Lecture 5 Nuclear Theory

Microscopic description of nuclear structure

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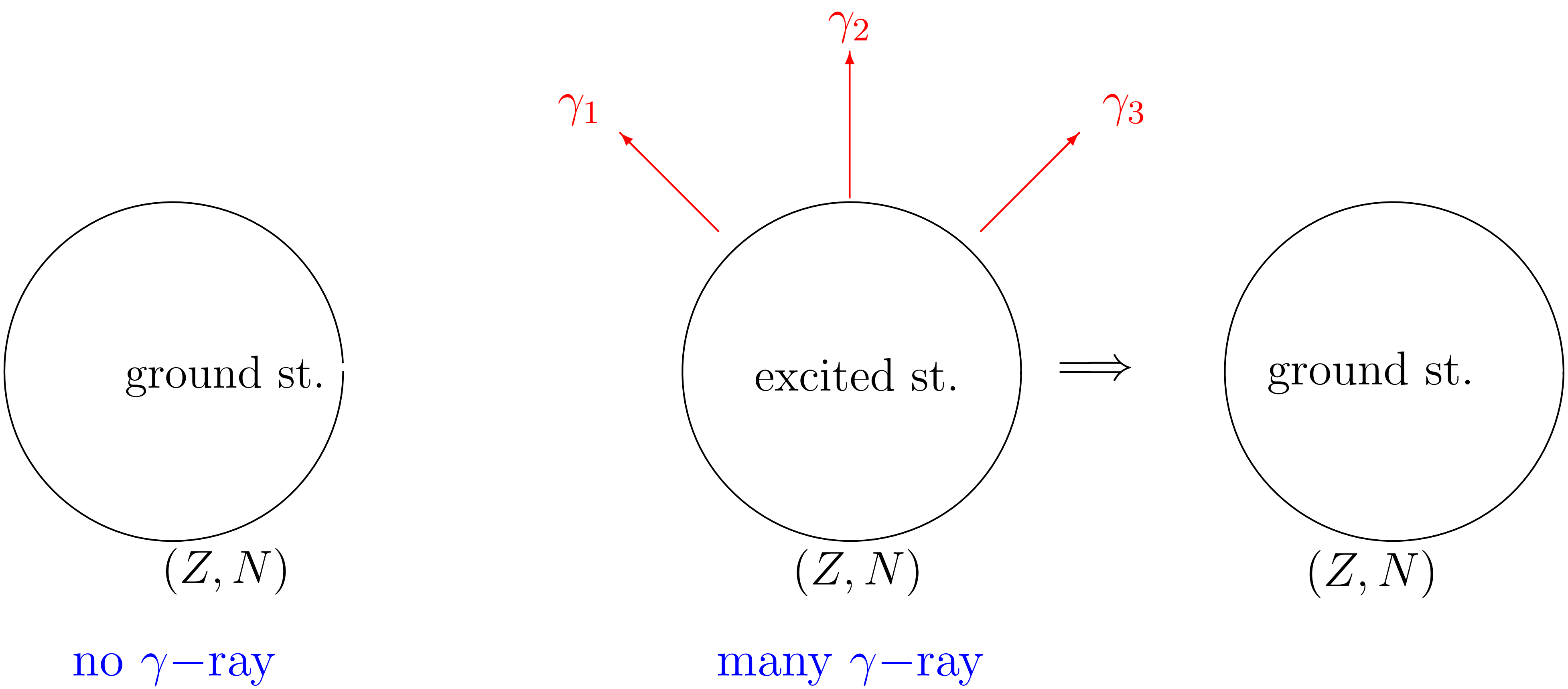
1. **Introduction**
2. **Magic numbers and shell structure in nuclei**
3. **One- and two-particle nuclei**
4. **Effective interaction between identical particles**
5. **Proton-neutron interaction and isomers**
6. **Summary**

1. Introduction — γ spectroscopy —



from γ−ray, we can know the energy levels.

after γ_1

$$\begin{cases}\gamma_2 & \text{no}\\ \gamma_3 & \text{yes}\end{cases}$$

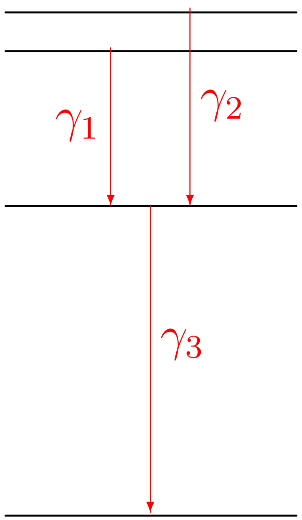
after γ_2

$$\begin{cases}\gamma_1 & \text{no}\\ \gamma_3 & \text{yes}\end{cases}$$

after γ_3

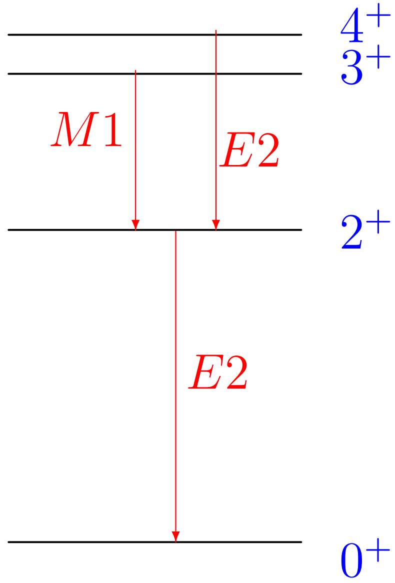
$$\begin{cases}\gamma_1 & \text{no}\\ \gamma_2 & \text{no}\end{cases}$$

$\Longrightarrow$



from coincidence

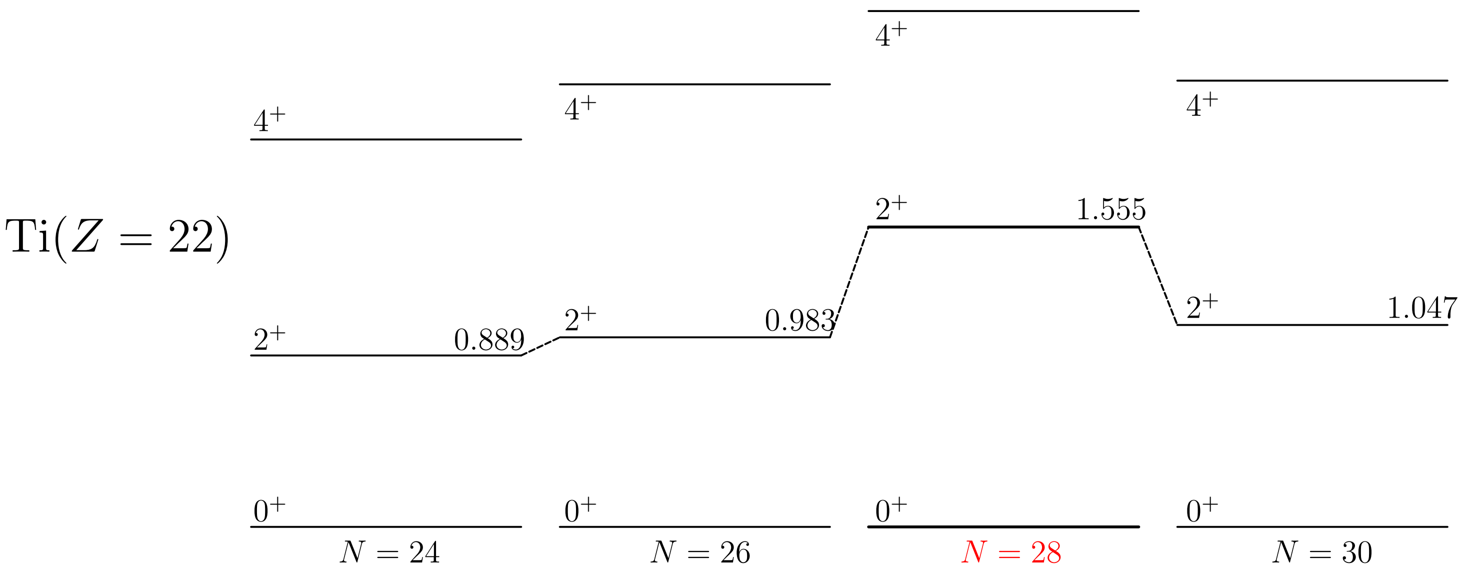
$\Longrightarrow$



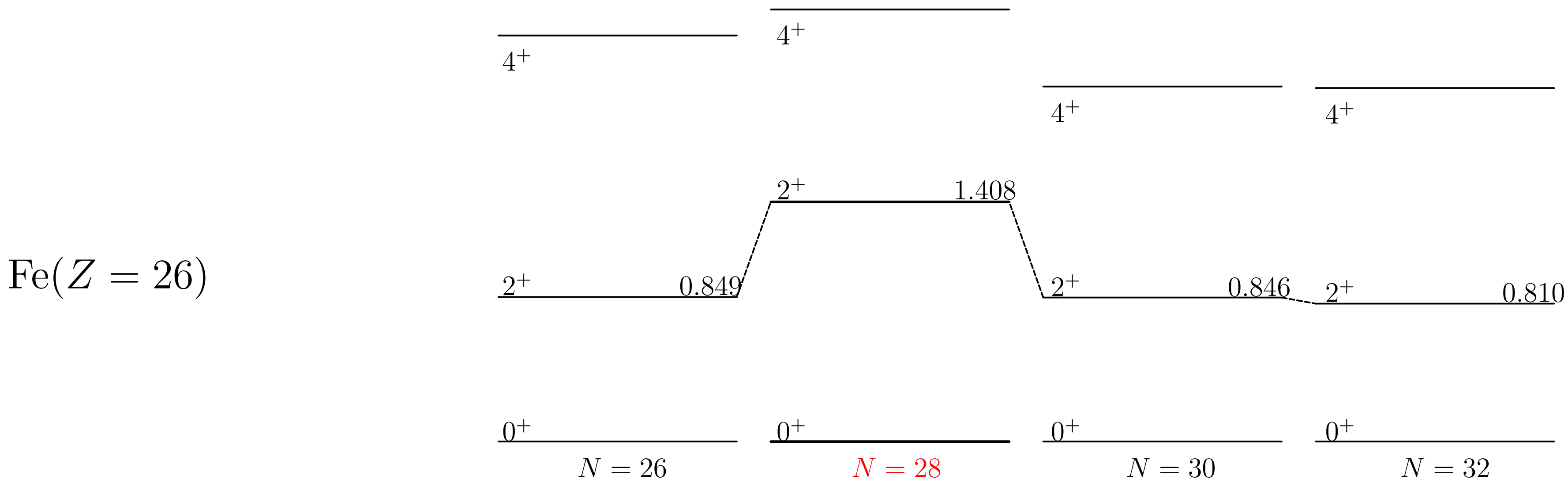
from multipolarity

2. Magic numbers

Energy spectra of Ti-isotopes



Energy spectra of Fe-isotopes



Stability of $N = 28$!!

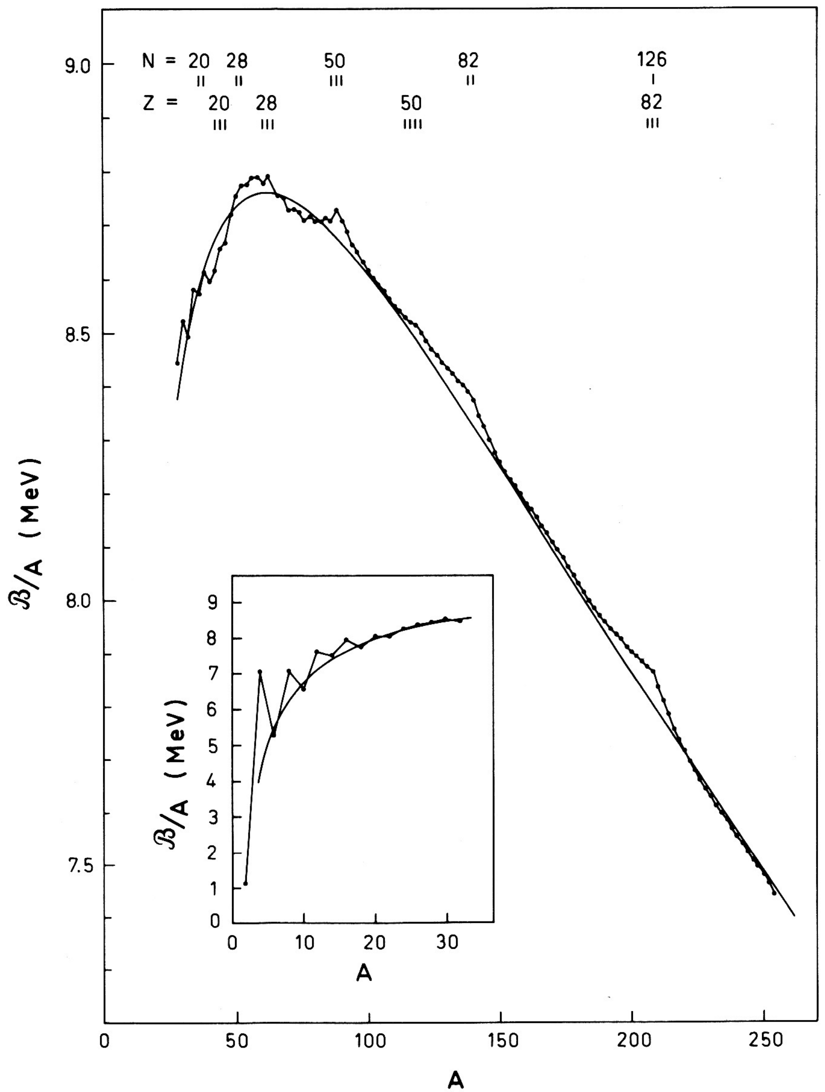
Nuclear Binding Energy

$$\mathcal{B}(Z,N) = ZM_p + NM_n - \mathcal{M}(Z,N)$$

Comparison of $\mathcal{B}_{exp}$ and $\mathcal{B}_{theory}$(liquid drc

$$\Downarrow$$

obvious discrepancy at $N, Z = 20, 28, 50, \cdot$



Magic Numbers

$$Z = 2,\ \ 8,\ \ 20,\ \ 28,\ \ 50,\ \ 82$$

$$N = 2,\ \ 8,\ \ 20,\ \ 28,\ \ 50,\ \ 82,\ \ 126$$

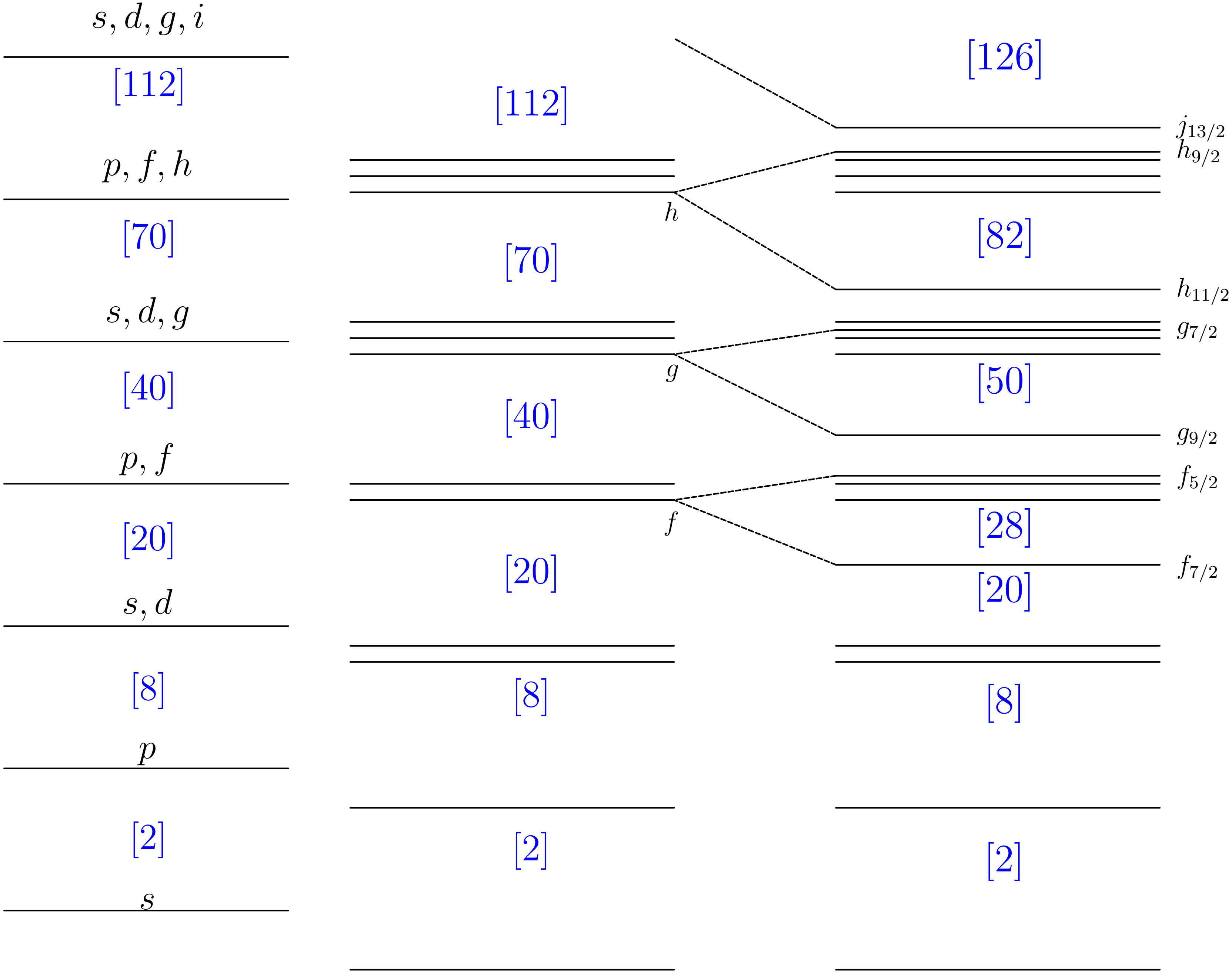
We needed 40 years for understanding these magic numbers !!

Hint: noble gas in atomic system $Z = 2, 10, 18, 36, 54, \cdots$

$$\Downarrow$$

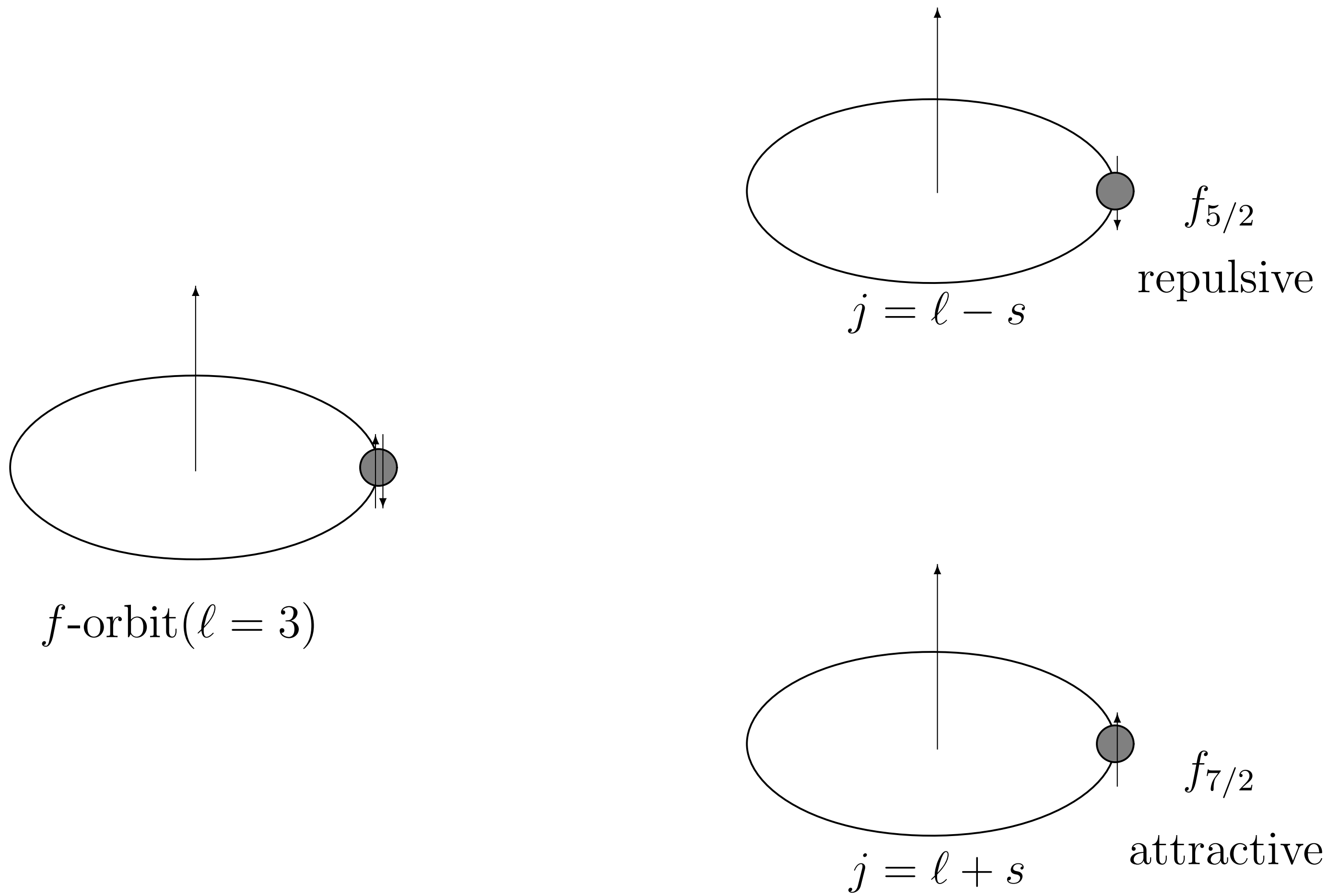
shell structure in nuclei!!(1949 M.G.Mayer and J.H.D.Jensen)

Magic numbers in nuclei



Spin-orbit Splitting

$$-\xi(\boldsymbol{\ell} \cdot \boldsymbol{s})$$



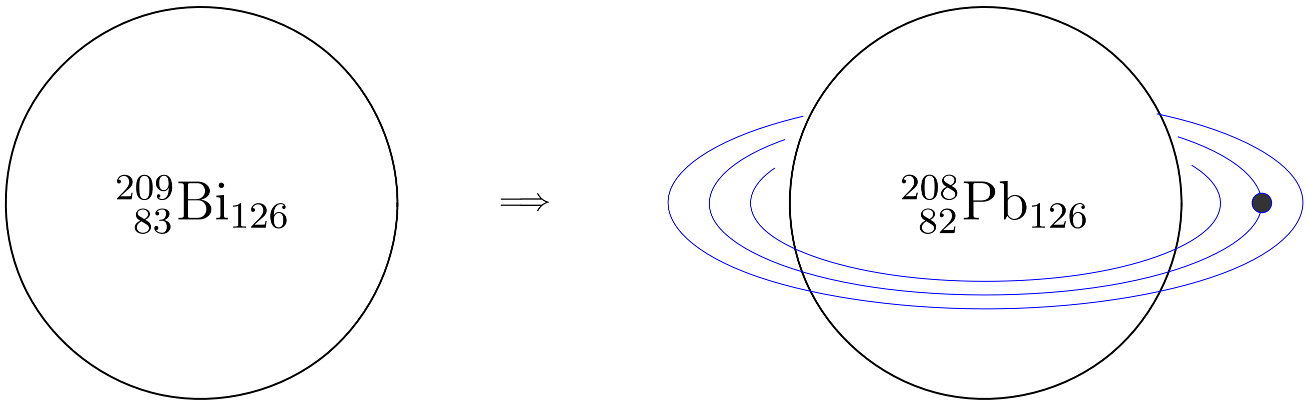
3. One- and Two-particle nuclei

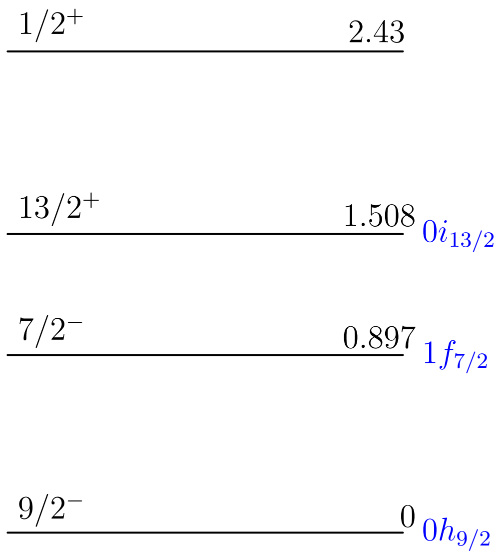
Magic numbers provide us inert core nuclei,

$$\text{e.g. } {}^{4}_{2}\mathrm{He}_{2},\quad {}^{16}_{8}\mathrm{O}_{8},\quad {}^{40}_{20}\mathrm{Ca}_{20},\quad {}^{48}_{20}\mathrm{Ca}_{28},\quad {}^{208}_{82}\mathrm{Pb}_{126}$$

• one-particle nuclei

$${}^{209}_{83}\mathrm{Bi}_{126} \Longrightarrow {}^{208}_{82}\mathrm{Pb}_{126} + p$$

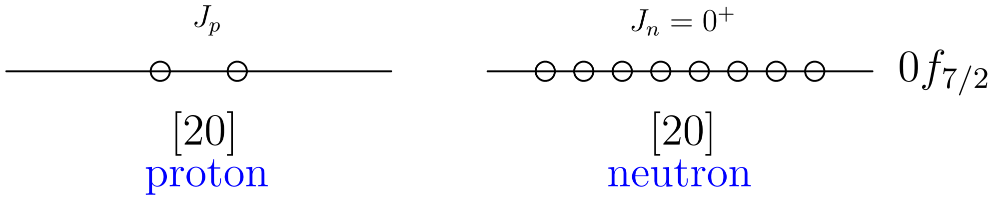




single-particle states

• two-particle nuclei

$${}^{50}_{22}\mathrm{Ti}_{28} \implies {}^{48}_{20}\mathrm{Ca}_{28} \;+\; 2p$$



Possible spin-states J in $(0f_{7/2})^2$-configuration $\implies$ $J_p = 0^+,\ 2^+,\ 4^+,\ 6^+$

$\vec{J} = \vec{7/2} + \vec{7/2}$

classical mechanic	$J = 0.0 \sim 7.0$
quantum mechanics	$J = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7$
fermion statistics	$J = 0,\ 2,\ 4,\ 6$

m-scheme for fermion system

Example $(d_{3/2})^2$

m=3/2	1/2	-1/2	-3/2	M
$\times$	$\times$			2
$\times$		$\times$		1
$\times$			$\times$	0
	$\times$	$\times$		0
	$\times$		$\times$	-1
		$\times$	$\times$	-2

$$M = 2,\ 1,\ 0,\ -1,\ -2 \quad \Longrightarrow \quad J = 2^+$$

$$M = 0 \quad \Longrightarrow \quad J = 0^+$$

m-scheme for boson system

Example $(d)^2$

m=2	1	0	-1	-2	M
$\times\times$					4
$\times$	$\times$				3
$\times$		$\times$			2
$\times$			$\times$		1
$\times$				$\times$	0
	$\times\times$				2
	$\times$	$\times$			1
	$\times$		$\times$		0
	$\times$			$\times$	-1
		$\times\times$			0
		$\times$	$\times$		-1
		$\times$		$\times$	-2
			$\times\times$		-2
			$\times$	$\times$	-3
				$\times\times$	-4

Problem: Derive all possible J in the $f^2_{7/2}$-configuration.

m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

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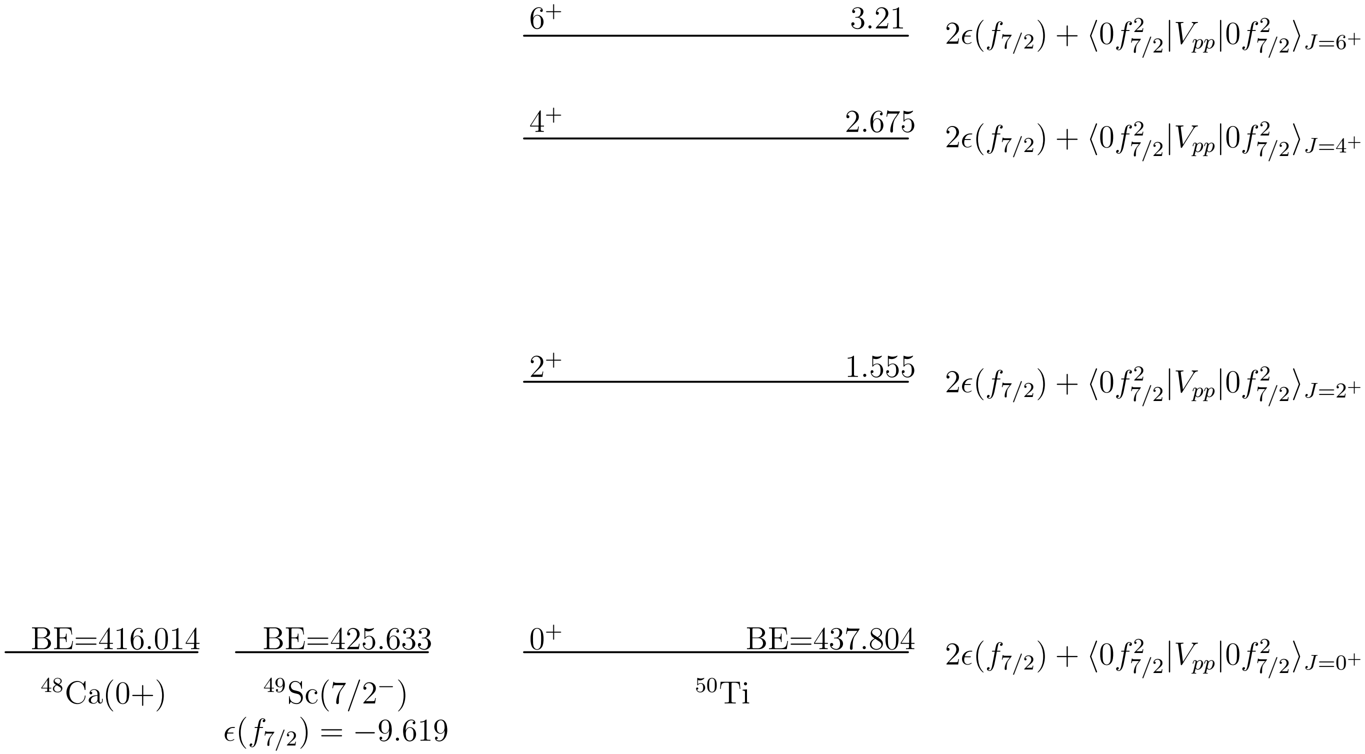
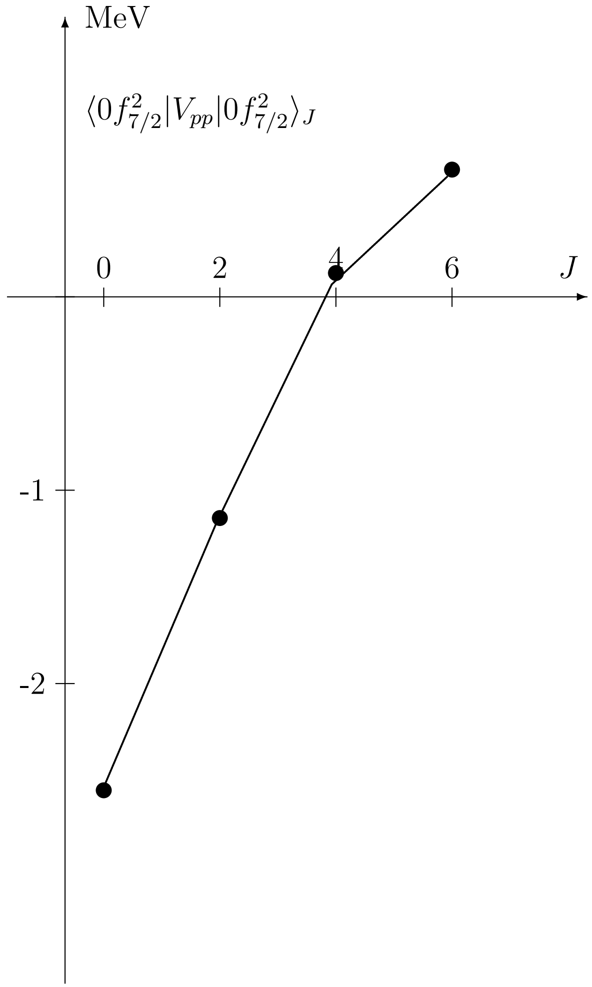
m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

$$ {}^{50}_{22}\mathrm{Ti}_{28} \Longrightarrow {}^{48}_{20}\mathrm{Ca}_{28} + 2p $$





Problem: Derive experimental values of $\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_J$ for $J = 0^+,\ 2^+,\ 4^+,\ 6^+$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=0^+} =$$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=2^+} =$$

$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=4^+} =$$

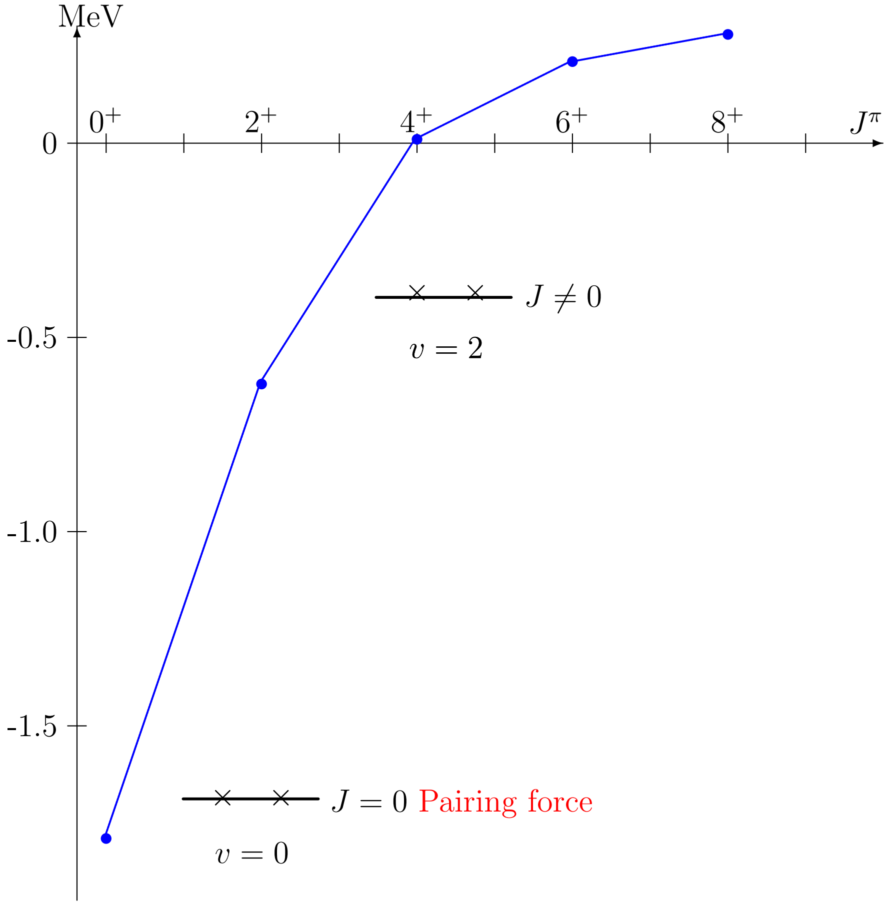
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=6^+} =$$

4. Effective two-body interaction between identical particles

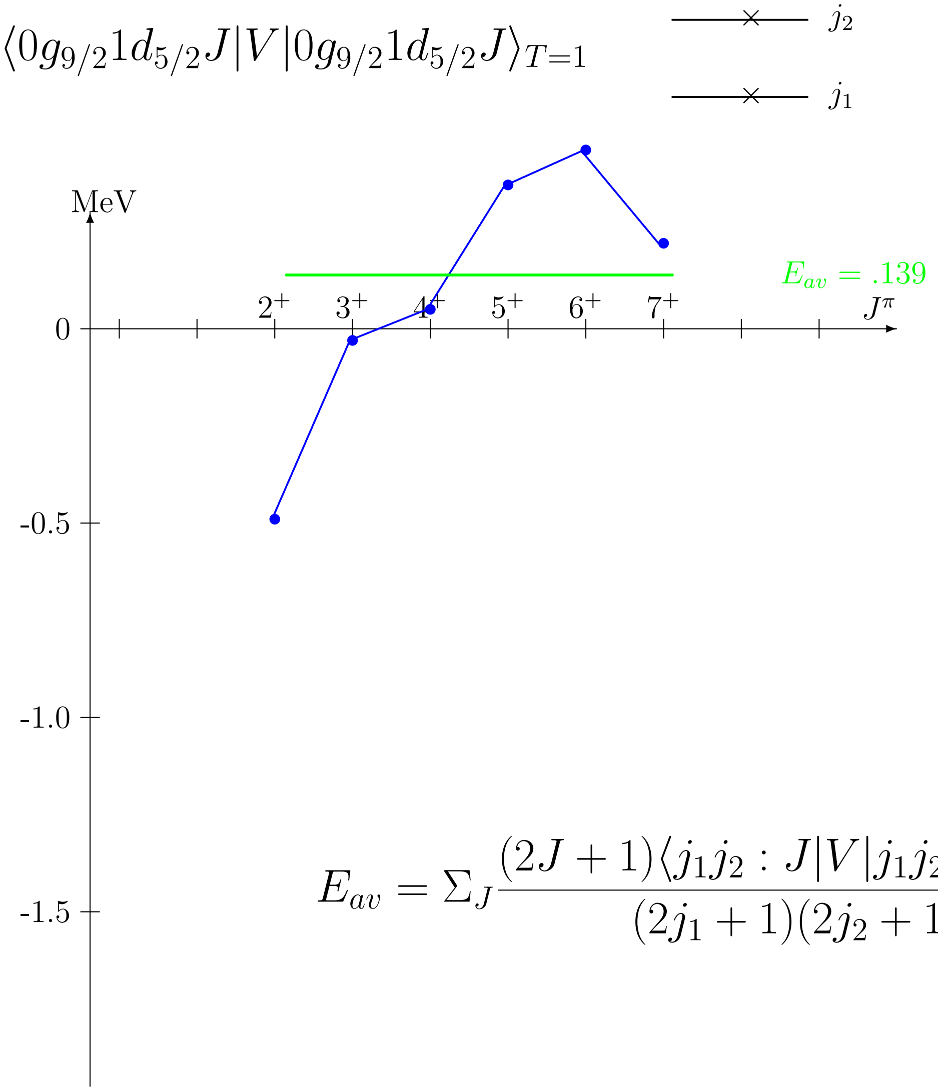
—(proton-proton or neutron-neutron interactions)

$$\langle (0g_{9/2})^2 J | V | (0g_{9/2})^2 J \rangle_{T=1}$$

v:seniority



$$\langle 0g_{9/2} 1d_{5/2} J | V | 0g_{9/2} 1d_{5/2} J \rangle_{T=1}$$



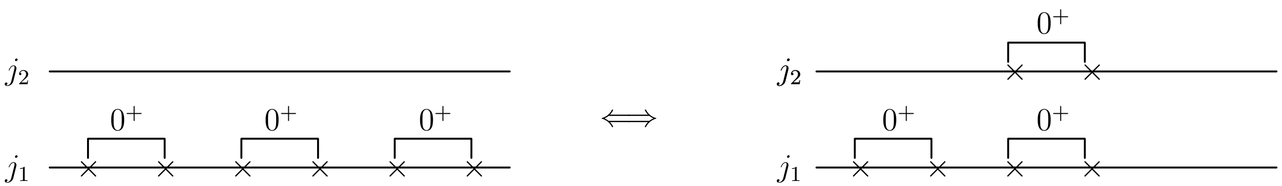
$$E_{av} = \Sigma_J \frac{(2J+1)\langle j_1 j_2 : J | V | j_1 j_2 : J \rangle_{T=1}}{(2j_1+1)(2j_2+1)}$$

Pairing property of effective interaction between identical particles

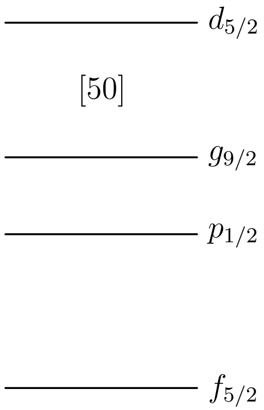
Strong attractive force $\langle j^2 J = 0^+ | V | j^2 J = 0^+ \rangle_{T=1}$

Large matrix element $\langle j^2 J = 0^+ | V | j'^2 J = 0^+ \rangle_{T=1}$

- Such property is reproduced by short-range force like $-V_0 \delta(r)$
- Application of BCS theory to nuclear system



Structure of ^{90}Zr



$^{90}_{40}\mathrm{Zr}_{50}$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \;=\; (g_{9/2}, p_{1/2})_p^2$

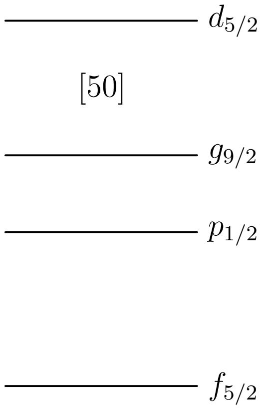
Neutron: $N = 50 \quad$ (closed shell) $\quad J_n = 0^+$

$$(g_{9/2})^2 \quad \to \quad J =$$

$$(g_{9/2} p_{1/2}) \quad \to \quad J =$$

$$(p_{1/2})^2 \quad \to \quad J =$$

Structure of ^{90}Zr



$^{90}_{40}\mathrm{Zr}_{50}$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \; = \; (g_{9/2}, p_{1/2})_p^2$

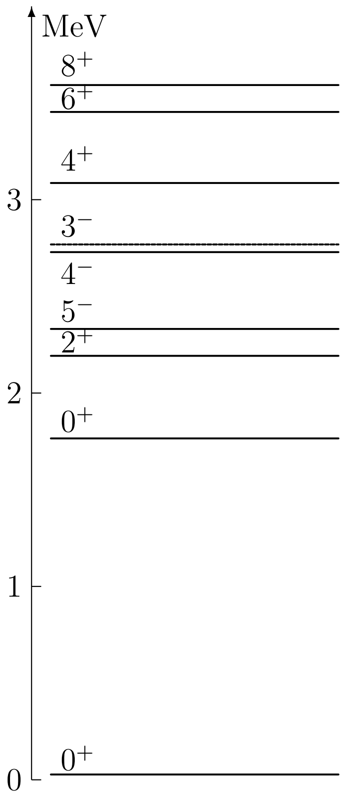
Neutron: $N = 50 \quad$ (closed shell) $\quad J_n = 0^+$

$$(g_{9/2})^2 \quad \rightarrow \quad J = 0^+,\ 2^+,\ 4^+,\ 6^+,\ 8^+$$

$$(g_{9/2}p_{1/2}) \quad \rightarrow \quad J = 4^-,\ 5^-$$

$$(p_{1/2})^2 \quad \rightarrow \quad J = 0^+$$

^{90}Zr



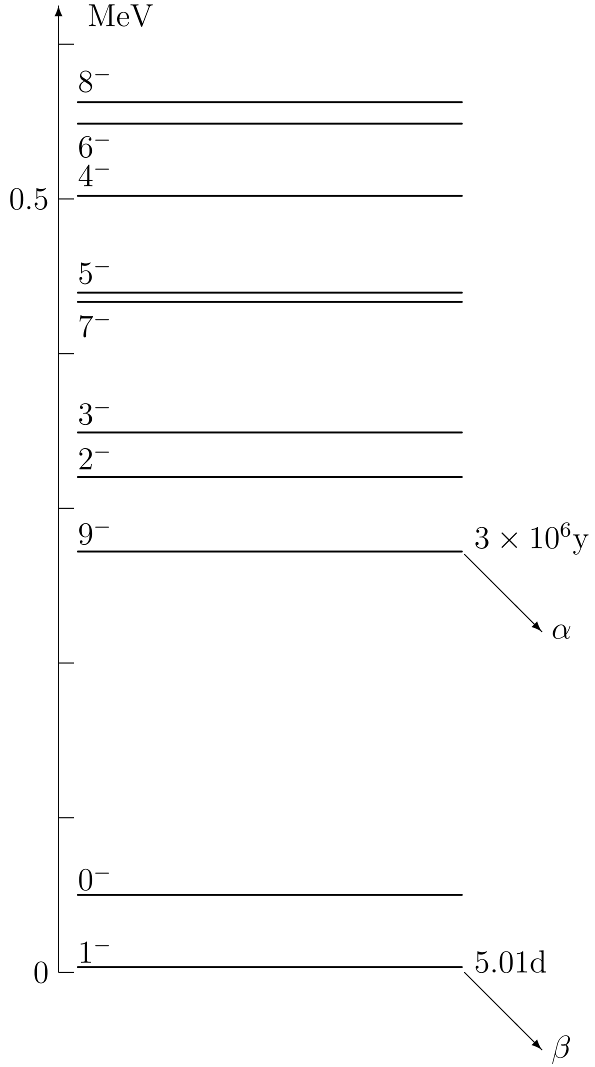
$$\longleftarrow \quad 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 8^+ | V_{pp} | g_{9/2}^2 J = 8^+ \rangle$$

$$\longleftarrow \quad \epsilon(g_{9/2}) + \epsilon(p_{1/2}) + \langle g_{9/2} p_{1/2} J = 5^- | V_{pp} | g_{9/2} p_{1/2} J = 5^- \rangle$$

$$\left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \begin{pmatrix} 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 0^+ | V | g_{9/2}^2 J = 0^+ \rangle & \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \\ \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle & 2\epsilon(p_{1/2}) + \langle p_{1/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Eigen-value problem

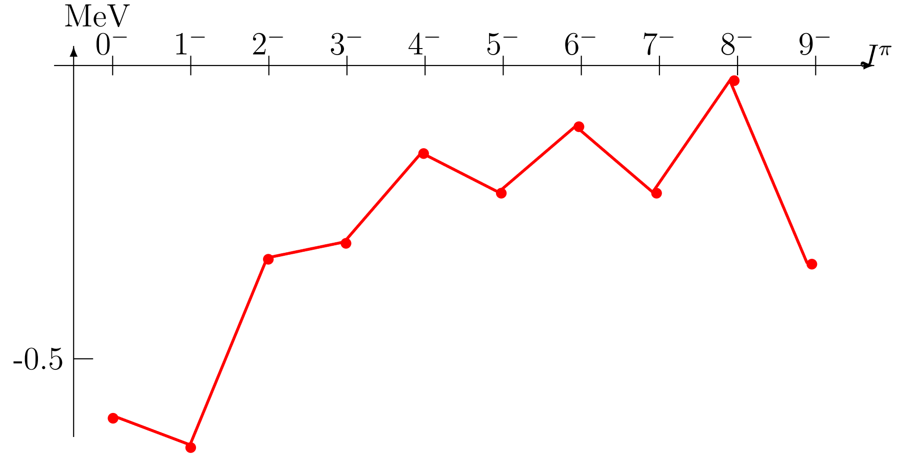
5. Proton-neutron interaction and isomer



$^{210}_{83}\mathrm{Bi}_{127} \Longrightarrow {}^{208}_{82}\mathrm{Pb} + \mathrm{p} + \mathrm{n}$

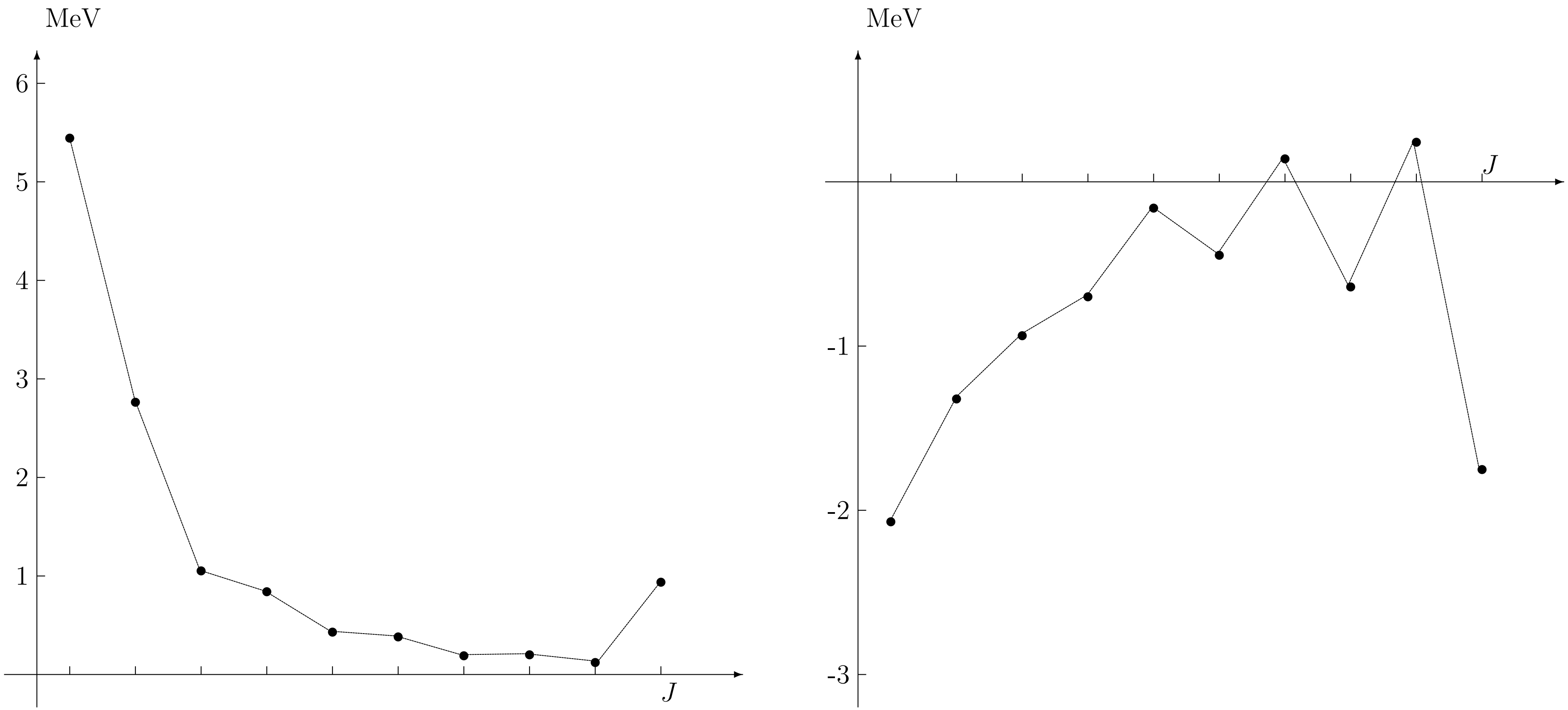
$^{210}_{83}\mathrm{Bi}_{127} \quad (0h_{9/2})_p \times (1g_{9/2})_n \;\; J = 0^-,\; 1^-,\; \cdots, 9^-$

$$\langle (0h_{9/2})_p (1g_{9/2})_n J | V_{pn} | (0h_{9/2})_p (1g_{9/2})_n J \rangle$$

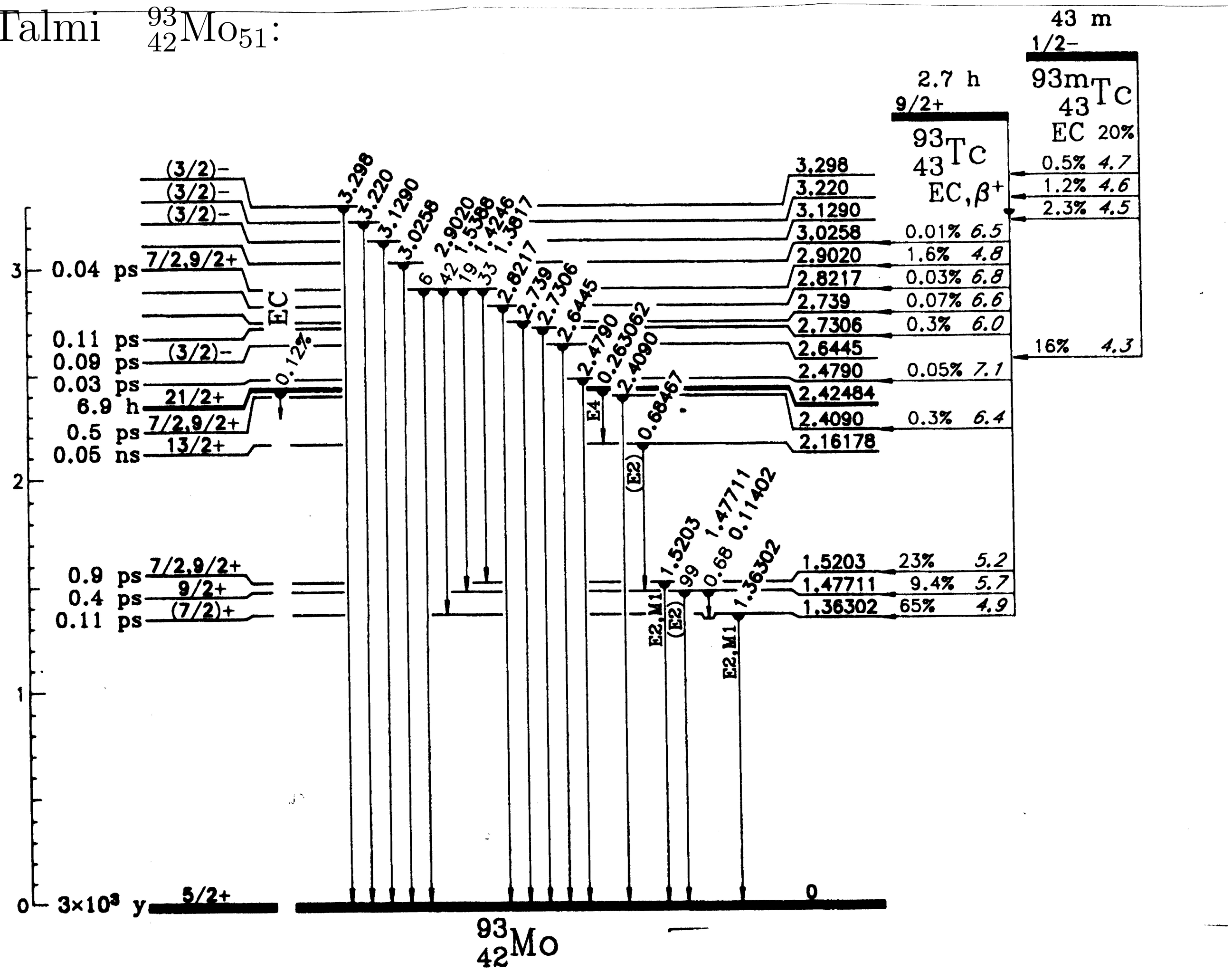


$$\langle (0g_{9/2})_p (0g_{9/2})_n^{-1} : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n^{-1} : J \rangle$$

$$\langle (0g_{9/2})_p (0g_{9/2})_n : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n : J \rangle$$



I. Talmi $^{93}_{42}\mathrm{Mo}_{51}$:



$$(0g^2_{9/2})_p J_p = 8^+ \ \times \ (1d_{5/2})_n J_n = \tfrac{5}{2}; J = 21/2^+$$

High spin isomers in nuclei near drip-line

Direct One-Proton Emission

First observation: $^{53}_{27}\mathrm{Co}_{26}(\frac{19}{2}^-) \rightarrow {}^{52}_{26}\mathrm{Fe}_{26}(0^+)$

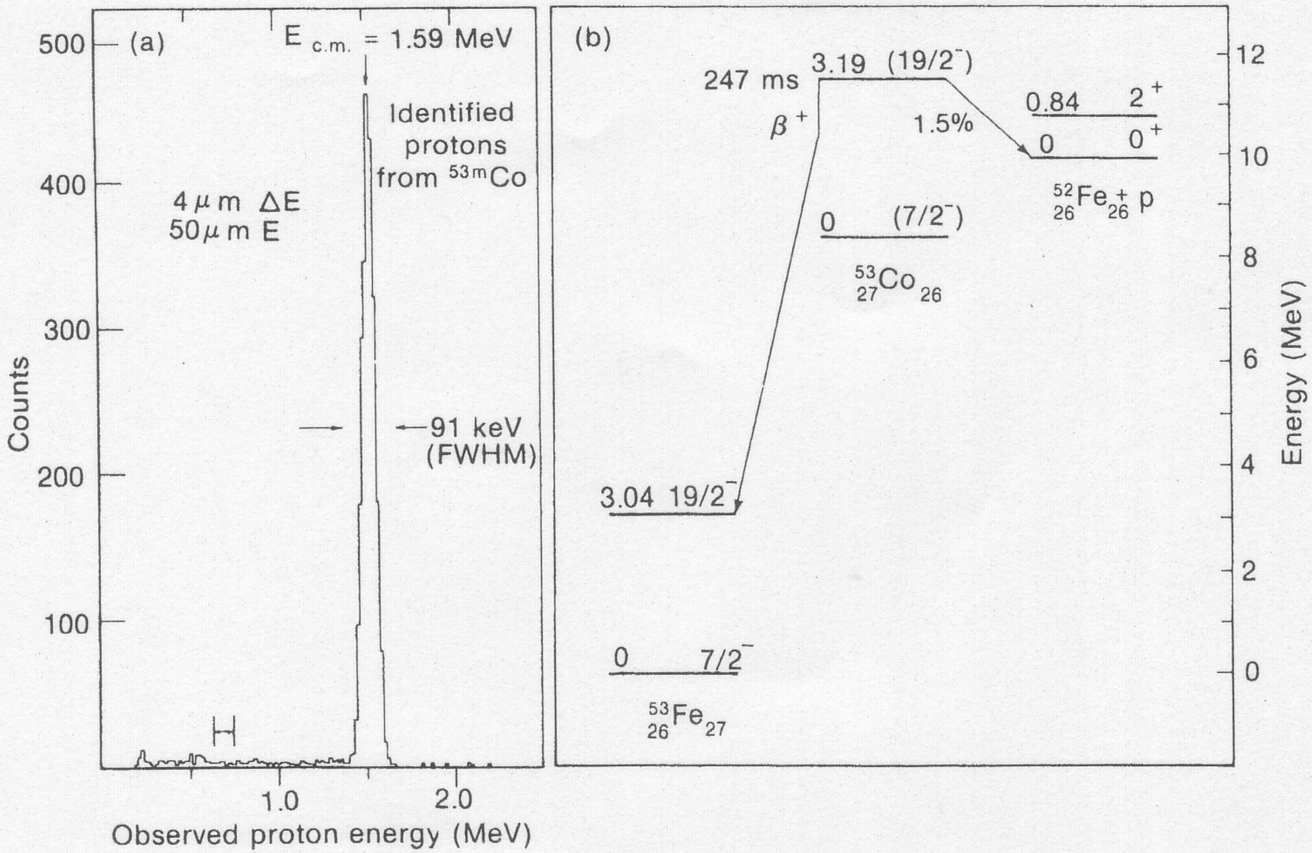


Figure 30. (a) Proton energy spectrum from the decay of ^{53m}Co produced by the ^{54}Fe(p, 2n) reaction. (b) Decay scheme of ^{53m}Co.

Shell-model calculations of high-spin ispmers in neutron-defficient $1g_{9/2}$-shell nuclei
K. Ogawa: Phys. Rev. C28(1983)958

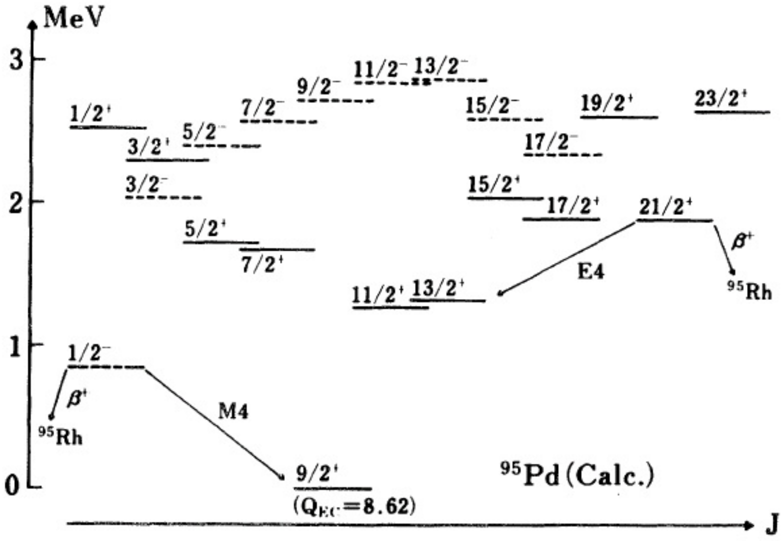


FIG. 1. Calculated energy levels of ^{95}Pd. The lowest levels with each spin-parity state below 3 MeV are shown by solid lines for positive-parity states and by dashed lines for negative-parity states. The transitions from the isomeric states are indicated by arrows.

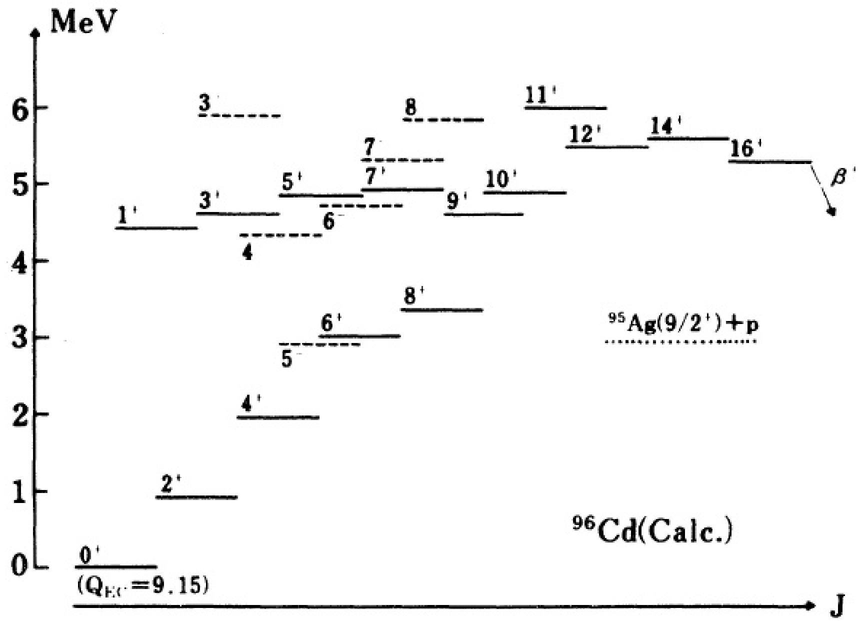
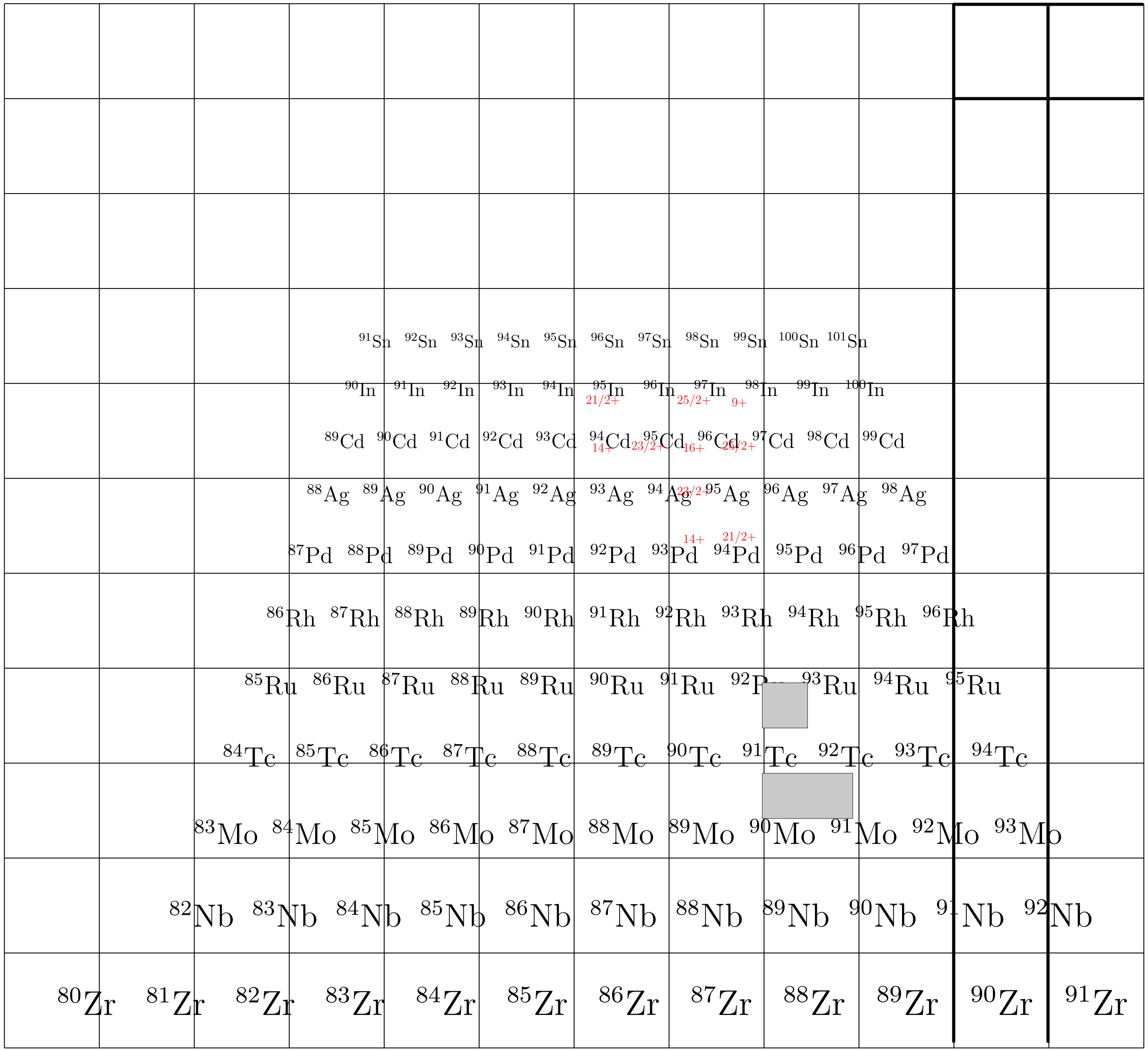


FIG. 3. Calculated energy levels of ^{96}Cd. The lowest levels with each spin-parity state below 6 MeV are shown.

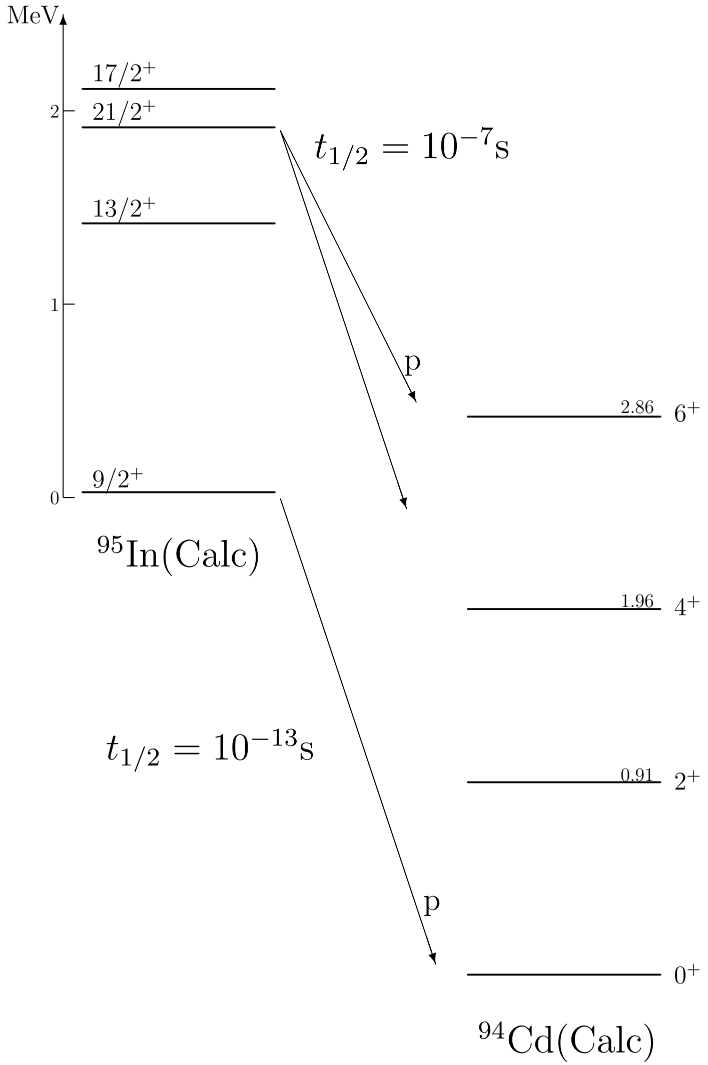
^{95}Pd

^{96}Cd



91Sn 92Sn 93Sn 94Sn 95Sn 96Sn 97Sn 98Sn 99Sn 100Sn 101Sn
90In 91In 92In 93In 94In 95In 96In 97In 98In 99In 100In
21/2+ 25/2+ 9+
89Cd 90Cd 91Cd 92Cd 93Cd 94Cd 95Cd 96Cd 97Cd 98Cd 99Cd
14+ 23/2+ 16+ 25/2+
88Ag 89Ag 90Ag 91Ag 92Ag 93Ag 94Ag 95Ag 96Ag 97Ag 98Ag
23/2+
14+ 21/2+
87Pd 88Pd 89Pd 90Pd 91Pd 92Pd 93Pd 94Pd 95Pd 96Pd 97Pd
86Rh 87Rh 88Rh 89Rh 90Rh 91Rh 92Rh 93Rh 94Rh 95Rh 96Rh
85Ru 86Ru 87Ru 88Ru 89Ru 90Ru 91Ru 93Ru 94Ru 95Ru
84Tc 85Tc 86Tc 87Tc 88Tc 89Tc 90Tc 91Tc 92Tc 93Tc 94Tc
83Mo 84Mo 85Mo 86Mo 87Mo 88Mo 89Mo 90Mo 91Mo 92Mo 93Mo
82Nb 83Nb 84Nb 85Nb 86Nb 87Nb 88Nb 89Nb 90Nb 91Nb 92Nb
80Zr 81Zr 82Zr 83Zr 84Zr 85Zr 86Zr 87Zr 88Zr 89Zr 90Zr 91Zr

Stability of $^{95}_{49}$In



High-spin isomers in unstable nuclei

6. Summary

- Using $\gamma-$spectroscopy, we know the energy levels of each nucleus.
- Magic numbers provide us a simple description of nuclei.
- From nuclei near magic number, we know the effective interactions between nucleons.
- Effective interaction between identical particles(pp or nn) $\rightarrow$ pairing property
- Effective interaction between proton and neutron $\rightarrow$ high-spin state $\rightarrow$ isomers
- Isomers $\rightarrow$ new stability and new decay modes