Nishina School
RIKEN
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(Magnetic) Spectrometer

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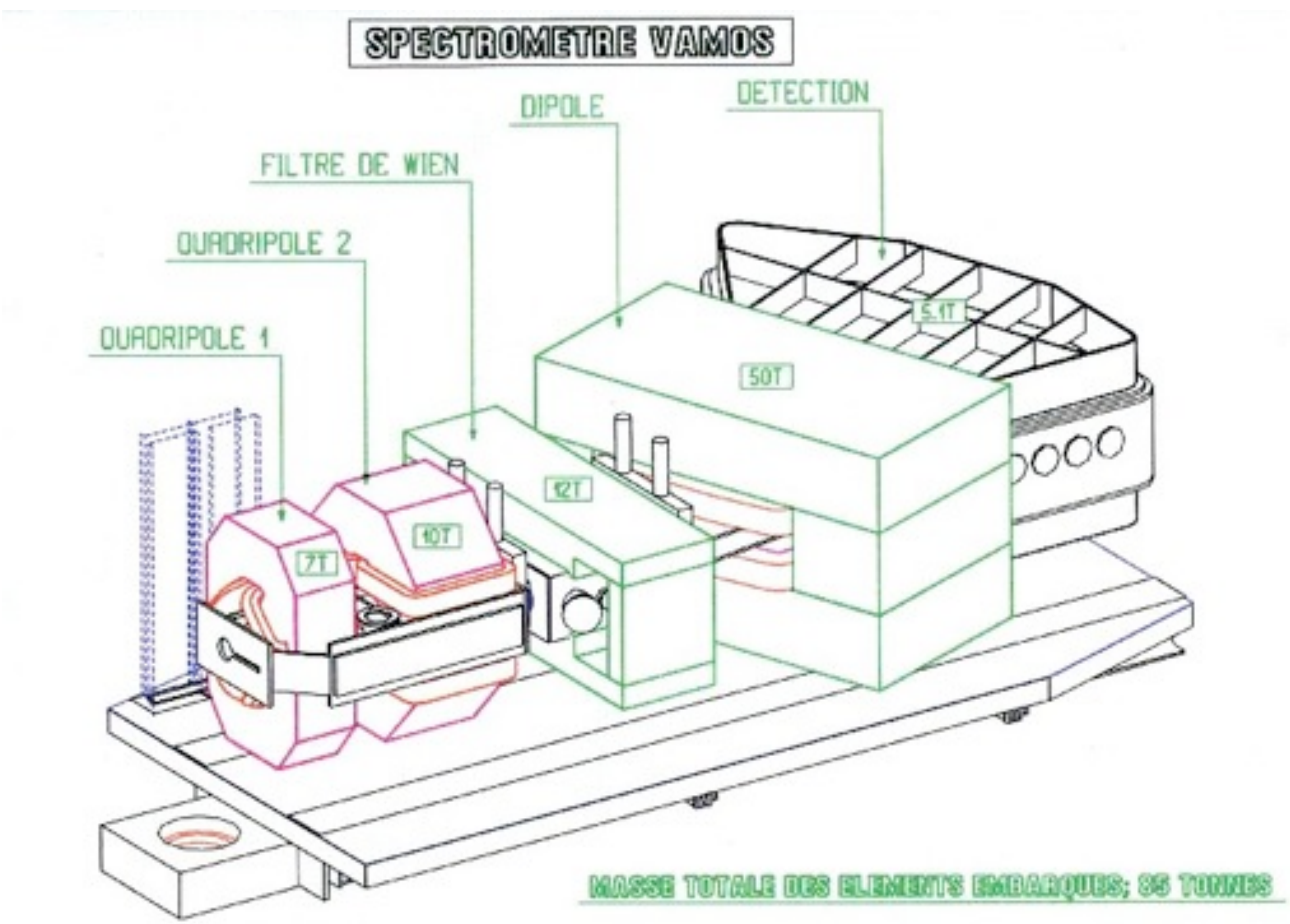
What is a magnetic spectrometer?

A device to measure momentum of charged particles (*p*, HI, *e*. . .)

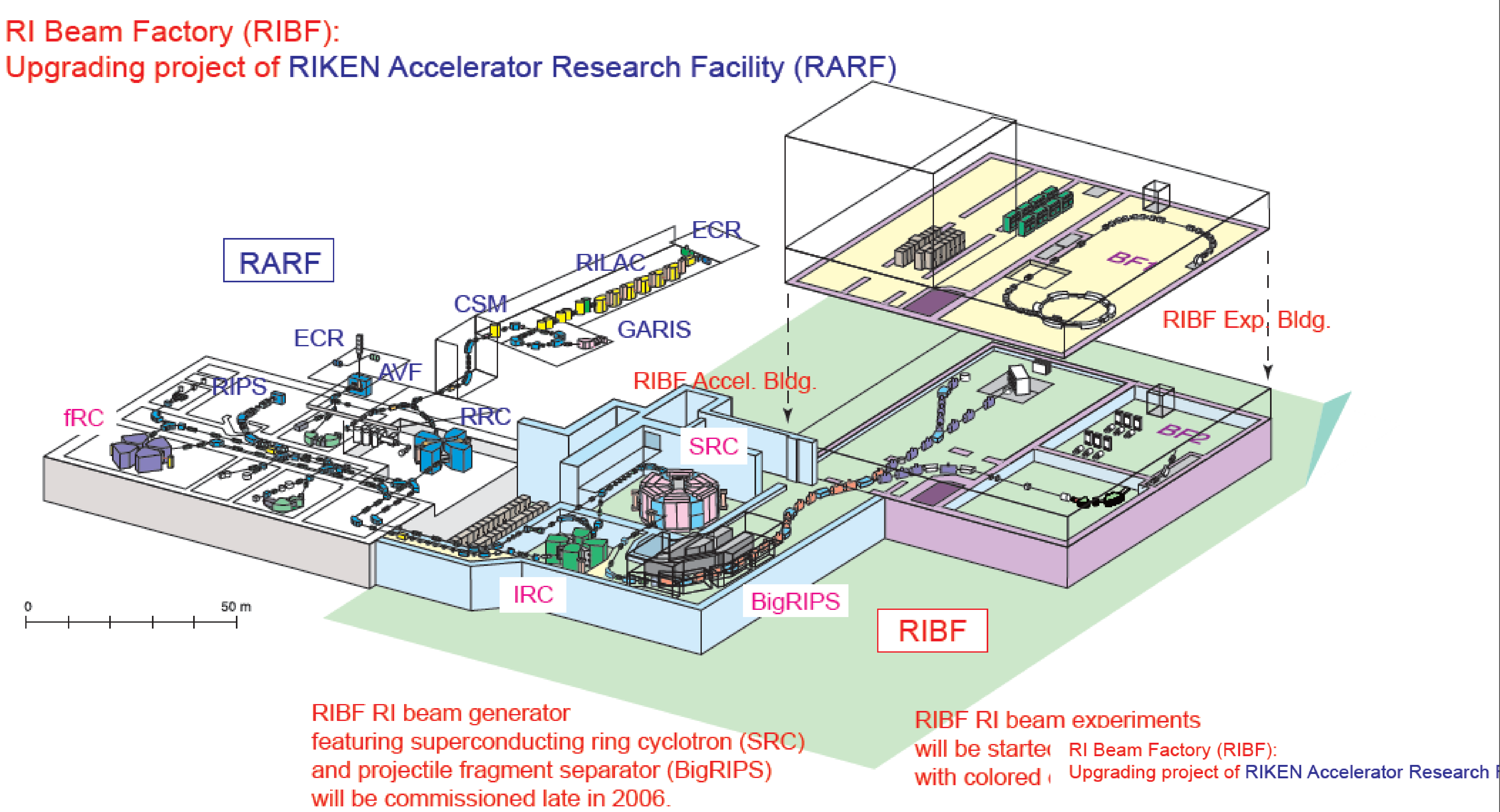
- High resolution and/or large acceptance
- Non-destructive
- Applicability to high energies

VAMOS spectrometer @GANIL



Alpha magnetic spectrometer

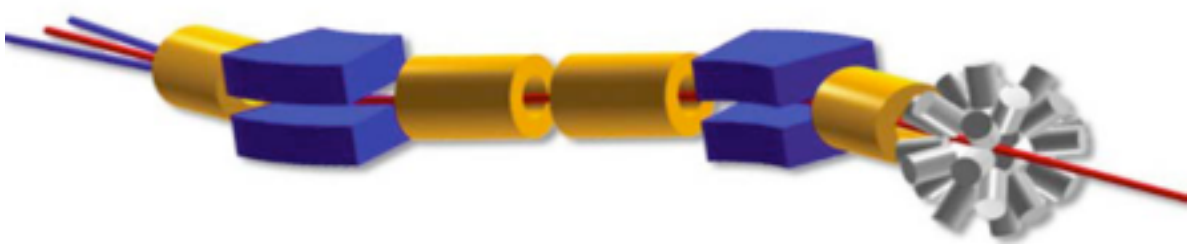
Magnetic spectrometers @ RIBF



Magnetic spectrometers @ RIBF

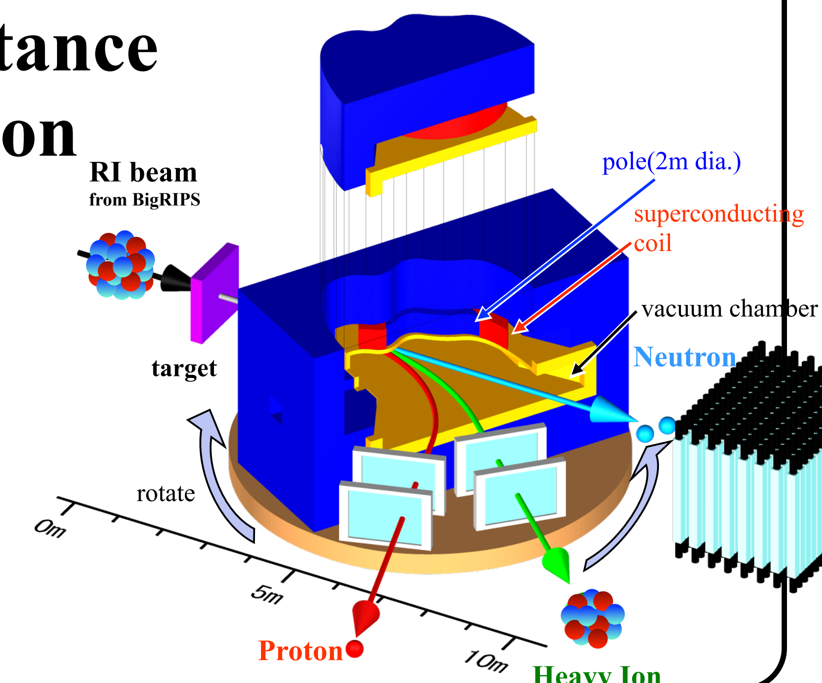
Zero-degree

Multi-purpose
Medium resolution
Medium acceptance



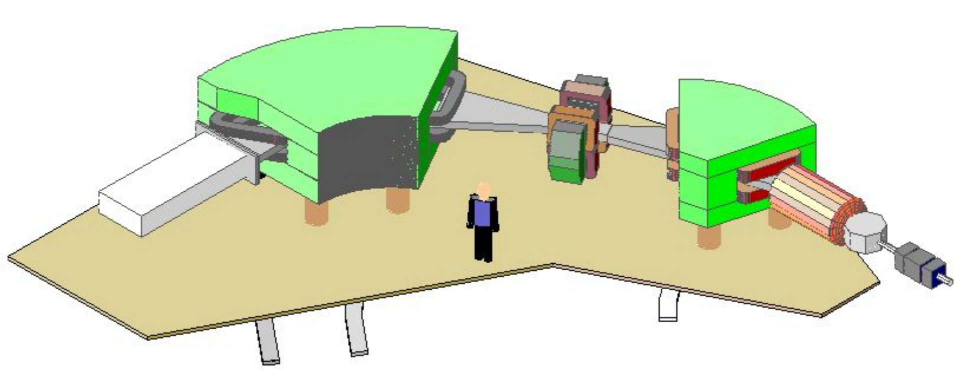
SAMURAI

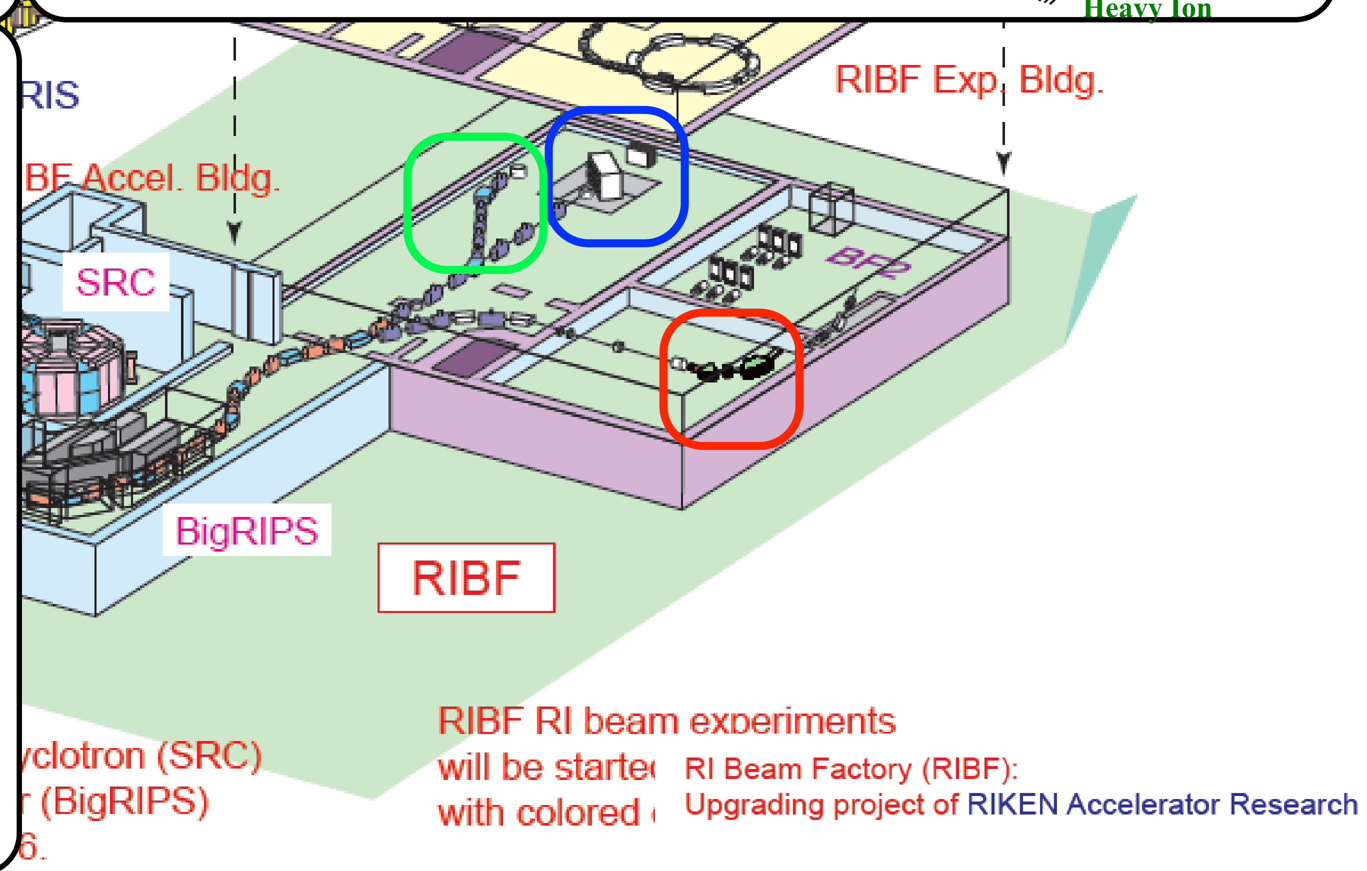
Large acceptance
Low resolution



SHARAQ

High-resolution (p and θ)
Small acceptance



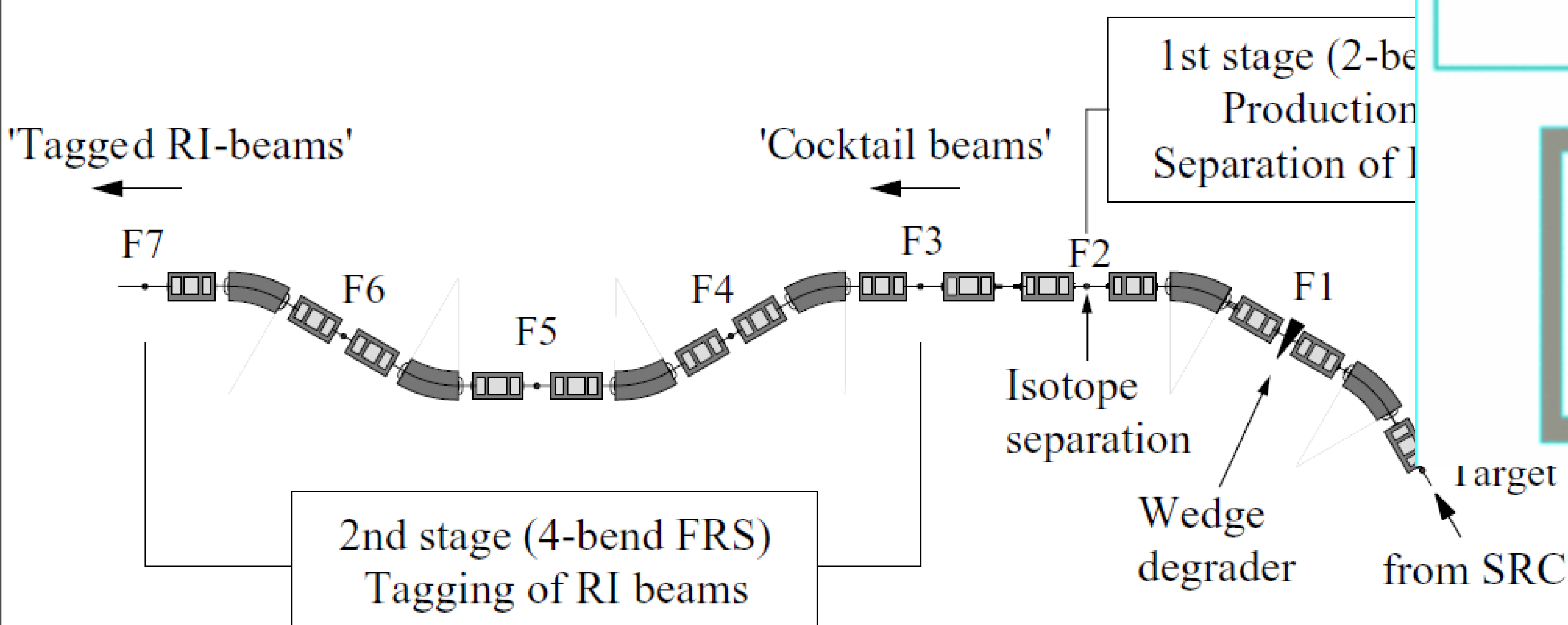


RI-beam separator

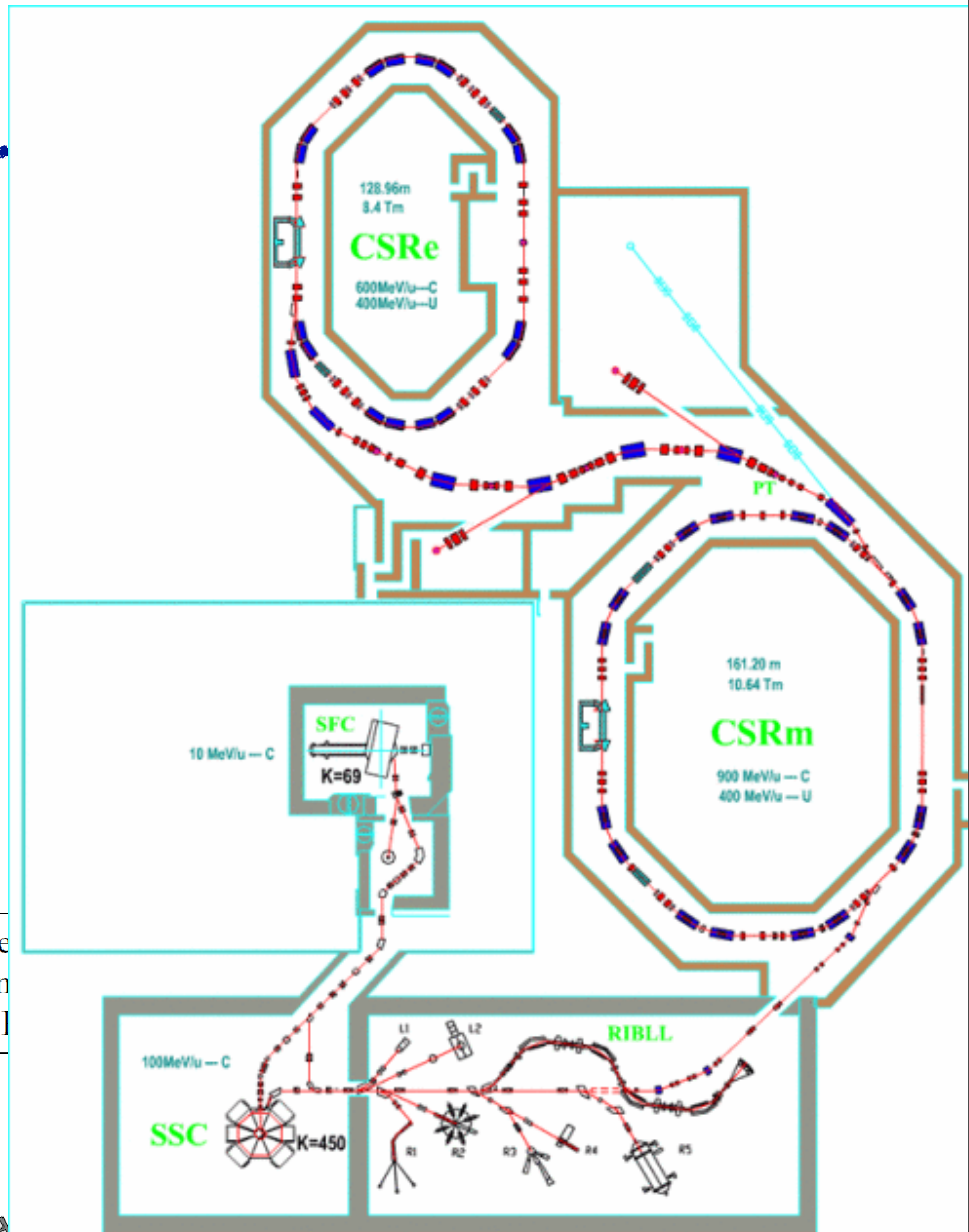
A RI-beam separator is a kind of spectrometer.

- Beam quality (size, purity)
- Particle identification

BigRIPS : Tandem (Two-stage) Separator



TOF, Bρ, ΔE → Z, A/Q (A, Q), P



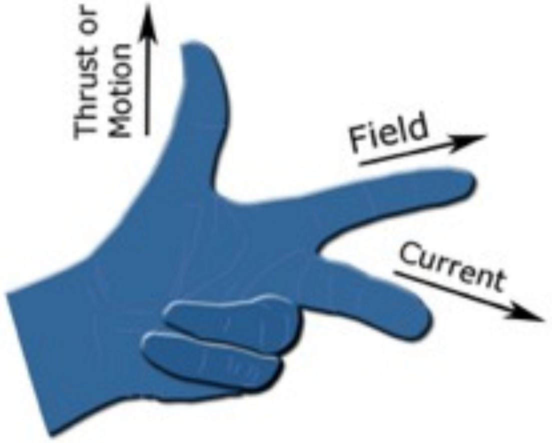
Ion optics

Framework to describe ion trajectories in magnetic systems

Analogy to "light" optics
concepts of "focus", "dispersion", "magnification"

"MUST" knowledge to manipulate charged particles
in spectrometers and RI-beam separators

Lorentz force

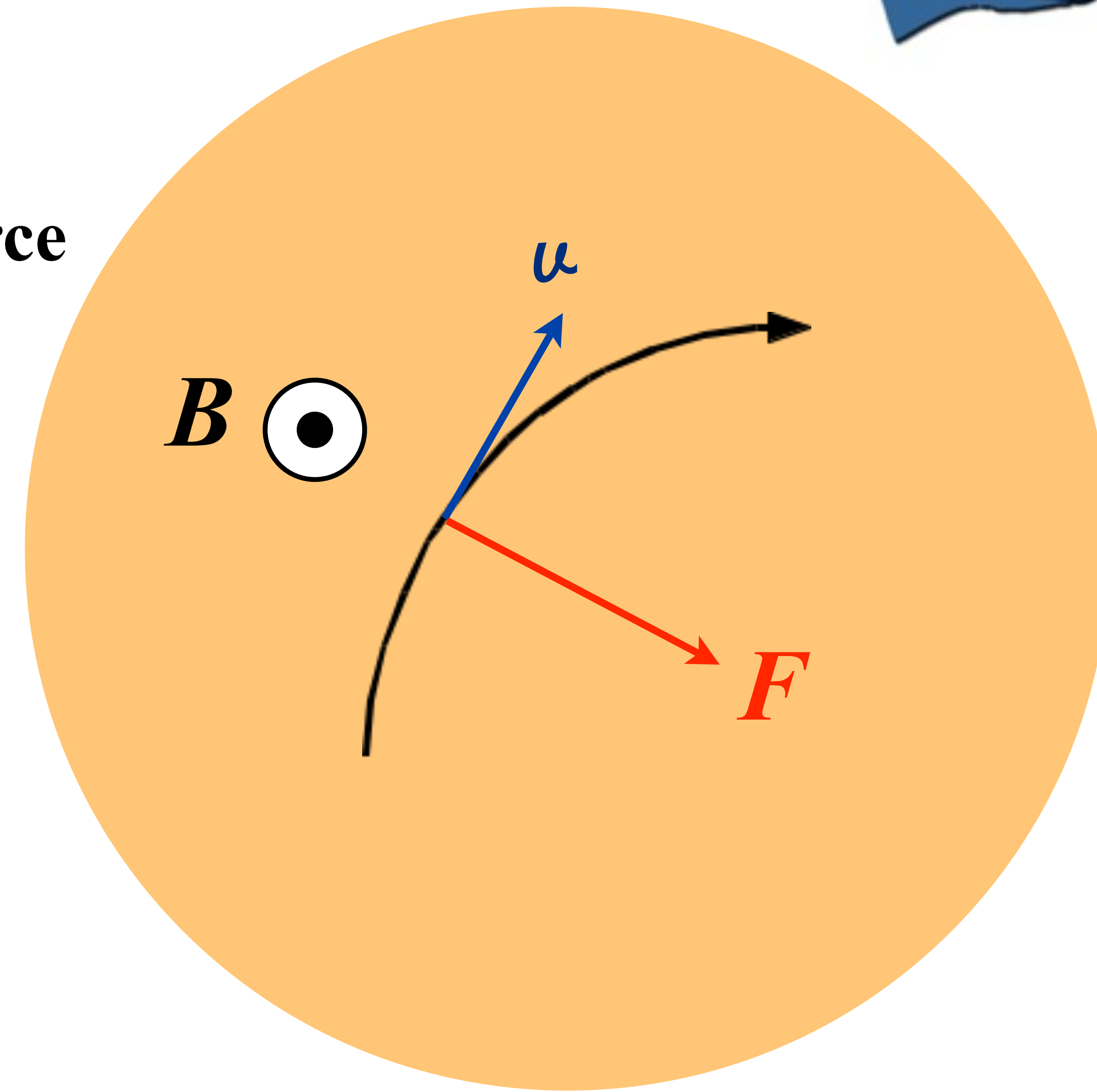


$$F = q\vec{v} \times \vec{B}$$

Centrifugal force

$$qvB = m\frac{v^2}{\rho}$$

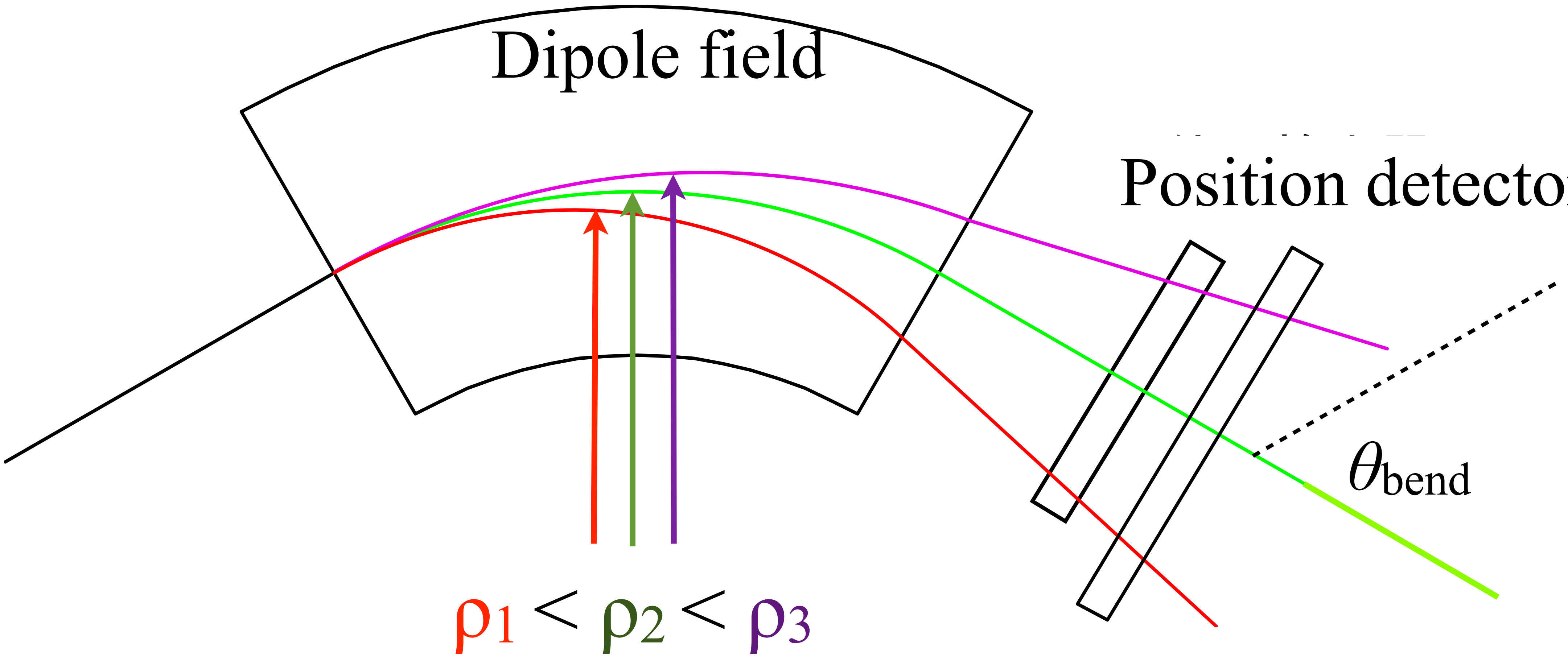
$$= \frac{pv}{\rho}$$



$$\frac{p}{q} = B\rho$$: **Magnetic rigidity**

Scaling factor in ion-optics

Momentum measurement (primitive way)



$$\theta_{\mathbf{bend}} \propto q/p = 1/B\rho_{\mathbf{particle}}$$

$$\frac{\Delta\theta}{\theta_{\mathbf{bend}}} = \frac{\Delta p}{p}$$

Ex. $\theta_{bend} = 60\text{deg} \sim 1$ radian

$\Delta\theta \sim 10^{-3}$ radian

$$\frac{\Delta p}{p} \sim 10^{-3}$$

Calculation of magnetic rigidity

^{78}Ni 200 MeV/nucleon (β ~ 0.6)

1) Momentum/nucleon

$m_N = 931.5$ MeV/c^2

$p_N = \sqrt{2 m_N T_N} = 610$ MeV/c

2) Total momentum of ^{78}Ni

$P = A\, p_N$

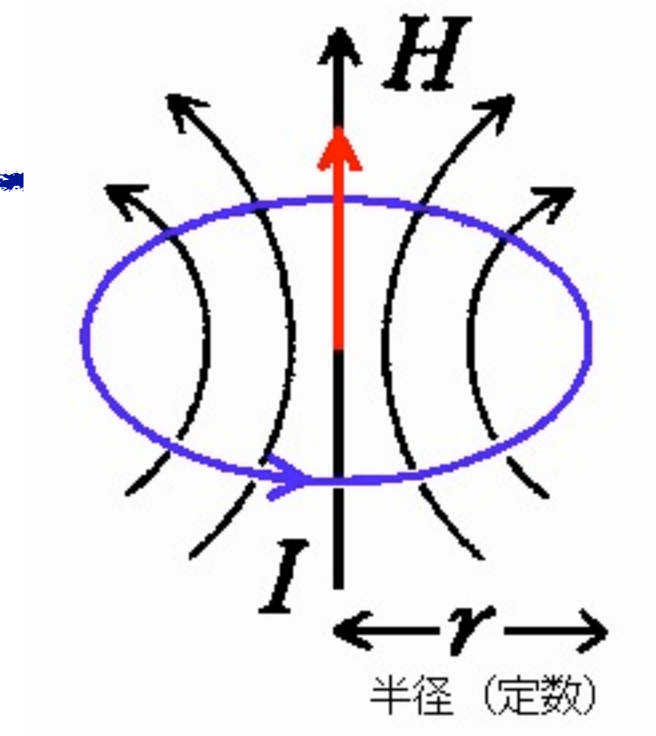
3) Magnetic rigidity of ^{78}Ni

Bρ = P/Z = 78×610 MeV/c / 28 = 1700 MeV/c

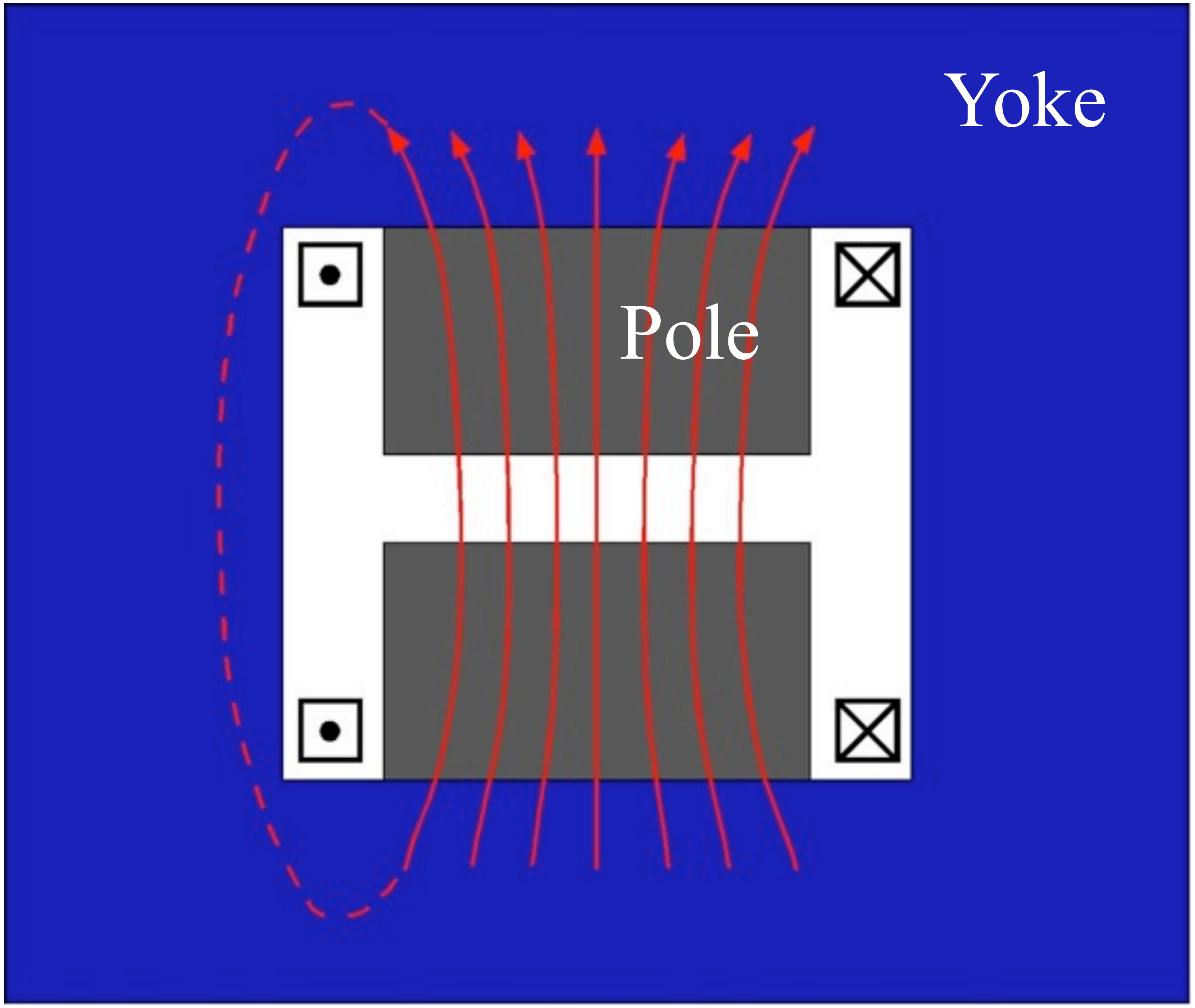
= 5.7 Tm

$$\mathbf{0.3}B\rho\ [\mathrm{Tm}] = p\ [\mathrm{GeV/c}]/q$$

Dipole magnet



How we can produce a dipole magnetic field?



Ampere's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I$$

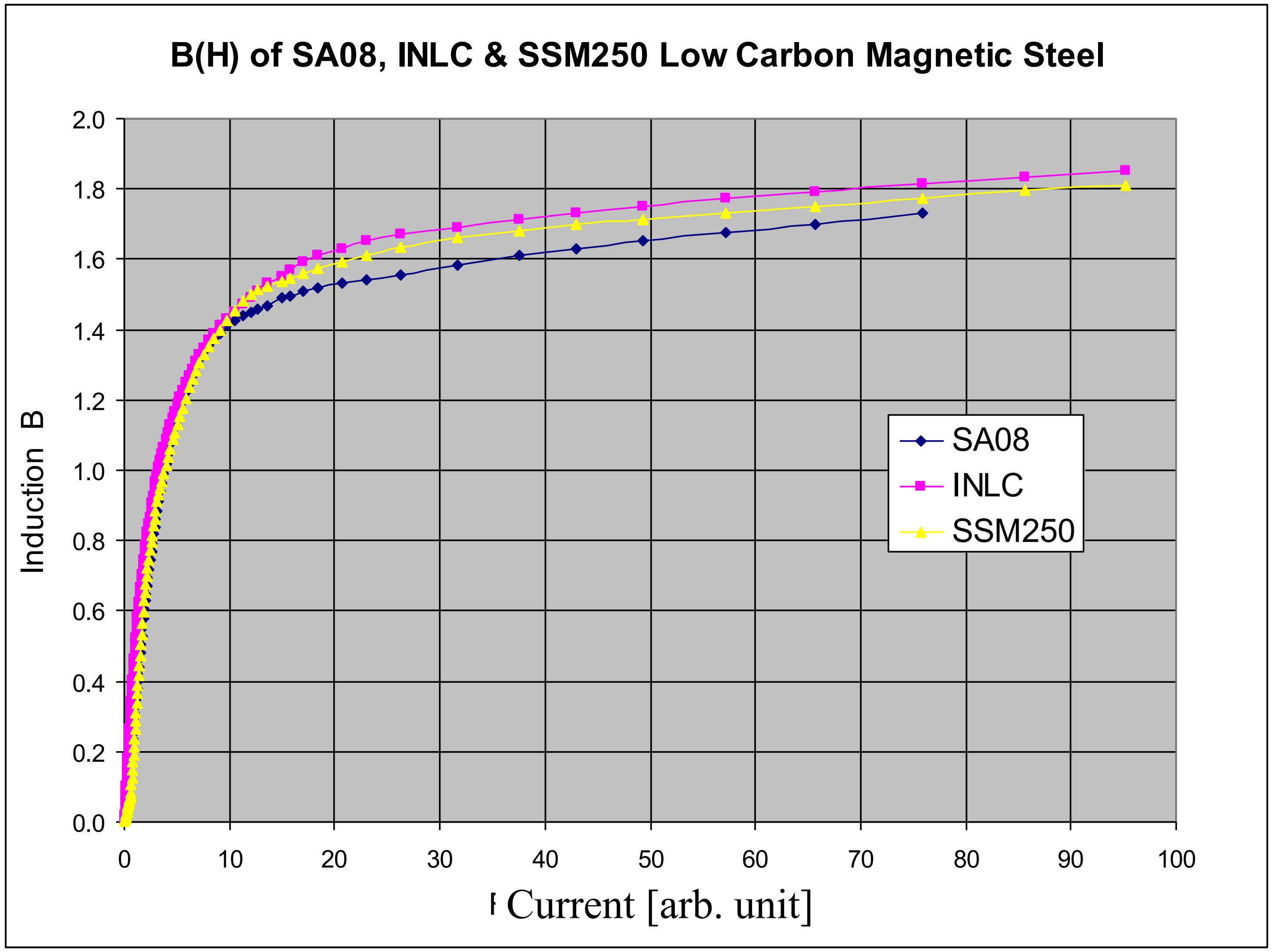
QUESTION:
Why the yoke helps us to produce a higher magnetic field?

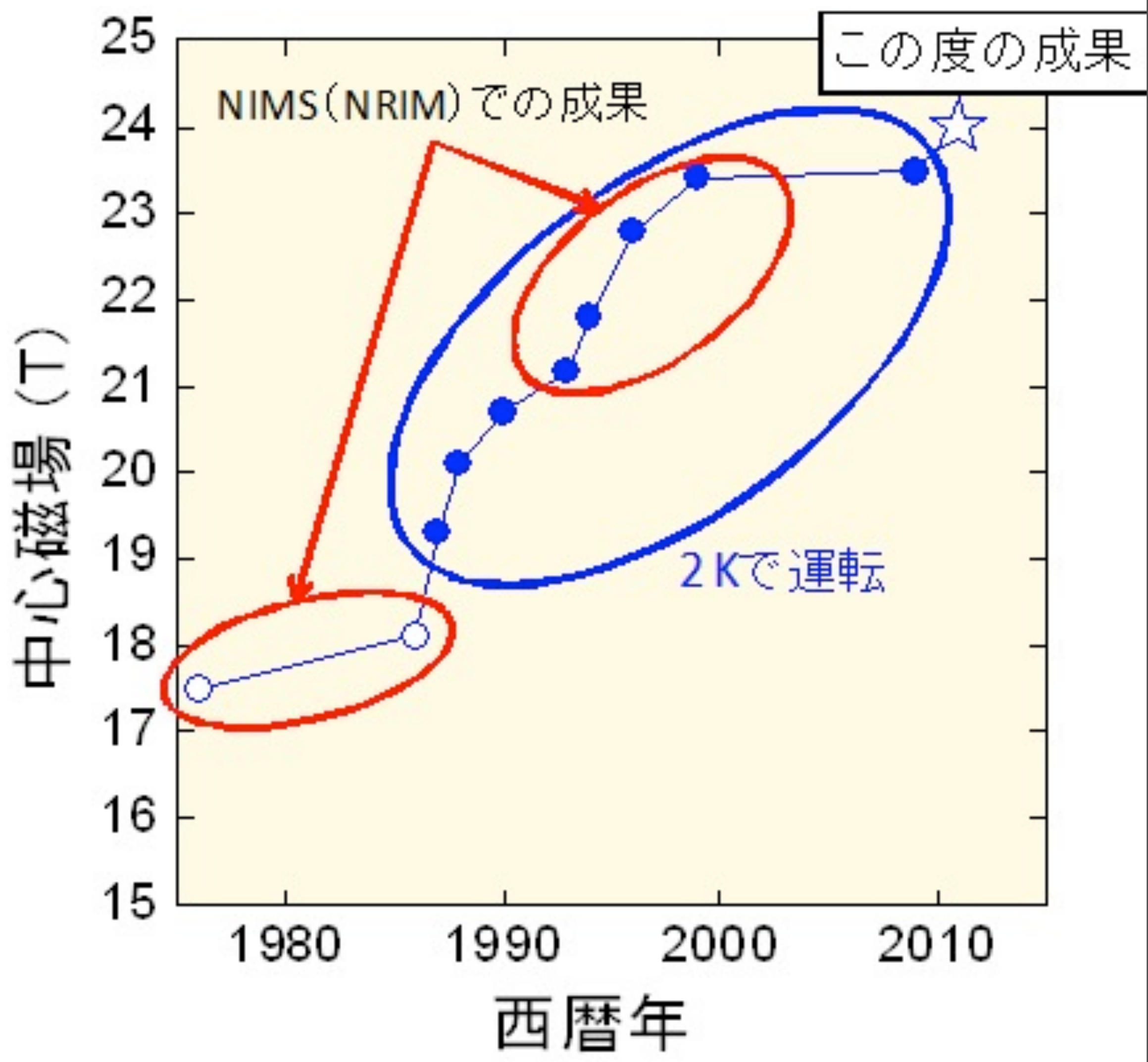
Magnetic field strength

QUESTION: How strong is the geomagnetism?	~50 μT
How strong is the magnetic field of SRC?	~4 T

Normal conducting magnet with iron core

Superconducting magnet





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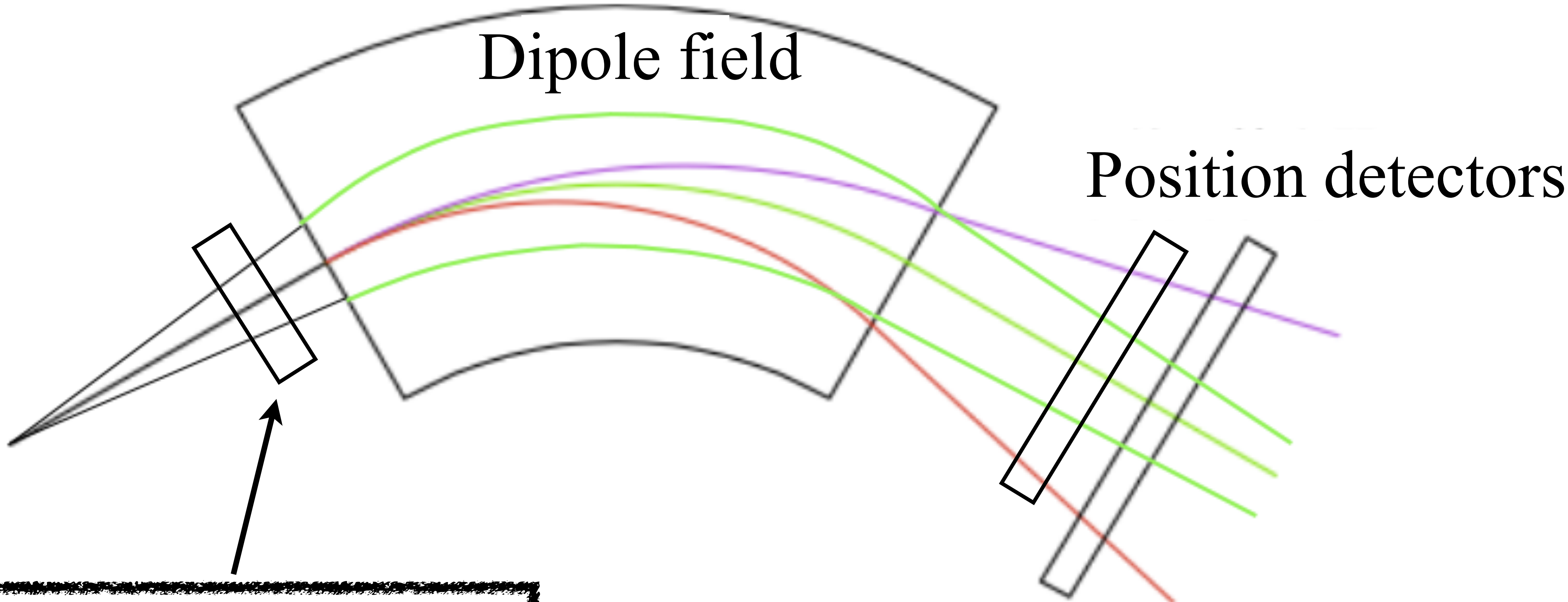
$= 5.7$ Tm

$$0.3 B\rho\ [\mathrm{Tm}] = p\ [\mathrm{GeV/c}]/q$$

Normal-conducting magnet: $B_{max} \sim 1.8$ T

→ ρ ~ 3.2 m

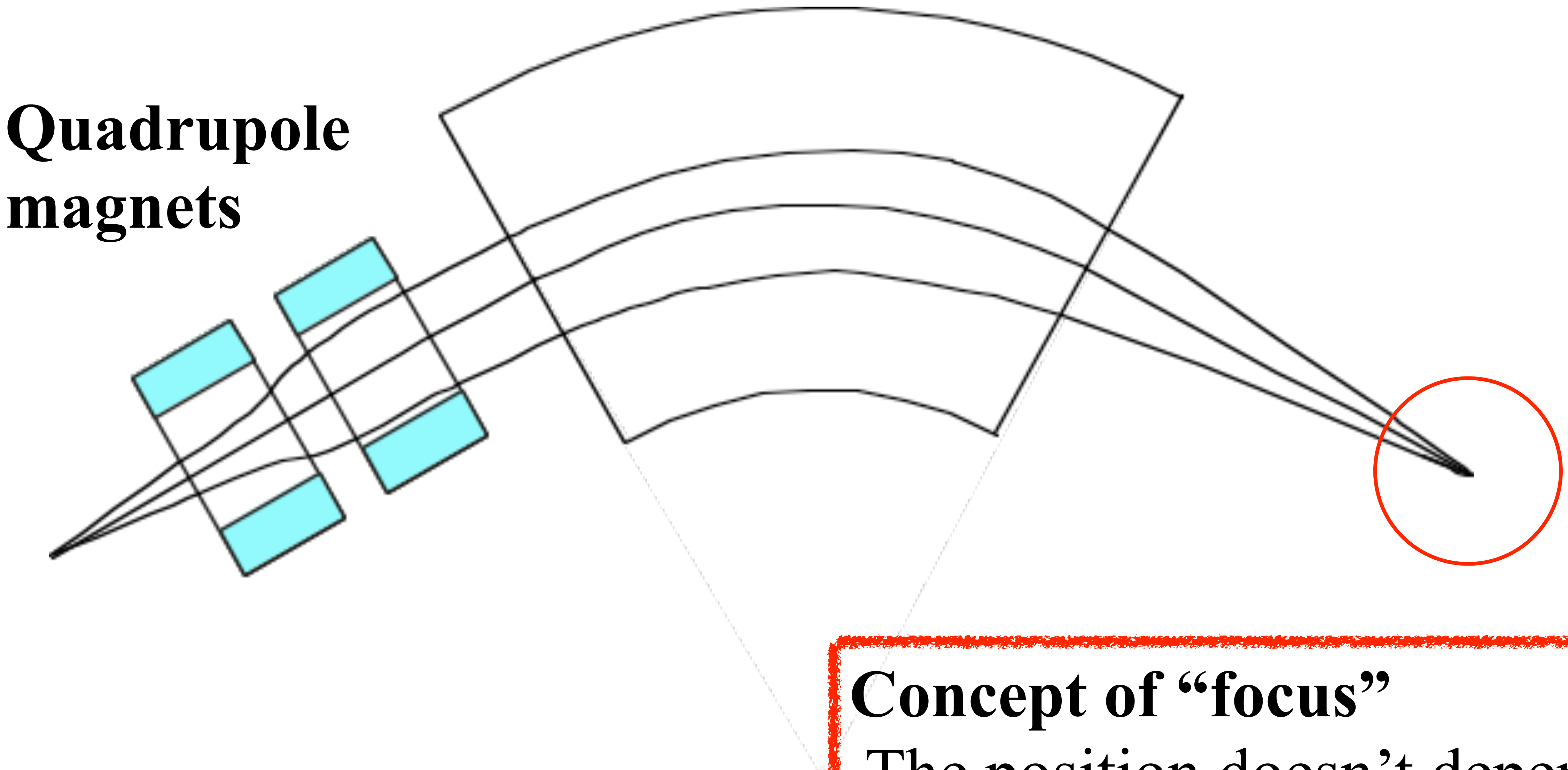
Momentum measurement (primitive way)



Detector(s) to determine incident angle is necessary

The momentum resolution is usually less than 10^{-3}

Momentum measurement (sophisticated way)



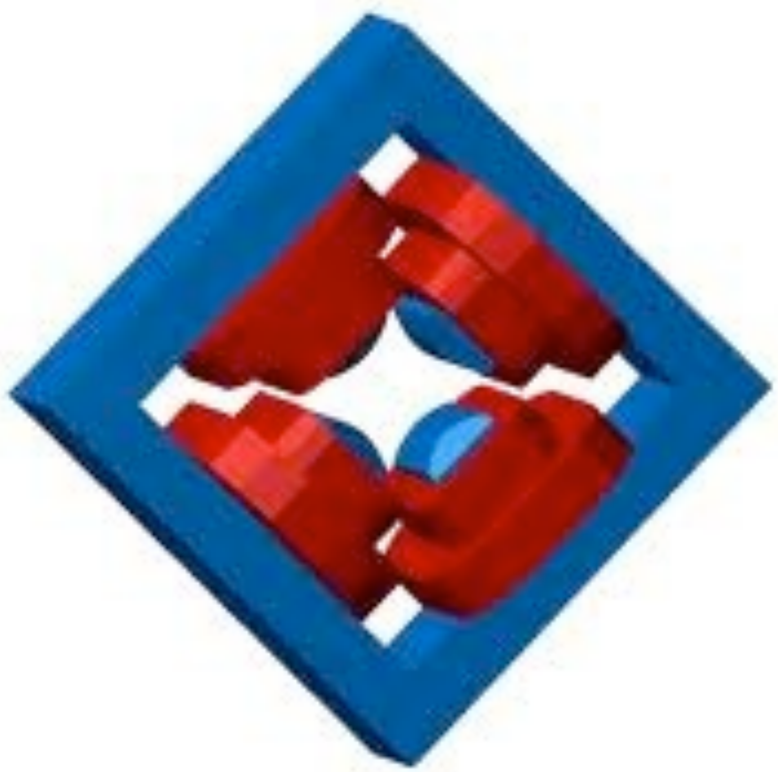
Concept of "focus"

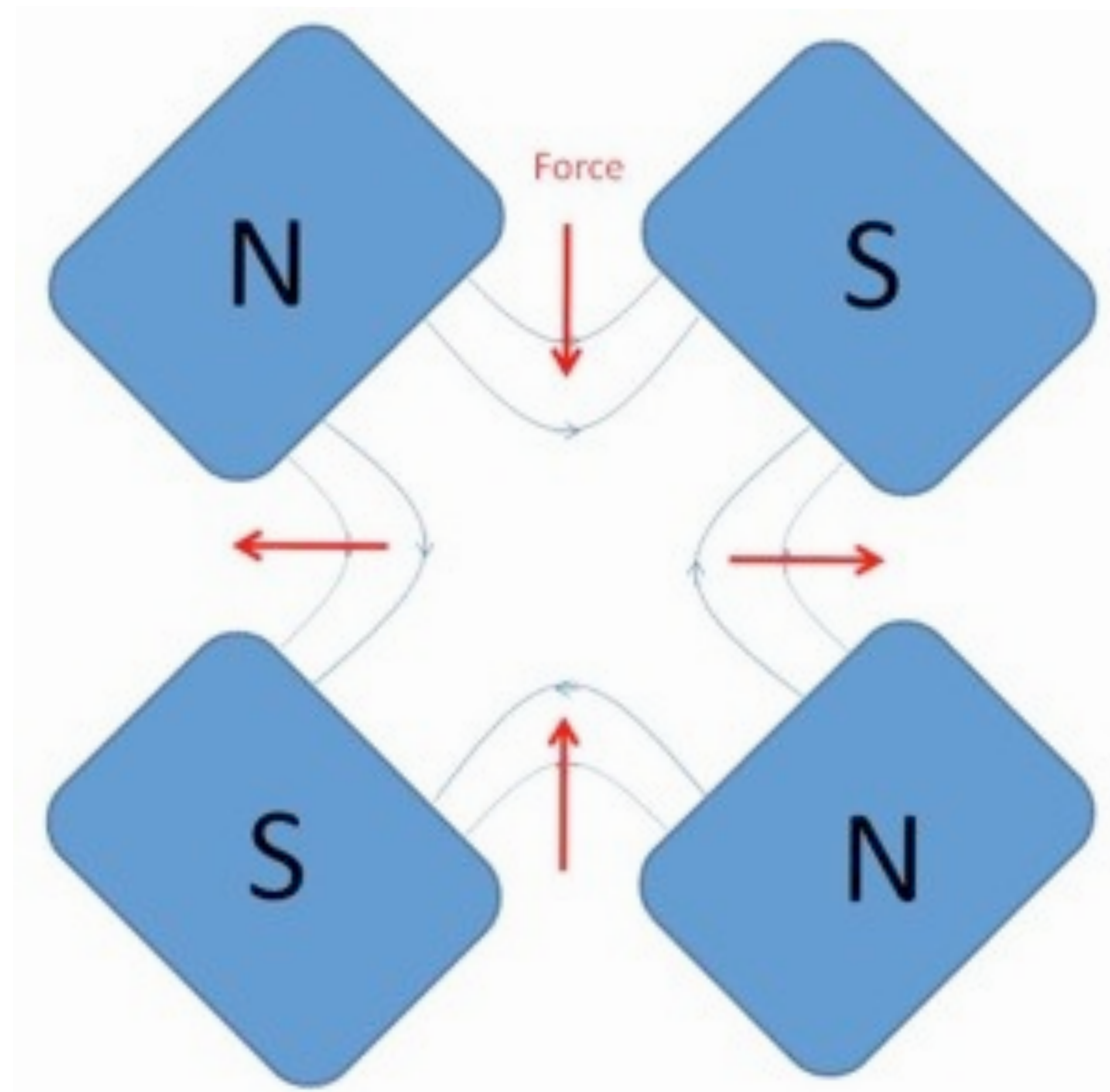
The position doesn't depend on the beam injection angle.

Quadrupole magnet

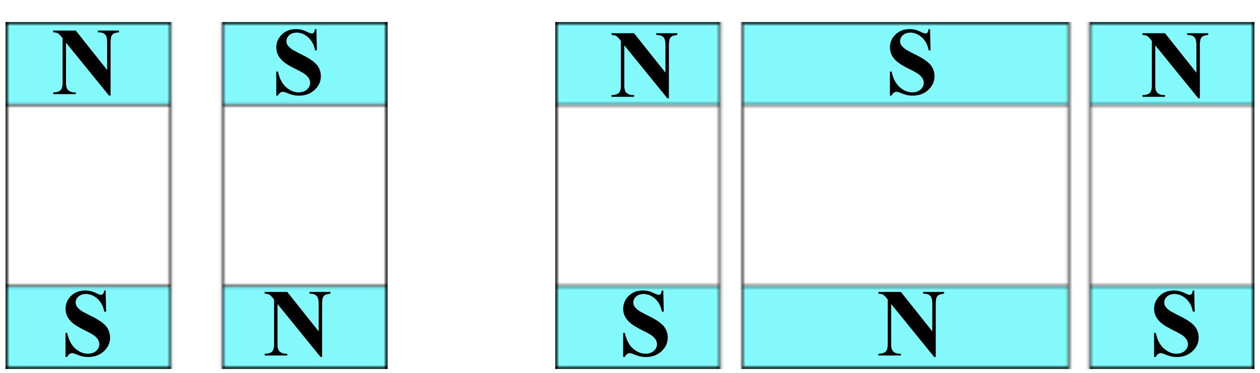
functions as lens

focus charged particles in horizontal or vertical direction

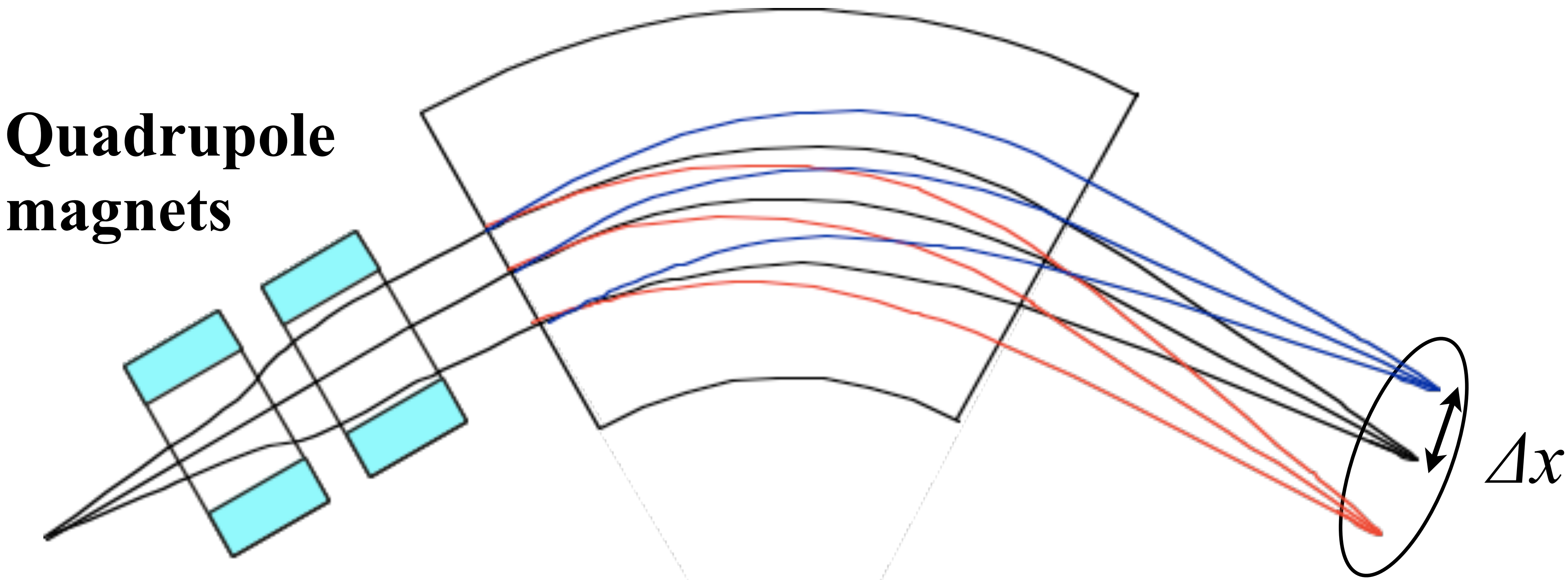




usually used as doublet or triplet



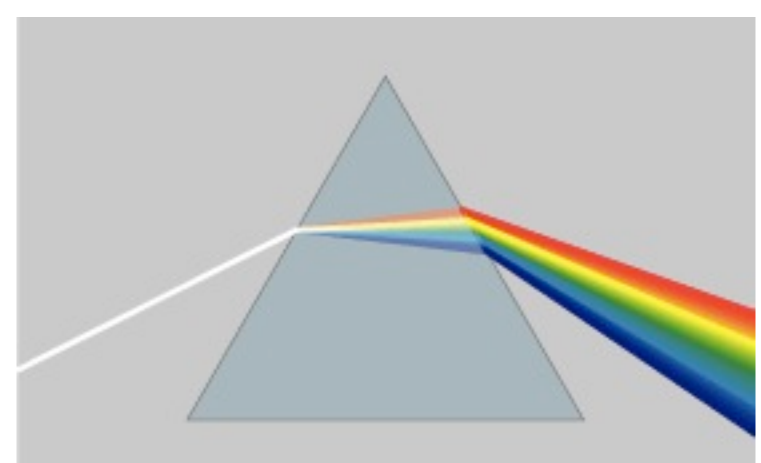
Momentum measurement (sophisticated way)



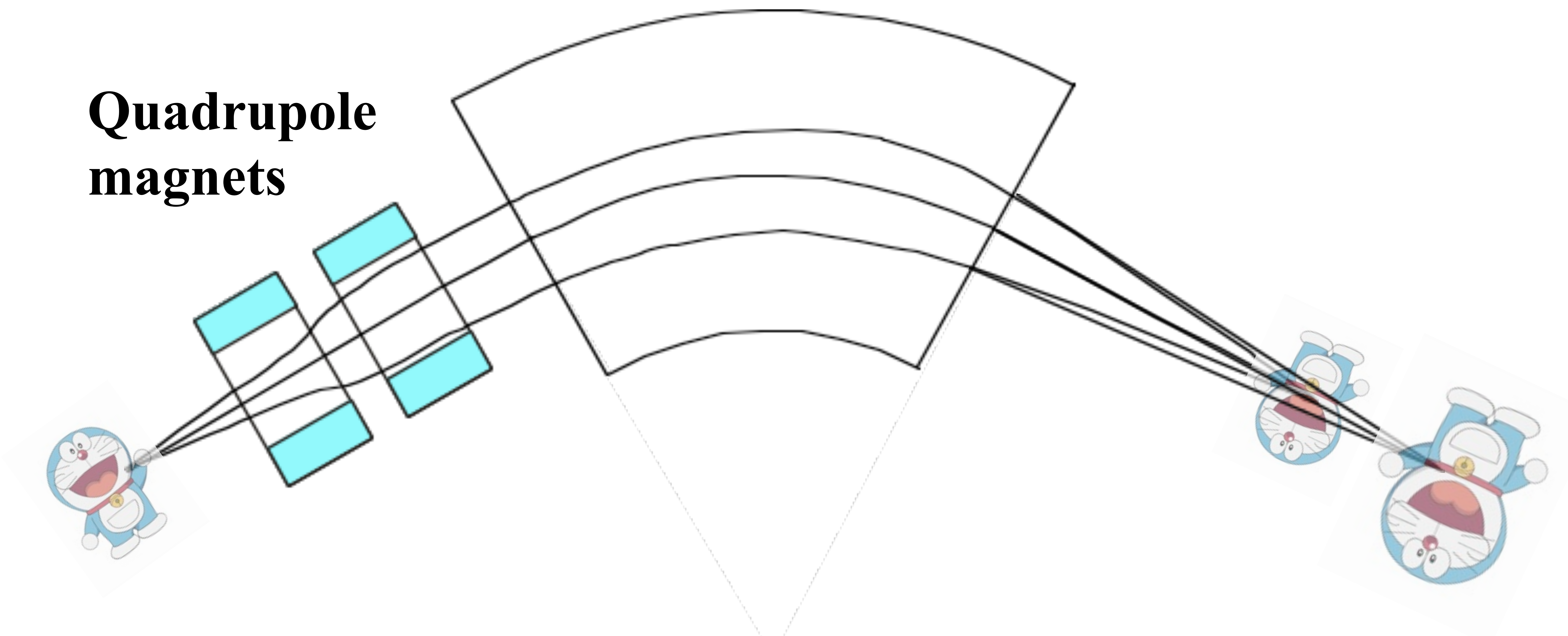
Concept of "dispersion"

position deviation per unit momentum deviation

$$D = \frac{\mathbf{\Delta} x}{\mathbf{\Delta} p}$$

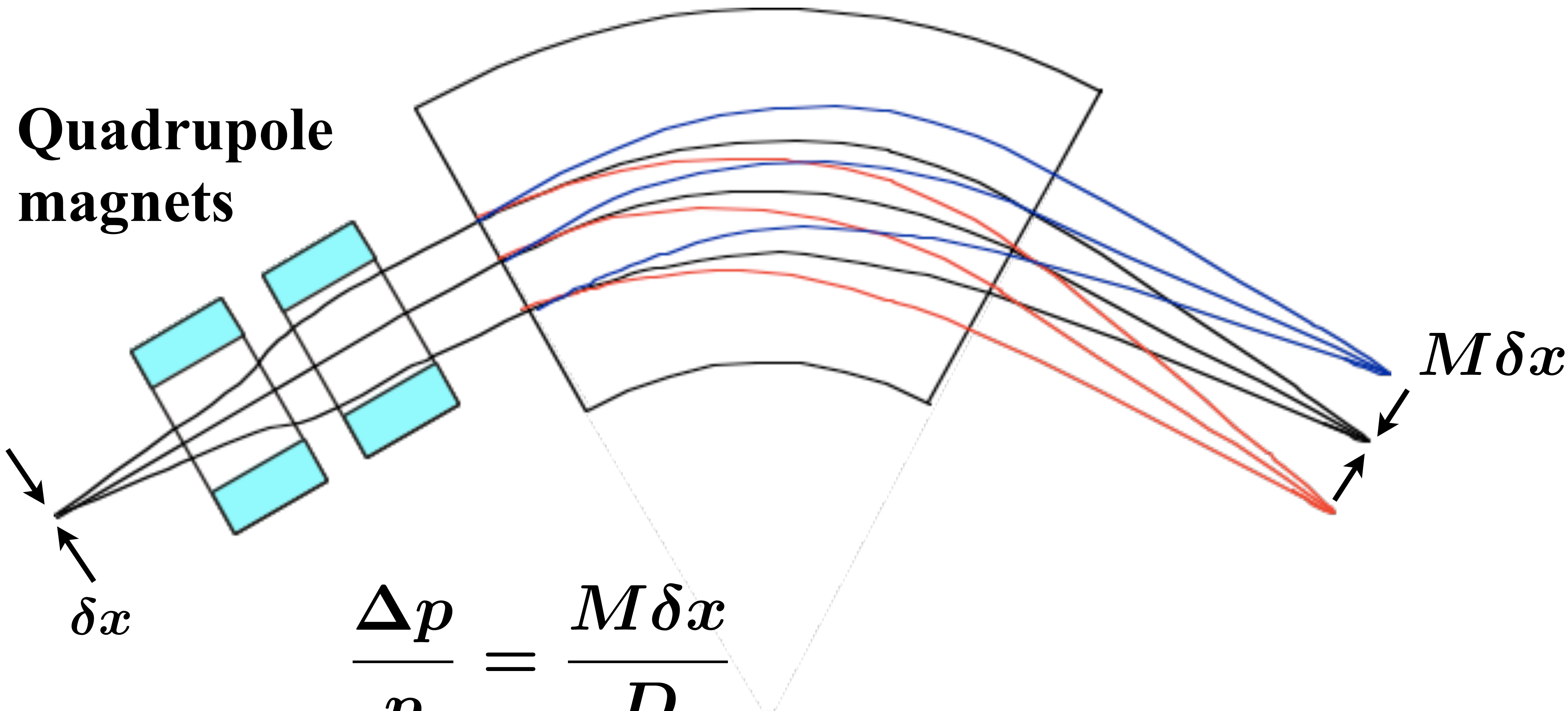


Momentum measurement (sophisticated way)



Concept of “magnification”
Ratio of initial and final image-sizes

Momentum Resolution

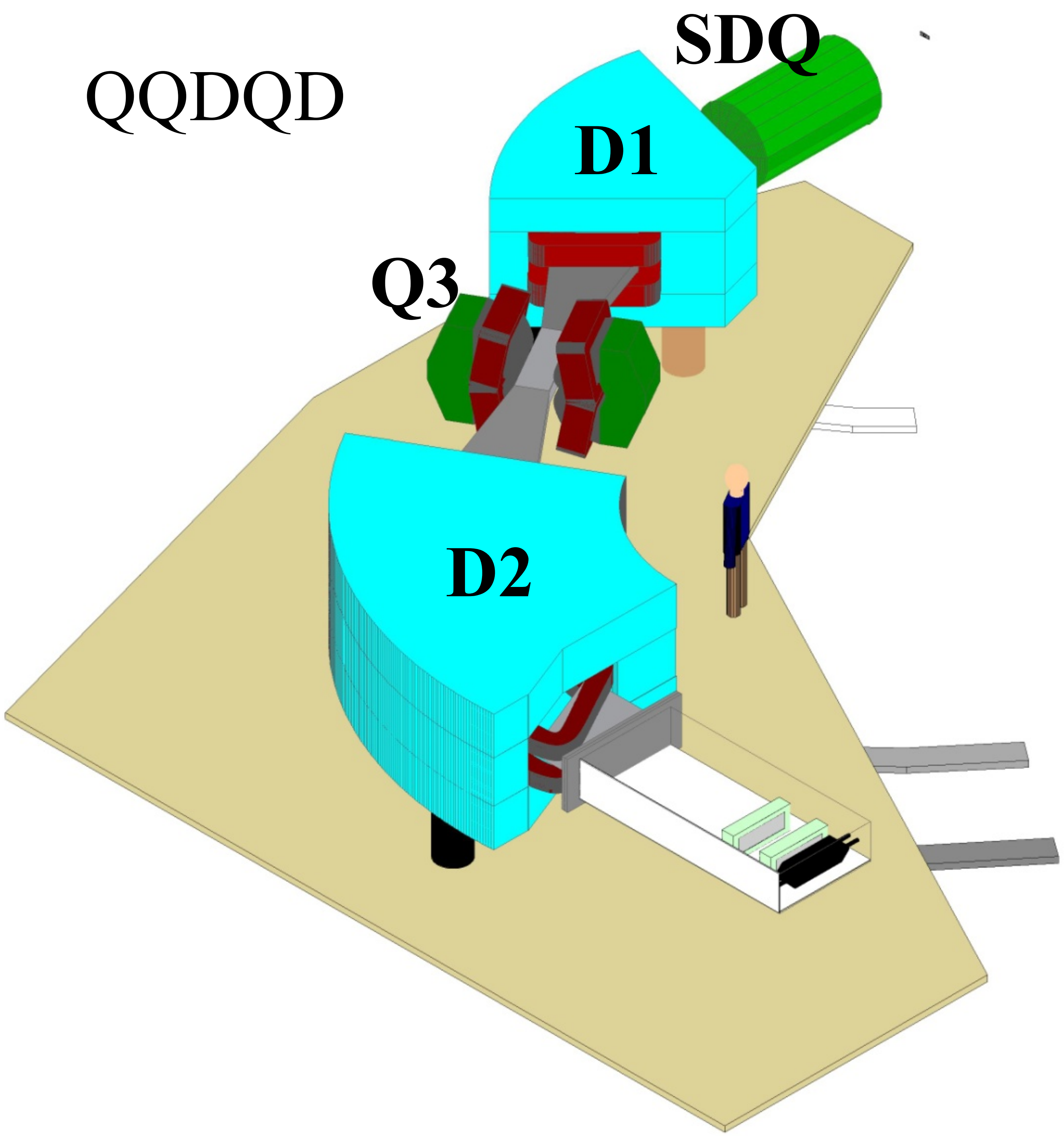


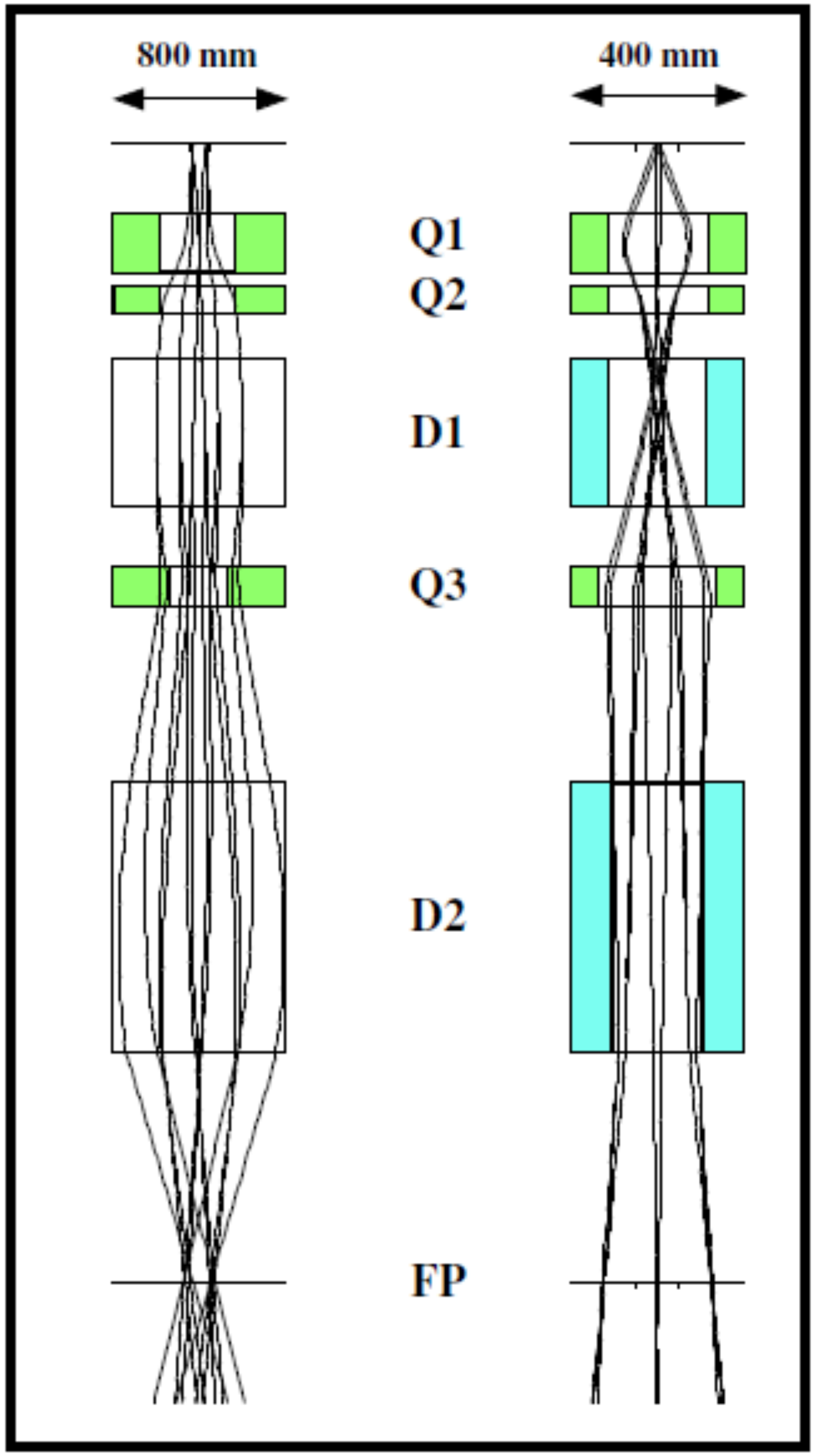
$$\frac{\Delta p}{p} = \frac{M\delta x}{D}$$

Large dispersion and small magnification result in a HIGH momentum resolution

better than 10^{-4} is possible

Actual example : SHARAQ spectrometer





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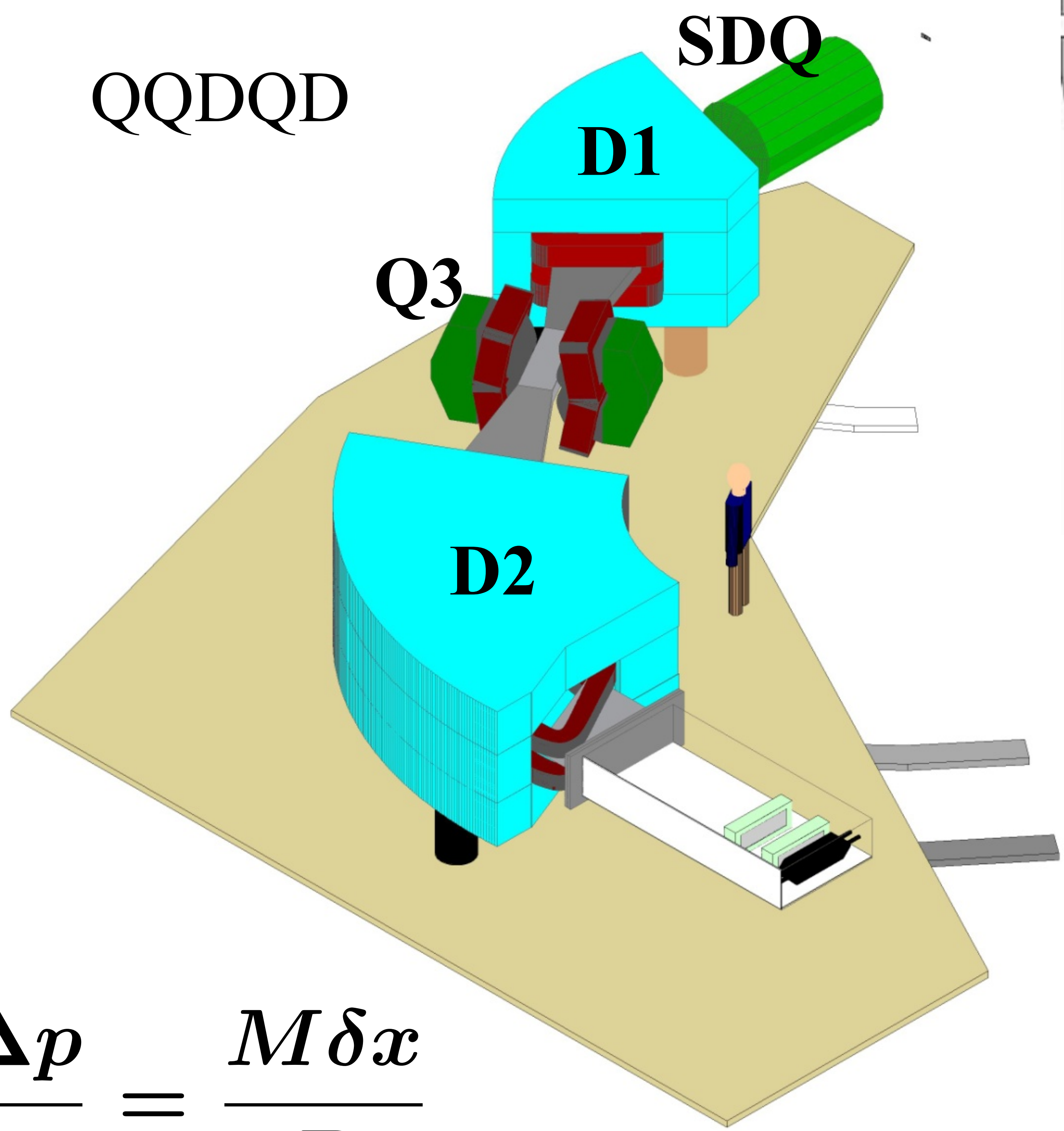
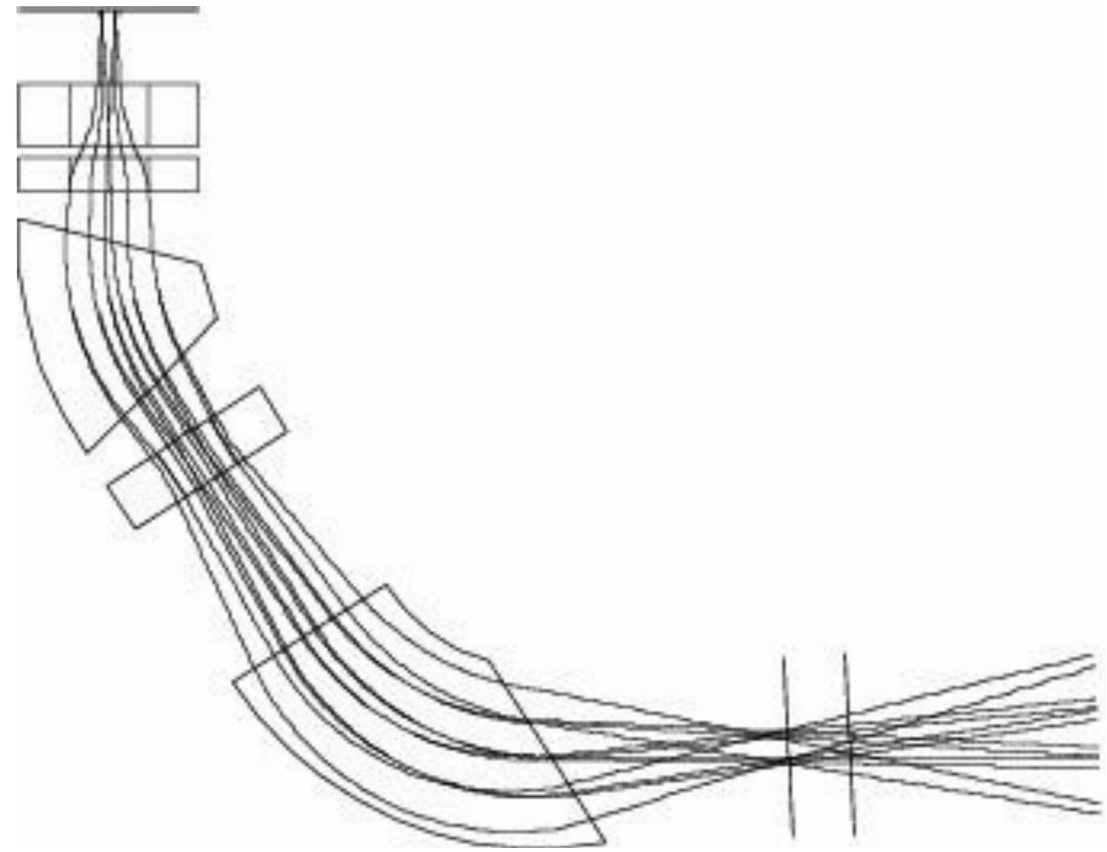




Dispersion D=5.85 m
Magnification M=0.397

Momentum resolution
Δp/p = 1/14700
(δx=1 mm)

$$\frac{\Delta p}{p} = \frac{M\delta x}{D}$$

More general description

$$\begin{pmatrix} x_f \\ \theta_f \\ y_f \\ \phi_f \\ t_f \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & 0 & 0 & 0 & R_{16} \\ R_{21} & R_{22} & 0 & 0 & 0 & R_{26} \\ 0 & 0 & R_{33} & R_{34} & 0 & 0 \\ 0 & 0 & R_{43} & R_{44} & 0 & 0 \\ R_{51} & R_{52} & 0 & 0 & 1 & R_{56} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_i \\ \theta_i \\ y_i \\ \phi_i \\ t_i \\ \delta \end{pmatrix}$$

$\boldsymbol{R_{11}}$

(horizontal) magnification

$\boldsymbol{R_{16}}$

dispersion

$\boldsymbol{R_{12} = 0}$

focus condition

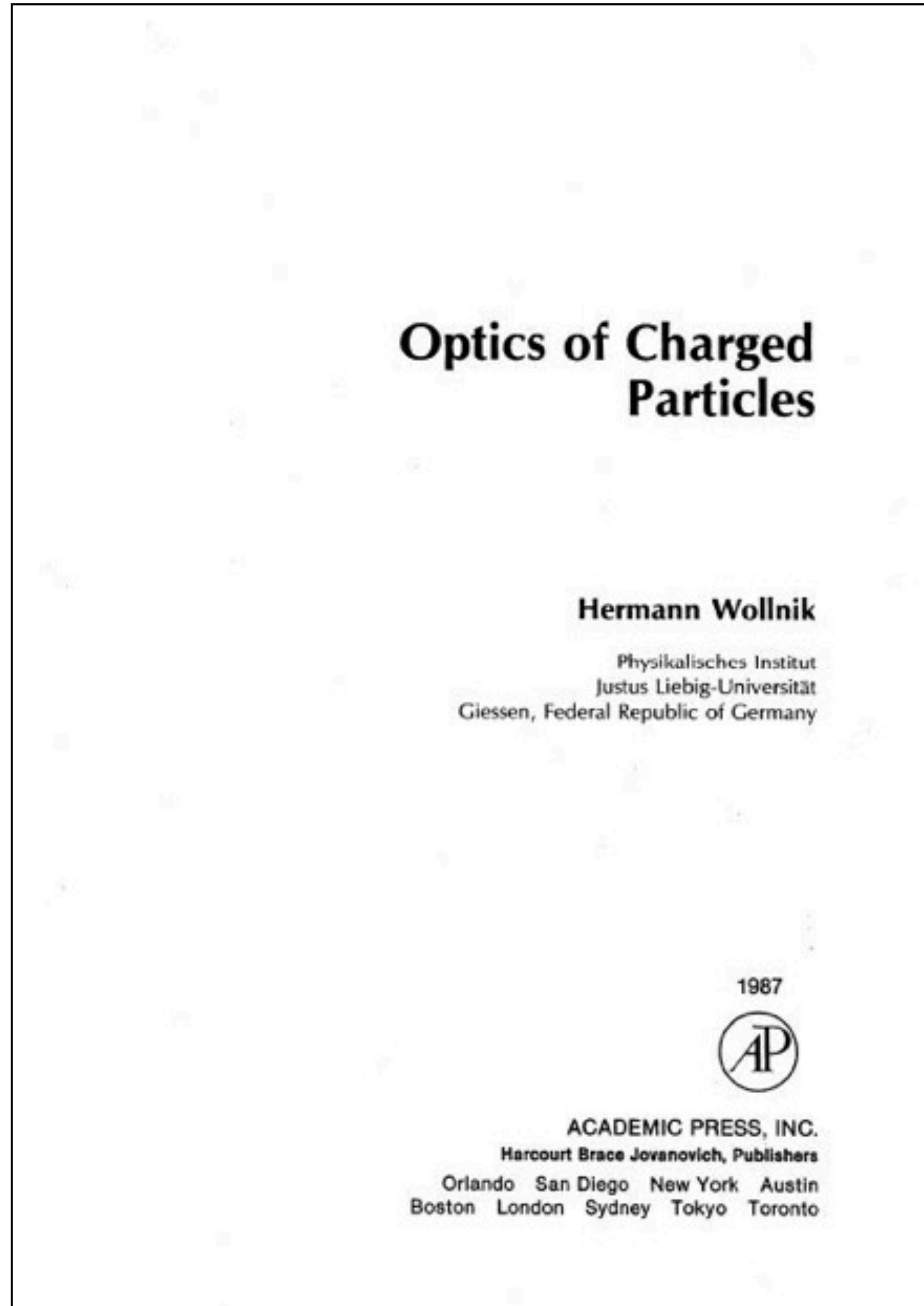
Matrix of SHARAQ

$$\begin{pmatrix} -0.3974 & -0.0000 & 0.0000 & 0.0000 & 0.0000 & -5.8582 \\ -0.7727 & -2.5164 & 0.0000 & 0.0000 & 0.0000 & 0.6608 \\ 0.0000 & 0.0000 & -0.0000 & -2.3039 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.4340 & -0.1971 & 0.0000 & 0.0000 \\ -0.2948 & -0.9073 & 0.0000 & 0.0000 & 1.0000 & -0.0279 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Further reading

"Optics of charged particles"

Hermann Wollnik


Optics of Charged Particles

Hermann Wollnik

Physikalisches Institut
Justus Liebig-Universität
Giessen, Federal Republic of Germany

1987

ACADEMIC PRESS, INC.
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Orlando San Diego New York Austin
Boston London Sydney Tokyo Toronto

If you want to make ion-optics calculations

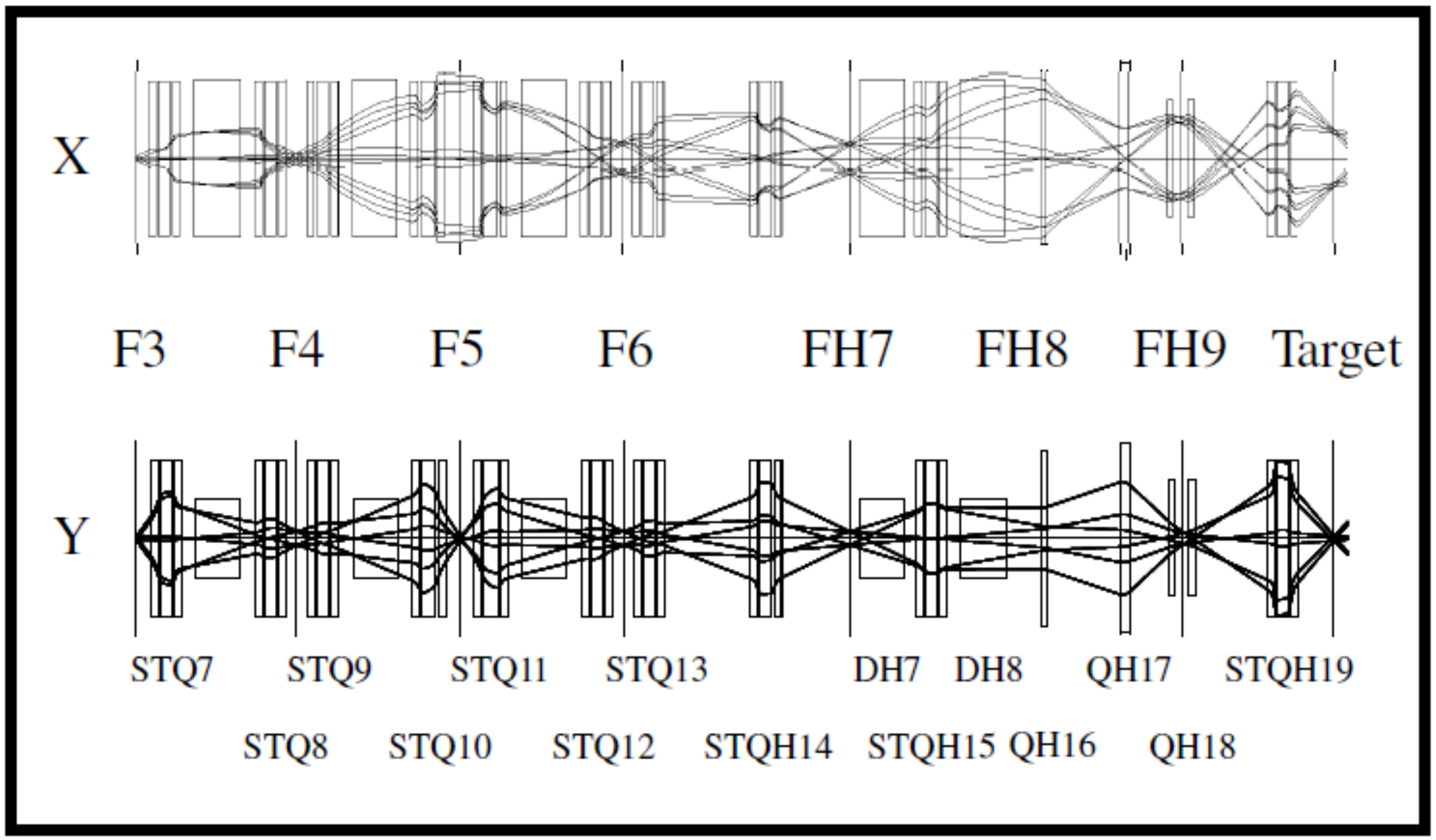
GIOS, GICOSY

http://www-linux.gsi.de/~weick/gios/

http://www-linux.gsi.de/~weick/gicosy/

COSY Infinity

http://bt.pa.msu.edu/index_cosy.htm



Messages

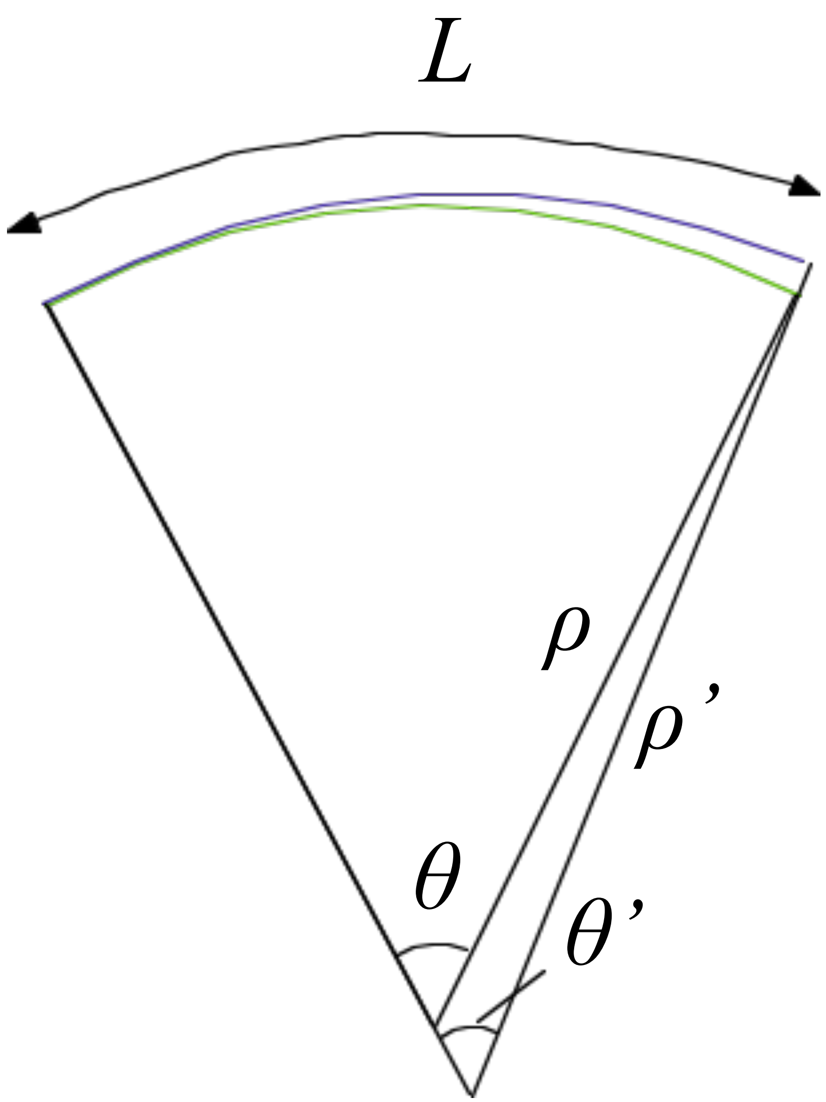
Ion-optical (magnetic) analysis is a versatile technique
in nuclear and particle physics experiments.

Once you're familiar with it, you can use it in a variety of occasions as

High-resolution momentum measurement with a spectrometer
RI-beam production & tagging with a RI-beam separator
Beam acceleration & transport with an accelerator&beam-line

Addendum

Proof of “ $\theta_{\text{bend}} \propto q/p = 1/B\rho_{\text{particle}}$ ”



The lower(higher)-momentum particle is bended with a bending radius of ρ (ρ’).

Assumed that the length of arc L is same for the two particles, the bending angles are written for the particles as

$$\theta = \frac{L}{2\pi\rho}$$

$$\theta' = \frac{L}{2\pi\rho'}$$

Since magnetic field strength B is common for two particles, the bending angle is inversely proportional to the magnetic rigidity.

$$\frac{\theta'}{\theta} = \frac{B\rho}{B\rho'}$$