Small-x phenomenology

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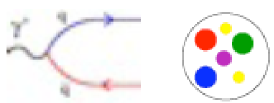
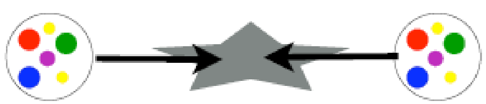
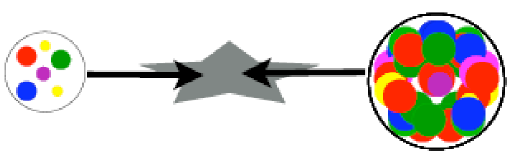
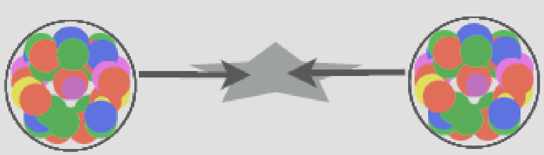
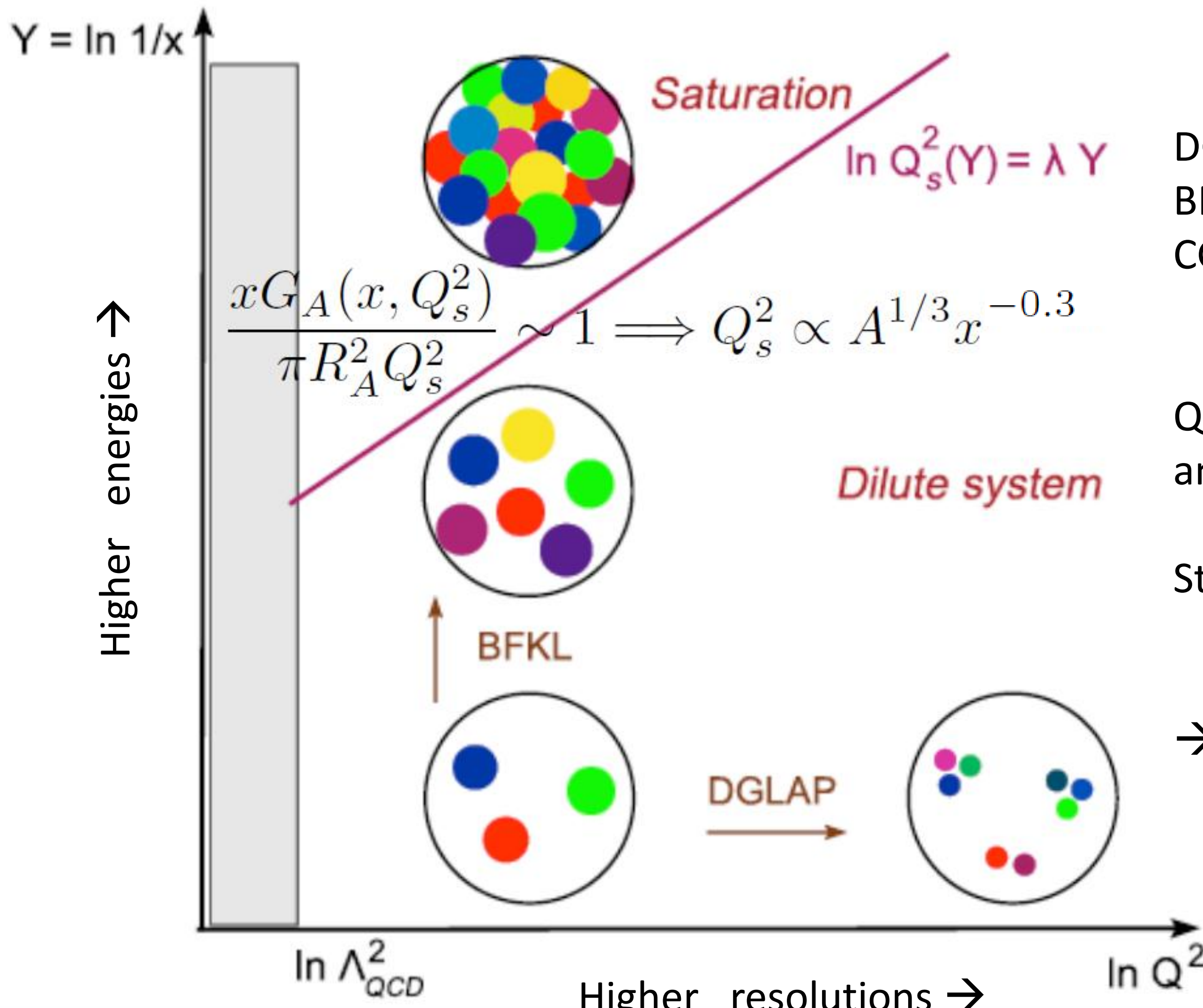
KEK theory center
21st Oct. 2011 @ RIKEN
"Future directions of high-energy QCD"

Plan

- Introduction of small-x physics and CGC
-  electron-proton (HERA)
-  proton-proton (RHIC, LHC)
-  proton(d)-nucleus (RHIC)

 nucleus-nucleus (RHIC, LHC)

"Phase diagram" of a proton/nucleus



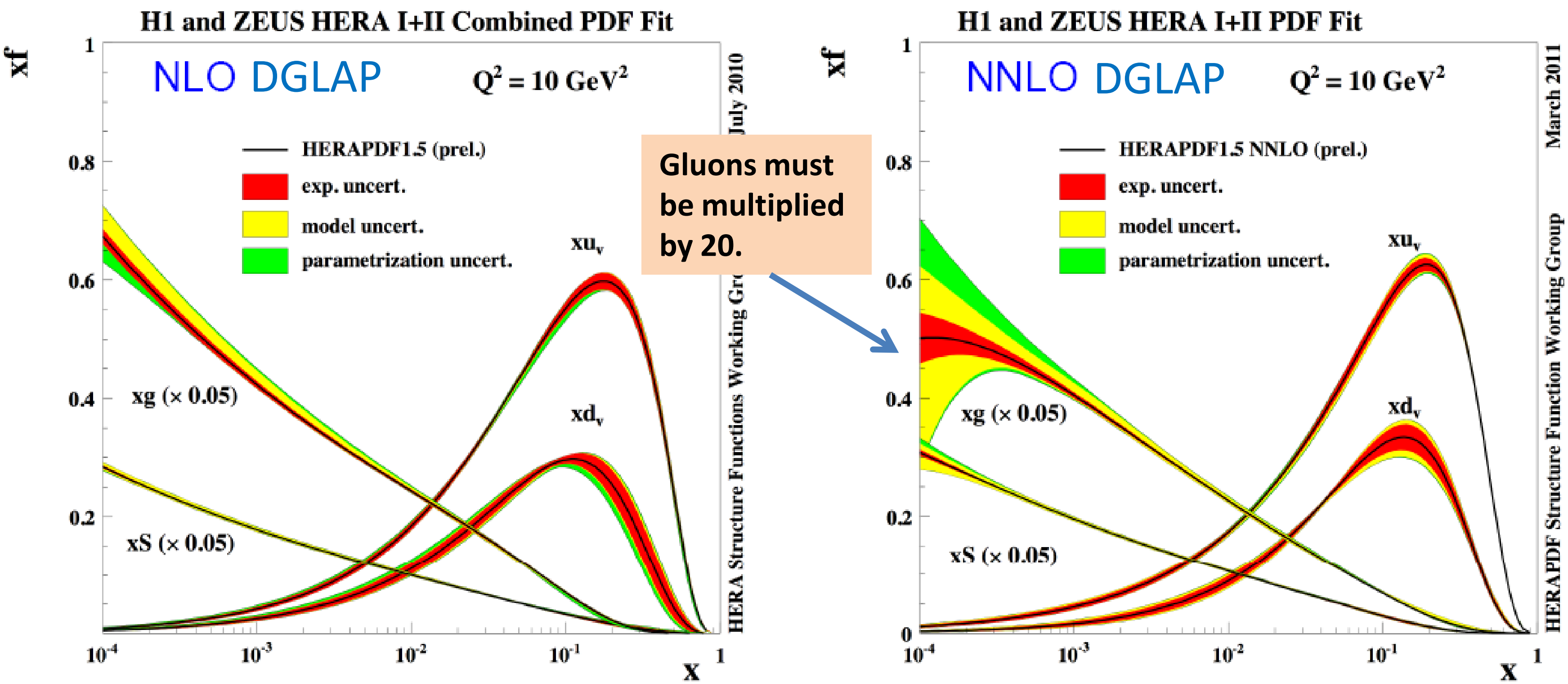
DGLAP : summation of ln Q^2
BFKL : summation of ln 1/x
CGC : BFKL + nonlinear effects
(strong classical fields)

Qs(x,A) : boundary btw saturated and NON-saturated regimes

Structure/topology will not change with improved descriptions (beyond LO)
→ slope, straight /curve, etc

High density gluons are indeed seen in a proton

HERA H1 + ZEUS combined results



H1prelim-10-142 / ZEUS-prel-10-018

H1prelim-11-042 / ZEUS-prel-11-002

At small-x, dominant degrees of freedom are not valence quarks, but GLUONS .

Geometric Scaling: existence of Qs

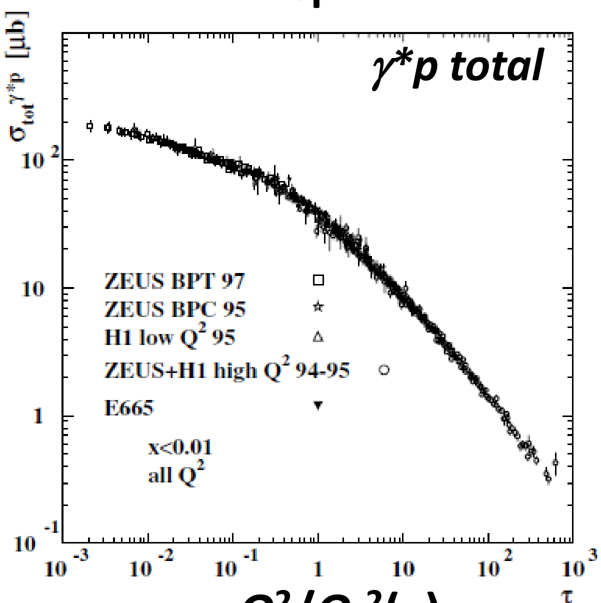
DIS (ep, eA) cross sections scale with Q^2/Qs^2

Stasto, Golec-Biernat, Kwiecinski
PRL 86 (2001) 596

Freund, Rummukainen, Weigert, Schafer
PRL 90 (2003) 222002

Marquet, Schoeffel
Phys. Lett. B639 (2006) 471

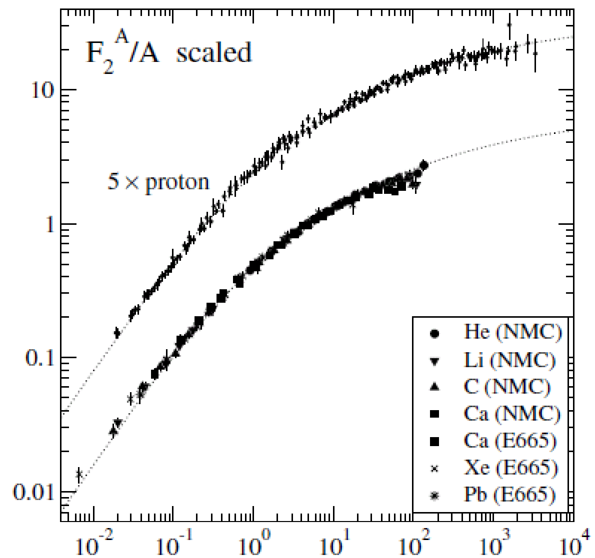
ep



$Q^2/Q_s^2(x)$

FIG. 1. Experimental data on $\sigma_{\gamma^* p}$ from the region $x < 0.01$ plotted versus the scaling variable $\tau = Q^2 R_0^2(x)$.

eA



$Q^2/Q_s^2(x,A)$

FIG. 3. Scaling behavior of NMC and E665 F_2^A data vs $\tau = (\frac{x_{\rm bj}}{x_0})^{2\lambda} \frac{Q^2}{A^{1/3}}$. The vertical axis corresponds to the left-hand side of Eq. (5). The dashed line corresponds to the geometric scaling curve obtained from HERA data. These are shown offset by a factor of 5.

Diffractive ep

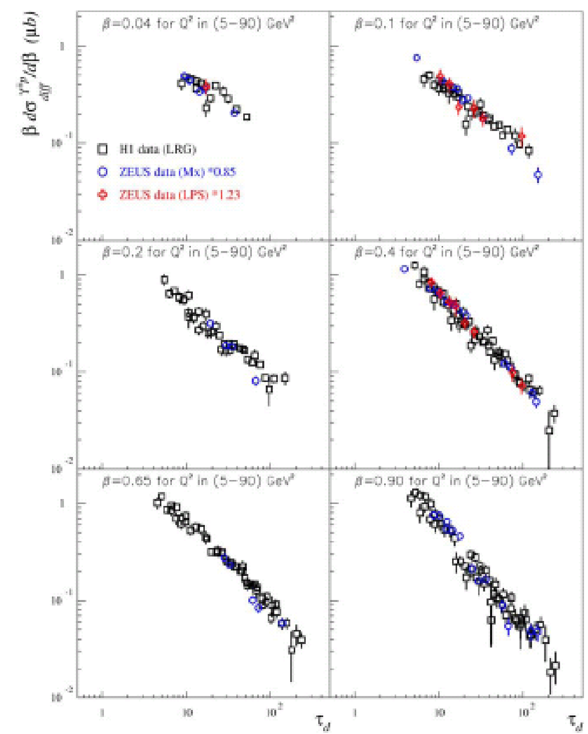


Fig. 2. The diffractive cross-section $\beta d\sigma_{\rm diff}^{\gamma^* p \to Xp}/d\beta$ from H1 and ZEUS measurements, as a function of τ_d in bins of β for Q^2 values in the range [5; 90] GeV^2 and for $x_{\mathbb{P}} < 0.01$. Only statistical uncertainties are shown.

$Q^2/Q_s^2(x_P)$

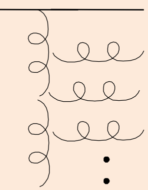
- **Existence of saturation scale Qs**
- **Can determine x and A dependences of Qs**
- **Extends outside of the saturation regime** $k_t < Q_s^2/\Lambda_{QCD}$ (Iancu,Itakura,McLerran)

Going up higher energies: evolution eqs.

<u>Evolution wrt x (or rapidity $y = \ln 1/x$)</u>

Multiple gluon emissions

$N_g \sim e^{\omega \ln 1/x}$



- **BFKL** (LO : $(\alpha_s \ln 1/x)^n$, NLO: $\alpha_s (\alpha_s \ln 1/x)^n$)

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k_t})}{\partial \ln(\mathbf{x_0}/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k_t})$$

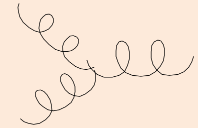
K : gluon splitting g→ gg

ϕ : unintegrated gluon distr.

Recombination of gluons

$N_g \leq 1$

Unitarity



- **BK** (includes the nonlinear effects)

$$\frac{\partial \phi(\mathbf{x}, \mathbf{k_t})}{\partial \ln(\mathbf{x_0}/\mathbf{x})} \approx \mathcal{K} \otimes \phi(\mathbf{x}, \mathbf{k_t}) - \phi(\mathbf{x}, \mathbf{k_t})^2$$

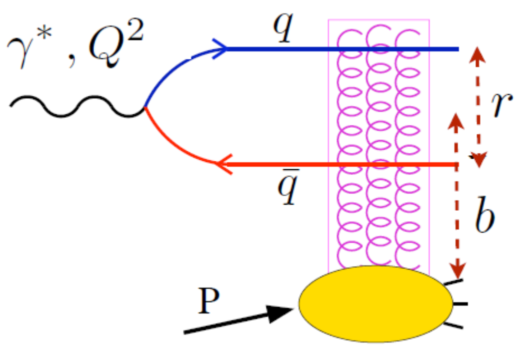
Known up to full NLO accuracy. [Balitsky, Chirilli 2008]

But for practical purposes, we use BK with running coupling → "rcBK" [Balitsky, Gardi et al. , Kovchegov-Weigert]

$$K^{\rm run}(\mathbf{r}, \mathbf{r_1}, \mathbf{r_2}) = \underbrace{\frac{N_c\, \alpha_s(r^2)}{2\pi^2} \left[\frac{r^2}{r_1^2\, r_2^2}\right.}_{\text{LO}} \left. + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right]$$

Phenomenology in DIS at small-x

- **Dipole formalism**



LC wf : known

$$\sigma_{T,L}^{\gamma^* P}(x, Q^2) = \int_0^1 dz \int d^2\mathbf{r} \left| \Psi_{T,L}^{\gamma^* \to q\bar{q}}(z, Q, r) \right|^2 \sigma^{dip}(x, r)$$

$$\sigma^{dip}(x, r) = 2 \int d^2 b \boxed{\mathcal{N}(x, b, r)}$$

dipole cross section

Dipole scatt. amplitude
← Solution to BK eq

- **Phenomenology** ("global" fit, $x < 0.01$) :

How to parametrize dipole cross section?

- use approximate solution to BK

LO-BK → IIM [Iancu-KI-Munier 2004], Soyez (with heavy quarks) [2007]
u, d, s + c, b

parameters : Energy dependence and magnitude of Qs, radius of a proton
(theory ambiguity) (non-perturbative effects)

- use numerical solution to BK

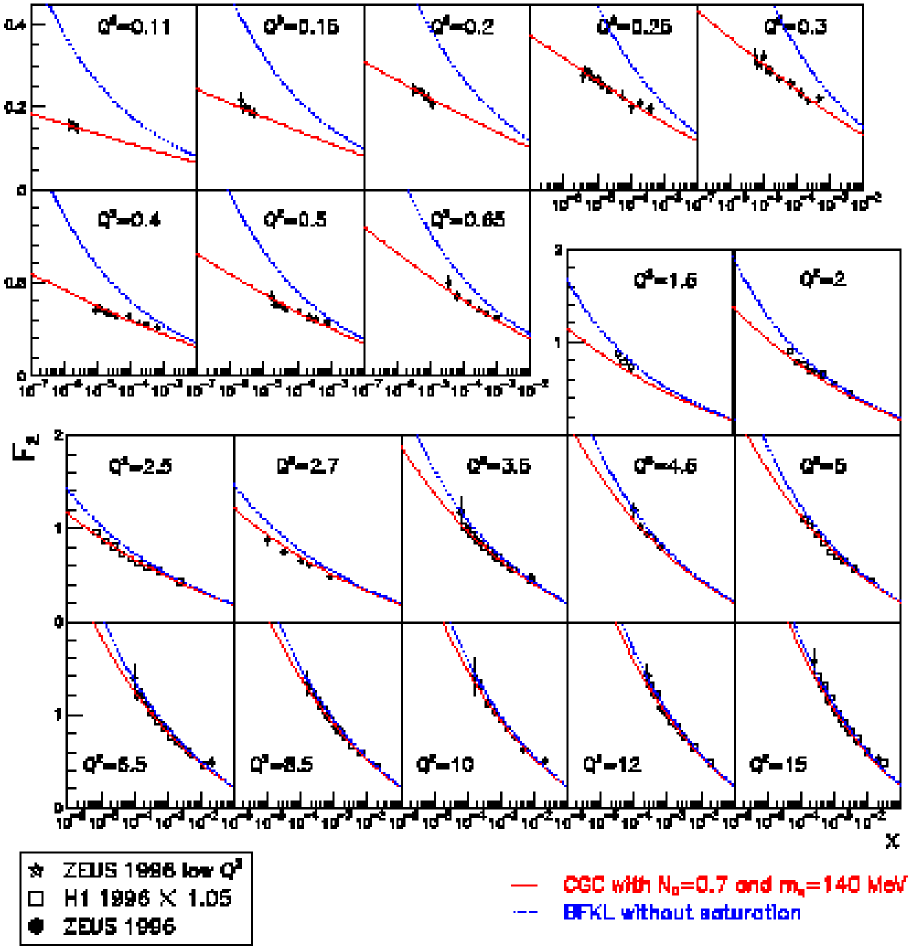
rcBK → AAMQS [Albacete-Armesto-Milhano-Quiroga-Salgado,2009,2011]

parameters : characterizing initial conditions (MV, modified MV)

Fit to HERA data: "IIM" model

[Iancu,KI,Munier '04]

$F_2(x, Q^2)$



- **Fit to the data with small *x* and moderate Q^2**

  $x < 0.01$ & $0.045 < Q^2 < 45$ GeV^2

- **Analytic solutions to BK built in:** <u>**geometric scaling**</u> **&** <u>**its violation**</u>**,** <u>**saturation**</u>**.**

- **Only 3 parameters:**

  proton radius R, x_0 (nonpert.) and λ for $Q_S{}^2(x)=(x_0/x)^\lambda$ GeV^2

- **Good agreement with the data**

  $x_0 = 0.26 \times 10^{-4}$, $\lambda = 0.25$

- **Also works well for vector meson (ρ, ϕ) production, diffractive F_2, F_L**

  [Forshaw et al, Goncalves, Machado '04]

Red line : the CGC fit

Blue line : BFKL w/o saturation

Fit to HERA data: AAMQS$_{2011}$

- Initial Conditions : modified GBW/MV models

$x_0 = 0.00893$ or 0.008

$$\begin{cases} \mathcal{N}^{\rm GBW}(r, x=x_0) = 1 - \exp\left[-\dfrac{(r^2 Q_{s0}^2)^\gamma}{4}\right], \\ \mathcal{N}^{\rm MV}(r, x=x_0) = 1 - \exp\left[-\dfrac{(r^2 Q_{s0}^2)^\gamma}{4} \ln\left(\dfrac{1}{r\Lambda} + e\right)\right] \end{cases}$$

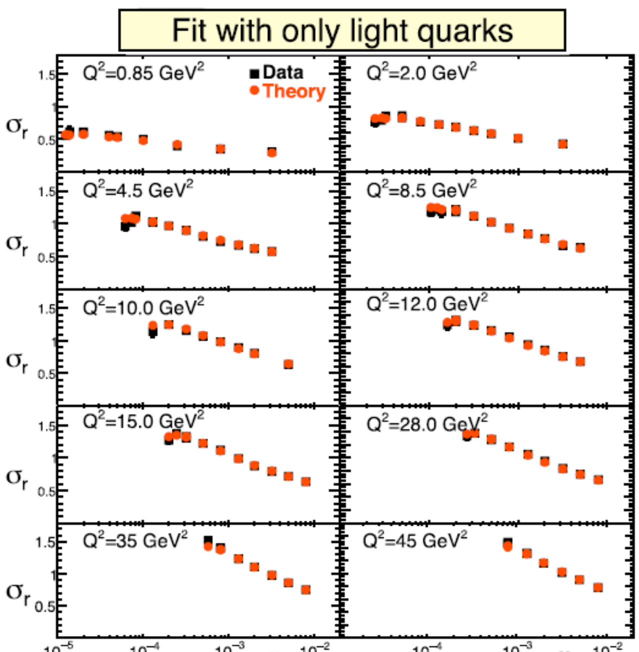
(γ=1 : ordinary GBW)

(γ=1 : ordinary MV)

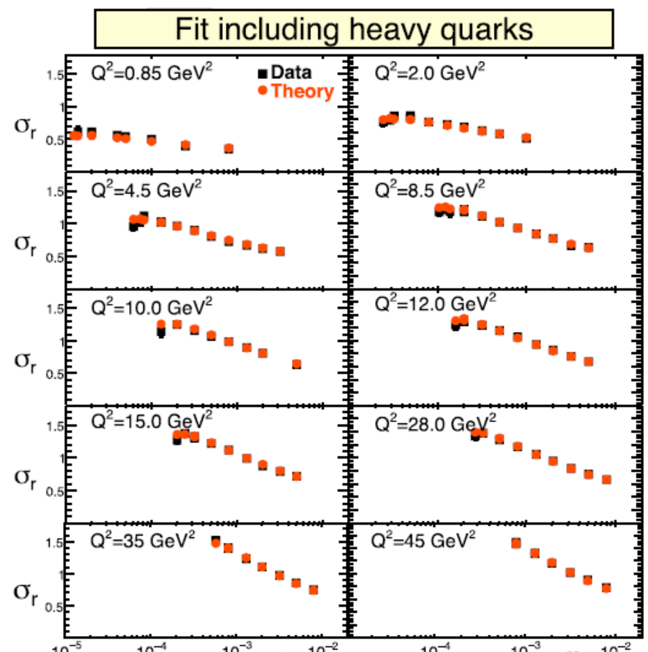
- IR regularization for 1-loop running coupling

$$\alpha_{s,n_f}(r^2) = \frac{4\pi}{\beta_{0,n_f} \ln\left(\frac{4C^2}{r^2 \Lambda_{n_f}^2}\right)}$$

freeze the coupling at $\alpha_s^{\rm fr} = 0.7$



m_{uds}=140MeV



m_c=1.27GeV, m_b=4.2GeV

← Modified GBW

(Left) γ=0.971
Qs_0^2=0.241

(Right) γ=0.959
Qs_0^2=0.240

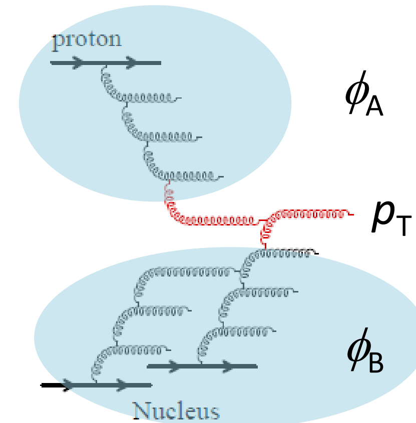
There are two more parameters (C,σ_0)

Hadron collisions (pp/pA): two formulae for single hadron spectra

- ***k*$_t$ factorization**

$$\frac{d\sigma^{A+B\to g}}{dy d^2p_T d^2X} \sim K \frac{\alpha_s}{p_T^2} \phi_A(k_1, x_1, b) \otimes \phi_B(k_2, x_2, X-b)$$

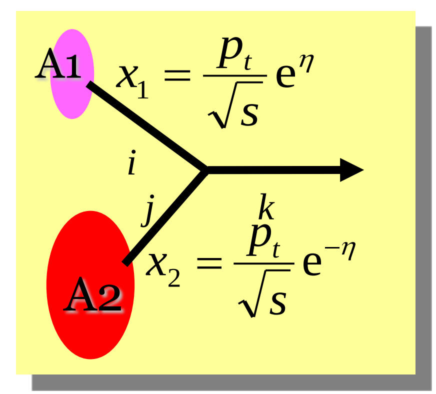
- proved for pp, pA at LO
- good when both A and B are saturated (mid rapidity at very high energy)
- used in various calculations e.g. multiplicity distribution, etc



- **DHJ formalism** [Dumitru-Hayashigaki-Jalilian--Marian 2006]

$$\frac{dN}{dy_h d^2p_T} = \frac{K}{(2\pi)^2} \sum_{ijk} \int_{x_F}^{1} \frac{dz}{z^2} x_1 f_{i/p}(x_1, p_T^2) \tilde{N}_j\left(\frac{p_T}{z}, x_2\right) D_{h/k}(z, p_T^2)$$

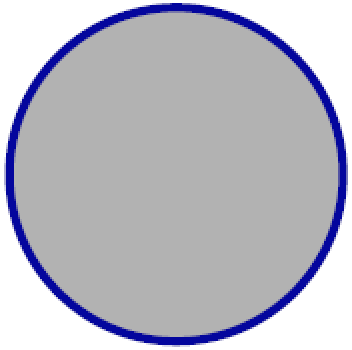
- "Large-x / small-x" reactions: valid at forward rapidity x_1~1, x_2 <<1
- $f_{i/p}$(x) : pdf for valence partons in a projectile
- $D_{h/k}$(z) : frag. func. for outgoing hadron h from a parton k
- N : un-integrated gluon distribution in a target



How to treat nuclei?

MC modeling for a nucleus:

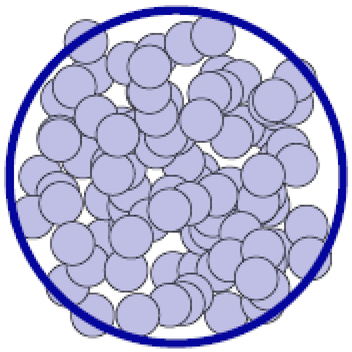
- The simplest will be a homogeneous disk
  no impact parameter dependence
  an additional parameter Q^2_{s0A} needed



may use a simple parametrization by KLN,
or numerical solution to rcBK

- Random nucleons w/ Woods-Saxon dist.
  fluctuating density $\Rightarrow$ b-dependence
  $Q^2_{s0A} = Q^2_{s0p} \times N$ w/o additional parameter

Drescher-Nara



-Nucleons are described as disks or Gaussians.

- Can be used for IC in AA collisions

Towards better description of pA at forward rapidities

- Xsec formula vs nuclear modeling
- how to parametrize gluon distribution

  → KLN or rcBK

→ forward

	kt factorization	DHJ formalism
Homogeneous disk	"KT/KLN" Kharzeev, et al.	"DHJ/rcBK" Albacete-Marquet 2010
MC model (randomly generated)	"MC-KT/KLN" Drescher-Nara, Albacete- Dumitru-Nara	"MC-DHJ/rcBK" Fujii-KI-Kitadono-Nara 2011

↓ Less parameters

In the DHJ formalism (looking at forward rapidity), target nucleus is generated randomly, and the gluon distribution is given by rcBK

DHJ/rcBK

[Albacete-Marquet 2010]

- Single hadron spectra at forward rapidity in RHIC

$$\frac{dN_h}{dy_h\, d^2 p_t} = \frac{K}{(2\pi)^2} \sum_q \int_{x_F}^{1} \frac{dz}{z^2} \Big[x_1 f_{q/p}(x_1, p_t^2)\, \tilde{N}_F\left(x_2, \frac{p_t}{z}\right) D_{h/q}(z, p_t^2) \quad \text{quark}$$

$$+ x_1 f_{g/p}(x_1, p_t^2) \tilde{N}_A\left(x_2, \frac{p_t}{z}\right) D_{h/g}(z, p_t^2) \Big] \quad \text{gluon}$$

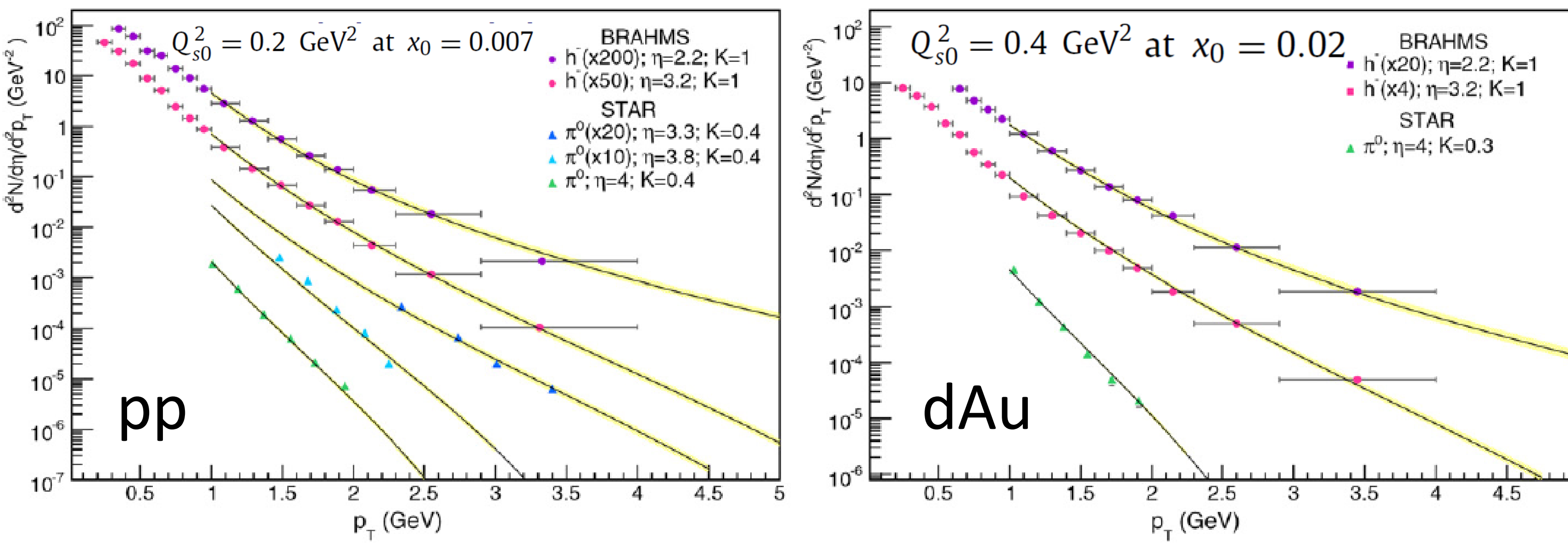
For $f_i(x,p_t)$, use CTEQ6 NLO pdf

For $D_i(z,p_t)$, use DSS NLO FF

For $N_{F/A}$, use solution to rcBK with MV model (x_0,Q_{s0}) as I.C.

$$\underline{\mathcal{N}_A(r,Y)} = 2\mathcal{N}_F(r,Y) - \mathcal{N}_F^2(r,Y)$$

gluon scattering amplitude



Very good agreement with the data. But they fit pp and dAu independently.

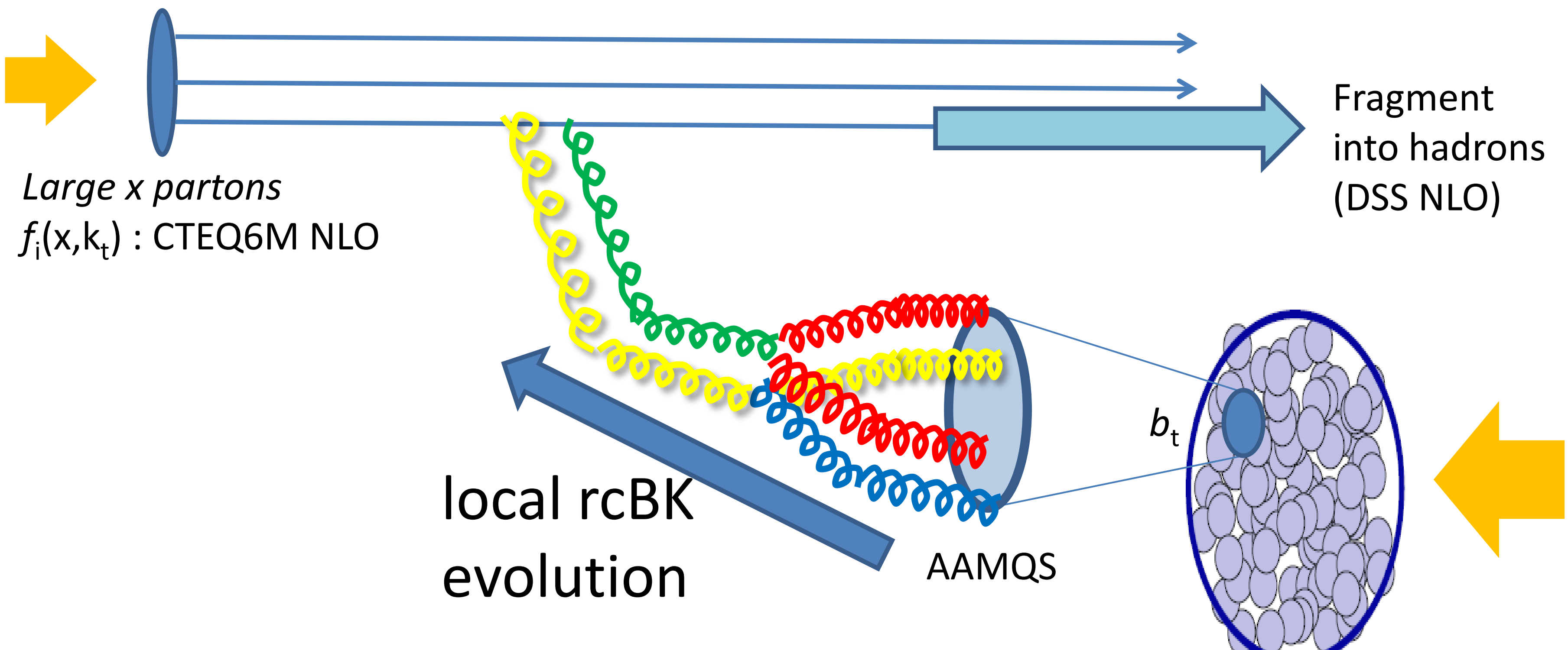
MC-DHJ/rcBK

[Fujii,KI,Kitadono,Nara, arXiv:1107.1333]

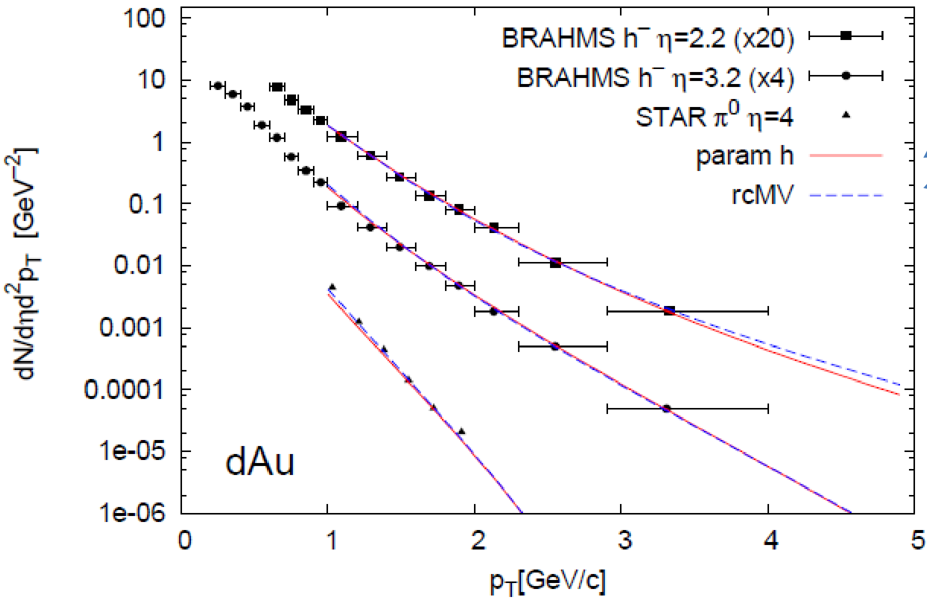
To reduce ambiguity

- construct a nucleus by randomly placing nucleons
- use AAMQS parameters for proton IC optimized for DIS at small-x
- quantum evolution is performed "locally" in b space

(to avoid IR div. in b-dep BK)



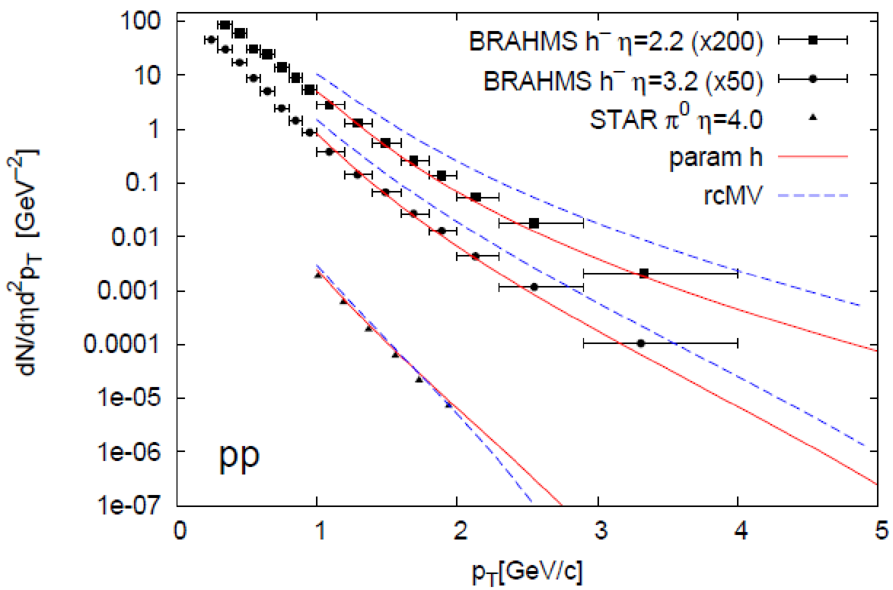
MC-DHJ/rcBK : results



modified MV model ($\gamma = 1.118$)

"running coupling" version of MV model [Iancu-KI-Triantafylopoulos] : to be consistent with rcBK evolution

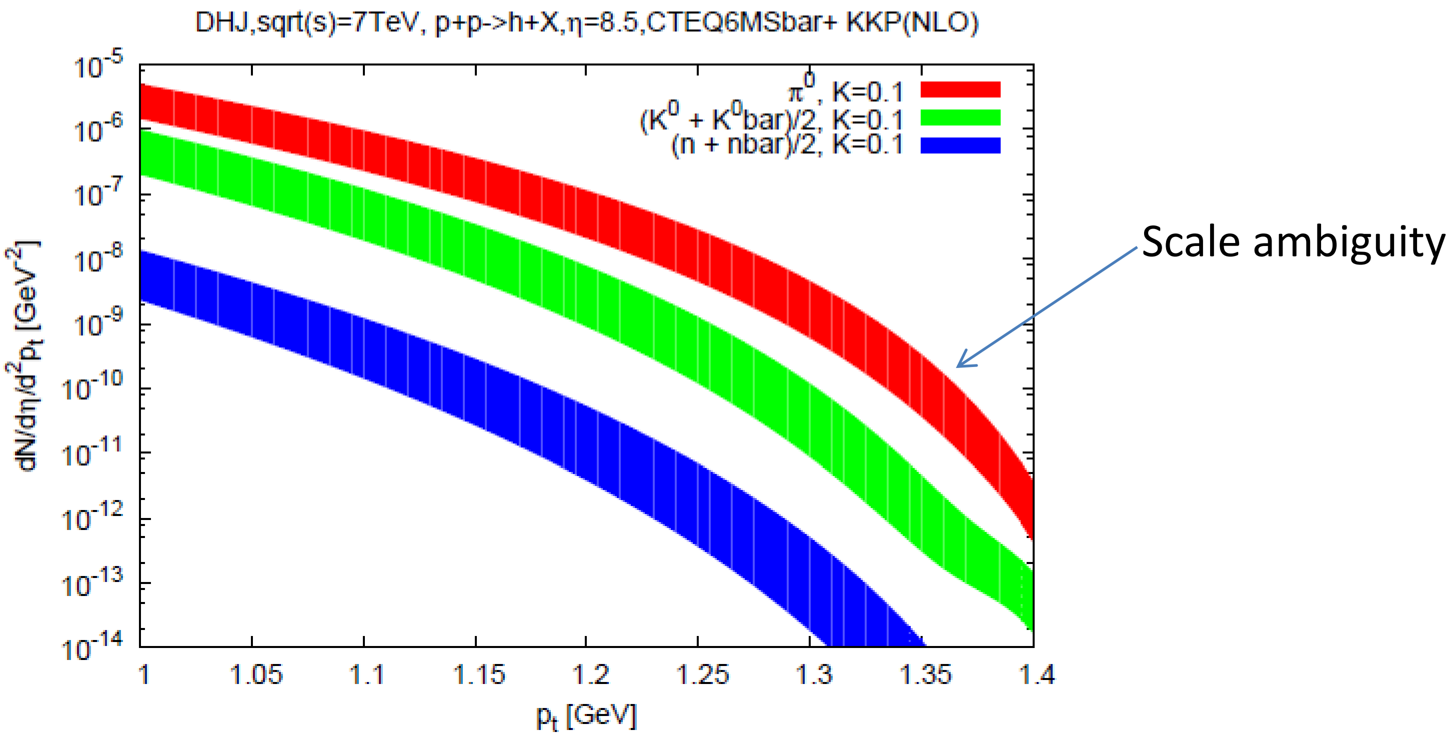
- Set *h* works well even in *pp*, but not as good as Albacete-Marquet
- rcMV is not "tuned" (similar param as MV)
- However, both work quite well in dAu (IC dependence reduces at high rapidity)



- reproduce the data nicely
- AAMQS set h and rcMV for $\mathcal{N}(r, y)$
- Q^2_{s0A} fixed by MC; no additional parameter

Best results from theoretical point of view, but still needs better (global) description including pp data (tuning of rcMV is necessary)

MC-DHJ/rcBK extrapolated to LHC



- Hadron productions (π^0, K^0 and n) at $\boxed{\eta = 8.5 \text{ at } 7\text{ TeV}}$ (LHCf) is being studied in this framework

Very forward region could be dominated by soft interaction, but still necessary to understand how much hard contribution exists.

Towards further improvements?

- Two formula (KT and DHJ) are derived in LO

  ← not consistent with the use of rcBK

- Running-coupling corrections to LOKT [Horowitz-Kovchegov, 2011]

LO

$$
\left\{
\begin{aligned}
\frac{d\sigma}{d^2k_T\,dy} &= \frac{2\,\alpha_s}{C_F}\,\frac{1}{\boldsymbol{k}^2}\int d^2q\,\phi_p(\boldsymbol{q},y)\,\phi_A(\boldsymbol{k}-\boldsymbol{q},Y-y)\\
\phi_A(\boldsymbol{k},y) &= \frac{C_F}{\alpha_s\,(2\pi)^3}\int d^2b\,d^2r\,e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\;\nabla_{\boldsymbol{r}}^2\,N_G(\boldsymbol{r},\boldsymbol{b},y)
\end{aligned}
\right.
$$

"rcKT"

$$
\left\{
\begin{aligned}
\frac{d\sigma}{d^2k_T\,dy} &= \frac{2\,C_F}{\pi^2}\,\frac{1}{\boldsymbol{k}^2}\int d^2q\,\overline{\phi}_p(\boldsymbol{q},y)\,\overline{\phi}_A(\boldsymbol{k}-\boldsymbol{q},Y-y)\,\frac{\alpha_s\left(\Lambda_{\rm coll}^2\,e^{-5/3}\right)}{\alpha_s\left(Q^2\,e^{-5/3}\right)\,\alpha_s\left(Q^{*\,2}\,e^{-5/3}\right)}\\
\overline{\phi}_A(\boldsymbol{k},y) &= \frac{C_F}{(2\pi)^3}\int d^2b\,d^2r\,e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\;\nabla_{\boldsymbol{r}}^2\,N_G(\boldsymbol{r},\boldsymbol{b},y)
\end{aligned}
\right.
$$

Some scale defined by ***k,q***

so far, there is no phenomenological anaysis based on this.

- Next, we need running-coupling DHJ !!

Summary

- Theoretical description of high-energy hadron scattering based on CGC is now (almost) established up to leading log accuracy with running coupling corrections. → rcBK paradigm

- In particular, phenomenological analysis with rcBK has been making a progress enough to be compared with experimental data. → HERA DIS at small-x, RHIC dAu at forward rapidity

- Nontrivial steps (I didn't mention):

  → multiparticle (dihadron) correlations

  → AA collisions (Better description of the dAu at forward rapidity provides useful information for IC of AA collisions.)