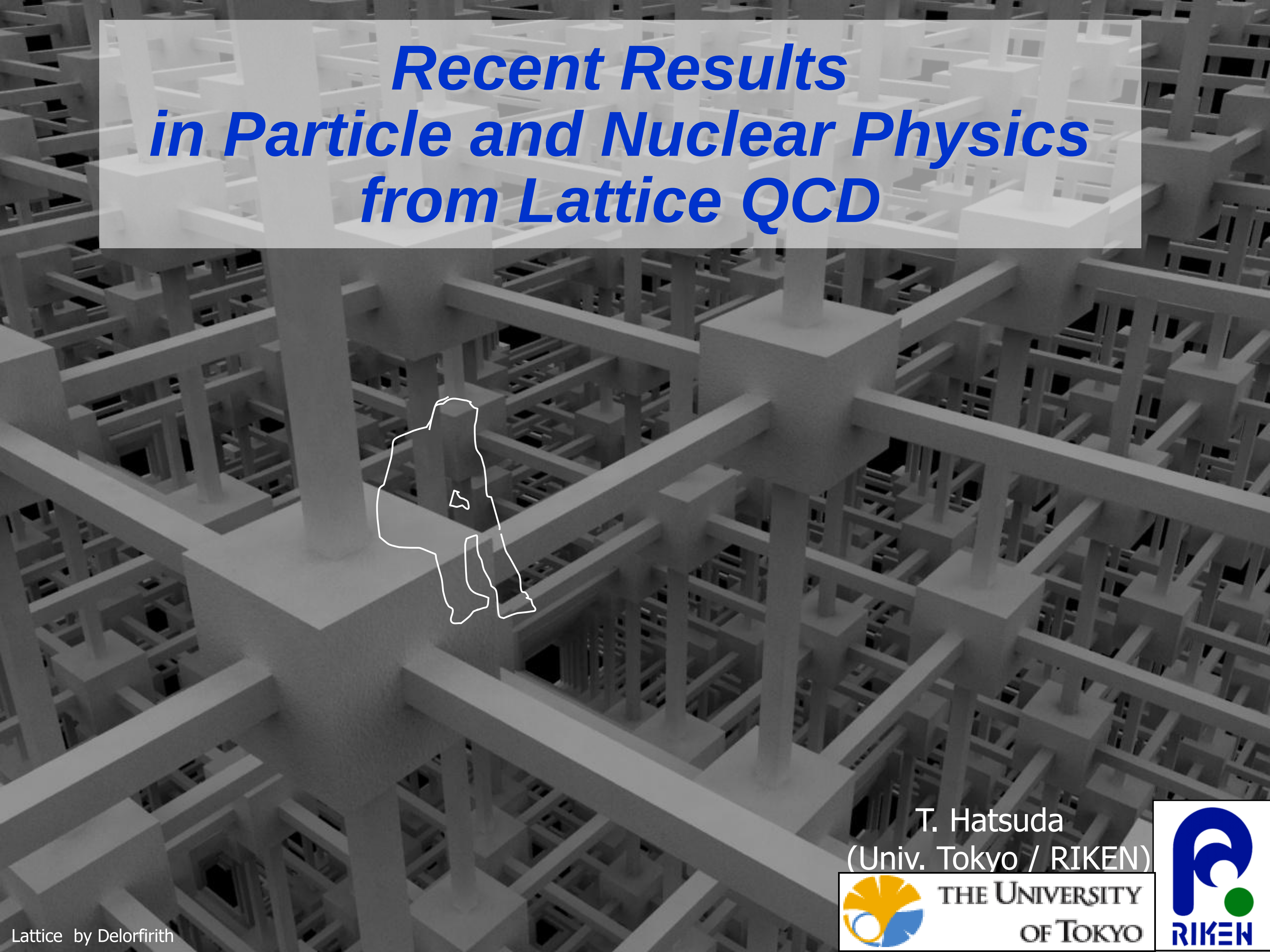
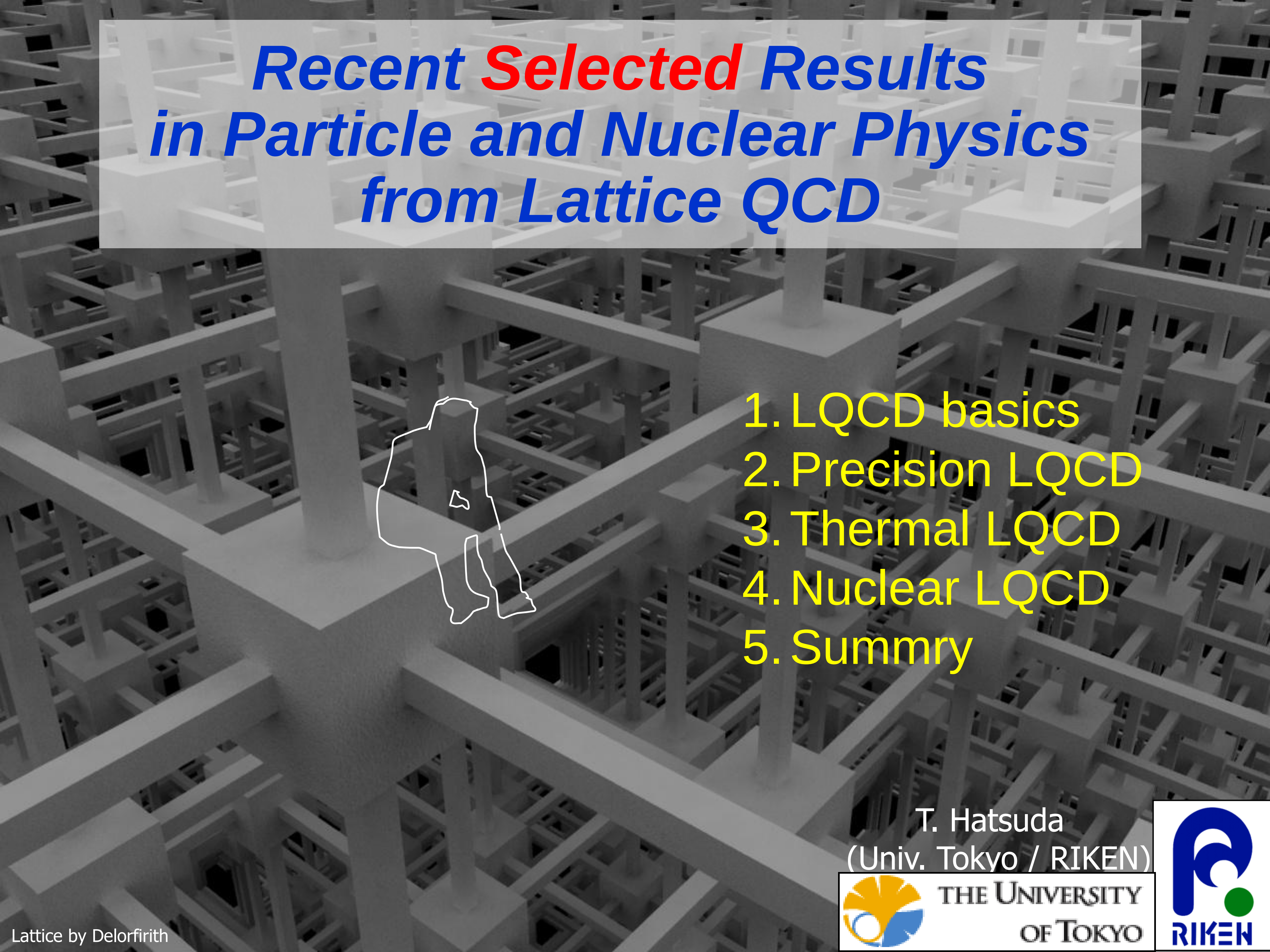
Recent Results in Particle and Nuclear Physics from Lattice QCD



Lattice by Delorfirith

T. Hatsuda
(Univ. Tokyo / RIKEN)

Recent Selected Results in Particle and Nuclear Physics from Lattice QCD



1. LQCD basics
2. Precision LQCD
3. Thermal LQCD
4. Nuclear LQCD
5. Summry

T. Hatsuda
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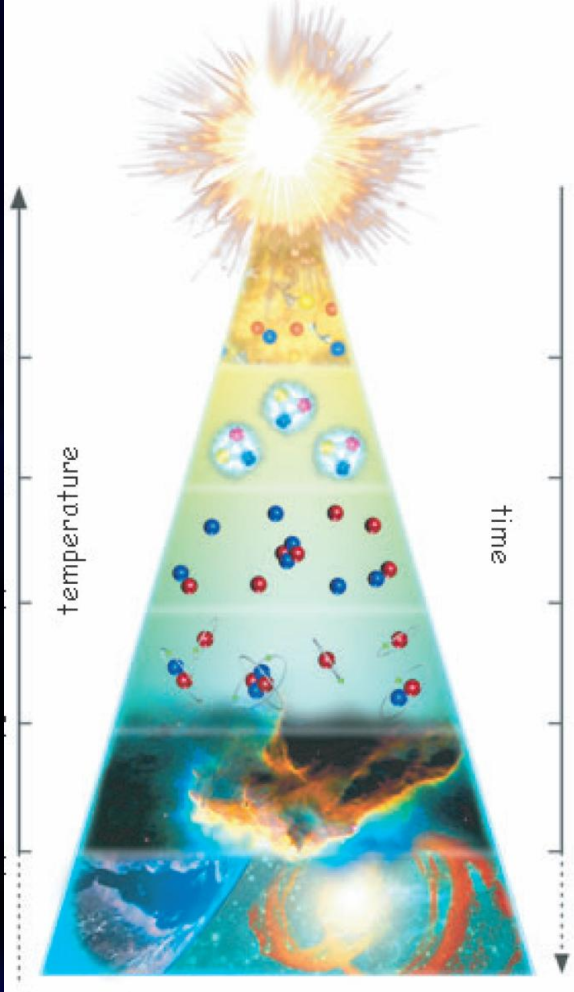
Lattice by Delorfirith

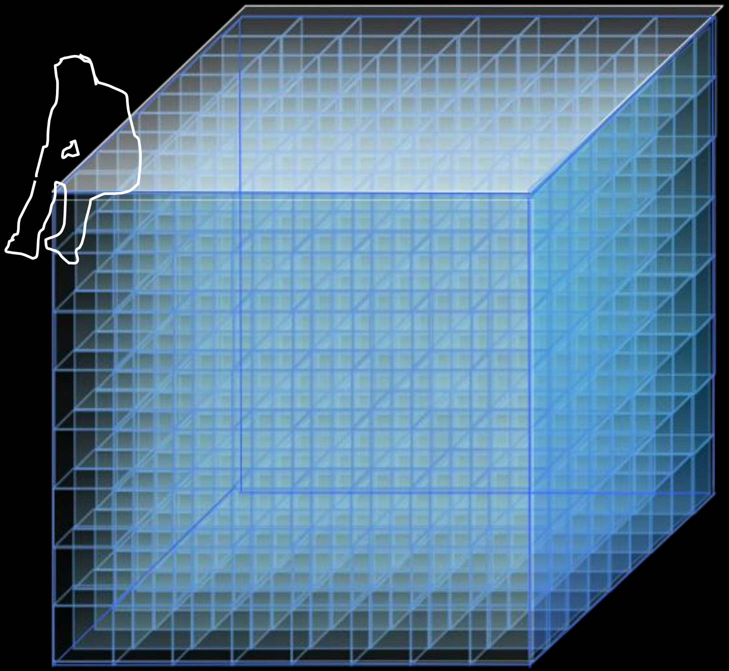
Current challenges in QCD



- Primordial form of matter
  quark-gluon plasma , hadron structure
- Origin of heavy elements
  in explosive astrophysical phenomena
- Super dense matter
  neutron star, exotic matter, …
- Inputs for “new physics” search
  dark matter, …

Lattice QCD provides
(1) precision calculations & (2) qualitative pictures

Lattice QCD basics



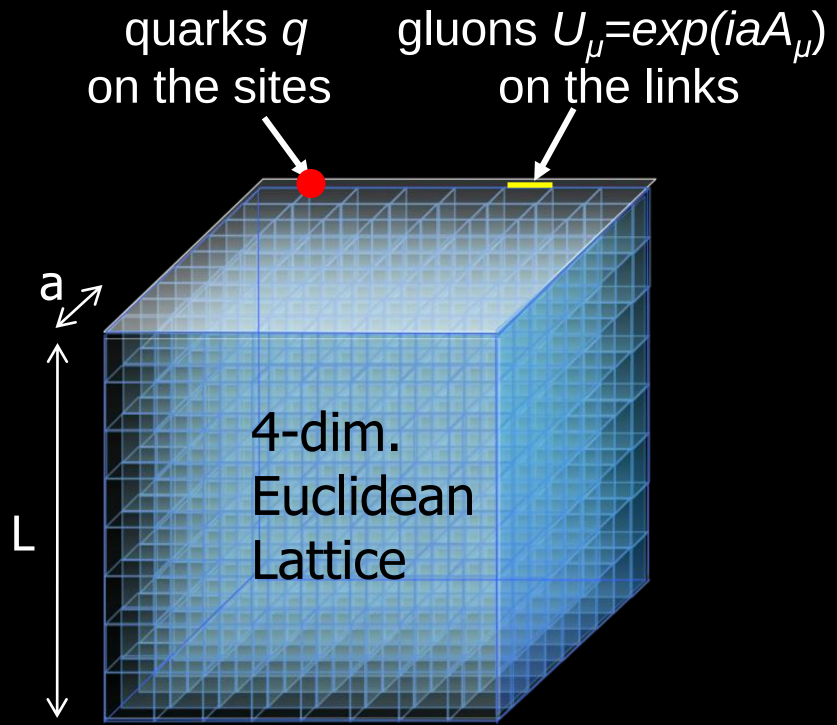
$$\boxed{\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \bar{q}\gamma^\mu(i\partial_\mu - g t^a A^a_\mu)q - m\bar{q}q}$$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

quarks *q*
on the sites

gluons $U_\mu = exp(iaA_\mu)$
on the links

$$\boxed{\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G_a^{\mu\nu} + \bar{q}\gamma^\mu(i\partial_\mu - g t^a A^a_\mu)q - m\bar{q}q}$$

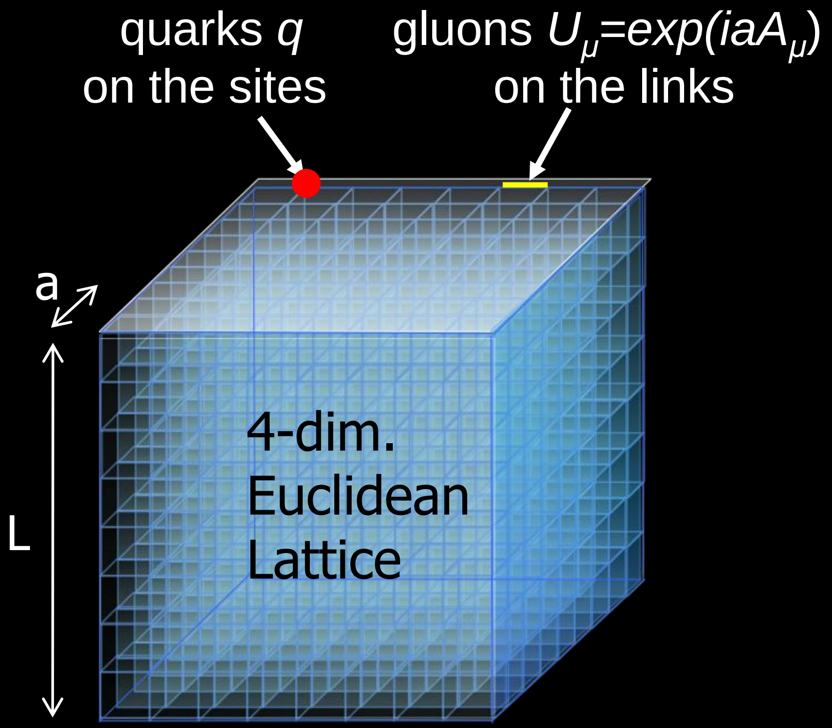


$$\boxed{Z = \int [dU][dqd\bar{q}]\ \exp\left[-\int d\tau d^3x \mathcal{L}_{\mathrm{E}}\right]}$$

Monte Carlo method

Observable
=O(g, m, a, L)

$$\mathcal{L} = -\frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g t^{a} A^{a}_{\mu}) q - m\bar{q}q$$



What can be done

- hadron properties & interactions
- hot plasma in equilibrium

What is difficult

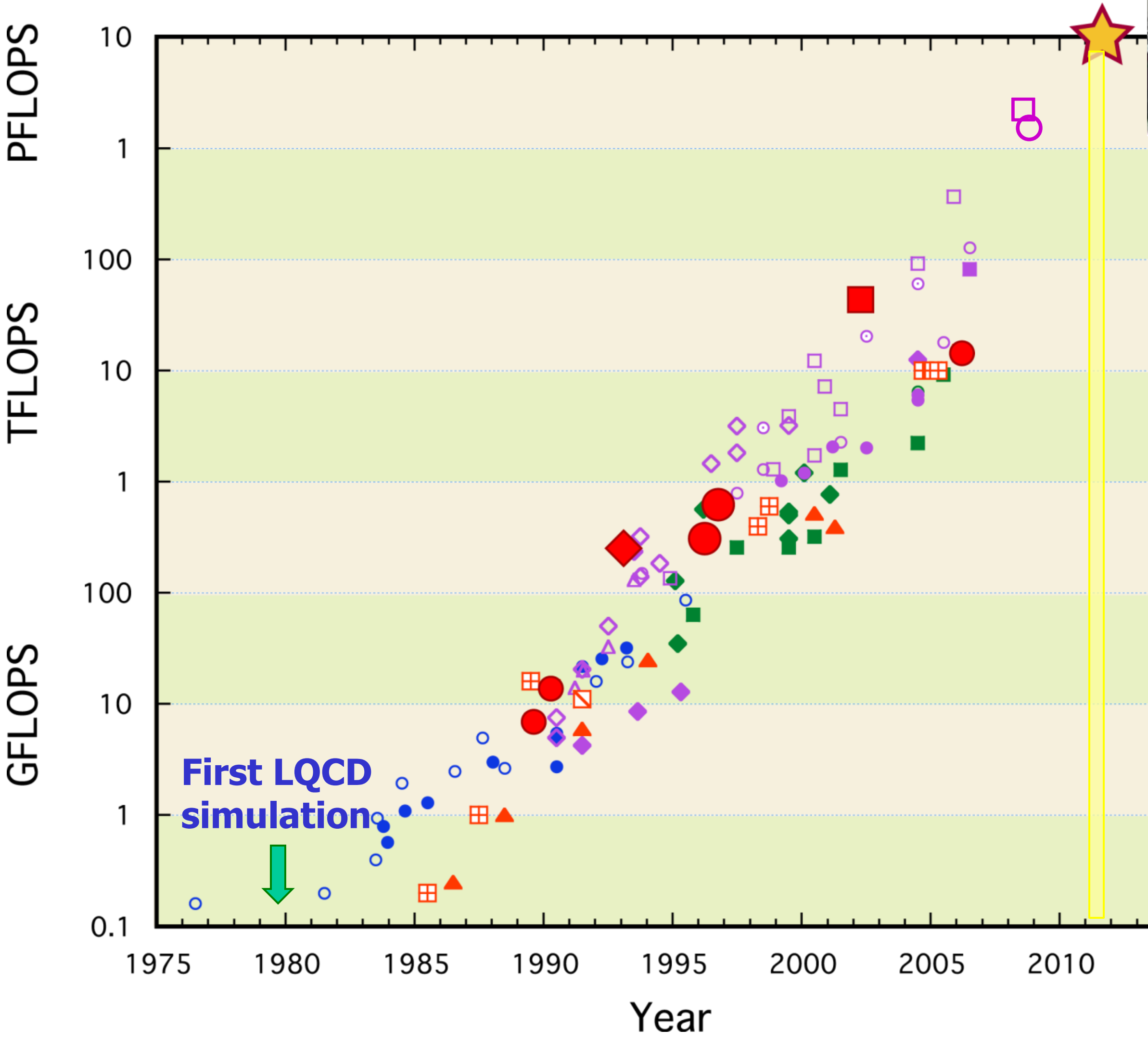
- cold plasma
- phenomena far from equilibrium

$$Z = \int [dU][dqd\bar{q}] \;\exp\left[-\int d\tau d^3x \mathcal{L}_{\mathrm{E}} \right]$$

Monte Carlo method

Observable
=O(g, m, a, L)

Supercomputer peak performance



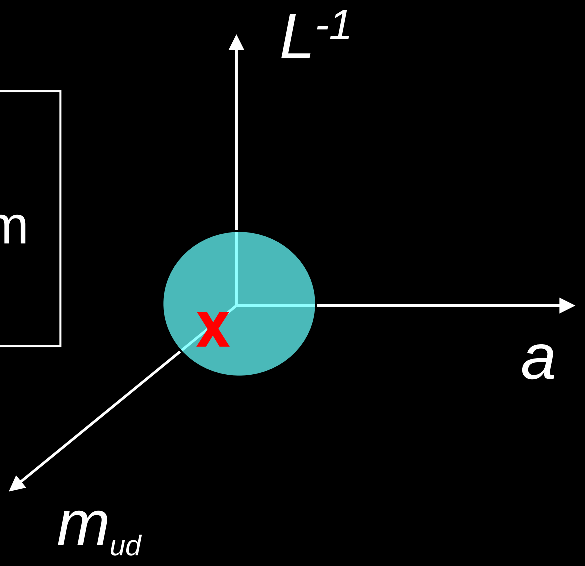
First LQCD simulation

10PFlops K computer (RIKEN)

original plot by A. Ukawa

Three Important limits

$L^{-1} \rightarrow 0$ (thermodynamics limit) : finite size scaling
$a \rightarrow 0$ (continuum limit) : asymptotic freedom
$m \rightarrow 0$ (chiral limit) : chiral pert. theory



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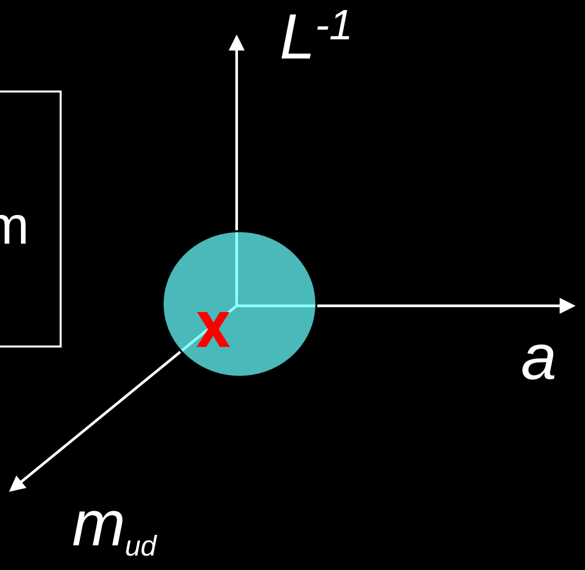


“Techniques”

Fermions:

Staggered, Wilson, Domain-wall, Overlap

different ways of handling chiral symmetry

Improved actions:

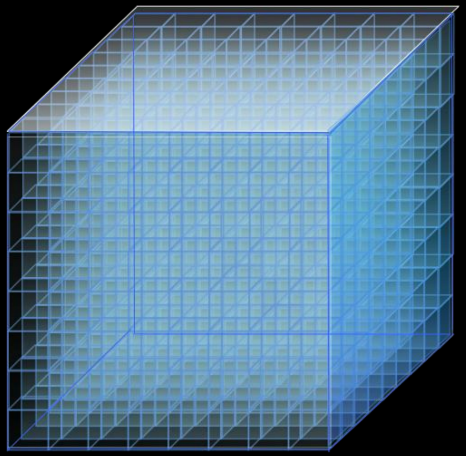
stout, HEX, asktad, HISQ, clover, ….

different ways of reducing the discretization error

Advanced algorithms:

RHMC, DDHMC, LMA, ….

techniques to make the simulations fast and reliable



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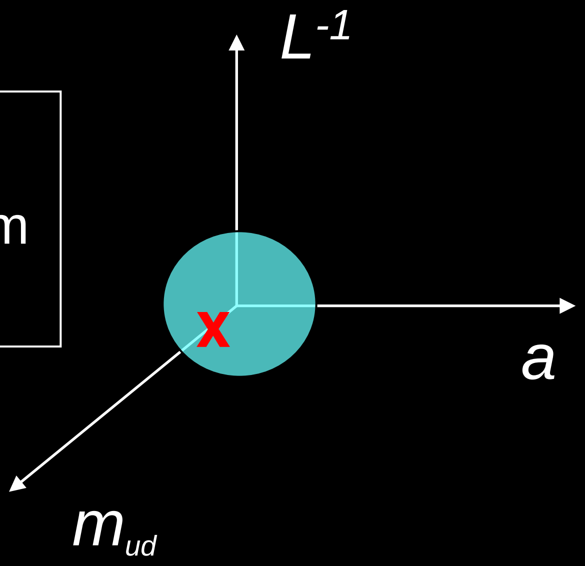

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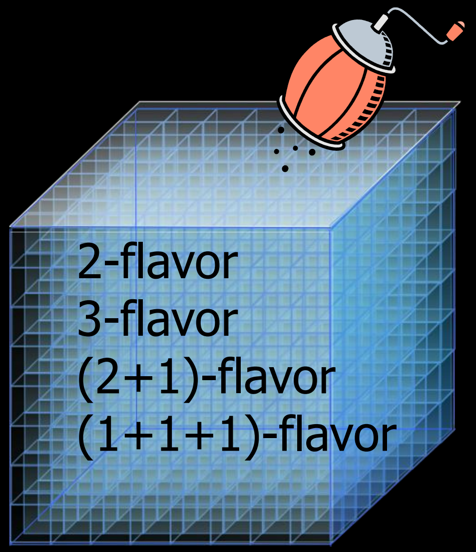
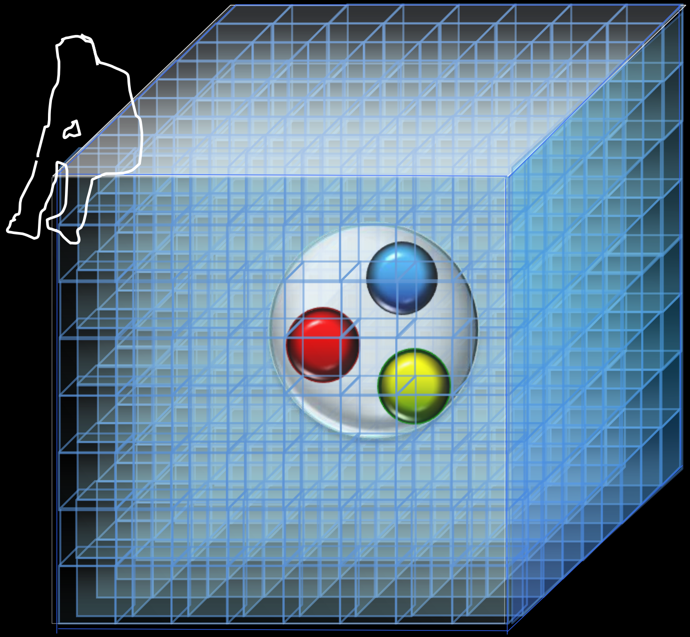
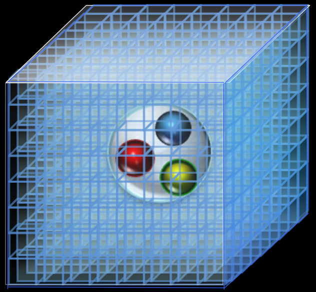


Precision Lattice QCD



Hadron masses @ 2009

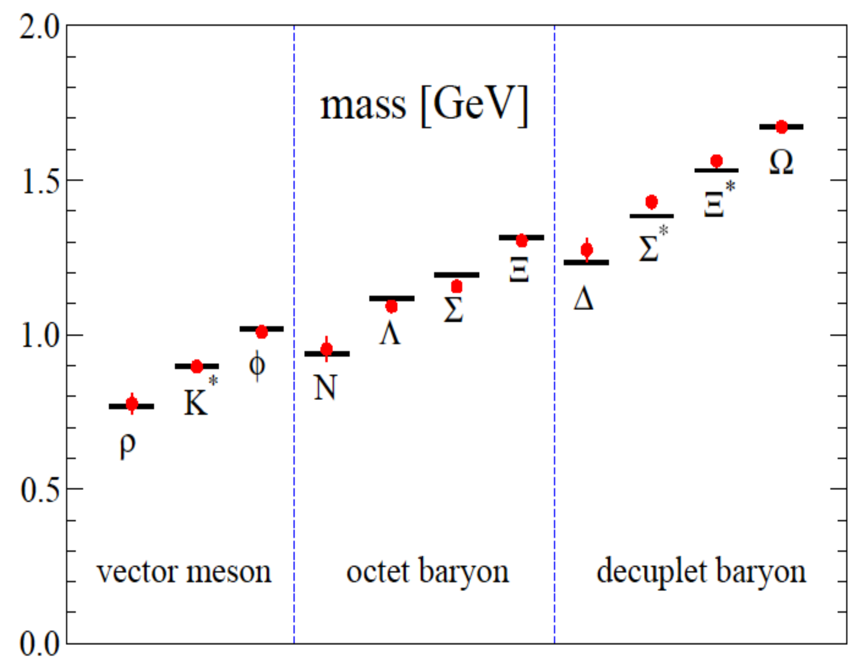


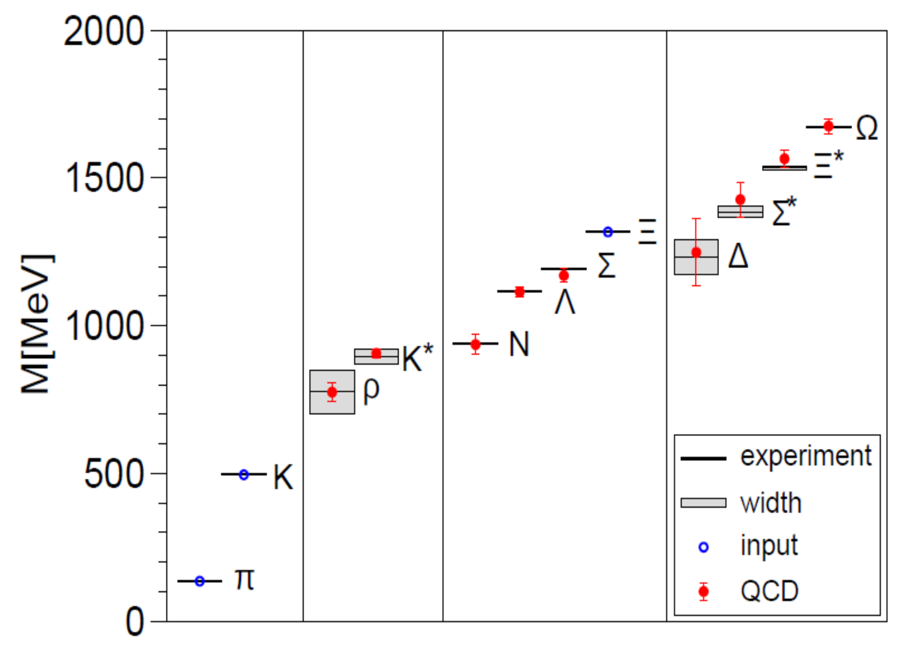
PACS-CS Collaboration,
Phys.Rev.D79(2009)034503

(2+1)-flavor, Wilson
L =2.9 fm, a =0.09 fm
m_π(min) =156 MeV

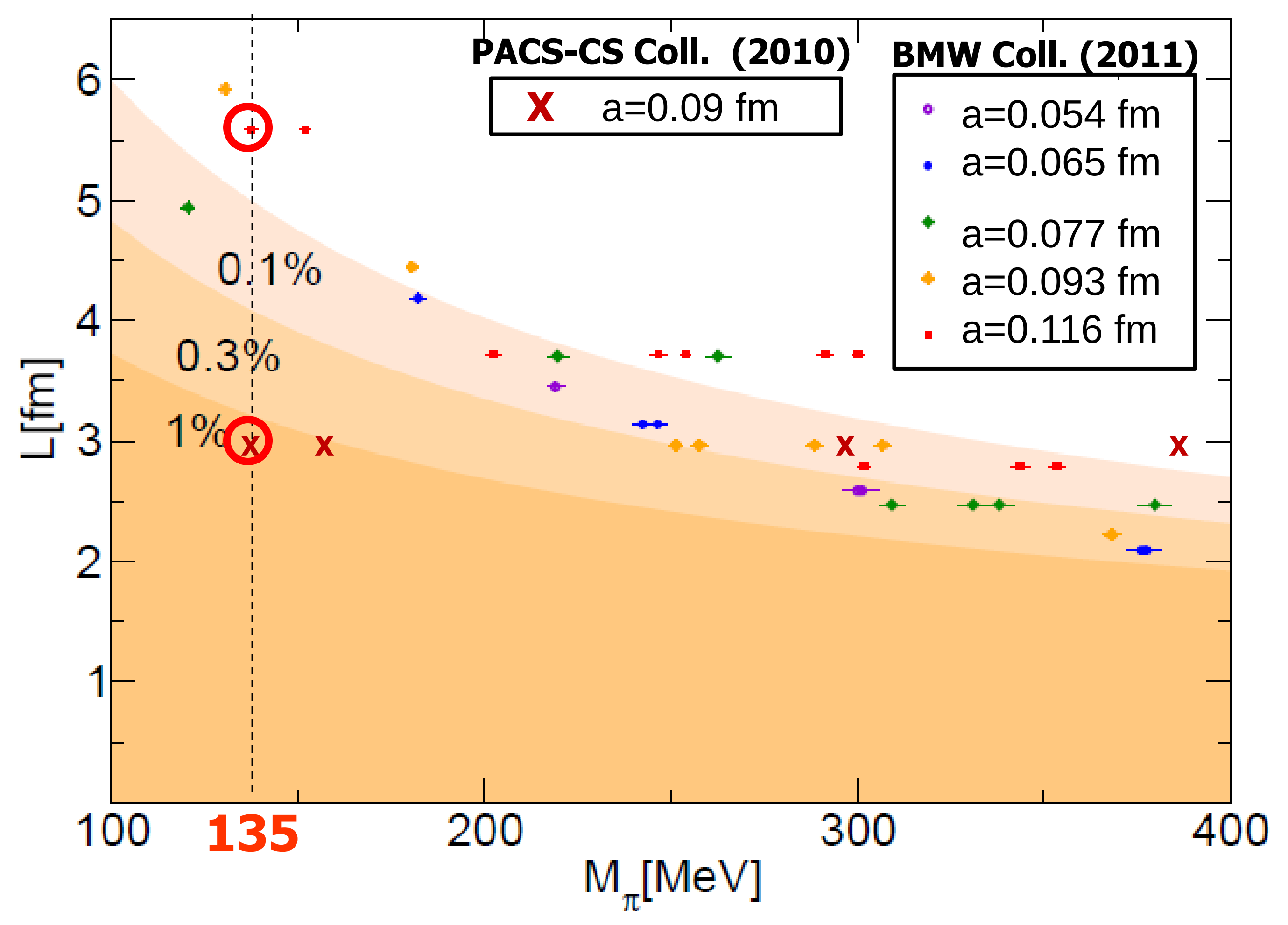
BMW Collaboration,
Science 322 (2008) 1224.

(2+1)-flavor, Wilson
L =(2.0- 4.1) fm, a =0.065, 0.085, 0.125 fm
m_π(min) =190 MeV



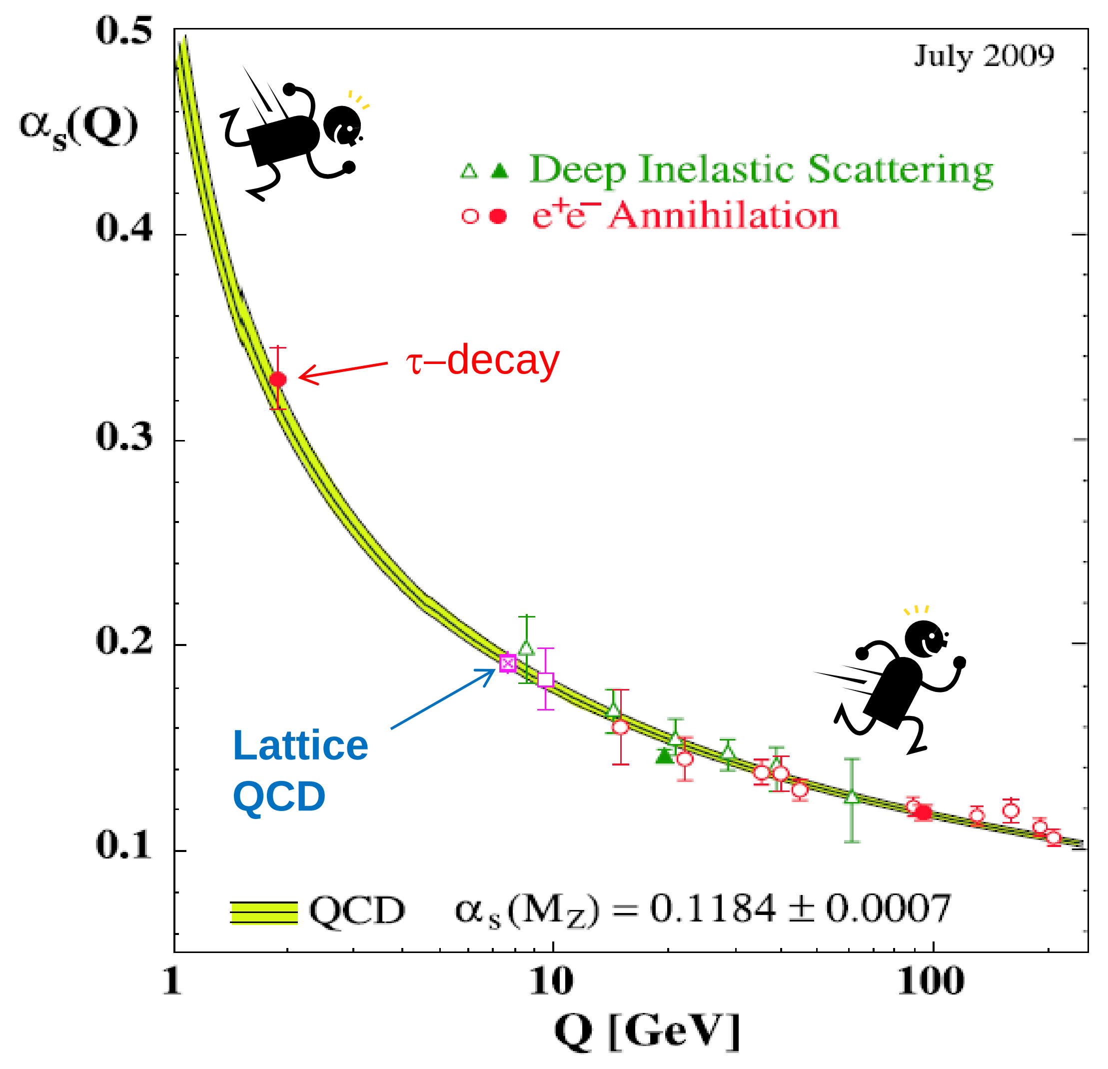


Hadron masses @ 2011
Physical point simulations in (2+1)-flavor QCD



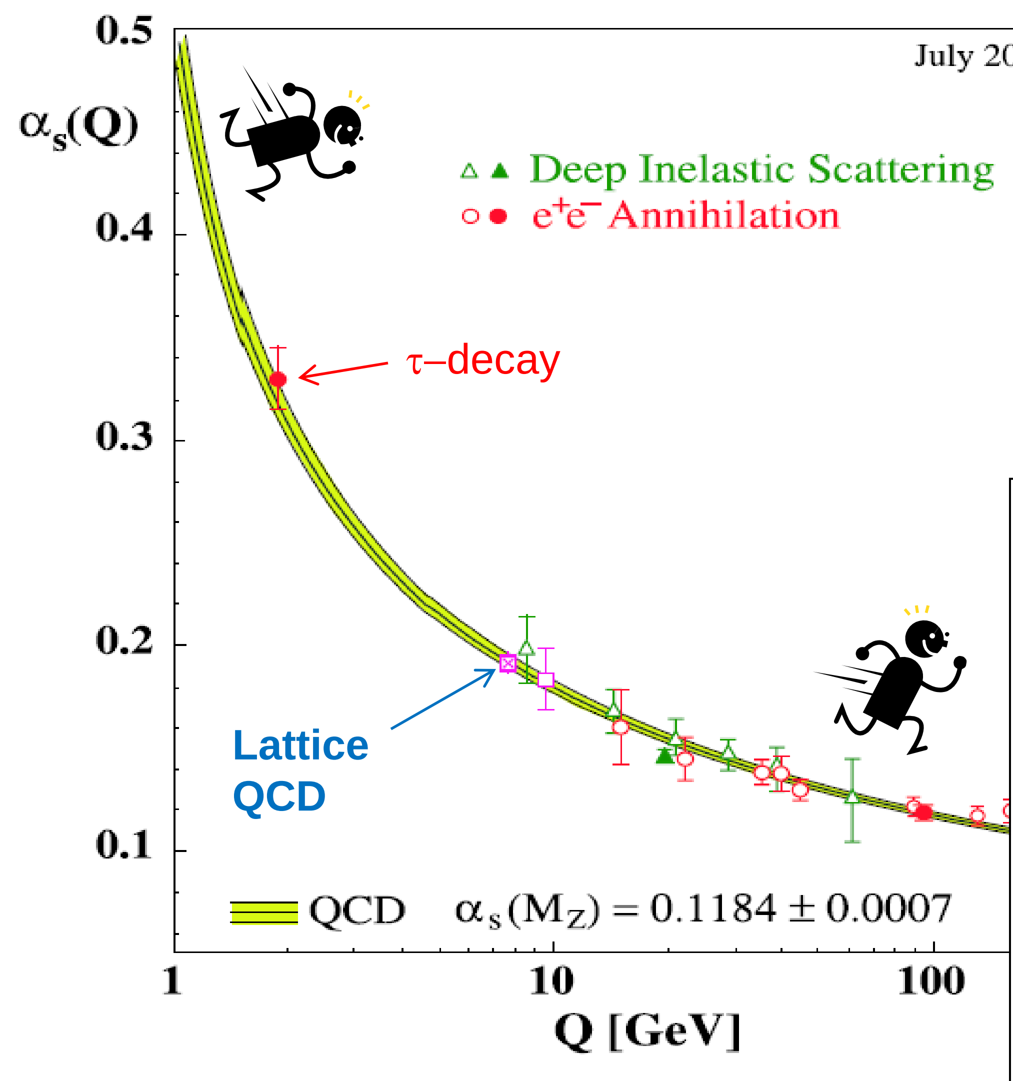
PACS-CS Coll.: Phys. Rev.D81 (2010) 074503
BMW Coll.: Phys. Lett. B701 (2011) 265

QCD running coupling



Bethke, Eur. Phys. J C(2009)64:689

QCD running coupling

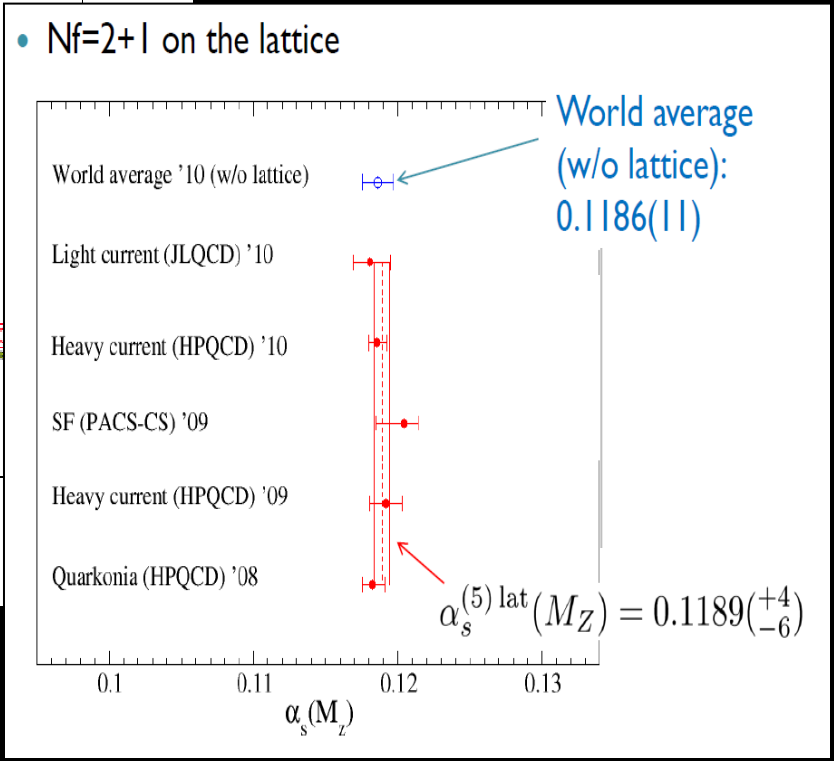


τ–decay

Lattice QCD

Bethke, Eur. Phys. J C(2009)64:689

Shintani @Lattice2011



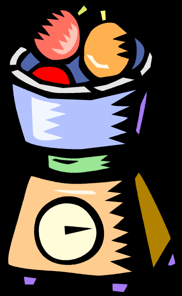
- Nf=2+1 on the lattice

World average (w/o lattice): 0.1186(11)

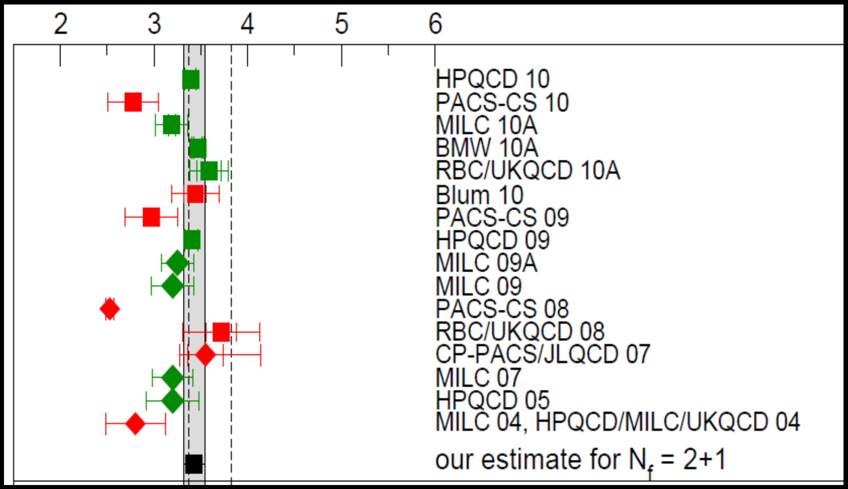
$$\alpha_s^{(5)\,\mathrm{lat}}(M_Z) = 0.1189(^{+4}_{-6})$$

Light quark masses (MSbar, @2GeV)

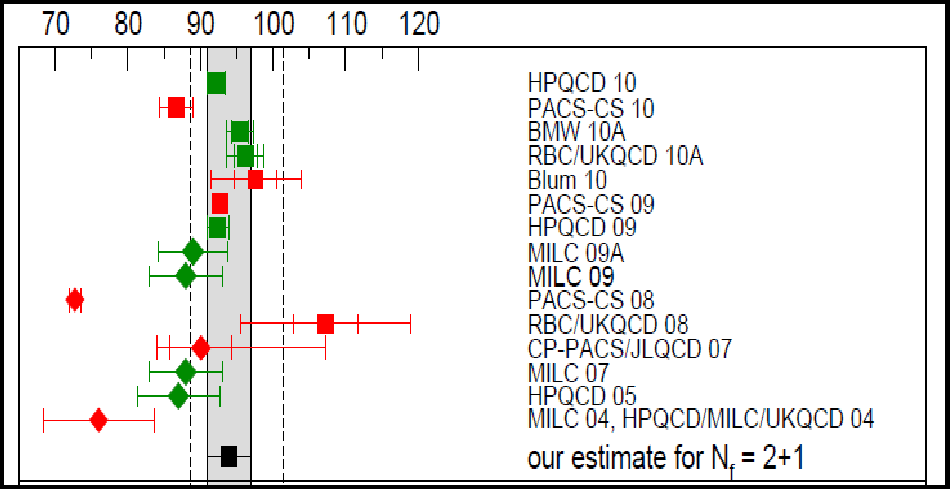
Summary by
FLAG working group
arXiv:1011.4408[hep-lat]



m_{ud} [MeV]



m_s [MeV]



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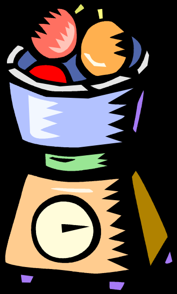
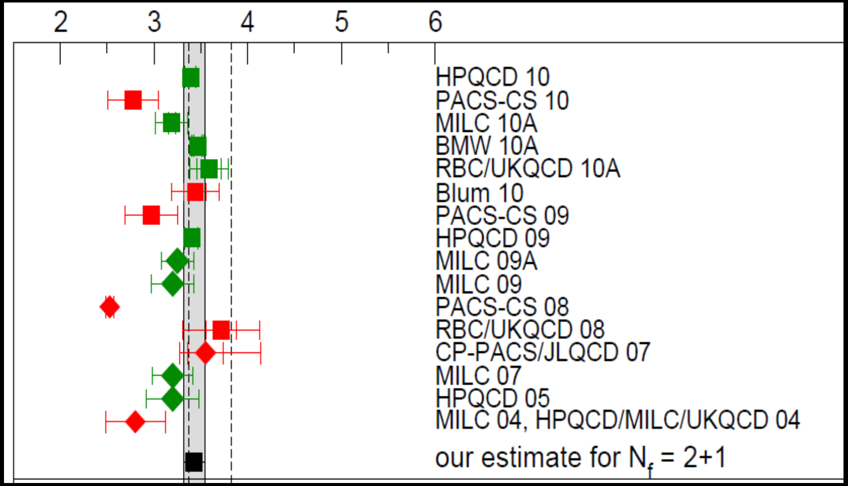
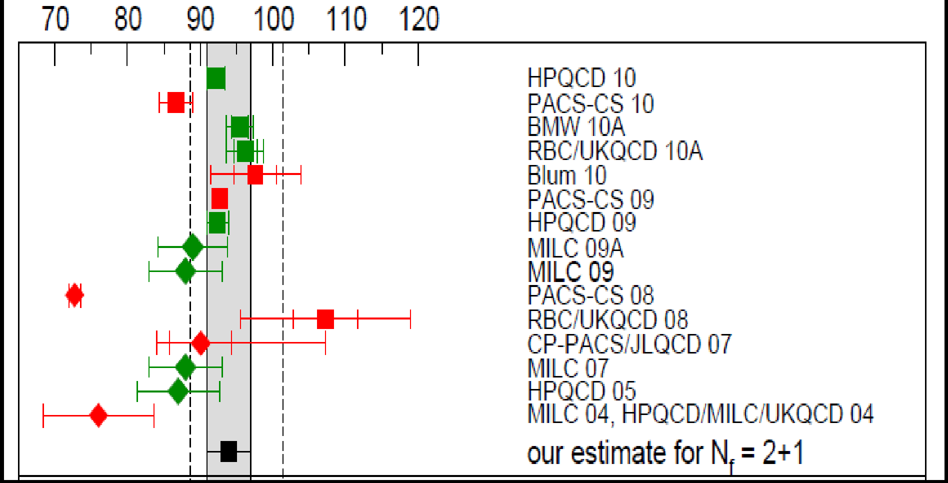


m_{ud} [MeV]



m_s [MeV]

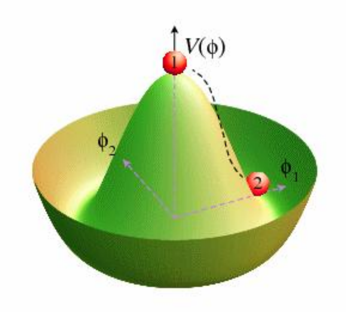


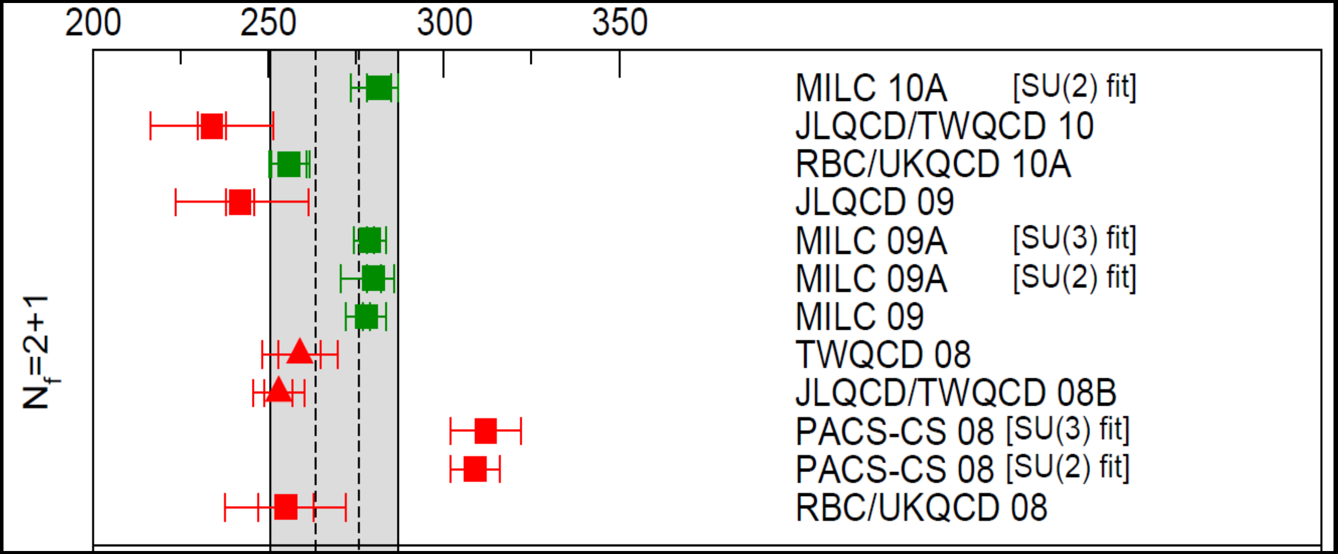
QCD(simulation)+QED(estimate)

N_f	m_u [MeV]	m_d [MeV]	m_{ud} [MeV]	m_s [MeV]	m_s/m_{ud}
2+1	2.19(15)	4.67(20)	3.42(11)	94(3)	27.4(4)

QCD+QED simulation has also been started
Blum et al., Phys. Rev. D82 (2010) 094508

Low energy constants



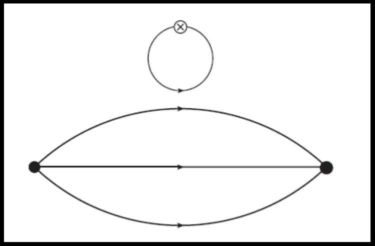


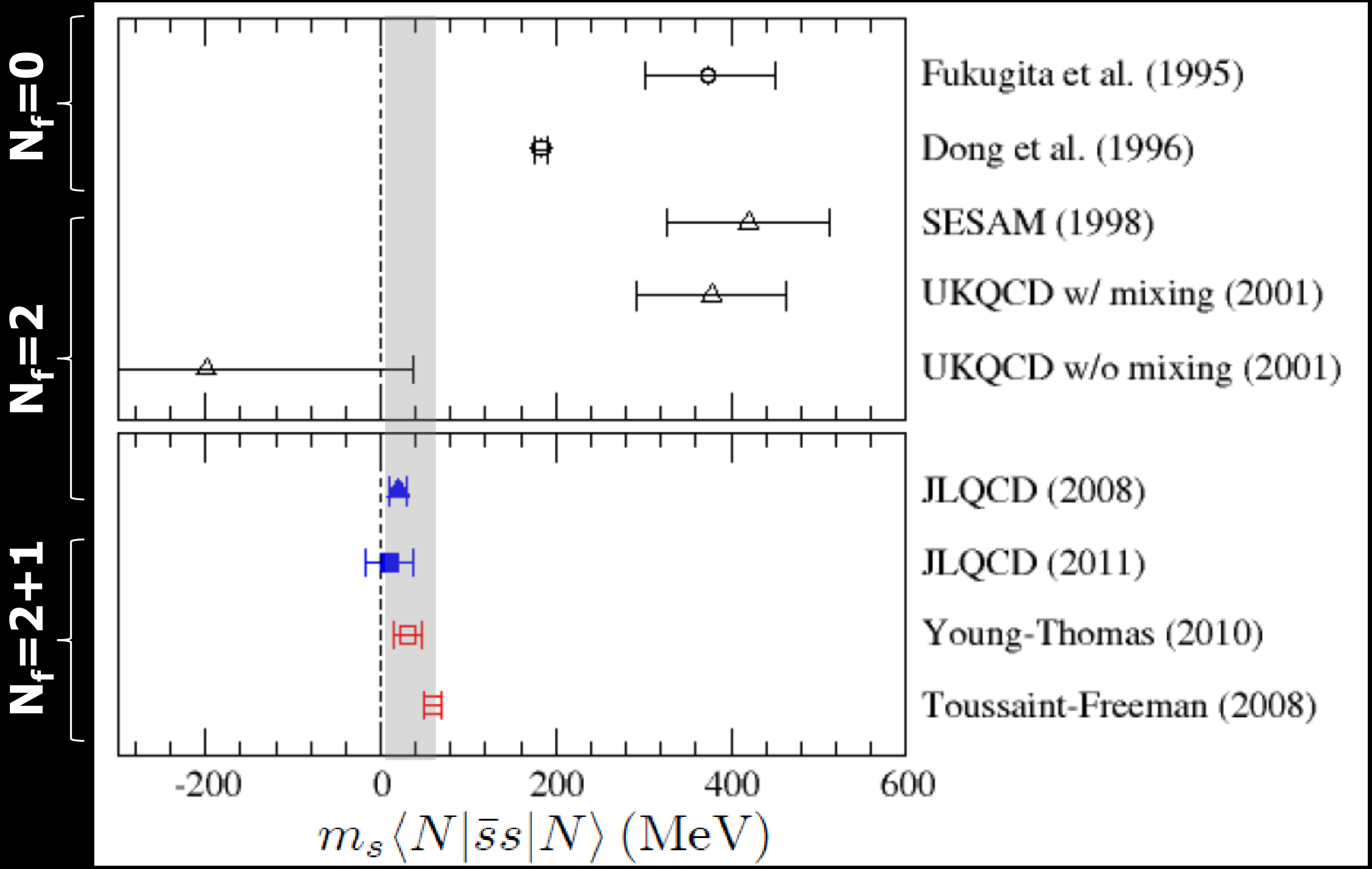
$$\boxed{[-\langle \bar{q}q\rangle_{2\mathrm{GeV}}]^{1/3} = (250 - 275)[\mathrm{MeV}]}$$

$$L_{4-8}, f_+(0), f_K/f_\pi, B_K, \mathrm{etc}$$

More in arXiv:1011.4408 [hep-lat]
(FLAG working group)

strangeness content of the proton

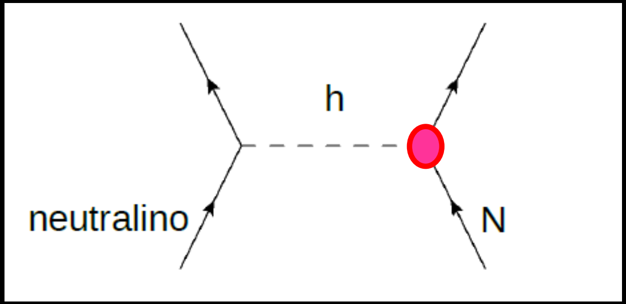




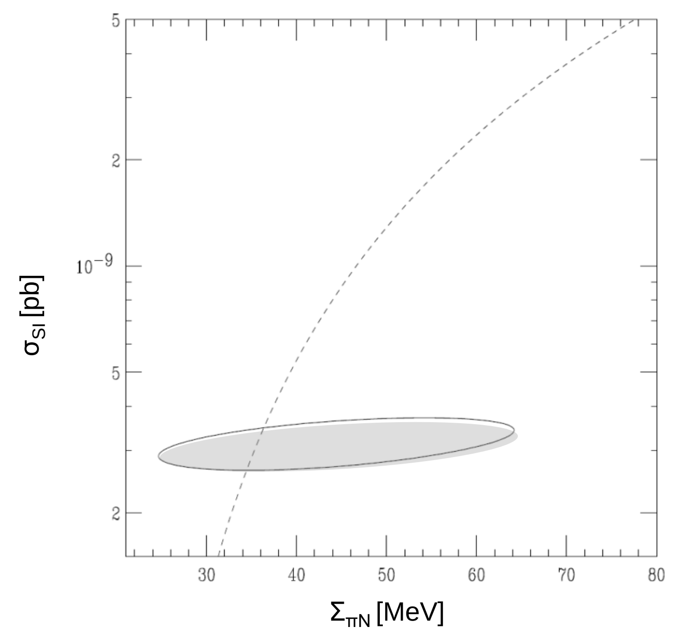
Takeda [JLQCD Coll.], Phys.Rev. D83 (2011) 114506

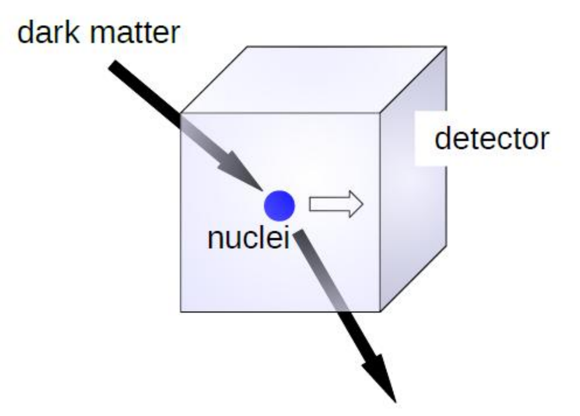
$$m_s\langle N|\bar{s}s|N\rangle < 60\ \mathrm{MeV} \quad \Leftrightarrow y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle} < 0.05$$

DM – Nucleon Interaction

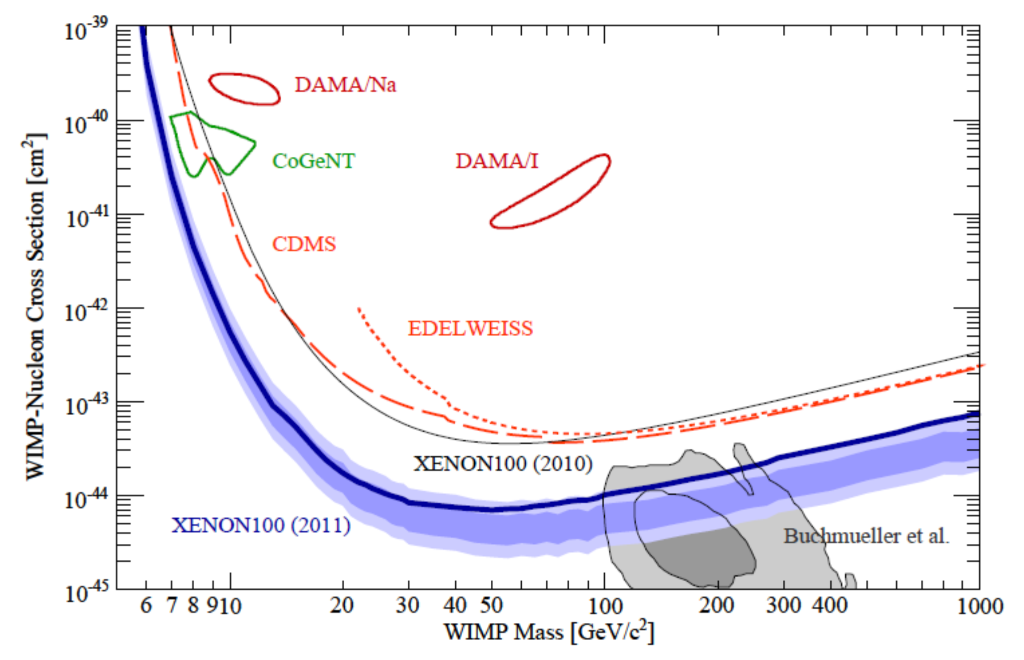


Giedt, Thomas, Young
Phys. Rev. Lett. 103 (2009) 201802

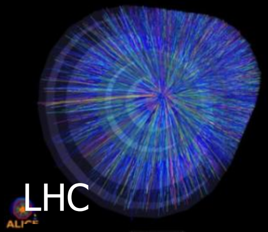




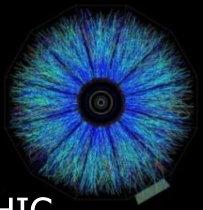
XENON100 Coll.:
Phys.Rev.Lett. 105 (2010) 131302
arXiv:1104.2549 [astro-ph.CO]
arXiv:1107.2155 [astro-ph.IM]



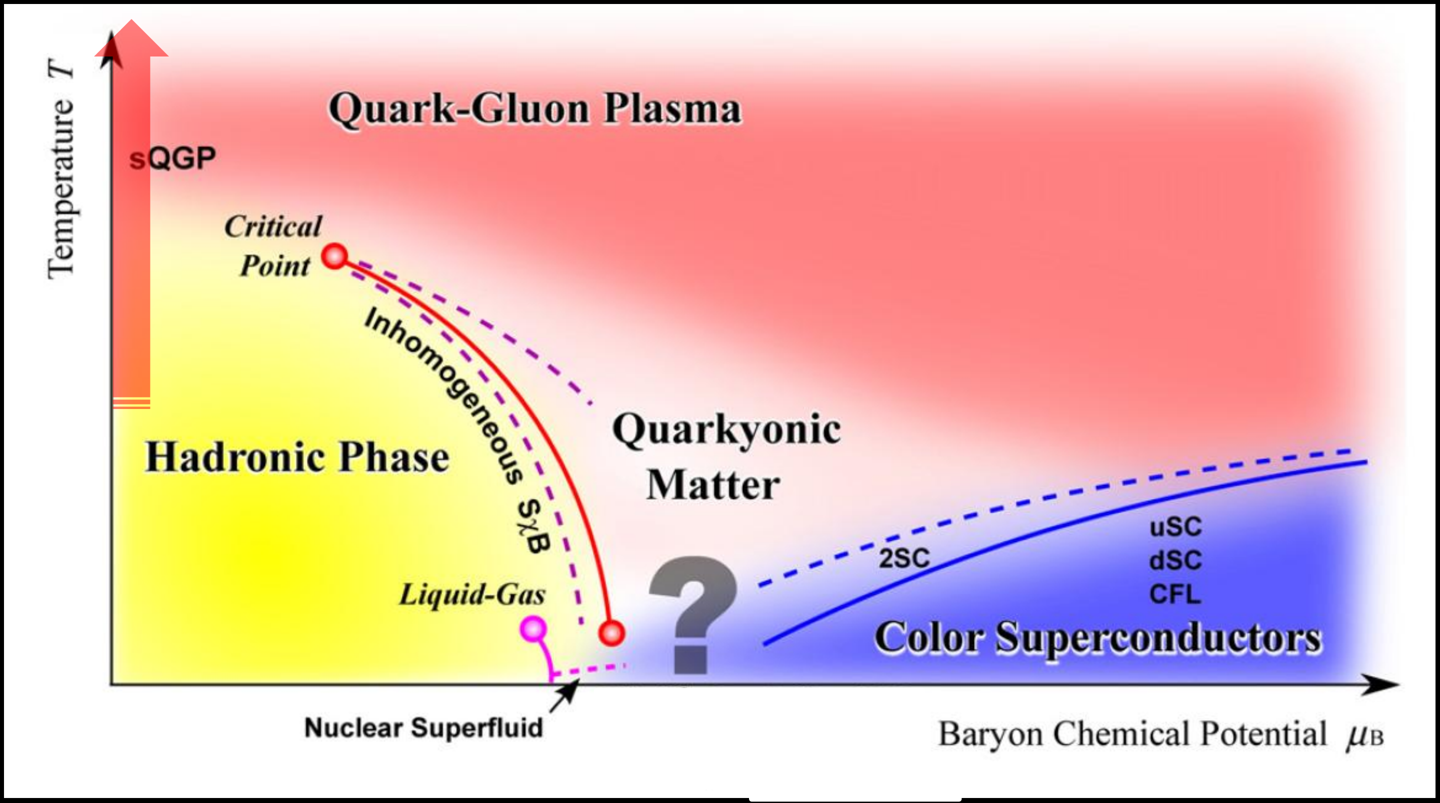
Thermal Lattice QCD



LHC



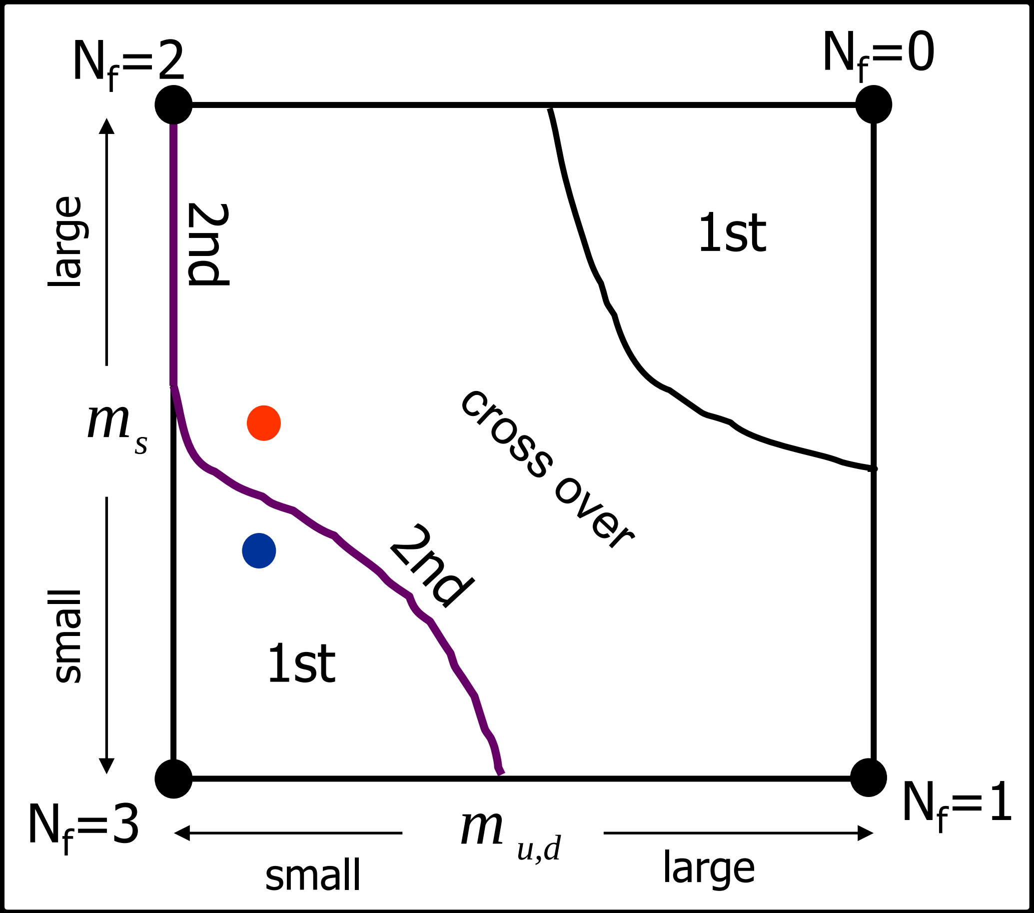
RHIC

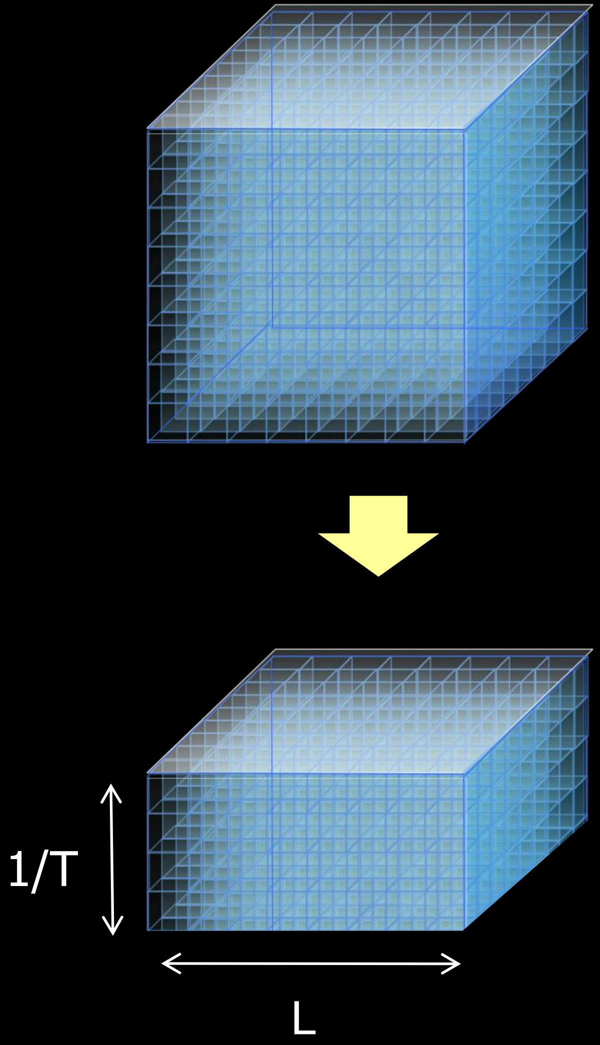


Fukushima and Hatsuda,
Rep. Prog. Phys.74 (2011)014001

Thermal QCD transition at μ=0

Columbia plot





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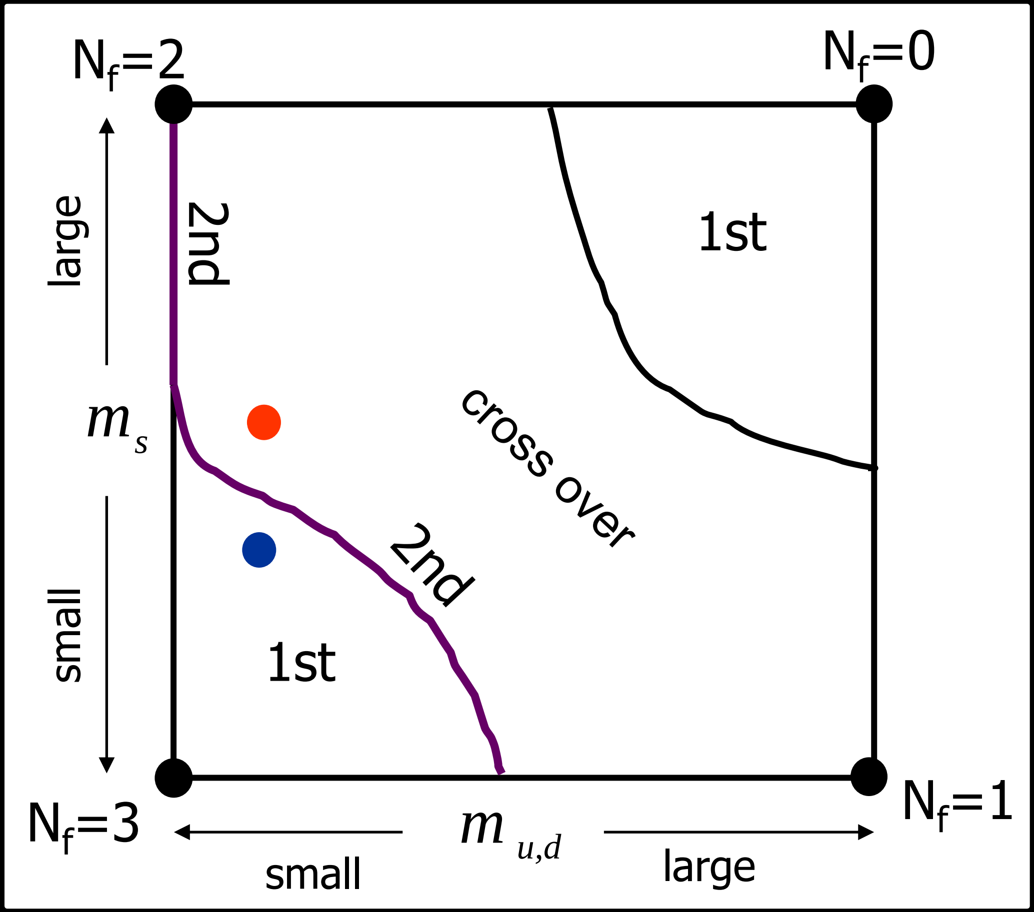
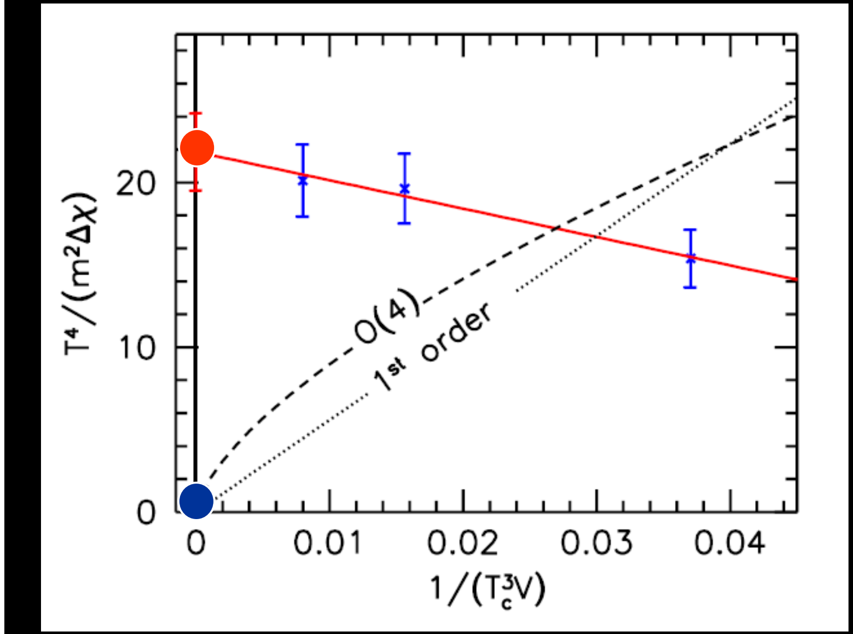


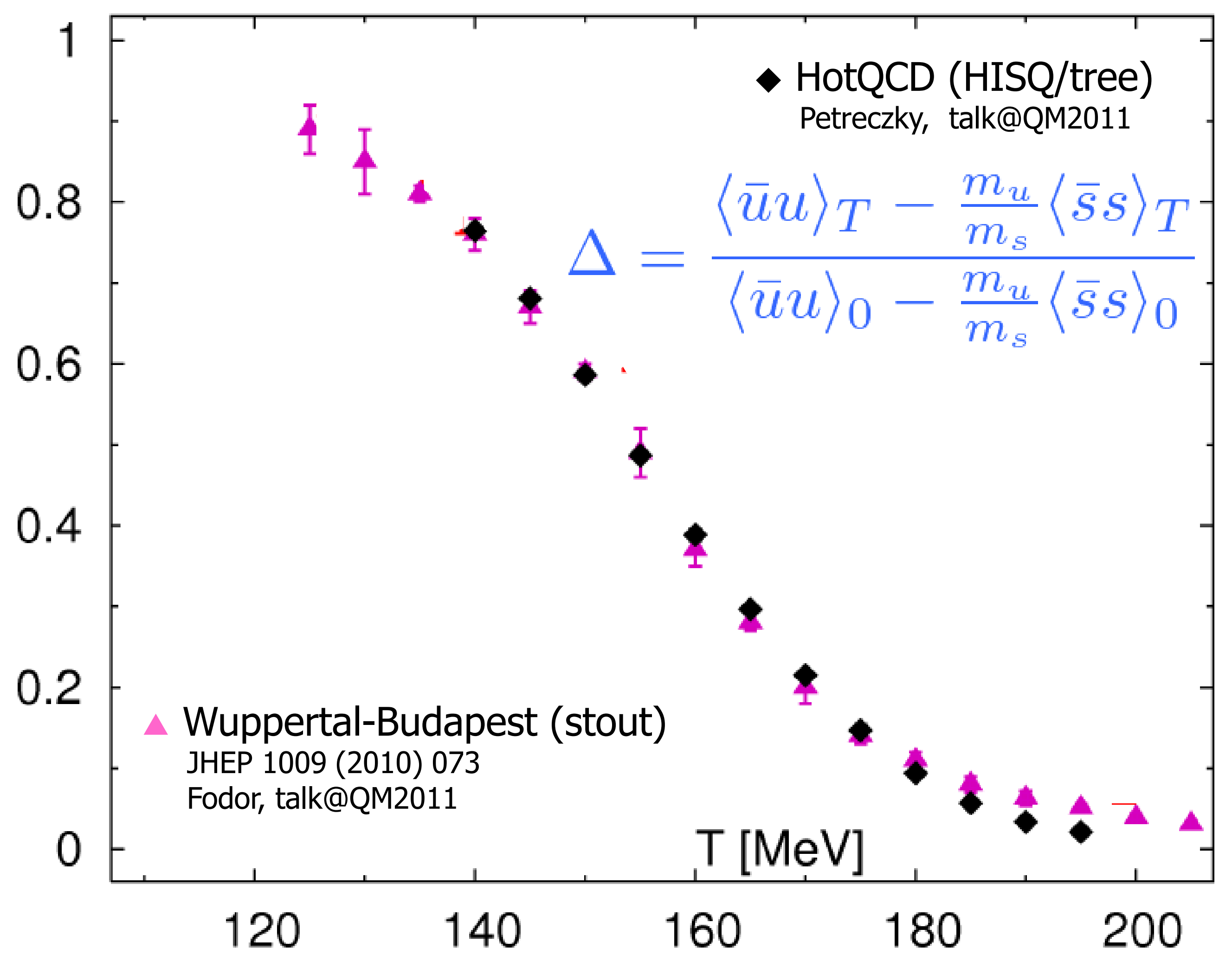
Finite size scaling

$$\chi_m = \frac{\partial^2 P}{\partial m_{\mathrm{ud}}^2} \sim \begin{cases} V & \text{1st order} \\ V^{2/3} & \text{2nd order} \\ V^0 & \text{crossover} \end{cases}$$



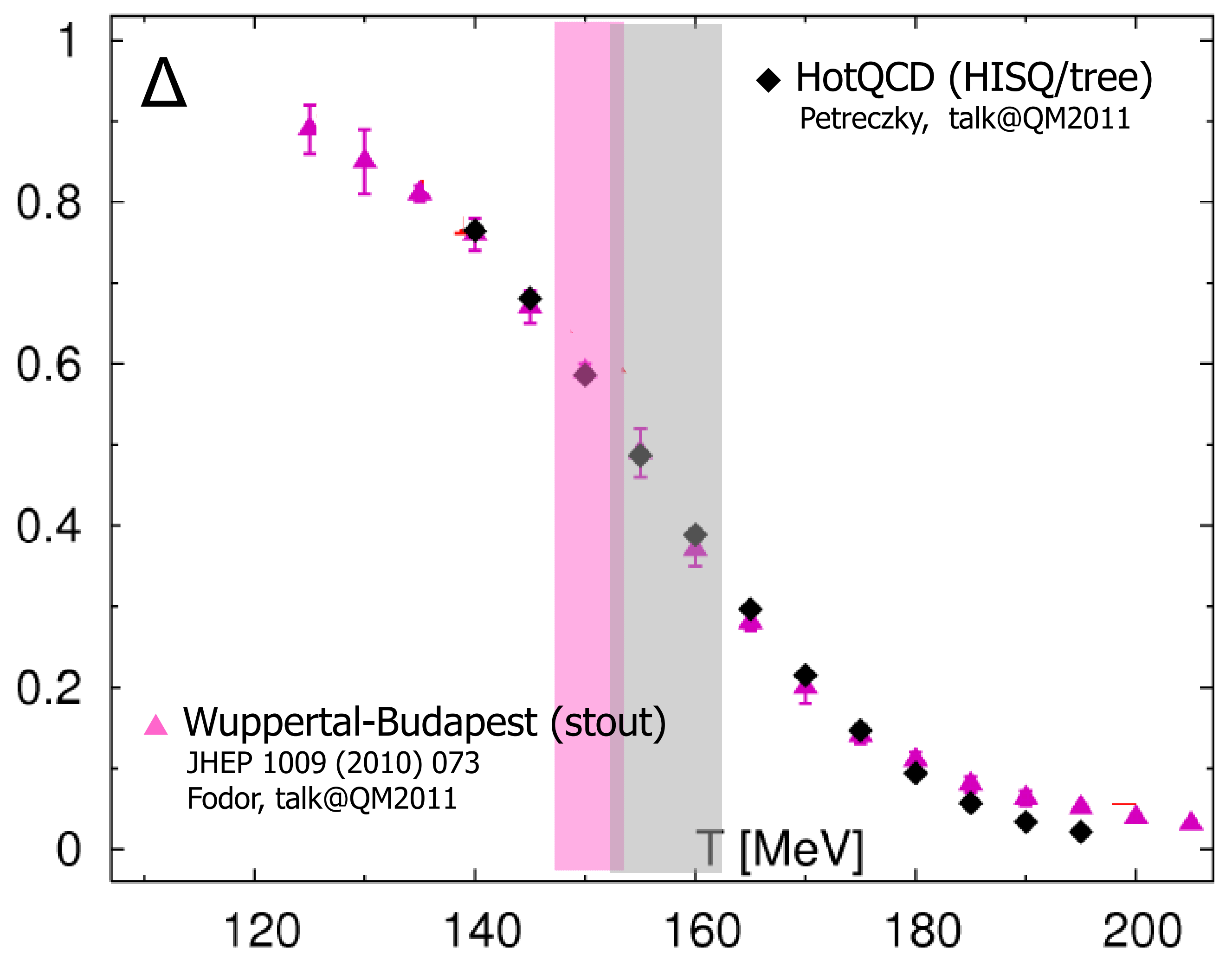
Budapest group, Nature 443 (2006) 675
Staggered, (2+1)-flavor, physical mass

Thermal chiral condensate at m_π=135 MeV



$$\Delta = \frac{\langle \bar{u}u\rangle_T - \frac{m_u}{m_s}\langle \bar{s}s\rangle_T}{\langle \bar{u}u\rangle_0 - \frac{m_u}{m_s}\langle \bar{s}s\rangle_0}$$

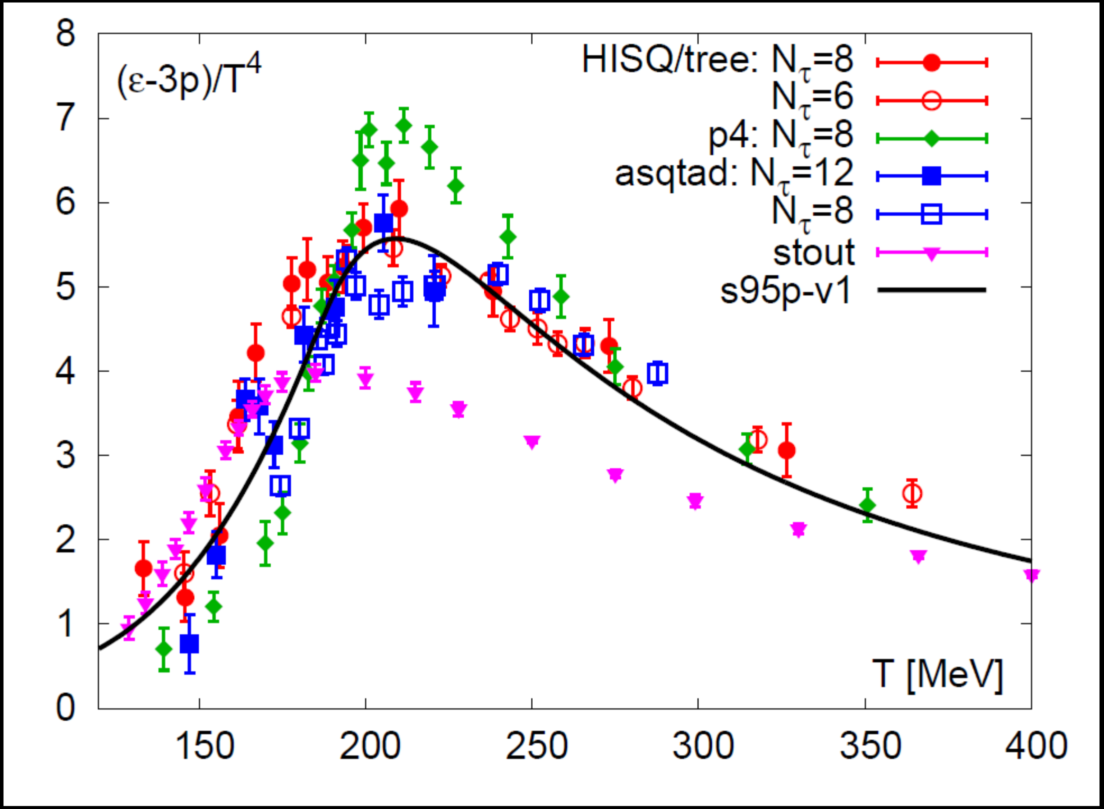
Pseudo-critical temperature T_{pc}



Chiral susceptibility peak (max. fluctuation) ⇒ T_{pc}=150-160MeV

Equation of state (EOS) : p(T), ε(T)

$$\frac{p(T)}{T^4} - \frac{P(T_0)}{T_0^4} = \int_{T_0}^{T} \frac{dT'}{T'} \frac{\epsilon(T') - 3p(T')}{T'^4}$$

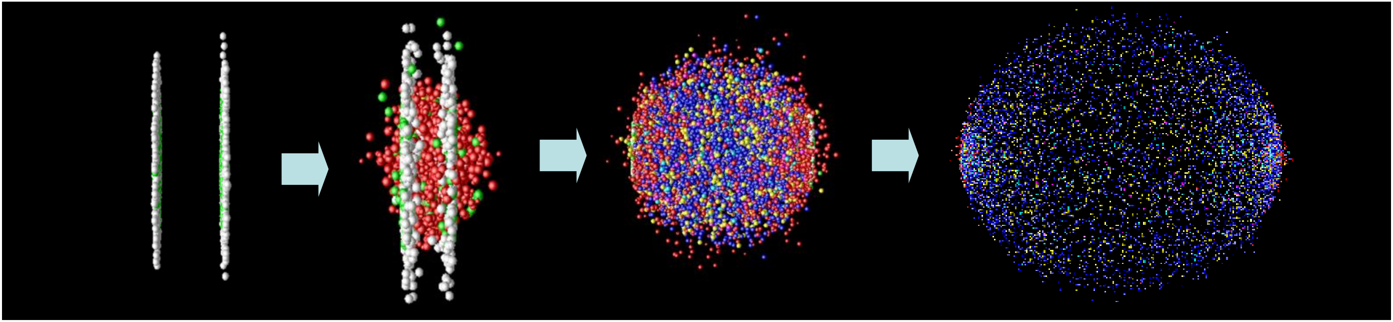


▼ Wuppertal-Budapest Coll.,
JHEP 1011 (2010) 77

● ● ■ HotQCD Coll.,
ArXiv: 1012.1257 [hep-lat]

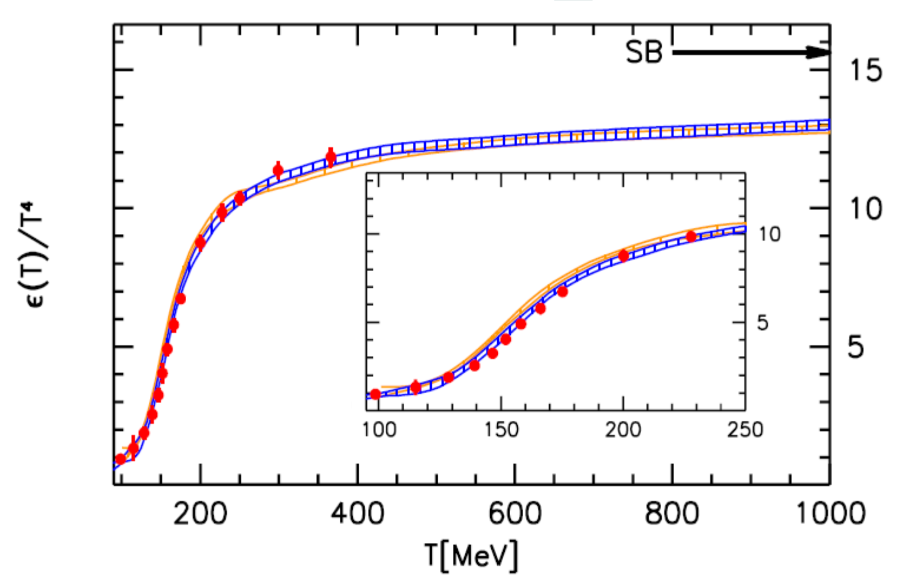
- Discrepancy exists among different improved staggered actions
- Hopefully resolved in a → 0 limit
- Need to be checked by Wilson, Domain-wall, Overlap

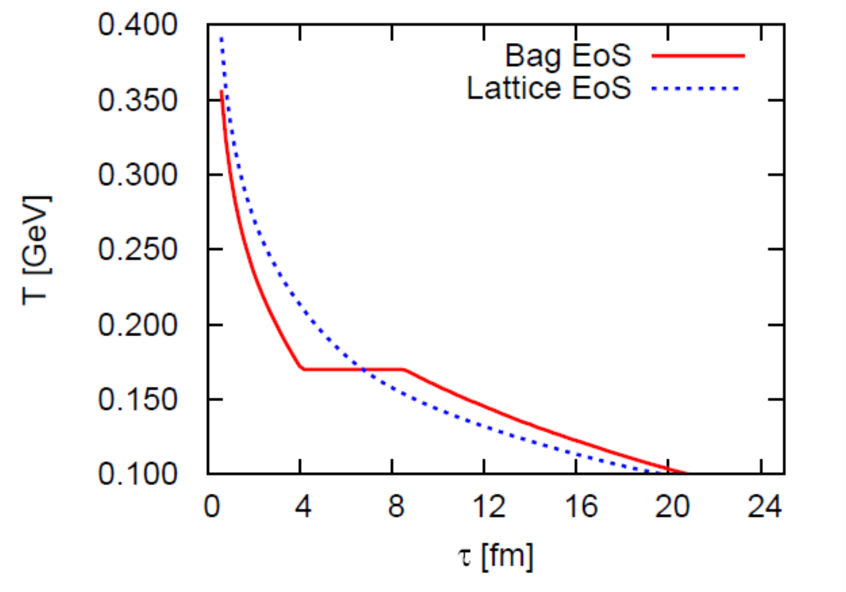
LQCD-EOS applied to RHIC



3D hydro.

$\partial_\mu T^{\mu\nu}(p, \varepsilon)=0$

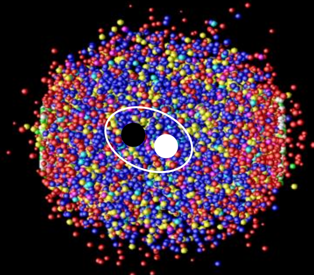




Wuppertal-Budapest's LQCD EOS
JHEP 1011 (2010) 77

Akamatsu, Hamagaki, Hirano, Hatsuda
arXiv:1107.36[nucl-th]

Heavy QQbar in QGP



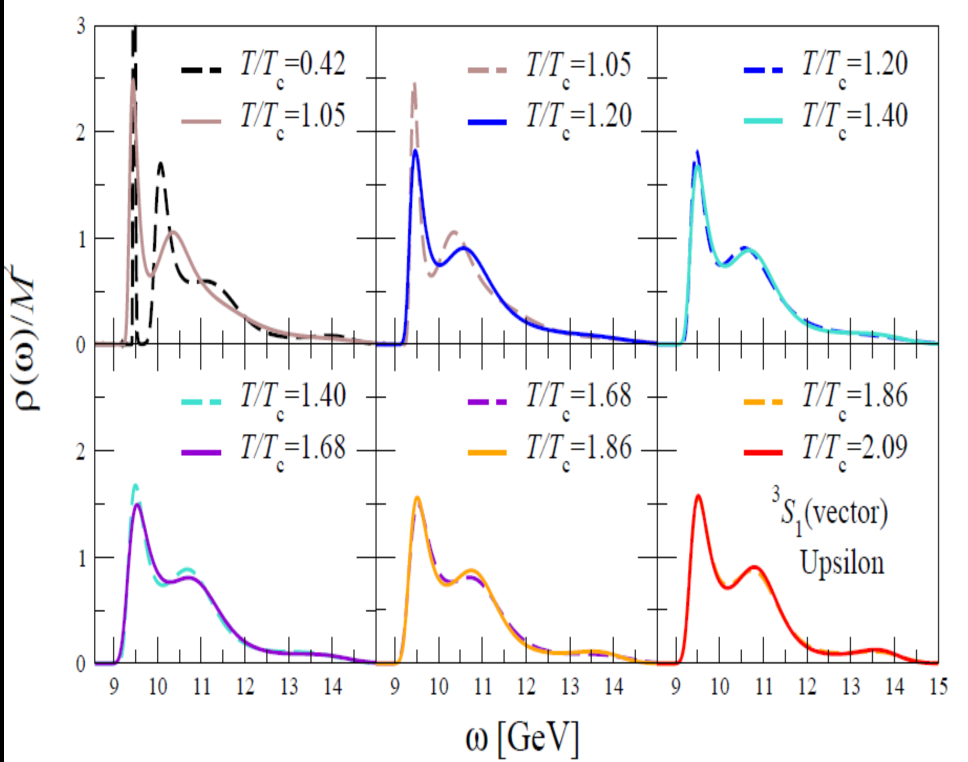
Matsui & Satz,
PLB (1986)

LQCD + Bayesian analysis ⇒ spectral function ⇒ dilepton rate

Asakawa, Nakahara, Hatsuda, Prog. Part. Nucl. Phys.46 (2001) 469

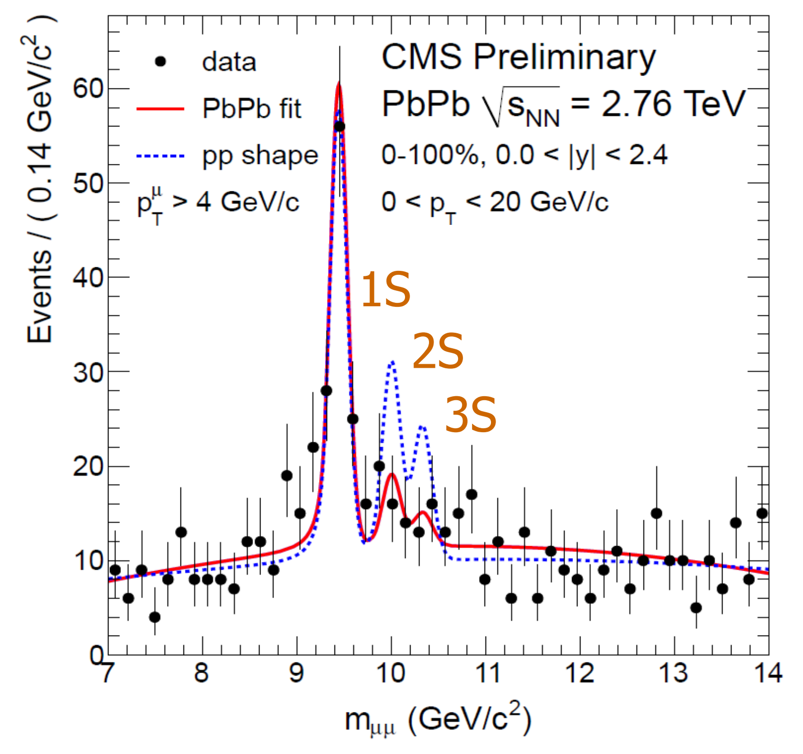
Υ spectral function on the lattice

$N_f=2$, $a_s=0.162$ fm, $\xi=6$, $L=1.94$ fm
Aarts et al., arXiv:1109.4496 [hep-lat]

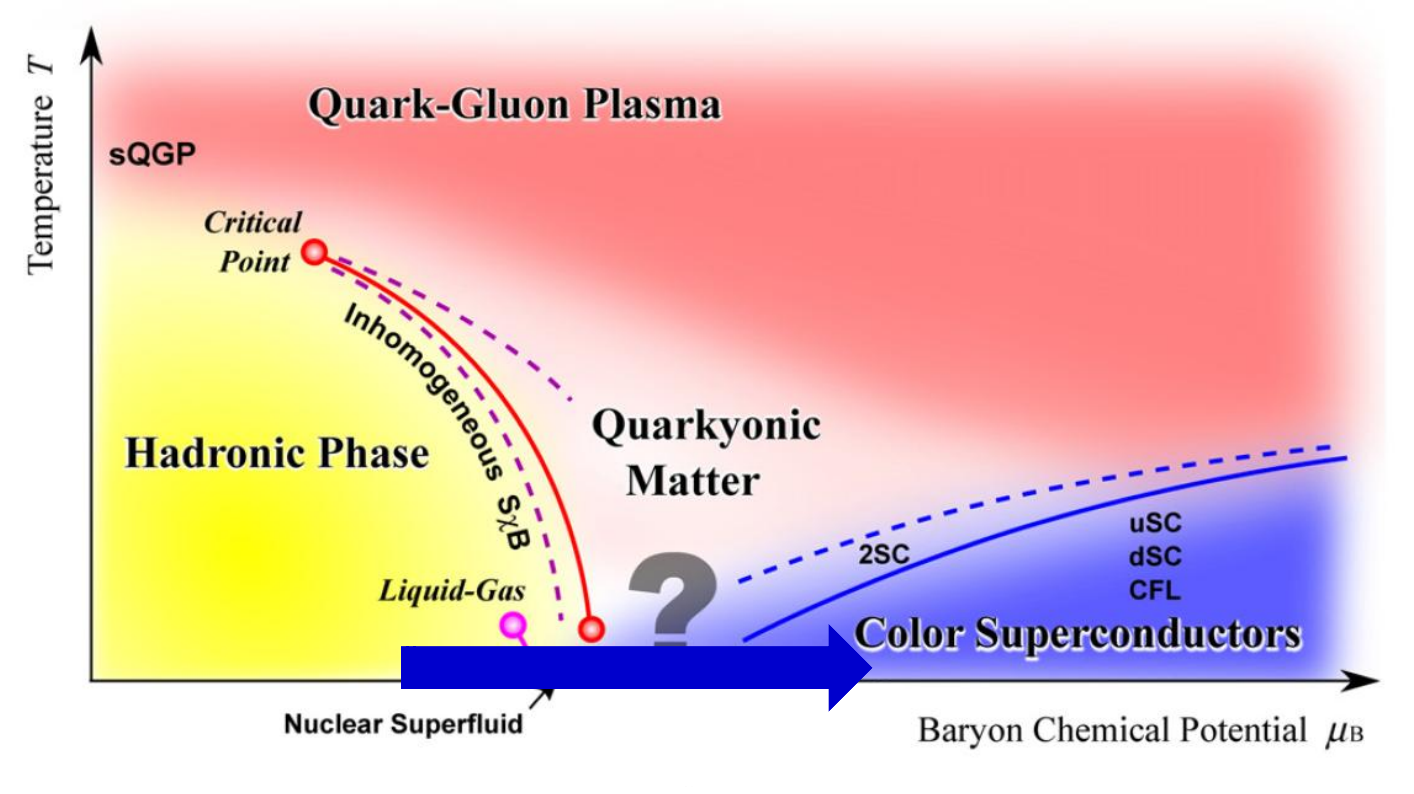


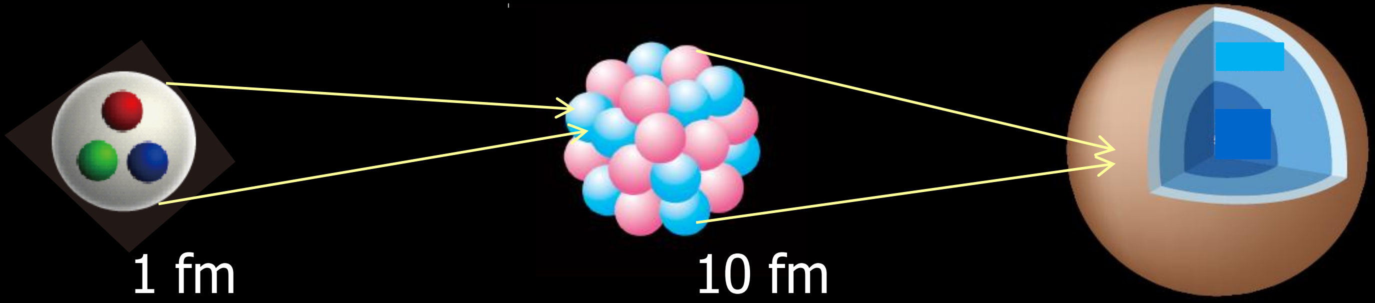
Υ in pp and PbPb collisions at LHC

CMS Coll., arXiv:1105.4894[nucl-ex]
Wyslouch (for CMS), arXiv:1107.2895[nucl-ex]



Nuclear Lattice QCD



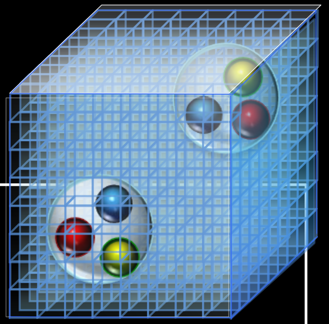


1 fm

10 fm

10 km

Nuclear Force from LQCD



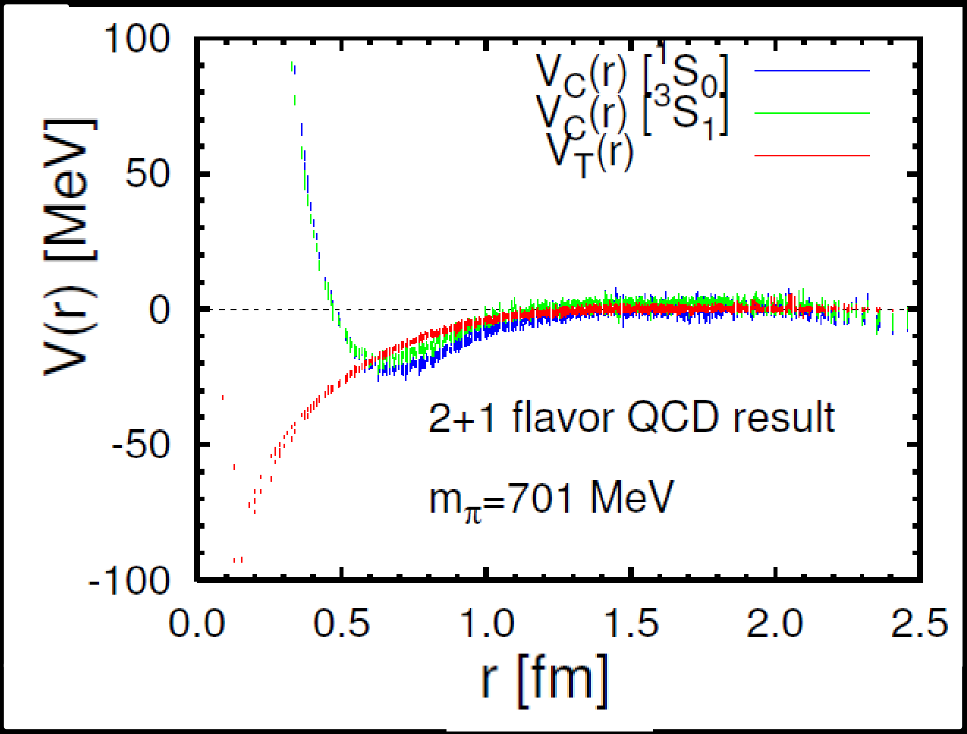
1. Low energy NN int. ⇔ NN potential

$$V(\vec{r},\nabla)=V_{\mathrm{C}}(r)+S_{12}V_{\mathrm{T}}(r)+\vec{\mathrm{L}}\cdot\vec{\mathrm{S}}\;V_{\mathrm{LS}}(r)+\{V_{\mathrm{D}}(r),\nabla^2\}+\cdots$$

2. NN potential from NN "wave function"

$$\phi(\vec{r})=\langle 0|N(\vec{x}+\vec{r})N(\vec{x})|6q\rangle$$

Ishii, Aoki, Hatusda,
Phys.Rev.Lett. 99 (2007) 022001



repulsive core

+ attractive well

+ tensor force

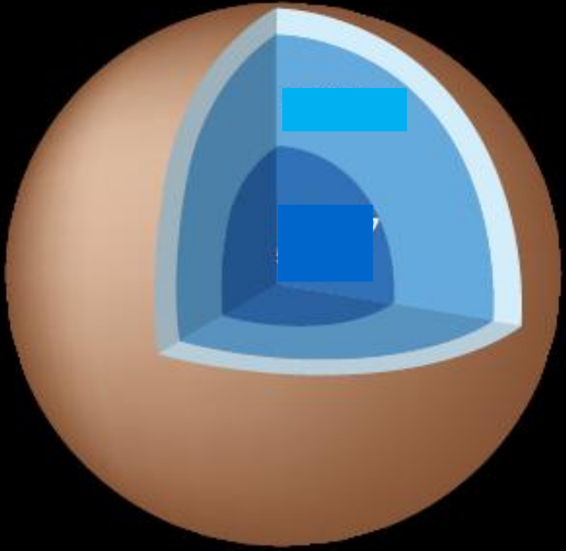
from LQCD

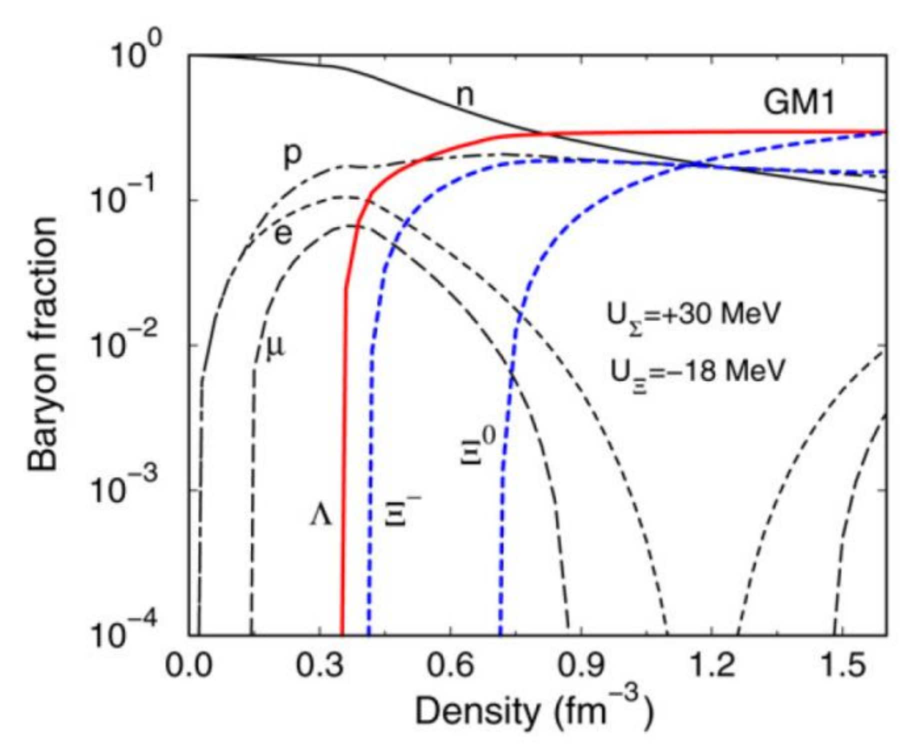
HAL QCD Coll., arXiv:1004.0405[hep-lat]

Inner core of neutron star

-- role of 2-body and 3-body forces --

Radius ∼ 10 km
Mass ∼ solar mass
Central density ∼ 10^{12} kg/cm^3



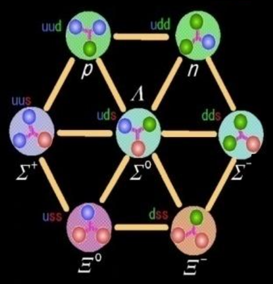


Schaffner-Bielich, Nucl. Phys.A 835, 279 (2010)

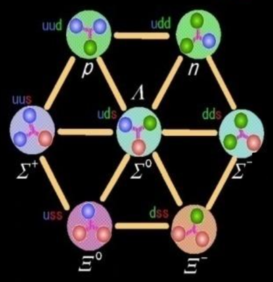
YN interaction ⇔ onset of hyperon mixture
NNN (BBB) interaction ⇔ large max mass (e.g. 1.97(4) $M_\odot$)

BB interactions in 3-flavor LQCD

1. Numerical experiments of YN & YY interactions
   (not easily accessible in laboratory experiments)
2. Physical origin of the short range NN repulsion
3. Fate of H-dibaryon



x

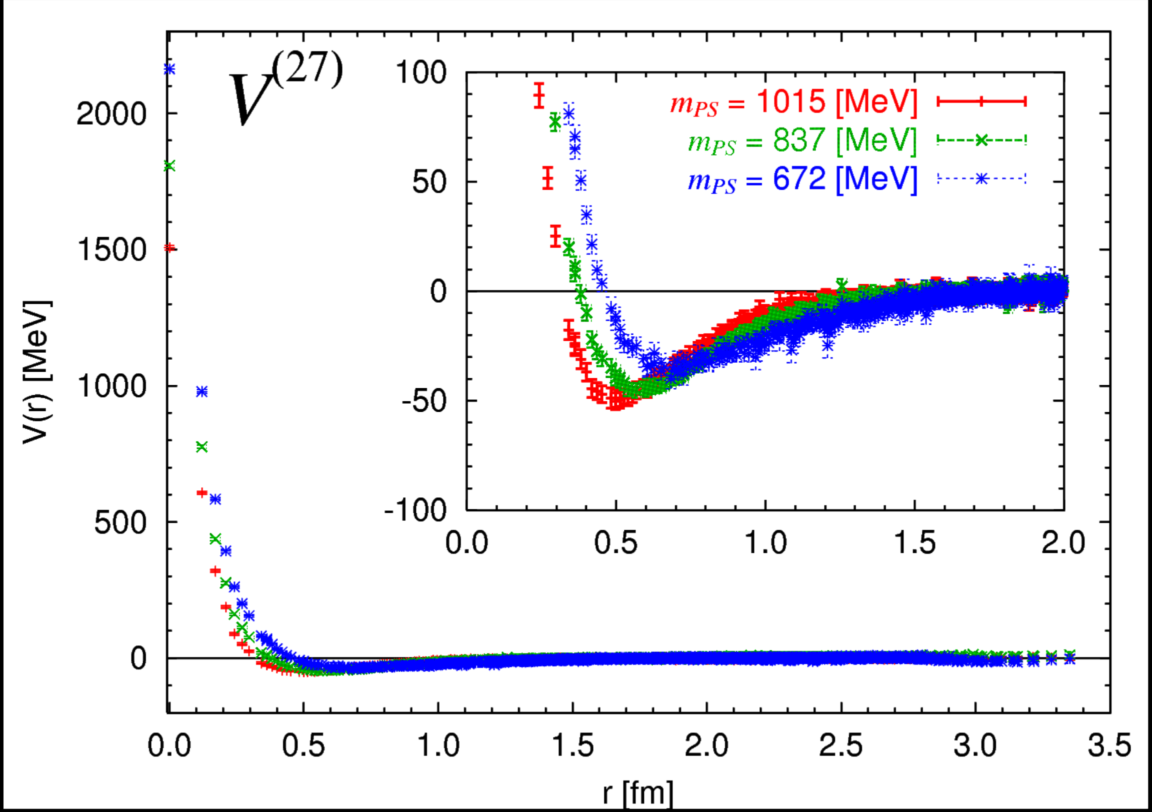


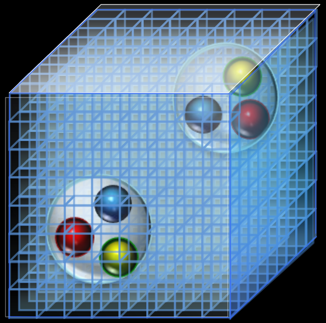
$$8 \times 8 = \underset{\text{Symmetric}}{\underline{27 + 8s + 1}} + \underset{\text{Anti-symmetric}}{\underline{10^* + 10 + 8a}}$$

Six independent potentials in the flavor-basis

BB potentials in flavor-basis

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002





BB potentials in flavor-basis

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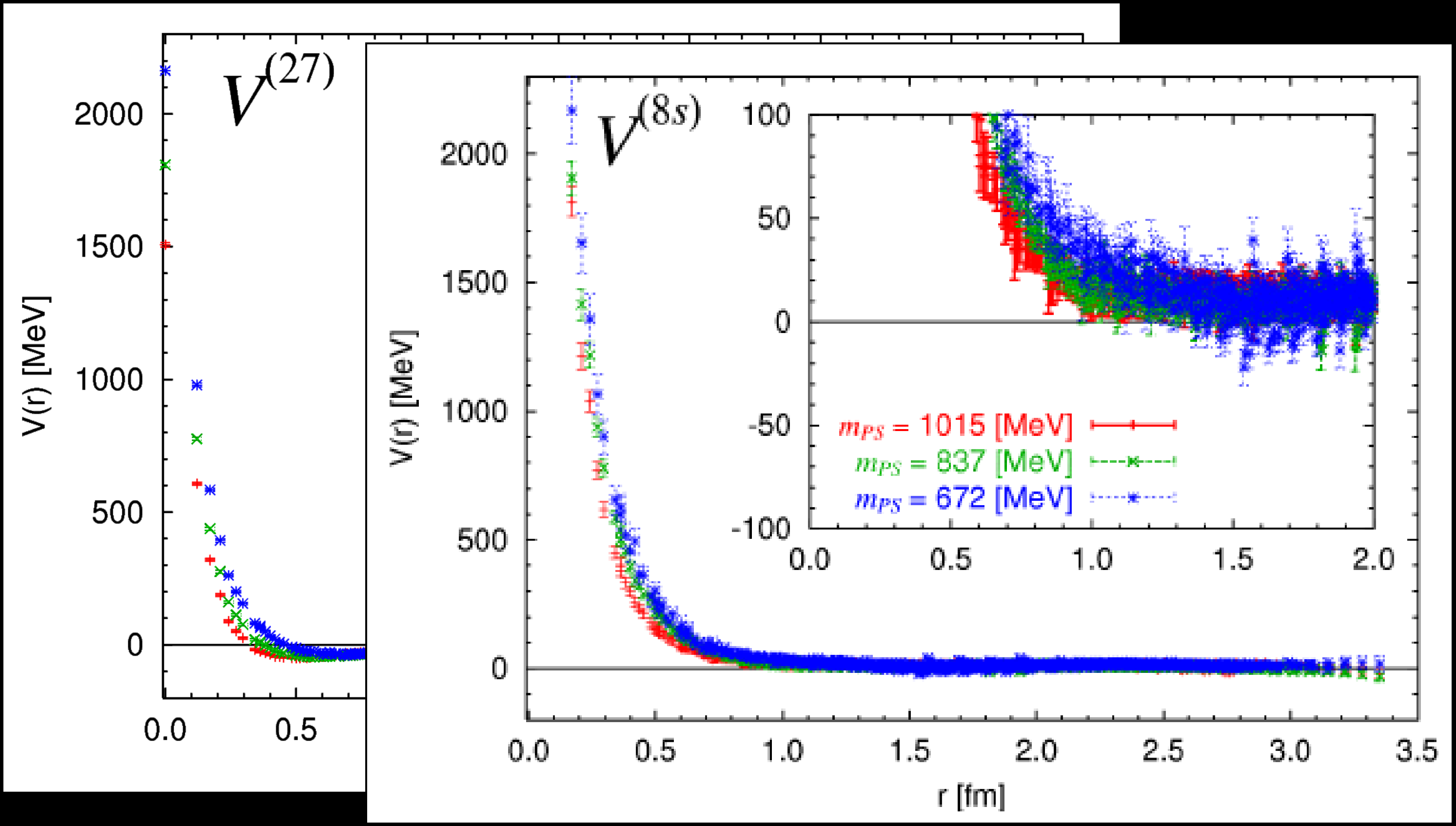
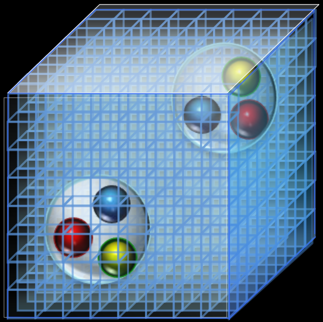
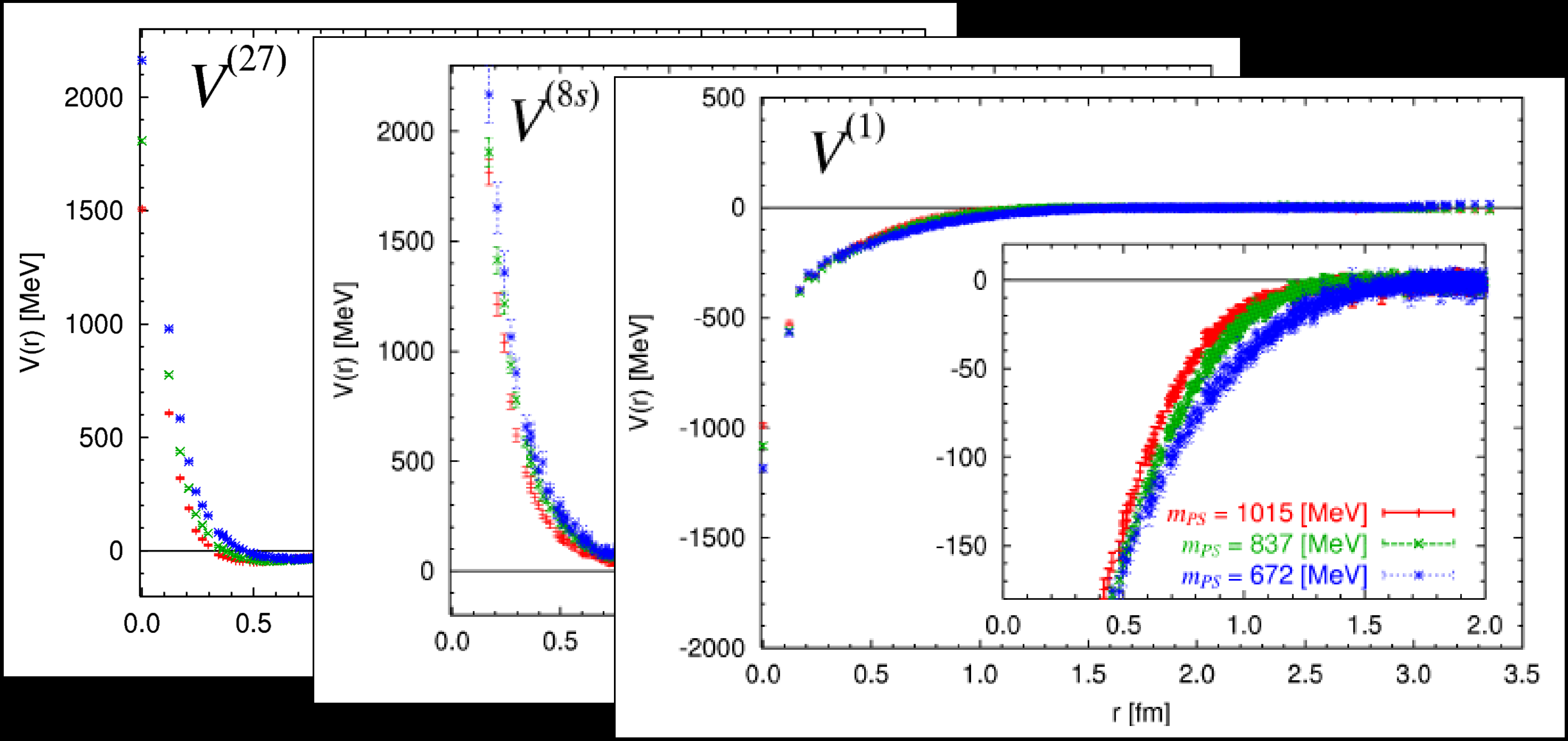




BB potentials in flavor-basis

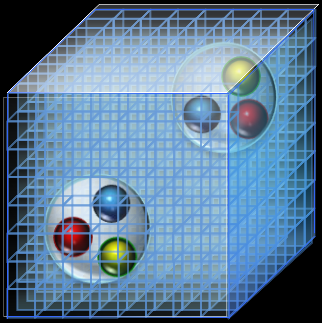
Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002



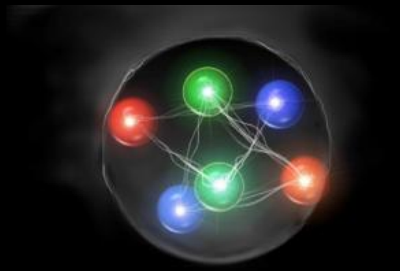
Short range BB int. ⇔ Quark Pauli principle

1 : allowed,
27 : partially blocked, 8_s : blocked

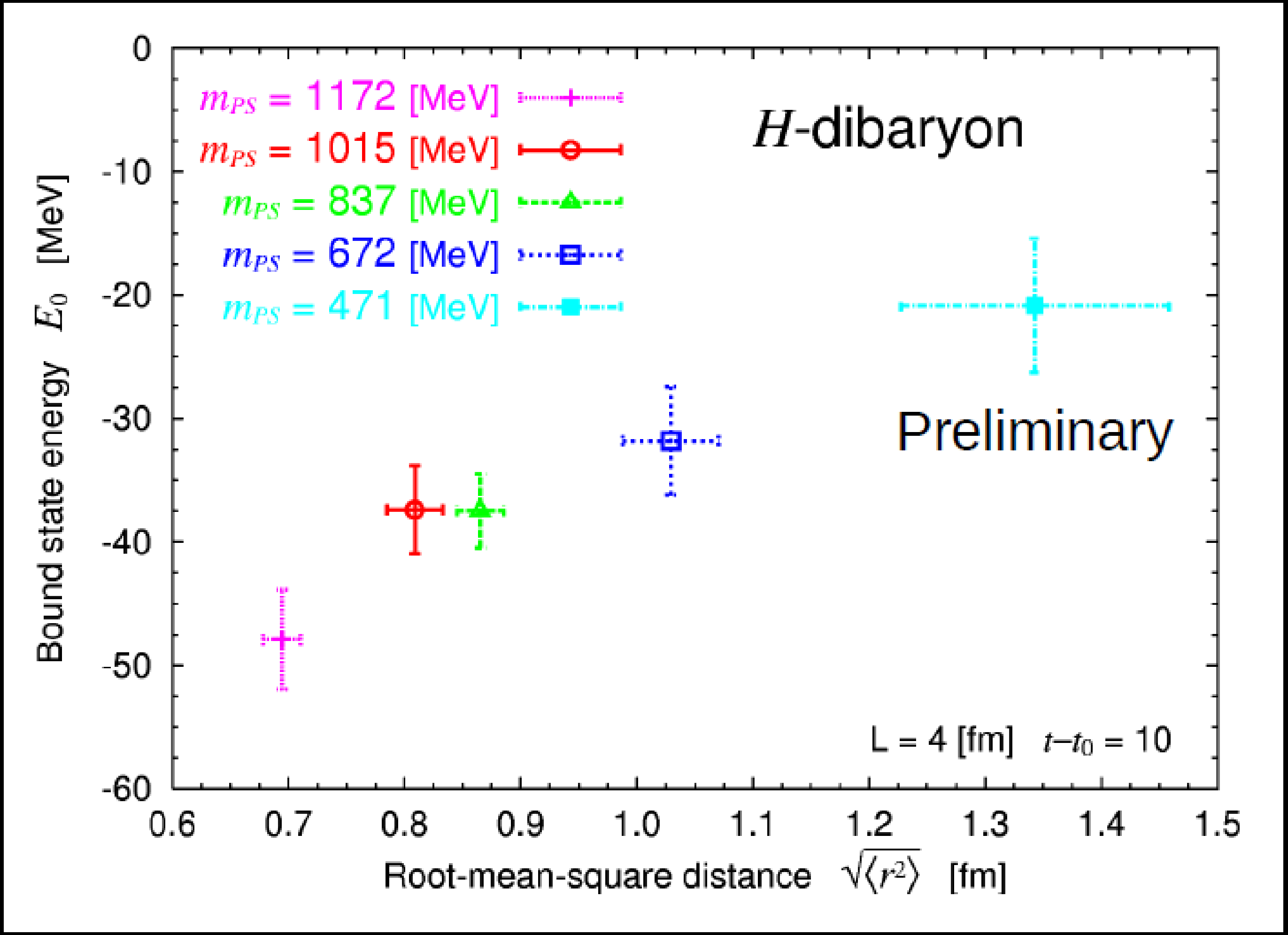
c.f. constituent quark model (Oka, Yazaki, Shimizu, ...)



H-dibaryon from LQCD
-- binding energy vs. size --

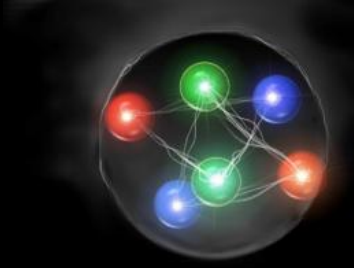


Jaffe, Phys. Rev. Lett. 38 (1977) 195

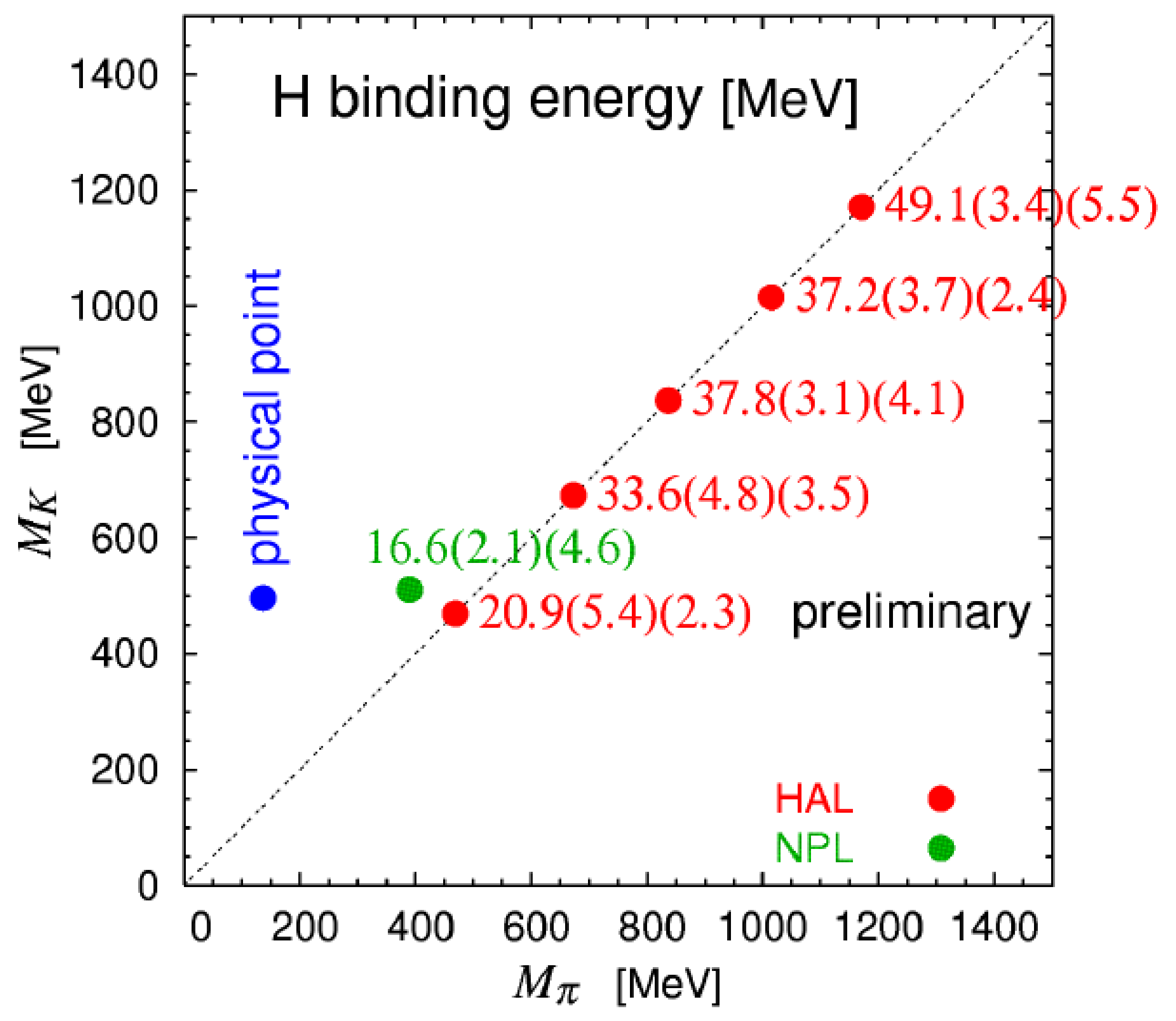


Inoue et al. [HAL QCD Coll.] Phys. Rev. Lett. 106 (2011) 162002

H dibaryon from LQCD



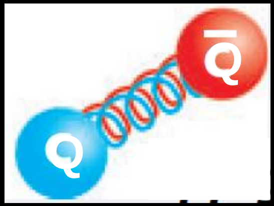
Jaffe, Phys. Rev. Lett. 38 (1977) 195



Inoue et al. [HAL QCD Coll.] Phys. Rev. Lett. 106 (2011) 162002

Beane et al. [NPLQCD Coll.] Phys.Rev.Lett. 106 (2011) 162001

“Constituent quark model” from LQCD ?



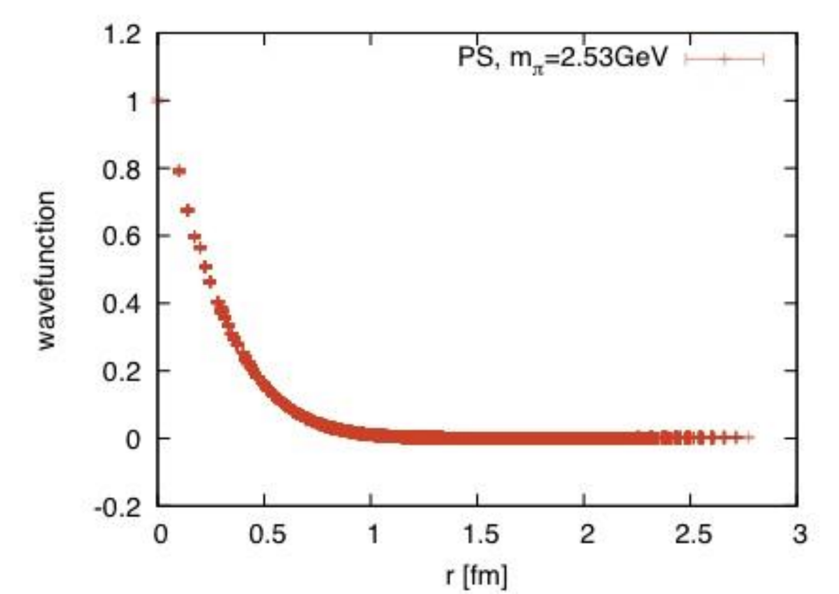
Quenched QCD: Coulomb gauge
L = 3.3 fm, a = 0.104 fm

Ikeda & Iida,1102.2097 [hep-lat]

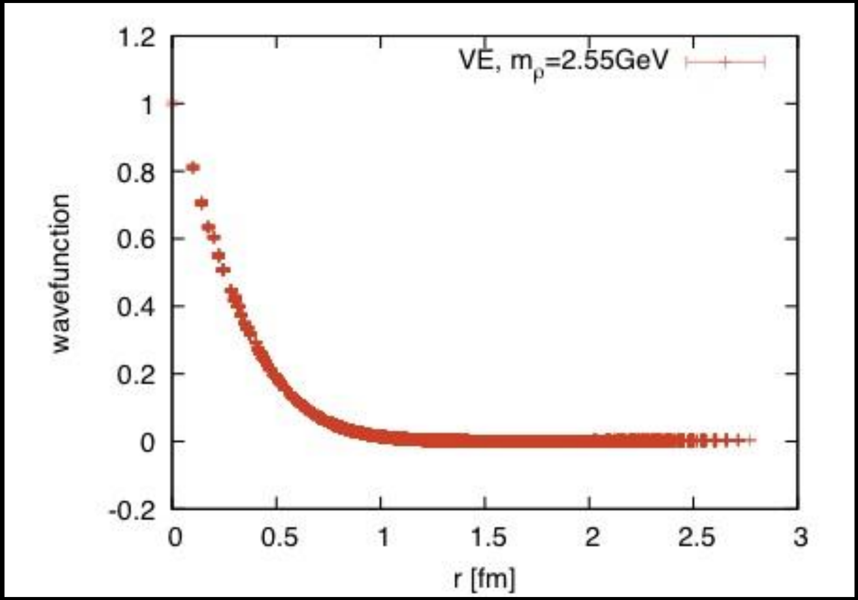
(2+1)-flavor QCD: Coulomb gauge
L = 3 fm, a = 0.09 fm

Kawanai & Sasaki,1102.3246 [hep-lat]
Kawanai @ Lattice2011

$J^P=0^-$ channel



$J^P=1^-$ channel



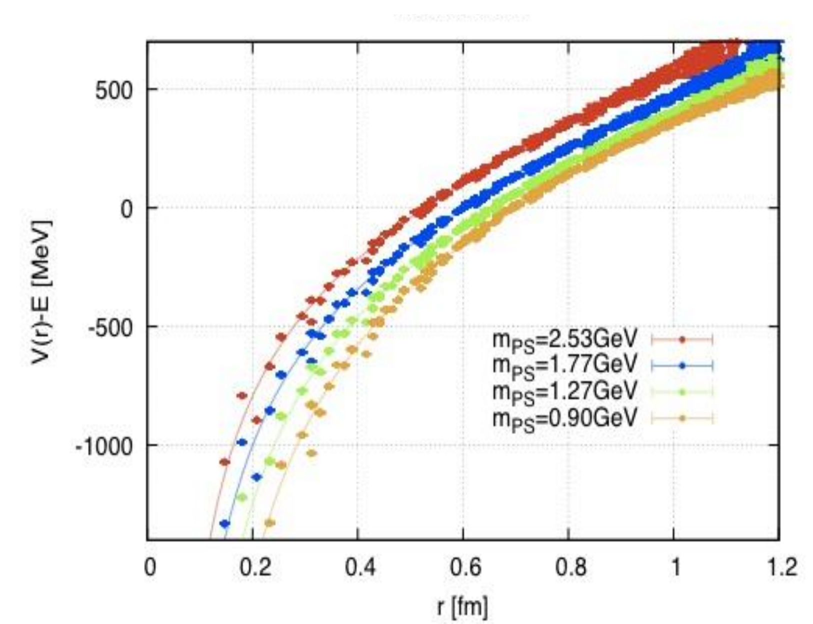
“Constituent quark model” from LQCD ?

Quenched QCD: Coulomb gauge
L = 3.3 fm, a = 0.104 fm
(2+1)-flavor QCD: Coulomb gauge
L = 3 fm, a = 0.09 fm

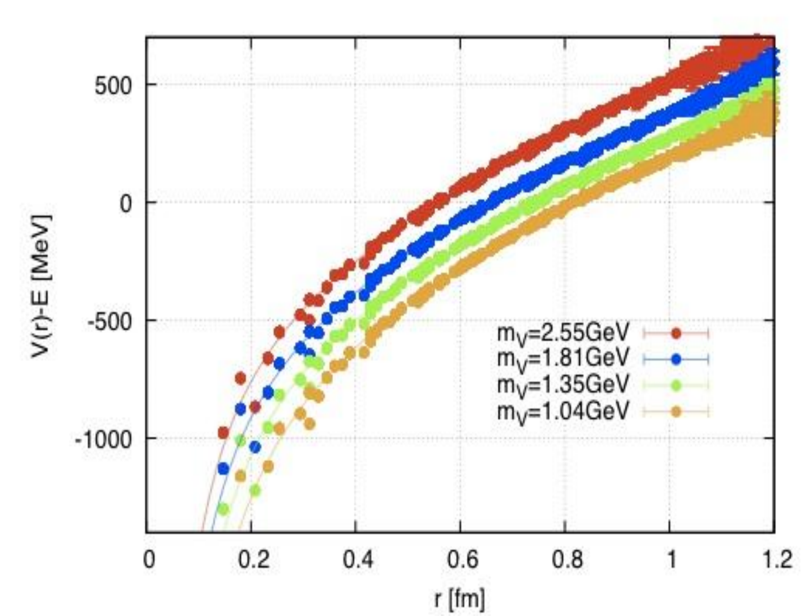
Ikeda & Iida,1102.2097 [hep-lat]

Kawanai & Sasaki,1102.3246 [hep-lat]
Kawanai @ Lattice2011

$J^P=0^-$ channel

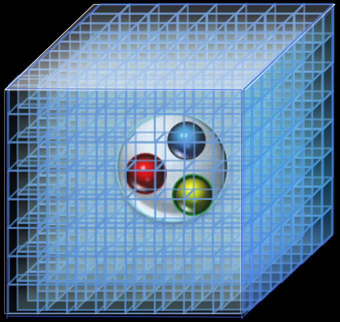


$J^P=1^-$ channel



Coulomb+linear+spin-dep. potential between dynamical quarks

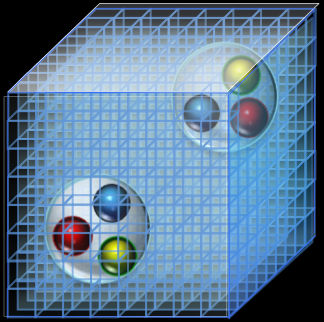
“Summary”



1. LQCD provides precision computations
   (2+1)-flavor, L=6fm, a=0.05fm, m_π=135MeV
   α_s , $m_{u,d,s,}$ low energy constants, …

2. LQCD provides inputs for PAN phenomenology
   - dark matter (e.g. ssbar in the nucleon)
   - quark-gluon plasma (EOS, spectral function,…)

3. LQCD provides qualitative pictures
   - nucleon and hyperon forces
   - the constituent quark model



“Future” (10 Pflops era from 2012)

At next PANIC (2014), we would (like to) hear

1. Physical point simulations for many observables

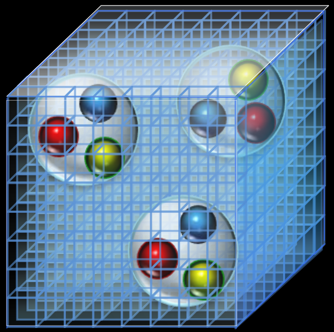
   no more chiral extrapolation

2. Simulations with “better” fermions

   staggered, Wilson → domain wall, overlap

3. Realistic BB, BBB forces & light nuclei

4. New ideas in $\mu/T > 1$?



Yamazaki et al. [PACS-CS Coll.], Phys.Rev.D81(2010)111504

T. Doi [HAL QCD Coll.], arXiv:1106.2276[hep-lat]

Backup slides

“Conclusion”

”The Scientist as Rebel”
by Freeman J. Dyson

It often happens that the understanding of the mathematical nature of an equation is impossible without a detailed understanding of its solutions. The black hole is a case in point.

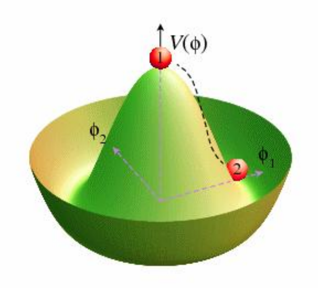
One could say without exaggeration that Einstein’s equations of general relativity were understood only at a very superficial level before the discovery of the black hole.

🟨 → String theory　　🟪 → D-brane

🟨 → QCD　　🟪 → Nucleon? QGP?

The progress of science requires the growth of understanding in both directions, downward from the whole to the parts and upward from the parts to the whole.

Chiral condensate



Gell-Mann-Oakes-Renner (GOR) formula (1968)

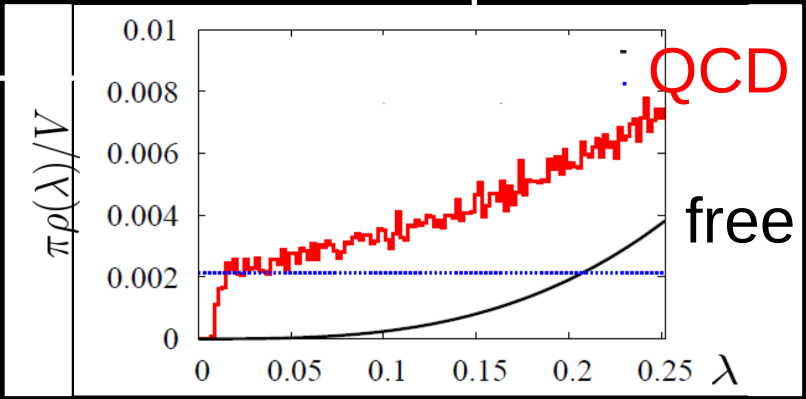
$$f_\pi^2 m_\pi^2 = -(m_u + m_d)\langle \bar{q}q \rangle_0 + O(m_{u,d}^2)$$

Di Vecchia-Veneziano formula (1980)

$$\chi_{\rm top} = \frac{-\langle \bar{q}q \rangle_0}{1/m_u + 1/m_d + 1/m_s} + O(m_u^2)$$

Banks-Casher relation (1980)

$$\langle \bar{q}q \rangle = -\lim_{m\to 0} \int_0^\infty d\lambda \frac{2m}{\lambda^2 + m^2} \rho(\lambda) = -\pi\rho(0)$$



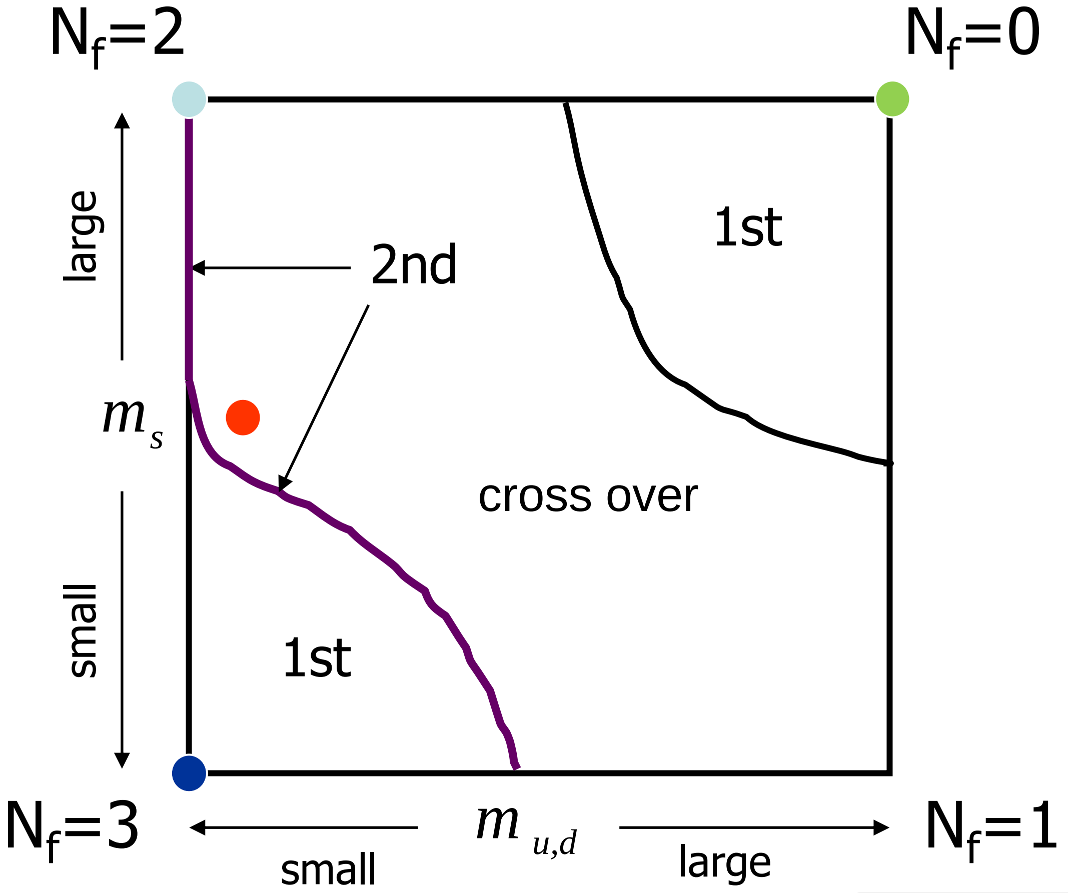
Order of the thermal QCD transition (μ=0)

Svetitsky & Yaffe, NPB210 ('82)

Pisarski and Wilczek, PRD29 ('84)

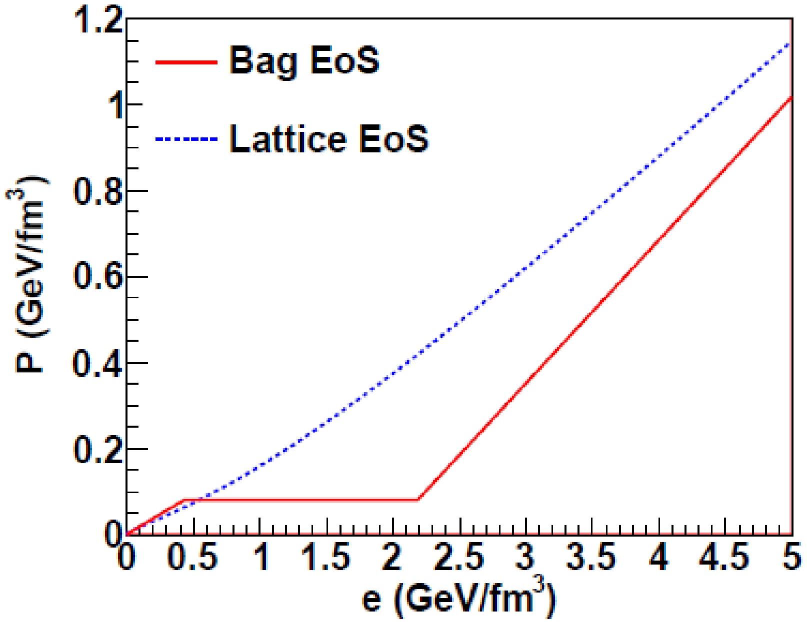
$$\mathcal{V} = \frac{a}{2}\phi^2 + \frac{b}{4}\phi^4 - h\phi$$

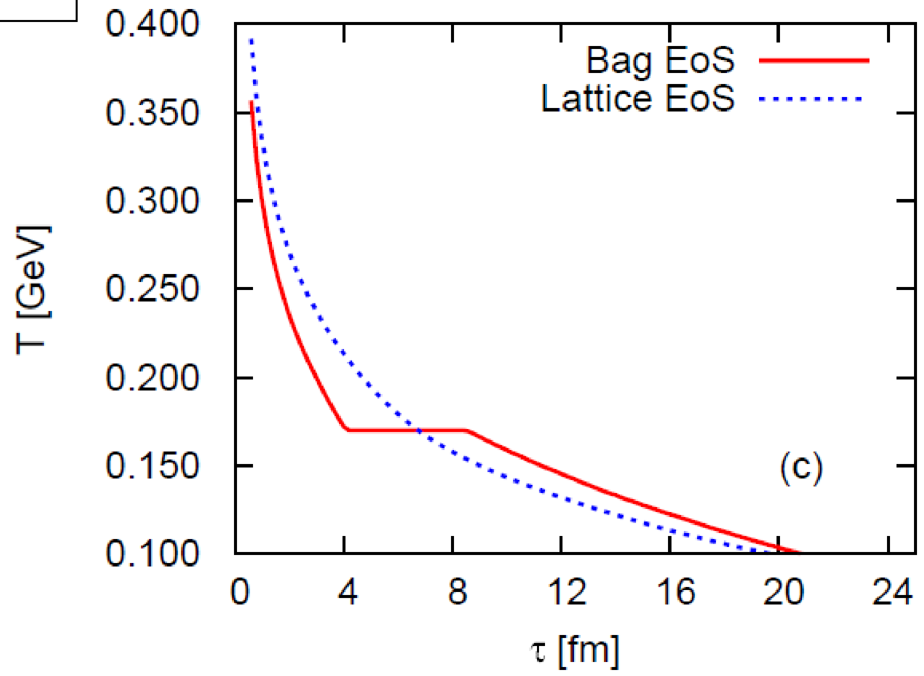
$$\mathcal{V} = \frac{a}{2}L^2 - \frac{c}{3}L^3 + \frac{b}{4}L^4 - hL$$



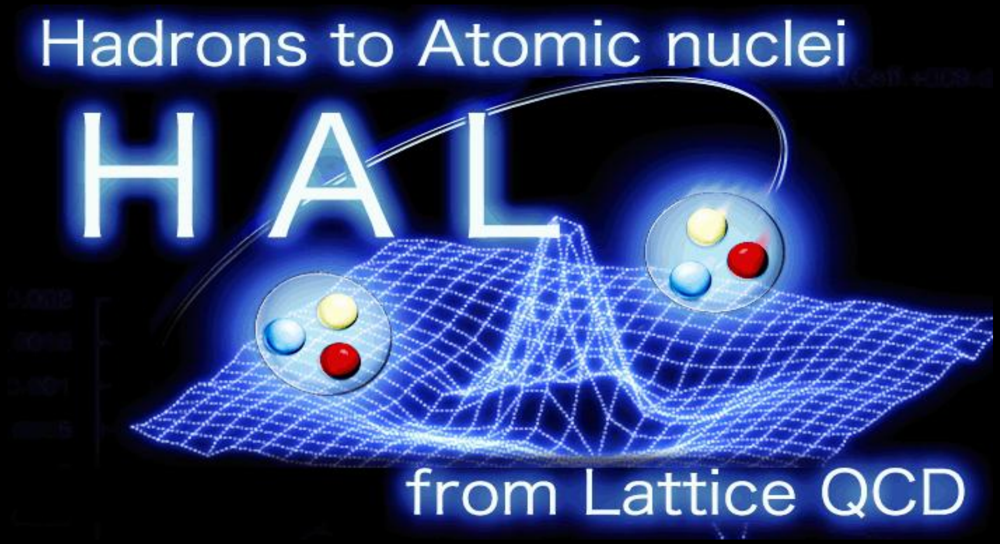
$$\mathcal{V} = \frac{a}{2}\sigma^2 - \frac{c}{3}\sigma^3 + \frac{b}{4}\sigma^4 - h\sigma$$

$$\mathcal{V} = -A\chi + \frac{a}{2}\chi^2 + \frac{b}{4}\chi^4$$





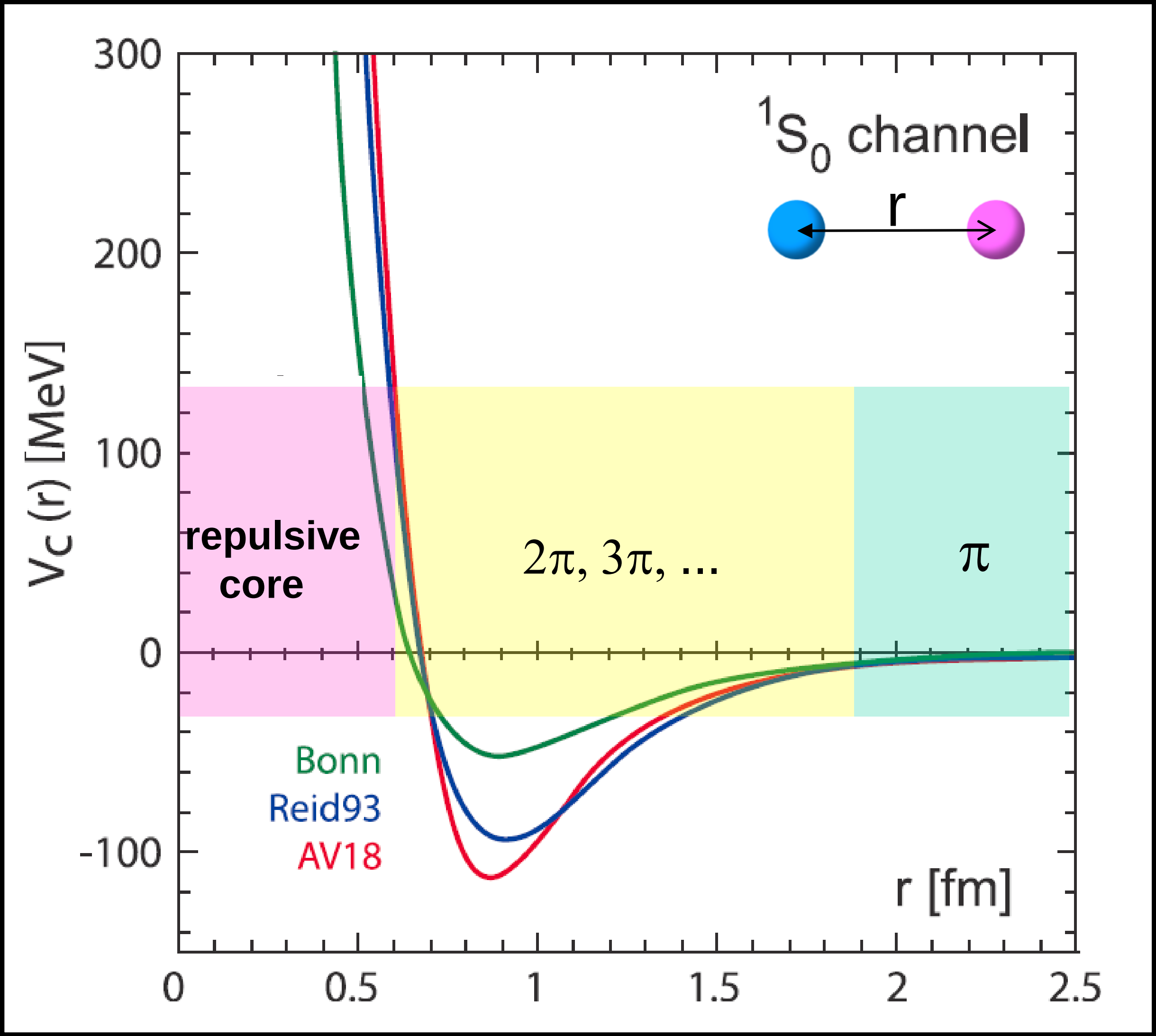
Akamatsu et al.,
arXiv:1107.36[nucl-th]



Tohoku Univ.	H. Nemura
Univ. Tsukuba	S. Aoki, N. Ishii, K. Sasaki
RIKEN	K. Murano, T. Hatsuda
Nihon Univ.	T. Inoue
Tokyo Inst. Tech.	Y. Ikeda
CNS, Univ. Tokyo	T. Doi

Phenomenological NN potentials

(~40 parameters to fit 5000 phase shift data)



One-pion exchange

by Yukawa (1935)

Multi-pions

by Taketani et al. (1951)

Repulsive core

by Jastrow (1951)

Key channels in NN scattering ($^{2s+1}L_J$)

$$V(\vec{r},\nabla) = V_{\rm C}(r) + S_{12}V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\;V_{\rm LS}(r) + \{V_{\rm D}(r),\nabla^2\} + \cdots$$

LO LO NLO NNLO

$^1S_0$ Central force ⟺ nuclear BCS pairing

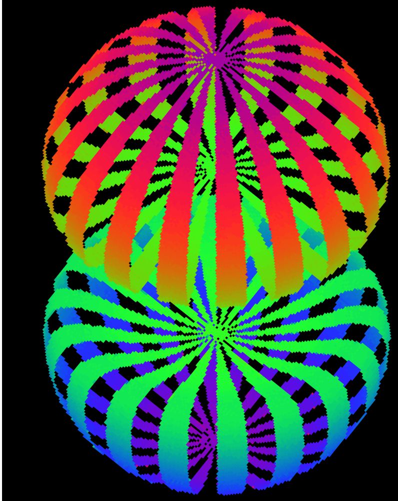
Bohr, Mottelson & Pines, Phys. Rev. 110 (1958)

$^3S_1$-$^3D_1$ Tensor force ⟺ deuteron binding

Pandharipande et al., Phys. Rev. C54 (1996)

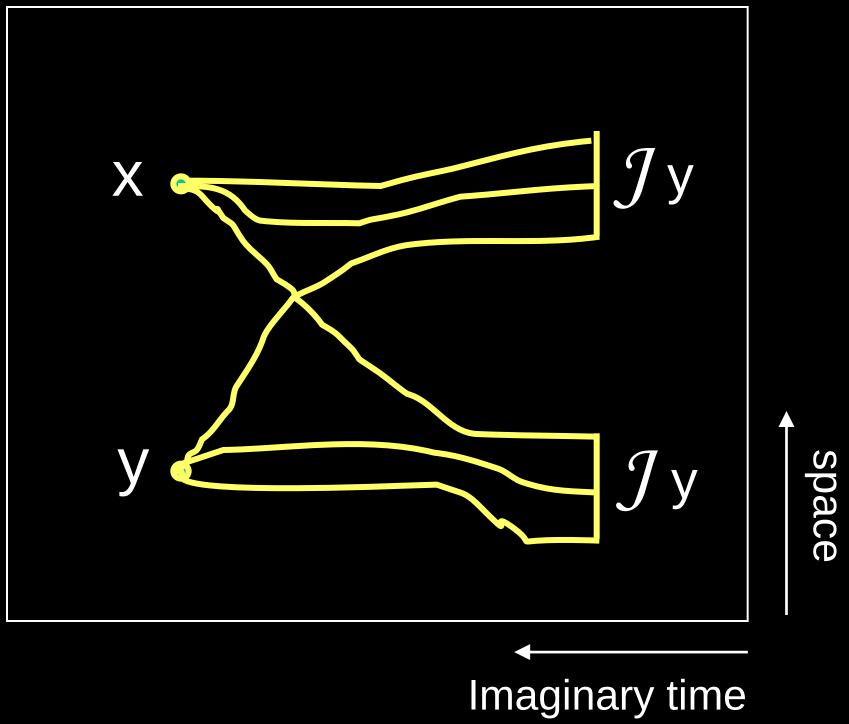
$^3P_2$-$^3F_2$ LS force ⟺ neutron superfluidity in neutron stars

Tamagaki, Prog. Theor. Phys. 44 (1970)



Density profile of the deuteron with $S_z=\pm1$

Equal-time NBS amplitude $\phi(\mathbf{r})$ in lattice QCD



+ all possible combinations

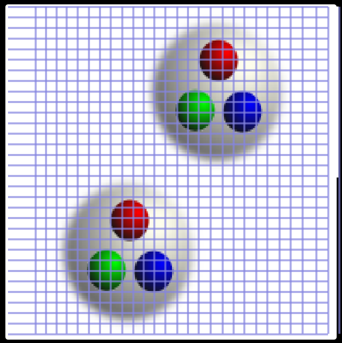
$$\begin{aligned} C_4(\mathbf{r};t) &= \langle N_1(\mathbf{x},t)N_2(\mathbf{y},t)\mathcal{J}_1^\dagger(0)\mathcal{J}_2^\dagger(0)\rangle \\ &= \sum_n \langle 0|N_1(\mathbf{x})N_2(\mathbf{y})|n\rangle A_n\, e^{-E_n t} \longrightarrow \phi(\mathbf{r})A_0 e^{-E_0 t} \end{aligned}$$

$\phi(r > R)$ → phase shift : Luscher, Nucl. Phys. B354 (1991) 531

$\phi(r < R)$ → potential : Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001

Methods to extract NN interaction from LQCD

Luscher, Nucl. Phys. B354 (1991) 531



[1] Temporal correlation : $E_{NN}(L)$ → NN phase shift

$$\frac{2Z_{00}(1,q)}{L\pi^{1/2}} = k\cot\delta_0(k)$$

- quenched QCD: CP-PACS Coll. (1995)
- full QCD: NPLQCD Coll. (2006-)

[2] Spatial correlation :

BS wave function → NN potential → observables

$$(E-H_0)\phi(\mathbf{r}) = \int U(\mathbf{r},\mathbf{r}')\phi(\mathbf{r}')d\mathbf{r}'$$

← half off-shell T-matrix

- π-π system : CP-PACS Coll. (2005)
- NN system (quenched QCD) : Ishii, Aoki & T.H., PRL 99, 022001 (2007).
- NN, YN systems (full QCD): HAL QCD Coll. (2008-)

Systematic procedure to define the NN potential in lattice QCD

Full details, see
Aoki, Ishii & Hatsuda, 0909.5585 [hep-lat]

(i) Choose your favorite operator: e.g. $N(x) = \epsilon_{abc} q^a(x) q^b(x) q^c(x)$

observables do not depend on the choice
yet the local operator is useful

Nishijima, Haag, Zimmermann (1958)

(ii) Measure the NBS amplitude: $\phi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 6q \rangle$

(iii) Define the non-local potential: $(E - H_0)\phi(\vec{r}) = \int U(r, \vec{r}\,')\phi(\vec{r}\,') d^3r'$

(iv) Velocity expansion : $U(\vec{r}, \vec{r'}) = V(\vec{r}, \nabla)\delta^3(\vec{r} - \vec{r}\,')$

$$\boxed{V(\vec{r}, \nabla) = V_{\rm C}(r) + S_{12} V_{\rm T}(r) + \vec{\rm L}\cdot\vec{\rm S}\ V_{\rm LS}(r) + \{V_{\rm D}(r), \nabla^2\} + \cdots}$$

Okubo-Marshak (1958), Tamagaki-Watari (1967)

(v) Calculate observables : phase shifts, binding energies etc

Properties of lattice NN potential *U(r,r')*

$$U(\vec{r},\vec{r}')=V(\vec{r},\nabla)\delta^3(\vec{r}-\vec{r}')$$

[1] *U*(*r*,*r'*) is *N(x)*-dependent

QM : (ψ, V) → observables
QFT : (asymptotic field, vertices) → observables
($N(x)$, $U(r,r')$) → observables

[2] *U*(*r*,*r'*) is *E*-independent

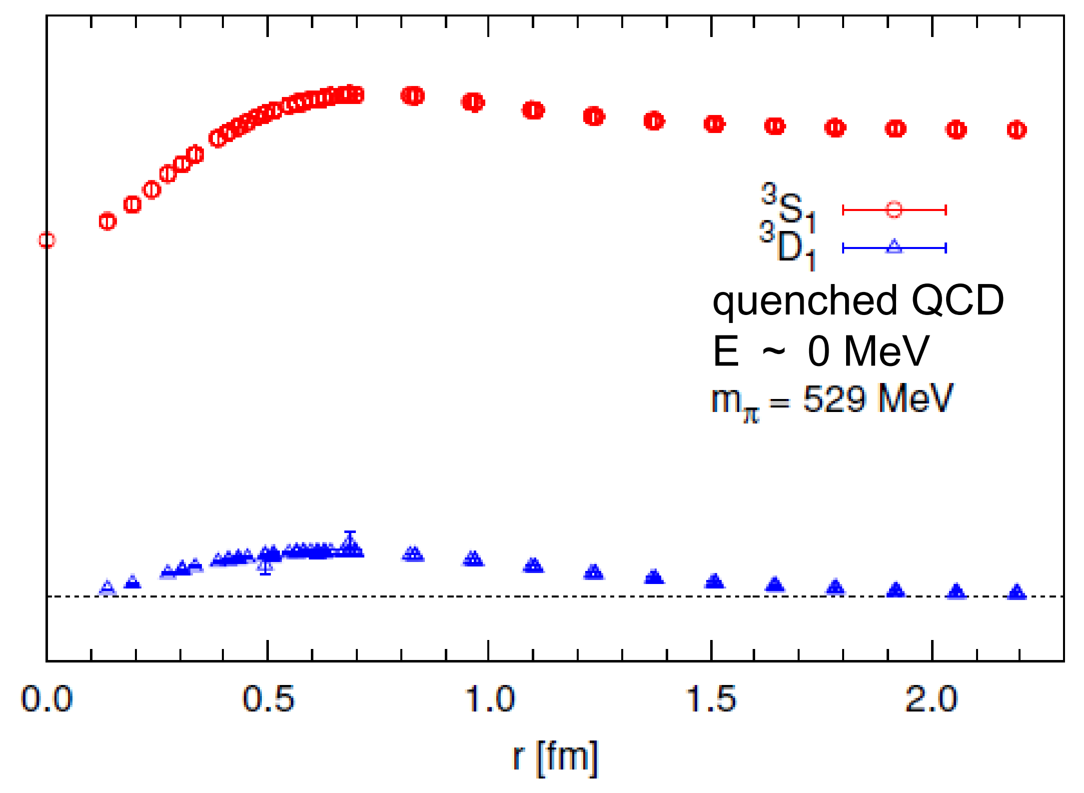
non-locality can be determined order by order

[3] *U*(*r*,*r'*) has minor volume dependence

Wave function is sensitive to the volume
Potential is insensitive to the volume
remember the deuteron !

Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Ishii & Hatsuda,
0909.5585 [hep-lat]



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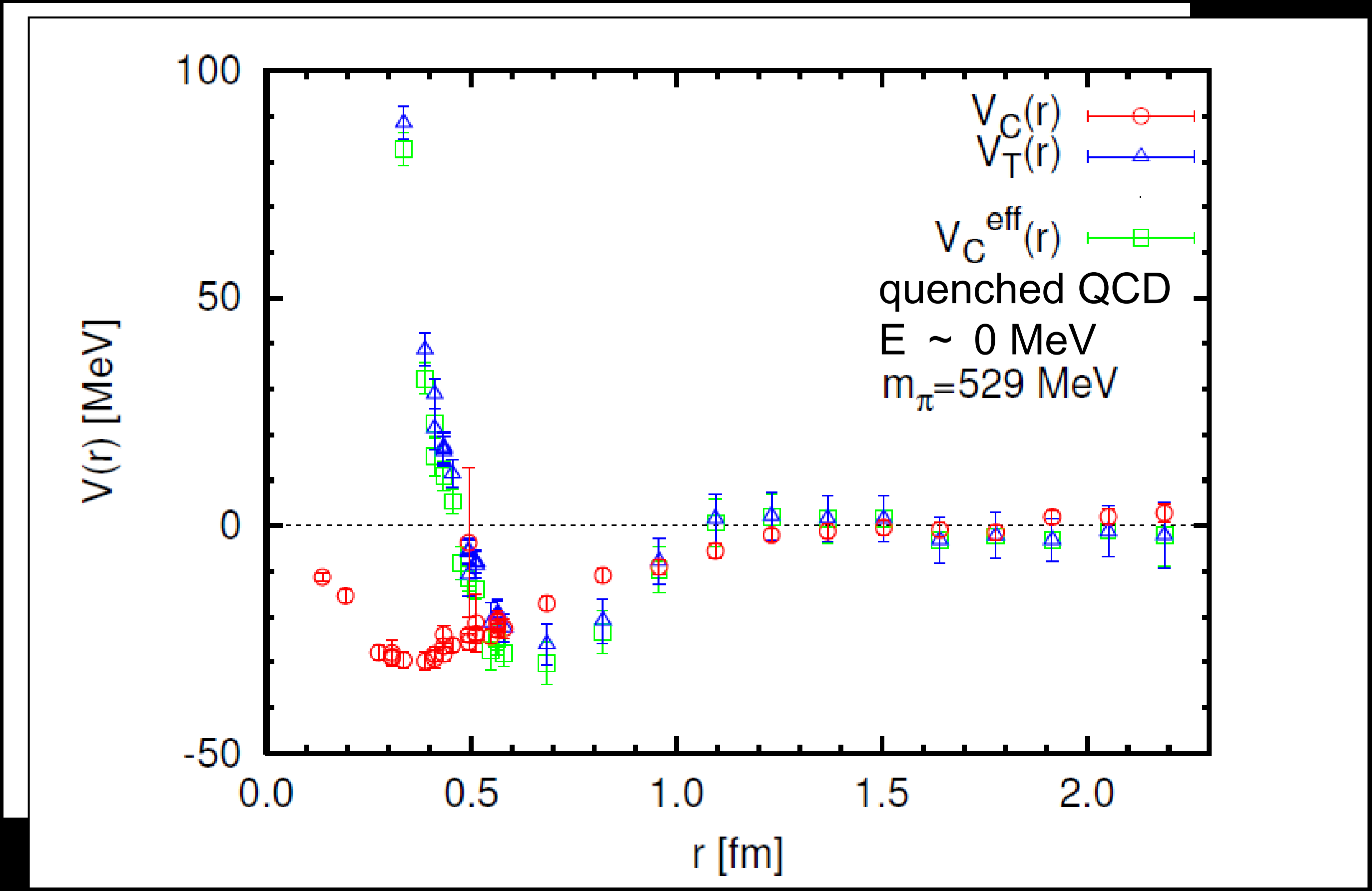
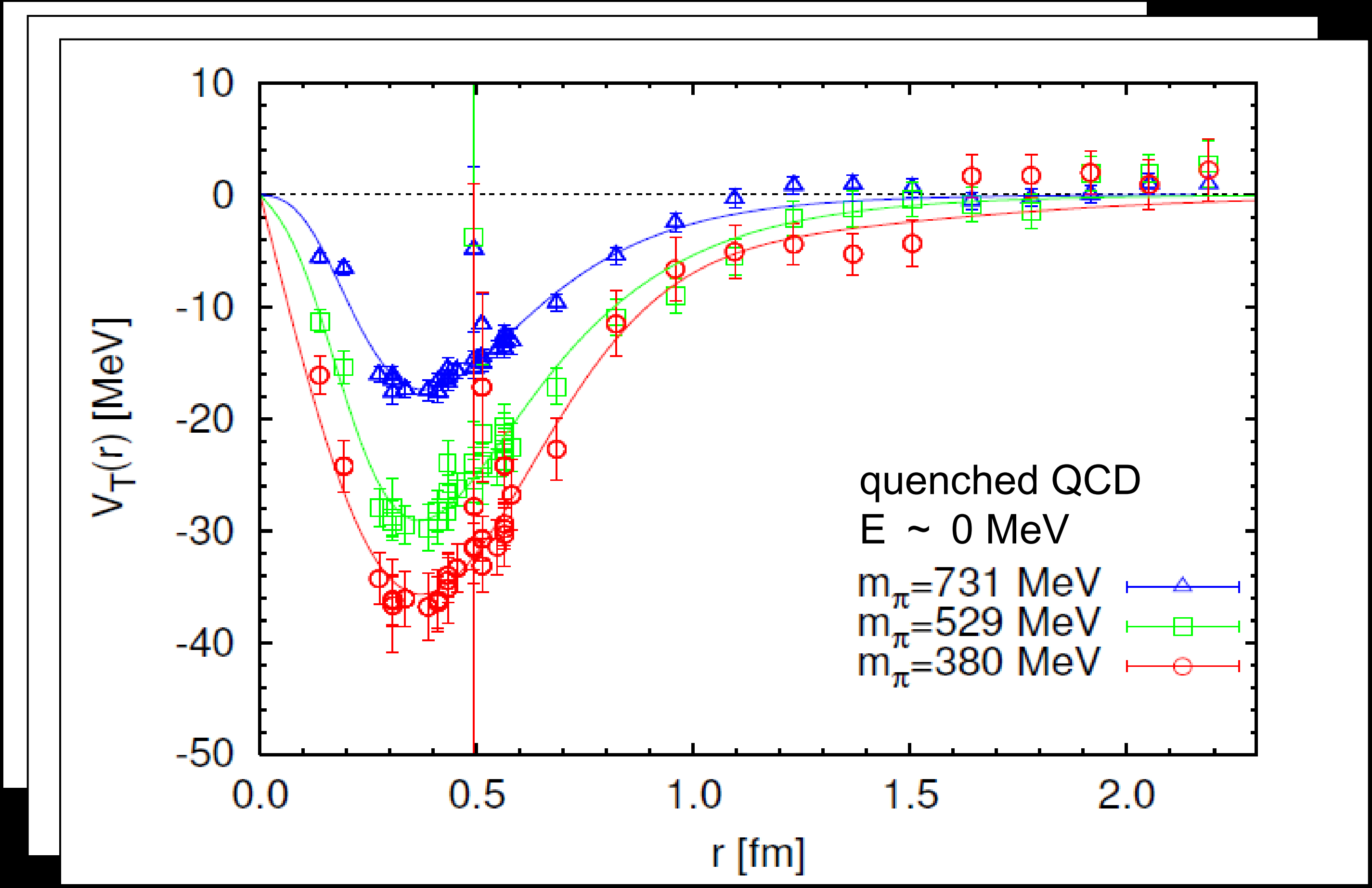
0909.5585 [hep-lat]



Central & tensor potentials : $V_C(r)$ & $V_T(r)$

Aoki, Ishii & Hatsuda,
0909.5585 [hep-lat]

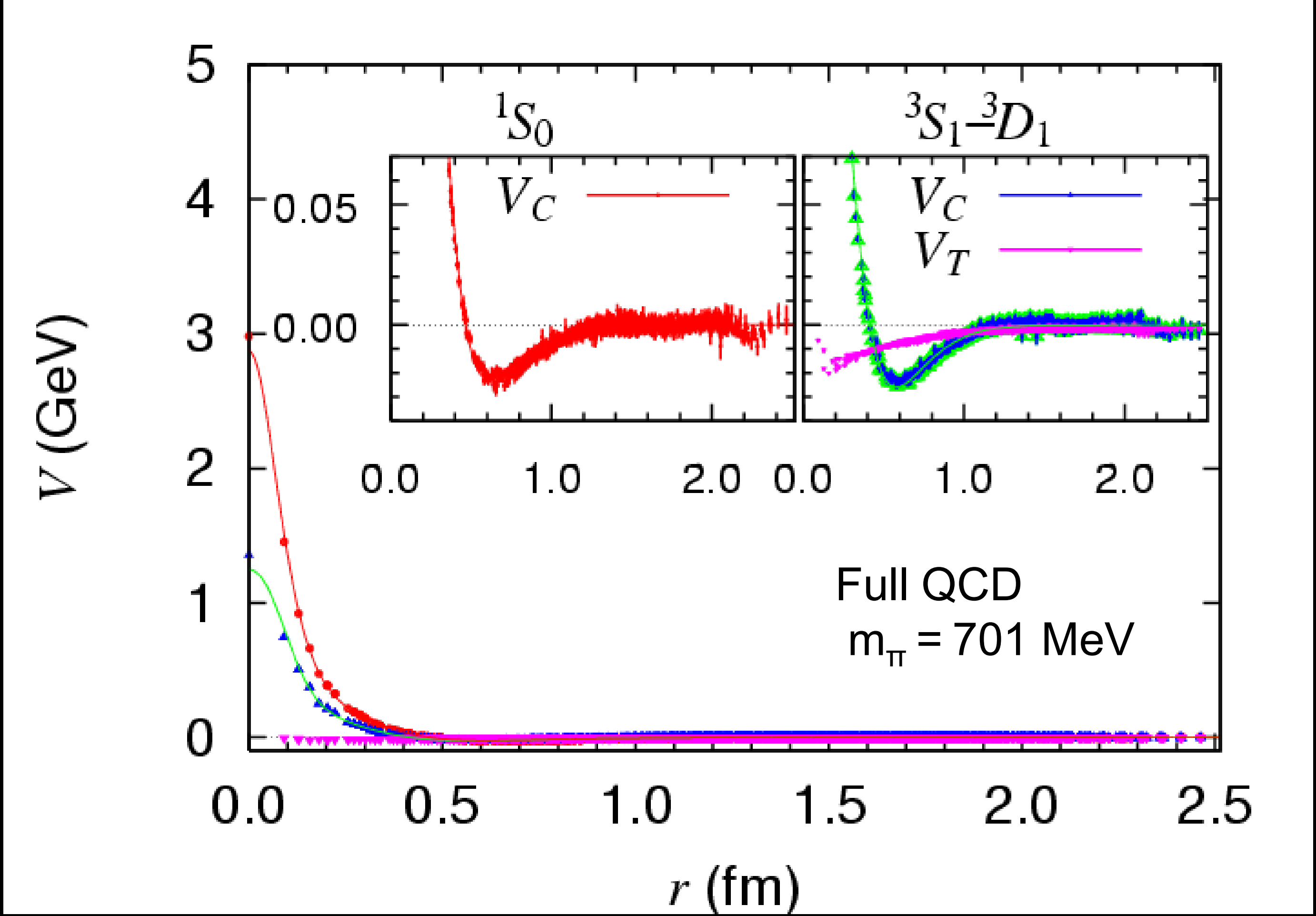


fit function

$$V_T(r) = b_1(1-e^{-b_2 r^2})^2 \left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r}$$
$$+b_3(1-e^{-b_4 r^2})^2 \left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r},$$

- Rapid quark-mass dependence of $V_T(r)$
- Evidence of the one-pion-exchange

ΛN interaction



- Repulsive core + attractive well
- Weak tensor force
- Overall attraction

Nemura et al. (HAL QCD Coll.)

irreducible BB source operator

$$\overline{BB^{(27)}} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi} \quad \text{or} \quad +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} + \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(8s)}} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB^{(1)}} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^+}\,\overline{\Sigma^-} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^0}\,\overline{\Sigma^0} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^-}\,\overline{\Sigma^+}$$

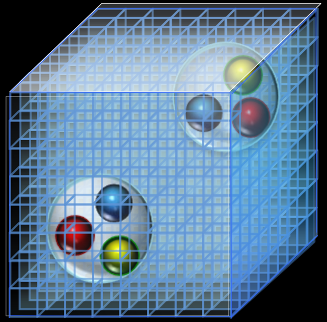
$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} + \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} - \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

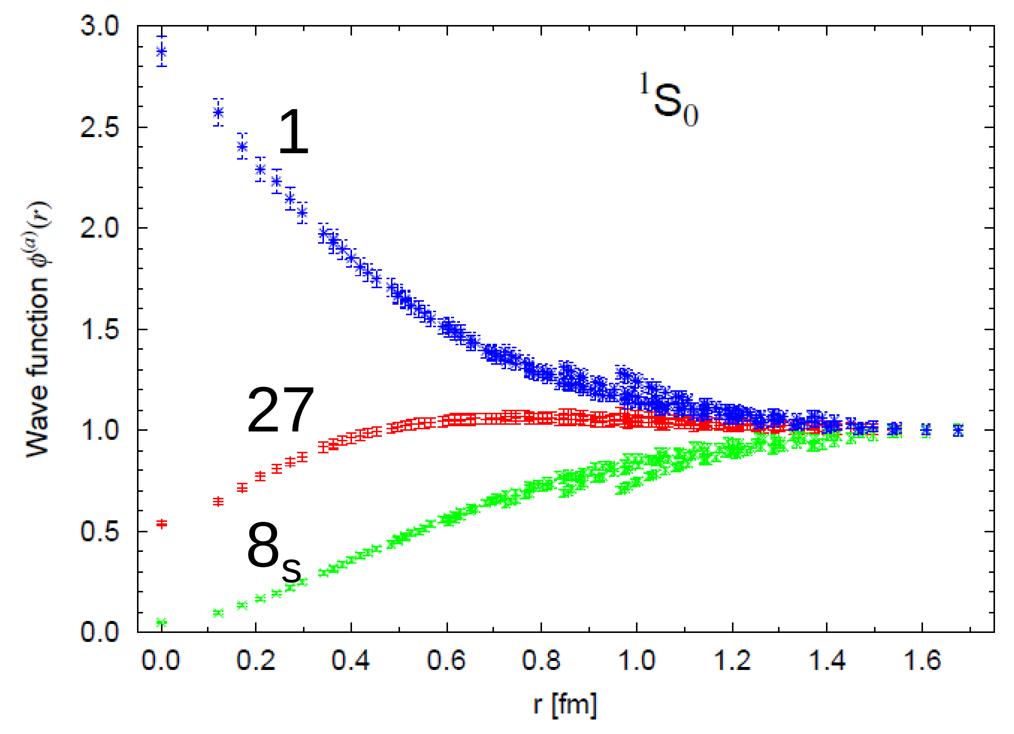
$$\overline{BB^{(10^*)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB^{(10)}} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^+} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^+}\,\overline{p}$$

$$\overline{BB^{(8a)}} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} - \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} + \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

BB wave functions in flavor-basis





Iwasaki + clover
(CP-PACS/JLQCD config.)
L=1.9 fm, a=0.12 fm, 16^3x32
m_π=835 MeV, m_B=1752 MeV

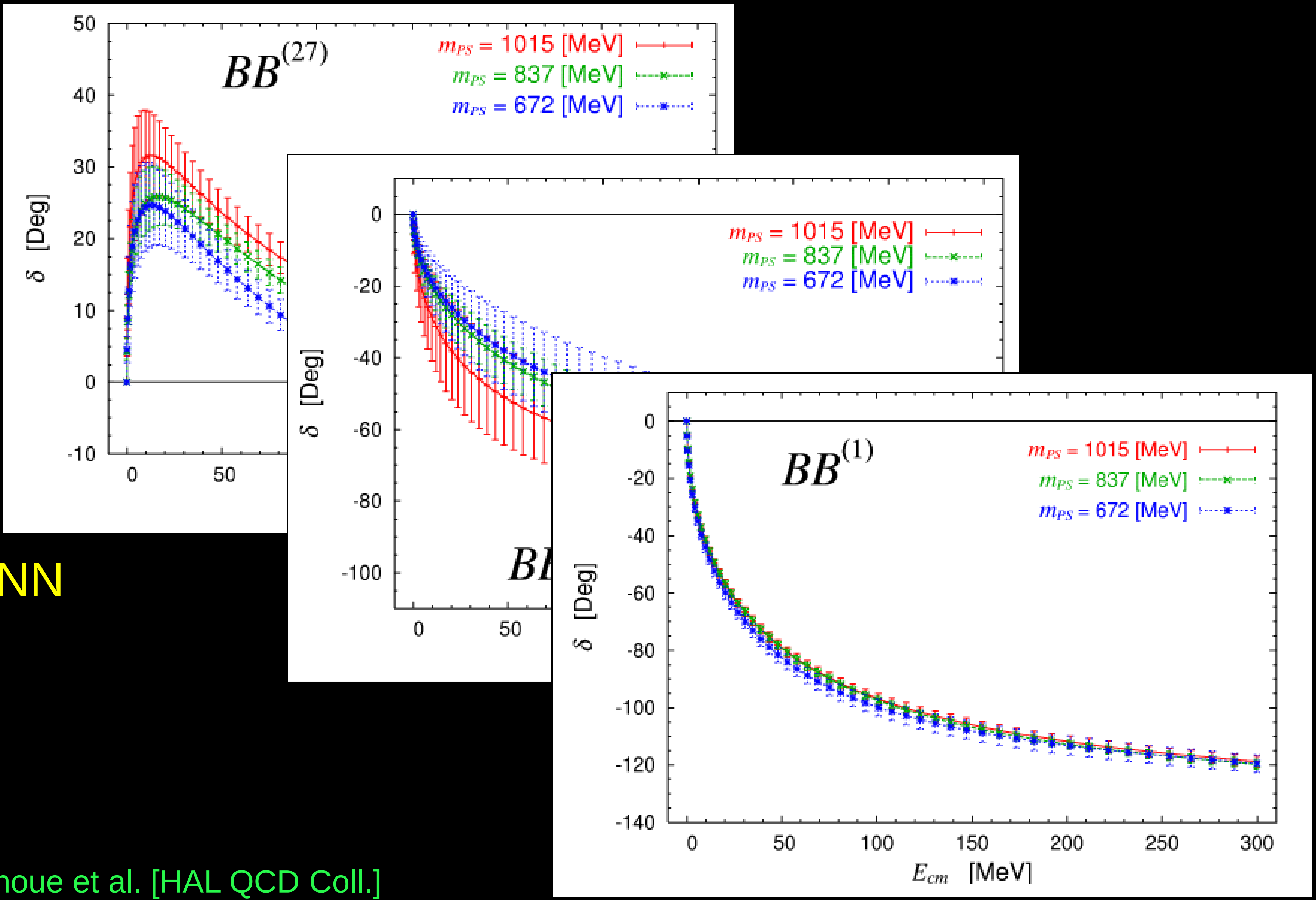
Inoue et al. (HAL QCD Coll.)
Prog. Theor. Phys. 124 (2010) 591

Origin of the short range BB int. ⇔ Quark Pauli principle !

1 : allowed, 27 : partially blocked, 8_s : blocked

c.f. constituent quark model (Oka, Yazaki, Shimizu, ...)

BB phase shifts in flavor-basis ($^1S_0$ channel)



NN

Inoue et al. [HAL QCD Coll.]
Phys. Rev. Lett. 106 (2011) 162002