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# di-hadron based transversity extraction or the importance of di-hadron fragmentation (DiFF or IFF)

#### Marco Radici



In collaboration with A. Bacchetta (Univ. Pavia) A. Bianconi (Univ. Brescia) A. Courtoy (Univ. Liege)

# Outline

• What are DiFF and Where to extract them

• Why do we need them ? the quest for transversity: Collins vs. IFF

- Who did what ? (= the present situation)
- Which are the latest "press news"?
- Perspectives



General framework

# Single-hadron fragmentation

K<sub>T</sub>-dependent fragmentation functions



# Single-hadron fragmentation

Integrate over the transverse momentum



Standard fragmentation functions

 $D_1^{q \to h}(z)$ 

No Collins fragmentation function





# Single-hadron fragmentation





# Di-hadron fragmentation

K<sub>T</sub>-dependent DiFF

from q-q correlator  $\Delta(z_1, z_2, K_T, R_T)$ project out :







# Di-hadron fragmentation

Integrate over the transverse momentum



$$\int d\mathbf{K}_T \left( \mathbf{S}_T^q \times \mathbf{K}_T \right) H_1^{\perp q \to h_1 h_2} + \left( \mathbf{S}_T^q \times \mathbf{R}_T \right) H_1^{\triangleleft q \to h_1 h_2} \longrightarrow \left( \mathbf{S}_T^q \times \mathbf{R}_T \right) H_1^{\triangleleft q \to h_1 h_2} (z_1, z_2, R_T^2)$$

# Di-hadron fragmentation





 $h_1$ 

 $2R_T$ 

$$d\mathbf{K}_T \left( \mathbf{S}_T^q \times \mathbf{K}_T \right) H_1^{\perp q \to h_1 h_2} + \left( \mathbf{S}_T^q \times \mathbf{R}_T \right) H_1^{\triangleleft q \to h_1 h_2} \longrightarrow \left( \mathbf{S}_T^q \times \mathbf{R}_T \right) H_1^{\triangleleft q \to h_1 h_2} (z_1, z_2, R_T^2)$$

Chiral-odd  $H_1^{\triangleleft q \to h_1 h_2}$  survives ! (memo: h<sub>1</sub>,h<sub>2</sub> must be distinguishable!)



### Where do DiFF occur?



## Where do DiFF occur?



#### Factorization

( at NLO & LL, same DGLAP as single-h case

F.Ceccopieri, M.R., A.Bacchetta, P.L. **B650**(07)

#### Universality



## Where do DiFF occur?



#### Invariant mass spectrum



*OPAL, ZP***C56** (92)

#### Non trivial !

# hadron collisions

#### Invariant mass spectrum





- In-medium modifications
- Mass shifts (Q)



in the

SIDIS

#### Invariant mass spectrum

HERMES, JHEPO6 (08)



the Why

# how to extract transversity: Collins vs. IFF

### The Collins mechanism

J. Collins, NP**B396** (93)



$$\mathbf{k} \times \mathbf{P}_h \cdot \mathbf{S}_T \propto \cos\left(\frac{\pi}{2} - \phi\right) = \sin\phi$$

transverse motion of hadron = spin analyzer of fragmenting quark

### The Collins mechanism

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transverse motion of hadron

spin analyzer of fragmenting quark

## Effects of TMD evolution



## Effects of TMD evolution



is it similar for Collins effect ? Need to check..





#### Comparison with models

 $\mathbf{x} \mathbf{h}_1(\mathbf{x})$ 



[0] M. Anselmino et al., arXiv:0812.4366

[1-8] models

#### TMD factorization $\rightarrow$ TMD evolution

- Convolution
- Soft factors
- Evolution and Sudakov form factors



#### is there a way to skip all this?

### TMD factorization $\rightarrow$ TMD evolution

- Convolution
- Soft factors
- Evolution and Sudakov form factors



#### is there a way to skip all this ?



### The IFF mechanism

#### Collins, Heppelman, Ladinsky, NP B420 (94)



azimuthal orientation of hadron pair

spin analyzer of fragmenting quark

### SIDIS SSA: Collins vs. IFF

 $\phi_h$ Indiron plane  $P_h$ 

M.R. et al., PR D65 (02); A. Bacchetta & M.R., PR D67 (03)

 $M_h^2$ 

relative to the lepton-scattering plane, of the target "↑" state. Twist-3 contribution polarized and unpolarized cross sections appear with different azimuthal depende

and

$$H_{1,q}^{\triangleleft}(z, M_{\pi\pi}, \cos\theta) \simeq H_{1,q}^{\triangleleft, sp}(z, M_{\pi\pi}) + H_{1,q}^{\triangleleft, pp}(z, M_{\pi\pi}) \cos\theta,$$

where the Legendre expansions are truncated to include only the s- and p-wave con assumed to be a valid approximation in the range of the invariant mas [43], which is typical of the present experiment.

In refs. [15, 37, 43], it was proposed to measure  $\sigma_{UU}$  and  $\sigma_{UT}$  integrated over  $\theta$ , which has the advantage that in the resulting expression for these cross sections fragmentation functions that appear are  $D_{1,q}(z, M_{\pi\pi})$  and  $H_{1,q}^{\triangleleft,sp}(z, M_{\pi\pi})$  (see eqs. However, this requires an experimental acceptance that is complete in  $\theta$ , which is to achieve, not only because of the geometrical acceptance of the detector, but als of the acceptance in the momentum of the detected pions. As the momentum  $|P_{\pi}| > 1$  GeV strongly influences the  $\theta$  distribution, the measured asymmetry kept differential in  $\theta$ .

The single-spin asymmetry  $A_{UT} \equiv \frac{1}{|S_T|} \sigma_{UT} / \sigma_{UU}$  Pontains components of a source of the model of the model of the model of the spin of the model interest here, which is related to the product of transversity and the fragmentation  $H_1^{\triangleleft,sp}$ , is defined as Carrering s θ do wedde sin (φρ

2 Ida

$$A_{UT}^{\sin(\phi_{R\perp}+\phi_{S})\sin\theta} \equiv \frac{2}{|S_{T}|} \frac{\int \operatorname{decs} \theta \, \mathrm{d}\varphi_{R\perp} \, \mathrm{d}\varphi_{S} \, \mathrm{d}\sigma_{UT}}{\int \operatorname{decs} \theta \, \mathrm{d}\varphi_{R\perp} \, \mathrm{d}\varphi_{S} \, \mathrm{d}\sigma_{U}}$$

$$\lim_{UT} (\varphi_{R}+\phi_{S})\sin\theta = -\frac{1-y}{1-y+y^{2}/2} \frac{|\mathbf{R}|^{2}}{M_{h}} \frac{\sum_{q} (\theta_{q}^{2} (h_{1}^{q}) \otimes \mathbf{H}_{1,q \to \pi}^{q} + \pi_{\pi}^{-1} (z, M_{h}^{2}))}{\sum_{q} (\theta_{q}^{2} - \theta_{T})^{2}} \frac{|\mathbf{R}|^{2}}{\sum_{q} (\theta_{q}^{2} (h_{1}^{q}) \otimes \mathbf{H}_{1,q \to \pi}^{q} + \pi_{\pi}^{-1} (z, M_{h}^{2}))|}{\int (1-y+y^{2})^{2}} \frac{|\mathbf{R}|^{2}}{M_{h}} \frac{\sum_{q} (\theta_{q}^{2} (h_{1}^{q}) \otimes \mathbf{H}_{1,q \to \pi}^{q} + \pi_{\pi}^{-1} (z, M_{h}^{2}))|}{\sum_{q} (1-y+y^{2})^{2}} \frac{|\mathbf{R}|^{2}}{2} \sqrt{1(z_{4} (M_{\pi\pi}^{2} + \theta_{T})^{2} + \theta_{T}^{2} + \theta_{T}$$

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$$\Lambda$$
 (m x M  $\phi$   $\phi$   $0) = 1 N^{\uparrow} - N^{\downarrow}$ 

### SIDIS SSA: Collins vs. IFF



 $1 \dots 0 M D D D ( \neg / 0 )$ 

relative to the lepton-scattering plane, of the target "\" state. Twist-3 contribution  
polarized and unpolarized cross sections appear with different azimuthal depend  
Both dihadron fragmentation functions 
$$D_{1,q}$$
 and  $H_{1,q}^{\triangleleft}$  can be expanded in  
Legendre functions of  $c g g q$  Rende  $[43], \otimes H_{1,q}^{\perp,q} \to h ]$   $(x, z, P_{h\perp}^2)$   
 $\overline{1D_{1,q}(yM_{\pi\pi},ygg)} = \overline{D_{1,q}(z,M_{\pi\pi})} e^2 q^3 f g (M_{\pi\pi}) D g (zM_{\pi\pi}) \frac{1}{4}(3\cos^2 q)$   
and

$$H_{1,q}^{\triangleleft}(z, M_{\pi\pi}, \cos\theta) \simeq H_{1,q}^{\triangleleft, sp}(z, M_{\pi\pi}) + H_{1,q}^{\triangleleft, pp}(z, M_{\pi\pi}) \cos\theta,$$

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The single-spin asymmetry  $A_{UT} \equiv \frac{1}{|S_T|} \sigma_{UT} / \sigma_{UU}$  Pontains components of a source ous Fourier and Legendre expansion. The amplitude  $A_{UT}^{\sin(\phi_R \perp \frac{P_R \phi_S}{h \phi_S}) \sin \theta}$  of the model. interest here, which is related to the product of transversity and the fragmentation  $H_1^{\triangleleft,sp}$ , is defined as

$$A_{UT}^{\sin(\phi_{R\perp}+\phi_{S})\sin\theta}(x,y,z,M_{h}^{2}) = -\frac{1-y}{1-y+y^{2}/2} \frac{|\mathbf{R}|}{M_{h}} \sum_{\substack{sin(\phi_{R\perp}+\phi_{S})\sin\theta}} \frac{2}{|\mathbf{S}_{T}|} \frac{\int d\cos\theta \, d\phi_{R\perp} \, d\phi_{S} \, d\phi_{R\perp} \, \phi_{S} \, d\sigma_{UT}^{7}/\sin\theta}{\int dcs\theta \, d\phi_{R\perp} \, d\phi_{S} \, d\sigma_{UT}^{7}/\sin\theta}}{\int dcs\theta \, d\phi_{R\perp} \, d\phi_{S} \, d\sigma_{UT}^{7}/\sin\theta}} |\mathbf{R}| = \frac{M_{h}}{2} \sqrt{1-\frac{4m_{\pi}^{2}}{M_{h}^{2}}}$$
Due to the factor  $e_{q}^{2}$ , the amplitude is expected to be  $up$ -quark dominated. The results reported here are extracted from the single-spin asymmetry

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$$A_{\text{res}}(m \approx M - \phi_{\text{res}}, \theta) = \frac{1}{N^{\uparrow} - N^{\downarrow}}$$

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 $H_1^{\triangleleft q}(z_1, z_2, M_h^2) \rightarrow z = z_1 + z_2 \qquad \approx H_{1,sp}^{\triangleleft q}(z, M_h^2) + \cos \theta H_{1,pp}^{\triangleleft q}$  $\zeta = \frac{z_1 - z_2}{z} = a + b \cos \theta$ 

$$\begin{split} & \sum_{q} e_{q}^{2} h_{1}^{q}(x) \ H_{1,q \to \pi^{+}\pi^{-}}^{\triangleleft}(z, M_{h}^{2}) \\ & \sum_{q} e_{q}^{2} f_{1}^{q}(x) \ D_{1,q \to \pi^{+}\pi^{-}}(z, M_{h}^{2}) \\ & = \frac{M_{h}}{2} \sqrt{1 - \frac{4m_{\pi}^{2}}{M_{h}^{2}}} \end{split}$$

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$$\begin{split} H_1^{\triangleleft q}(z_1,z_2,M_h^2) \to &z = z_1 + z_2 \\ \zeta &= \frac{z_1 - z_2}{z} = a + b \cos \theta \\ \text{partial wave expansion} \\ \text{in Legendre polinomials} \\ \text{of } \cos \theta \end{split} \approx H_{1,sp}^{\triangleleft q}(z,M_h^2) + \cos \theta H_{1,pp}^{\triangleleft q} \\ \swarrow \\ \chi \\ \text{weight of} \\ \text{interference} \\ (\pi^+\pi^-)_{\text{s}} \text{ and } (\pi^+\pi^-)_{\text{p}} \end{split}$$

$$\frac{\sum_{q} e_{q}^{2} h_{1}^{q}(x) H_{1,q \to \pi^{+}\pi^{-}}^{\triangleleft}(z, M_{h}^{2})}{\sum_{q} e_{q}^{2} f_{1}^{q}(x) D_{1,q \to \pi^{+}\pi^{-}}(z, M_{h}^{2})}$$

$$M_{h} \int_{-1}^{-1} 4m_{\pi}^{2}$$

 $\overline{M_h^2}$ 

2



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$$\begin{aligned} \text{SIDIS} & A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h^2) = -\frac{1 - y}{1 - y + y^2/2} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) \ H_{1,sp}^{\prec q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) \ D_1(z, M_h^2)} \\ C_q \ e_q^2 \ f_1^q(x) \ D_{1,q \to \pi^+ \pi^-}(z, M_h^2) \end{aligned}$$

$$= \frac{M_h}{2} \sqrt{1 - \frac{4m_\pi^2}{M_h^2}}$$



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X.Artru, J.Collins, ZPC **69** (96) D.Boer, R.Jakob, M.R., PR D**67** (03)



$$\begin{split} H_1^{\triangleleft q}(z_1,z_2,M_h^2) \to &z = z_1 + z_2 \\ \zeta &= \frac{z_1 - z_2}{z} = a + b \cos \theta \\ \text{partial wave expansion} \\ \text{in Legendre polinomials} \\ \text{of } \cos \theta \end{split} \approx H_{1,sp}^{\triangleleft q}(z,M_h^2) + \cos \theta H_{1,pp}^{\triangleleft q} \\ \swarrow \\ \chi \\ \text{weight of} \\ \text{interference} \\ (\pi^+\pi^-)_{\text{s}} \text{ and } (\pi^+\pi^-)_{\text{p}} \end{split}$$

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### Advantages of IFF mechanism

- Simple products instead of convolutions
- No complications in factorization
- Evolution equations understood
- Universality ok
- "cleaner" e+e- extraction (less background)



#### <2008 : the "model" era

*A. Bacchetta* & *M.R.*, *PR D***74** (06)



1.  $q \rightarrow QX_1 \rightarrow \pi^+ \pi^- X_1$ 2.  $q \rightarrow \omega X_2 \rightarrow \pi^+ \pi^- X_2$ 3.  $q \rightarrow \omega X'_3 \rightarrow \pi^+ \pi^- (\pi^0 X_3)$ 4.  $q \rightarrow \eta X_4 \rightarrow \pi^+ \pi^- X_4$ 5.  $q \rightarrow K^0 X_5 \rightarrow \pi^+ \pi^- X_5$ 6. All-(1.+2.+3.)=backgr 7. All

→ predict asymmetry




## 2008 : the "data" era HERMES

HERMES, JHEPO6 (08)



 $0.2 \le z$  $0.5 \le M_h \le 1 \text{ GeV}$ 

flavor symmetry:  $D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}}$ ;  $H_1^{\triangleleft u} = H_1^{\triangleleft \bar{d}} = -H_1^{\triangleleft d} = -H_1^{\triangleleft \bar{u}}$ 

Using Torino's transversity without errors

Model has to be reduced by a factor  $0.32\pm0.06 (\chi^2/d.o.f.=1.26)$ 

model prediction +
fitting normalization

A.Bacchetta, F.Ceccopieri, A.Mukherjee, M.R., PR D79 (09)



A. Vossen et al. (Belle), PRL 107 (11)



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## fitting the Belle data

A. Courtoy, A. Bacchetta, M.R., A. Bianconi, PRD 85 (12)

$$d\sigma$$
(two pairs) =  $\frac{1}{4\pi^2} d\sigma^0 \left(1 + \cos(\phi_R + \overline{\phi}_R) A\right)$ 

#### fitting the Belle data

A. Courtoy, A. Bacchetta, M.R., A. Bianconi, PRD 85 (12)

$$d\sigma$$
(two pairs) =  $\frac{1}{4\pi^2} d\sigma^0 \left(1 + \cos(\phi_R + \overline{\phi}_R) A\right)$ 

1. parametrize  $DiFF(z, M_h)$  at  $Q_0^2 = 1$  GeV inspired by model

2. evolve DiFF's at  $Q_{Belle}^2 = 100 \text{ GeV}$  (LO, no gluons)

3. integrate  $d\sigma^0$  to get  $d\sigma^0(1 \text{ pair}) \propto D_1(z, M_h)$ no unpol. data  $\Rightarrow$  fit output of PYTHIA Monte Carlo for  $(\pi^+, \pi^-)$  emission at Belle kin.

4. fit Belle data for asymmetry  $A \Rightarrow extract H_1^{\triangleleft q}$ 



 $\overline{P}_1$ 

 $P_2 \pi - \varphi_R$ 

 $\overline{P}_h$ 





•  $\pounds_{MC} = 647.26 \text{ pb}^{-1} \iff >2M \text{ events } \sim 2 n_{\pi+\pi}$  no cuts in acceptance •  $40(z) \times 50(Mh) \times 4 \text{ flavors } \times 4 \text{ channels } = 32K \text{ bins}$   $(u,d,s,c) \quad (Q,\omega,K^0 \text{ decays } + \text{ continuum})$ •  $2m_{\pi} \le M_h \le 1.3 \text{ GeV}$   $0.2 \le z$   $1 >> 2M_h/zQ$  ( $\Rightarrow 31585 \text{ bins}$ ) • isospin symmetry + charge conjugation:  $u = \overline{u} = d = \overline{d}$   $s = \overline{s}$   $c = \overline{c}$  $(except K^0 \rightarrow \pi^+\pi^-)$ 

• general form: parameters  $17(\text{continuum}) + 20(Q) + 20(\omega) + 22(K^0) = 79$  $1.69 \quad 1.28 \quad 1.68 \quad 1.85 \quad 1.62$ 





•  $\mathcal{L}_{MC} = 647.26 \text{ pb}^{-1} \iff >2M \text{ events } \sim 2 \text{ n}_{\pi+\pi}$  no cuts in acceptance •  $40(z) \times 50(Mh) \times 4 \text{ flavors } \times 4 \text{ channels } = 32K \text{ bins}$ (u,d,s,c) (Q, $\omega$ ,K<sup>0</sup> decays + continuum) •  $2m_{\pi} \leq M_{h} \leq 1.3 \text{ GeV}$  0.2  $\leq z$  1  $>> 2M_{h}/zQ$  ( $\Rightarrow$  31585 bins) • isospin symmetry + charge conjugation: u =  $\overline{u} = d = \overline{d}$  s =  $\overline{s}$  c =  $\overline{c}$ (except K<sup>0</sup>  $\Rightarrow \pi^{+}\pi^{-}$ ) • general form:  $D_{1}^{q}(z, M_{h}) \sim z^{\alpha_{1}}(1-z)^{\alpha_{2}} 2|\mathbf{R}|^{\beta} \text{ BW}(M_{h}) \exp[d_{\{\delta\}}(z) + h_{\{\lambda\}}(M_{h}) + f_{\{\gamma\}}(zM_{h})]$ parameters 17(continuum) + 20(Q) + 20( $\omega$ ) + 22(K<sup>0</sup>) = 79 • 1.69 1.28 1.68 1.85 1.62





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## results for unpolarized DiFF $D_1^q$



A. Courtoy, A. Bacchetta, M.R., A. Bianconi, PRD 85 (12)

 $A(\cos\theta_2, z, M_h^2, \overline{z}, \overline{M}_h^2) = \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \frac{|\mathbf{R}| \sin\theta}{M_h} \frac{|\mathbf{\overline{R}}| \sin\overline{\theta}}{\overline{M}_h} \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) H_{1,sp}^{\triangleleft \overline{q}}(\overline{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) D_1^{\overline{q}}(\overline{z}, \overline{M}_h^2)}$ 

$$A(\cos\theta_2, z, M_h^2, \overline{z}, \overline{M}_h^2) = \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \frac{|\mathbf{R}| \sin\theta}{M_h} \frac{|\overline{\mathbf{R}}| \sin\overline{\theta}}{\overline{M}_h} \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) H_{1,sp}^{\triangleleft \overline{q}}(\overline{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) D_1^{\overline{q}}(\overline{z}, \overline{M}_h^2)}$$

< > average bin value of each angle  $\# (\pi^+, \pi^-) \text{ pairs} \qquad n_q(Q^2) = \int dz \int dM_h^2 D_1^q(z, M_h^2, Q^2)$  $\# \text{ pol.} (\pi^+, \pi^-) \text{ pairs} \qquad n_q^{\uparrow}(Q^2) = \int dz \int dM_h^2 \frac{|\mathbf{R}|}{M_h} H_{1,sp}^{\triangleleft q}(z, M_h^2, Q^2)$ 

$$A(\cos\theta_2, z, M_h^2, \overline{z}, \overline{M}_h^2) = \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \frac{|\mathbf{R}|\sin\theta}{M_h} \frac{|\overline{\mathbf{R}}|\sin\overline{\theta}}{\overline{M}_h} \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) H_{1,sp}^{\triangleleft \overline{q}}(\overline{z}, \overline{M}_h^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) D_1^{\overline{q}}(\overline{z}, \overline{M}_h^2)}$$

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$$\# (\pi^+, \pi^-) \text{ pairs } n_q(Q^2) = \int dz \int dM_h^2 D_1^q(z, M_h^2, Q^2)$$
  
# pol.  $(\pi^+, \pi^-)$  pairs  $n_q^{\uparrow}(Q^2) = \int dz \int dM_h^2 \frac{|\mathbf{R}|}{M_h} H_{1,sp}^{\triangleleft q}(z, M_h^2, Q^2)$ 

$$A(z, M_h^2, Q^2) = \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \frac{|\mathbf{R}| \langle \sin \theta \rangle \langle \sin \overline{\theta} \rangle}{M_h} \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) n_{\overline{q}}^{\uparrow}(Q^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) n_q(Q^2)}$$

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isospin symmetry + charge conjugation  $u = -d = -\overline{u} = \overline{d}$ 





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< > average bin value of each angle

$$\# (\pi^{+}, \pi^{-}) \text{ pairs } n_{q}(Q^{2}) = \int dz \int dM_{h}^{2} D_{1}^{q}(z, M_{h}^{2}, Q^{2})$$

$$\# \text{ pol. } (\pi^{+}, \pi^{-}) \text{ pairs } n_{q}^{\uparrow}(Q^{2}) = \int dz \int dM_{h}^{2} \frac{|\mathbf{R}|}{M_{h}} H_{1,sp}^{\triangleleft q}(z, M_{h}^{2}, Q^{2})$$

$$A(z, M_h^2, Q^2) = \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \frac{|\mathbf{R}| \langle \sin \theta \rangle \langle \sin \overline{\theta} \rangle}{M_h} \frac{\sum_q e_q^2 H_{1,sp}^{\triangleleft q}(z, M_h^2) n_{\overline{q}}^{\uparrow}(Q^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) n_q(Q^2)}$$

isospin symmetry + charge conjugation  $u = -d = -\overline{u} = \overline{d}$ 

u — 
$$\sim \pi^+$$

$$- \pi^{-}$$

$$\mathcal{A}(z, M_h^2, Q^2) \approx -\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \left\langle \sin \theta \right\rangle \left\langle \sin \overline{\theta} \right\rangle \frac{5}{9} \frac{\frac{|\mathbf{R}|}{M_h} H_{1, sp}^{\triangleleft \, u}(z, M_h^2) \, n_u^{\uparrow}(Q^2)}{\sum_q e_q^2 \, D_1^q(z, M_h^2) \, n_q(Q^2)}$$

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$$\int dz \int dM_h^2 A(z, M_h^2, Q^2) \times \left[ \sum_q e_q^2 D_1^q(z, M_h^2) n_q(Q^2) \right] \times \left[ -\frac{\langle 1 + \cos^2 \theta_2 \rangle}{\langle \sin^2 \theta_2 \rangle} \frac{1}{\langle \sin \theta \rangle \langle \sin \overline{\theta} \rangle} \frac{9}{5} \right] = \left[ n_u^{\uparrow}(Q^2) \right]^2$$

$$\begin{split} A(z, M_h^2, Q^2) \approx -\frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \left\langle \sin \theta \right\rangle \left\langle \sin \overline{\theta} \right\rangle \frac{5}{9} \frac{\frac{|\mathbf{R}|}{M_h} H_{1,sp}^{\triangleleft \, u}(z, M_h^2) n_u^{\uparrow}(Q^2)}{\sum_q e_q^2 D_1^q(z, M_h^2) n_q(Q^2)} \\ \int dz \int dM_h^2 A(z, M_h^2, Q^2) \times \left[ \sum_q e_q^2 D_1^q(z, M_h^2) n_q(Q^2) \right] \times \left[ -\frac{\langle 1 + \cos^2 \theta_2 \rangle}{\langle \sin^2 \theta_2 \rangle} \frac{1}{\langle \sin \theta \rangle \langle \sin \overline{\theta} \rangle} \frac{9}{5} \right] = \left[ n_u^{\uparrow}(Q^2) \right] \end{split}$$

• general form at  $Q_0^2 = 1$ :  $H_{1,sp}^{\triangleleft q}(z, M_h) \sim (1-z) 2 |\mathbf{R}| \operatorname{BW}(M_h) \exp[d_{\{\delta\}}(z) + h_{\{\lambda\}}(M_h)] f_{\{\gamma\}}(zM_h)$ 

- 9 parameters
- chiral-odd LO evolution with *ad-hoc* modified HOPPET (M.Guagnelli-Pavia)
- $8(z) \times 8(M_h) \{z \in [0.8, 1], M_h \in [1.5, 2]\} = 46$  bins
- $\chi^2/dof = 0.57$
- errors dominated by Belle exp. A

# results for polarized DiFF $H_1^{\triangleleft q}$

A. Courtoy, A. Bacchetta, M.R., A. Bianconi, PRD 85 (12)



A.Bacchetta, A. Courtoy, M.R., PRL 107 (11)

$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h^2) = -\frac{1 - y}{1 - y + y^2/2} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_{1,sp}^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

$$\begin{array}{c} D_{1}^{u} = D_{1}^{d} = D_{1}^{\bar{u}} = D_{1}^{d} \\ D_{1}^{s} = D_{1}^{\bar{s}} \\ D_{1}^{c} = D_{1}^{\bar{c}} \end{array}$$
recall
$$\begin{array}{c} D_{1}^{c} = D_{1}^{\bar{c}} \\ D_{1}^{c} = D_{1}^{\bar{c}} \end{array}$$

$$H_1^{\triangleleft u} = -H_1^{\triangleleft d} = -H_1^{\triangleleft \bar{u}} = H_1^{\triangleleft \bar{d}}$$
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A.Bacchetta, A. Courtoy, M.R., PRL 107 (11)

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$$A_{UT} \approx -\langle C(y) \rangle \frac{xh_1^{u_v}(x,Q^2) - \frac{1}{4}xh_1^{d_v}(x,Q^2)}{xf_1^{u+\overline{u}}(x,Q^2) + \frac{1}{4}xf_1^{d+\overline{d}}(x,Q^2) + \frac{1}{4}\frac{n_s(Q^2)}{n_u(Q^2)}xf_1^{s+\overline{s}}(x,Q^2)} \frac{n_u^{\uparrow}(Q^2)}{n_u(Q^2)}$$

A. Bacchetta, A. Courtoy, M.R., PRL 107 (11)

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$$A_{UT} \approx -\langle C(y) \rangle \frac{x h_1^{u_v}(x, Q^2) - \frac{1}{4} x h_1^{d_v}(x, Q^2)}{x f_1^{u + \overline{u}}(x, Q^2) + \frac{1}{4} x f_1^{d + \overline{d}}(x, Q^2) + \frac{1}{4} \frac{n_s(Q^2)}{n_u(Q^2)} x f_1^{s + \overline{s}}(x, Q^2)} \frac{n_u^{\uparrow}(Q^2)}{n_u(Q^2)} = -0.259$$
(11% error)

A. Bacchetta, A. Courtoy, M.R., PRL 107 (11)

$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h^2) = -\frac{1 - y}{1 - y + y^2/2} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_{1,sp}^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

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$$A_{UT} \approx -\langle C(y) \rangle \frac{xh_1^{u_v}(x,Q^2) - \frac{1}{4}xh_1^{d_v}(x,Q^2)}{xf_1^{u+\overline{u}}(x,Q^2) + \frac{1}{4}xf_1^{d+\overline{d}}(x,Q^2) + \frac{1}{4}\frac{n_s(Q^2)}{n_u(Q^2)}xf_1^{s+\overline{s}}(x,Q^2)} \frac{n_u^{\uparrow}(Q^2)}{n_u(Q^2)} = -0.259$$
(11% error)
from data
MSTW08LO
MSTW08LO

A. Bacchetta, A. Courtoy, M.R., PRL 107 (11)

$$A_{UT}^{\sin(\phi_R + \phi_S) \sin \theta}(x, y, z, M_h^2) = -\frac{1 - y}{1 - y + y^2/2} \frac{|\mathbf{R}|}{M_h} \frac{\sum_q e_q^2 h_1^q(x) H_{1,sp}^{\triangleleft q}(z, M_h^2)}{\sum_q e_q^2 f_1^q(x) D_1^q(z, M_h^2)}$$

 $D_1^u = D_1^d = D_1^{\bar{u}} = D_1^d$  $D_1^s = D_1^{\bar{s}}$  $D_1^c = D_1^{\bar{c}}$ 

$$\begin{aligned} H_1^{\triangleleft u} &= -H_1^{\triangleleft d} = -H_1^{\triangleleft \bar{u}} = H_1^{\triangleleft \bar{d}} \\ H_1^{\triangleleft s} &= -H_1^{\triangleleft \bar{s}} = H_1^{\triangleleft c} = -H_1^{\triangleleft \bar{c}} = 0 \end{aligned}$$



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(11% error)
(11% error)
(11% error)

nem

#### 2011 : the "collinear transversity" era

A. Bacchetta, A. Courtoy, M.R., PRL 107 (11)



based on data from



uncertainty band from Collins effect

M.Anselmino et al., NP **B191** (Proc.Supp.) (09)

# 2012 : COMPASS data officially released © COMPASS & HERMES

2002-4 Deuteron Data

2007 Proton Data



 $0.2 \leq z$  $0.28 \leq M_h \leq 1.2 \text{ GeV}$ 

#### Pavia model prediction

A. Bacchetta & M.R., PR D74 (06)

# Which are the latest "press" news ?

## New extraction : proton data

$$A_{UT} \approx -\langle C(y) \rangle \frac{xh_1^{u_v}(x,Q^2) - \frac{1}{4}xh_1^{d_v}(x,Q^2)}{xf_1^{u+\overline{u}}(x,Q^2) + \frac{1}{4}xf_1^{d+\overline{d}}(x,Q^2) + \frac{1}{4}\frac{n_s(Q^2)}{n_u(Q^2)}xf_1^{s+\overline{s}}(x,Q^2)} \frac{n_u^{\uparrow}(Q^2)}{n_u(Q^2)} \frac{n_u^{\uparrow}(Q^2)}{n_u(Q^2)}$$

## New extraction : proton data

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from data

#### New extraction : proton data

$$A_{UT} \approx -\langle C(y) \rangle \frac{xh_1^{u+u}(x,Q^2) - \frac{1}{4}xh_1^{d+u}(x,Q^2)}{xf_1^{u+u}(x,Q^2) + \frac{1}{4}xf_1^{d+d}(x,Q^2) + \frac{1}{4}\frac{n_x(Q^2)}{n_u(Q^2)}xf_1^{s+s}(x,Q^2)} \frac{n_u^*(Q^2)}{n_u(Q^2)} = -0.208$$
(9% error)  
from data







## 2012 : the "collinear transversity fitting" era

combining proton and deuteron data  $\Rightarrow$  separate  $u_v$  and  $d_v$  components of  $h_1$  $\Rightarrow$  separately fit each component

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#### functional form

 $xh_1^{q_v}(x) = \tanh\left[\sqrt{x} \left(A_q + B_q x + C_q x^2\right)\right] \left[\operatorname{SB}_q(x) + \operatorname{SB}_{\bar{q}}(x)\right]$ 

Soffer bound  

$$SB_q(x) = \frac{1}{2} |f_1^q(x) + g_1^q(x)|$$

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MSTW08LO DSSV
### 2012 : the "collinear transversity fitting" era

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$$\operatorname{SB}_{q} + \operatorname{SB}_{\bar{q}} \rightarrow \infty x \rightarrow 0$$

$$\operatorname{Soffer}_{\text{grants finite and stable}}$$

$$\operatorname{SB}_{q}(x) = \frac{1}{2} |_{x}$$

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fer bound  $\frac{1}{2} |f_1^q(x) + g_1^q(x)|$ MSTW08LO DSSV

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### 2012 : the "collinear transversity fitting" era

combining proton and deuteron data  $\Rightarrow$  separate u<sub>v</sub> and d<sub>v</sub> components of h<sub>1</sub>  $\Rightarrow$  separately fit each component

#### functional form

$$xh_1^{q_v}(x) = \tanh\left[\sqrt{x} \left(A_q + B_q x + C_q x^2\right)\right] \left[\operatorname{SB}_q(x) + \operatorname{SB}_{\bar{q}}(x)\right]$$

 $SB_q+SB_{\bar{q}} \rightarrow \infty x \rightarrow 0$ grants finite and stable tensor charge

> "flexible" form (2 nodes) we tried also  $(A_q + B_q x)$  "rigid" form

Soffer bound  $SB_q(x) = \frac{1}{2} |f_1^q(x) + g_1^q(x)|$ MSTW08LO DSSV

#### New results from fitting both p and D data Comparison with extraction Messian method

#### **PROTON**

#### DEUTERON



sabato 10 novembre 2012

# New results for $h_1^{u_v}$ and $h_1^{d_v}$ **Our Flexible Functional Form 2nd Order polynomial** "flexible" form



# New results for $h_1^{u_v}$ and $h_1^{d_v}$ **Our Rigid Functional Form** *1st order polynomial* preparation "rigid" form



## New results from fitting both p and D data 2) Monte Carlo method A. Bacchetta, A. Courtoy, M.R., in preparation

1. generate N replicas of data with Gaussian noise at  $1\sigma$ 

- 2. choose N such that keep same mean and std. deviation of data
- 3. fit N times the data  $\Rightarrow$  N different transversities
- 4. take  $1\sigma$  confidence interval of the whole set (if Gaussian-distributed, it's = 68%)

## New results from fitting both p and D data 2) Monte Carlo method A. Bacchetta, A. Courtoy, M.R., in preparation

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A. Bacchetta, A. Courtoy, M.R., in preparation

"rigid" form  $x h_1^{u_V}(x)$ 0.6 0.5 0.4  $1\sigma$  error band from replicas @2.4 GeV<sup>2</sup> 0.3 0.2 0.1 0.0  $\mathbf{x} h_1^{d_V}(\mathbf{x})$ -0.1 0.01 0.1 0.2 х 0.1 0.0 -0.1 Best fit central curve @2.4 GeV<sup>2</sup> -0.2and standard  $1\sigma$  error band -0.30.01 0.1 х

sabato 10 novembre 2012

#### **Can we find "unforeseen" replica?**

Yes, here at 1GeV<sup>2</sup>





2.07397 1.75523

sabato 10 novembre 2012



but most replicas "want" to merge to lower Soffer bound, driven by deuteron data





• Compass 2007 p + 2004 D

#### • Compass 2010 p + 2004 D



Soffer bound evolved at  $Q^2=10 \text{ GeV}^2$  including error estimate  $\Delta g_1$  from

> De Florian, Sassot, Stratmann, Vogelsang, PR D80 (09)

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Compass 2010 p + 2004 D

## is there anything going on ?

#### see Ralston, arXiv:0810.0871

A confining gauge theory violates the completeness of asymptotic states held as foundation points of the *S*-matrix. Spin-dependent experiments can yield results that appear to violate quantum mechanics. The point is illustrated by violation of the Soffer bound in *QCD*....





A. Bacchetta, A. Courtoy, M.R., in preparation

#### **Tensor Charge**

#### where we have data









A. Bacchetta, A. Courtoy, M.R., in preparation

#### **Tensor Charge**

#### full range 10<sup>-10</sup>- 1



Torino result @ different scale (0.8 GeV<sup>2</sup>)

1-flexible 2-hybrid 3-rigid





# Future ?

### PHENIX data

 $\phi$ 

R.Yang, Beijing Transversity Workshop (08)

work on predictions for  $pp^{\uparrow} \rightarrow (\pi^{+}\pi^{-})X$  still in progress..



# Status of transversity studies