.1}{}^{+3.0}_{-3.0}$	$1.09^{+0.23}_{-0.08}{}^{+0.22}_{-0.23} \pm 0.09$

Number of events at the LHC

$\mathcal{B}(B^+ \to D^{*\pm} D^{\mp} K^+]) \sim 6 \times 10^{-4}$

BaBar, Phys. Rev. D 83, 032004(2011)

$B^+ \to D^{*\pm} D^{\mp} K^+ \sim 2 \times 10^3$

LHCb, Phys. Rev. Lett. 133,131902(2024) 9fb^{-1}

A factor of 1000 smaller

The state of our interests

$B^+ \to [X(4310) \to D^{*\pm} D^{\mp}] K^+ \sim 5 \times 10^{-7}$

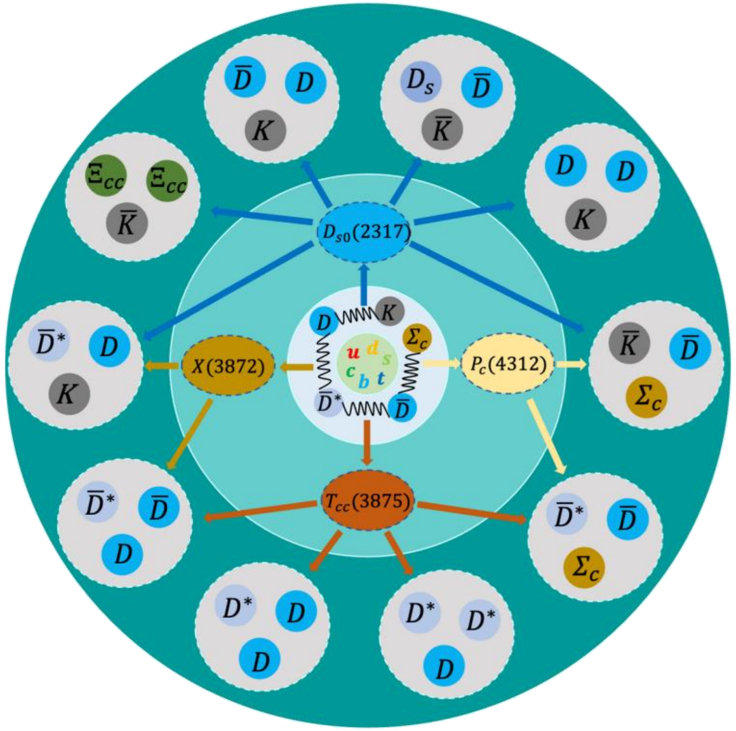
LHC integrated luminosity: 50 fb^{-1} Events: 10

350 fb^{-1} 100

Summary and outlook

- Λ(1405) and $D_{s0}^*(2317)$ do not easily fit into the naïve CQM, but can be understood as $\bar{K}N$ and DK hadronic molecules
- **We proposed two novel ways to validate their molecular nature**
  - The light quark mass evolution of the Λ(1405) serves as a nontrivial way to test its $\bar{K}N$ molecular and two-pole nature.
  - The nature of the $D_{s0}^*(2317)$ as a DK molecule can be checked by the existence of a unique three-body bound state $\bar{D}_s DK$.
- These ideas can be extended to study other exotic hadrons

Thanks for your attention



Science Bulletin 67 (2022) 1735–1738