## di-hadron based

## transversity extraction

## or <br> the importance of di-hadron fragmentation (DiFF or IFF)

Marco Radici


IN N Paxia
Istituto Nazionale di Fisica Nucleare

In collaboration with
A. Bacchetta (Univ. Pavia)
A. Bianconi (Univ. Brescia)
A. Courtoy (Univ. Liege)

## Outline

- What are DiFF and Where to extract them
-Why do we need them ? the quest for transversity: Collins vs. IFF
- Who did what? (= the present situation)
- Which are the latest "press news" ?
- Perspectives


# the What and the Where 

## General framework

## Single-hadron fragmentation

- $\mathrm{K}_{\mathrm{T}}$-dependent fragmentation functions
from q-q correlator $\Delta$ project out :


$$
\operatorname{Tr}\left[\Delta \gamma^{-}\right] \quad \longrightarrow D_{1}^{q \rightarrow h}\left(z, K_{T}^{2}\right)
$$

$\operatorname{Tr}\left[\Delta i \sigma^{i-} \gamma_{5}\right] \longrightarrow H_{1}^{\perp q \rightarrow h}\left(z, K_{T}^{2}\right)$


## Single-hadron fragmentation

- Integrate over the transverse momentum

- Standard fragmentation functions

$$
D_{1}^{q \rightarrow h}(z)
$$

- No Collins fragmentation function

$$
H_{1}^{\perp q \rightarrow h}(z)
$$



## Single-hadron fragmentation

- Integrate over the transverse momentum

- Standard fragmentation functions

$$
D_{1}^{q \rightarrow h}(z)
$$

- No Collins fragmentation function



## Di-hadron fragmentation

- $\mathrm{K}_{\mathrm{T}}$-dependent DiFF
from q-q correlator $\Delta\left(\mathrm{z}_{1}, \mathrm{z}_{2}, \mathbf{K}_{\mathrm{T}}, \mathbf{R}_{\mathrm{T}}\right)$ project out :

$\operatorname{Tr}\left[\Delta \gamma^{-}\right] \quad \longrightarrow \quad D_{1}^{q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, K_{T}^{2}, R_{T}^{2}, \mathbf{K}_{T} \cdot \mathbf{R}_{T}\right)$



## Di-hadron fragmentation

- Integrate over the transverse momentum



## Di-hadron fragmentation

- Integrate over the transverse momentum


$$
|\boldsymbol{R}|=\frac{M_{h}}{2} \sqrt{1-\frac{4 m_{\pi}^{2}}{M_{h}^{2}}}
$$

$$
\int d \mathbf{K}_{T} D_{1}^{q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, K_{T}^{2}, R_{T}^{2}, \mathbf{K}_{T} \cdot \mathbf{R}_{T}\right) \quad \longrightarrow \quad D_{1}^{q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, R_{T}^{2}\right)
$$

$$
\int d \mathbf{K}_{T}\left(\mathbf{S}_{T}^{q} \times \mathbf{K}_{T}\right) H_{1}^{\perp q \rightarrow h_{1} h_{2}}+\left(\mathbf{S}_{T}^{q} \times \mathbf{R}_{T}\right) H_{1}^{\varangle q \rightarrow h_{1} h_{2}} \quad \longrightarrow \quad\left(\mathbf{S}_{T}^{q} \times \mathbf{R}_{T}\right) H_{1}^{\varangle q \rightarrow h_{1} h_{2}}\left(z_{1}, z_{2}, R_{T}^{2}\right)
$$

- Chiral-odd $H_{1}^{\varangle q \rightarrow h_{1} h_{2}}$ survives! (memo: $\mathrm{h}_{1}, \mathrm{~h}_{2}$ must be distinguishable!)



## Where do DiFF occur?



SIDIS


$$
e^{-} e^{+} \text {to pions }
$$


$p-p$ to pions

## Where do DiFF occur?

## Factorization

at NLO \& LL, same DGLAP as single-h case
F.Ceccopieri, M.R., A.Bacchetta, P.L.B650(07) )

Universality


SIDIS


$$
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## Where do DiFF occur?

Factorization
(at NLO \& LL, same DGLAP as single-h case
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Universality


$$
e^{-} e^{+} \text {to pions }
$$

## $e+e-$

## - Invariant mass spectrum


OPAL, ZPC56 (92)

Non trivial!

## hadron collisions

- Invariant mass spectrum


STAR, PRL92 (04)

- In-medium modifications
- Mass shifts (@)
- Jet quenching


## SIDIS

- Invariant mass spectrum

HERMES, JHEPO6 (08)


## the Why

## how to extract transversity: Collins vs. IFF

## The Collins mechanism


$\mathbf{k} \times \mathbf{P}_{h} \cdot \mathbf{S}_{T} \propto \cos \left(\frac{\pi}{2}-\phi\right)=\sin \phi$
transverse motion of hadron
=
spin analyzer of fragmenting quark

## The Collins mechanism


$\mathbf{k} \times \mathbf{P}_{h} \cdot \mathbf{S}_{T} \propto \cos \left(\frac{\pi}{2}-\phi\right)=\sin \phi$
transverse motion of hadron
spin analyzer of fragmenting quark

## Effects ofTMD evolution



## Effects ofTMD evolution



## is it similar for Collins effect ? Need to check..

SIDIS
(2.5 GeV²)


## Comparison with models


[0] M. Anselmino et al., arXiv:0812.4366
[1-8] models

## TMD factorization $\rightarrow$ TMD evolution

- Convolution
- Soft factors
- Evolution and Sudakov form factors

is there a way to skip all this ?


## TMD factorization $\rightarrow$ TMD evolution

- Convolution
- Soft factors
- Evolution and Sudakov form factors

is there a way to skip all this ?



## The IFF mechanism

Collins, Heppelman, Ladinsky, NP B420 (94)

azimuthal orientation of hadron pair
spin analyzer of fragmenting quark

## SIDIS SSA: Collins vs. IFF

$$
A_{U T}^{\sin \left(\phi_{h}+\phi_{S}\right)}\left(x, y, z, P_{h \perp}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{\sum_{q} e_{q}^{2}\left[h_{1}^{q} \otimes H_{1, q \rightarrow h}^{\perp}\right]\left(x, z, P_{h \perp}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1, q \rightarrow \pi}(z)}
$$

M.R.et al., PR D65 (02); A. Bacchetta \& M.R., PR D67 (03)


$$
\begin{gathered}
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, q \rightarrow \pi^{+} \pi^{-}}^{\varangle}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1, q \rightarrow \pi^{+} \pi^{-}}\left(z, M_{h}^{2}\right)} \\
|\boldsymbol{R}|=\frac{M_{h}}{2} \sqrt{1-\frac{4 m_{\pi}^{2}}{M_{h}^{2}}}
\end{gathered}
$$

## SIDIS SSA: Collins vs. IFF

$$
A_{U T}^{\sin \left(\phi_{h}+\phi_{S}\right)}\left(x, y, z, P_{h \perp}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{\sum_{q} e\left(h_{1}^{q} \otimes H_{1, q \rightarrow h}^{\perp}\left(x, z, P_{h \perp}^{2}\right)\right.}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1, q \rightarrow \pi}(z)}
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|\boldsymbol{R}|=\frac{M_{h}}{2} \sqrt{1-\frac{4 m_{\pi}^{2}}{M_{h}^{2}}}
\end{gathered}
$$

## one technical detail, first..

| $\begin{aligned} & \pi^{+} \pi^{-} \mathrm{CM} \\ & \text { frame } \end{aligned}$ | $A P_{\pi^{+}}$ |
| :---: | :---: |
| $P_{\pi^{-}}$ | $P_{h}$ |

$$
\begin{aligned}
H_{1}^{\varangle q}\left(z_{1}, z_{2}, M_{h}^{2}\right) \rightarrow z & =z_{1}+z_{2} \\
\zeta & =\frac{z_{1}-z_{2}}{z}=a+b \cos \theta
\end{aligned}
$$

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$$
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$$
\begin{array}{cc}
H_{1}^{\varangle q}\left(z_{1}, z_{2}, M_{h}^{2}\right) \rightarrow z=z_{1}+z_{2} & \approx H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)+\cos \theta H_{1, p p}^{\varangle q} \\
\zeta=\frac{z_{1}-z_{2}}{z}=a+b \cos \theta & \\
\text { partial wave expansion } & \text { weight of } \\
\text { in Legendre polinomials } & \text { interference } \\
\text { of } \cos \theta & \left(\pi^{+} \pi^{-}\right)_{\mathrm{s}} \text { and }\left(\pi^{+} \pi^{-}\right)_{\mathrm{p}}
\end{array}
$$

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$$

SIDIS

$$
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)}
$$

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$$

$e^{+} e^{-}$

$$
A^{\cos \left(\phi_{R}+\bar{\phi}_{R}\right)}\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\quad \mathrm{a}_{12}=\quad \text { notation of Belle paper }
$$

$$
\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{z}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}
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\end{array}
$$

SIDIS $A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)}$
$e^{+} e^{-}$ $A^{\cos \left(\phi_{R}+\bar{\phi}_{R}\right)}\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\mathrm{a}_{12}=\quad$ notation of Belle paper
need two pairs for polariz. IFF

$$
\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}
$$

## Advantages of IFF mechanism

- Simple products instead of convolutions
- No complications in factorization
- Evolution equations understood
- Universality ok
" "cleaner" e+e- extraction (less background)


## Who did what?

## $<2008$ : the "model" era



1. $\mathrm{q} \rightarrow \mathrm{O} \mathrm{X}_{1} \rightarrow \pi^{+} \pi \cdot \mathrm{X}_{1}$
2. $q \rightarrow \omega X_{2} \rightarrow \pi^{+} \pi X_{2}$
3. $q \rightarrow \omega \mathrm{X}^{\prime}{ }_{3} \rightarrow \pi^{+} \pi^{-}\left(\pi^{0} \mathrm{X}_{3}\right)$
4. $q \rightarrow \eta X_{4} \rightarrow \pi^{+} \pi-X_{4}$
5. $q \rightarrow K^{0} \mathrm{X}_{5} \rightarrow \pi^{+} \pi \cdot \mathrm{X}_{5}$
6. All-(1.+2.+3.) =backgr
7. 

parameters tuned to HERMES MC
$\rightarrow$ predict asymmetry


## 2008 : the "data" era HERMES


$0.2 \leq \mathrm{z}$
$0.5 \leq M_{h} \leq 1 \mathrm{GeV}$

- flavor symmetry:

$$
D_{1}^{u}=D_{1}^{d}=D_{1}^{\bar{u}}=D_{1}^{\bar{d}} ; \quad H_{1}^{\varangle u}=H_{1}^{\varangle \bar{d}}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}
$$

- Using Torino's transversity without errors
- Model has to be reduced by a factor


## model prediction + fitting normalization

A.Bacchetta, F.Ceccopieri, A.Mukherjee, M.R., PR D79 (09)

## 2011 : the "parametrization" era

 $\mathrm{e}^{+} \mathrm{e}^{-}$Belle dataA. Vossen et al. (Belle), PRL 107 (11)


## fitting the Belle data

A.Courtoy, A.Bacchetta, M.R., A.Bianconi, PRD 85 (12)

$$
d \sigma(\text { two pairs })=\frac{1}{4 \pi^{2}} d \sigma^{0}\left(1+\cos \left(\phi_{R}+\bar{\phi}_{R}\right) A\right)
$$

## fitting the Belle data

$$
d \sigma(\text { two pairs })=\frac{1}{4 \pi^{2}} d \sigma^{0}\left(1+\cos \left(\phi_{R}+\bar{\phi}_{R}\right) A\right)
$$

1. parametrize $\operatorname{DiFF}\left(z, \mathrm{M}_{\mathrm{h}}\right)$ at $\mathrm{Q}_{0}{ }^{2}=1 \mathrm{GeV}$ inspired by model
2. evolve DiFF's at $\mathrm{Q}_{\text {Belle }}{ }^{2}=100 \mathrm{GeV}$ (LO, no gluons)
3. integrate $\mathrm{d} \sigma^{0}$ to get $\mathrm{d} \sigma^{0}(1$ pair $) \propto \mathrm{D}_{1}\left(\mathrm{z}, \mathrm{M}_{\mathrm{h}}\right)$ no unpol. data $\Rightarrow$ fit output of PYTHIA Monte Carlo for $\left(\pi^{+}, \pi^{-}\right)$emission at Belle kin.
4. fit Belle data for asymmetry $\mathrm{A} \Rightarrow$ extract $H_{1}^{\varangle q}$
fitting $\mathrm{D}_{1}$ from M.C.

$$
\frac{d \sigma^{0}}{d z d M_{h}}=\frac{4 \pi \alpha^{2}}{Q^{2}} \sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}\right)
$$


fitting $\mathrm{D}_{1}$ from M.C.

$$
\frac{d \sigma^{0}}{d z d M_{h}}=\frac{4 \pi \alpha^{2}}{Q^{2}} \sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}\right)
$$



- $\sum_{\mathrm{MC}}=647.26 \mathrm{pb}^{-1} \leftrightarrow>2 \mathrm{M}$ events $\sim 2 \mathrm{n}_{\pi+\pi-} \quad$ no cuts in acceptance
- $\mathbf{4 0}(\mathrm{z}) \times \mathbf{5 0}(\mathrm{Mh}) \times 4$ flavors $\times 4$ channels $=\mathbf{3 2 K}$ bins

$$
(\mathrm{u}, \mathrm{~d}, \mathrm{~s}, \mathrm{c}) \quad\left(\varrho, \omega, \mathrm{K}^{0} \text { decays }+ \text { continuum }\right)
$$

- $2 \mathrm{~m}_{\pi} \leq \mathrm{M}_{\mathrm{h}} \leq 1.3 \mathrm{GeV} \quad 0.2 \leq \mathrm{z} \quad 1 \gg 2 \mathrm{M}_{\mathrm{h}} / \mathrm{zQ} \quad$ ( $\Rightarrow 31585$ bins $)$
- isospin symmetry + charge conjugation: $\mathrm{u}=\overline{\mathrm{u}}=\mathrm{d}=\overline{\mathrm{d}} \quad \mathrm{s}=\overline{\mathrm{s}} \quad \mathrm{c}=\overline{\mathrm{c}}$

$$
\left(\text { except } \mathrm{K}^{0} \rightarrow \pi^{+} \pi^{-}\right)
$$

- general form:
parameters
$\begin{array}{ccc}\mathbf{1 7}(\text { continuum }) \\ 1.69 & \mathbf{1 . 2 8}(\mathrm{Q})+\underset{1.68}{20}(\omega)+\underset{1.85}{22}\left(\mathrm{~K}^{0}\right)= & \mathbf{7 9} \\ 1.62\end{array}$
fitting $\mathrm{D}_{1}$ from M.C.

$$
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- general form: $D_{1}^{q}\left(z, M_{h}\right) \sim z^{\alpha_{1}}(1-z)^{\alpha_{2}} 2|\mathbf{R}|^{\beta} \operatorname{BW}\left(M_{h}\right) \exp \left[d_{\{\delta\}}(z)+h_{\{\lambda\}}\left(M_{h}\right)-f_{\{\gamma\}}\left(z M_{h}\right)\right\}$ parameters $17($ continuum $)+20(\varrho)+20(\omega)+22\left(\mathrm{~K}^{0}\right)=79$ $\begin{array}{lllll}1.69 & 1.28 & 1.68 & 1.85 & 1.62\end{array}$
fitting $\mathrm{D}_{1}$ from M.C.

$$
\frac{d \sigma^{0}}{d z d M_{h}}=\frac{4 \pi \alpha^{2}}{Q^{2}} \sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}\right)
$$



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$$
(\mathrm{u}, \mathrm{~d}, \mathrm{~s}, \mathrm{c}) \quad\left(\varrho, \omega, \mathrm{K}^{0} \text { decays }+ \text { continuum }\right)
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- $2 \mathrm{~m}_{\pi} \leq \mathrm{M}_{\mathrm{h}} \leq 1.3 \mathrm{GeV} \quad 0.2 \leq \mathrm{z} \quad 1 \gg 2 \mathrm{M}_{\mathrm{h}} / \mathrm{zQ} \quad$ ( $\Rightarrow 31585$ bins $)$
- isospin symmetry + charge conjugation: $\mathrm{u}=\overline{\mathrm{u}}=\mathrm{d}=\overline{\mathrm{d}} \quad \mathrm{s}=\overline{\mathrm{s}} \quad \mathrm{c}=\overline{\mathrm{c}}$ (except $\mathrm{K}^{0} \rightarrow \pi^{+} \pi^{-}$)
- general form: $D_{1}^{q}\left(z, M_{h}\right) \sim z^{\alpha_{1}}(1-z)^{\alpha_{2}} 2|\mathbf{R}|^{\beta} \mathrm{BW}\left(M_{h}\right) \exp \left[d_{\{\delta\}}(z)+h_{\{\lambda\}}\left(M_{h}\right)-f_{\{\gamma\}}\left(z M_{h}\right)\right]$ parameters $\quad 17$ (continuum) $+20(\mathrm{Q})+20(\omega)+22\left(\mathrm{~K}^{0}\right)=79$
$\begin{array}{lllllll}-\chi_{\mathrm{ch}}^{2}=\sum_{q} \sum_{i j} \frac{\left(N_{i j}^{\mathrm{ch}, q}-\mathcal{L}_{M C}\left(d \sigma_{\mathrm{ch}}^{0, q}\right)_{i j}\right)^{2}}{\mathcal{L}_{M C}\left(d \sigma_{\mathrm{ch}}^{0 . q}\right)_{i j}} & 1.69 & 1.28 & 1.68 & 1.85 & 1.62\end{array}$


## results for unpolarized $\operatorname{DiFF} \quad \mathrm{D}_{1} \mathrm{q}$

$$
\begin{aligned}
& \mathrm{Q}_{0}{ }^{2}=1 \\
& \mathrm{D}_{1}{ }^{\mathrm{q}}
\end{aligned}
$$





A.Courtoy, A.Bacchetta, M.R., A.Bianconi, PRD 85 (12)

## fitting Belle Asymmetry

$$
A\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}
$$

## fitting Belle Asymmetry

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$<>$ average bin value of each angle
$\#\left(\pi^{+}, \pi^{-}\right)$pairs $\quad n_{q}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} D_{1}^{q}\left(z, M_{h}^{2}, Q^{2}\right)$
\# pol. $\left(\pi^{+}, \pi\right)$ pairs $n_{q}^{\uparrow}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} \frac{|\mathbf{R}|}{M_{h}} H_{1, s p}^{\triangleleft q}\left(z, M_{h}^{2}, Q^{2}\right)$

## fitting Belle Asymmetry

$A\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}$
$<>$ average bin value of each angle
$\#\left(\pi^{+}, \pi^{-}\right)$pairs $\quad n_{q}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} D_{1}^{q}\left(z, M_{h}^{2}, Q^{2}\right)$
\# pol. $\left(\pi^{+}, \pi^{-}\right)$pairs $n_{q}^{\uparrow}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} \frac{|\mathbf{R}|}{M_{h}} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}, Q^{2}\right)$

$$
A\left(z, M_{h}^{2}, Q^{2}\right)=\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle} \frac{|\boldsymbol{R}|\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle}{M_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) n_{\bar{q}}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)}
$$

## fitting Belle Asymmetry

$A\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}$
$<>$ average bin value of each angle
$\#\left(\pi^{+}, \pi^{-}\right)$pairs $\quad n_{q}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} D_{1}^{q}\left(z, M_{h}^{2}, Q^{2}\right)$
\# pol. $\left(\pi^{+}, \pi^{-}\right)$pairs $n_{q}^{\uparrow}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} \frac{|\mathbf{R}|}{M_{h}} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}, Q^{2}\right)$

$$
A\left(z, M_{h}^{2}, Q^{2}\right)=\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle} \frac{|\boldsymbol{R}|\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle}{M_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) n_{\bar{q}}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)}
$$

isospin symmetry + charge conjugation $\mathrm{u}=-\mathrm{d}=-\overline{\mathrm{u}}=\overline{\mathrm{d}}$


## fitting Belle Asymmetry

$A\left(\cos \theta_{2}, z, M_{h}^{2}, \bar{z}, \bar{M}_{h}^{2}\right)=\frac{\sin ^{2} \theta_{2}}{1+\cos ^{2} \theta_{2}} \frac{|\boldsymbol{R}| \sin \theta}{M_{h}} \frac{|\overline{\boldsymbol{R}}| \sin \bar{\theta}}{\bar{M}_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) H_{1, s p}^{\varangle \bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) D_{1}^{\bar{q}}\left(\bar{z}, \bar{M}_{h}^{2}\right)}$
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$\#\left(\pi^{+}, \pi^{-}\right)$pairs $\quad n_{q}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} D_{1}^{q}\left(z, M_{h}^{2}, Q^{2}\right)$
\# pol. $\left(\pi^{+}, \pi^{-}\right)$pairs $n_{q}^{\uparrow}\left(Q^{2}\right)=\int d z \int d M_{h}^{2} \frac{|\mathbf{R}|}{M_{h}} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}, Q^{2}\right)$

$$
A\left(z, M_{h}^{2}, Q^{2}\right)=\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle} \frac{|\boldsymbol{R}|\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle}{M_{h}} \frac{\sum_{q} e_{q}^{2} H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right) n_{\bar{q}}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)}
$$

isospin symmetry + charge conjugation $\mathrm{u}=-\mathrm{d}=-\overline{\mathrm{u}}=\overline{\mathrm{d}}$

$$
A\left(z, M_{h}^{2}, Q^{2}\right) \approx-\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle \frac{5}{9} \frac{\frac{|\boldsymbol{R}|}{M_{h}} H_{1, s p}^{\varangle u}\left(z, M_{h}^{2}\right) n_{u}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)}
$$

## continued

$$
A\left(z, M_{h}^{2}, Q^{2}\right) \approx-\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle \frac{5}{9} \frac{\frac{|\boldsymbol{R}|}{M_{h}} H_{1, s p}^{\varangle u}\left(z, M_{h}^{2}\right) n_{u}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)}
$$

## continued

$$
A\left(z, M_{h}^{2}, Q^{2}\right) \approx-\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle \frac{5}{9} \frac{5 \frac{|\boldsymbol{R}|}{M_{h}} H_{1, s p}^{\varangle}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{\top}\left(z, M_{h}^{2}\right)} n_{q}\left(Q^{2}\right)
$$

## continued

$$
\begin{gathered}
A\left(z, M_{h}^{2}, Q^{2}\right) \approx-\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle \frac{5}{9} \frac{\frac{|\boldsymbol{R}|}{M_{h}} H_{1, s p}^{\varangle u}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right)} n_{u}^{\uparrow}\left(Q^{2}\right) \\
\int d z \int d Q_{h}^{2} A\left(z, M_{h}^{2}, Q^{2}\right) \times\left[\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)\right] \times\left[-\frac{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}{\left\langle\sin ^{2} \theta_{2}\right\rangle} \frac{1}{\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle} \frac{9}{5}\right]=\left[n_{u}^{\uparrow}\left(Q^{2}\right)\right]^{2}
\end{gathered}
$$

continued

$$
\begin{gathered}
A\left(z, M_{h}^{2}, Q^{2}\right) \approx-\frac{\left\langle\sin ^{2} \theta_{2}\right\rangle}{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle \frac{5 \frac{|\boldsymbol{R}|}{M_{h}} H_{1, s p}^{\varangle u}\left(z, M_{h}^{2}\right)}{9} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{\sum_{q} e_{q}^{2} D_{1}^{\varphi}\left(z, M_{h}^{2}\right)} n_{q}\left(Q^{2}\right) \\
\int d z \int d M_{h}^{2} A\left(z, M_{h}^{2}, Q^{2}\right) \times\left[\sum_{q} e_{q}^{2} D_{1}^{q}\left(z, M_{h}^{2}\right) n_{q}\left(Q^{2}\right)\right] \times\left[-\frac{\left\langle 1+\cos ^{2} \theta_{2}\right\rangle}{\left\langle\sin ^{2} \theta_{2}\right\rangle} \frac{1}{\langle\sin \theta\rangle\langle\sin \bar{\theta}\rangle} \frac{9}{5}\right]=\left[n_{u}^{\uparrow}\left(Q^{2}\right)\right]^{2}
\end{gathered}
$$

- general form at $\mathrm{Q}_{0}{ }^{2}=1: \quad H_{1, s p}^{\varangle q}\left(z, M_{h}\right) \sim(1-z) 2|\mathbf{R}| \operatorname{BW}\left(M_{h}\right) \exp \left[d_{\{\delta\}}(z)+h_{\{\lambda\}}\left(M_{h}\right)\right] f_{\{\gamma\}}\left(z M_{h}\right)$
- 9 parameters
- chiral-odd LO evolution with ad-hoc modified HOPPET (M.Guagnelli-Pavia)
- $8(\mathrm{z}) \times 8\left(\mathrm{M}_{\mathrm{h}}\right)-\left\{\mathrm{z} \in[0.8,1], \mathrm{M}_{\mathrm{h}} \in[1.5,2]\right\}=46$ bins
- $\chi^{2} /$ dof $=0.57$
- errors dominated by Belle exp. A


## results for polarized DiFF $H_{1}^{\varangle q}$

A.Courtoy, A.Bacchetta, M.R., A.Bianconi, PRD 85 (12)

$$
\begin{aligned}
& \mathrm{Q}_{0}{ }^{2}=1 \\
& \frac{|\mathbf{R}|}{M_{h}} \frac{H_{1}^{\varangle u}}{D_{1}^{u}}
\end{aligned}
$$



$$
0.27 \leq \mathrm{z} \leq 0.33
$$

$\mathrm{Q}^{2}=100$
A


## first glances at transversity via HERMES data

A.Bacchetta, A. Courtoy, M. R., PRL 107 (11)

$$
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)}
$$

$$
D_{1}^{u}=D_{1}^{d}=D_{1}^{\bar{u}}=D_{1}^{\bar{d}}
$$

$$
D_{1}^{s}=D_{1}^{\bar{s}}
$$

$$
D_{1}^{c}=D_{1}^{\bar{c}}
$$

$$
\begin{gathered}
H_{1}^{\varangle u}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}=H_{1}^{\varangle \bar{d}} \\
H_{1}^{\varangle s}=-H_{1}^{\varangle \bar{s}}=H_{1}^{\varangle c}=-H_{1}^{\varangle \bar{c}}=0
\end{gathered}
$$

## first glances at transversity via HERMES data

A.Bacchetta, A. Courtoy, M. R., PRL 107 (11)

$$
\begin{gathered}
\qquad \begin{array}{c}
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)} \\
D_{1}^{u}=D_{1}^{d}=D_{1}^{\bar{u}}=D_{1}^{\bar{d}} \\
D_{1}^{s}=D_{1}^{\bar{s}} \\
D_{1}^{c}=D_{1}^{\bar{c}} \\
\text { recall }
\end{array} \\
H_{1}^{\varangle u}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}=H_{1}^{\varangle \bar{d}} \\
H_{1}^{\varangle s}=-H_{1}^{\varangle \bar{s}}=H_{1}^{\varangle c}=-H_{1}^{\varangle \bar{c}}=0
\end{gathered}
$$

## first glances at transversity via HERMES data

A.Bacchetta, A. Courtoy, M. R., PRL 107 (11)

$$
\begin{gathered}
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)} \\
D_{1}^{u}=D_{1}^{d}=D_{1}^{\bar{u}}=D_{1}^{\bar{d}} \\
D_{1}^{s}=D_{1}^{\bar{s}} \\
D_{1}^{c}=D_{1}^{\bar{c}} \\
\text { recall } \\
H_{1}^{\varangle u}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}=H_{1}^{\varangle \bar{d}} \\
H_{1}^{\varangle s}=-H_{1}^{\varangle \bar{s}}=H_{1}^{\varangle c}=-H_{1}^{\varangle \bar{c}}=0
\end{gathered}
$$

## first glances at transversity via HERMES data

$$
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)}
$$

recall

$$
\begin{aligned}
D_{1}^{u}=D_{1}^{d} & =D_{1}^{\bar{u}}=D_{1}^{\bar{d}} \\
D_{1}^{s} & =D_{1}^{\bar{s}} \\
D_{1}^{c} & =D_{1}^{\bar{c}}
\end{aligned}
$$

$$
H_{1}^{\varangle u}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}=H_{1}^{\varangle \bar{d}}
$$


evolved from


$$
H_{1}^{\varangle s}=-H_{1}^{\varangle \bar{s}}=H_{1}^{\varangle c}=-H_{1}^{\varangle \bar{c}}=0
$$ down to

$A_{U T} \approx-\langle C(y)\rangle \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)-\frac{1}{4} x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+\frac{1}{4} x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{1}{4} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}=-0.259$ (11\% error)
from data


## 1 <br> MSTW08LO

## first glances at transversity via HERMES data

$$
A_{U T}^{\sin \left(\phi_{R}+\phi_{S}\right) \sin \theta}\left(x, y, z, M_{h}^{2}\right)=-\frac{1-y}{1-y+y^{2} / 2} \frac{|\boldsymbol{R}|}{M_{h}} \frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1, s p}^{\varangle q}\left(z, M_{h}^{2}\right)}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1}^{q}\left(z, M_{h}^{2}\right)}
$$

from data

recall

$$
\begin{aligned}
D_{1}^{u}=D_{1}^{d} & =D_{1}^{\bar{u}}=D_{1}^{\bar{d}} \\
D_{1}^{s} & =D_{1}^{\bar{s}} \\
D_{1}^{c} & =D_{1}^{\bar{c}}
\end{aligned}
$$

$$
H_{1}^{\varangle u}=-H_{1}^{\varangle d}=-H_{1}^{\varangle \bar{u}}=H_{1}^{\varangle \bar{d}}
$$

evolved from


$$
H_{1}^{\varangle s}=-H_{1}^{\varangle \bar{s}}=H_{1}^{\varangle c}=-H_{1}^{\varangle \bar{c}}=0
$$ down to

$$
A_{U T} \approx-\langle C(y)\rangle \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)-\frac{1}{4} x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+\frac{1}{4} x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{1}{4} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}=-0.259
$$



$$
<Q^{2}>=2.5
$$

## 2011 : the "collinear transversity" era


based on data from

uncertainty band from Collins effect
M.Anselmino et al.,

NP B191 (Proc.Supp.) (09)

## 2012: COMPASS data officially released

C.Adolph et al. (Compass), PL B713 (12)

2002-4 Deuteron Data

2007 Proton Data


Pavia model prediction
A. Bacchetta \& M.R., PR D74 (06)

## Which are the latest "press" news?

## New extraction : proton data

$$
A_{U T} \approx-\langle C(y)\rangle \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)-\frac{1}{4} x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+\frac{1}{4} x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{1}{4} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}
$$

## New extraction : proton data

$A_{U T} \approx-\langle C(y)\rangle \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)-\frac{1}{4} x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+\frac{1}{4} x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{1}{4} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}$
from data


## New extraction : proton data

$$
A_{U T} \approx-\langle C(y)\rangle \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)-\frac{1}{4} x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+\frac{1}{4} x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{1}{4} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}=-0.208
$$

from data


## New extraction : proton data



## uncertainty band from Collins effect

M.Anselmino et al.,

NP B191 (Proc.Supp.) (09)

## New extraction : deuteron data

$$
A_{U T} \approx-\langle C(y)\rangle \frac{3}{5} \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)+x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{2}{5} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}=-0.208 \text { (9\% error) }
$$

from data



## uncertainty band from Collins effect

M.Anselmino et al.,

NP B191 (Proc.Supp.) (09)

## New extraction: deuteron data

$$
A_{U T} \approx-\langle C(y)\rangle \frac{3}{5} \frac{x h_{1}^{u_{v}}\left(x, Q^{2}\right)+x h_{1}^{d_{v}}\left(x, Q^{2}\right)}{x f_{1}^{u+\bar{u}}\left(x, Q^{2}\right)+x f_{1}^{d+\bar{d}}\left(x, Q^{2}\right)+\frac{2}{5} \frac{n_{s}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)} x f_{1}^{s+\bar{s}}\left(x, Q^{2}\right)} \frac{n_{u}^{\uparrow}\left(Q^{2}\right)}{n_{u}\left(Q^{2}\right)}=-0.208 \quad(9 \% \text { error })
$$

from data



## uncertainty band from Collins effect

M.Anselmino et al.,

NP B191 (Proc.Supp.) (09)

## 2012 : the "collinear transversity fitting" era

combining proton and deuteron data
$\Rightarrow$ separate $u_{v}$ and $d_{v}$ components of $h_{1}$
$\Rightarrow$ separately fit each component

## 2012 : the "collinear transversity fitting" era

combining proton and deuteron data $\Rightarrow$ separate $u_{v}$ and $d_{v}$ components of $h_{1}$ $\Rightarrow$ separately fit each component
functional form

$$
x h_{1}^{q_{v}}(x)=\tanh \left[\sqrt{x}\left(A_{q}+B_{q} x+C_{q} x^{2}\right)\right]\left[\mathrm{SB}_{q}(x)+\mathrm{SB}_{\bar{q}}(x)\right]
$$

Soffer bound

$$
\mathrm{SB}_{q}(x)=\frac{1}{2}\left|f_{1}^{q}(x)+g_{1}^{q}(x)\right|
$$

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$$

$\mathrm{SB}_{\mathrm{q}}+\mathrm{SB}_{\bar{q}} \rightarrow \infty \quad \mathrm{x} \rightarrow 0$
grants finite and stable tensor charge

Soffer bound
$\mathrm{SB}_{q}(x)=\frac{1}{2}\left|f_{1}^{q}(x)+g_{1}^{q}(x)\right|$

## 2012 : the "collinear transversity fitting" era

combining proton and deuteron data $\Rightarrow$ separate $u_{v}$ and $d_{v}$ components of $h_{1}$
$\Rightarrow$ separately fit each component
functional form

$$
x h_{1}^{q_{v}}(x)=\tanh \left[\sqrt{x}\left(A_{q}+B_{q} x+C_{q} x^{2}\right)\right]\left[\mathrm{SB}_{q}(x)+\mathrm{SB}_{\bar{q}}(x)\right]
$$

$\mathrm{SB}_{\mathrm{q}}+\mathrm{SB}_{\bar{q}} \rightarrow \infty \quad \mathrm{x} \rightarrow 0$ grants finite and stable tensor charge

"flexible" form (2 nodes)
we tried also
$\left(\mathrm{A}_{\mathrm{q}}+\mathrm{B}_{\mathrm{q}} \mathrm{x}\right) \quad$ "rigid" form

Soffer bound
$\mathrm{SB}_{q}(x)=\frac{1}{2}\left|f_{1}^{q}(x)+g_{1}^{q}(x)\right|$


DSSV

## New results from fitting both p and D data

 1) Hessian methodPROTON

flexible functional form
$\chi^{2} /$ dof $\sim 1.1$



DEUTERON


## New results for $h_{1}{ }^{u_{v}}$ and $h_{1}{ }^{d_{v}}$ <br> 1) Hessian method <br> A. Bacchetta, A.Courtoy, M.R., in preparation

"flexible" form


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## New results from fitting both p and D data 2) Monte Carlo method

1. generate N replicas of data with Gaussian noise at $1 \sigma$
2. choose N such that keep same mean and std. deviation of data
3. fit N times the data $\Rightarrow \mathrm{N}$ different transversities
4. take $1 \sigma$ confidence interval of the whole set (if Gaussian-distributed, it's $=68 \%$ )

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$1 \sigma$ error band from replicas @2.4 GeV²

Best fit central curve @2.4 GeV² and standard $1 \sigma$ error band


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Can we find "unforeseen" replica?

Yes, here at $\mathbf{G e V}^{2}$


$X^{2} /$ dof
1.56557
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Yes, here at $1 \mathrm{GeV}^{2}$


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# but most replicas "want" to merge 

 to lower Soffer bound, driven by deuteron data


- Compass 2007 p +2004 D
- Compass $2010 p+2004$ D



工 Soffer bound evolved at $\mathrm{Q}^{2}=10 \mathrm{GeV}^{2}$ including error estimate $\Delta \mathrm{g}_{1}$ from

De Florian, Sassot, Stratmann, Vogelsang, PR D80 (09)

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## is there anything going on ?

see Ralston, arXiv:0810.0871

A confining gauge theory violates the completeness of asymptotic states held as foundation points of the $S$-matrix. Spin-dependent experiments can yield results that appear to violate quantum mechanics. The point is illustrated by violation of the Soffer bound in QCD....

## tensor charges

A. Bacchetta, A.Courtoy, M.R., in preparation

## Tensor Charge



## 1-flexible <br> 2-hybrid <br> 3-rigid

## where we have data

$$
\delta q=\int_{6.4 \times 10^{-3}}^{0.28} d x h_{1}^{q}(x)
$$

Truncated tensor charge d at $1 \mathrm{GeV}^{2}$


## tensor charges

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## Tensor Charge

## full range $10^{-10}-1$



Future?

## PHENIX data

## work on predictions for $p p^{\uparrow} \rightarrow\left(\pi^{+} \pi^{-}\right) X$ still in progress..

R.Yang, Beijing Transversity Workshop (08)


## Status of transversity studies



