Fragmentation and gauge/string duality

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Disclaimer

The following results pertain to strongly coupled N=4 supersymmetric Yang-Mills theory. They are not immediately applicable to QCD.

Outline

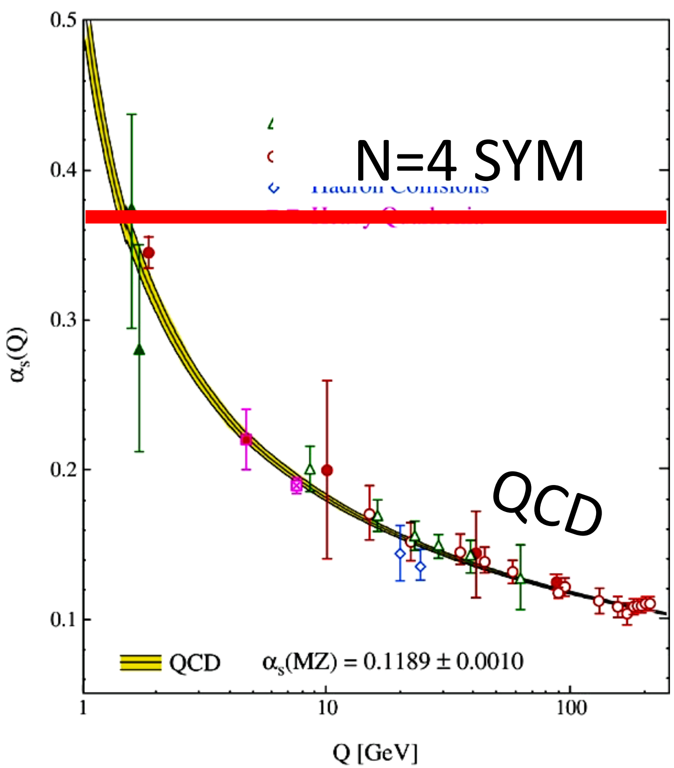
- Fragmentation at weak coupling
- Fragmentation at strong coupling
- Thermal hadron production
- Soft photon puzzle

YH, Iancu, Mueller, JHEP 0805 (2008)
YH, Matsuo, PLB 670 (2008)
YH, Matsuo, PRL 102 (2009)
YH, Ueda, NPB 837 (2010)
YH, Iancu, Mueller, Triantafyllopoulos, 1210.1534 ← New!

N=4 supersymmetric Yang-Mills

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_{i=1}^{4} \bar{\psi}^i_\alpha (\bar{\sigma} \cdot D)_{\alpha\beta} \psi^i_\beta + 2\sqrt{2} g f^{abc} \sum_{1 \le i < j \le 4} Re \left(\phi^{ij}_a \psi^{i\alpha}_b \psi^j_{c\alpha} \right)$$

$$+ \frac{1}{2} \sum_{1 \le i < j \le 4} \left(D_\mu \phi^{ij} \right)^\dagger D^\mu \phi^{ij} - \frac{g^2}{4} \sum_{\substack{1 \le i < j \le 4 \\ 1 \le k < l \le 4}} \left| f_{abc} \phi^{ij}_b \phi^{kl}_c \right|^2$$



Gauge boson (“gluon”),
4 Weyl fermions (“quarks”),
6 scalars, all in the adjoint rep. of “color” SU(Nc)

Global SU(4) R-symmetry

The beta function is zero.

Fragmentation at weak coupling: Energy distribution

Lowest order timelike anomalous dimension in N=4 SYM

$$\gamma(j) = \frac{\lambda}{4\pi^2}\Big(\psi(1) - \psi(j-1)\Big)$$

$$\lambda = g^2 N_c$$

`t Hooft coupling

DGLAP evolution of the fragmentation function:

$$x^2 D(x, Q^2/\mu^2) = \int \frac{\mathrm{d}j}{2\pi i}\, \mathrm{e}^{(j-2)\ln(1/x) + \gamma(j)\ln(Q^2/\mu^2)}$$

Saddle point at j=2

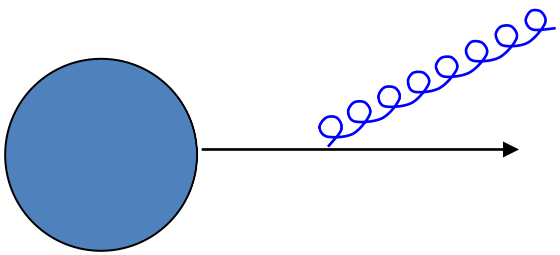
Where the energy is concentrated

$$\ln\frac{1}{x_c} = -\gamma'(2)\ln\frac{Q^2}{\mu^2} = \frac{\lambda}{24}\ln\frac{Q^2}{\mu^2}.$$

$$x_c = \left(\frac{\mu}{Q}\right)^{\lambda/12}$$

Number distribution

$$n(Q) \propto \left(\frac{Q^2}{\mu^2}\right)^{\gamma_T(1)}$$

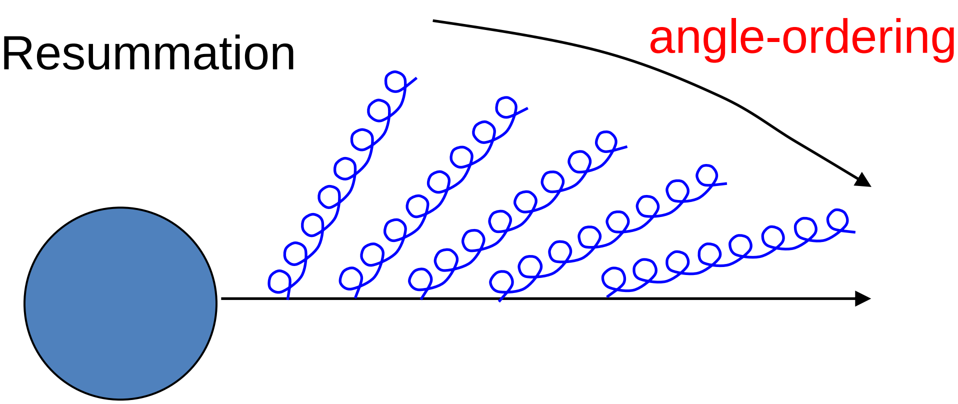


$$\sim \frac{1}{x} \longleftrightarrow \gamma_T(j) \sim \frac{\alpha_s}{j-1}$$

Soft singularity

$$\gamma_T(1) = \infty \ !$$

Resummation

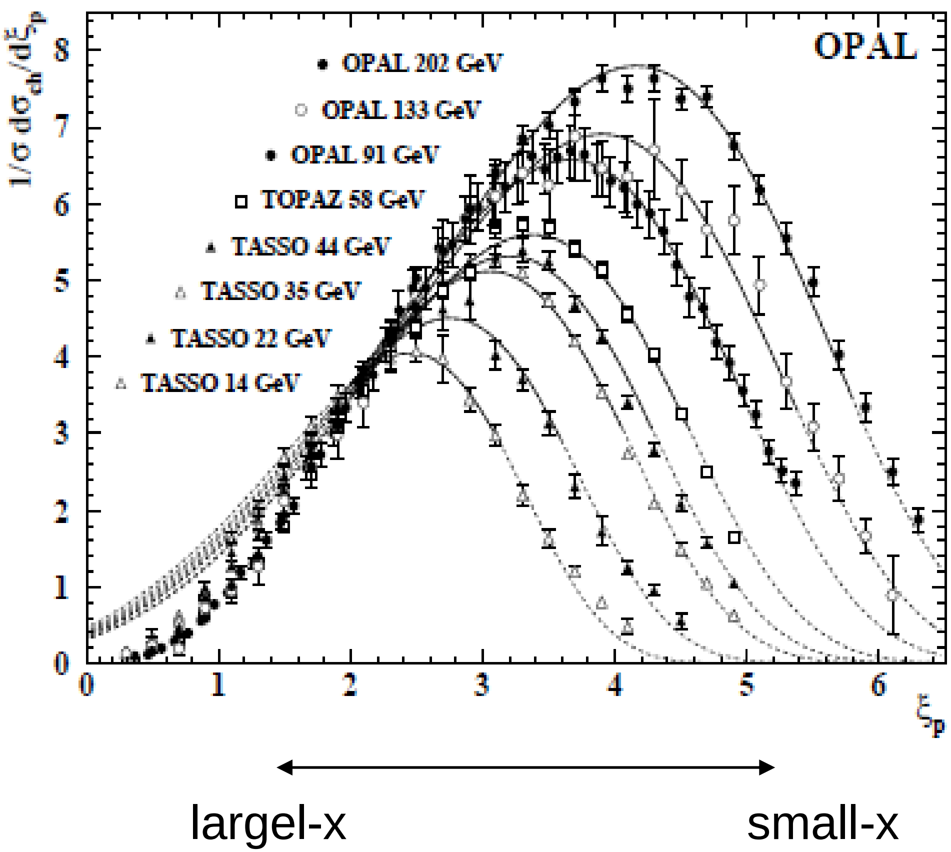


$$\gamma_T(j) = \frac{1}{4}\left[\sqrt{(j-1)^2 + \frac{2\lambda}{\pi^2}} - (j-1)\right]$$

$$\gamma_T(1) = \sqrt{\frac{\lambda}{8\pi^2}}$$

Mueller (1981)

Inclusive spectrum



"hump-backed" distribution peaked at

$$x_c = \left(\frac{\mu}{Q}\right)^{1/2}$$

Double logs + QCD coherence. Structure of jets well understood in pQCD.

Fragmentation at strong coupling

Why AdS/CFT?

— Conceptual interest

- Strong coupling → very fast fragmentation, presumably via wide angle splittings.
- No jets, events more or less spherical
- No pointlike partons.

— Phenomenology

- Physics beyond SM (Strassler)
- Nonperturbative aspects of fragmentation in QCD?

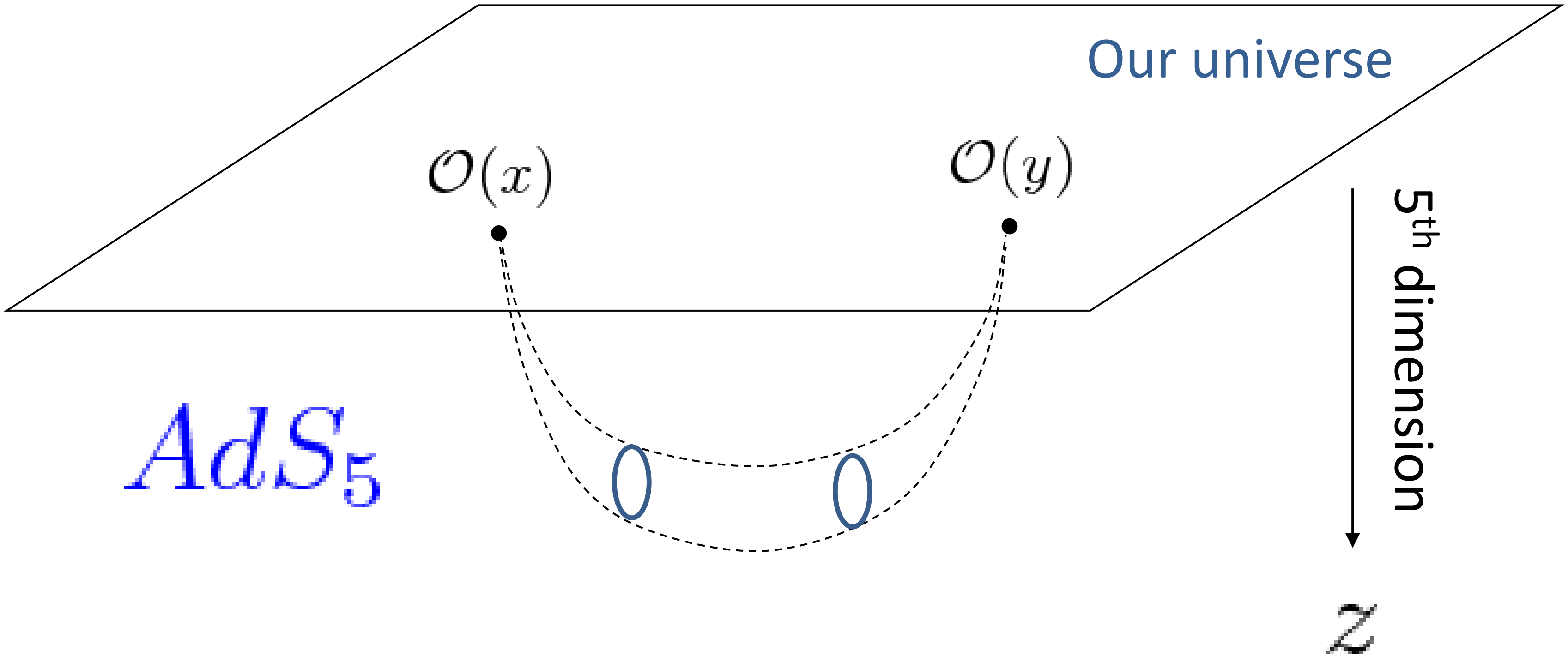
The AdS/CFT correspondence

Maldacena (1998)

- Take the limits $\lambda \to \infty$ and $N_C \to \infty$
- N=4 SYM at strong coupling is dual to weak coupling type IIB superstring theory on $AdS_5 \times S^5$

$$ds^2 = R^2 \frac{\overbrace{-dt^2 + d\vec{x}^2}^{\text{our universe}} + dz^2}{z^2} + R^2 d\Omega_5^2$$

CFT		string
(anomalous) dimension	⟷	mass
`t Hooft parameter λ	⟷	curvature radius R^4/α'^2
number of colors $1/N_C$	⟷	string coupling constant g_s



mass spectrum

$$m^2 = \frac{8}{\alpha'}$$

$$m^2 = \frac{4}{\alpha'}$$

$$m^2 = 0$$

Supergravity (SUGRA) limit

$$\lambda \sim 1/\alpha'^2 \to \infty$$

Energy correlation functions

Hofman, Maldacena (2008)

Energy 1-point function spherical for any λ

$$\langle \mathcal{E}(\Omega) \rangle = \frac{Q}{4\pi}$$

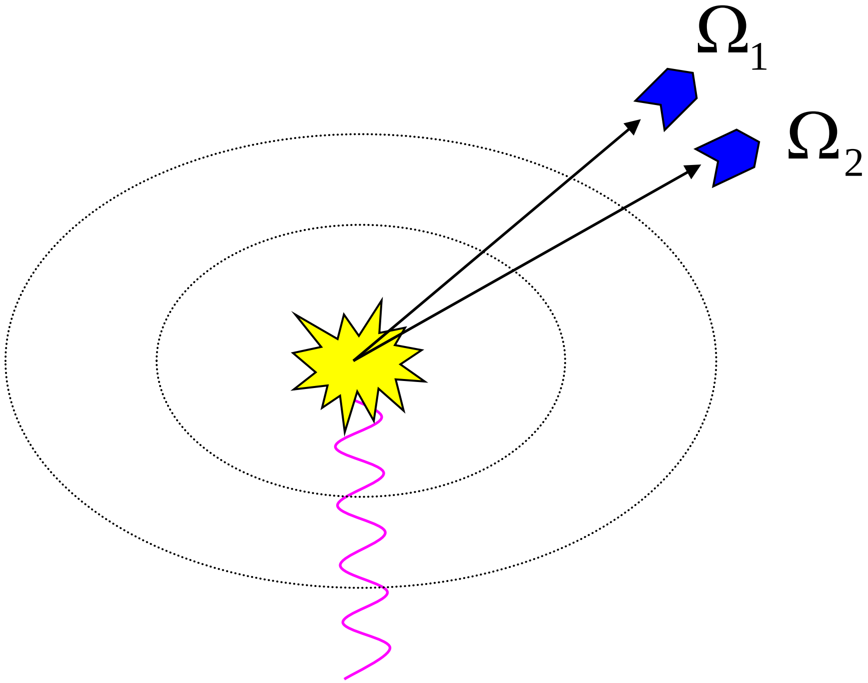
Fermions	$1 + \cos^2\theta$
Scalars	$\sin^2\theta = 1 - \cos^2\theta$

Energy 2-point function

$$\langle \mathcal{E}(\Omega_1)\mathcal{E}(\Omega_2) \rangle \sim \frac{1}{|\theta_{12}|^{2+2\gamma_S(3)}}$$

$$\gamma_S(3) = O(\lambda) << 1$$ weak coupling

$$= -\lambda^{1/4}/\sqrt{2}$$ strong coupling



Gribov-Lipatov reciprocity

DGLAP equation $$\frac{\partial}{\partial \ln Q^2} D_{S/T}(j, Q^2) = \gamma_{S/T}(j) D_{S/T}(j, Q^2)$$

An intriguing relation in DLA $$\gamma_T(j) = \gamma_S(j + 2\gamma_T(j))$$ Mueller (1983)

The two anomalous dimensions derive from a single function

$$\gamma_S(j) = f(j - \gamma_S(j))$$
$$\gamma_T(j) = f(j + \gamma_T(j))$$

Dokshitzer, Marchesini, Salam (2005)

Nontrivial check up to three loops (!) in QCD Mitov, Moch, Vogt (2006)

Fragmentation at strong coupling: Multiplicity

$$\gamma_S(j) = \frac{j}{2} - \frac{1}{2}\sqrt{2\sqrt{\lambda}(j - j_0)} \quad \overset{\text{crossing}}{\longleftrightarrow} \quad \gamma_T(j) = -\frac{1}{2}\left(j - j_0 - \frac{j^2}{2\sqrt{\lambda}}\right)$$

Lipatov et al. (2005)

$$n(Q) \propto (Q/\mu)^{2\gamma_T(1)} = (Q/\mu)^{1-3/2\sqrt{\lambda}}$$

YH, Matsuo (2008)

c.f. in perturbation theory, $n(Q) \propto \left(\frac{Q}{\mu}\right)^{\sqrt{\frac{\lambda}{2\pi^2}}}$

c.f. heuristic argument $n(Q) \propto Q$ YH, Iancu, Mueller (2008)

Energy distribution

$$x^2 D(x,\mu^2) = x^2 \left(\frac{Q}{\mu}\right)^{j_0} \exp\left(-\frac{\sqrt{\lambda}(\ln xQ/\mu)^2}{2\ln Q/\mu}\right)$$

Strongly peaked at

$$x_c = \left(\frac{\mu}{Q}\right)^{1-\frac{2}{\sqrt{\lambda}}} \simeq \frac{\mu}{Q}$$

kinematical lower limit !

cf. weak coupling $x_c = \left(\frac{\mu}{Q}\right)^{\lambda/12}$

Note: the limit $\lambda \to \infty$ is subtle.

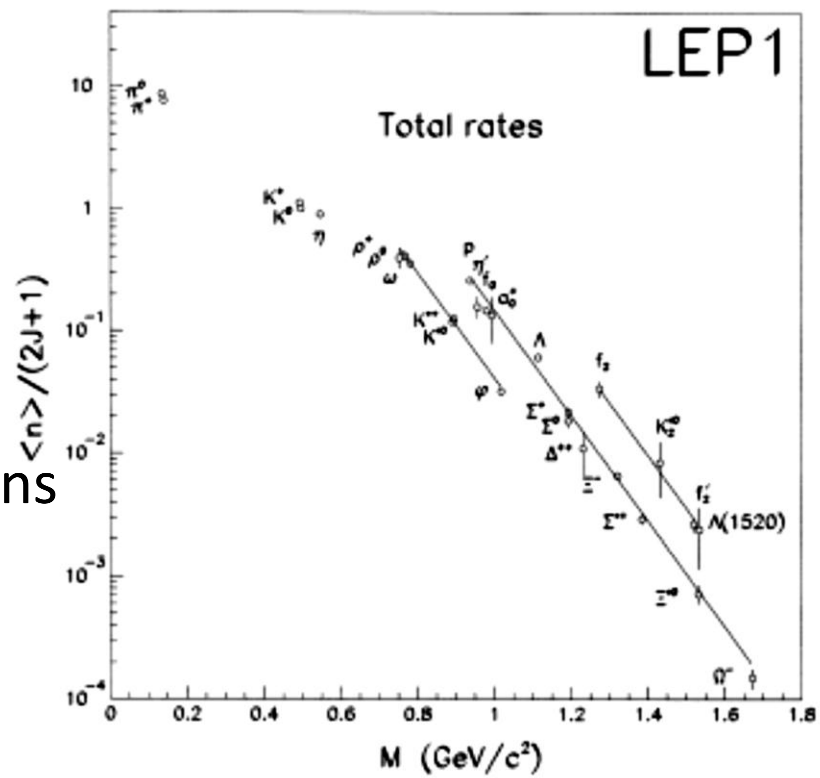
Thermal hadron production

Identified particle yields are well described by a thermal model

$$\frac{N^*}{N} \propto \exp\left(-\frac{M^* - M}{T}\right)$$

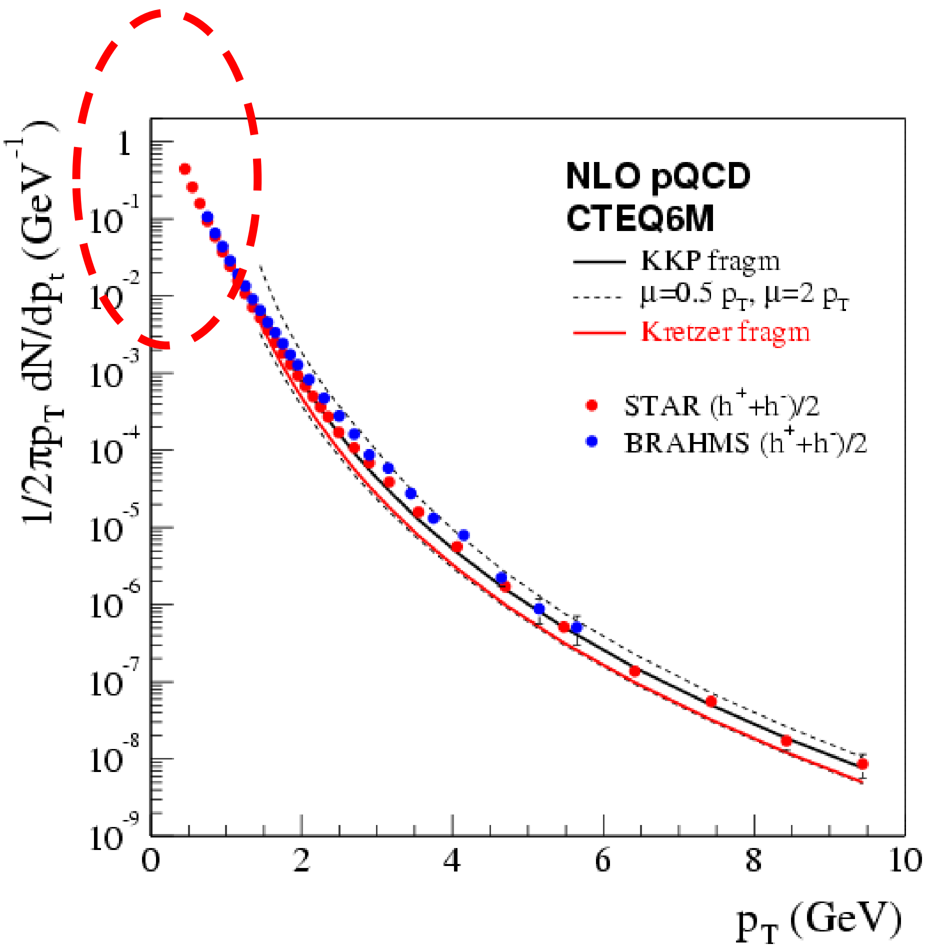
$T \sim 170\ \mathrm{MeV}$

The model works in e+e- annihilation, hadron collisions, and heavy-ion collisions

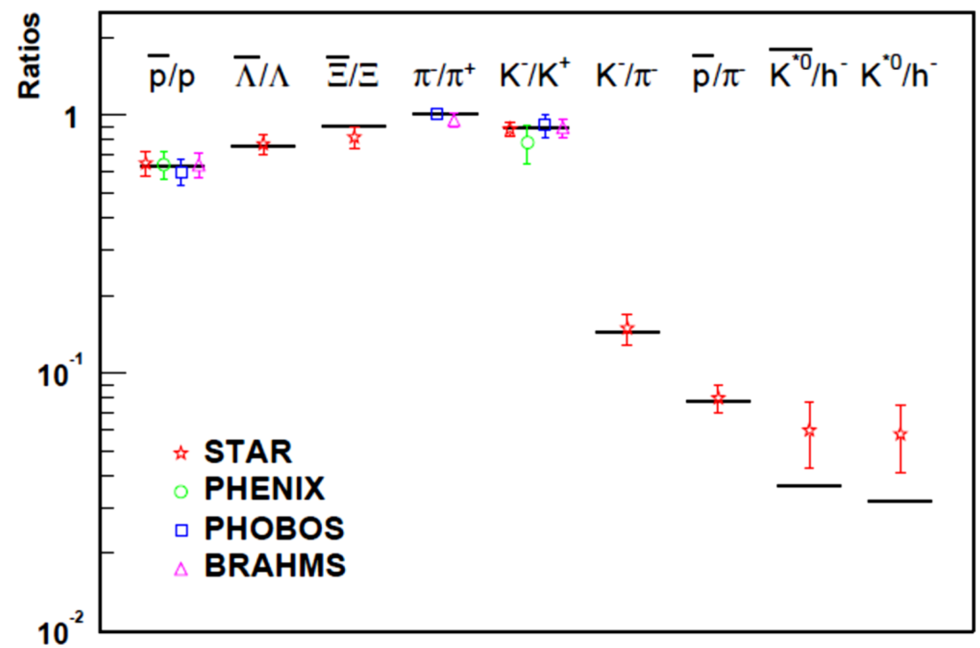


Becattini,
Chliapnikov,
Braun-Munzinger et al.

Thermal hadron production at RHIC



Charged hadrons in pp at 200 GeV



Multiplicity ratios in AA

Thermal production from gauge/string duality

Inclusive production vs. n-particle production

Bjorken and Brodsky (1970)

$$\langle 0|j^\mu(0)|p_1,...p_n\rangle\langle p_1,...p_n|j^\nu(0)|0\rangle \\ \to a_n(q^\mu q^\nu - g^{\mu\nu}q^2)\,e^{-\beta Q}$$

$$\Rightarrow \quad 2E\frac{dN}{d^3p} \sim e^{-\beta E}$$

$$\langle 0|\epsilon\cdot j(0)|p_1,...p_n\rangle \\ \sim \frac{g_c^{n+1}}{\alpha' g_c^2}\int dz\,d\Omega_5\sqrt{-G}F(\alpha'\partial^2)(\Phi)^n A_\mu$$

string amplitude

5D hadron

5D photon

$$A_\mu \propto H_1^{(1)}(Qz) \sim e^{iQz}$$

Saddle point at $z_s \sim i\beta$

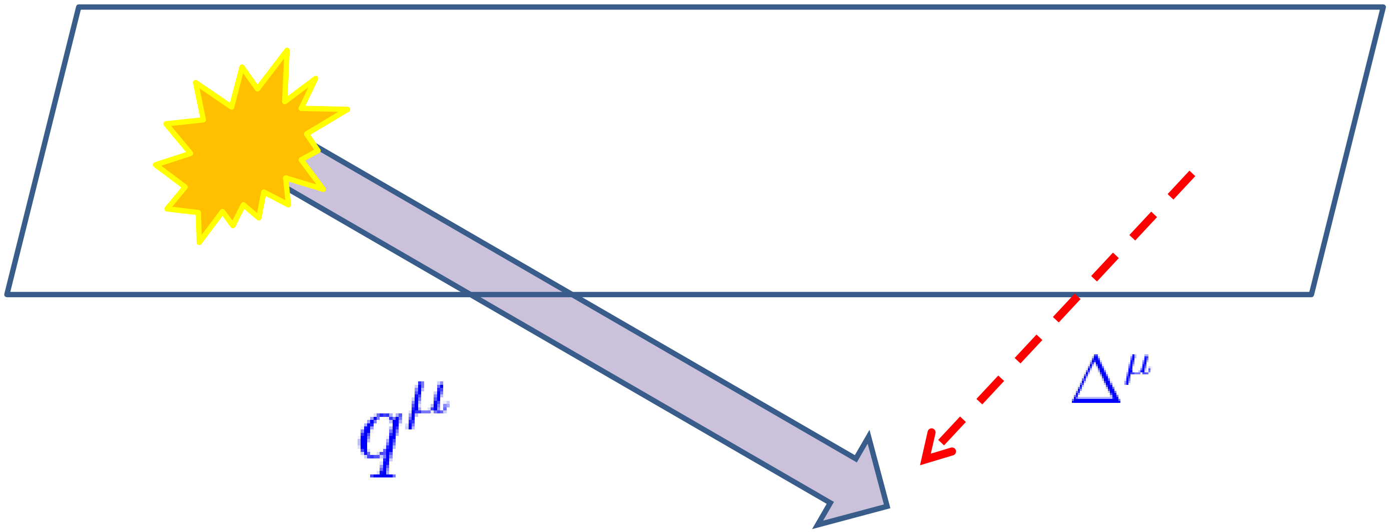
$$e^{iQz_s} \sim e^{-\beta Q}$$

exponential !

YH, Matsuo (2009)

DIS off a jet

YH, Iancu, Mueller, Triantafyllopoulos, 1210.1534



Send a spacelike signal at large time τ to probe the spacetime structure of a well-evolved jet.

$$\tau = xQ/\mu^2$$

Pointlike partons?

``Jet structure function”

$$\int \mathrm{d}^4x \, \mathrm{e}^{-iq\cdot x} \left\langle \hat{J}_\mu(x) \, \hat{J}_+(\tau, \Delta) \, \hat{J}_+(\tau, -\Delta) \, \hat{J}_\nu(0) \right\rangle$$

$$\sim K_2^2(\Delta_\perp \tau/\gamma) \sim e^{-2\Delta_\perp \tau/\gamma}$$

No structure below the scales

$$\delta x_\perp \sim \tau/\gamma \qquad \delta x_- \sim \gamma/\tau^2$$

→ The size of the whole system !

→ No pointlike partons

Fragmentation into photons

Wise men said….

$$\frac{dN_\gamma}{d^3\vec{k}} = \frac{\alpha}{(2\pi)^2}\frac{1}{E_\gamma}\int d^3\vec{p}_1 ... d^3\vec{p}_N \sum_{i,j} \eta_i\eta_j \frac{-(P_iP_j)}{(P_iK)(P_jK)}\frac{dN_{hadrons}}{d^3\vec{p}_1...d^3\vec{p}_N}$$

Soft photon production in hadronic collisions is given by the QED Bremsstrahlung formula

Landau, Pomeranchuk, Gribov, Low,…

Soft photon puzzle

Evidence for an excess of soft photons in hadronic decays of Z^0

The DELPHI Collaboration

Eur.Phys.J. C47 (2006) 273-294

Abstract. Soft photons inside hadronic jets converted in front of the DELPHI main tracker (TPC) in events of $q\bar{q}$ disintegrations of the Z^0 were studied in the kinematic range $0.2 < E_\gamma < 1\,\mathrm{GeV}$ and transverse momentum with respect to the closest jet direction $p_T < 80\,\mathrm{MeV/c}$. A clear excess of photons in the experimental data as compared to the Monte Carlo predictions is observed. This excess (uncorrected for the photon detection efficiency) is $(1.17 \pm 0.06 \pm 0.27) \times 10^{-3}$ γ/jet in the specified kinematic region, while the expected level of the inner hadronic bremsstrahlung (which is not included in the Monte Carlo) is $(0.340 \pm 0.001 \pm 0.038) \times 10^{-3}$ γ/jet. The ratio of the excess to the predicted bremsstrahlung rate is then $(3.4 \pm 0.2 \pm 0.8)$, which is similar in strength to the anomalous soft photon signal observed in fixed target experiments with hadronic beams.

$$\frac{dN}{dk} \sim \frac{A}{k}$$

Factor 3-5 discrepancy between the data and theory.

Observed also in $K^+p,\ \pi^{\pm}p,\ pp, \ldots$ since `80s.

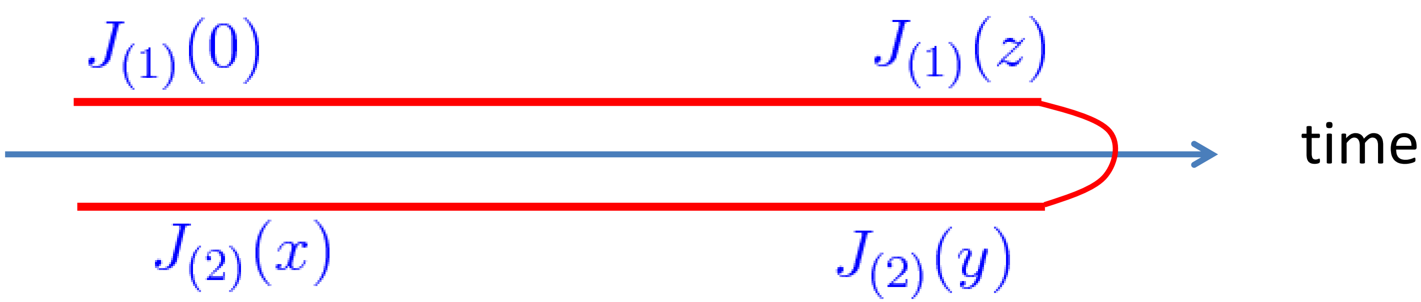
Soft photon production in AdS/CFT

YH, Ueda (2010)

$$k\frac{dN}{d^3\vec{k}} \equiv \frac{k}{\sigma_{\rm tot}}\frac{d\sigma}{d^3\vec{k}}$$

$$= \frac{e^2}{\pi^2 N_c^2 Q^4}(p_\mu p'_{\mu'} + p_{\mu'} p'_\mu - \eta_{\mu\mu'} p\cdot p')\sum_{\rm pol}\int d^4x\, d^4y\, d^4z\, e^{-iq\cdot x+ik\cdot(y-z)}$$

$$\times \langle 0|\mathrm{T}_C\{J^\mu_{(2)}(x)\varepsilon_k\cdot J_{(2)}(y)\varepsilon^*_k\cdot J_{(1)}(z)J^{\mu'}_{(1)}(0)\}|0\rangle,$$



Compute the four-point function using the Keldysh (closed time path) formalism

5D SUGRA action

$$S = \frac{1}{2\kappa^2}\int d^5x\,\sqrt{-g}\left(\Re - \frac{4}{3}\partial_m\phi\partial^m\phi\right) - \frac{1}{4g_{YM}^2}\int d^5x\,\sqrt{-g}\,F^a_{mn}F_a^{mn}e^{-\frac{4}{3}\phi}$$

graviton dilaton gauge boson

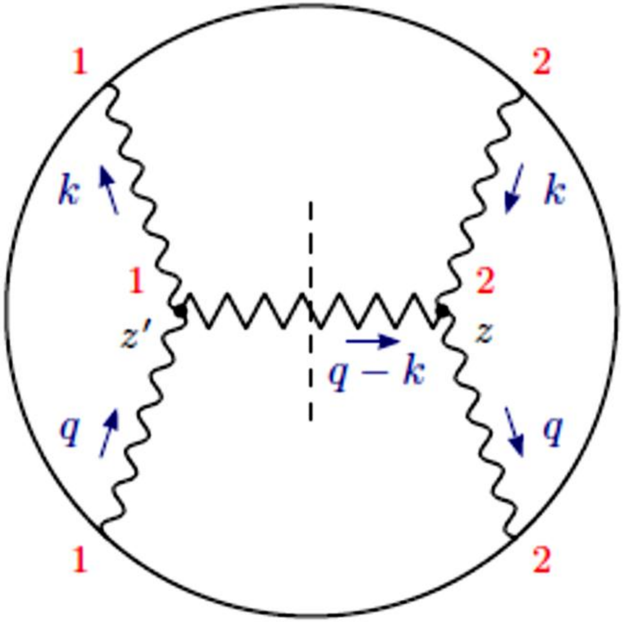
Strong coupling

$$k\frac{dN}{d^3\vec{k}} = k\frac{dN_G}{d^3\vec{k}} + k\frac{dN_A}{d^3\vec{k}} + k\frac{dN_\phi}{d^3\vec{k}} = \frac{\alpha_{em}}{8\pi^2k^2}\left(1 - \frac{k}{3Q}(7+\cos^2\theta)\right)$$

$$\approx \frac{\alpha_{em}}{8\pi^2k^2}.$$

Exact!

Bremsstrahlung !



Cf. weak coupling

$$k\frac{dN}{d^3\vec{k}} = \frac{\alpha_{em}}{2\pi^2k^2}\ln\frac{Q^2}{m^2},$$

Conclusions

- Strong coupling → No coherence, no collinear singularity, no pointlike partons. Energy distribution peaked at kinematical lower limit.
- Hadron spectrum exponential
- Novel source of soft photons.
- Need more work (cheating?) to tackle on QCD.