PREDICTING HYPERNUCLEI BASED ON CHIRAL INTERACTIONS

15th International Conference on Hypernuclear and Strange Particle Physics (HYP2025)

Oct 1st I Andreas Nogga

- YN & YY interactions
- Tools: Faddeev-Yakubovsky, J-NCSM & SRG
- Determination of CSB contact interactions and Λn scattering length

  & Application to $A=7$ and 8 hypernuclei
- Uncertainty of Λ separation energies & chiral YNN interactions
- $S=-2$ hypernuclei: predictions for $A \leq 7$
- Conclusions & Outlook

in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner

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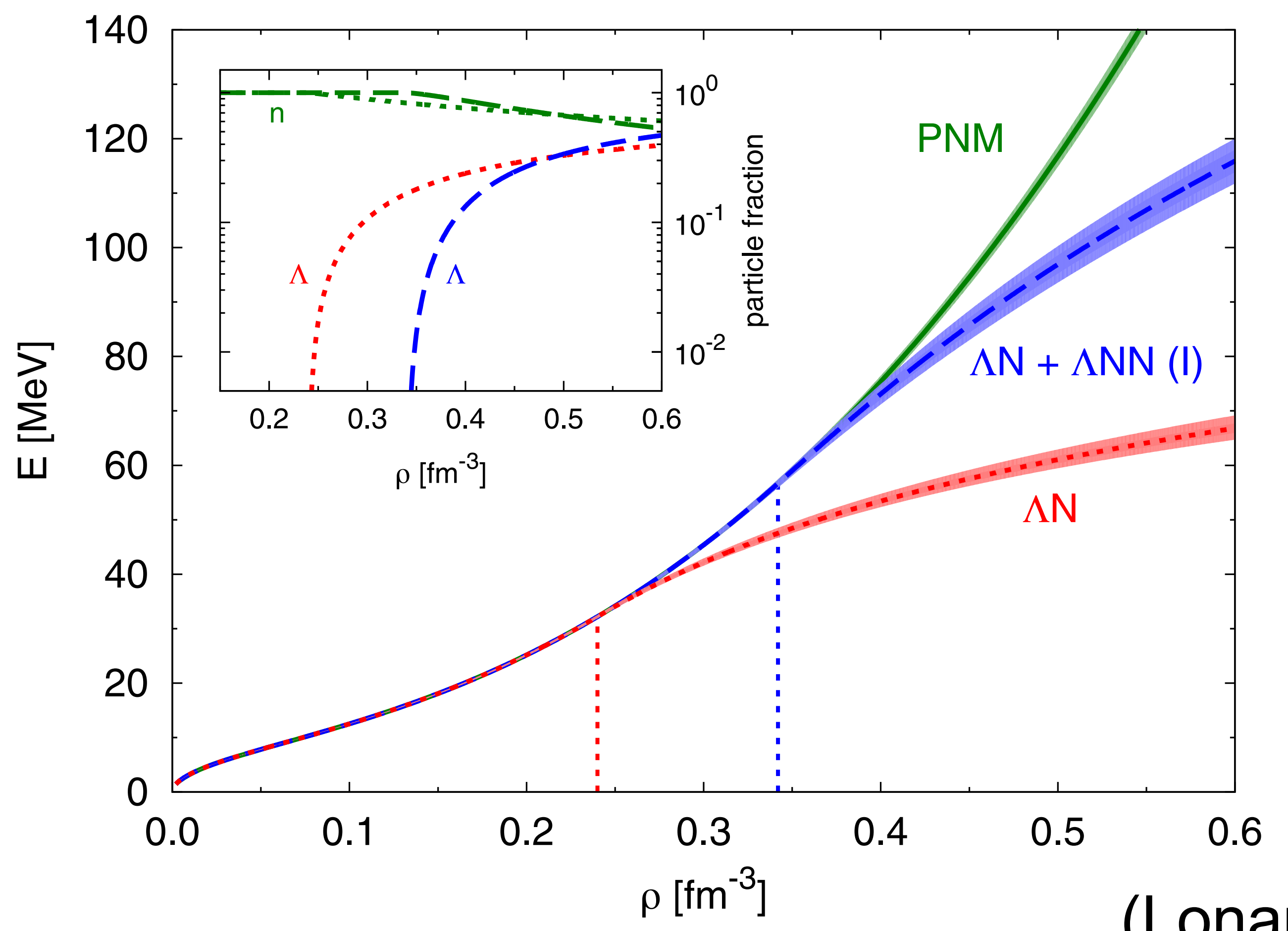
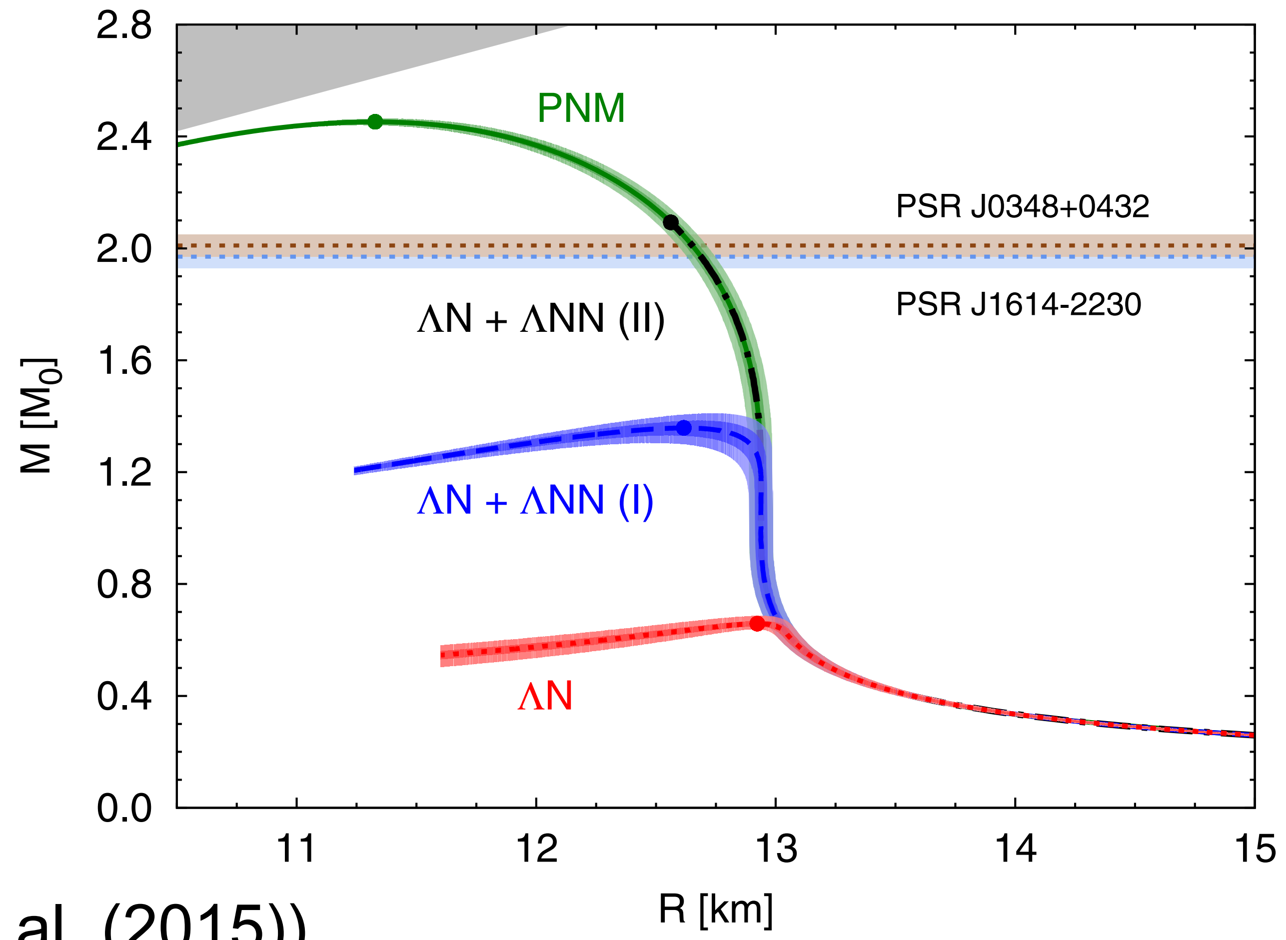
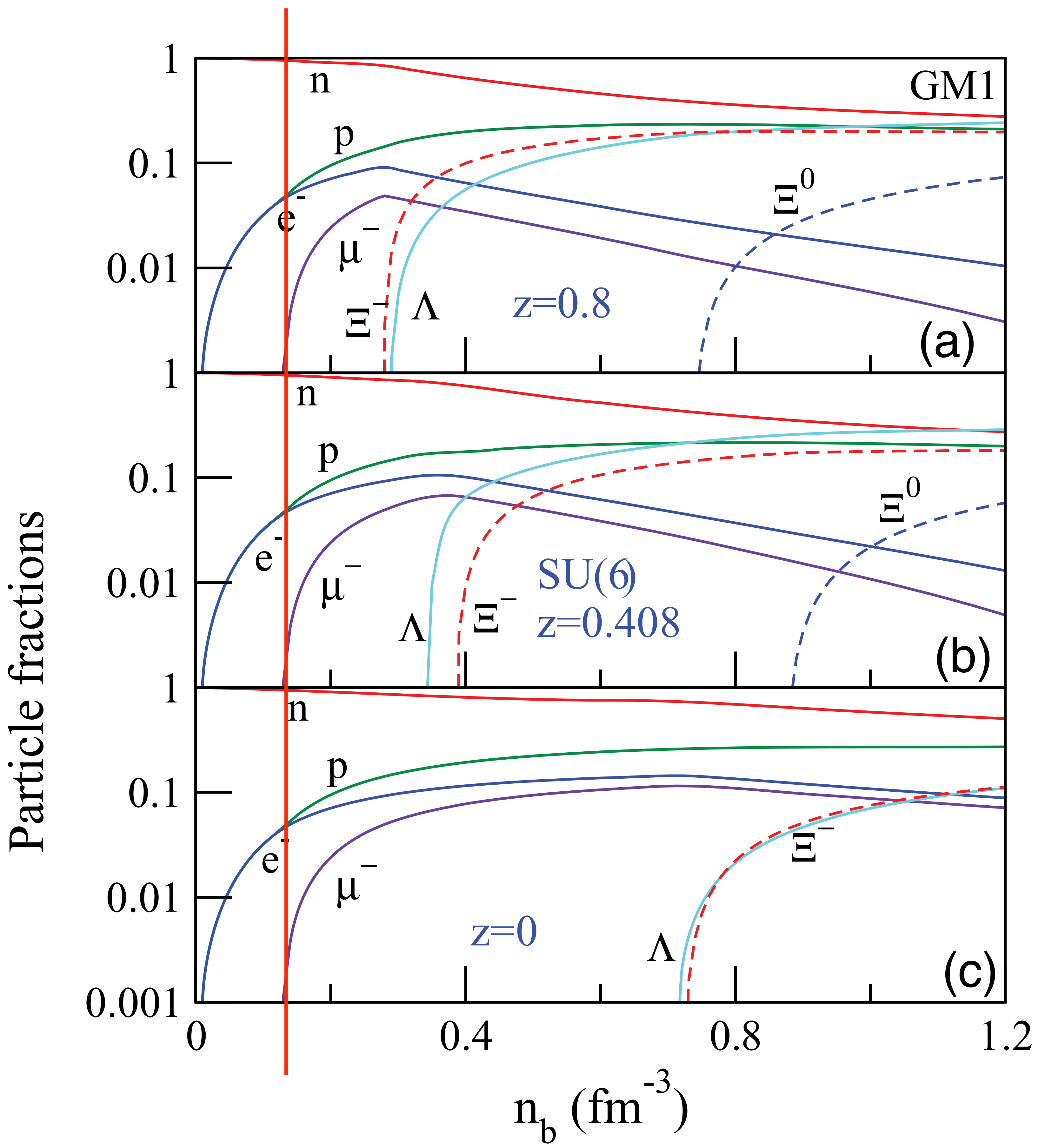
HYPERNUCLEAR INTERACTIONS

- **Why is understanding hypernuclear interactions interesting?**
  - hyperon contribution to the EOS, neutron stars, supernovae
  - "hyperon puzzle"
  - Λ as probe to nuclear structure
  - **flavor dependence of baryon-baryon interactions**





(Lonardoni et al. (2015))



(Weissenborn et al. 2012)

HYPERNUCLEI

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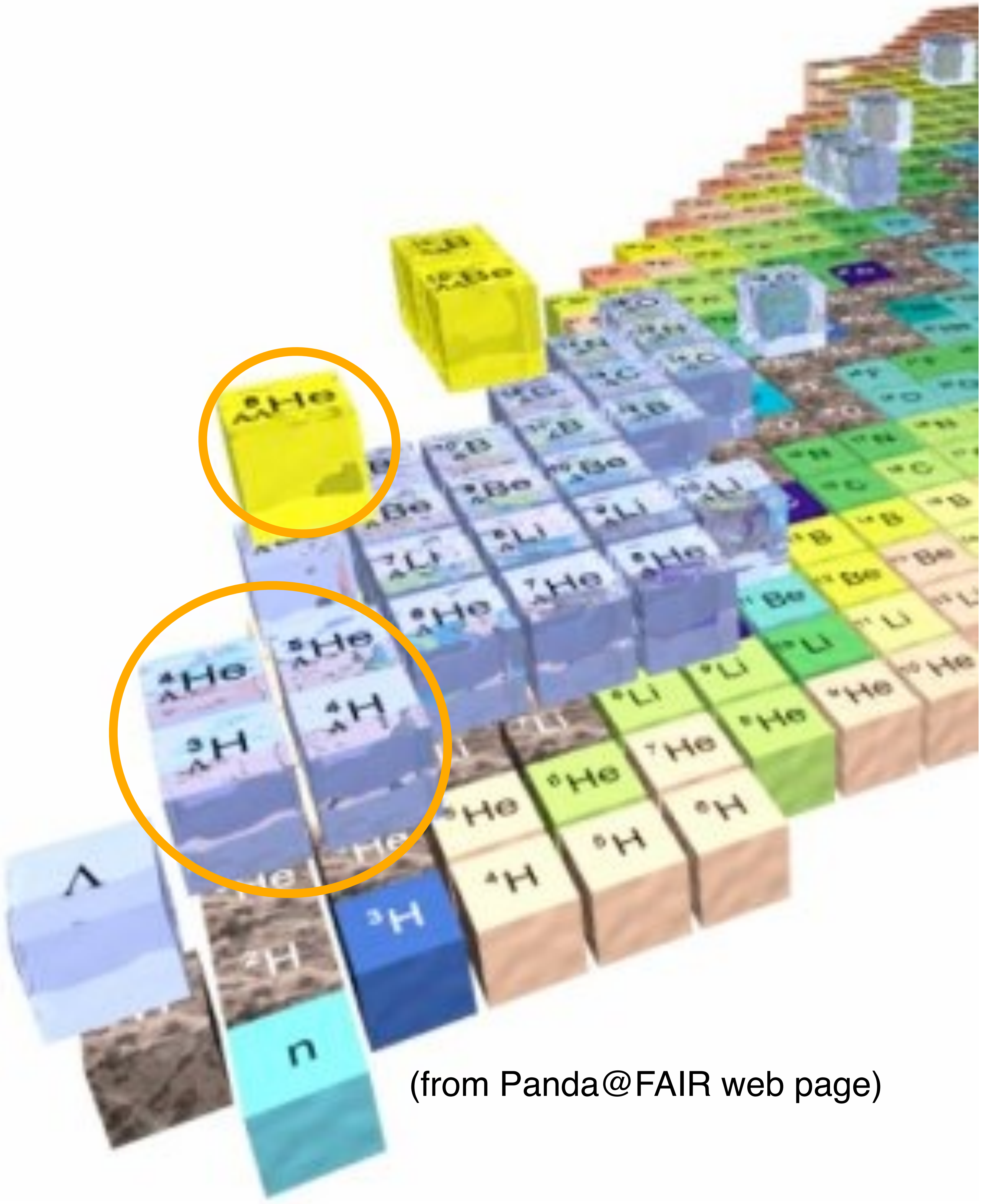
Only few YN data. Hypernuclear data provides additional constraints.

- ΛN interactions are generally weaker than the NN interaction
  - naively: **core nucleus + hyperons**
  - separation energies **quite** independent from NN(+3N) interaction

- no Pauli blocking of Λ in nuclei
  - good to study nuclear structure
  - even light hypernuclei exist in **several spin states**

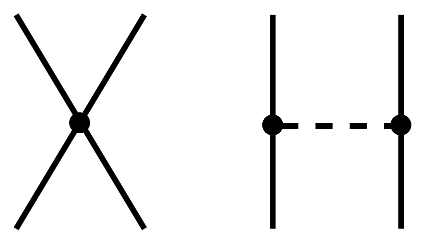
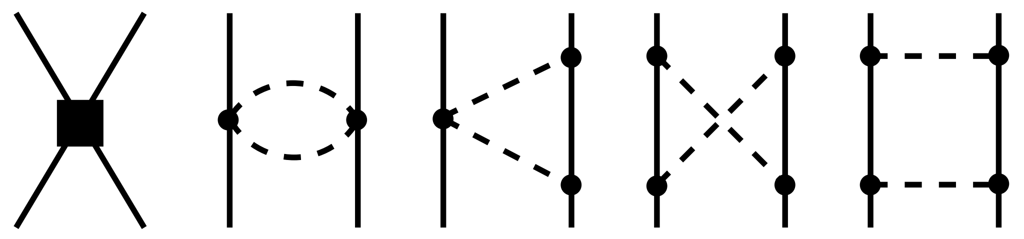
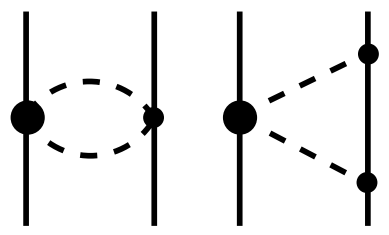
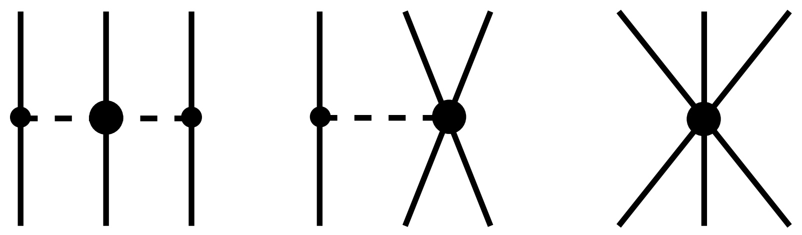
- **non-trivial constraints** on the YN interaction
- size of **YNN** interactions? → need to include **Λ-Σ conversion**!



(from Panda@FAIR web page)

CHIRAL NN & YN & YY INTERACTIONS

- Chiral EFT implements **chiral symmetry of QCD**
- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N^{2}LO (for YN)

	BB force	3B force	4B force	
LO		—	—	**5(+1) NN/YN (YY)** short range parameters
NLO		—	—	**23(+5) NN/YN (YY)** short range parameters
N^{2}LO			—	no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

Retain flexibility to adjust to data due to counter terms — LECs are not uniquely determined yet

Regulator required — cutoff/different orders often used to estimate uncertainty

$\Lambda - \Sigma$ ($\Lambda\Lambda - \Sigma\Sigma - \Xi N$) **conversion** is explicitly included (3BFs only in N^{2}LO)

Hypertriton used to determine spin dependence

Emulsion: talk by Kasagi (Tuesday, parallel)

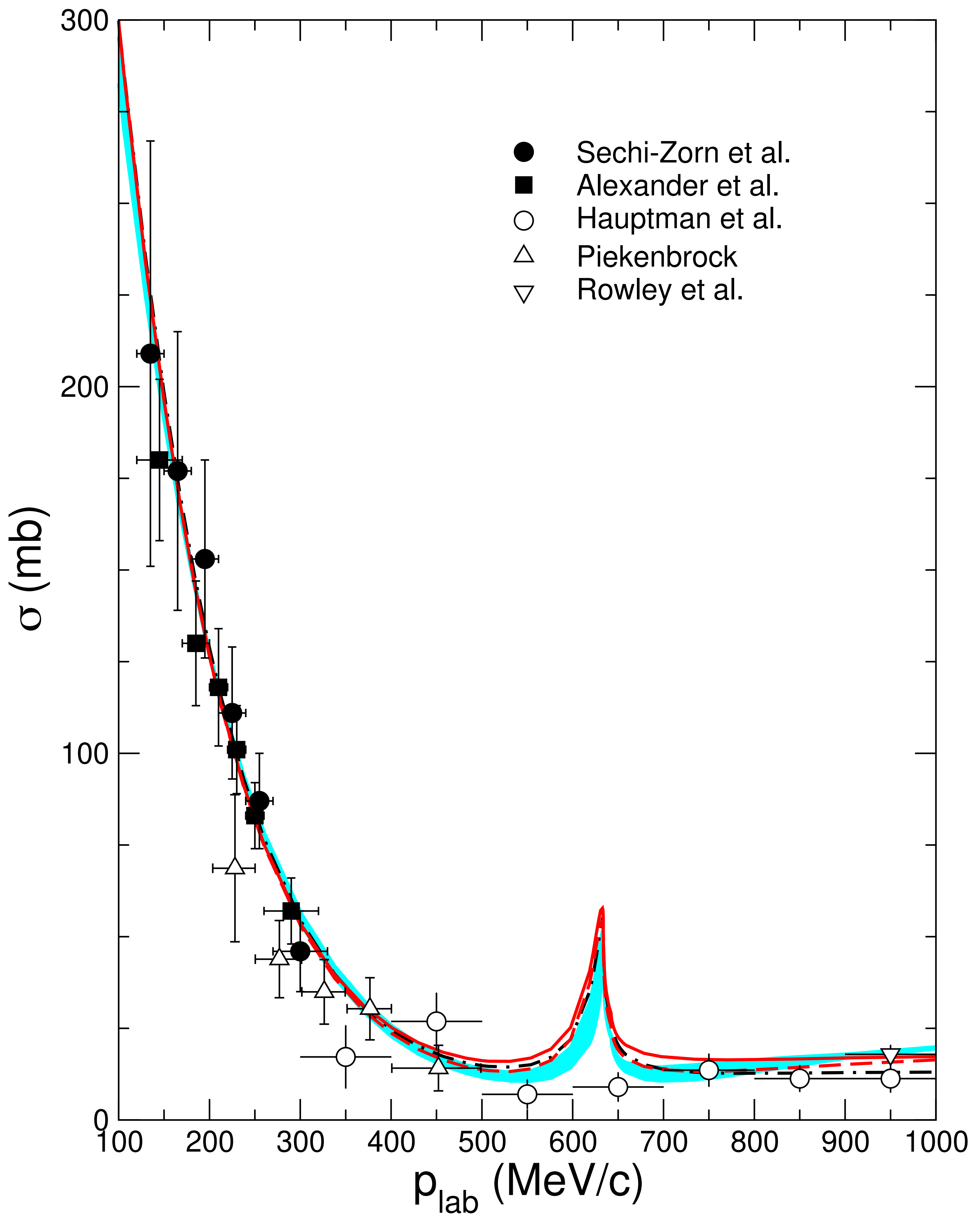
MAMI: talk by Kino (Tuesday, parallel)

SMS NLO/N^{2}LO INTERACTION

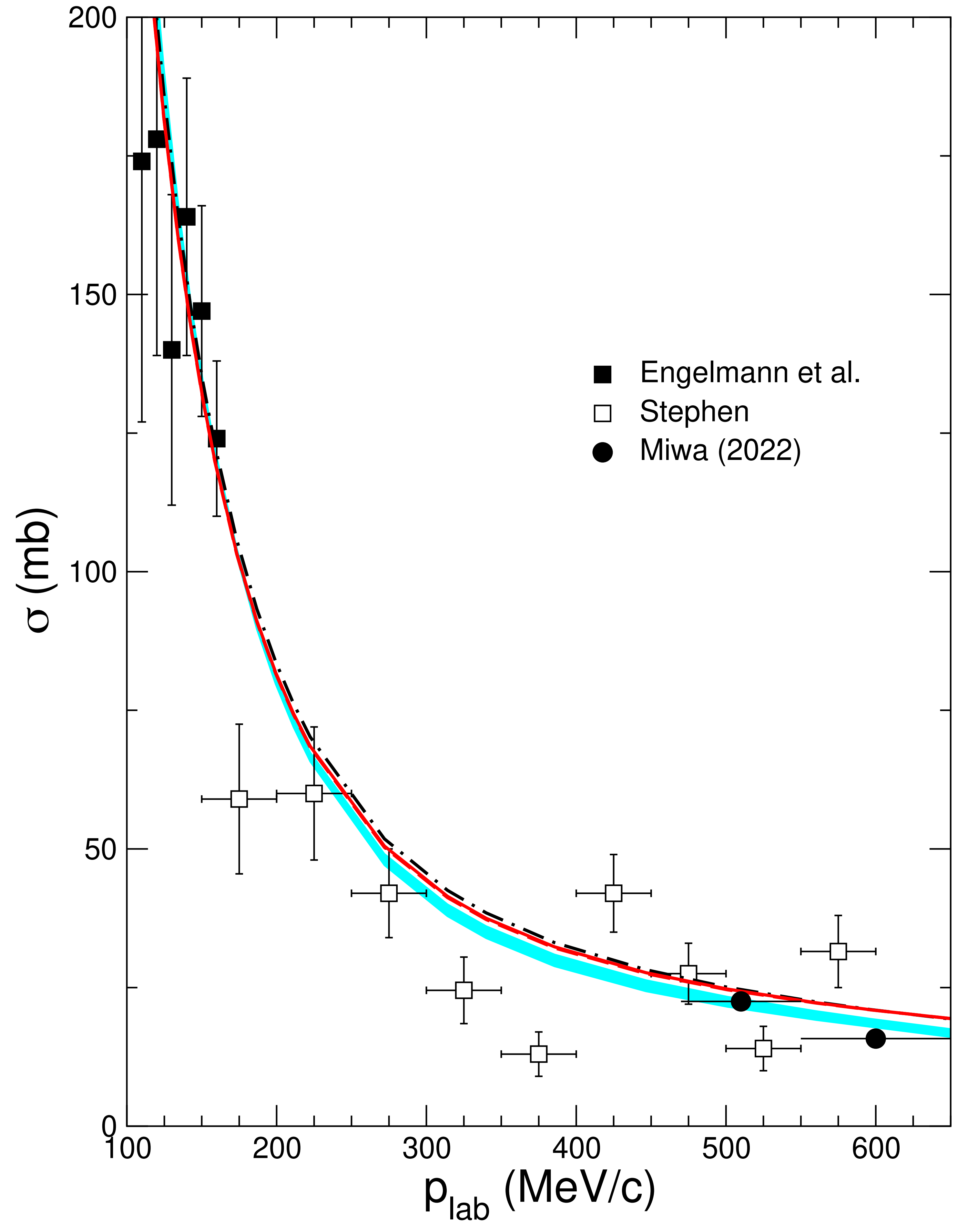
selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)

- total cross sections very similar in NLO and N^{2}LO
- similar to NLO19 (and NLO13)
- alternative fit
- new data provides **new constraints**!

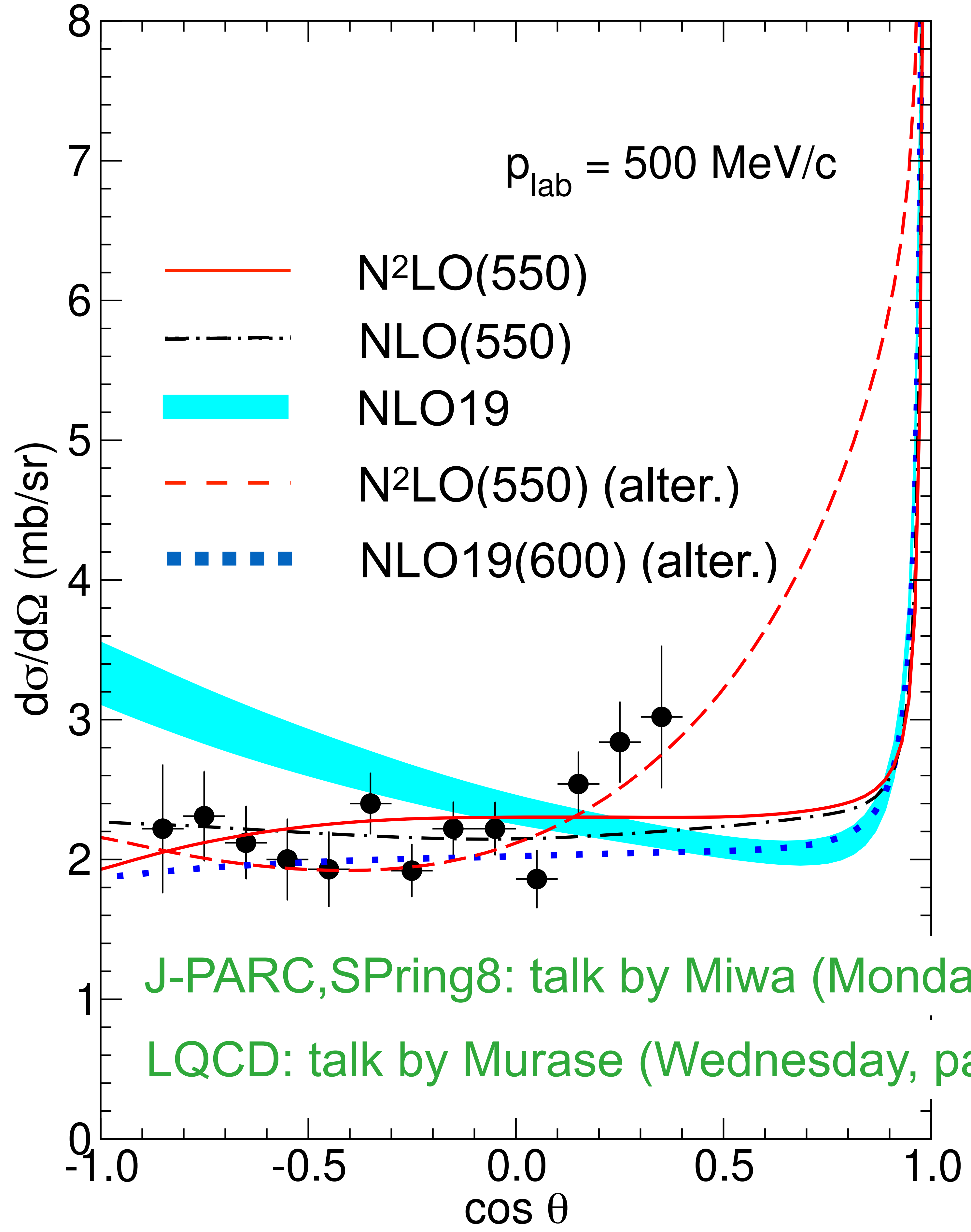
$\Lambda p \to \Lambda p$



$\Sigma^- p \to \Lambda n$



$\Sigma^+ p \to \Sigma^+ p$



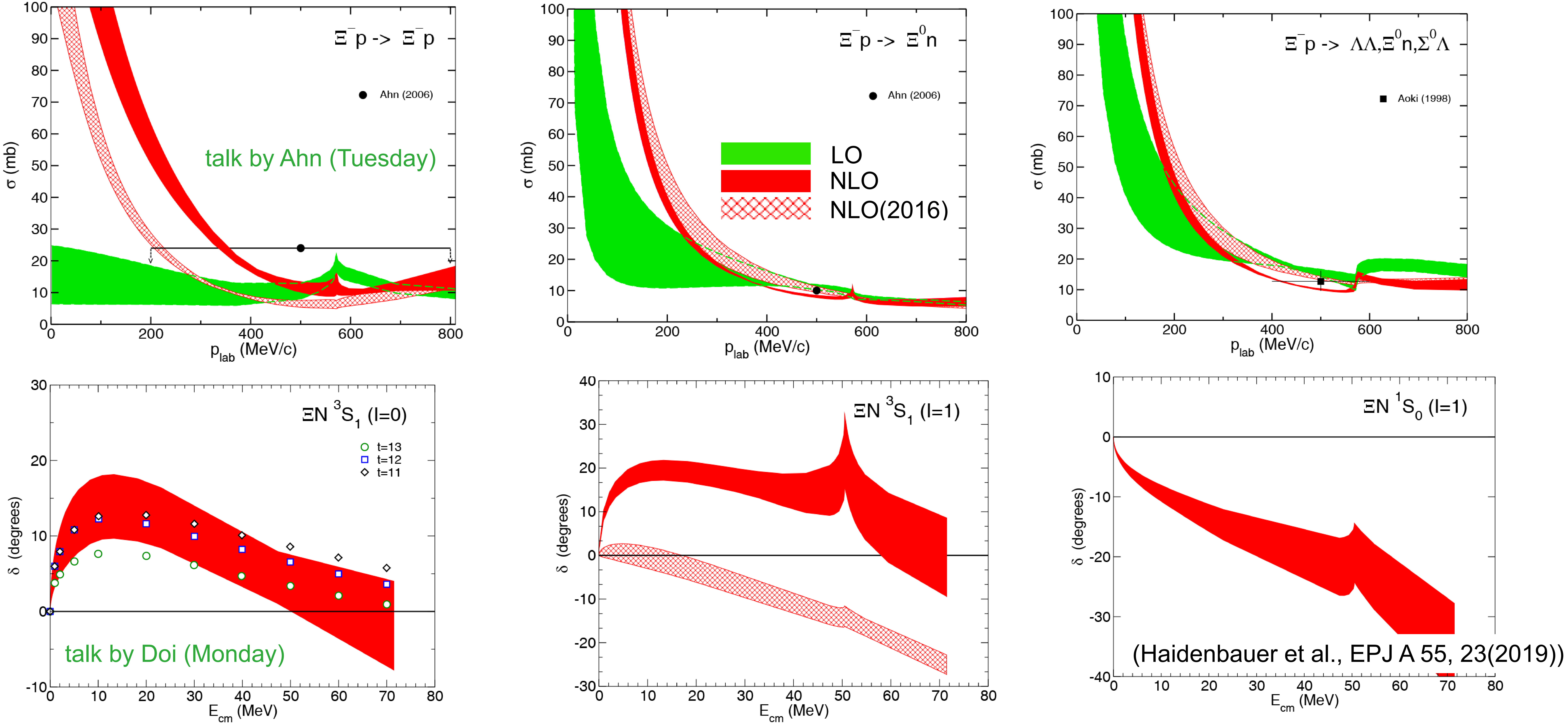
J-PARC,SPring8: talk by Miwa (Monday)

LQCD: talk by Murase (Wednesday, parallel)

J. Haidenbauer et al. EPJ A 59, 63 (2023)

YY INTERACTION

- adjusted to scarce data & LQCD (HAL QCD)
- updated version consistent with Ξ-nuclei (only change in $\Xi N\ ^{33}S_1$)



(Haidenbauer et al., EPJ A 55, 23(2019))

TOOLS

Need reliable predictions for hypernuclei to further constrain interactions

Faddeev-Yakubovsky (FY) equations for $A = 3$ and 4 (momentum space)

- long distance tails of wave functions can be well represented
- uses Jacobi coordinates separating off CM motion
- chiral interactions can be directly used
- hugh linear eigenvalue problem (dimension 10^9x10^9) even for A=4 systems
- is feasible only for A ≤ 4

(see Haidenbauer et al. arXiv:2508.05243 [nucl-th] (2025))

Jacobi-no core shell model (J-NCSM) for $A \geq 4$ (HO space)

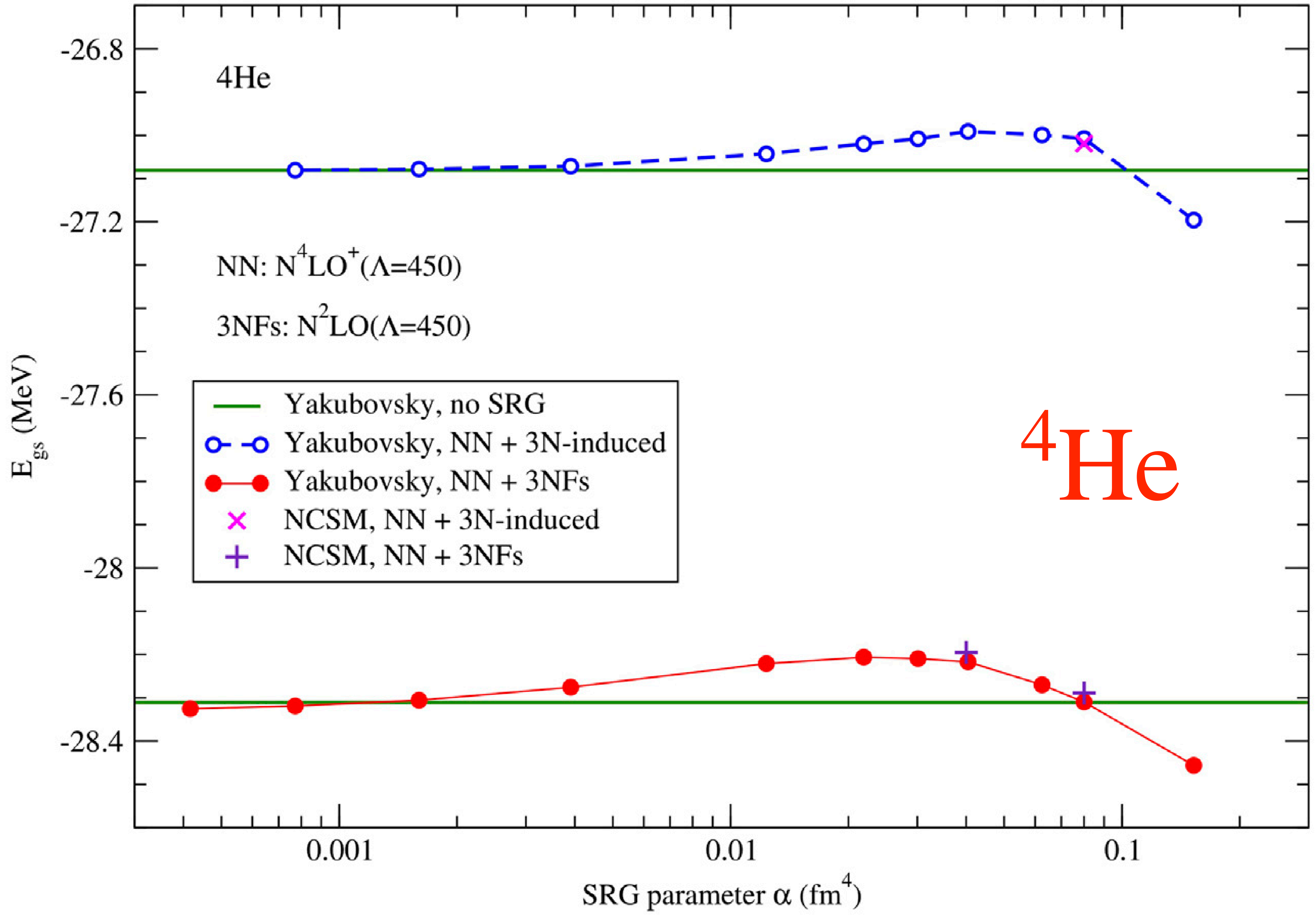
- smaller dimensions allow to tackle p-shell nuclei
- exact antisymmetrization of wave functions can be prepared
- uses Jacobi coordinates separating off CM motion
- chiral interactions require **similarity renormalization group (SRG)** evolution
- long distance wave functions require large HO model spaces

(see Liebig et al. EPJ A (2016); Le et al., EPJA 56,301 (2020); Le et al., EPJA (2021))

Many more approaches: talks by Hildenbrand, Tian, Umeya (Monday,parallel), Hiyama (Tuesday), Kamada (Tuesday,parallel), Gazda (Wednesday,parallel)

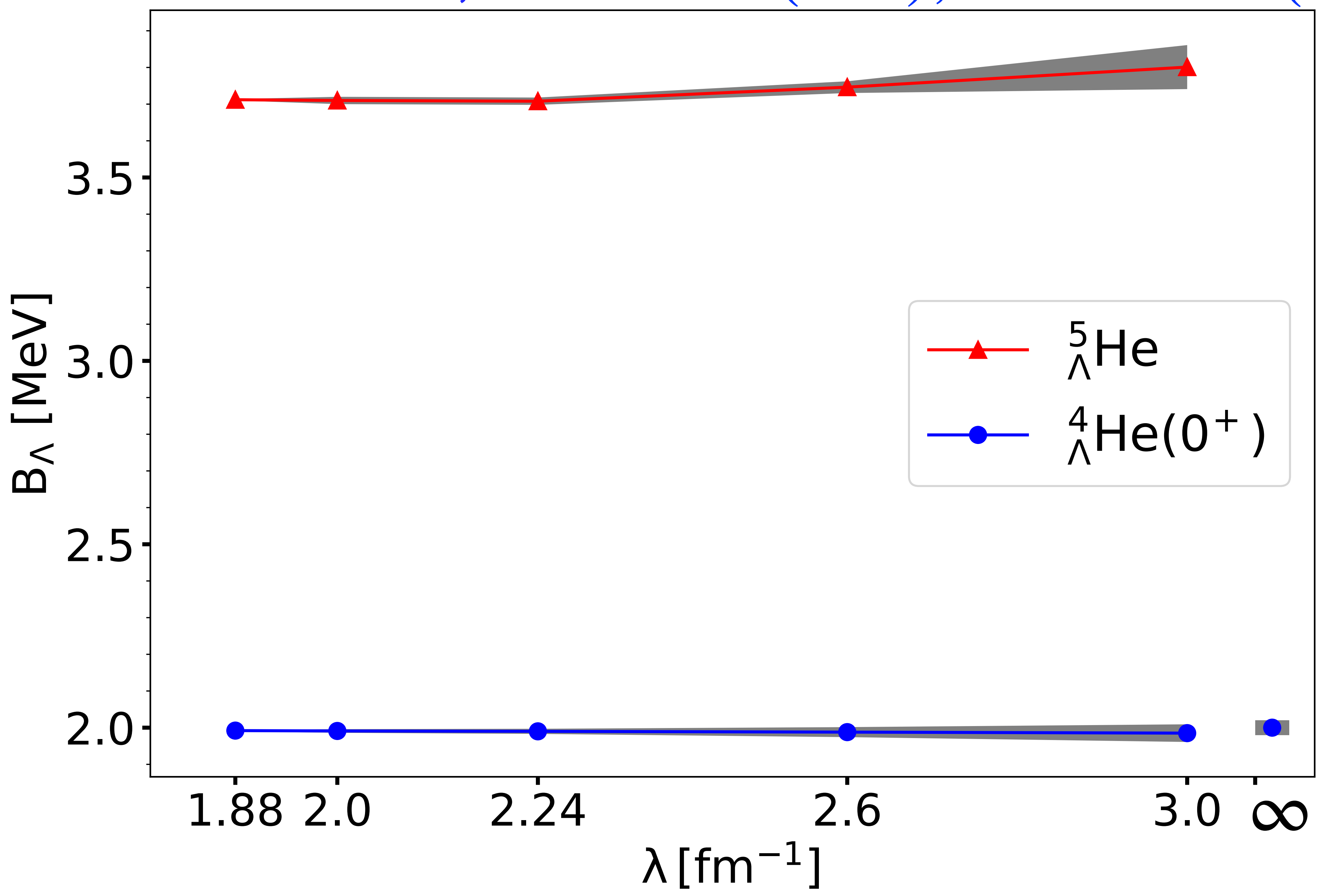
SRG DEPENDENCE OF RESULTS

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100 - 200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris et al. Front. Phys. 11,1098262 (2023))

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)

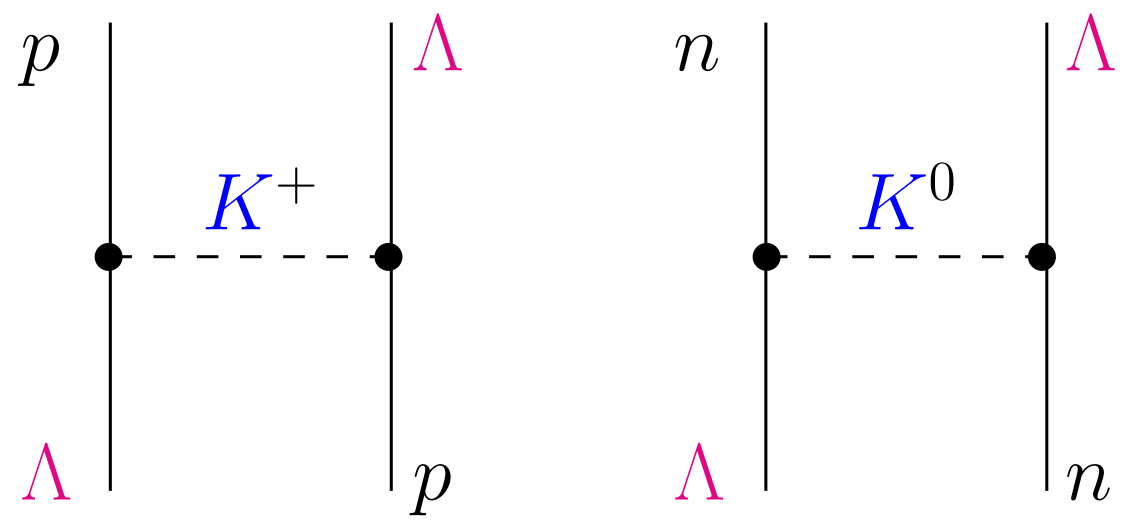


(see Le et al. PRC 102,024002 (2023))

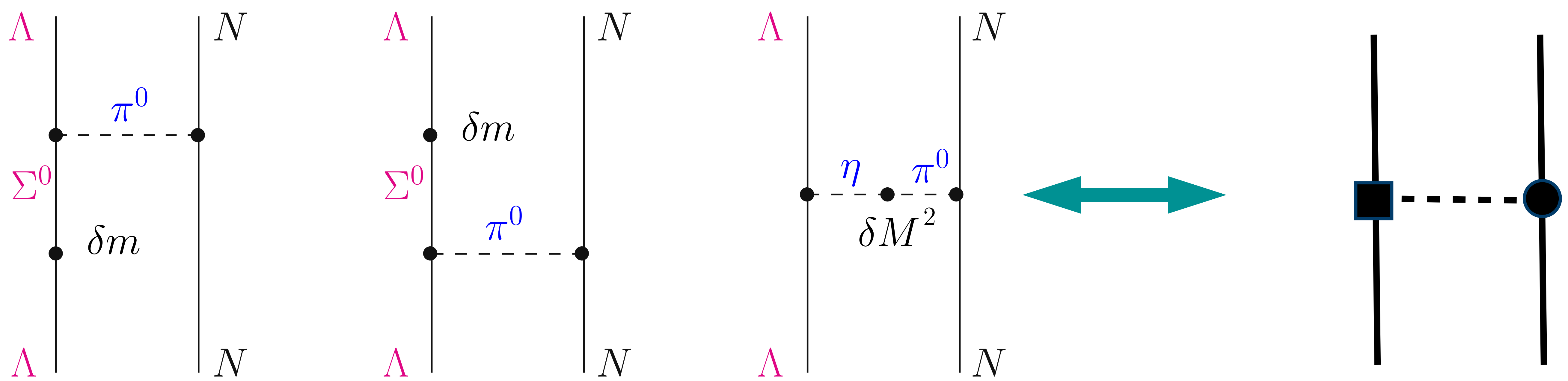
For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

CSB CONTRIBUTIONS TO YN INTERACTIONS

- **formally leading** contributions: Goldstone boson mass difference
  — very small due to the small relative difference of kaon masses

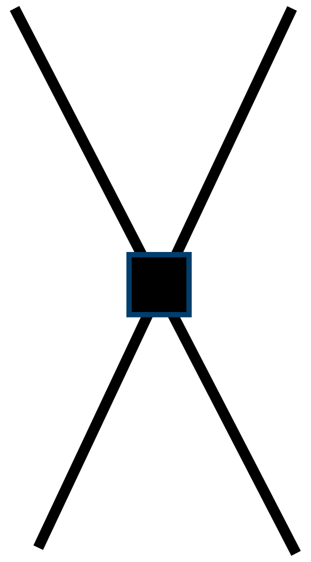


- **subleading but most important** — effective CSB ΛΛπ coupling constant (Dalitz, von Hippel, 1964)



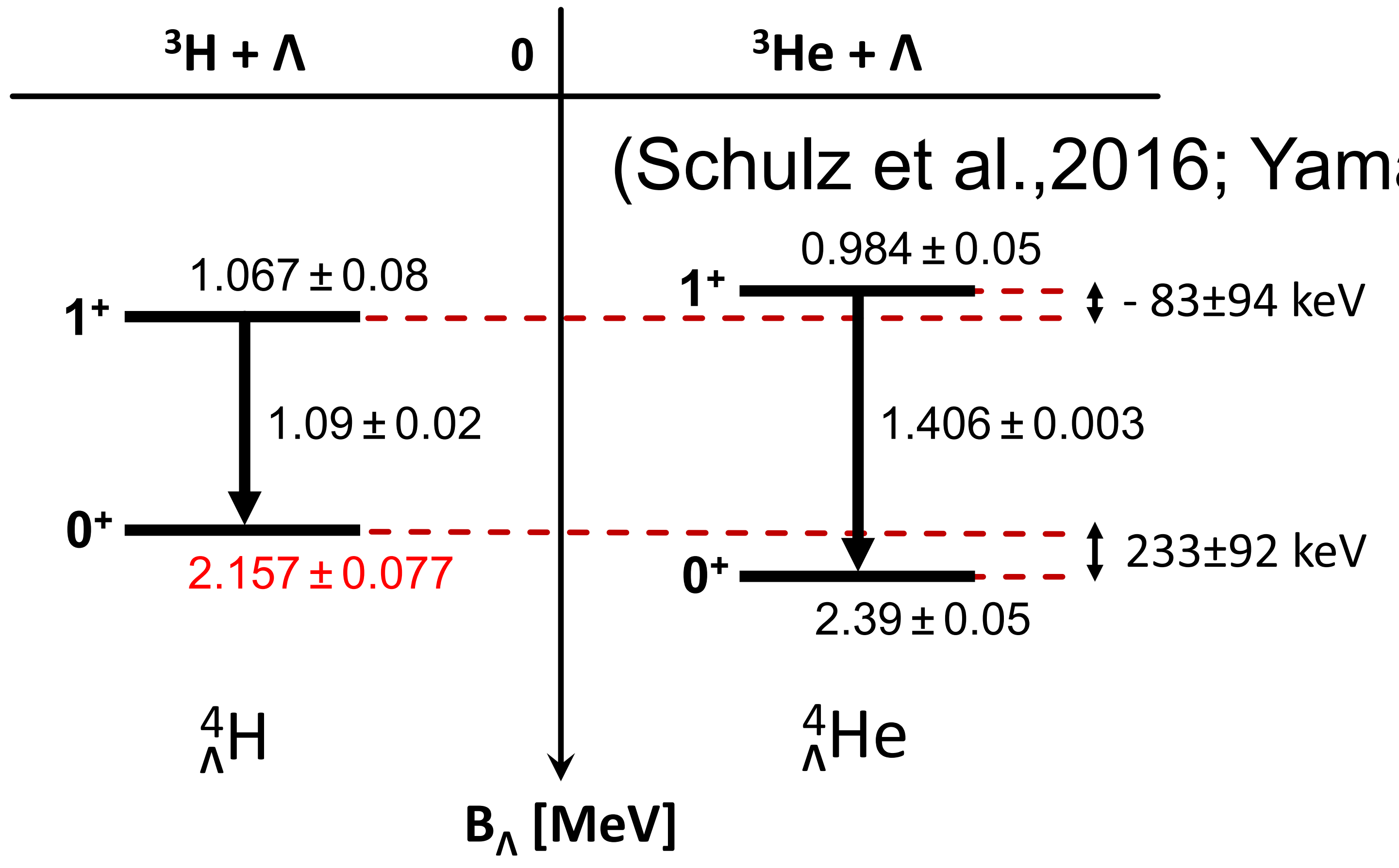
$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$

- **so far less considered but necessary for proper renormalization** — 2 CSB contact interactions



Need to determine two unknown CSB LECs and predict Λn scattering

FIT OF CSB CONTACT INTERACTIONS



(Schulz et al.,2016; Yamamoto, 2015)

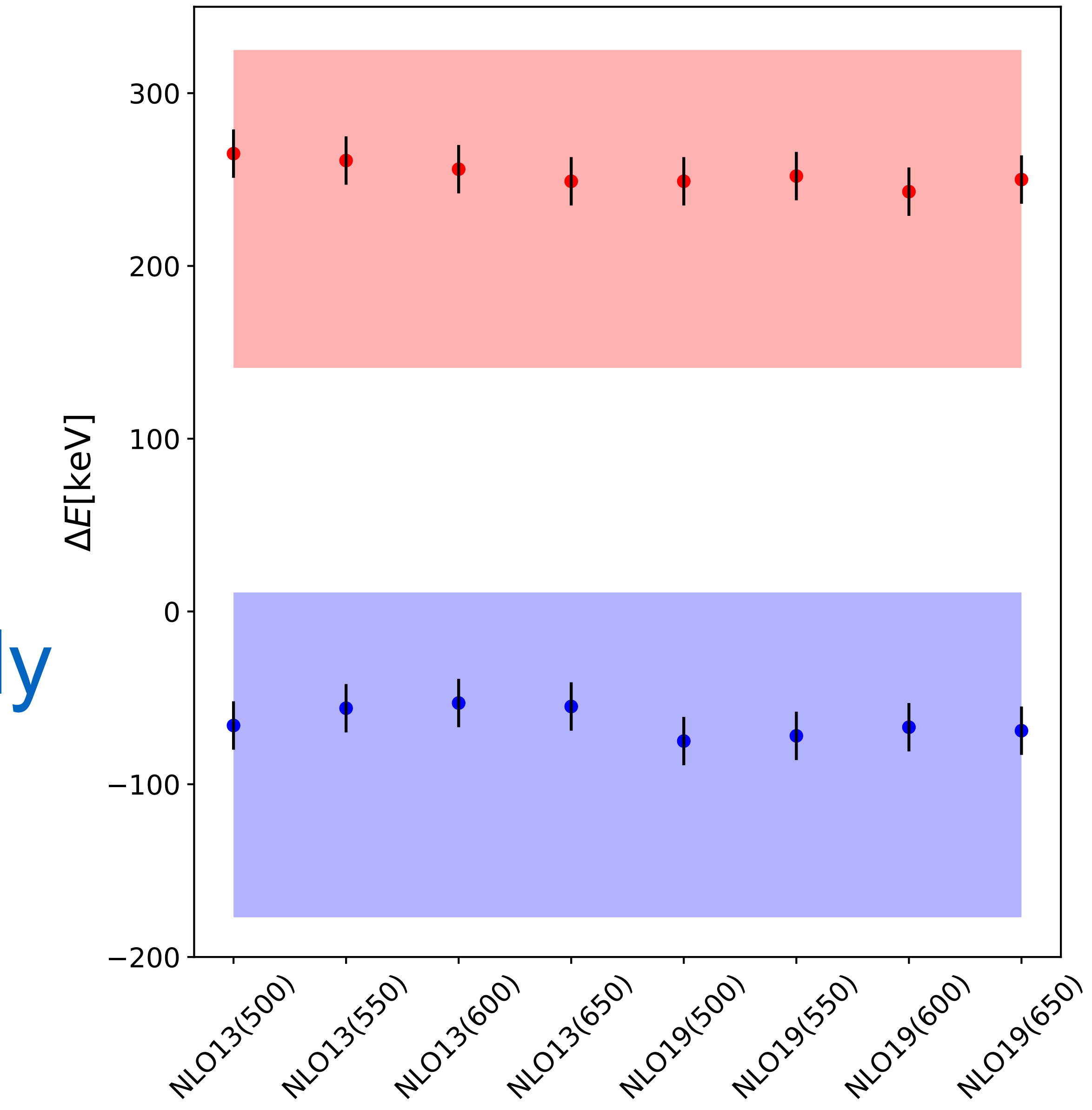
- Adjust the two CSB contact interactions to one main scenario (**CSB1**)
- update: Mainz average of CSB including new star data: 178 ± 55 keV/ -139 ± 58 keV

This was not used here.

- Fit of counter terms to data: size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d}\left(\frac{M_\pi}{\Lambda}\right)^2 \quad C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4\,\mathrm{GeV}^{-2} \propto 6 \cdot 10^{-3} \cdot 10^4\,\mathrm{GeV}^{-2}$$

(see Haidenbauer et al. FBS 62,105 (2021))



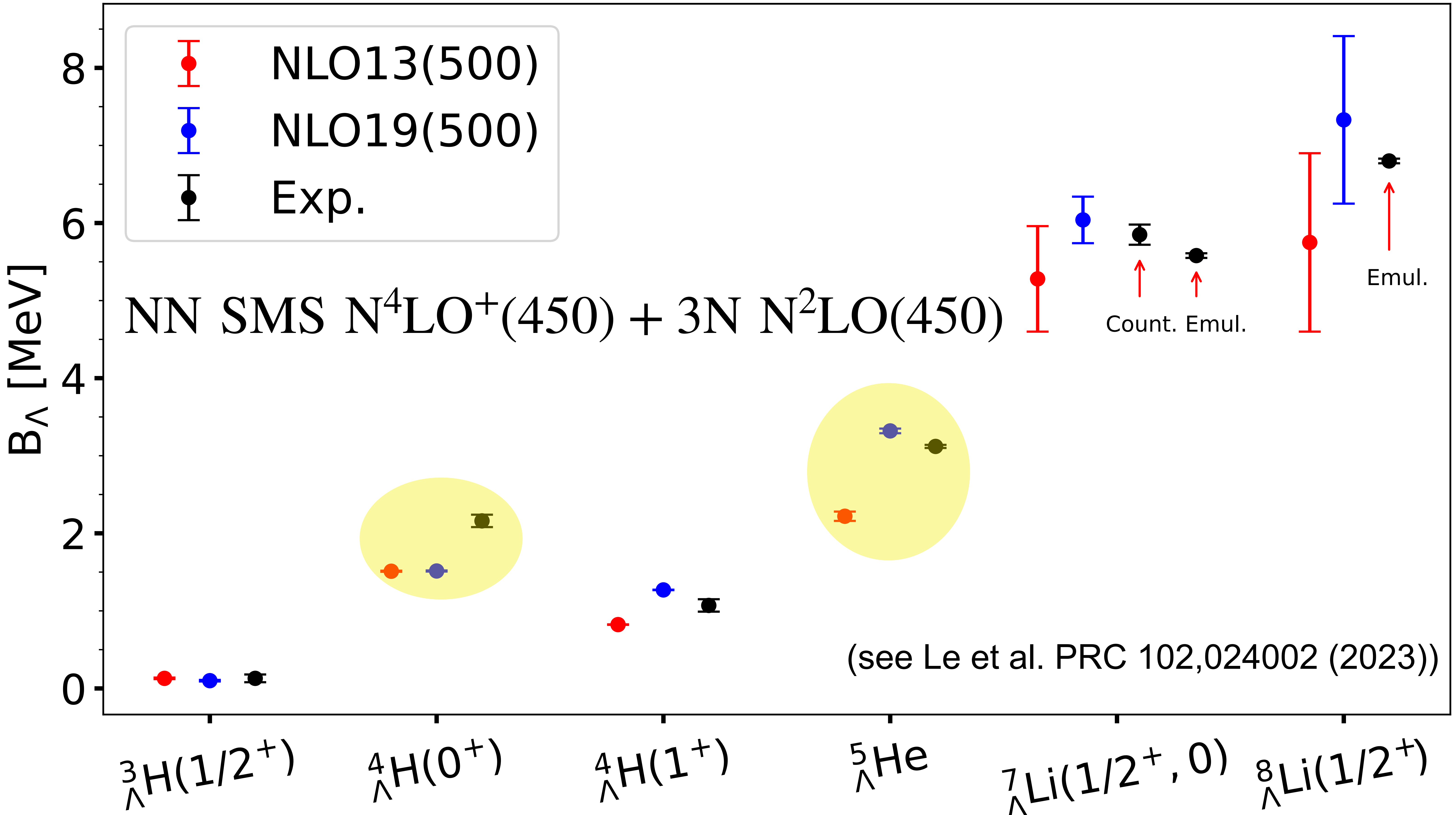
- problem: large experimental uncertainty of experiment
  → later adjustment of CSB predictions is likely

- up to now: **fit to central values** to test theoretical uncertainties

APPLICATION TO $A = 7$ AND 8

Status of Λ-separation energies for $A = 3 - 7$ (non CSB)

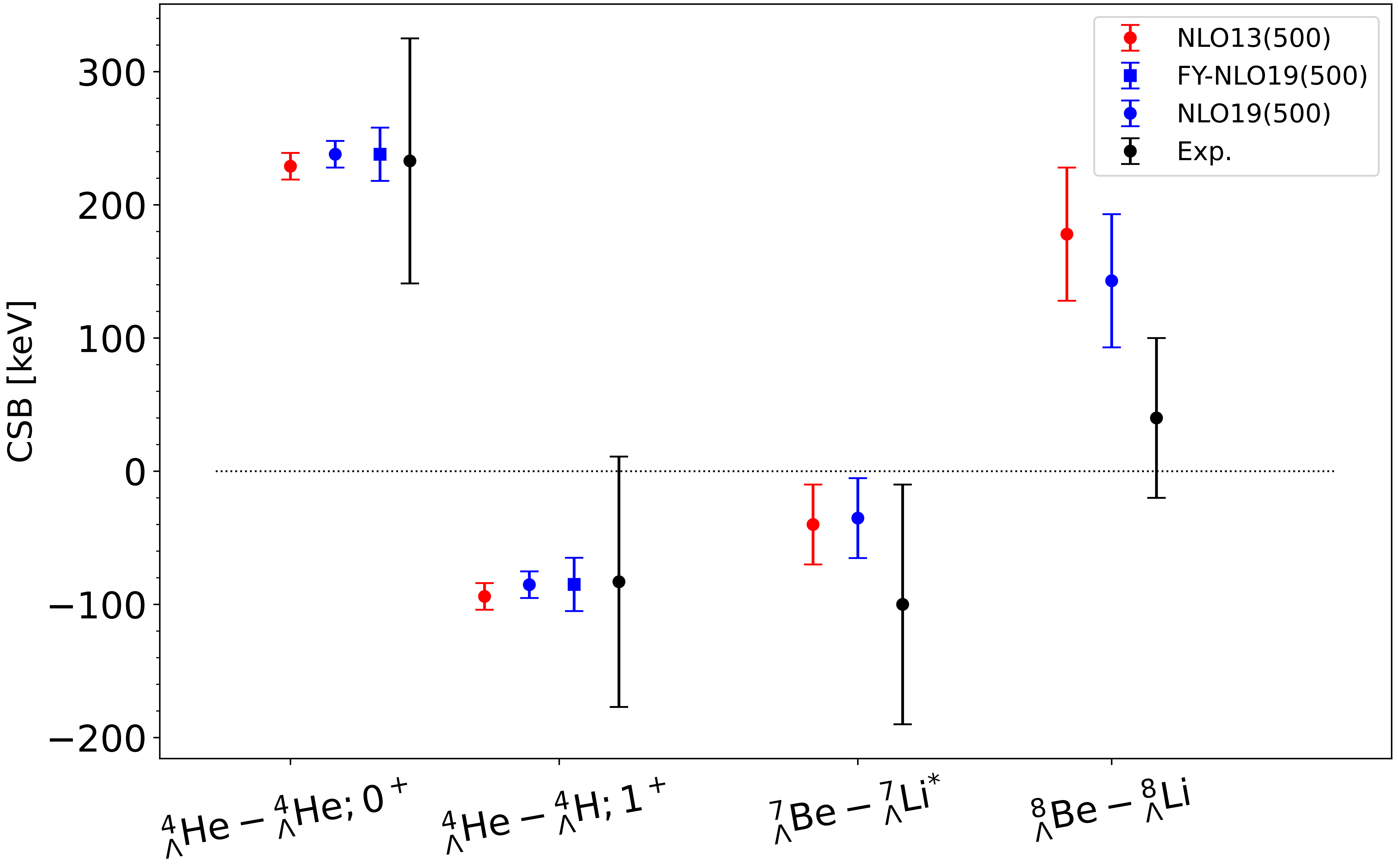
- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^{4}_{\Lambda}\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^{5}_{\Lambda}\mathrm{He}$ and heavier hypernuclei — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



PREDICTIONS FOR $A = 2{,}7$ AND 8

- CSB scattering length predicted **independent of the realization**

- keep in mind: CSB still not fixed — experimental uncertainty is large
- scenario studied here is only **marginally consistent** with CSB in $A = 8$



(Le et al. PRC 102,024002 (2023))

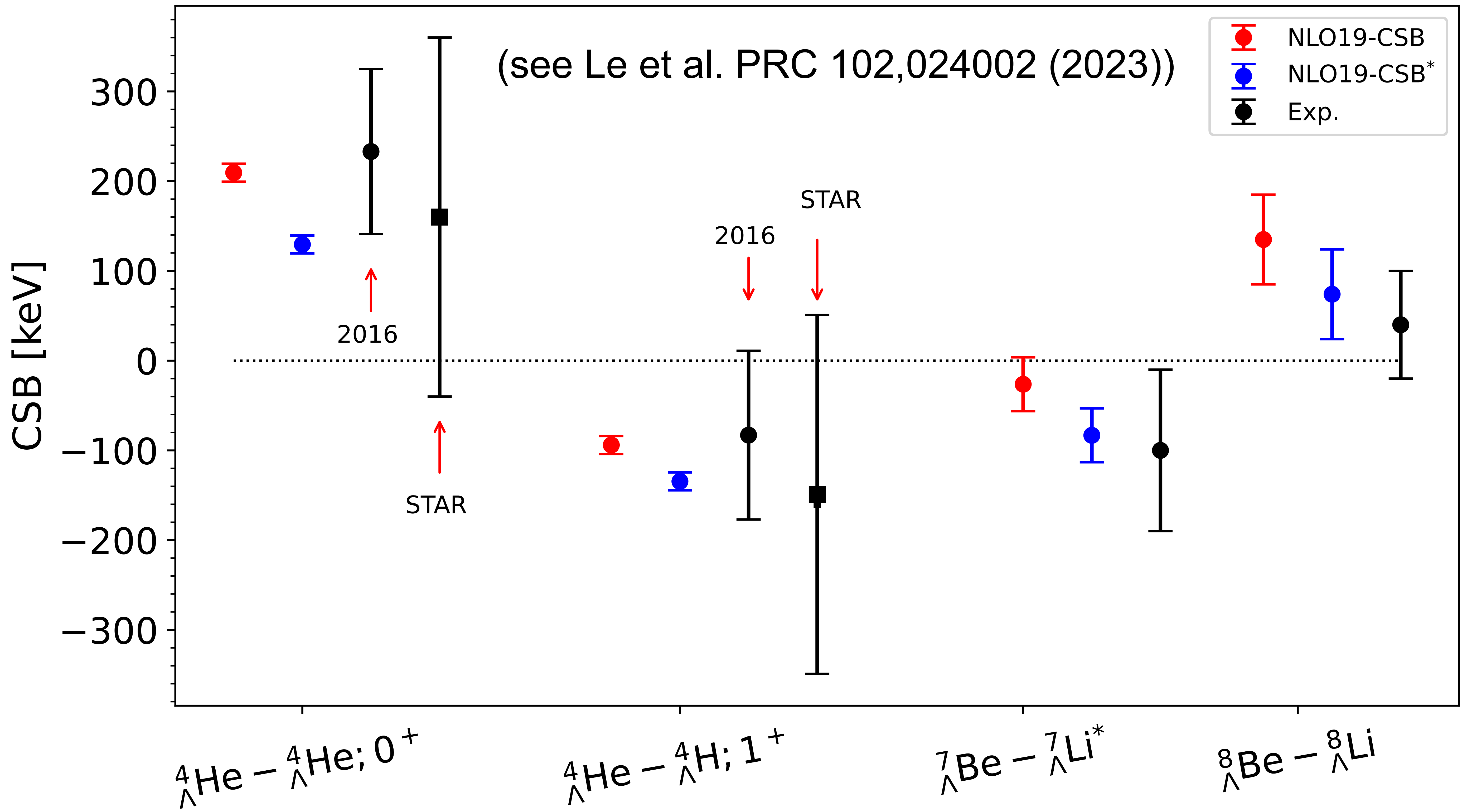
	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$
NLO13(500)	-2.604	-1.647	-3.267	-1.561
NLO13(550)	-2.586	-1.551	-3.291	-1.469
NLO13(600)	-2.588	-1.573	-3.291	-1.487
NLO13(650)	-2.592	-1.538	-3.271	-1.452
NLO19(500)	-2.649	-1.580	-3.202	-1.467
NLO19(550)	-2.640	-1.524	-3.205	-1.407
NLO19(600)	-2.632	-1.473	-3.227	-1.362
NLO19(650)	-2.620	-1.464	-3.225	-1.365

(Haidenbauer et al. FBS 62,105 (2021))

DFT: talk by Tanimura (Monday,parallel)

USING STAR DATA FOR $A = 4$ CSB ...

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

updated STAR data: talk by Shao (Tuesday,parallel)

UNCERTAINTY ANALYSIS FOR $A = 3$ TO 5

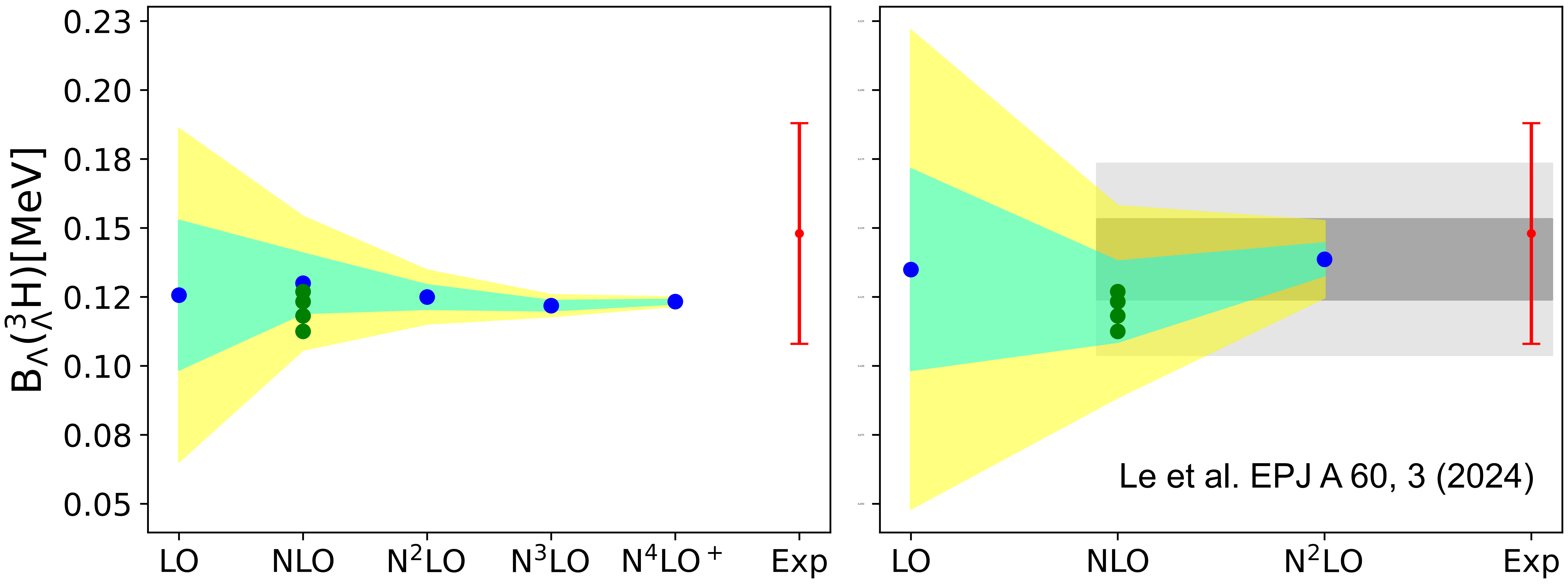
Results for **different orders** enable uncertainty estimate:

SMS YN interactions up order N²LO, but neglected YNN interaction here

- Ansatz for the order by order convergence (Melendez et al. PRC 2017,2019):

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, } \ldots)$$

- **Extracting** c_k for $k \leq K$ **from calculation** ⟶ **probability distributions** for c_k ⟶ $\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$



APPLICATION TO $^{5}_{\Lambda}\mathrm{He}$ AND SUMMARY

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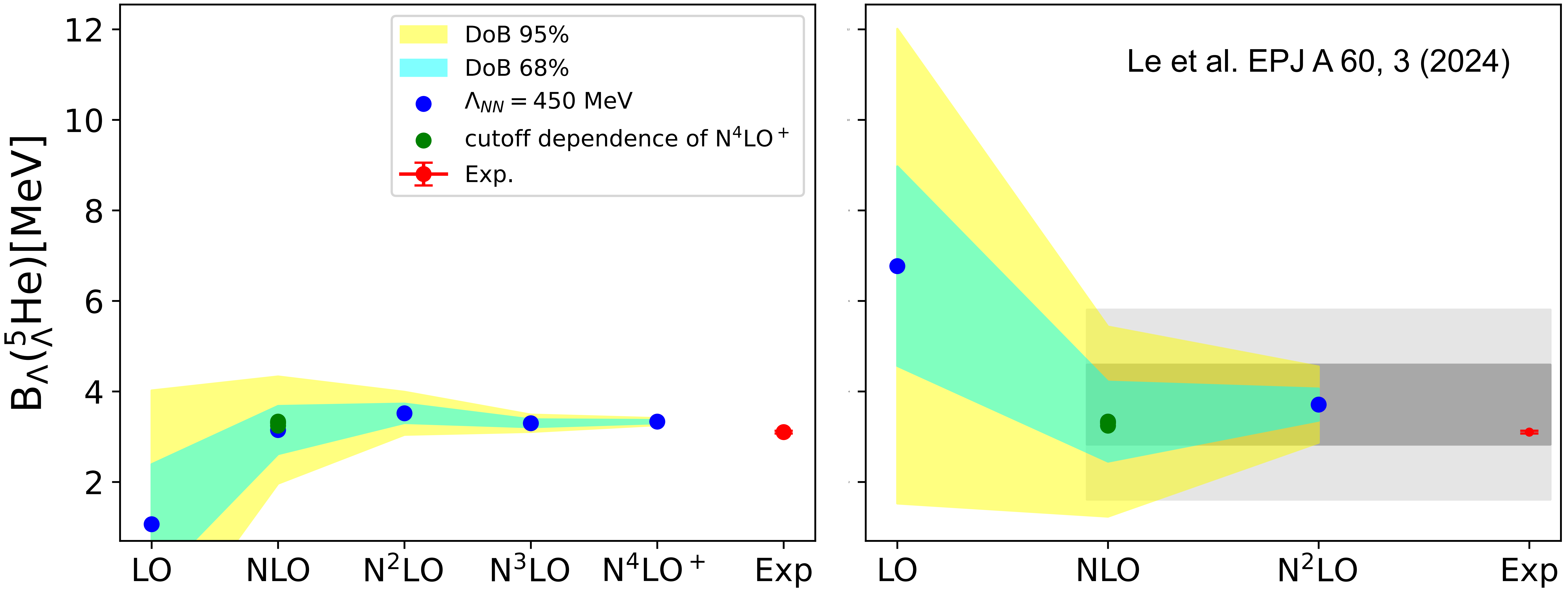
- **without YNN**: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$
- at the same time: obtain estimate of YNN contribution!

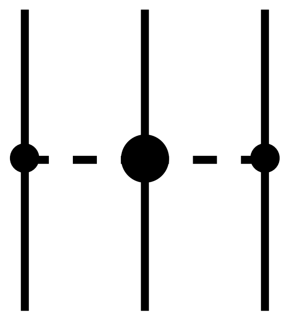
nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
$^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
$^{4}_{\Lambda}\mathrm{He}\,(0^{+})$	0.157	0.239
$^{4}_{\Lambda}\mathrm{He}\,(1^{+})$	0.114	0.214
$^{5}_{\Lambda}\mathrm{He}$	0.529	0.881



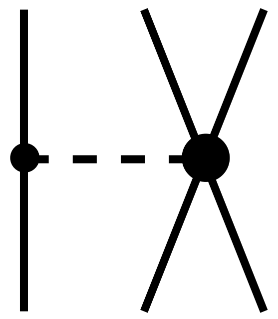
YNN (ΛNN) INTERACTIONS

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

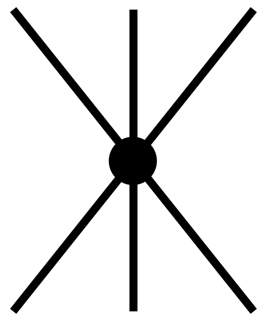
ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN
(up to 10)



2 LECs in ΛNN
(up to 14)

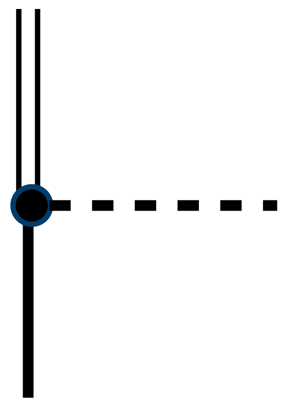


3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

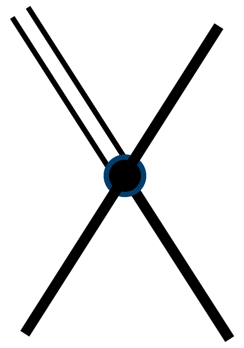
only few data → need to keep the **# of LECs** small

Decuplet baryons $(\Sigma^* \ldots)$ might enhance YNN partly to NLO (Petschauer et al., NPA 957, 347 (2017))

By decuplet saturation all LECs can be related to the following leading octet-decuplet transitions
(Petschauer et al. Front. Phys. 8,12 (2020))



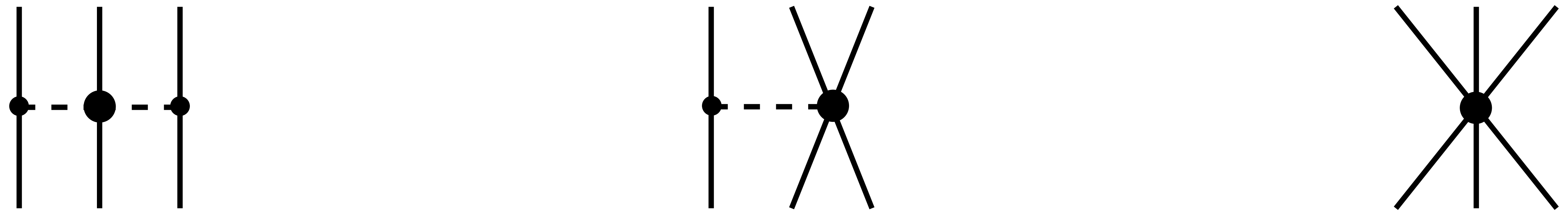
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) INTERACTIONS

Decuplet saturation relates all LECs to G_1 and G_2



$\propto C^2$ $\quad\quad\quad$ $\propto CG_1, CG_2$ $\quad\quad\quad$ $\propto (G_1)^2, (G_2)^2, G_1G_2$

$\propto C^2$ $\quad\quad\quad$ $\propto C(G_1 + 3G_2)$ $\quad\quad\quad$ $\propto (G_1 + 3G_2)^2$ $\quad$ for ΛNN: 1 LEC

→ density dependent BB interactions (Petschauer et al., NPA 957, 347 (2017))

→ application to nuclear matter (Haidenbauer et al., EPJ A 53, 121 (2017))

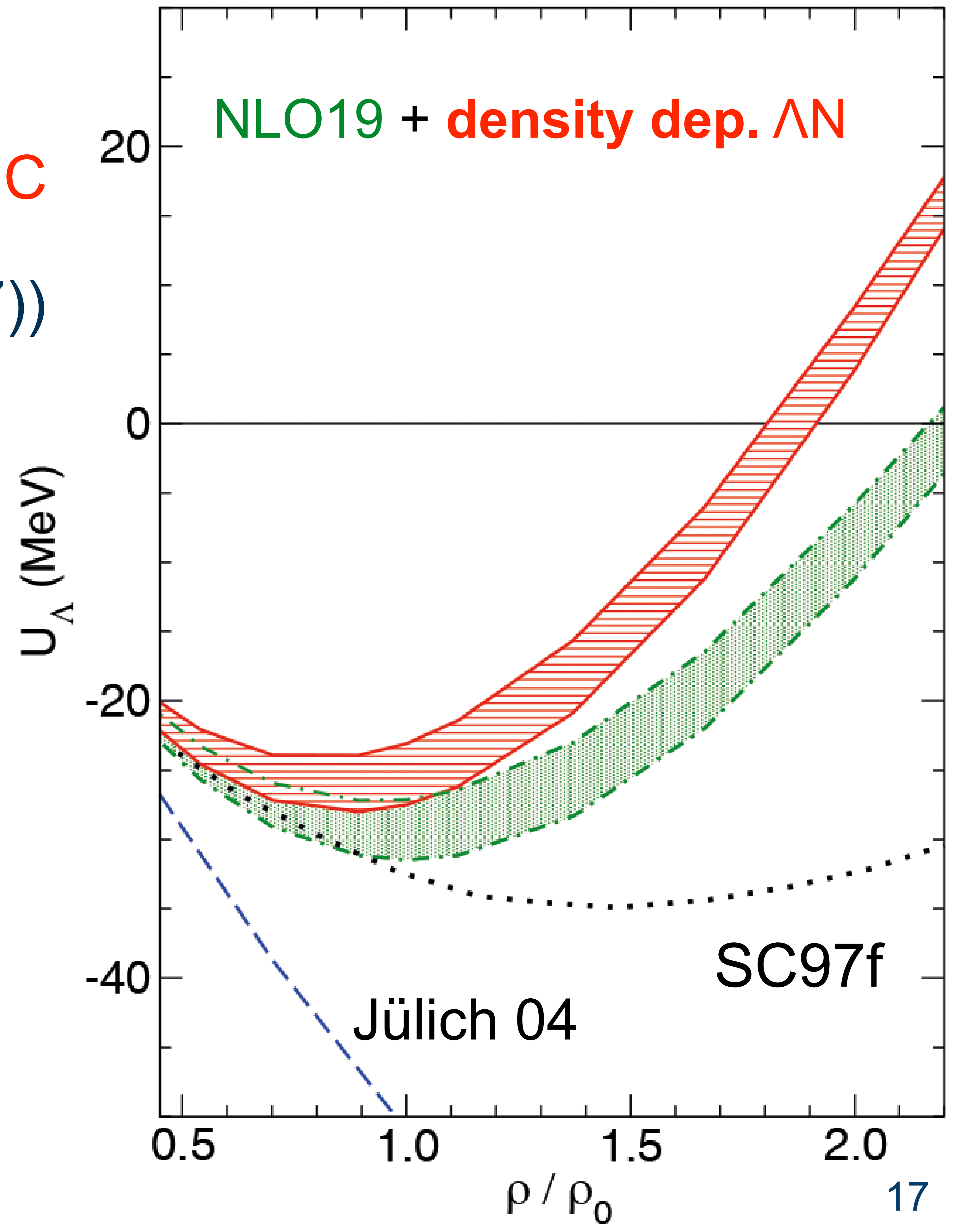
neutron stars (Logoteta et al., EJA 55, 207 (2019))

- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of 2π and 1π exchange

(fixes sign of $G_1 + 3G_2$!)

Consistent with light hypernuclei?

Overcome approximation of density dependent YN interactions



YNN (ΛNN) INTERACTIONS IN PRACTICE

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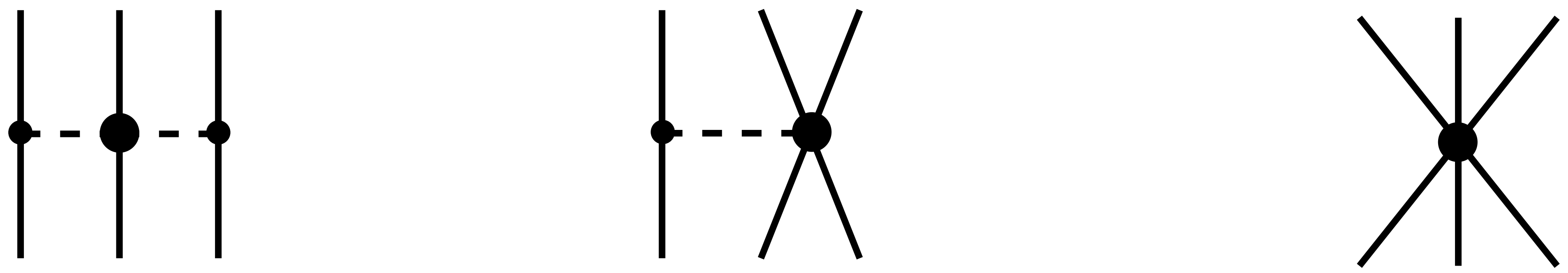
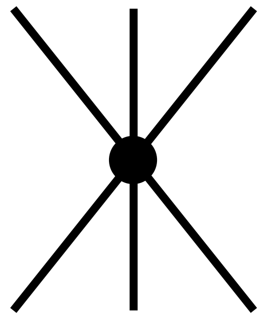
Decuplet approximation in YNN



$\propto C^2$ $\quad\quad \propto CG_1, CG_2 \quad\quad \propto (G_1)^2, (G_2)^2, G_1 G_2$

→ fit shows: not sufficient to fix spin dependence

Decuplet approximation for ΛNN contact terms



ΛNN $\propto C_1', C_2', C_3'$

$$C_1' = C_3' = \frac{(G_1 + 3G_2)^2}{72\Delta} \qquad C_2' = 0$$

***ad hoc* choice:** alter C_2:

$$V_{\Lambda NN} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

C_2' introduces a spin dependent interaction in the most relevant particle channel

YNN (ΛNN) INTERACTIONS IN PRACTICE

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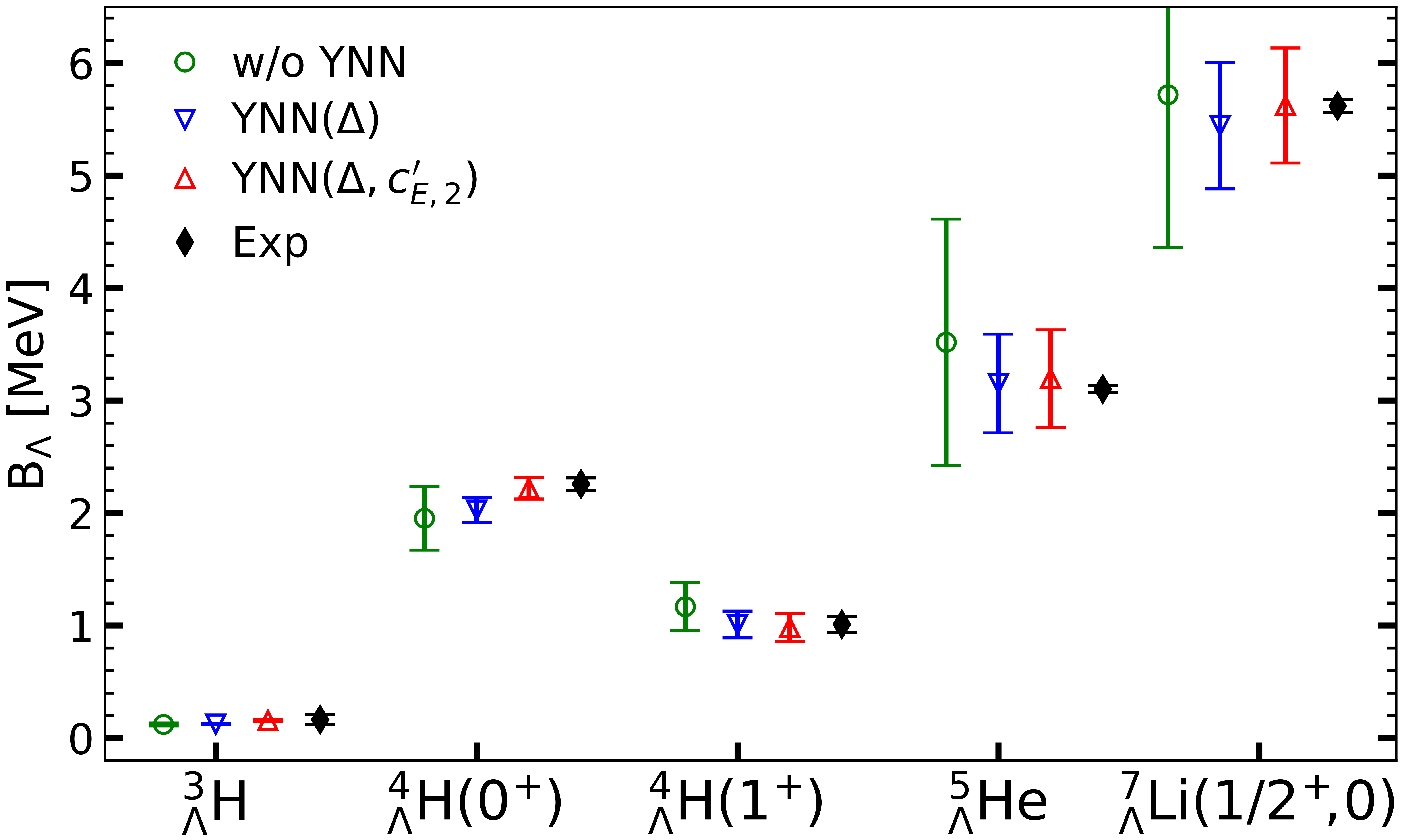
- Fit to 0^+ and 1^+ state of ${}^4_\Lambda\mathrm{He}$ and/or ${}^5_\Lambda\mathrm{He}$
- spin-dependence in $A = 4$ not well explained by decuplet saturation
- C_2' term improves 0^+ of ${}^4_\Lambda\mathrm{He}$ and $1/2^+$ of ${}^7_\Lambda\mathrm{Li}$
- agreement generally much better than $\mathrm{N^2LO}$ uncertainty



YNN PREDICTION FOR SPECTRUM OF $^{7}_{\Lambda}\mathrm{Li}$

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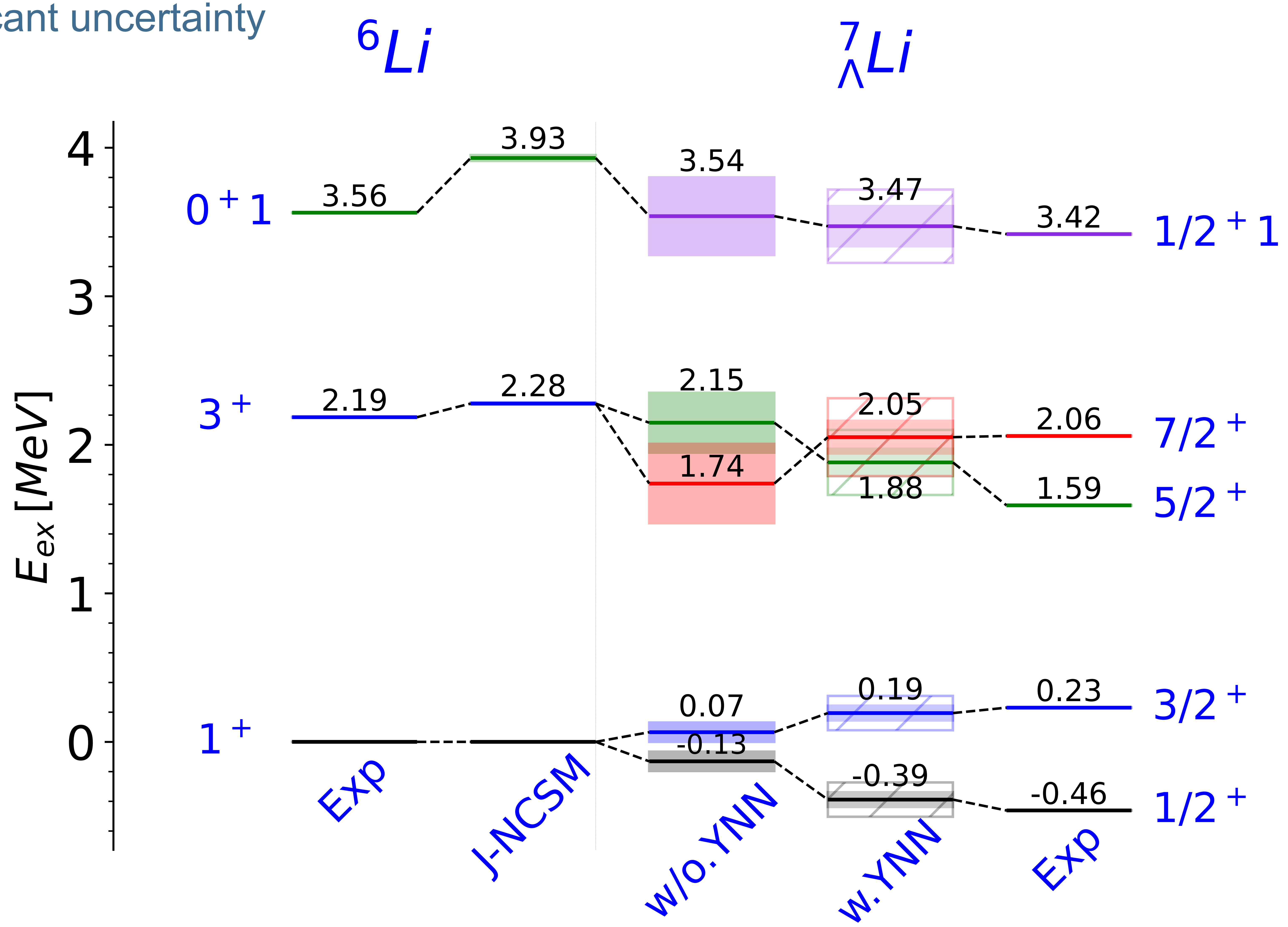
- good agreement
- C_2' term included, but not very important (not shown)
- higher states have significant uncertainty



$S = -2$ HYPERNUCLEI — ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

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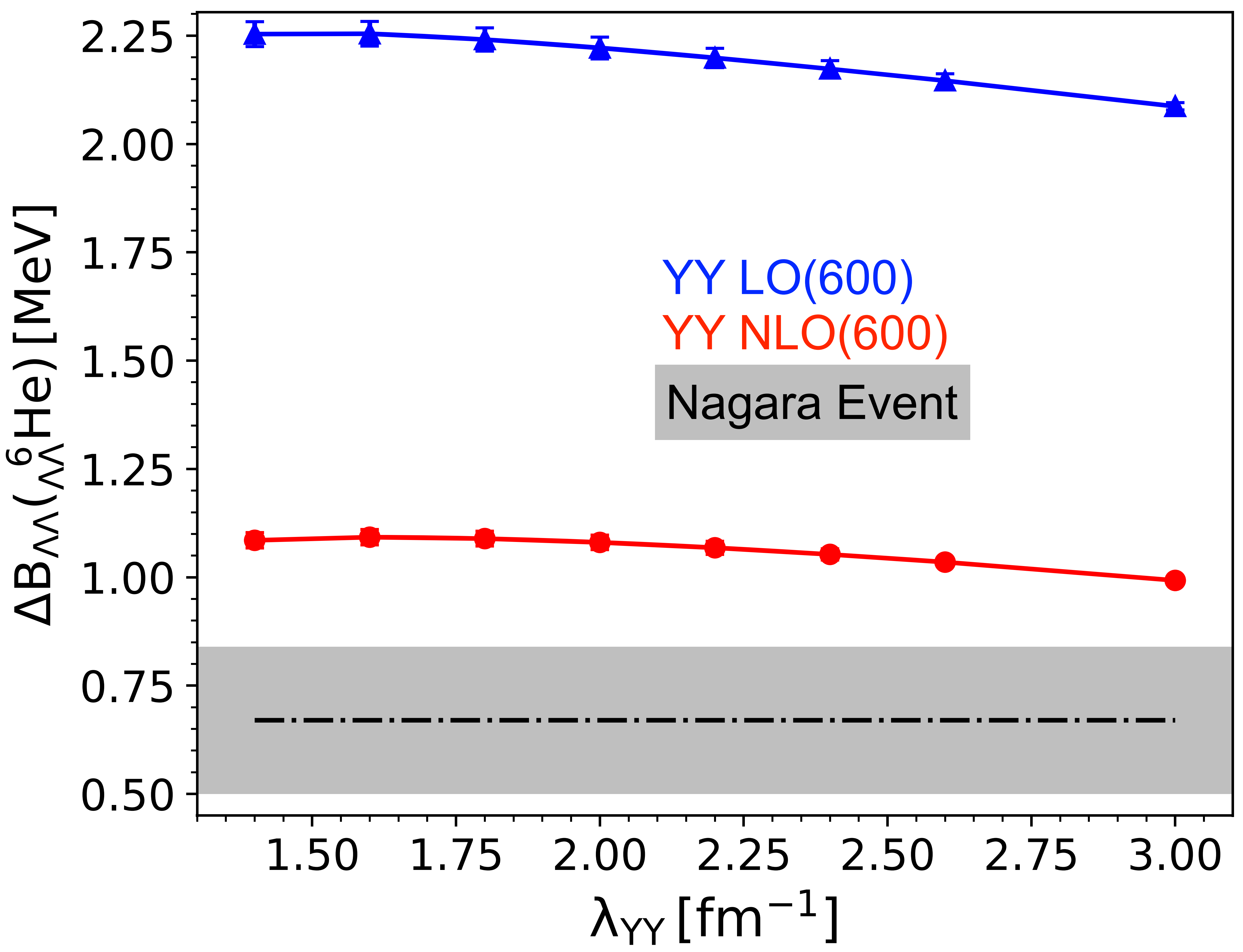
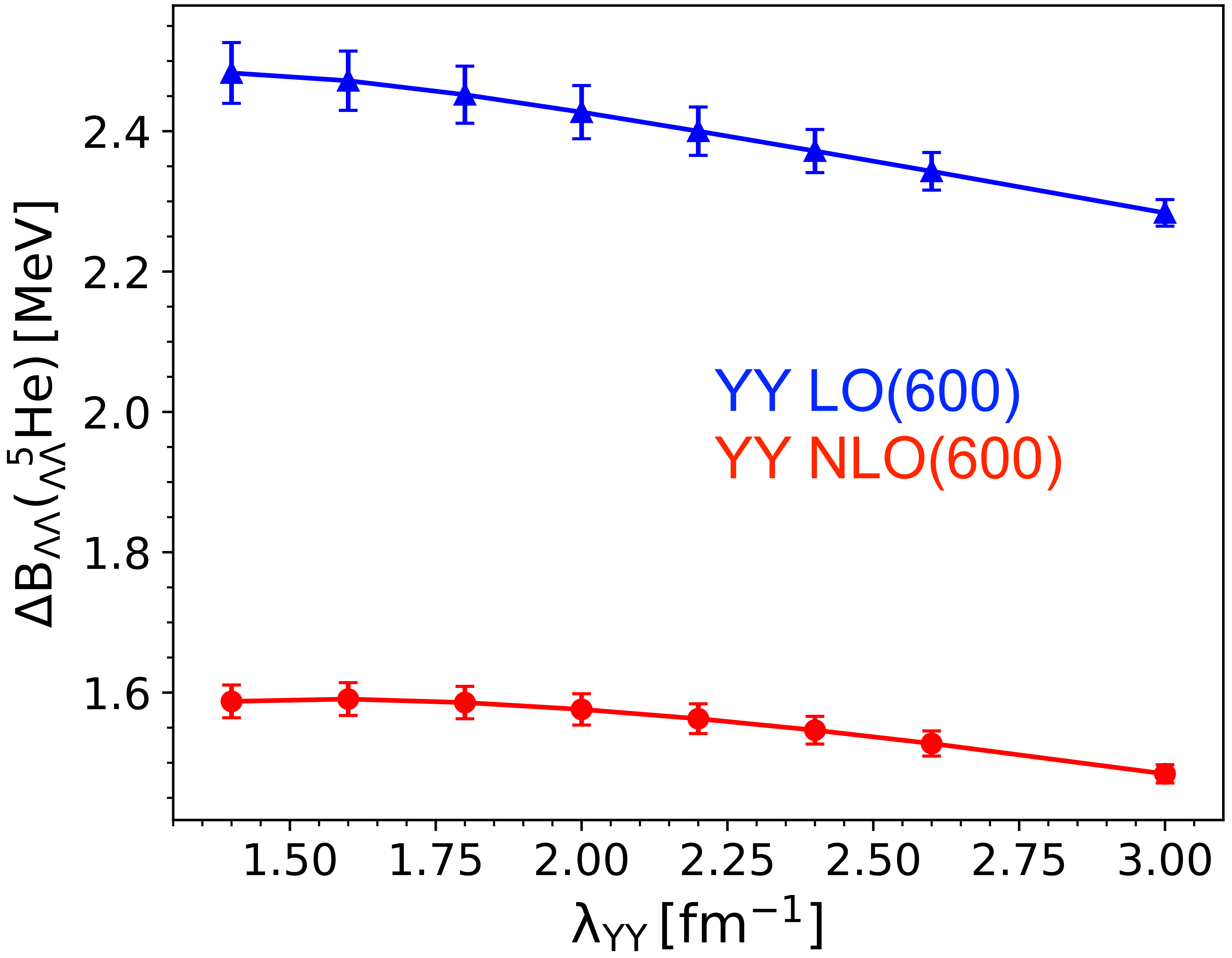
- $\Lambda\Lambda$ excess binding energy $\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda} = 2E\left({}^{A-1}_{\Lambda}X\right) - E\left({}^{A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$
- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernucle
- small λ_{YY} dependence
- uncertainties are numerical only!
- LO overbinds YY
- NLO predicts binding fairly well

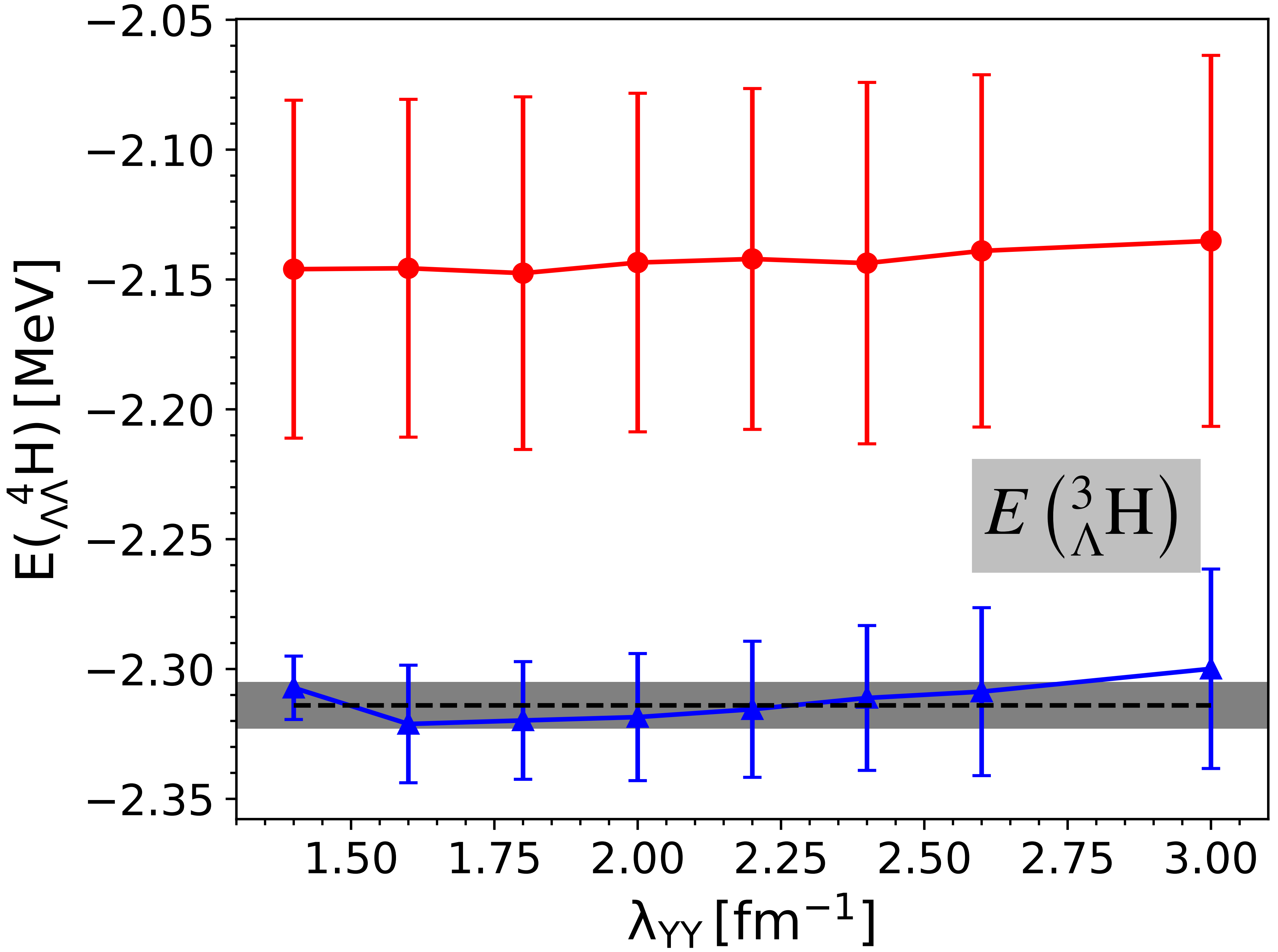


(Le et al., EPJA 57,217 (2021))

Can an $S = -2$ bound state for $A = 4,5$ be expected?

$S = -2$ HYPERNUCLEI — ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ & ${}^{4}_{\Lambda\Lambda}\mathrm{H}$





- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., PLB 797, 134893 (2019))
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., PRC 65, 041001 (2002)!)

$S = -2$ bound state for $A = 5$ can be expected, for $A = 4$ less likely but not ruled out!

Ξ HYPERNUCLEI

- experimentally accessible: Ξ^- capture process (experimental data for ${}^{15}_{\Xi}\mathrm{C}$ and ${}^{12}_{\Xi}\mathrm{Be}$)
- $\Xi N - \Lambda\Lambda$ conversion channel open: possibly short life times/difficult calculations
- HAL QCD & chiral YY interactions: suppression $\Xi N - \Lambda\Lambda$ transition
- ΞN interaction relevant: Ξ is often the second hyperon to appear in neutron matter

see talk by Gal (Wednesday,parallel)

Identify possibly interesting states:

calculations based on chiral interactions neglecting $\Xi N - \Lambda\Lambda$ transitions

(keeping $\Xi N - \Lambda\Sigma, \Sigma\Sigma$) $\longrightarrow$ states are bound states

finetuning of ${}^{11}S_0$ interaction to correct for missing $\Lambda\Lambda$ channel

neglect YN interaction to avoid transitions to $\Lambda\Lambda$

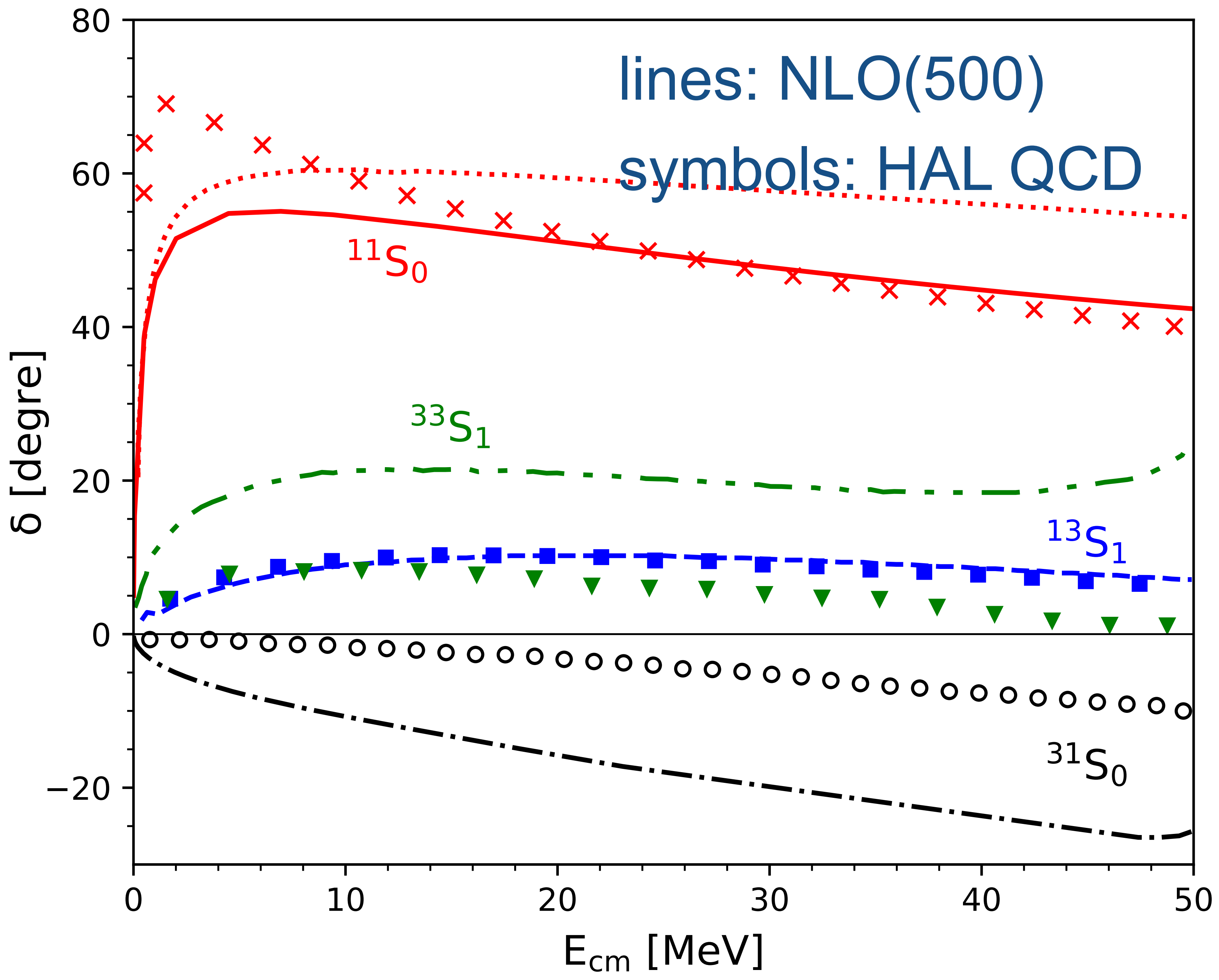
perturbative width estimates indicate small widths ✔

Here: look at ${}^{7}_{\Xi}\mathrm{H}$ (exp. expected), ${}^{5}_{\Xi}\mathrm{H}$, ${}^{4}_{\Xi}\mathrm{H}$ and ${}^{4}_{\Xi}\mathrm{n}$ explore possible bound states and their widths

Ξ HYPERNUCLEI

Ξ separation energies (NLO(500) and SMS N⁴LO⁺(450))

	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	–	–
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2



	$V^{S=-2}$			
	$^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	− 1.95	0.02	− 0.7	− 2.31
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	− 0.6	0.25	− 0.004	− 0.74
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	− 0.02	0.16	− 0.13	− 1.14
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	− 0.002	0.08	− 0.01	− 0.006
${}^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$	− 0.96	0.94	− 0.58	− 3.63
${}^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$	− 1.23	1.79	− 0.79	− 6.74

(Le et al. EPJ A 57, 339 (2021))

CONCLUSIONS & OUTLOOK

- **YN/YY interactions not well understood**
  - scarce YN/YY data
  - more information necessary to solve "hyperon puzzle"
- **Hypernuclei provide important constraints**
  - ${}^1S_0$ ΛN scattering length & ${}^3_\Lambda\mathrm{H}$
  - ${}^1S_0$ $\Lambda\Lambda$ scattering length & ${}^{\ 6}_{\Lambda\Lambda}\mathrm{He}$ & predictions for A=4,5
  - CSB of ΛN scattering & ${}^4_\Lambda\mathrm{He}$ / ${}^4_\Lambda\mathrm{H}$
  - Light $A = 4 - 7$ Ξ-hypernuclei could help to determine spin/isospin dependence in $\Xi\mathrm{N}$
- **New SMS YN interactions**
  - order LO, NLO and N²LO allow **uncertainty quantification**
  - have a **no unique** determination of contact interactions (**data necessary**)
- **Chiral 3BF**
  - decuplet saturation sufficient for overall strength / spin-dependent $\Lambda\mathrm{NN}$ leads to further improvement
  - study cutoff dependence / application to more p-shell hypernuclei necessary