Workshop on Fragmentation Functions and QCD 2012 (Fragmentation 2012)

Friday 09 November 2012 - Sunday 11 November 2012

RIKEN Wako, Nishina Hall

Precision Measurement of Charged Pion and Kaon Multiplicities in e^+e^- Annihilation at Belle

Improving Knowledge of Nucleon Structure/ Hadronization by Measuring at

1) Low Q^2

2) High z, for

3) Identified Hadrons

Martin Leitgab,
University of Illinois at Urbana-Champaign,
for the Belle collaboration

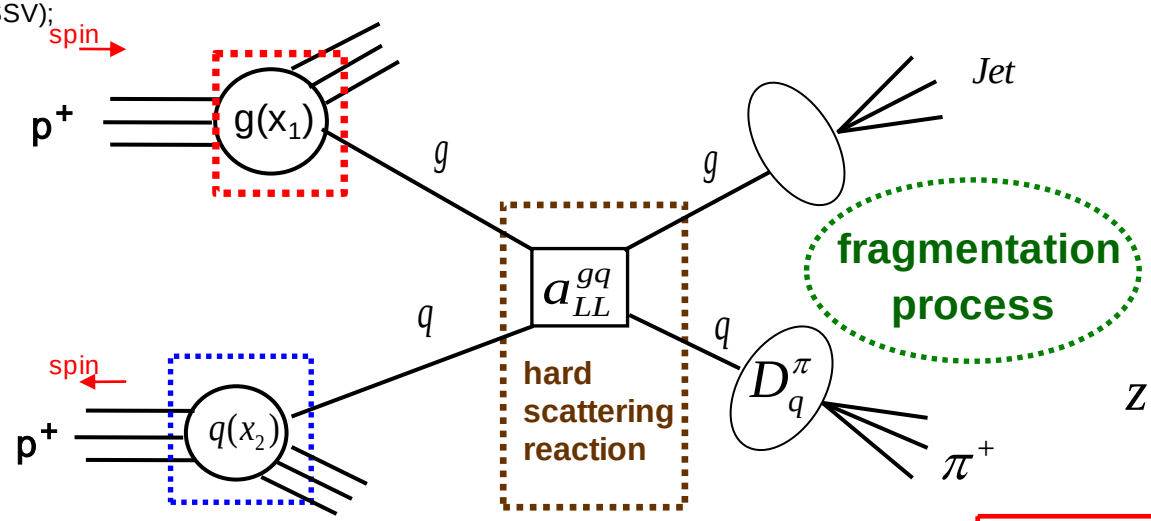
1. Application of Measurement: Use of Unpolarized Fragmentation Functions

- Direct Insight into Non-perturbative Hadronization Mechanism
- FF input quantities for analyses of nucleon structure:
  - extraction of **quark** & **antiquark, gluon helicity** distributions in **SIDIS** (Hermes, Compass, JLab 12 GeV) and **pp** (RHIC experiments)

de Florian, Sassot, Stratmann, Vogelsang (DSSV);
Phys. Rev. Lett. **101**, 072001 (2008)
and
Phys. Rev. D **80**, 034030 (2009)

Leader, Sidorov, Stamenov (LSS);
Phys. Rev. D **82**, 114018 (2010)

$$p^{\uparrow} + p^{\downarrow} \rightarrow \pi^{+} + X$$



$$z \equiv \frac{E_h^{cms}}{\sqrt{s}/2}$$

Fragmentation Function

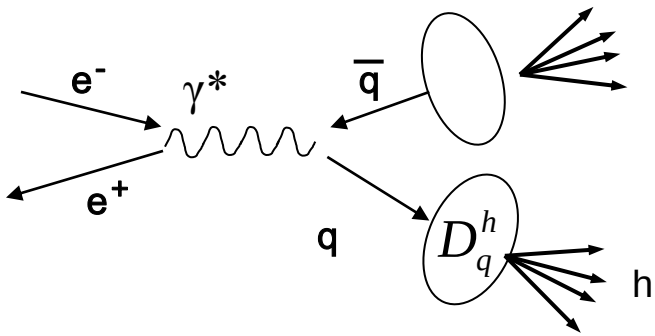
measured at RHIC

$$\frac{d^3\Delta\sigma(p^{\downarrow}p^{\uparrow} \rightarrow \pi^{+}X)}{dx_1 dx_2 dz} \propto \Delta g(x_1) \cdot \Delta q(x_2) \times \frac{d^2\hat{\sigma}(gq \rightarrow gq)}{dx_1 dx_2} \times D_q^{\pi^+}(z) + \ldots$$

To be extracted — SIDIS — pQCD

2. Theory Introduction: Extractions of FFs from Experimental Data

- In LO: FF D_i^h describes probability for a parton i to fragment into a hadron h



- FF at different energy scales relatable by DGLAP evolution equations
- FFs D_i^h can be extracted from e+e- data in pQCD analysis:

$$N^h(z,Q^2) = \frac{1}{\sigma_{\text{tot had}}} \frac{d\sigma(e^+e^- \to hX)}{dz} \qquad z \equiv \frac{E_h^{cms}}{\sqrt{s}/2}$$

measured: hadron multiplicity

$$\overset{\text{NLO QCD}}{=} \sum_{i=q,\bar{q},g} C_i^{NLO}(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

Simultaneous pQCD fit for several $N^h(z,Q^2)$

extracted: FFs

2. Theory Introduction: Extractions of FFs from Experimental Data

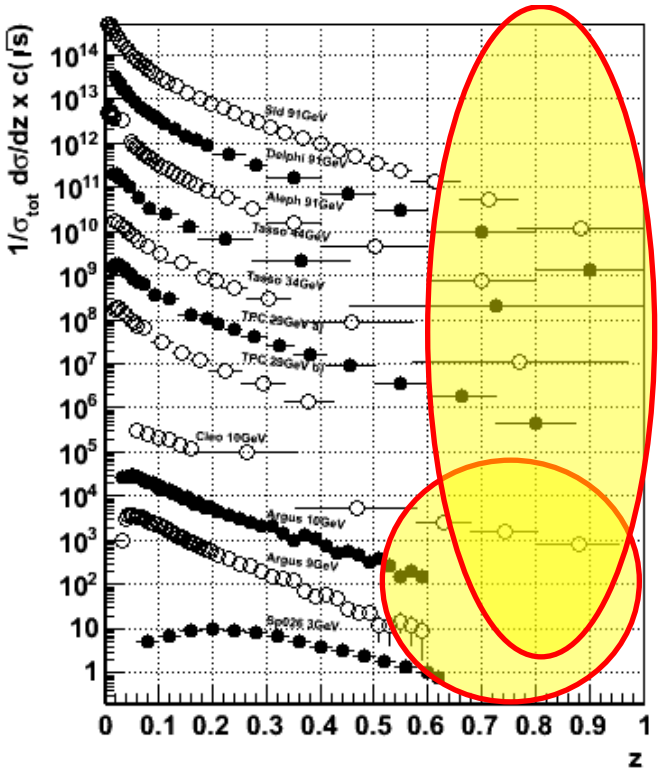
Recent FF extraction including uncertainties (e+e-): - Albino, Kniehl, Kramer; Nucl. Phys. B **725**, 181 (2005)

- Hirai, Kumano, Nagai, Sudoh ; Phys. Rev. D **75**, 094009 (2007)

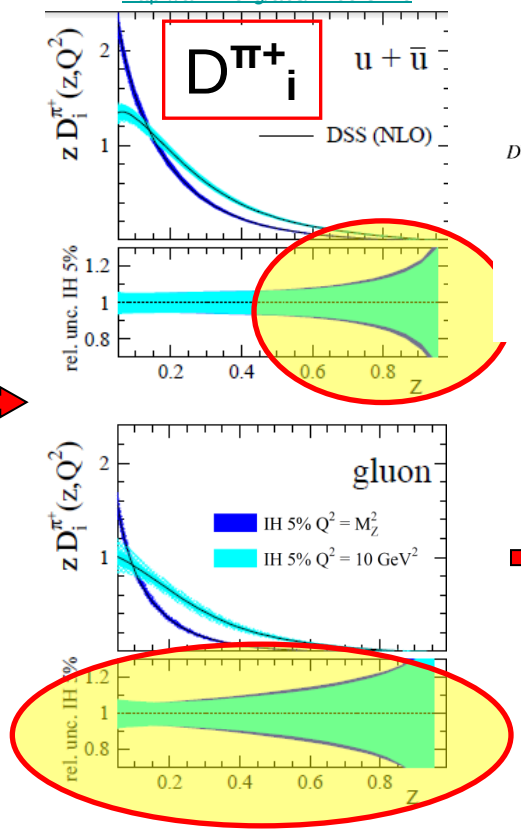
'Global' Analyses: - Albino, Kniehl, Kramer (e+e-, pp); Nucl. Phys. B **803**, 42 (2008)

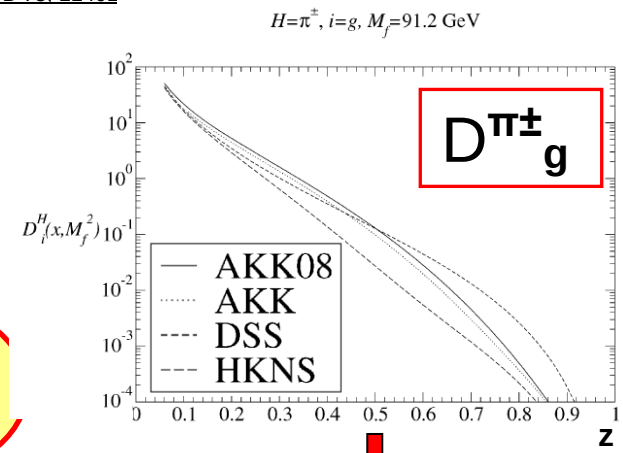
- de Florian, Sassot, Stratmann (e+e-, SIDIS, pp); Phys. Rev. D 75, 114010 (2007); 76, 074033 (2007)

World Data (Sel.) for Pion Multiplicities



http://arxiv.org/abs/1209.3240





Large uncertainties (esp. gluon FF) **due to**:

- Lack of **precise data at low energy scales** (far from LEP/SLC)

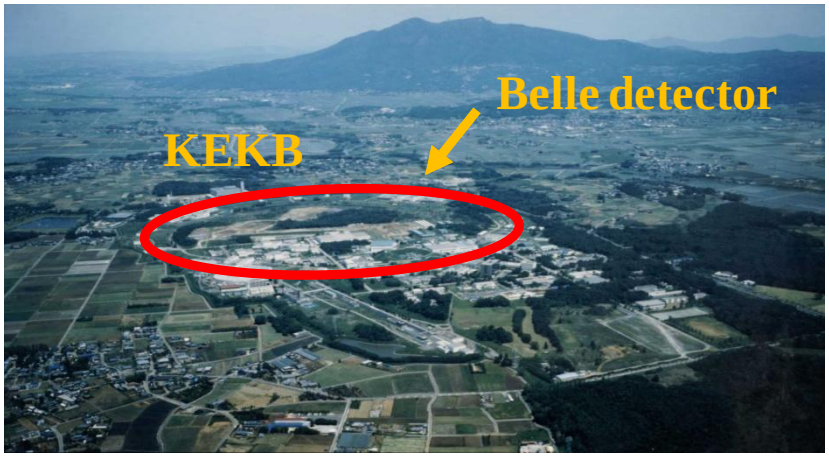
- Lack **of precise data at high z**

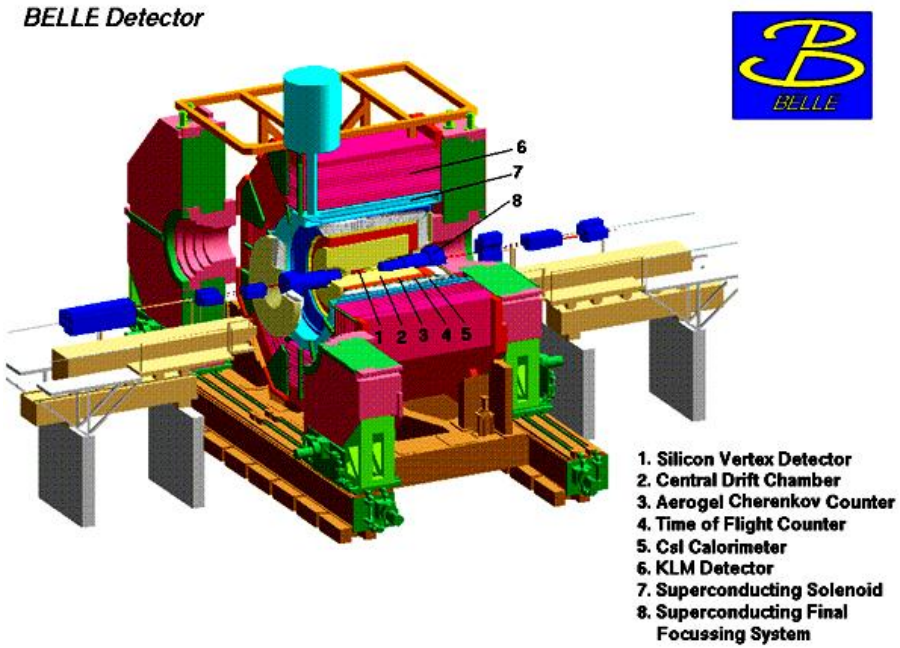
Improve knowledge of FF and nucleon structure via **high precision** hadron measurement **at low Q^2**

3. Belle Experiment

- KEK-B: asymmetric e^+ (3.5 GeV) e^- (8 GeV) collider:
  - **$\sqrt{s}$ = 10.58 GeV,** $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
  - **$\sqrt{s}$ = 10.52 GeV,** e^+e^- -> qqbar (u,d,s,c) 'continuum production'

- ideal detector for high precision measurements:
  - tracking acceptance θ [17 °;150°]: Silicon Vertex Detector, Drift Chamber
  - particle identification (PID): dE/dx, Cherenkov, ToF, EMcal, Mu detector
  - available data:
    - ~1.8 *10^9 events at 10.58 GeV,
    - ~220 *10^6 events at 10.52 GeV





4. Measurement and Corrections
Particle Misidentifcation

- Particle misidentification expected to be largest uncertainty:

  particle identification probabilities $p_{(i \to j)}$:
  probability that particle of species ***i*** PID-selected as particle of species ***j***.

Physical particle: π

→ Belle PID likelihood information from: Drift Chamber (dE/dx), Cherenkov, ToF, Calorimeter, Muon Detector

→ Reconstructed particle:

- $p_{(\pi \to e)}$ → e
- $p_{(\pi \to \mu)}$ → μ
- $p_{(\pi \to \pi)}$ → π
- $p_{(\pi \to K)}$ → K
- $p_{(\pi \to p)}$ → p

$$\vec{N}_x = \begin{pmatrix} N_e \\ N_\mu \\ N_\pi \\ N_K \\ N_p \end{pmatrix}$$

$$[P]_{ij} = \begin{pmatrix} p_{(e\to e)} & p_{(\mu\to e)} & p_{(\pi\to e)} & p_{(K\to e)} & p_{(p\to e)} \\ p_{(e\to \mu)} & p_{(\mu\to \mu)} & p_{(\pi\to \mu)} & p_{(K\to \mu)} & p_{(p\to \mu)} \\ p_{(e\to \pi)} & p_{(\mu\to \pi)} & p_{(\pi\to \pi)} & p_{(K\to \pi)} & p_{(p\to \pi)} \\ p_{(e\to K)} & p_{(\mu\to K)} & p_{(\pi\to K)} & p_{(K\to K)} & p_{(p\to K)} \\ p_{(e\to p)} & p_{(\mu\to p)} & p_{(\pi\to p)} & p_{(K\to p)} & p_{(p\to p)} \end{pmatrix}$$

$$\vec{\tilde{N}}_j = \hat{P}\,\vec{N}_i$$

$$\vec{N}_i = \hat{P}^{-1}\,\vec{\tilde{N}}_j :$$

correction through inversion of matrix.

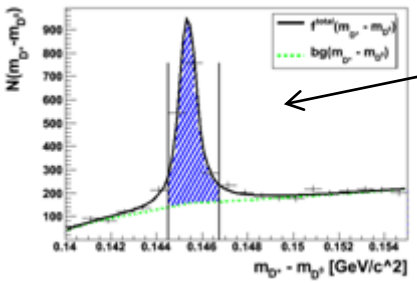
4. Measurement and Corrections
Particle Misidentifcation

- Experimental data based extraction of PID probabilities by decay sample study

e.g. $D^* \rightarrow D^0$, $D^0 \rightarrow K^-$, $D^0 \rightarrow \pi^+_{fast}$, $D^* \rightarrow \pi^+_{slow}$

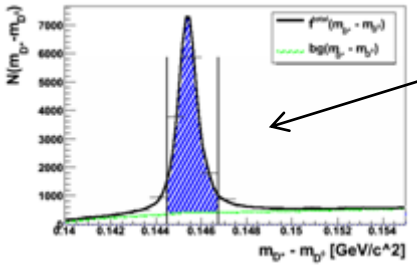
a) Kinematically reconstruct D*

b) extract PID probability from invariant mass plots



$m_{D^*} - m_{D^0}$ for **K^- tracks** with p_{lab} in [1.4; 1.6] GeV/c, $\cos\theta_{lab}$ in [0.02; 0.21], **reconstructed as π**

$$p_{(K^- \to \pi^-)} = \frac{\quad}{\quad}$$

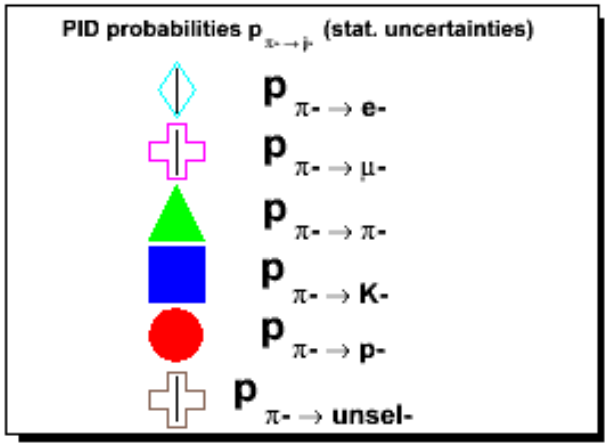


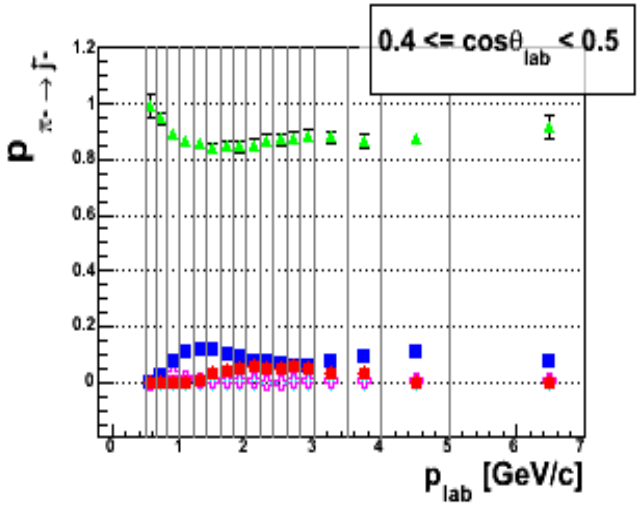
$m_{D^*} - m_{D^0}$ for **K^- tracks** with p_{lab} in [1.4; 1.6] GeV/c, $\cos\theta_{lab}$ in [0.02; 0.21]

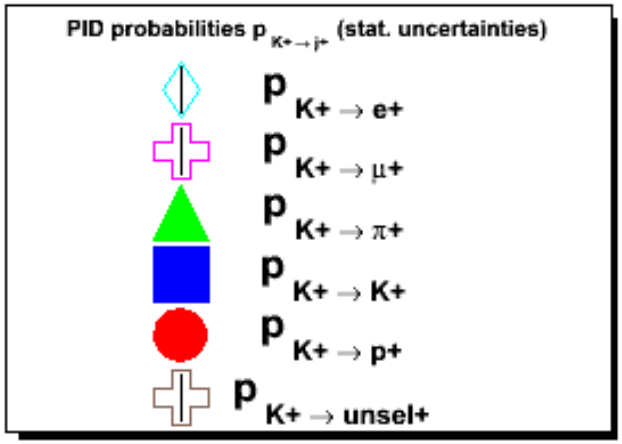
$$\boxed{p_{(K^- \to \pi^-)} \approx 0.111 \pm 0.004}$$

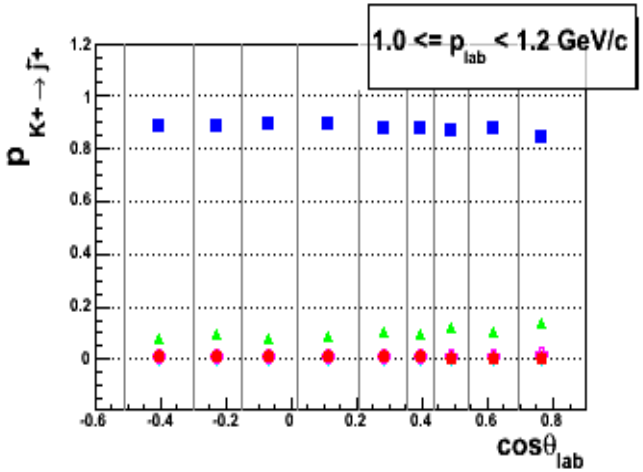
4. Measurement and Corrections
Particle Misidentifcation

sample PID probabilities from D* decay studies









completed extensive data-based PID calibration by extraction of probabilities

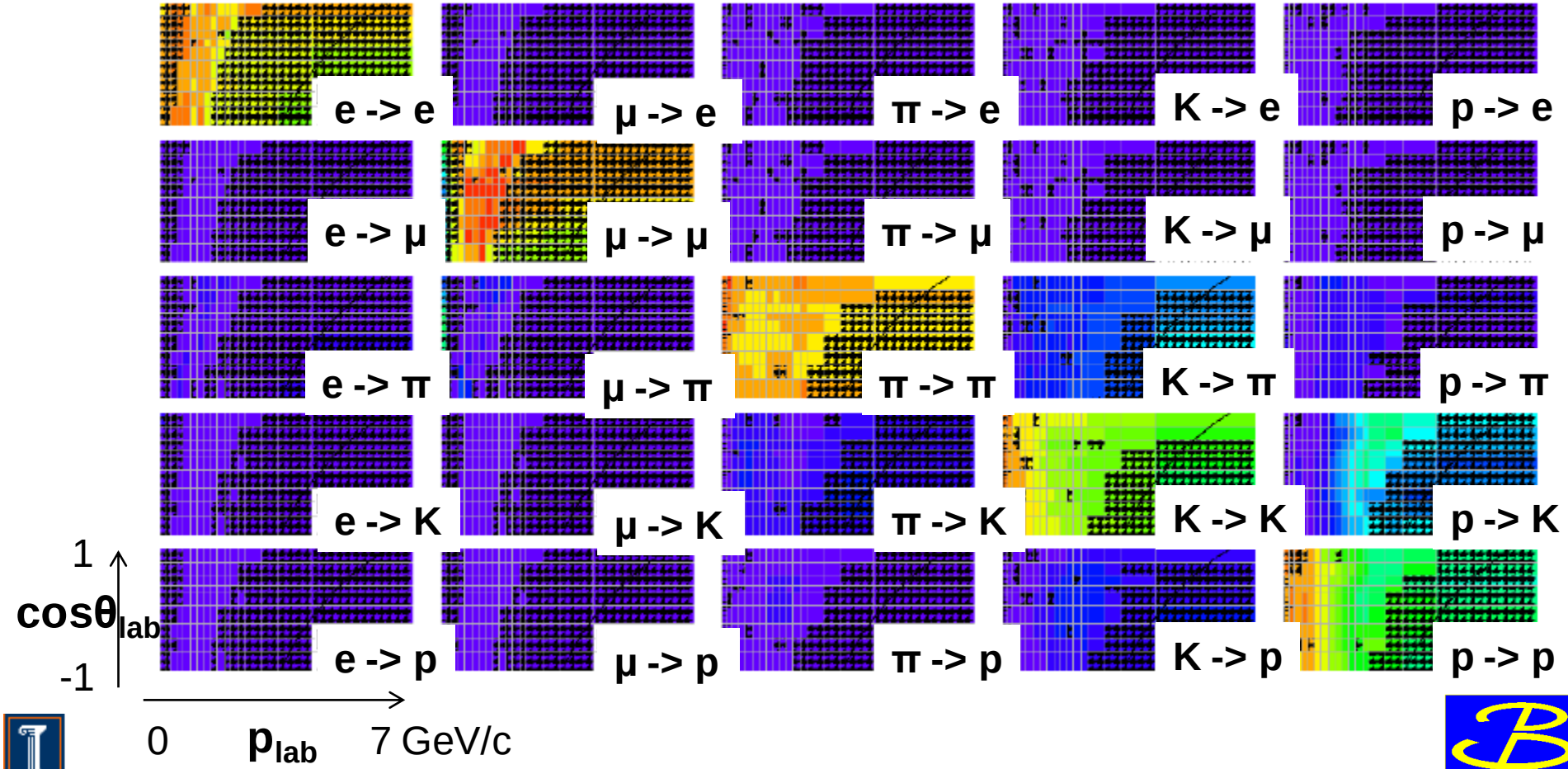
$p_{(\pi, K \to j)}$ from D* decay sample,

$p_{(\pi, p \to j)}$ from Λ decay sample,

$p_{(e, \mu \to j)}$ from J/ψ decay sample.

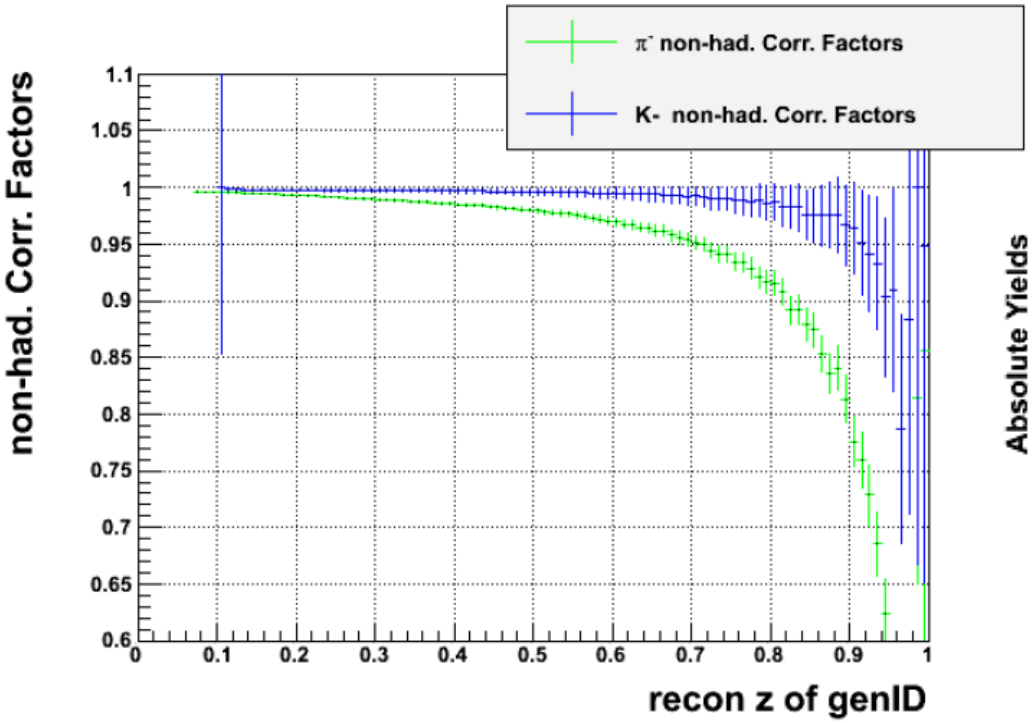
4. Measurement and Corrections
Particle Misidentifcation

- All PID probabilities for negative track charge (~3500).
- Bins with fill pattern has extrapolated probabilities (tendency of experimental data & MC)



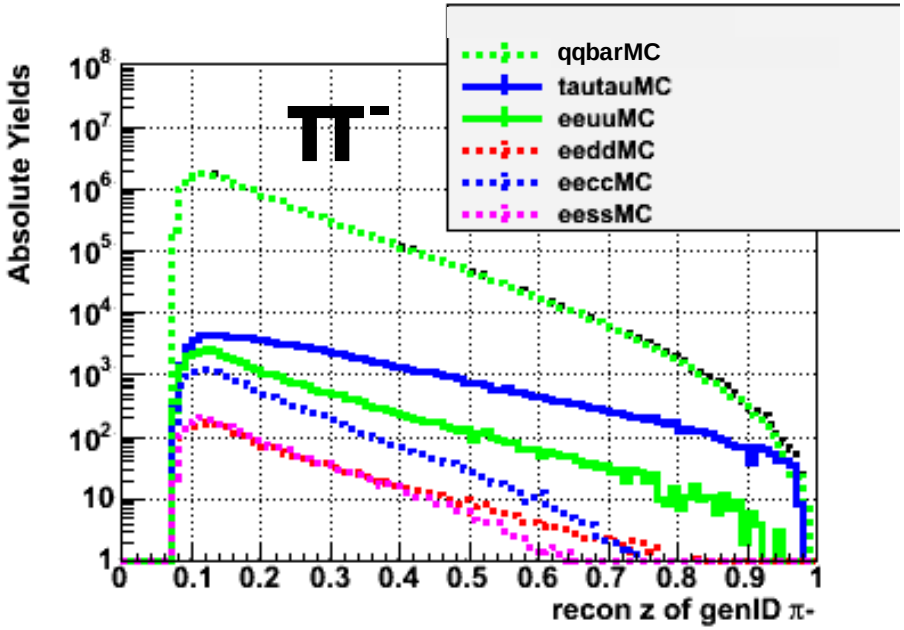
4. Measurement and Corrections
Measurement Sample Impurities

- For same luminosity, compare $q\bar{q}$, τ τ, 2γ Monte Carlo samples generated by resp. cross sections after analysis cuts
- At high z, main impurities for pions from τ events: up to 35%.



Yields from $q\bar{q}$ events relative to total yields for π^-, K^-

Plots from about 430 * 10^6 Monte Carlo Events.



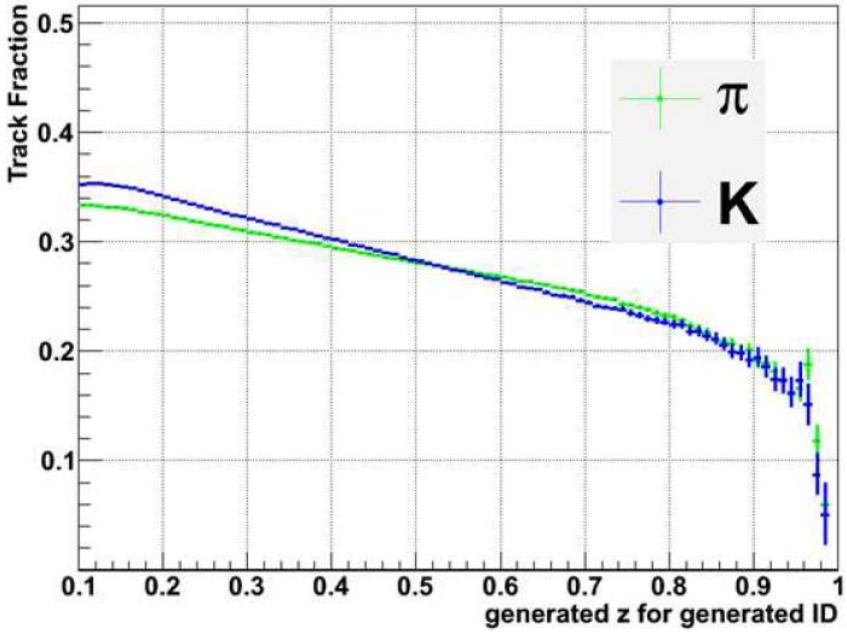
Absolute yields from different event types for π^-

4. Measurement and Corrections
Initial/Final State Treatment

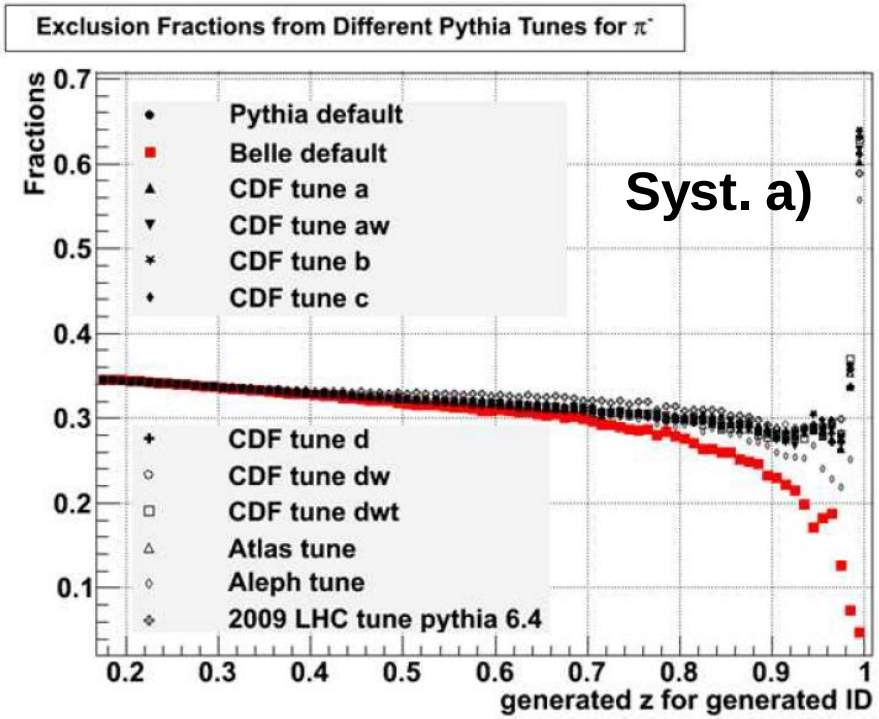
- Measurement result should only contain events closer to nominal sqrt(s) than O(~1%) (= theoretical uncertainty on Q^2 evolution).
- In this analysis, all tracks from events further than 0.5% to sqrt(s) excluded, exclusion fractions from generic Belle MC.
- Exclusion fractions dependent on shape of MC- studied 12 tunes (100M events each), assign spread of exclusion fractions as systematic uncertainty (insignificant to max. z).

generic Belle MC, ,
particles from Evtgen & GEANT.

Fraction of tracks from high ISR/FSR events, Belle MC

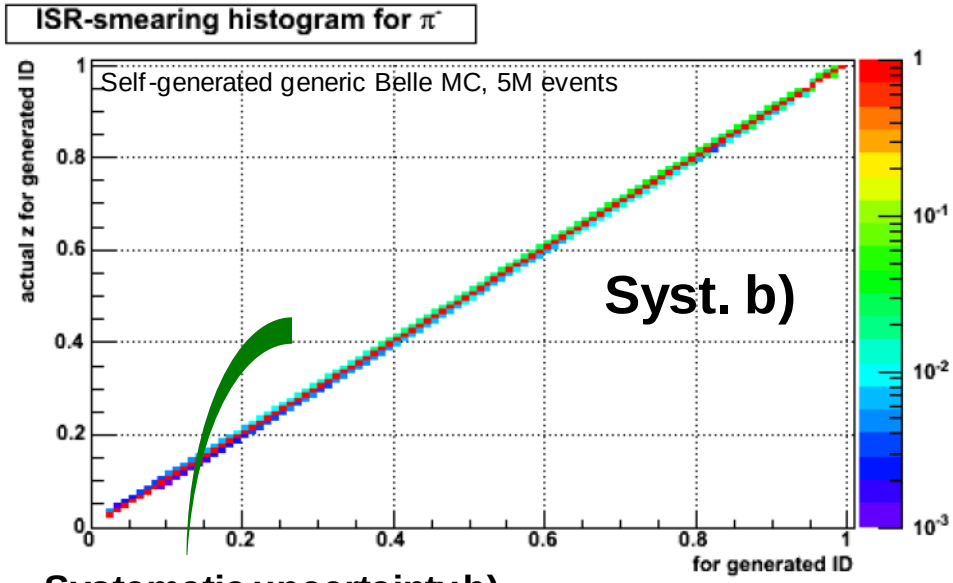


Self-generated Pythia tunes, no GEANT, 100M events each.

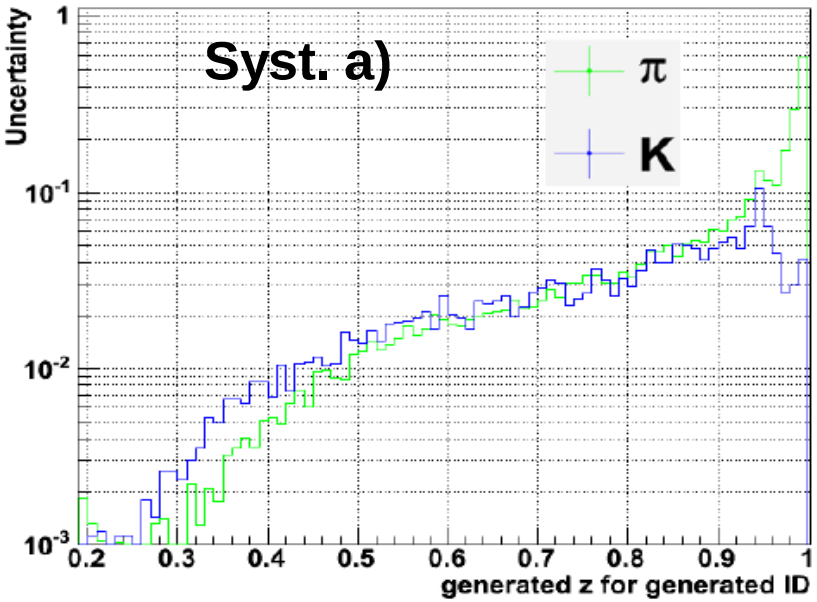


4. Measurement and Corrections
Initial/Final State Treatment

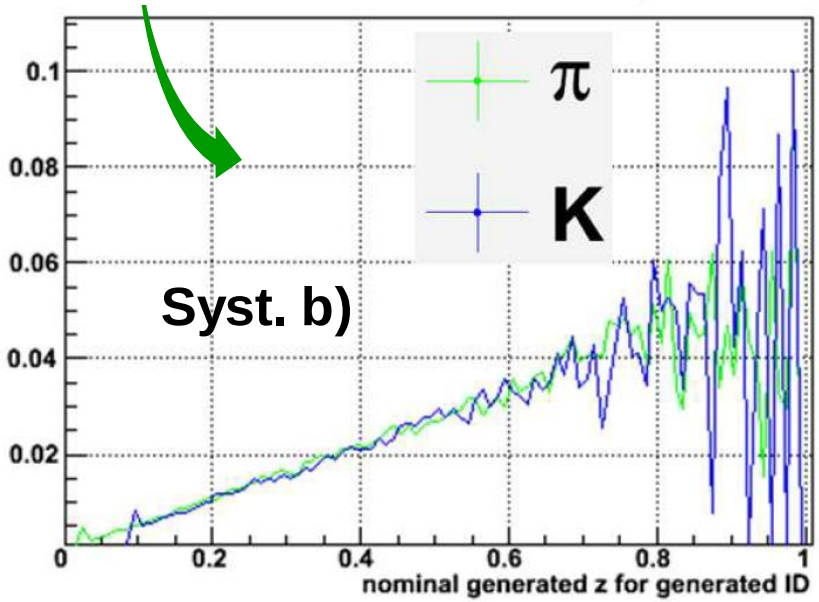
- Remaining ISR/FSR: causes smearing.
- Calculated actual versus nominal z (with correct boost)
- Systematic uncertainty from integrated bin-by-bin overall smearing.



Systematic uncertainty a) from MC shape

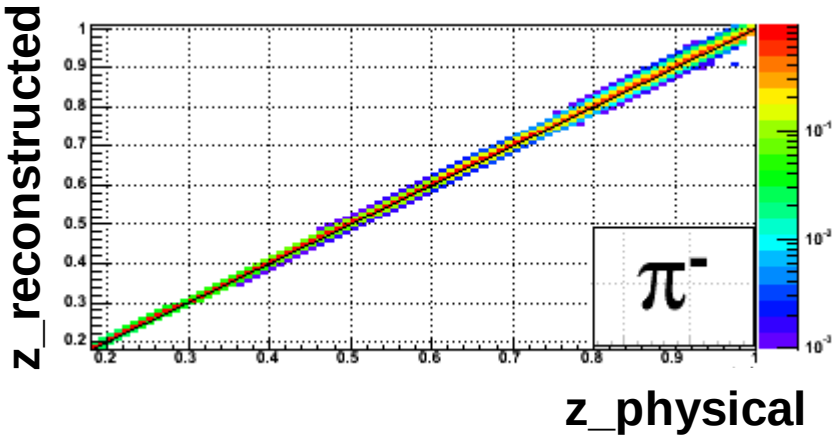


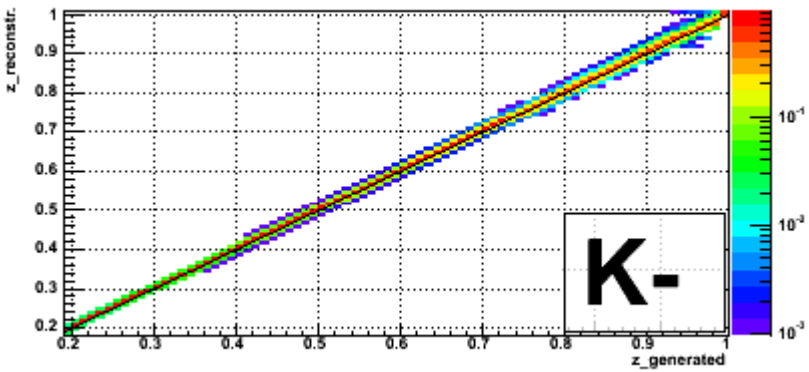
Systematic uncertainty b) from remaining ISR/FSR smearing



4. Measurement and Corrections
Additional Corrections

- Monte Carlo-based correction for kinematical smearing.



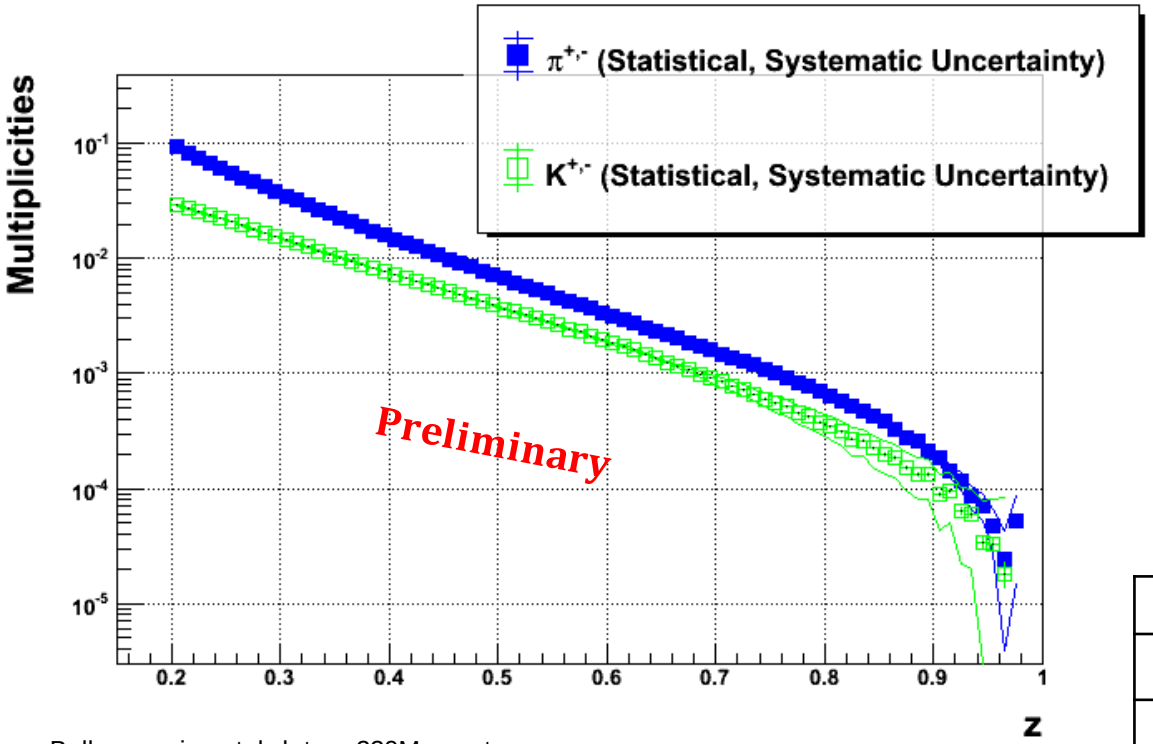


10^9 Monte Carlo events after analysis cuts

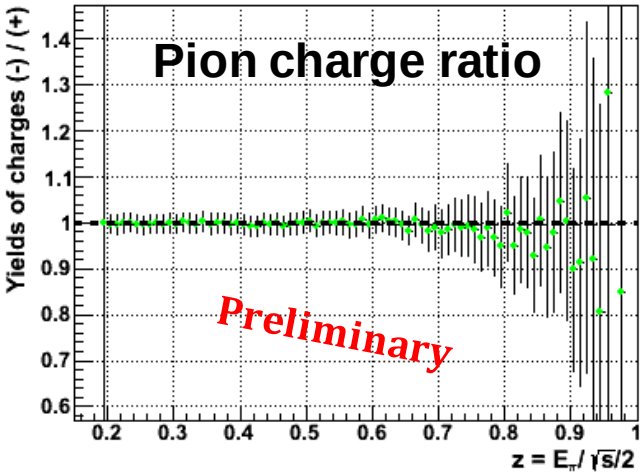
- Further corrections:
  - Decay-in-flight,
  - Detector Interaction/ shower particles,
  - Detector/tracking efficiencies,
  - Analysis acceptance

5. Results and Discussion
Preliminary Results

- Binning in z: width = 0.01; yields normalized to hadronic cross section
- Systematic uncertainties: z ~0.6: 1% (2%) for π (K);
  z ~0.9: 14% (50%) for π (K)



Belle experimental data, ~220M events

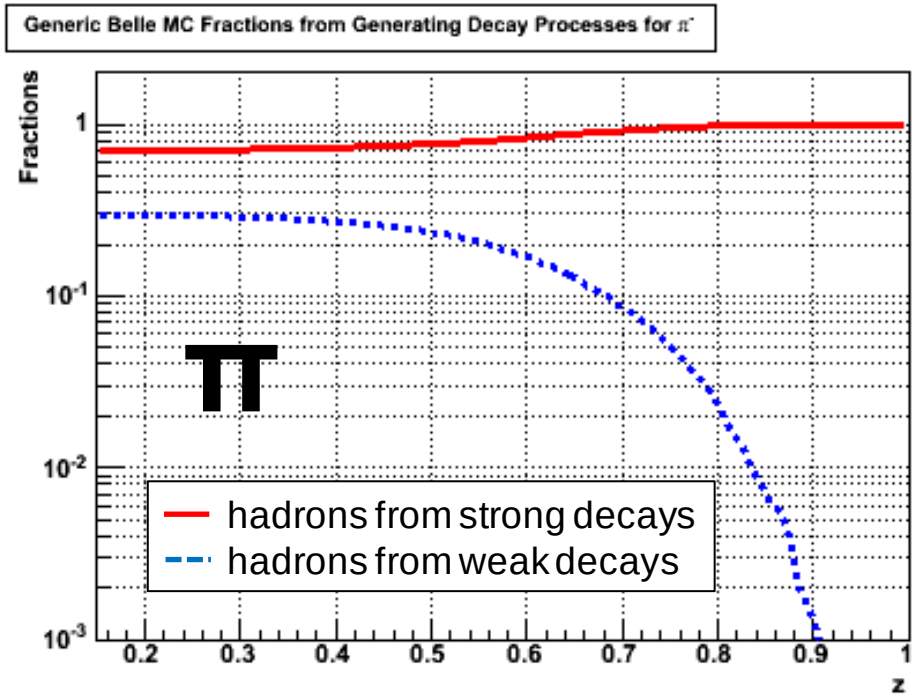


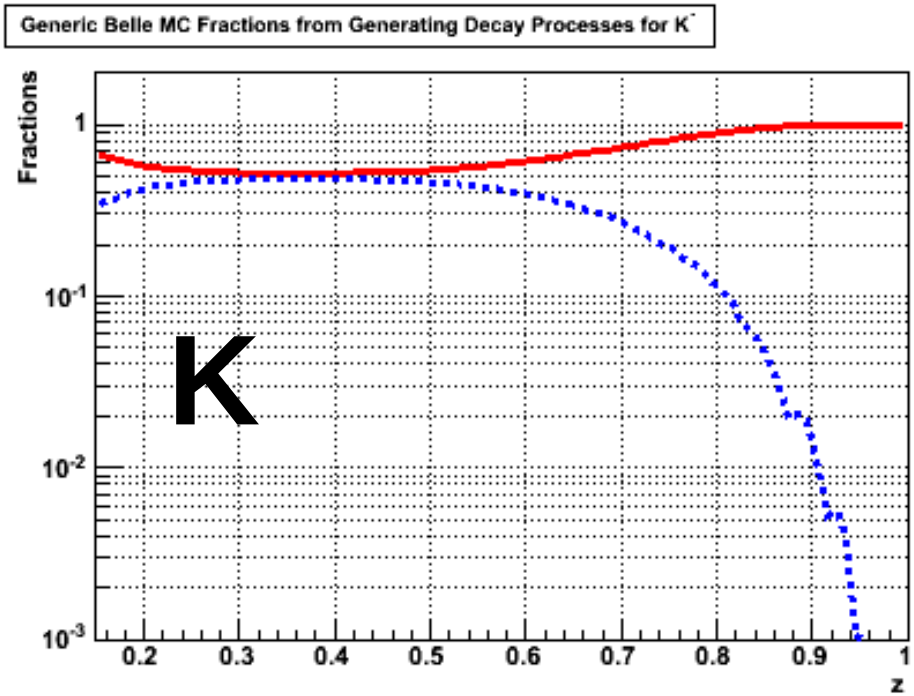
Species	Constant Results	X^2/NDF
π	0.9974 ± 0.0033	3.97/76 = 0.05
K	0.9991 ± 0.0037	5.43/76 = 0.07

Improvement of precision: cf. slide 16, 17.

5. Results and Discussion
Hadrons from Strong and Weak Decays

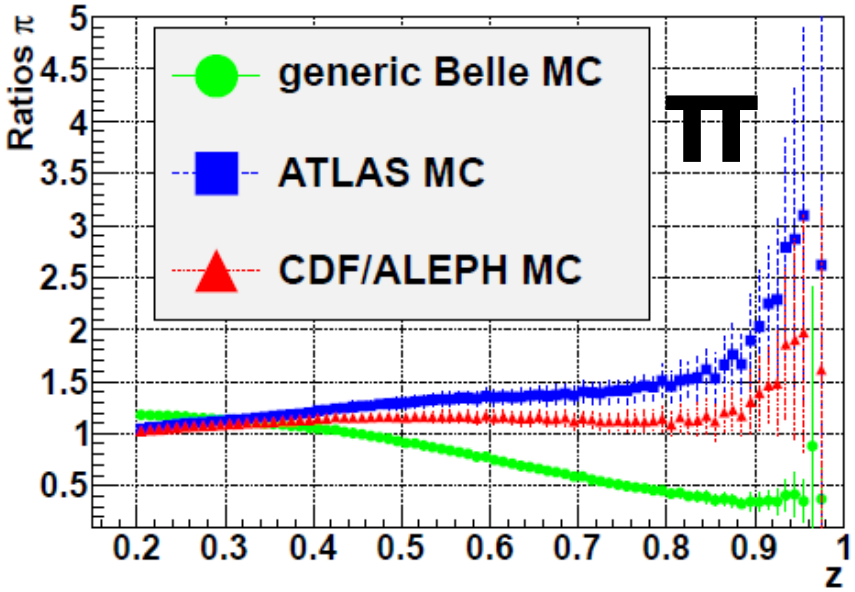
- Preliminary results include hadrons from all strong and weak decays (e.g. recovered decay-in-flight)
- Additional Belle MC-based relative fractions of strong vs. weak decay hadrons quoted.

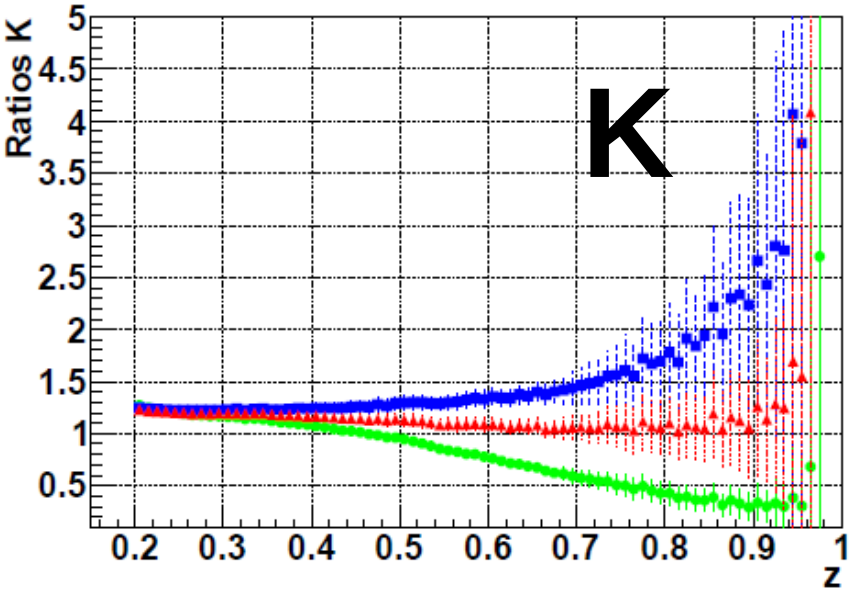




5. Results and Discussion
Comparison to Monte Carlo

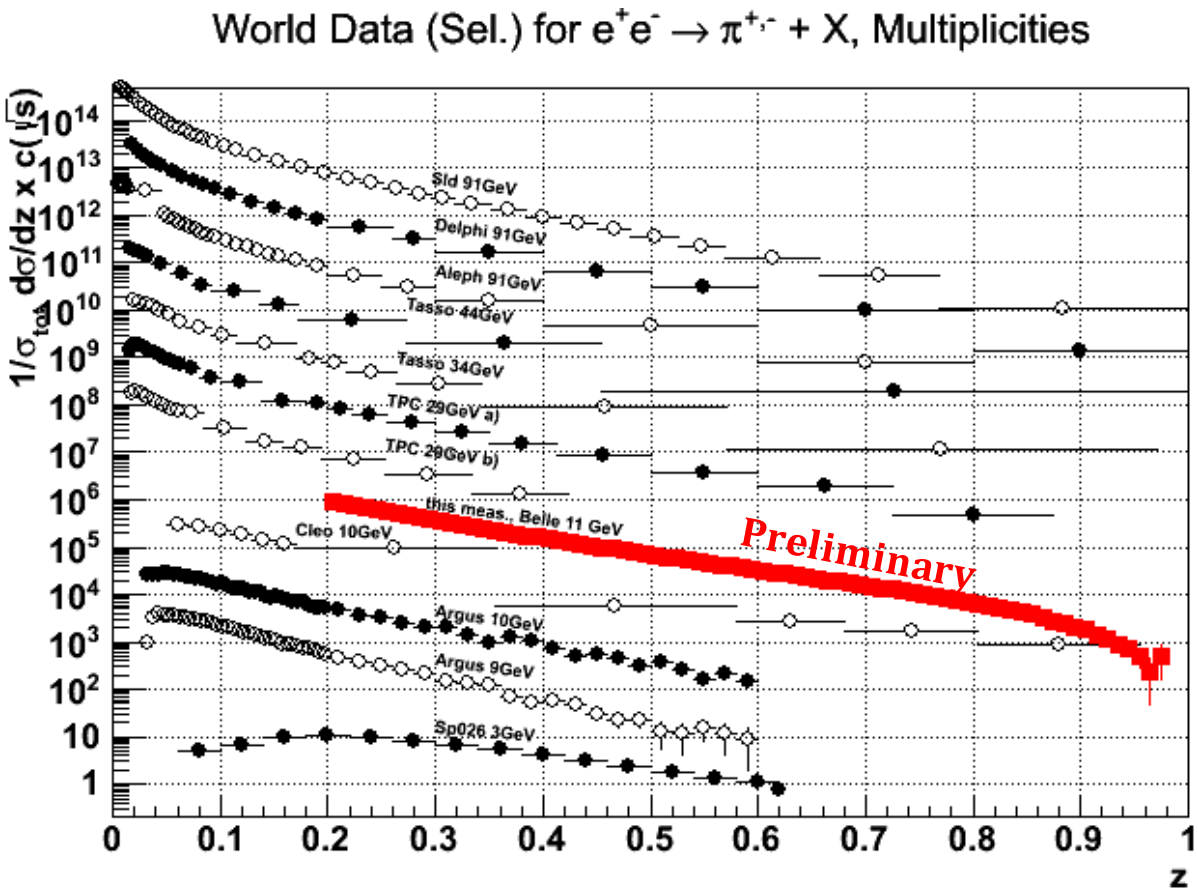
- Ratio of samples from generic Belle MC and self-generated CDF, Atlas, Aleph, general LHC tunes over experimental data, all for events e^+e^- -> qqbar (u,d,s,c) at sqrt(s) = 10.6 GeV.
- Pions and kaons:
  - Belle tune undershoots experimental multiplicities by ~1/3 at high z.
  - general LHC and Aleph tune overshoot experimental data by factor 5 at max z.
  - Atlas and CDF tunes stay within 20% of experimental data for all z.
- Spread of ratios above and below experimental data suggests consistency of measurement.

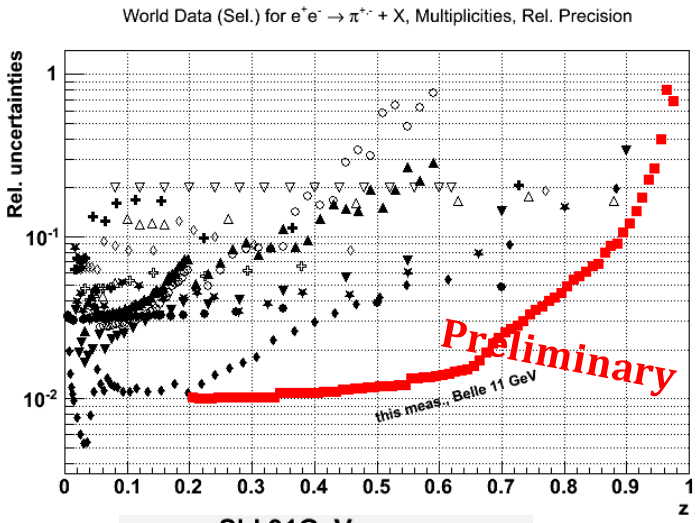




5. Results and Discussion
Comparison to World Data

- With BaBar provides first measurements of z dependence of hadron multiplicities at high z > 0.7.
- comparable or better precision than all compared datasets despite high z resolution, especially for measurements at low energy scales.





- Sld 91GeV
- Delphi 91GeV
- Aleph 91GeV
- Tasso 44GeV
- Tasso 34GeV
- TPC 29GeV a)
- TPC 29GeV b)
- this meas., Belle 11 GeV
- Cleo 10GeV
- Argus 10GeV
- Argus 9GeV
- Sp026 3GeV

5. Results and Discussion
Comparison to World Data

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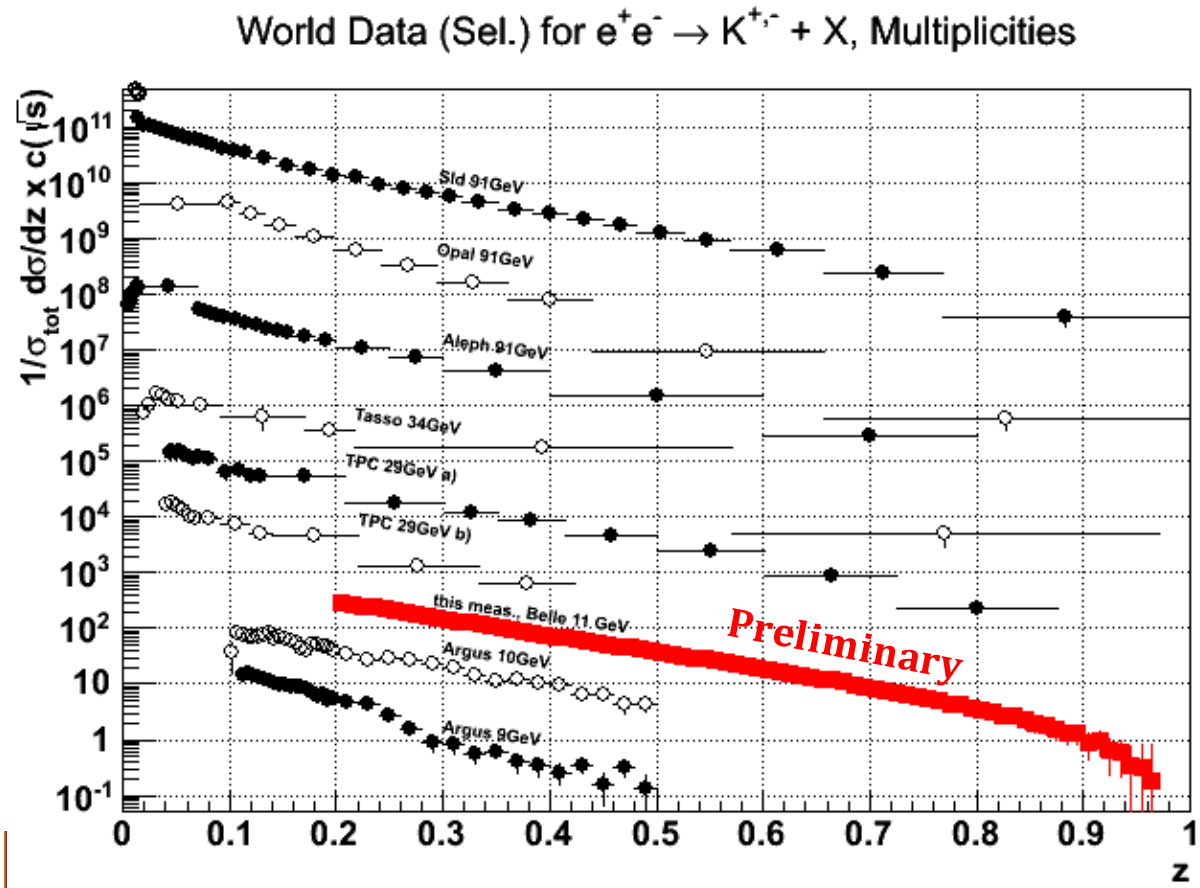
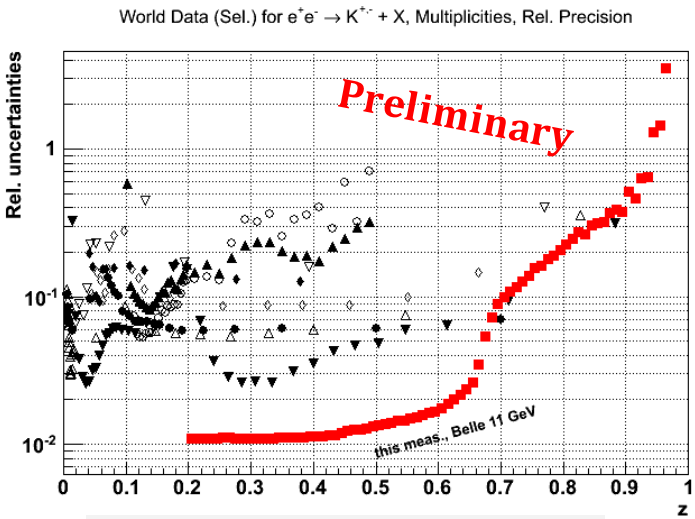




- Sld 91GeV
- Opal 91GeV
- Aleph 91GeV
- Tasso 34GeV
- TPC 29GeV a)
- TPC 29GeV b)
- this meas., Belle 11 GeV
- Argus 10GeV
- Argus 9GeV

6. Unpolarized FFs at Belle- Summary & Outlook

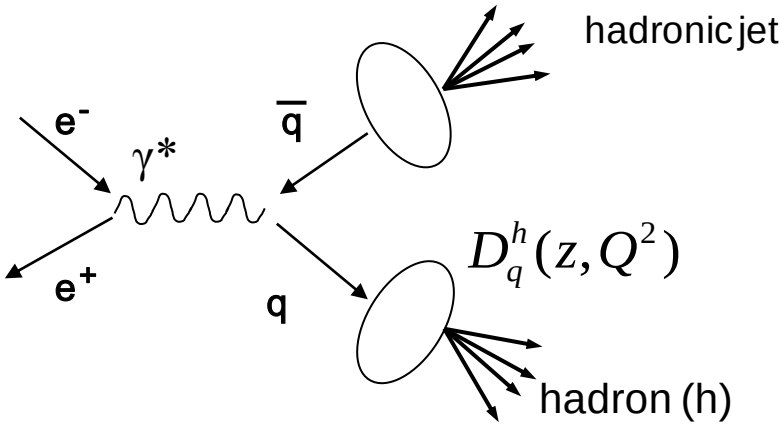
- Completed precision measurement of charged pion and kaon multiplicities at low sqrt(s) up to high z at Belle.
- Improvements of extractions of unpolarized FFs expected.
- Systematic studies finished, now working on publication.
- Preliminary results with uncertainties available for interested groups.

Backup

1. Introduction to Fragmentation Functions (FF)

- Fragmentation: creating color-charge neutral objects from parton in final state of hard scattering reactions.

- e^+e^- annihilation into quark-antiquark pair:



$$z \equiv \frac{E_h^{cms}}{\sqrt{Q^2}/2}$$

$$Q^2 = q^2 = (p_{e^-} + p_{e^+})^2 = s$$

- In LO QCD: FF D_i^h describes probability for a parton i to fragment into a hadron h

$$\frac{d\sigma(e^+e^- \to hX)}{dz} \overset{\text{LO QCD}}{\propto} \sum_{i=q,\bar{q}} e_i^{\,2} \cdot \boxed{D_i^h(z,Q^2)}$$

- Sum rules:

$$\sum_h \int_0^1 z \cdot D_q^h(z,Q^2) = 1$$

$$\sum_q \int_{z_{\min}}^1 \left[D_q^h(z,Q^2) + D_{\bar{q}}^h(z,Q^2) \right] dz = n_h$$

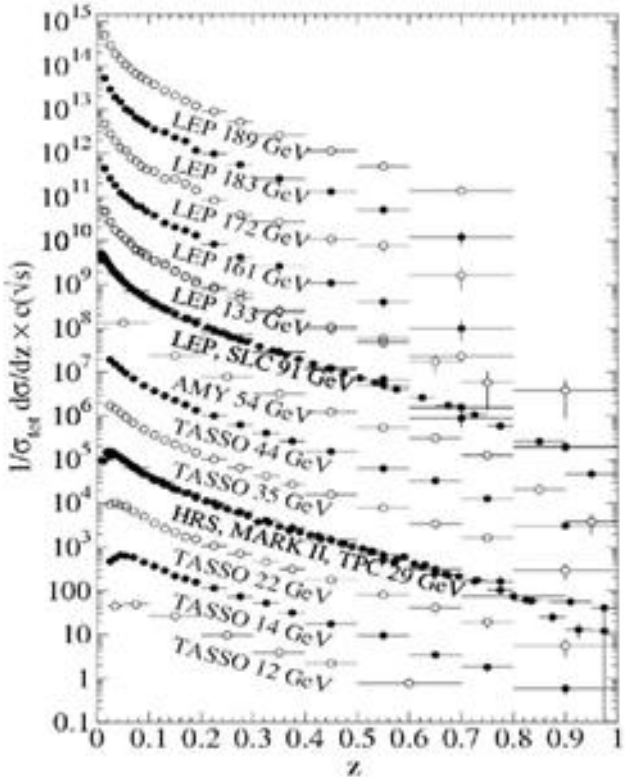
1. Introduction to Fragmentation Functions- Evolution

- Fragmentation Funtions from different energy scales can be related by evolution equations (DGLAP) from first principles QCD:

$$\frac{d}{d\ln Q^2} D_\Sigma^h(z,Q^2) \propto P_{qq} \otimes D_\Sigma^h + P_{gq} \otimes D_g^h, \qquad D_\Sigma^h \equiv \sum_q (D_q^h + D_{\bar{q}}^h)$$

- Evolution enables to relate fragmentation functions/ cross sections from different energy scales/ experiments:

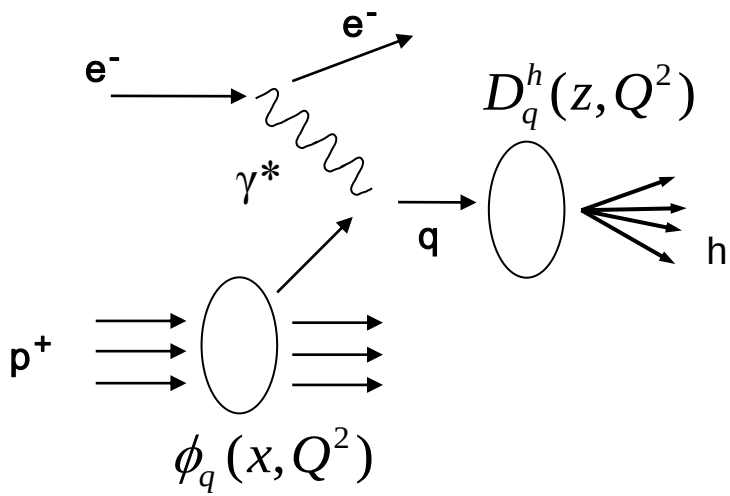
$$\frac{d\sigma(e^+e^- \to hX)}{dz} \overset{\text{LO QCD}}{\propto} \sum_{i=q,\bar{q}} e_i^2 \cdot D_i^h(z,Q^2)$$



World e^+e^- -> h+- X data

1. Introduction to Fragmentation Functions- Universality

- Fragmentation Funtions also found in deep inelastic scattering (DIS) processes:



$$x \equiv \frac{p_q}{p_{p^+}}$$

$$Q^2 = -q^2,\ q^2 = (p_{e^-,in} - p_{e^-,\mathrm{fin}})^2$$

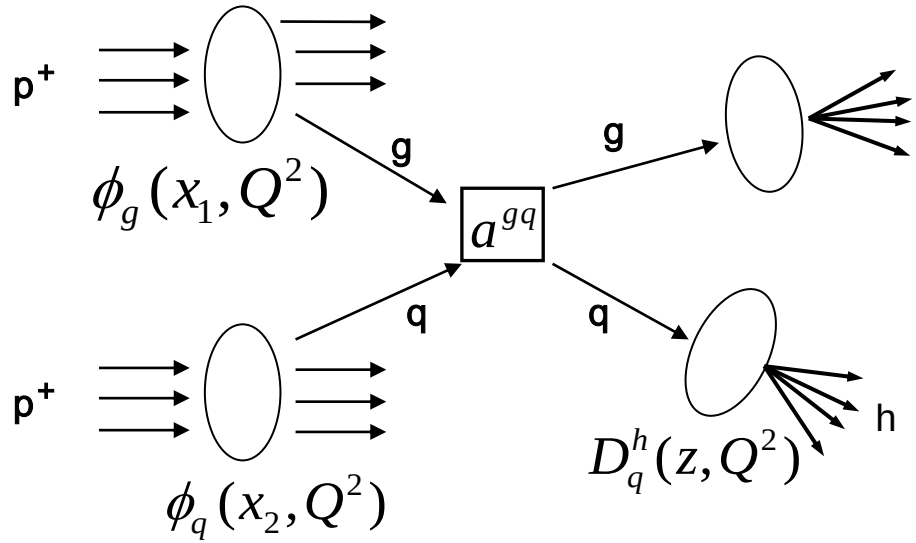
- Parton distribution functions (PDF) Φi gives probability to find parton in proton with momentum fraction x.

$$\frac{d^2\sigma(e^- p^+ \to hX)}{dxdz} \overset{\text{LO QCD}}{\propto} \sum_{i=q,\bar{q}} \phi_i(x,Q^2) \cdot e_i^2 \cdot \boxed{D_i^h(z,Q^2)}$$

- FFs same in e+e- and SIDIS (universality) -> determination of nucelon structure/PDFs via semi-inclusive DIS (SIDIS, e.g. HERMES experiment) will benefit from precise knowledge of fragmentation functions.

1. Introduction to Fragmentation Functions- Universality

- Fragmentation Funtions also show up in proton proton collisions:



$$\frac{d^3\sigma(p^+p^+\to hX)}{dx_1dx_2dz} \propto \phi_g(x_1,Q^2)\cdot\phi_q(x_2,Q^2)\cdot\frac{d^2\hat{\sigma}(gq\to gq)}{dx_1dx_2}\boxed{D_q^h(z,Q^2)}+\ldots$$

- Also present for spin dependent case:

$$\frac{d^3\Delta\sigma(p^{+\uparrow}p^{+\downarrow}\to hX)}{dx_1dx_2dz} \propto \Delta\phi_g(x_1,Q^2)\cdot\Delta\phi_q(x_2,Q^2)\cdot\frac{d^2\hat{\sigma}(gq\to gq)}{dx_1dx_2}\boxed{D_q^h(z,Q^2)}+\ldots$$

1. Introduction to Fragmentation Functions- Summary

- Fragmentation Funtions describe formation of hadrons from partons
- Need good knowledge of fragmentation functions to enable best extractions of quark and gluon distribution functions from experimental data:

$\sigma_{\text{SIDIS, pp}}$ → QCD analysis → $\phi_q(x,Q^2)$, $\Delta G(x,Q^2)$, $\Delta q(x,Q^2)$

measured

$D_q^h(z,Q^2)$

- Extraction of fragmentation functions: use relations with measured (normalized) cross sections from e^+e^-, SIDIS, pp

$\sigma_{e^+e^-}$, σ_{SIDIS}, σ_{pp} → QCD analysis → $D_q^h(z,Q^2)$

measured

2. Extraction of FF from experimental data- History

- Fits of **e^+e^-** annihilation data:

  - Kretzer, Phys. Rev. D **62**, 054001 (2000)
  - Kniehl, Kramer, Poetter, Nucl. Phys. **B582**, 514 (2000)
  - Albino, Kniehl, Kramer, Nucl. Phys. **B725**, 181 (2005)
    Nucl. Phys. **B734**, 50 (2006)

- Fits of **e^+e^-** annihilation data fully propagating experimental **uncertainties**:

  - Hirai, Kumano, Nagai, Sudoh (**HKNS**), KEK, Phys. Rev. D **75**, 094009 (2007)

- Fits of **e^+e^-** annihilation, **SIDIS** and **pp** data with uncertainties:

  - de Florian, Sassot, Stratmann (**DSS**), BNL, Phys. Rev. D **75**,114010 (2007)
    Phys. Rev. D **76**, 074033 (2007)

2. Extraction of FF from experimental data- Extraction Scheme

- **Extract FF** via NLO global **pQCD** analysis. e.g. e^+e^- annihilation:

$$N^h(z,Q^2) = \frac{1}{\sigma_{\text{tot had}}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

measured: hadron multiplicity

$$\overset{\text{LO QCD}}{=} \sum_{i=q,\bar{q}} e_i^{\,2} \cdot D_i^h(z,Q^2) \Big/ \sum_{i=q,\bar{q}} e_i^{\,2}$$

$$\overset{\text{NLO QCD}}{=} \sum_{i=q,\bar{q},g} C_i^{\,NLO}(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

pQCD fit

extracted: Parton FFs

- parameterize of D_i^h at initial scale $Q_0^{\,2}$

HKNS $$D_i^h(z,Q_0^{\,2}) = M_{HKNS,i}^{\,h} z^{\alpha_i^\eta} (1-z)^{\beta_i^\eta}$$

DSS $$D_i^h(z,Q_0^{\,2}) = M_{DSS,i}^{\,h} z^{\alpha_i^\eta} (1-z)^{\beta_i^\eta} \left[1+\gamma_i^h (1-z)^{\delta_i^h}\right]$$

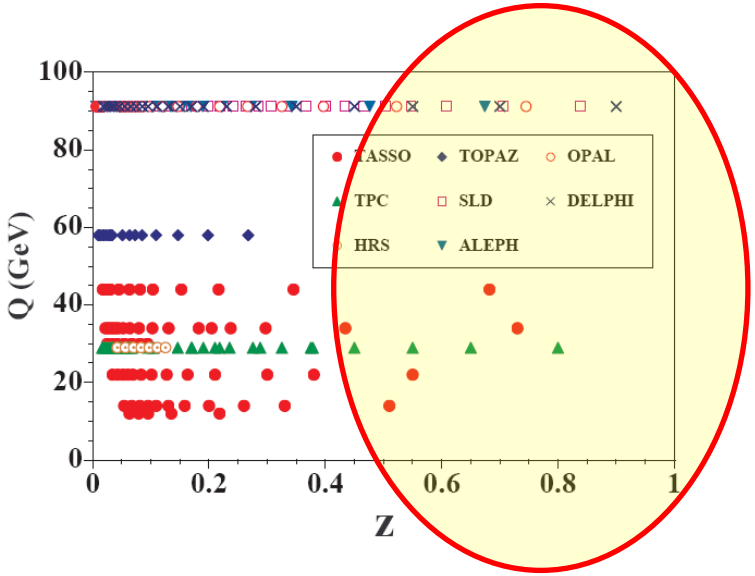
- Evolve expressions for normalized cross sections to Q^2 of fitted datasets via DGLAP equations and simultaneously fit all datasets.

2. Extraction of FF from experimental data- Limitations I

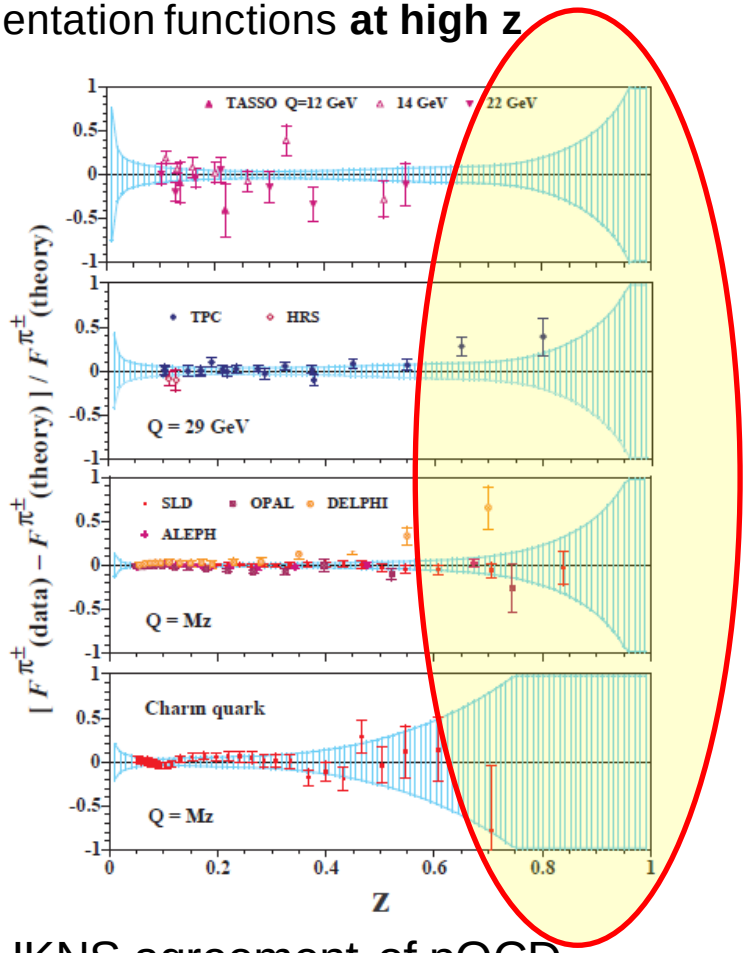
- **Lack of experimental data** and high uncertainties on existing data **at high z**

Effect: high z behaviour of hadron multiplicities weakly constrained -

-> **large uncertainties** on extracted fragmentation functions **at high z**



HKNS experimental data



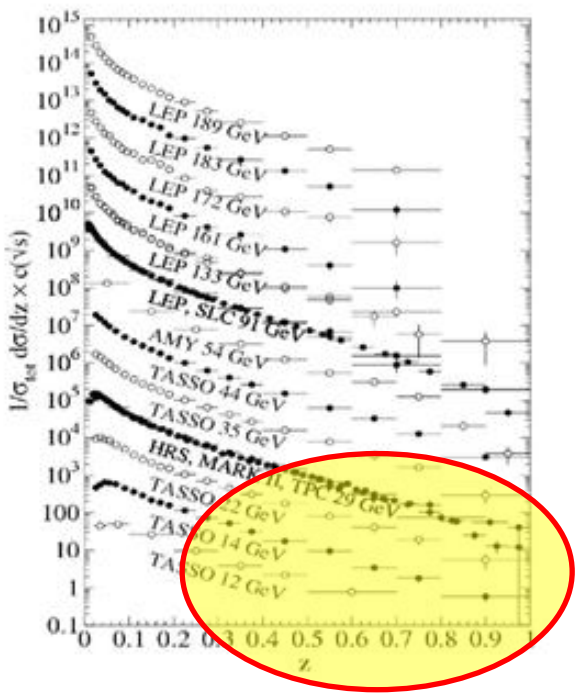
HKNS agreement of pQCD fits with data

2. Extraction of FF from experimental data- Limitations II a)

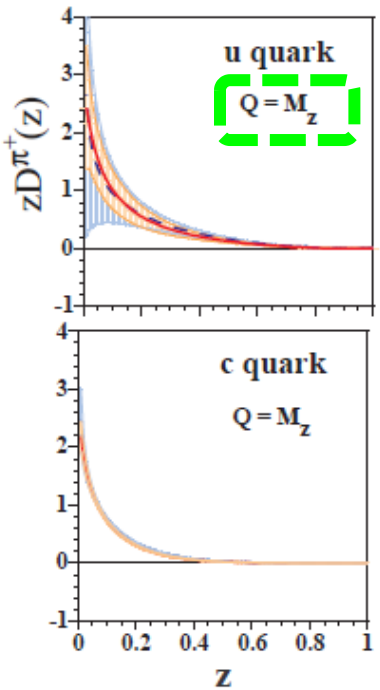
- **<u>Lack of precise data at low energy scales</u>** (far from LEP)

Effects:

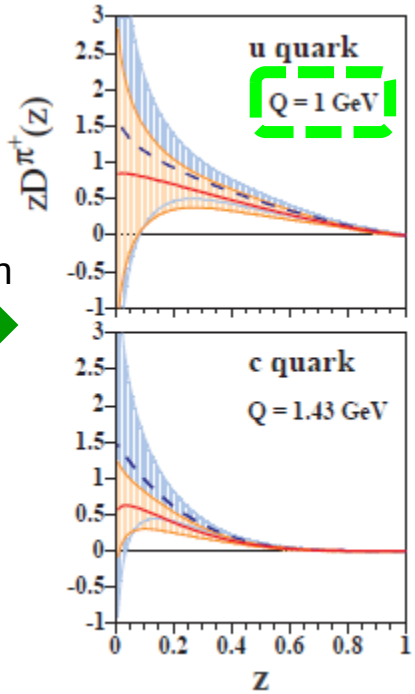
a) **Large uncertainties after evolution to small energy scales** (fragmentation functions needed as input in SIDIS and pp measurements)



World e+e- -> h+- X data (PDG)



evolution



HKNS extracted fragmentation functions

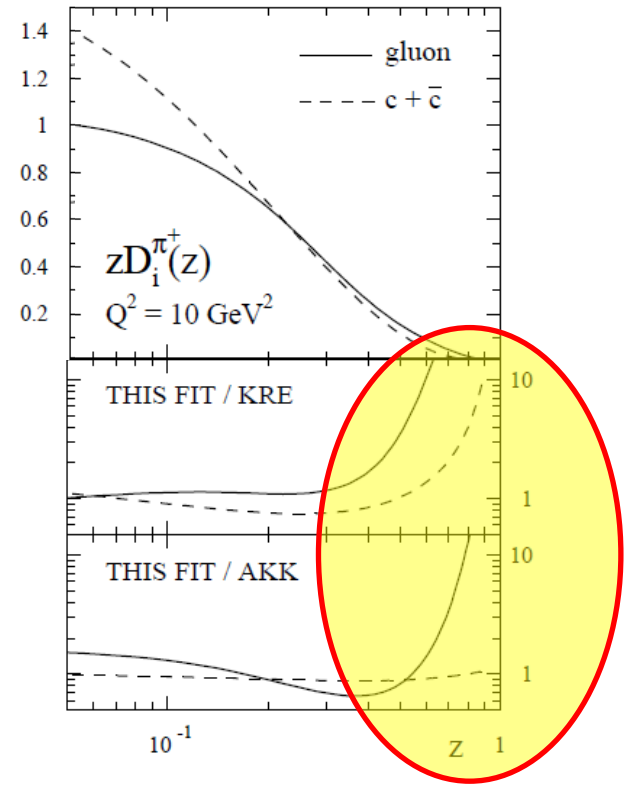
2. Extraction of FF from experimental data- Limitations II b)

- **<u>Lack of precise data at low energy scales</u>** (far from LEP)

Effects:

b) **gluon fragmentation function:** quark-gluon mixing in evolution weak & only enters cross section weekly at NLO -> **only loose constraints**

$$\frac{d}{d\ln Q^2} D_\Sigma^h(z,Q^2) \propto P_{qq} \otimes D_\Sigma^h + P_{qq} \otimes D_g^h$$



DSS extracted fragmentation functions and agreement with previous extractions

2. Extraction of FF from experimental data- Summary

- Most recent extractions of fragmentation functions suffer from :

  - Lack of **precise data at low energy scales** (far from LEP)

  - Lack **of precise data at high z**

- **Motivation of present analysis:** improve knowledge of fragmentation functions by

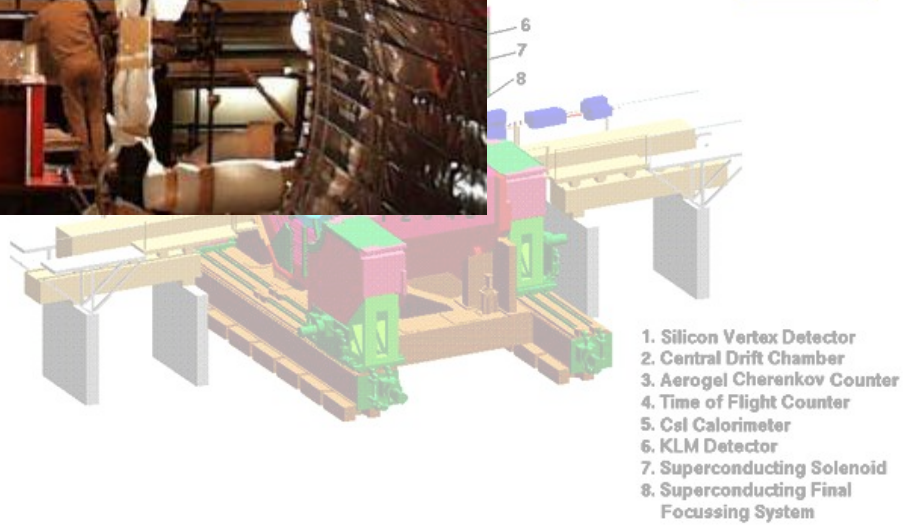
high precision
multiplicity measurement
at low Q^2

→ BELLE

3. The Belle Experiment

- KEK-B: asymmetric e+ (3.5 GeV) e- (8 GeV) collider:
  - **$\sqrt{s}$ = 10.58 GeV,** $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$
  - $\sqrt{s}$ = 10.52 GeV, e+e- -> qqbar (u,d,s,c) 'continuum production'

- ideal detector f
  - track Drift
  - partic kov, ToF,
  - availa

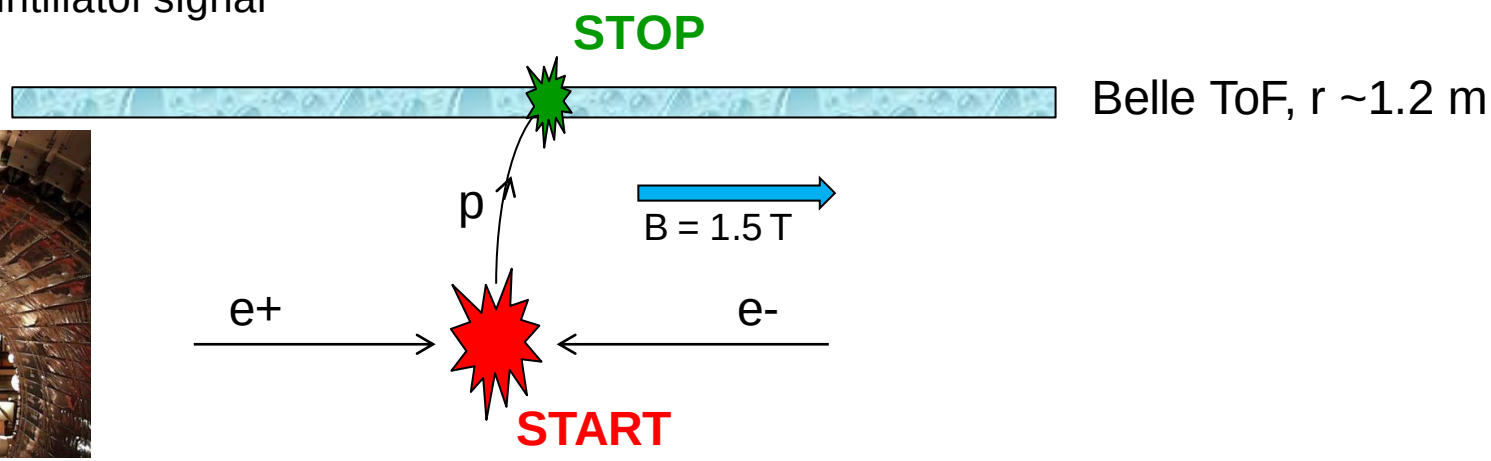


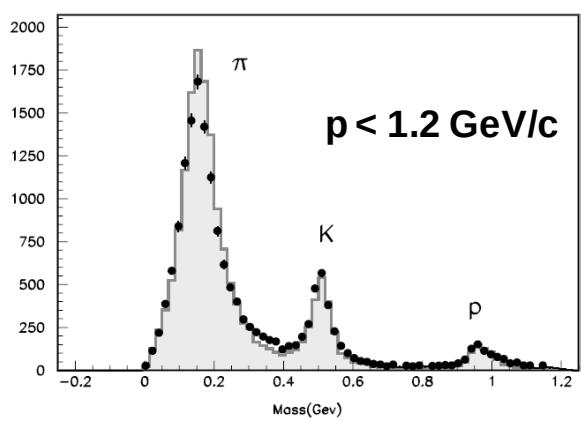
4. Particle Identification: Time-of-Flight Detector

• concept: measure time-of-flight of particle and combine with measured momentum from track curvature in magnetic field

$$v = \frac{\Delta l}{\Delta t_{ToF}}, \quad p = m\gamma\, \mathrm{v}$$

• at Belle: start signal for timing measurement given by collider RF clock, stop by plastic scintillator signal





Extract **PID likelihood information** for each track

4. Particle Identification: PID Probabilities

- not possible to unambiguously assign species to measured particle because of likelihood character of PID

Physical particle

π → Belle PID likelihood information from: Drift Chamber (dE/dx), Cherenkov, ToF, Calorimeter, Muon Detector →

Reconstructed particle

$p(\pi \to \tilde{e}) \to e$

$p(\pi \to \tilde{\mu}) \to \mu$

$p(\pi \to \tilde{\pi}) \to \pi$

$p(\pi \to \tilde{K}) \to K$

$p(\pi \to \tilde{p}) \to p$

- particle identification probabilities $p(i \to \tilde{j})$:
  probability that particle of species i is reconstructed as particle of species $\tilde{j}$.

$$[P]_{ij} = \begin{pmatrix} p(e\to\tilde{e}) & p(\mu\to\tilde{e}) & p(\pi\to\tilde{e}) & p(K\to\tilde{e}) & p(p\to\tilde{e}) \\ p(e\to\tilde{\mu}) & p(\mu\to\tilde{\mu}) & p(\pi\to\tilde{\mu}) & p(K\to\tilde{\mu}) & p(p\to\tilde{\mu}) \\ p(e\to\tilde{\pi}) & p(\mu\to\tilde{\pi}) & p(\pi\to\tilde{\pi}) & p(K\to\tilde{\pi}) & p(p\to\tilde{\pi}) \\ p(e\to\tilde{K}) & p(\mu\to\tilde{K}) & p(\pi\to\tilde{K}) & p(K\to\tilde{K}) & p(p\to\tilde{K}) \\ p(e\to\tilde{p}) & p(\mu\to\tilde{p}) & p(\pi\to\tilde{p}) & p(K\to\tilde{p}) & p(p\to\tilde{p}) \end{pmatrix}$$

- off-diagonal elements ~ O(10%) -> correct for particle mis-identification

5. Particle Identification Correction

- correction by inversion of PID probability matrix:

$$\vec{N} = \begin{pmatrix} N_e \\ N_\mu \\ N_\pi \\ N_K \\ N_p \end{pmatrix}$$

$$\vec{\tilde{N}}_j = \hat{P}\,\vec{N}_i$$

$$\vec{N}_i = \hat{P}^{-1}\,\vec{\tilde{N}}_j$$

- for inversion have to determine all elements of PID probability matrix

  - ‘Monte Carlo Simulation’: detailed statistical description of detector, based on detector properties and material interaction physics

  - response of PID detectors only modeled to ~10% accuracy

→ Extract PID probabilities/ ‘calibrate’ Belle PID based on experimental data

5. Particle Identification Correction- PID Calibration from data

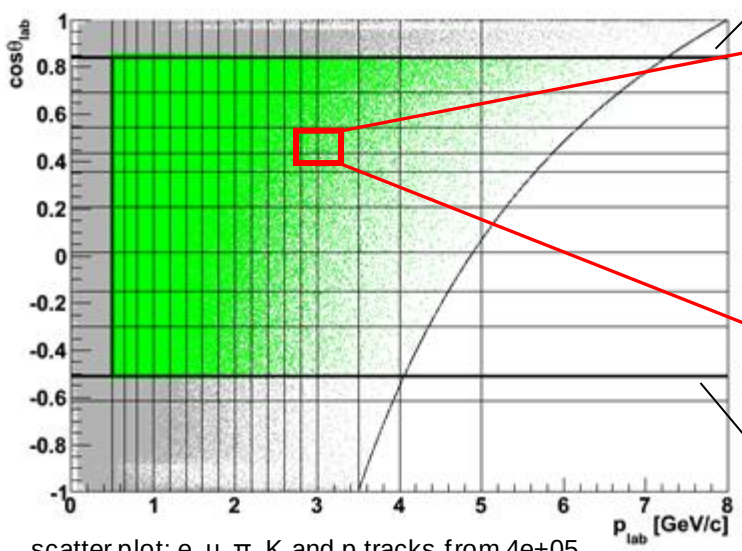
- Belle experimental data-based calibrations exist (to resolve B meson decay particles)

- limitations:
  - calibration available only for limited kinematics ($z \leq 0.5$)
  - only available for diagonal and largest off-diagonal elements

→ Intensive efforts needed to extend PID to full kinematic range

- Belle PID response depending on track kinematics – perform calibration on binning in laboratory frame momentum and scattering angle

ToF forward geometry acceptance limit



scatter plot: e, μ, π, K and p tracks from 4e+05 events

fill matrix of PID probabilities for each single bin from real data calibration- need large statistics

$$[P]_{ij\,(p_{lab},\ \cos\theta_{lab})} = \begin{pmatrix} p(e\to\tilde{e}) & p(\mu\to\tilde{e}) & p(\pi\to\tilde{e}) & p(K\to\tilde{e}) & p(p\to\tilde{e}) \\ p(e\to\tilde{\mu}) & p(\mu\to\tilde{\mu}) & p(\pi\to\tilde{\mu}) & p(K\to\tilde{\mu}) & p(p\to\tilde{\mu}) \\ p(e\to\tilde{\pi}) & p(\mu\to\tilde{\pi}) & p(\pi\to\tilde{\pi}) & p(K\to\tilde{\pi}) & p(p\to\tilde{\pi}) \\ p(e\to\tilde{K}) & p(\mu\to\tilde{K}) & p(\pi\to\tilde{K}) & p(K\to\tilde{K}) & p(p\to\tilde{K}) \\ p(e\to\tilde{p}) & p(\mu\to\tilde{p}) & p(\pi\to\tilde{p}) & p(K\to\tilde{p}) & p(p\to\tilde{p}) \end{pmatrix}$$

ToF backward geometry acceptance limit

Backup: 2. Extraction of Fragmentation Functions

$$N^h(z,Q^2) = \frac{1}{\sigma_{\text{tot had}}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

$$\overset{\text{LO QCD}}{=} \sum_{i=q,\bar{q}} e_i^{\,2} \cdot D_i^h(z,Q^2) \Big/ \sum_{i=q,\bar{q}} e_i^{\,2}$$

$$\overset{\text{NLO QCD}}{=} \sum_{i=q,\bar{q},g} C_i^{\,NLO}(z,\alpha_s) \otimes D_i^h(z,Q^2)$$

$$= \sum_{\mathrm{i}=q,\bar{q},g} \int_z^1 \frac{dz'}{z'} C_{\mathrm{i}}^{NLO}(z',\alpha_{\mathrm{s}}(\mu),\frac{s}{\mu^2}) D_{\mathrm{i}}^h(\frac{z}{z'},\mu^2) + O(\frac{1}{\sqrt{s}})$$

2c. FF Extractions by HKNS & DSS

- **Method of extraction:**

  - parameterization of D_i^h at initial scale Q_0^2

HKNS $$D_i^h(z,Q_0^2) = \frac{1}{B[2+\alpha_i^h, \beta_i^h+1]} M_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}$$

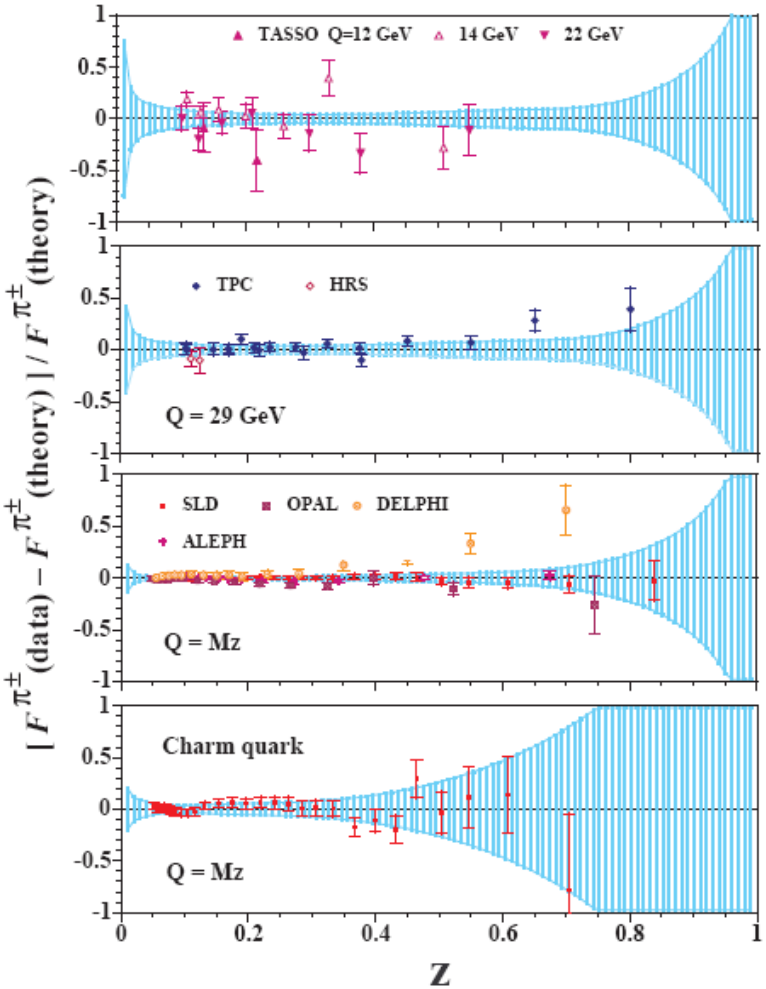
DSS $$D_i^h(z,Q_0^2) = \frac{1}{B[2+\alpha_i^h, \beta_i^h+1] + \gamma_i^h B[2+\alpha_i^h, \beta_i^h+\delta_i^h+1]} M_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h} [1+\gamma_i^h (1-z)^{\delta_i^h}]$$

- evolution of (scale-dependent) D_i^h to Q^2 of fitted datasets via pQCD DGLAP evoluation equations and simulataneous fit of all datasets.

$$\frac{d}{d\ln Q^2} \vec{D}^h(z,Q^2) = [\hat{P}^{(T)} \otimes \vec{D}^h](z,Q^2);$$

$$\vec{D}^h \equiv \begin{pmatrix} D_\Sigma^h \\ D_g^h \end{pmatrix}, \quad D_\Sigma^h \equiv \sum_q (D_q^h + D_{\bar{q}}^h), \quad \hat{P}^{(T)} \equiv \begin{pmatrix} P_{qq}^{(T)} & 2n_f P_{gq}^{(T)} \\ \frac{1}{2n_f} P_{qg}^{(T)} & P_{gg}^{(T)} \end{pmatrix}.$$

2c. Fit Results HKNS



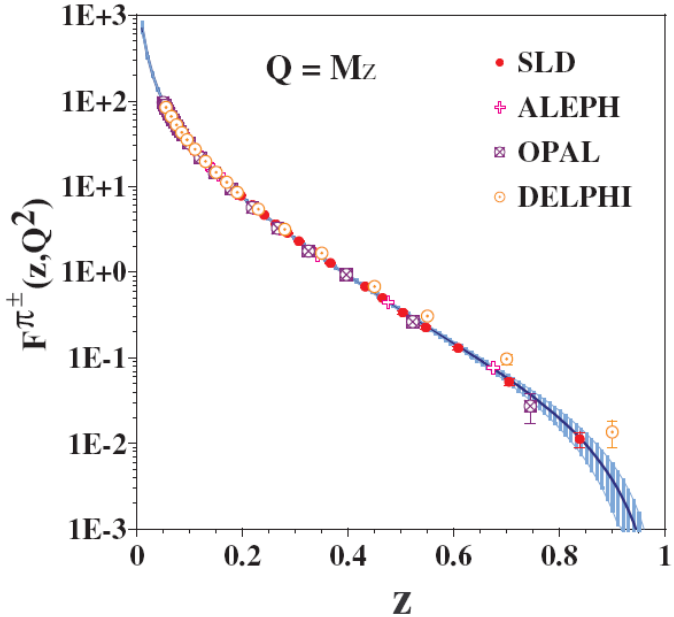
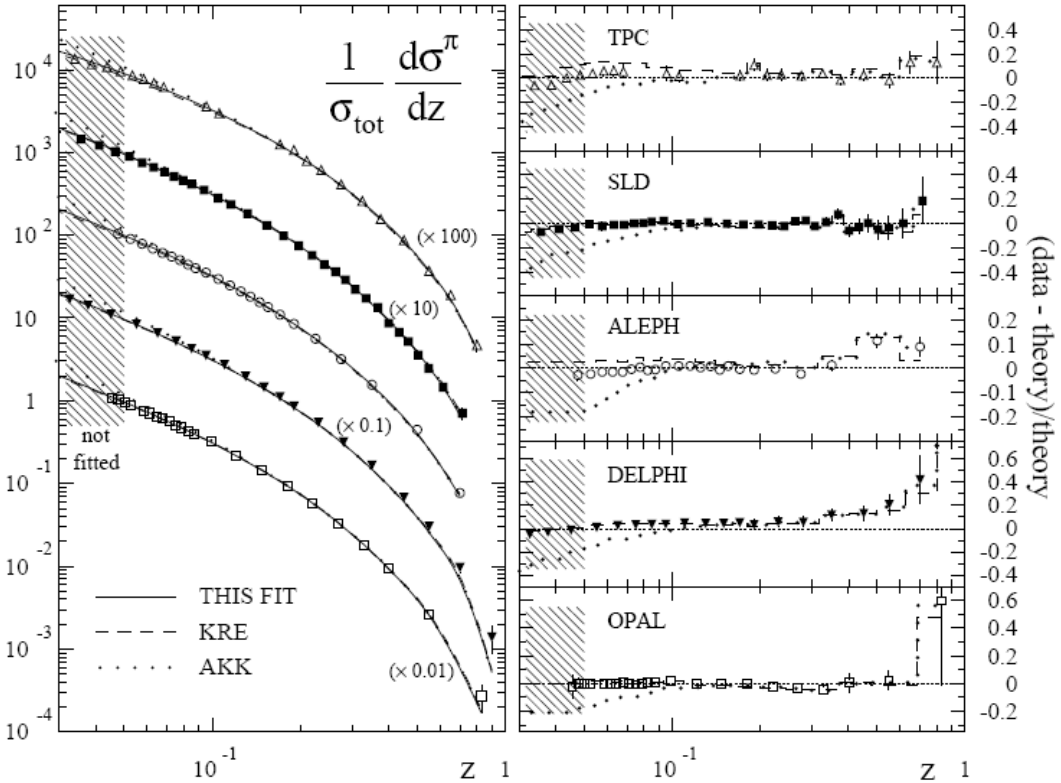


TABLE VII: Each χ^2 contribution in the pion analysis.

experiment	# of data	χ^2 (LO)	χ^2 (NLO)
TASSO	29	52.1	51.9
TPC	18	33.5	27.3
HRS	2	1.1	2.0
TOPAZ	4	2.6	2.6
SLD (all)	29	11.3	10.6
SLD (u,d,s)	29	46.0	36.4
SLD (c)	29	24.4	26.1
SLD (b)	29	71.2	66.4
ALEPH	22	22.8	24.0
OPAL	22	45.4	45.8
DELPHI (all)	17	48.3	48.6
DELPHI (u,d,s)	17	29.6	31.1
DELPHI (b)	17	64.9	60.8
total	264	453.2	433.5
(/d.o.f.)		(1.81)	(1.73)

BELLE

2c. Fit Results DSS

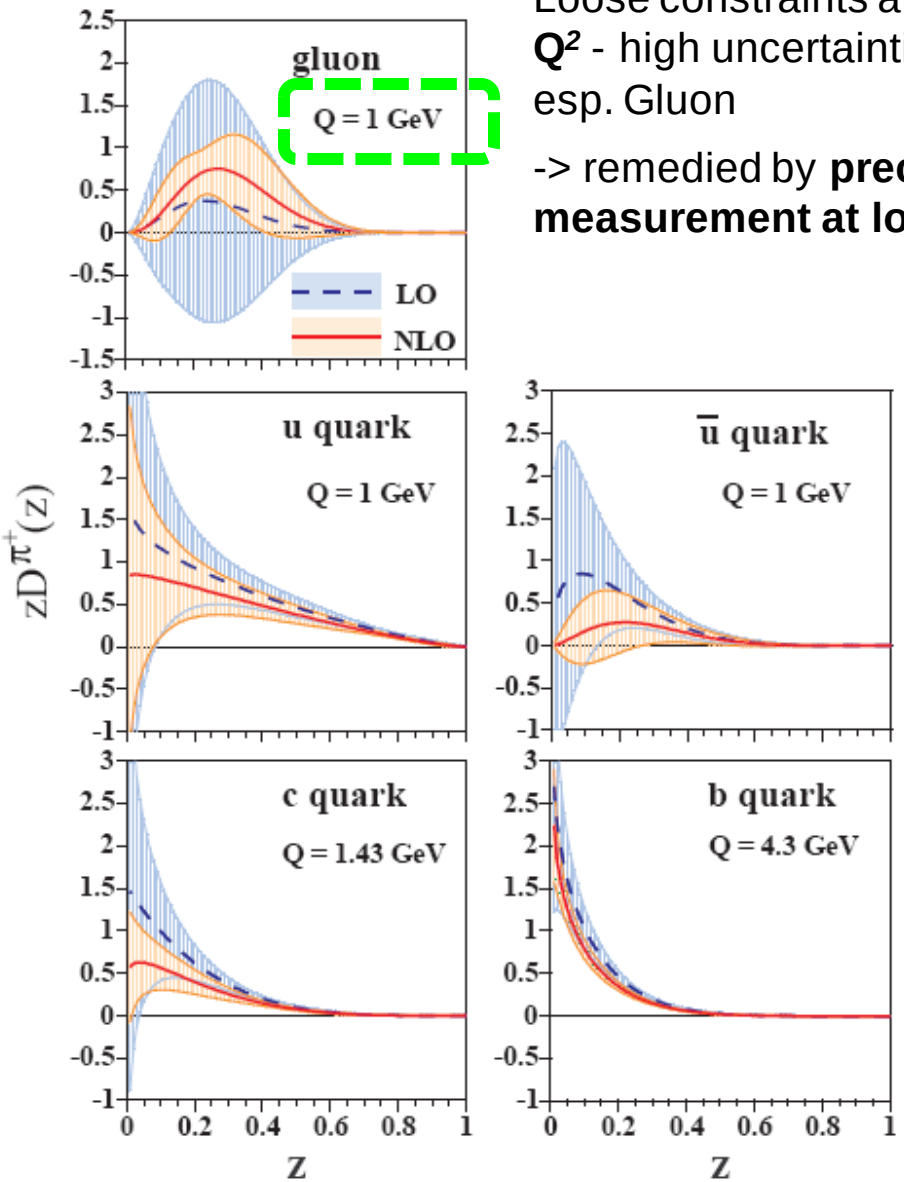


experiment	data type	rel. norm. in fit	data points fitted	χ^2
TPC [15]	incl.	0.94	17	18.5
	"*uds* tag"	0.94	9	1.9
	"*c* tag"	0.94	9	5.7
	"*b* tag"	0.94	9	7.4
TASSO [17]	incl. (34 GeV)	0.94	11	30.1
	incl. (44 GeV)	0.94	7	20.5
SLD [16]	incl.	1.008	28	14.0
	"*uds* tag"	1.008	17	11.6
	"*c* tag"	1.008	17	11.1
	"*b* tag"	1.008	17	33.2
ALEPH [11]	incl.	0.97	22	38.3
DELPHI [12]	incl.	1.0	17	42.3
	"*uds* tag"	1.0	17	26.4
	"*b* tag"	1.0	17	42.8
OPAL [13, 14]	incl.	1.0	21	9.2
	"*u* tag"	1.10	5	11.8
	"*d* tag"	1.10	5	9.0
	"*s* tag"	1.10	5	49.8
	"*c* tag"	1.10	5	38.3
	"*b* tag"	1.10	5	73.0
HERMES [18]	π^+	1.03	32	67.4
	π^-	1.03	32	120.8
PHENIX [19]	π^0	1.09	23	76.4
STAR [22]	π^0, $\langle\eta\rangle = 3.3$	1.05	4	3.4
	π^0, $\langle\eta\rangle = 3.7$	1.05	5	9.8
BRAHMS [21]	π^+, $\langle\eta\rangle = 2.95$	1.0	18	28.2
	π^-, $\langle\eta\rangle = 2.95$	1.0	18	43.0
TOTAL:			392	843.7

2c. Extracted FF HKNS

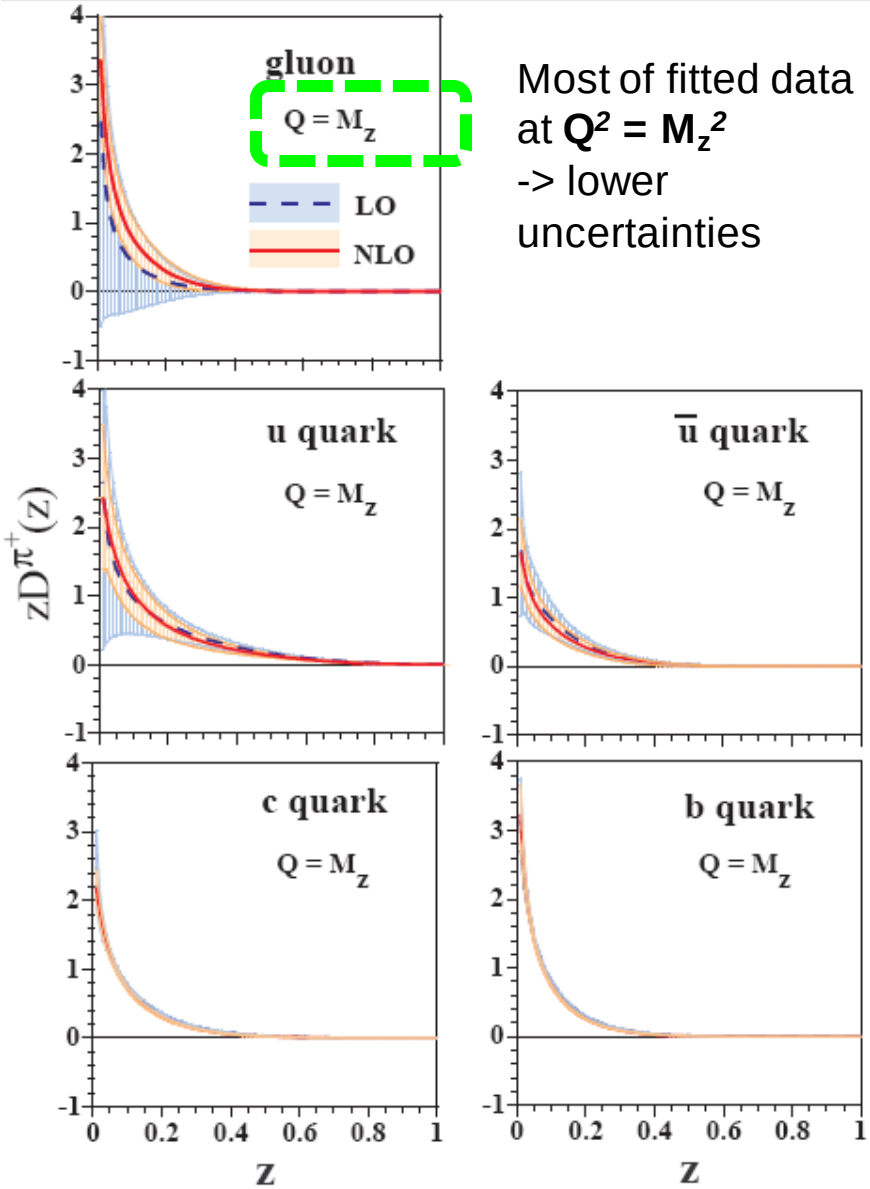
Loose constraints at low **Q^2** - high uncertainties, esp. Gluon

-> remedied by **precision measurement at low Q^2**



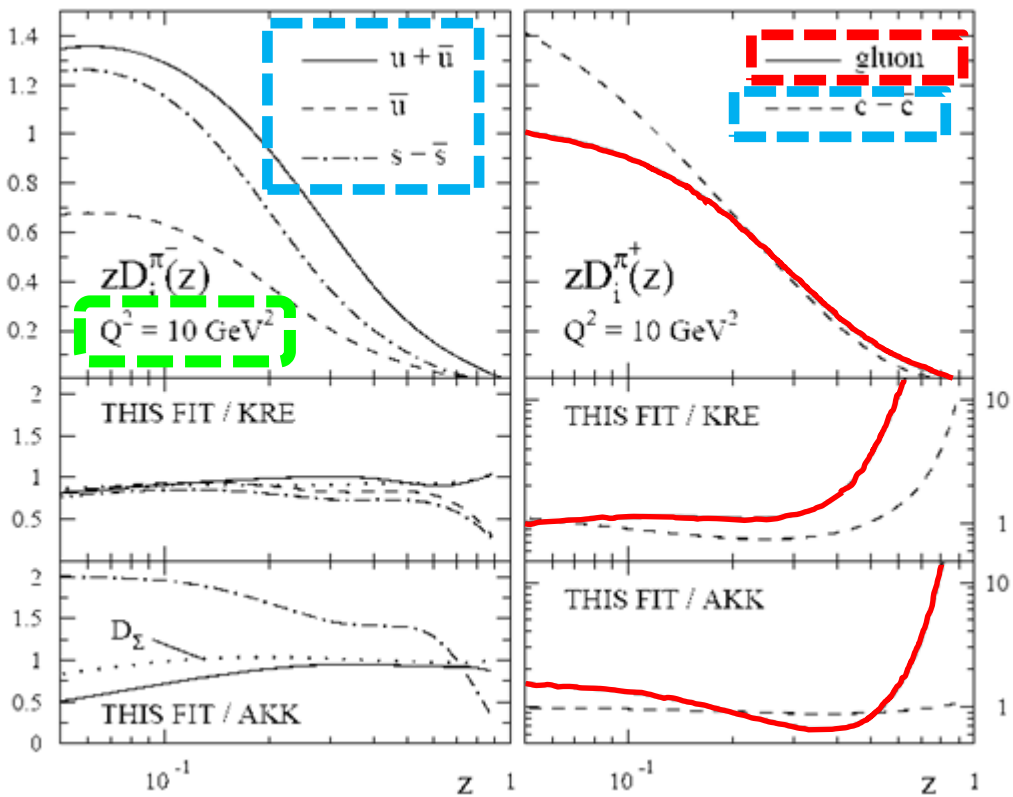
Most of fitted data at **$Q^2 = M_z^2$**

-> lower uncertainties

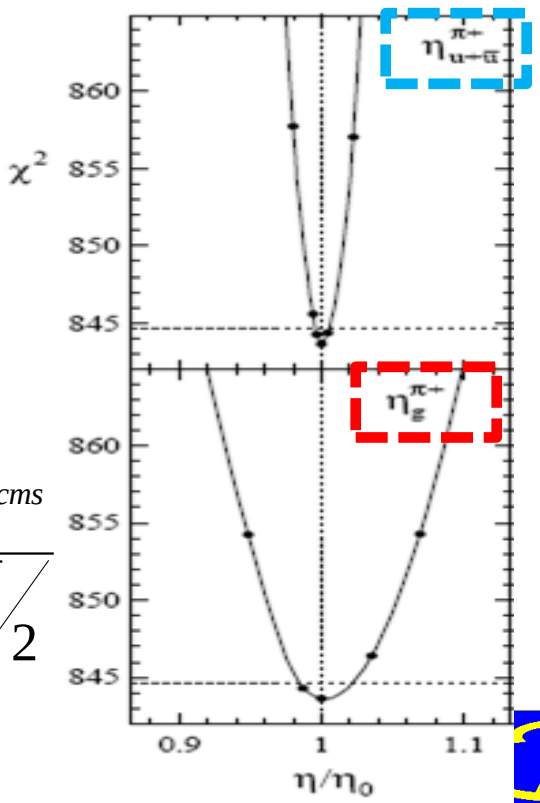


2c. Extracted FF DSS

- Inclusion of **pp** and **SIDIS** data provide better constraints at low **Q^2** & high **z**
- Gluon FF still loosely constrained because
  NLO corrections not large enough and
  quark-gluon mixing from scaling evolution requires precise data far enough from M_Z,
  large theoretical uncertainties in pp-
-remedied by **precision measurement at low Q^2**



$$z \equiv \frac{E_h^{cms}}{\sqrt{s}/2}$$



3. Particle Misidentification Correction– Lepton PID Probabilities from J/Ψ -> l^+l^-

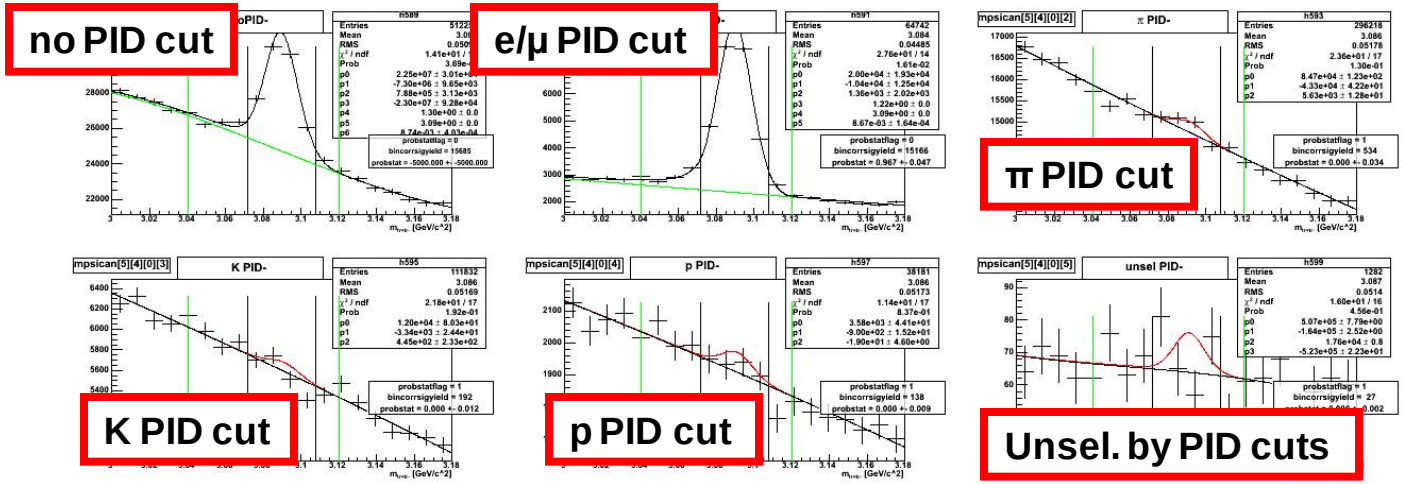
e-/ μ-

Real data exp[7;55], HadronBJ
onres ~590 fb^-1, cont ~70fb^-1

Example fits

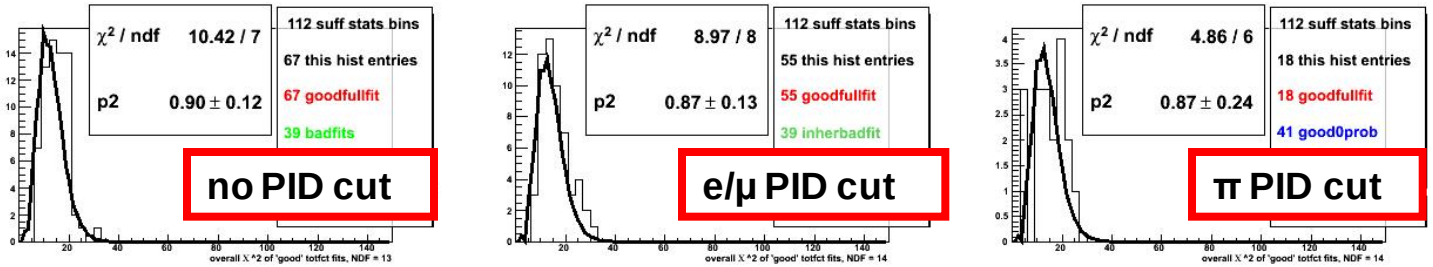
e-/ μ- with
p_{lab} in [1.4; 1.6] GeV/c,
$\cos\theta_{lab}$ in [0.21; 0.36]

e and μ components not resolvable with kinematics only.
S/N too low for KPID and pPID.



Cumulative Chisquare

distribution of all chisquares fitted with theoretical Chisquare distribution.

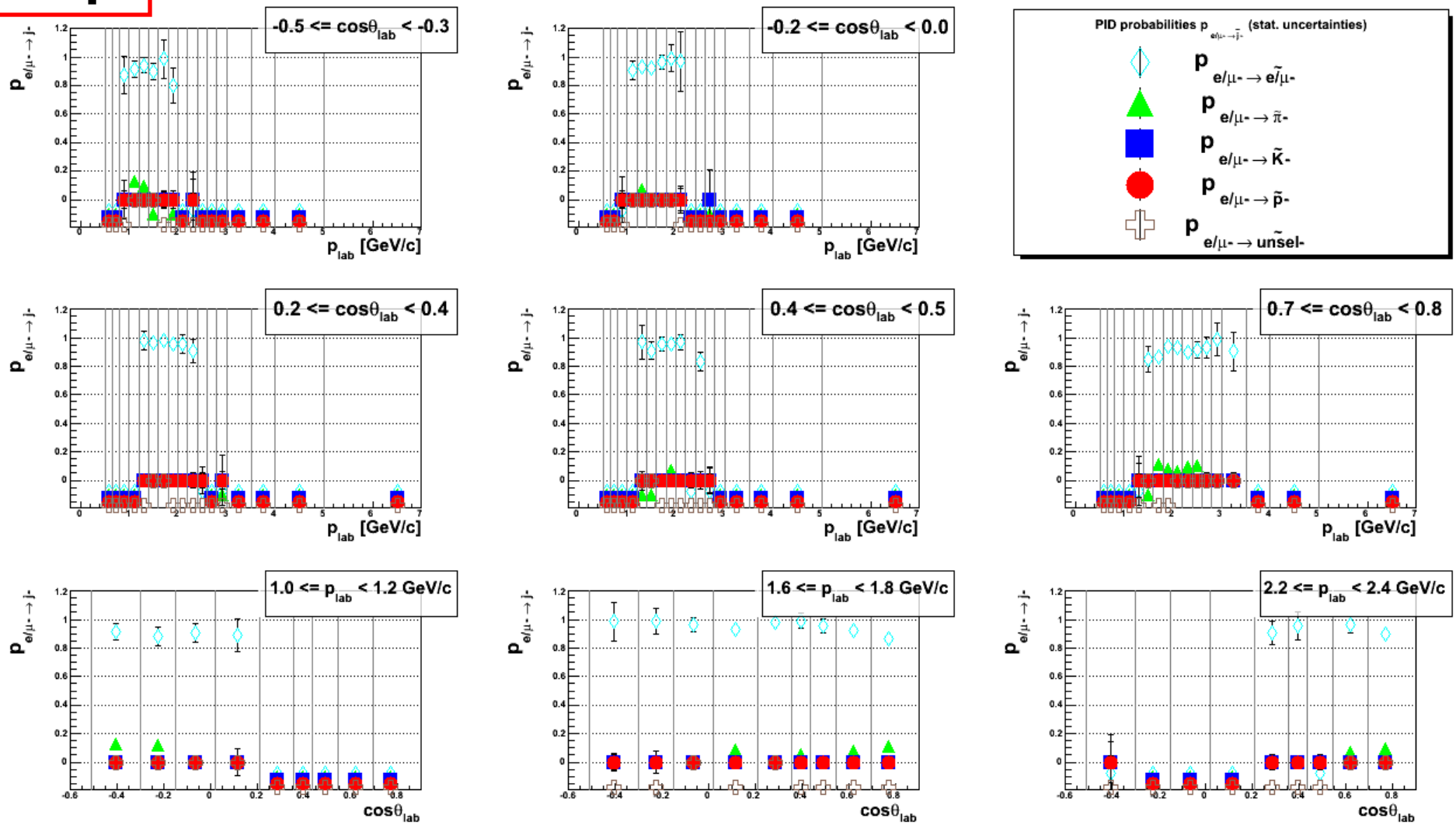


3. Particle Misidentification Correction– Lepton PID Probabilities from J/Ψ -> l^+l^-

- Lepton PID probability samples.
- vs p_{lab} and $\cos\theta_{lab}$.

Real data exp[7;55],
onres ~590 fb^-1, cont ~70fb^-1

e-/ μ-



3. Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

$D^* \rightarrow \pi_{slow} (D^\circ \rightarrow \pi_{fast} K)$

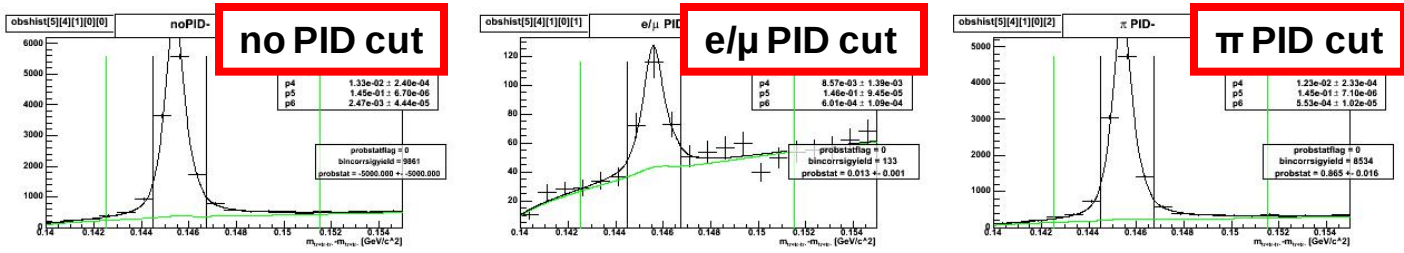
Real data exp[7;55], HadronBJ
onres ~590 fb^-1, cont ~70fb^-1

Example fits

π^-

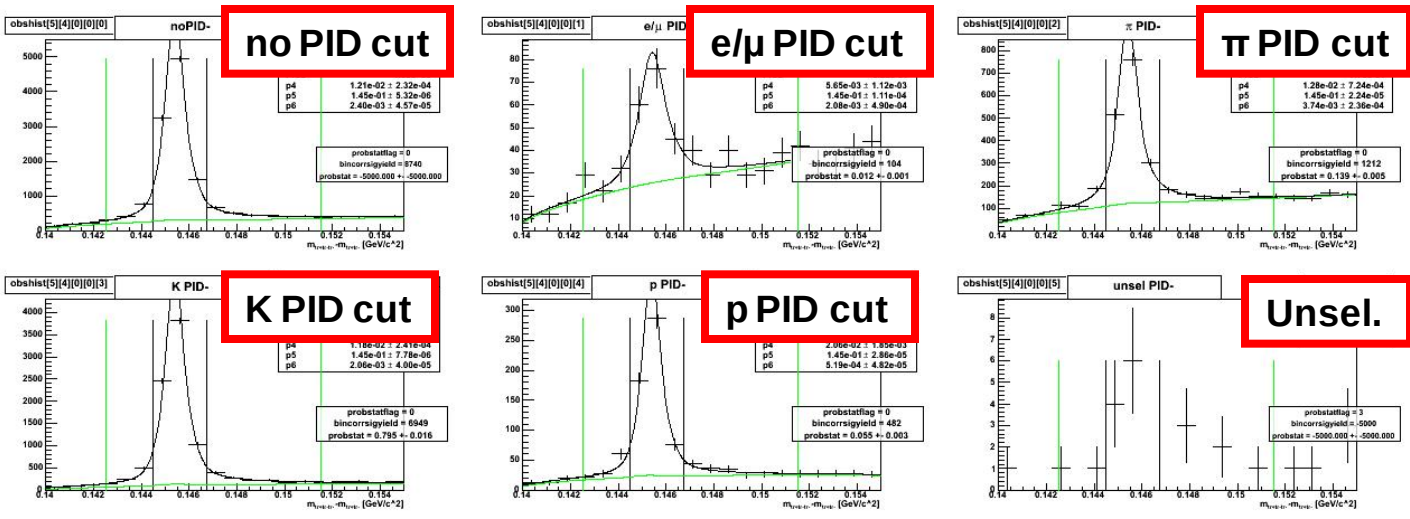
π^- with
p_{lab} in [1.4; 1.6] GeV/c,
$\cos\theta_{lab}$ in [0.21; 0.36]

candidate selection along BN 779, Nishida-san .
Good S/N.



K^-

K^- with
p_{lab} in [1.4; 1.6] GeV/c,
$\cos\theta_{lab}$ in [0.21; 0.36]



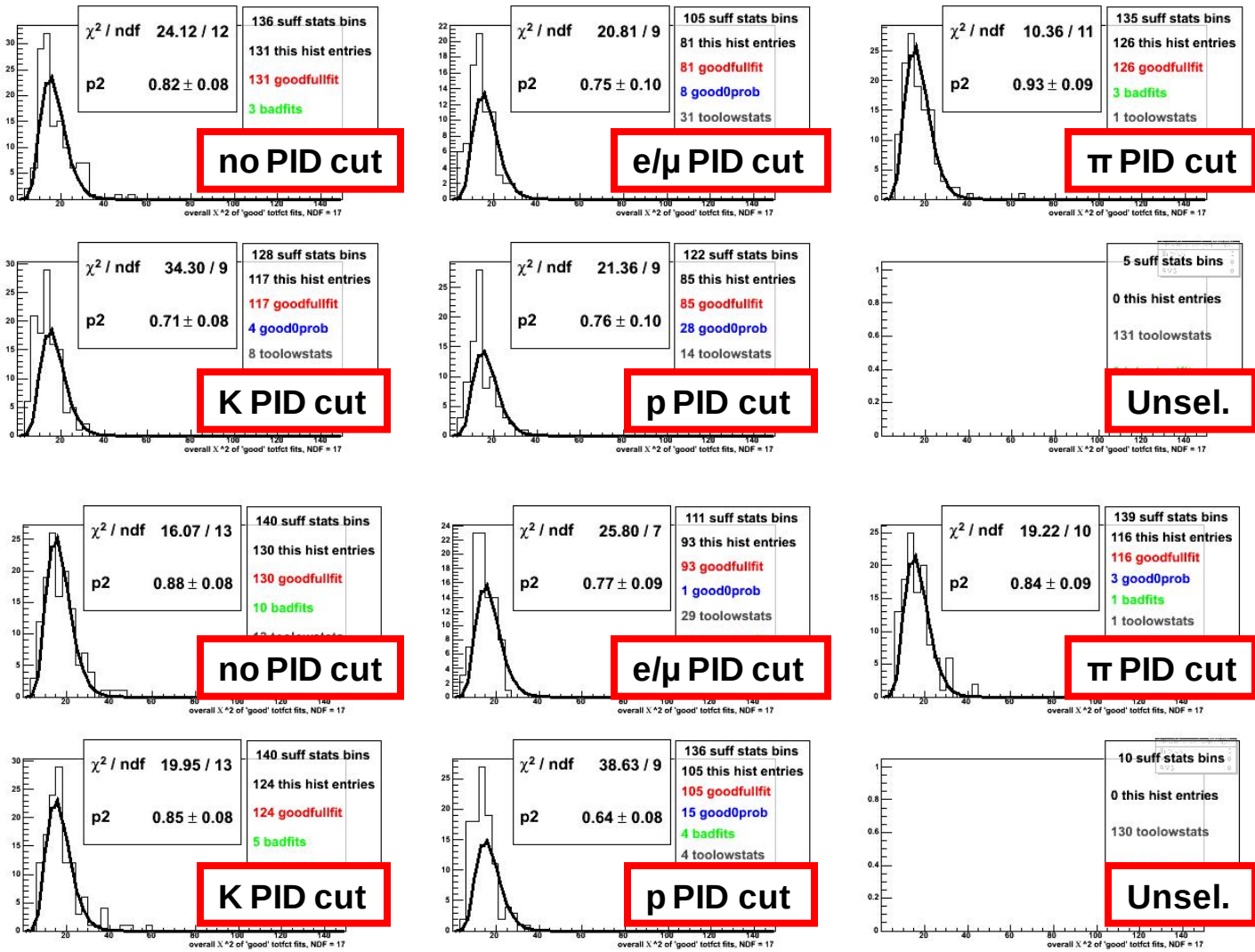
3. Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

Real data exp[7;55], HadronBJ
onres ~590 fb^-1, cont ~70fb^-1

Cumulative Chisquare

π^-

distribution of all chisquares fitted with theoretical Chisquare distribution.



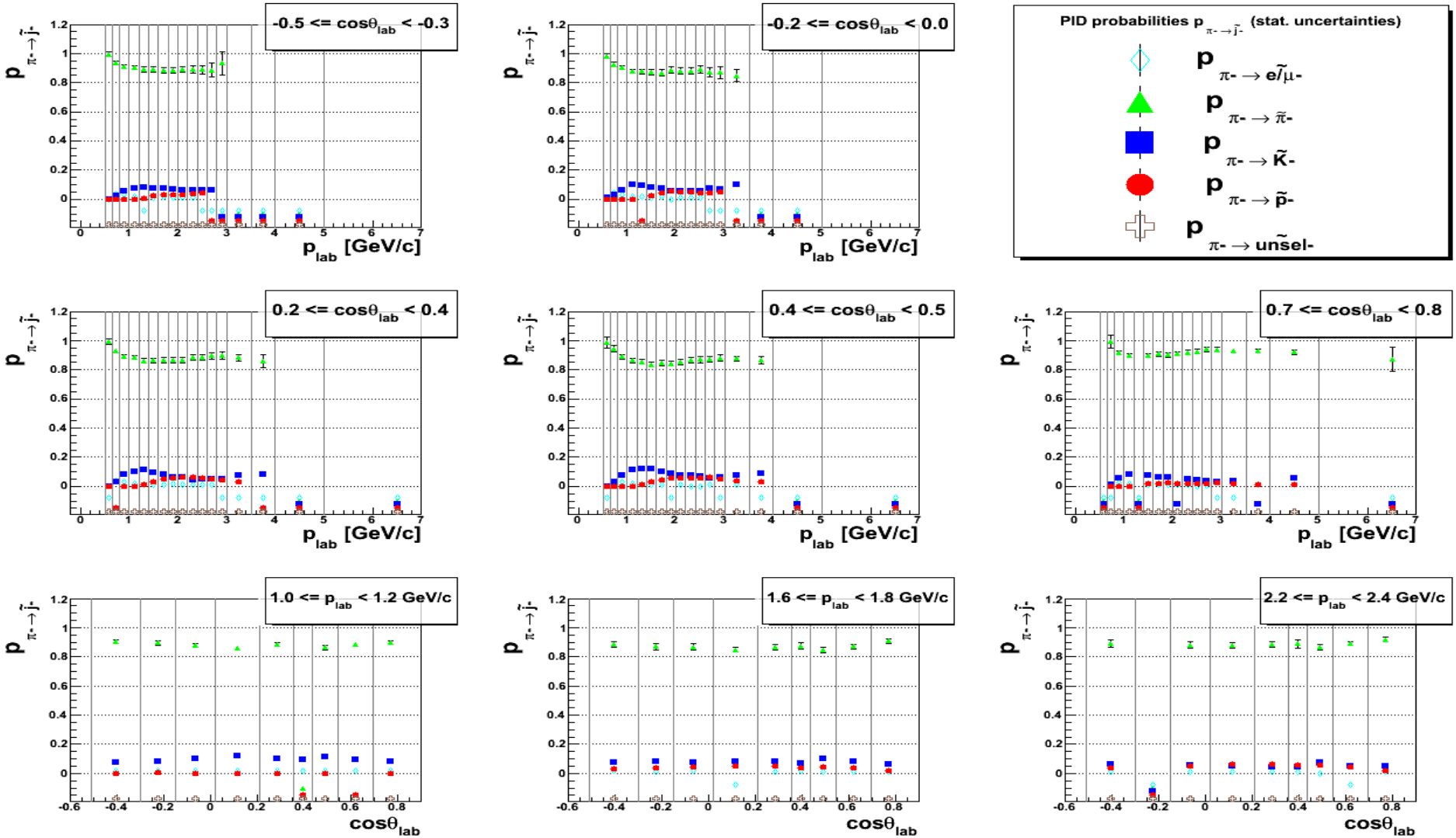
K^-

3. Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

π^-

- Pion PID probability samples.
- Good kinematic availability.

Real data exp[7;55],
onres ~580 fb^-1, cont ~65fb^-1

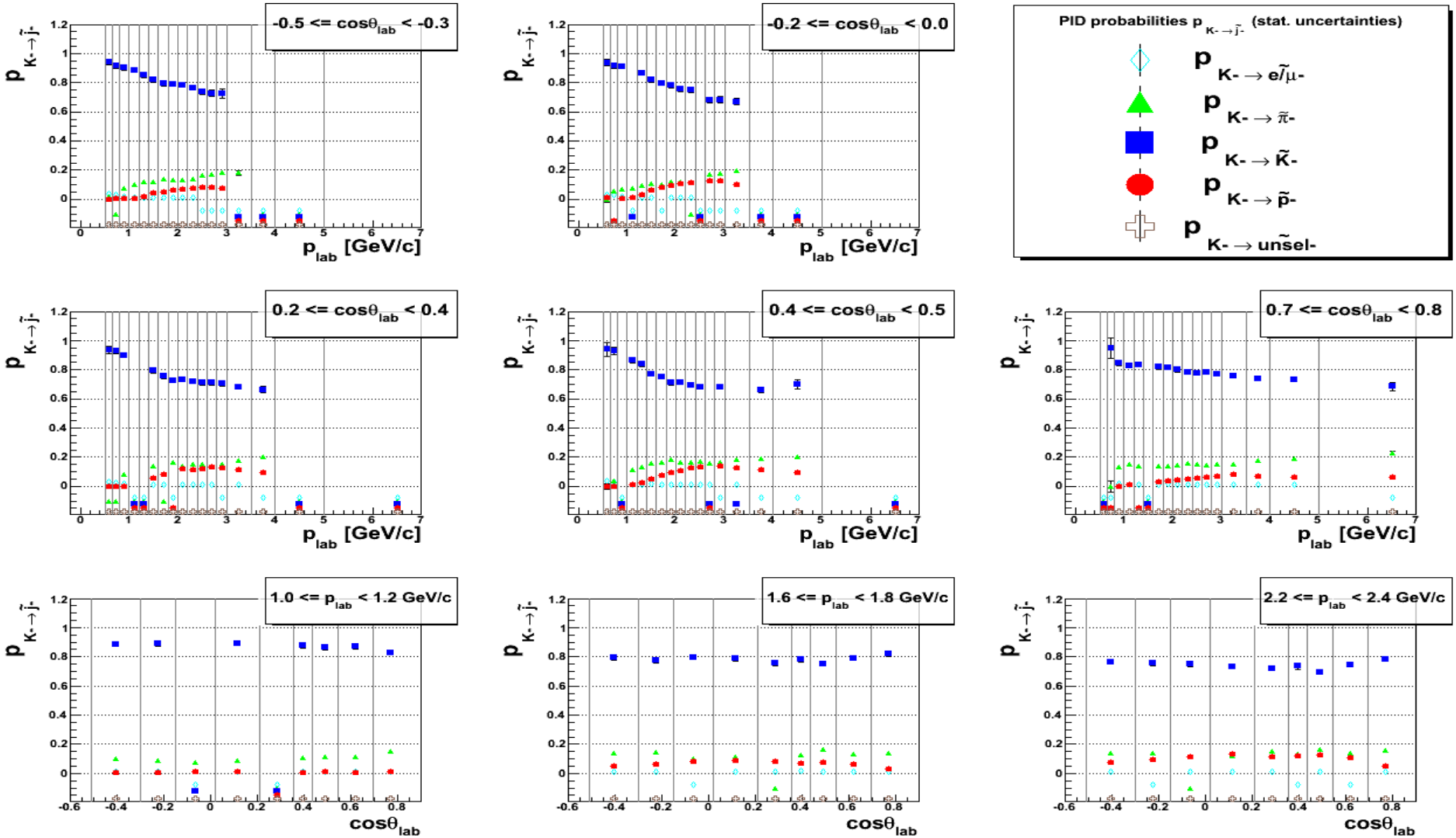


3. Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

K-

- Kaon PID probability samples.
- Good kinematic availability.

Real data exp[7;55],
onres ~580 fb^-1, cont ~65fb^-1



3. Particle Misidentification Correction– Proton PID Probabilities from Λ -> p π

p-

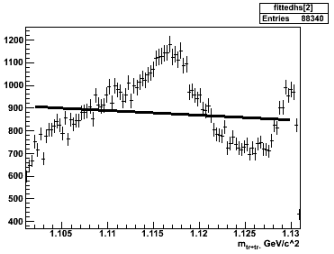
Real data exp[7;55], HadronBJ MDst_vee2;
onres ~590 fb^-1, cont ~70fb^-1

Example fits

p- with
p_{lab} in [1.4; 1.6] GeV/c,
$\cos\theta_{lab}$ in [0.21; 0.36]

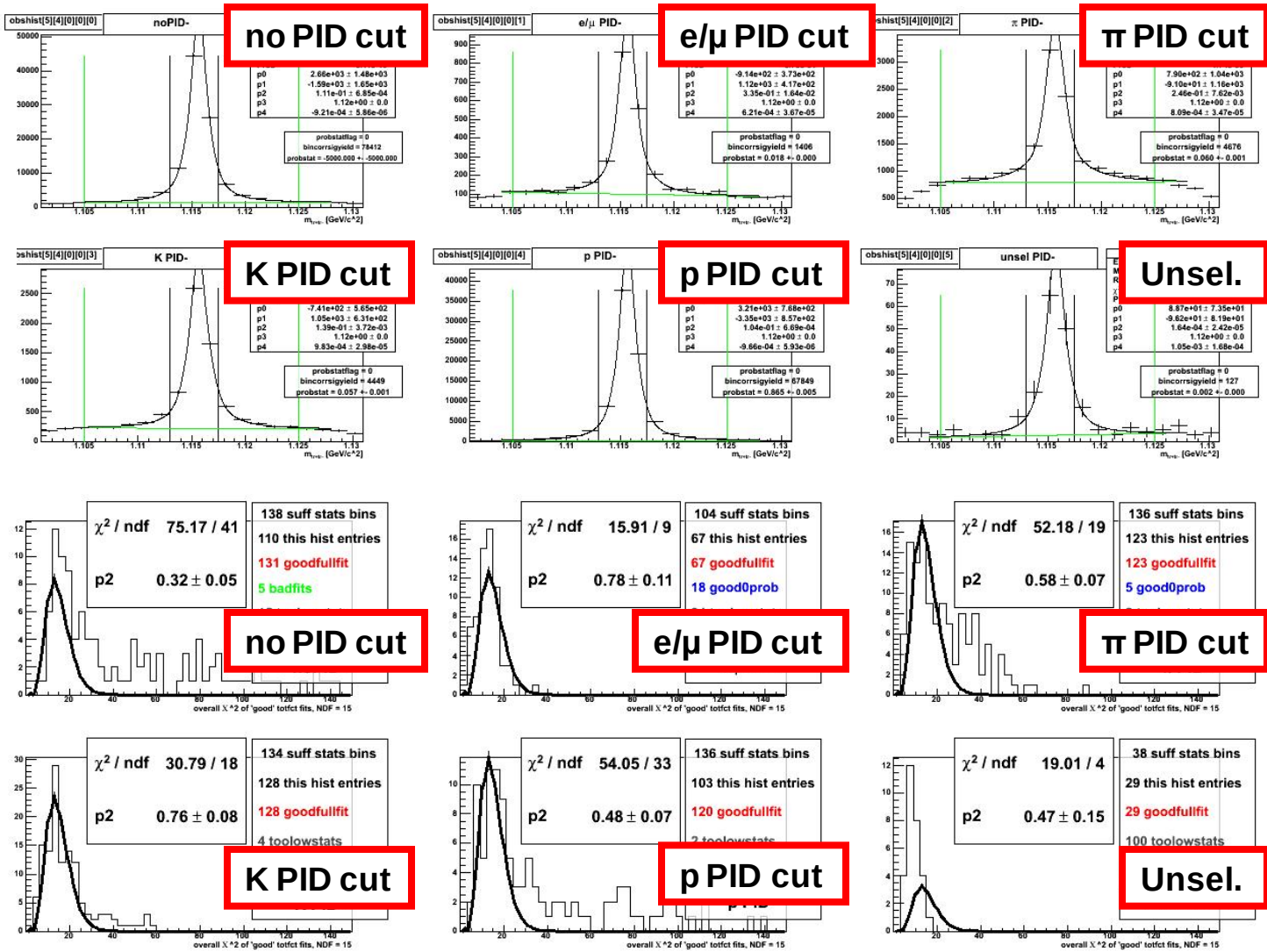
candidate selection along BN 670 (Majumder-san), BN 1126 (Yeh-san).

Good S/N, but candidate selection Ks- cuts
-> non-linear background.



Cumulative Chisquare

distribution of all chisquares fitted with theoretical Chisquare distribution.

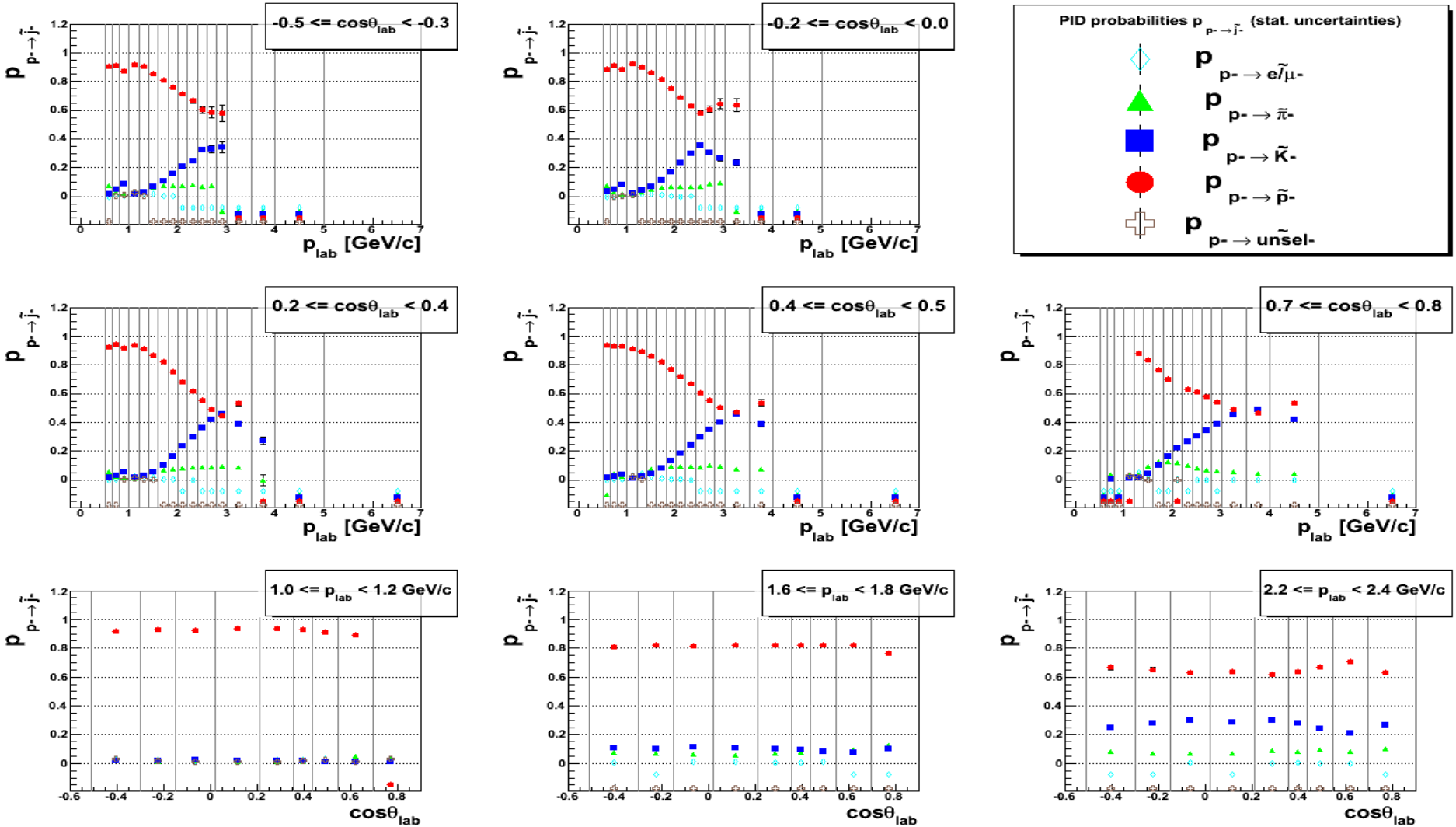


3. Particle Misidentification Correction– Proton PID Probabilities from Λ -> p π

p-

- Proton PID probability samples.
- vs p_{lab} and $\cos\theta_{lab}$.

Real data exp[7;55], HadronBJ
MDst_vee2;
onres ~590 fb^-1, cont ~70fb^-1

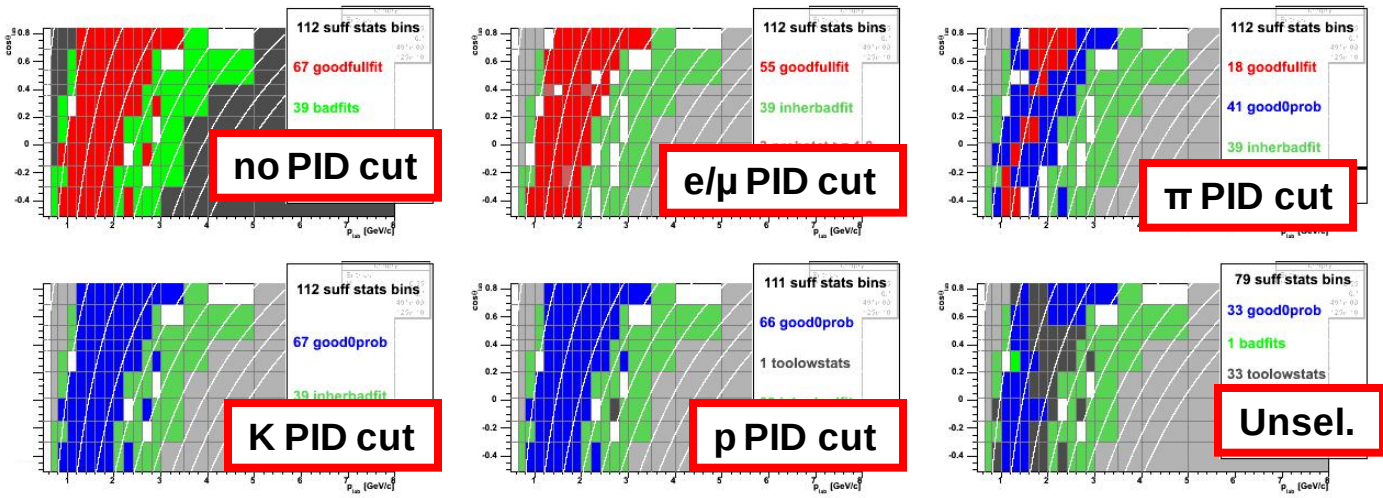


Backup: Particle Misidentification Correction– Lepton PID Probabilities from J/Ψ -> l^+l^-

e-/ μ-

Real data exp[7;55],
onres ~590 fb^-1, cont ~70fb^-1

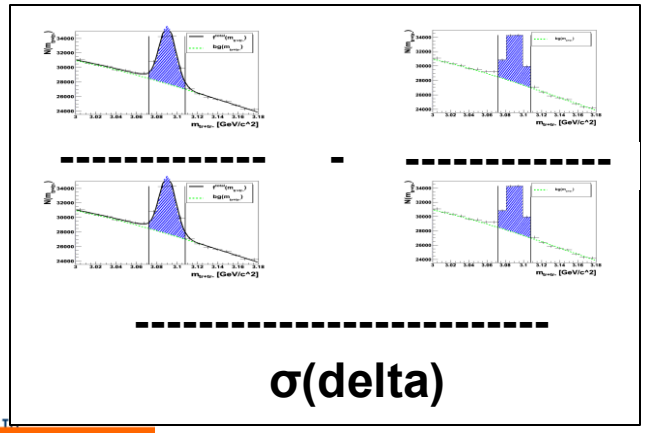
Kinematic extraction availability

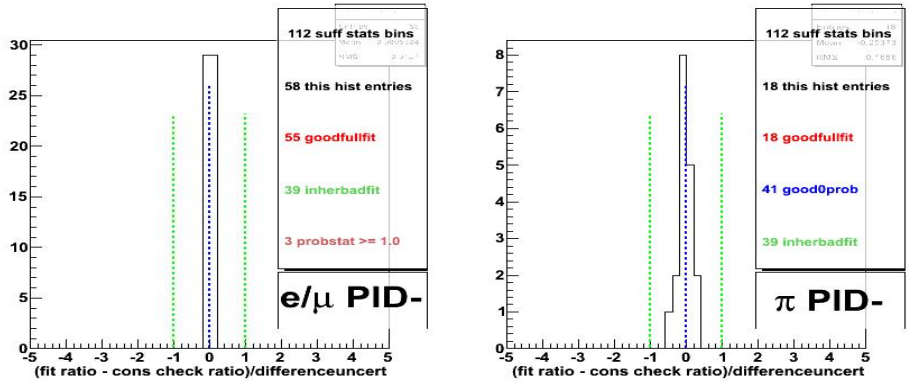


Red/**blue**: good fits.
Green: failed fits (no signal)
White: by-hand-rejected.

Restricted kinematic phase space for decay leptons (J/Ψ from B).

Consistency Check





Agreement between the two methods within 1 sigma for all extracted probabilities.

Backup: Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

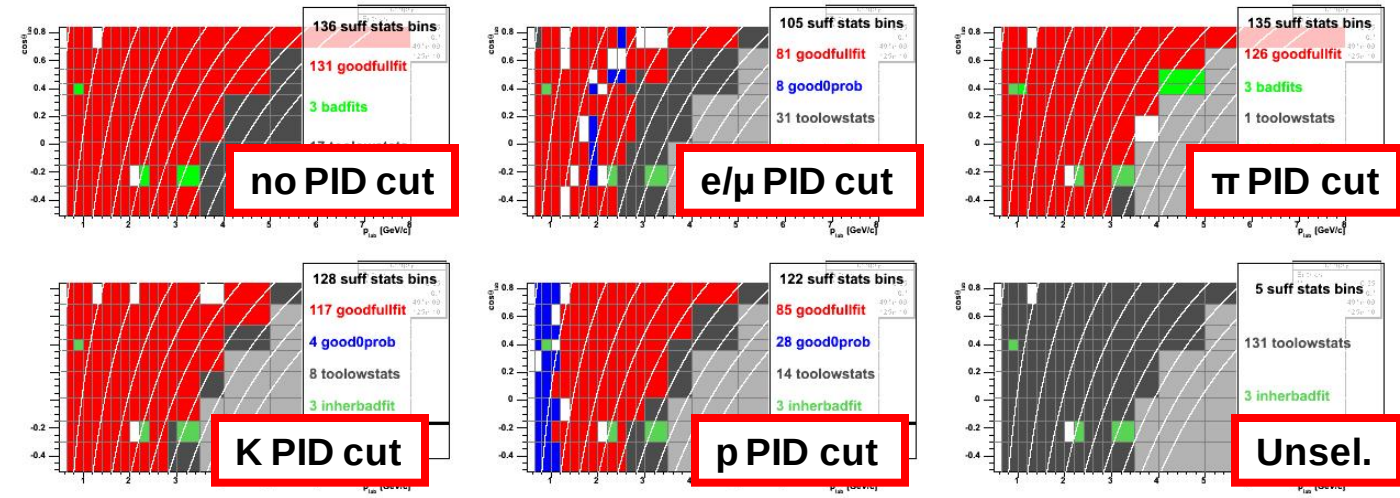
Kinematic extraction availability

Real data exp[7;55], HadronBJ
onres ~590 fb^-1, cont ~70fb^-1

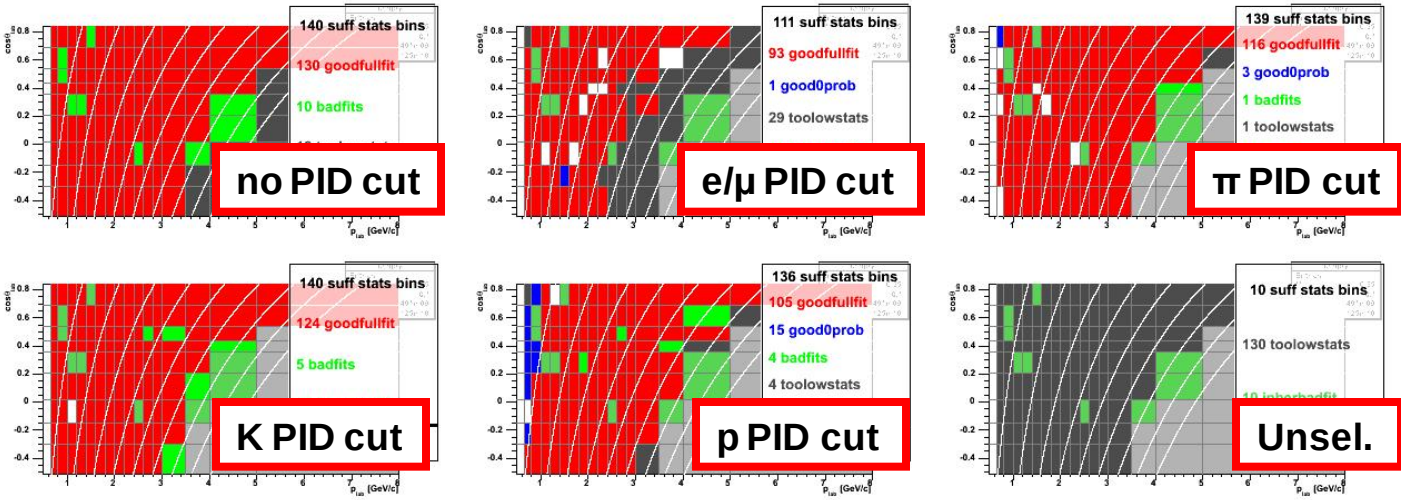
π^-

Red/**blue**: good fits.
Green: failed fits (no signal)
White: by-hand-rejected.

Good availability.



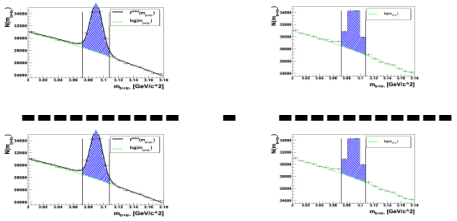
K^-



Backup: Particle Misidentification Correction– Pion and Kaon PID Probabilities from D*

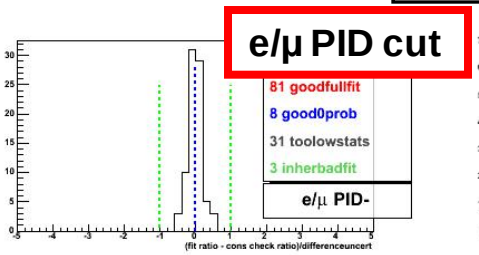
Consistency check

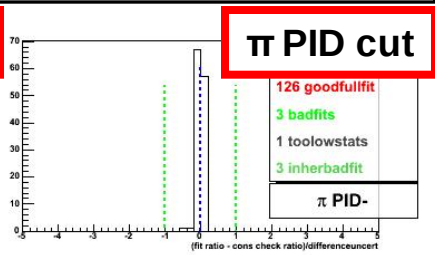
Real data exp[7;55], HadronBJ
onres ~590 fb^-1, cont ~70fb^-1



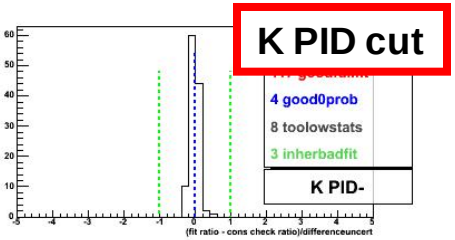
σ(delta)

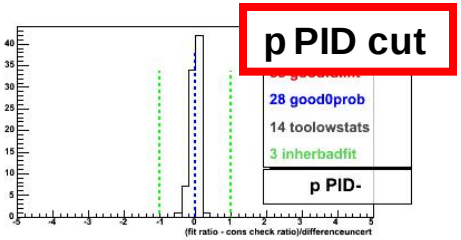
π^-

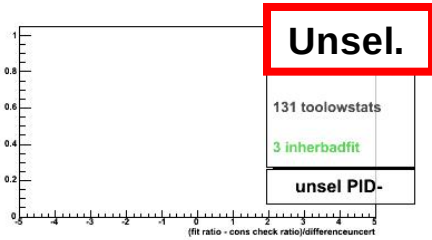




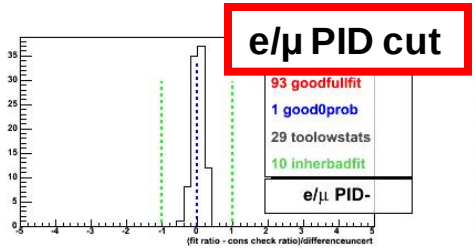
Agreement between the two methods within 1 sigma for all extracted probabilities.

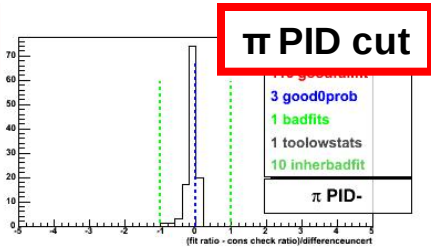


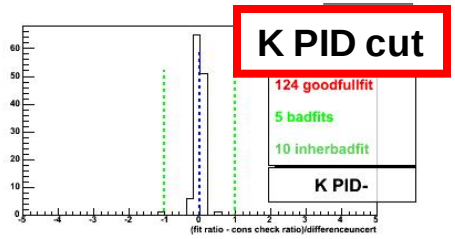


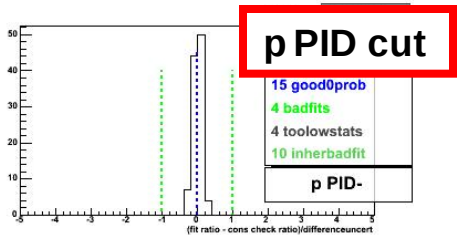


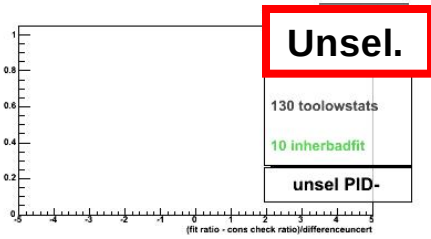
K^-











Backup: Particle Misidentification Correction– Proton PID Probabilities from Λ -> p π

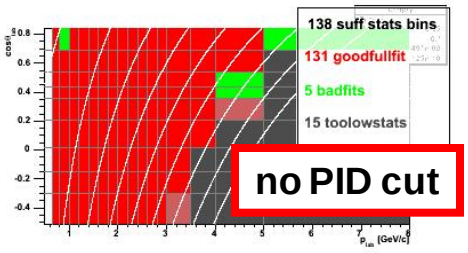
p-

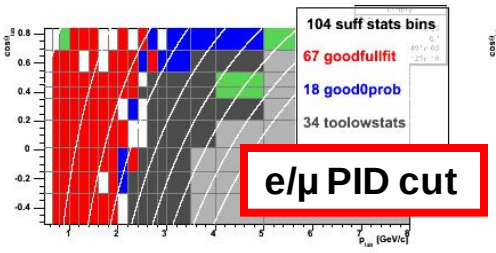
Real data exp[7;55], HadronBJ MDst_vee2;
onres ~590 fb^-1, cont ~70fb^-1

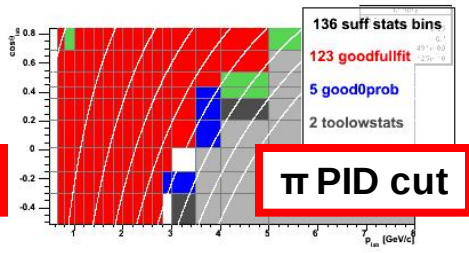
Kinematic extraction availability

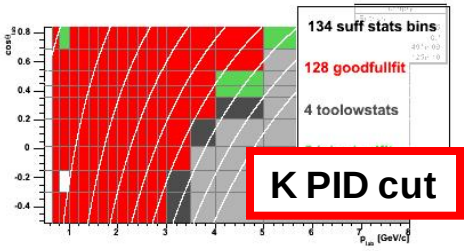
Red/**blue**: good fits.
Green: failed fits (no signal)
White: by-hand-rejected.

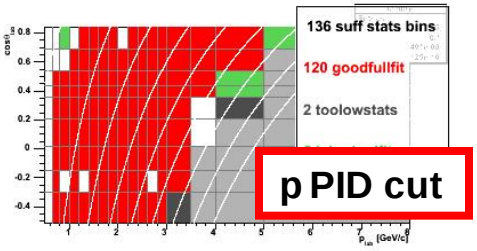
Good availability.

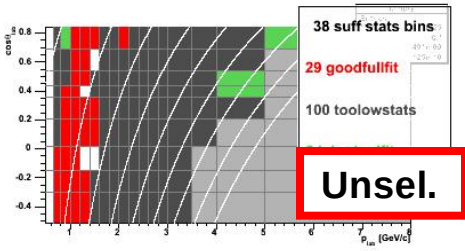




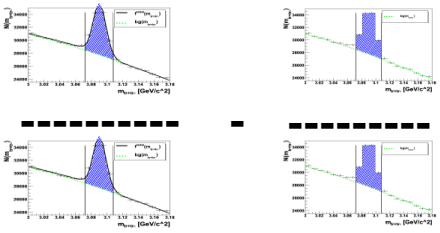




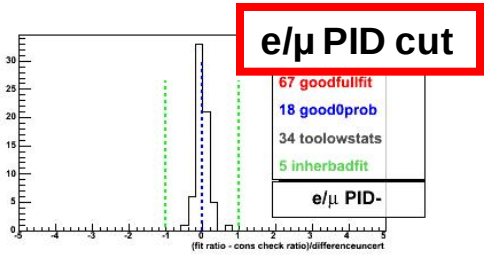


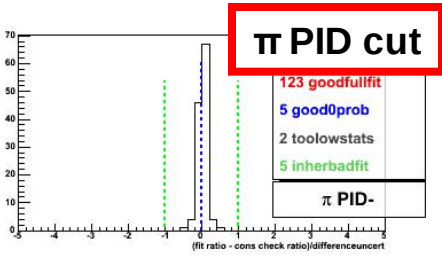


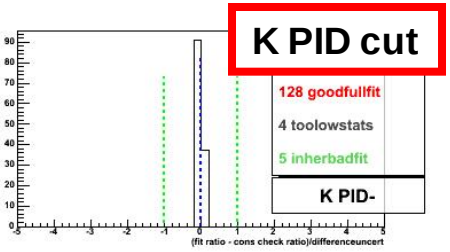
Consistency Check

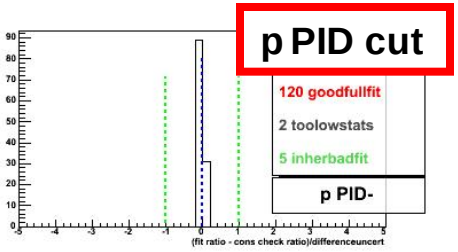


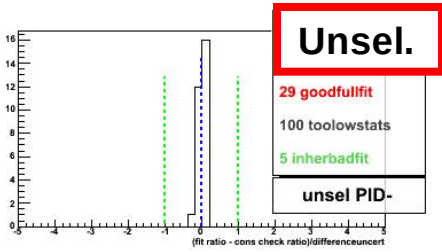
σ(delta)





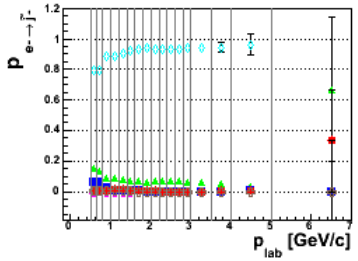


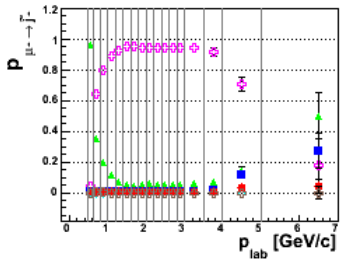




Backup: Particle Misidentification Correction- Obtain MC Probabilities from ‚MCHadronBJ' Sample

- For extrapolation techniques need MC PID probabilities
- If PID correction performed with probabilities from generic uds charm MC: unphysical corrected spectra:

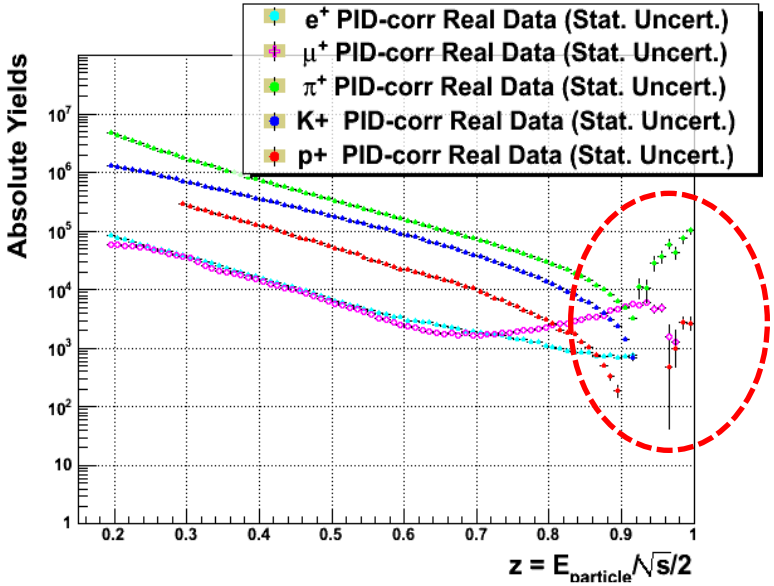




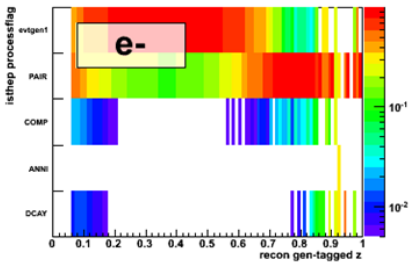
5 streams exp7-55 cont, ~335 fb^-1, ~600e06 events
IPdr < 13mm, IPdz < 4cm; N_SVD(rphi + z) >= 3; Evis > 7 GeV

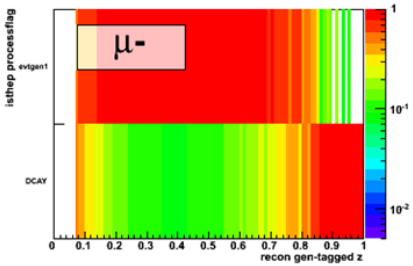
e- -> j-

mu- -> j-



uds charm generic MC cont exp7-55 3 streams, std cuts (Ipdr, Ipdz, NSVD, Evis)





In generic uds and charm MC,
most high z electrons are created in pair production,
most high z muons created in decay-in-flight
-> high fake rates -> unphysical PID-corr. spectra

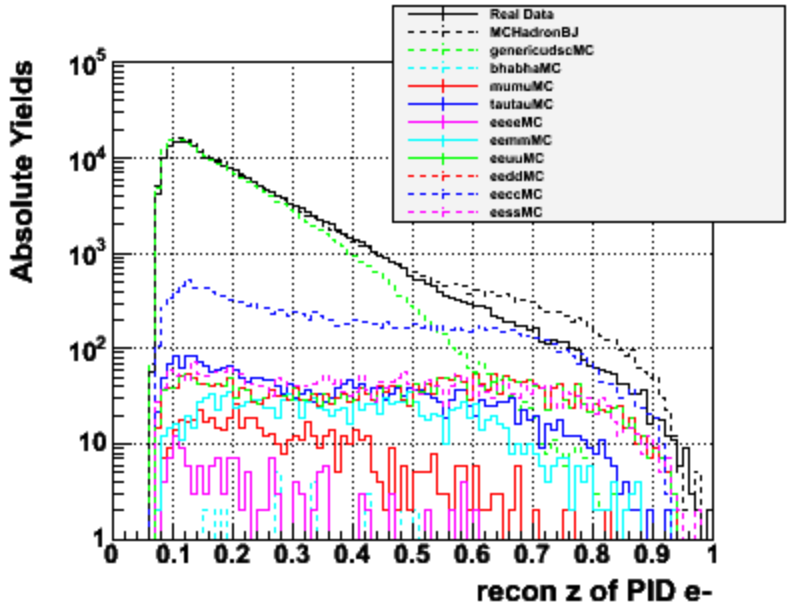
Backup: Particle Misidentification Correction- Obtain MC Probabilities from ‚MCHadronBJ' Sample

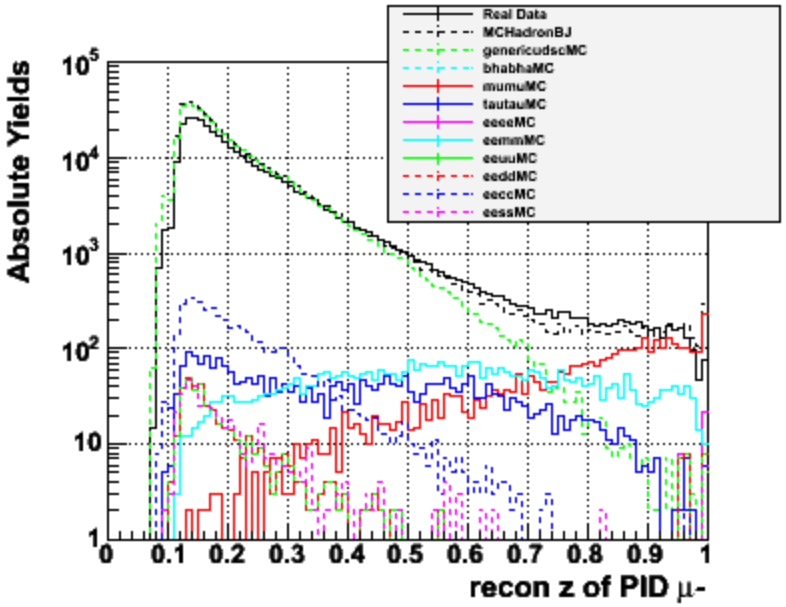
- Include non-qqbar processes entering HadronBJ skim in MC probability sample
- Otherwise MC PID probabilities produce unphyiscal spectra in correction

type	cross section (nb)
tautau (KKMC)	0.9187 (+-0.0003)
eeee	40.9
eemumu	18.9
eeuu	11.7
eedd	0.798
eess	0.227
eecc	0.030
eetautau	0.018
mumu (KKmc)	1.005 +- 0.001
bhabha (BHLUMI)	123.5 +- 0.2

After extensive discussion with Casey-san, Inami-san provided tau, two-photon, muon and Bhabha MC on the Nagoya cluster.

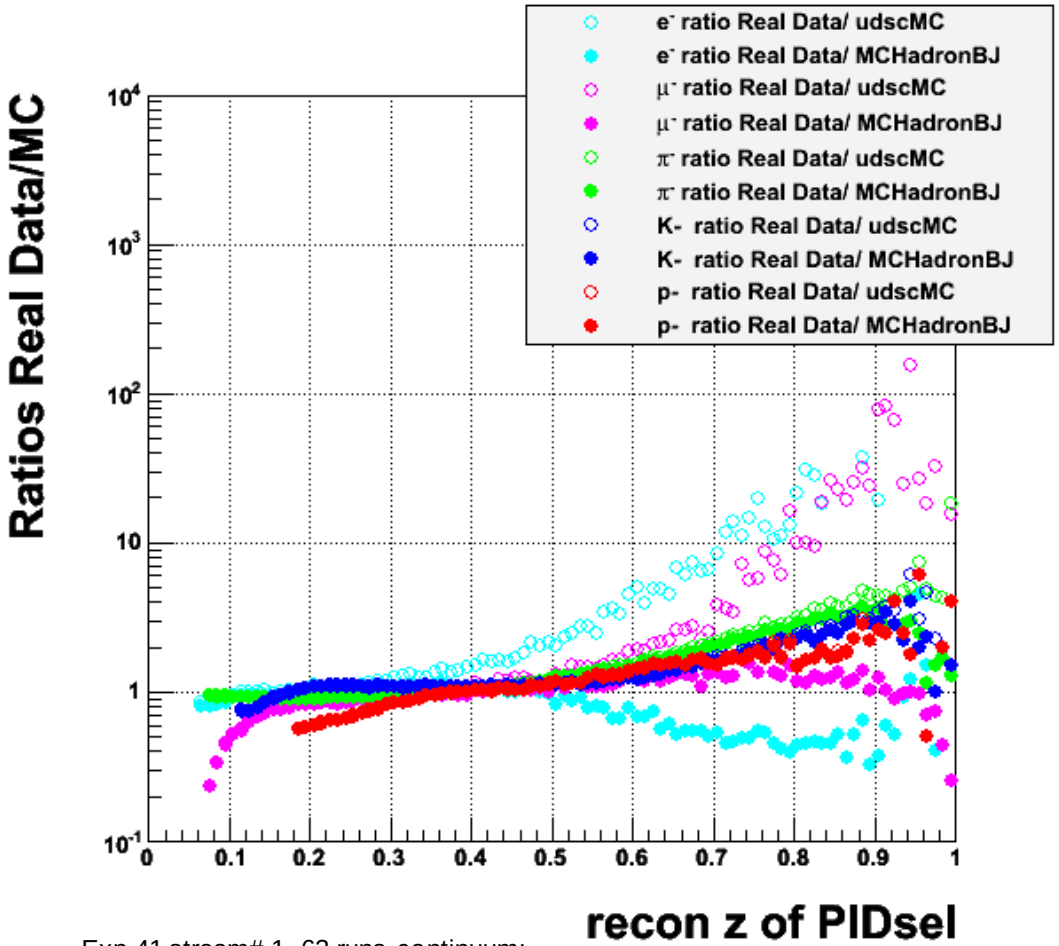
All processes are generated in amounts according to their cross sections at the respective energy levels.



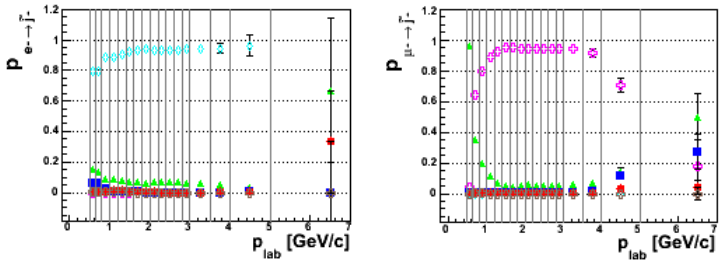


Backup: Particle Misidentification Correction- Obtain MC Probabilities from ‚MCHadronBJ' Sample

- Comparing ‚MCHadronBJ' to real data HadronBJ skim for selected runs in exp41

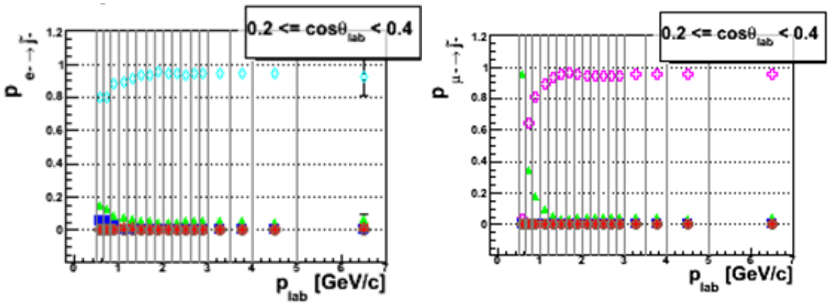


Exp 41 stream# 1, 63 runs continuum;
generic uds + charm + 'Nagoya MC'; real data HadronBJ skim
IPdr < 13mm, IPdz < 4cm; N_SVD(rphi + z) >= 3; Evis > 7 GeV



5 streams exp7-55 cont, ~335 fb^-1, ~600e06 events
IPdr < 13mm, IPdz < 4cm; N_SVD(rphi + z) >= 3; Evis > 7 GeV

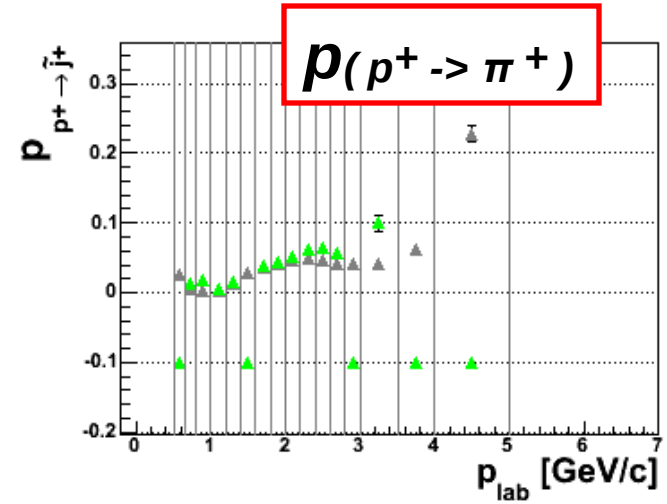
e- -> j- **mu- -> j-**

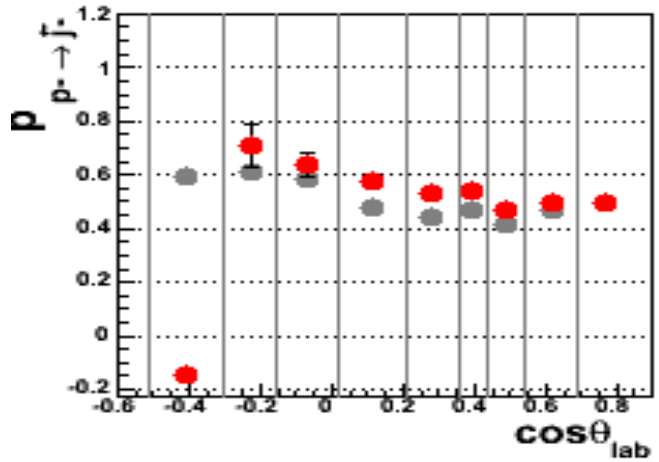


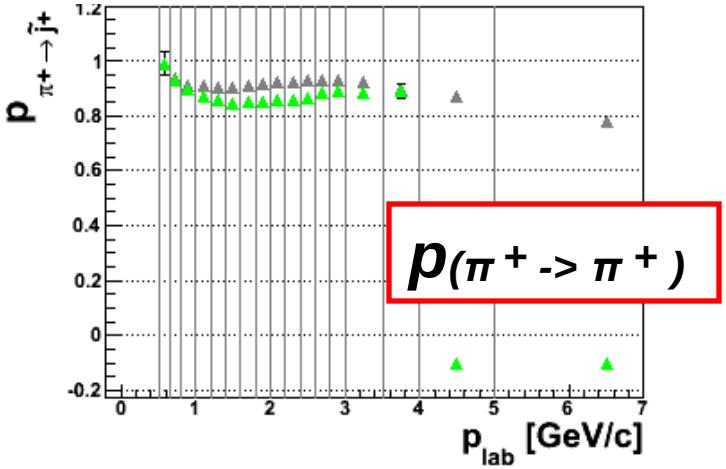
Exp 41 stream# 1 + 2, 63/64 runs continuum, 557/572 runs onres;
generic uds + charm + mixed + charged + 'Nagoya MC',
total ~100 fb^-1, 235e+06 events
IPdr < 13mm, IPdz < 4cm; N_SVD(rphi + z) >= 3; Evis > 7 GeV

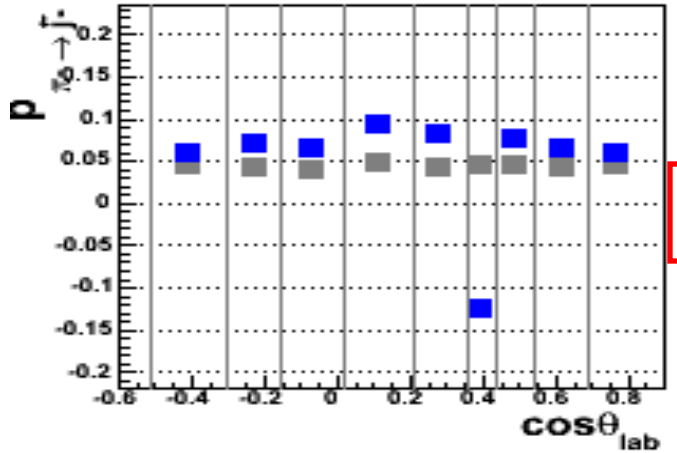
Backup:
Particle Misidentification Correction– Extrapolation of Real Data PID Probabilities

- Interpolate small gaps with real data PID probability
- Extrapolate high p_{lab} probabilities in high plab bins with Monte Carlo help.
- Color points: real data probabilities, gray points: generic MC probabilities.



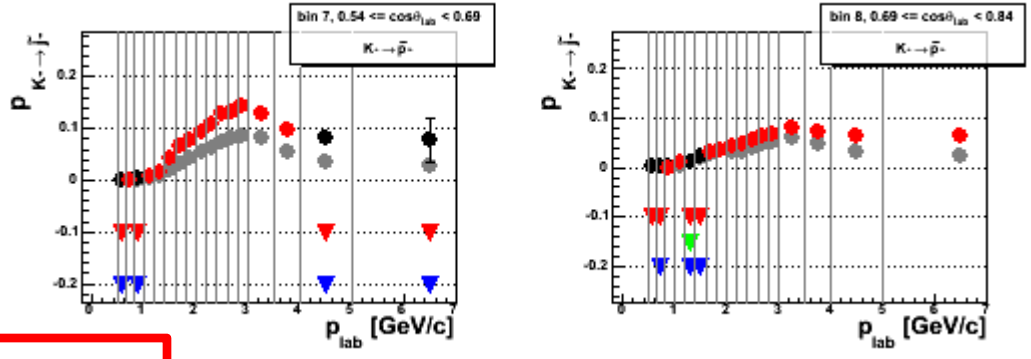






Backup: Particle Misidentification Correction- PID Correction & averaging between T1) and T2)

- **Proton turn-off at z~0.8**: caused by **fake rate K->p**.
  in D* sample fake rate K->p higher than in MCHadronBJ:

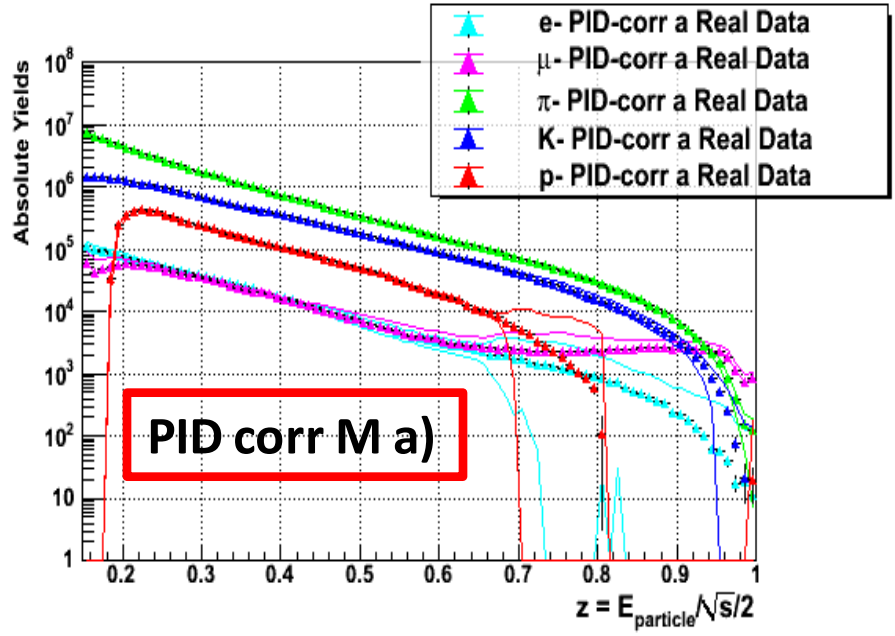


K- -> p-

Color: real data K- ->p- probs from K out of D* decays.

Gray: MCHadronBJ probabilities from all gen K reaching the detector after std cuts.

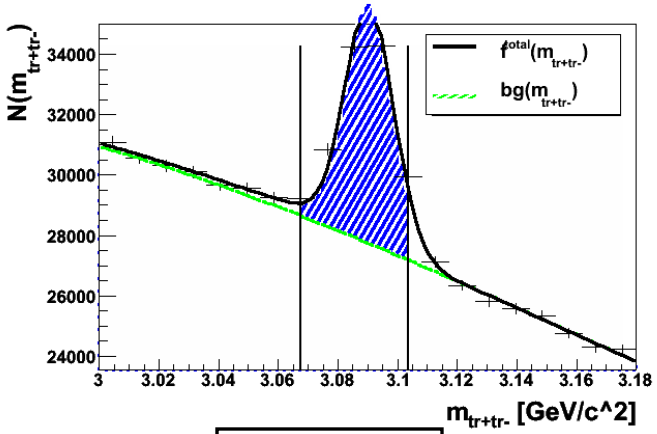
Black: method a) extrapolated probabilities.



Systematic uncertainties of PID correction consistent with positive proton yields at high z.

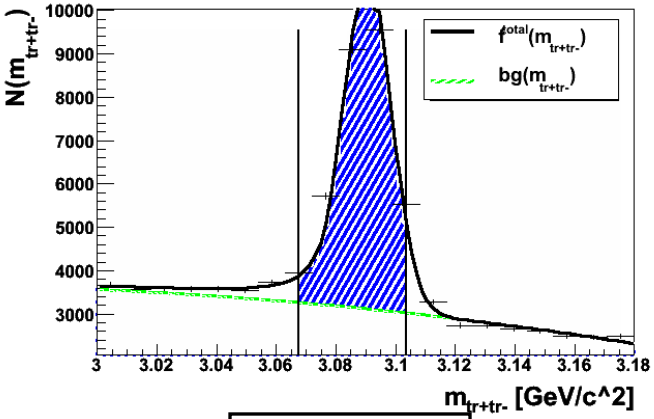
Backup. PID Probability Extraction

- Probability extraction p(i->j): building ratios between signal yields- integrated signal functions



no PID cut

N_i



e/μ PID cut

N_i->j

$$p_{i\to j} = \frac{N_{i\to j}}{N_i} = \frac{\dfrac{\int_{m_{low}}^{m_{high}} g^{sig}_{\mathfrak{D}^{i\to j}(m)}(\overrightarrow{par}_{\mathfrak{D}^{i\to j}(m)}, m)\, dm}{histbinwidth}}{\dfrac{\int_{m_{low}}^{m_{high}} f^{sig}_{\mathfrak{D}^{i}(m)}(\overrightarrow{par}_{\mathfrak{D}^{i}(m)}, m)\, dm}{histbinwidth}}.$$

Backup: HadronBJ Impurities

Process	σ (nb)
$b\bar{b}$	1.1
$q\bar{q}$ $(q = u,\ d,\ s,\ c)$	3.3
$\tau^+\tau^-$	0.93
QED $(25.551^o < \theta < 159.94^o)$	37.8
$\gamma\gamma \to q\bar{q}$ $(w > 500$ MeV$)$	11.1

- In initial test study, following BN 390,

  * QED background was neglected;

  * Beam-gas background was also neglected, since only tracks within the ToF geometry acceptance and with ***z*** < 0.75 were considered in our analysis.

- upper bound is estimated for tautau (gamma gamma) events:
  additional directed systematic uncertainty of -1.8% (-1.4%) per *z*-bin.

Physics Process	$\sqrt{s}_{\text{on res}} = 10.58$ GeV		$\sqrt{s}_{\text{cont}} = 10.52$ GeV
	Cross Section [nb]	Relative Number of Events	Relative Number of Events
$B\overline{B}$	1.09	27.9%	-
$q\bar{q}$	2.62	67.0%	92.9%
$\tau^+\tau^-$	0.05	1.3%	1.8%
QED	$3 \cdot 10^{-3}$	$5 \cdot 10^{-4}$	0.04%
$\gamma\gamma$	0.04	1.0%	1.4%
Beam-Gas	0.11	2.8%	3.9%

Backup - Binning for Particle Misidentification Correction

p_{lab} binning:

bin 0 = 0.5 - 0.65 GeV
bin 1 = 0.65 - 0.8 GeV

... widths of 200 MeV

bin 12 = 2.8 – 3.0 GeV
bin 13 = 3.0 – 3.5 GeV
bin 14 = 3.5 – 4.0 GeV
bin 15 = 4.0 – 5.0 GeV
bin 16 = 5.0 – 8.0 GeV

$\cos\theta_{lab}$ binning:

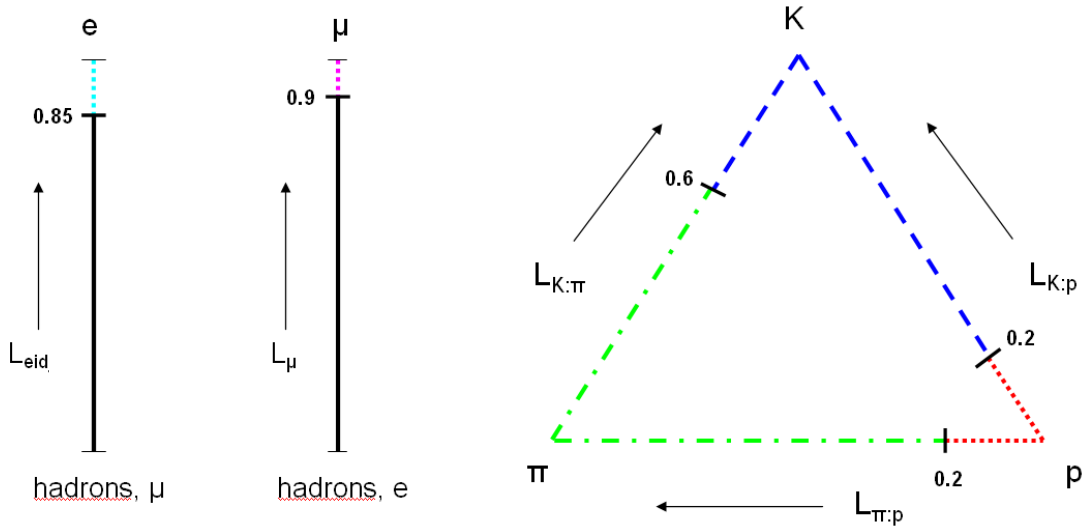
bin 0: -0.511 – -0.300
bin 1: -0.300 – -0.152
bin 2: -0.152 – 0.017
bin 3: 0.017 – 0.209
bin 4: 0.209 – 0.355
bin 5: 0.355 – 0.435
bin 6: 0.435 – 0.542
bin 7: 0.542 – 0.692
bin 8: 0.692 – 0.842

Θ_{lab} binning within ToF acceptance
ToF θ_{lab} [34;120)°,
$\cos\theta_{lab}$ in [0.829;-0.5).

Θ_{lab} binning reflects ACC hardware segmentation.

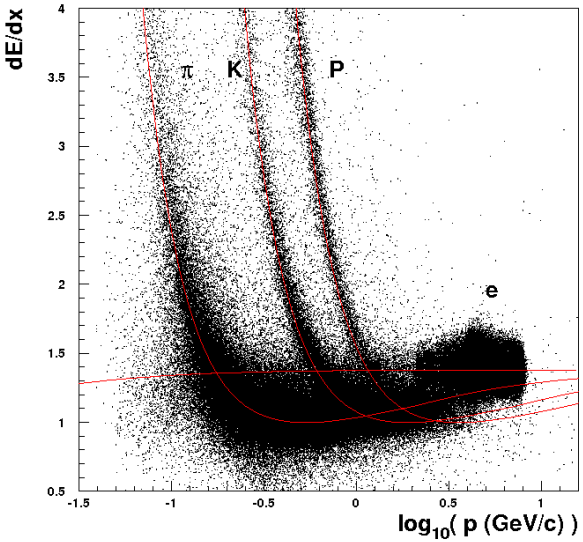
Backup – Likelihood Cuts for Particle Selection

Selected Species j	Cuts on Likelihood Quantities				
	L_{eid}	L_{μ}	$L_{K:\pi}$	$L_{K:p}$	$L_{\pi:p}$
e	$[0.85; 1.0]$	$[-5.0; 0.9)$	-	-	-
μ	$[0.0; 0.85)$	$[0.9; 1.0]$	-	-	-
π	$[0.0; 0.85)$	$[-5.0; 0.9)$	$[0.0; 0.6)$	$[0.0; 1.0]$	$[0.2; 1.0]$
K	$[0.0; 0.85)$	$[-5.0; 0.9)$	$[0.6; 1.0]$	$[0.2; 1.0]$	$[0.2; 1.0]$
p	$[0.0; 0.85)$	$[-5.0; 0.9)$	$[0.0; 0.6)$	$[0.0; 0.2)$	$[0.0; 0.2)$



Backup- CDC, ToF and ACC PID discrimination power

- CDC discrimination power via dE/dx



- ToF discrimination power via time-of-flight

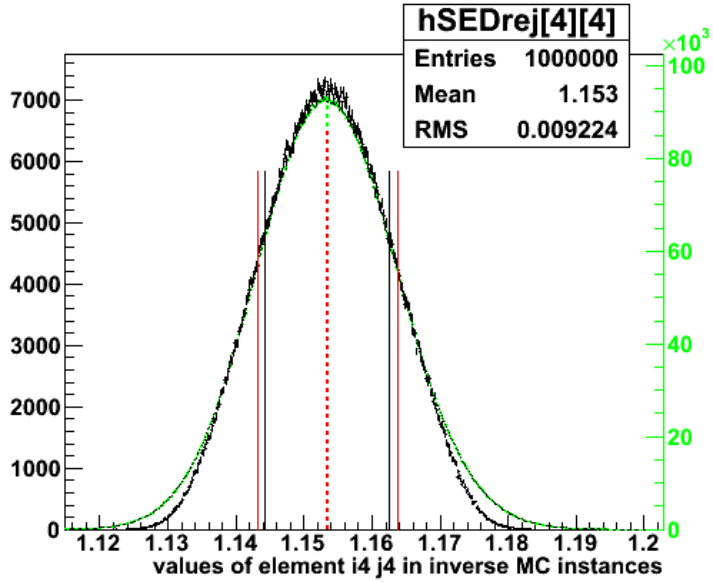
Track Momentum [GeV/c]	Times-of-Flight for Particle Species i [ns]				
	e	μ	π	K	p
1.0	4.004	4.209	4.274	4.892	5.572
2.0	4.003	4.008	4.013	4.123	4.421
3.0	4.003	4.005	4.007	4.057	4.194

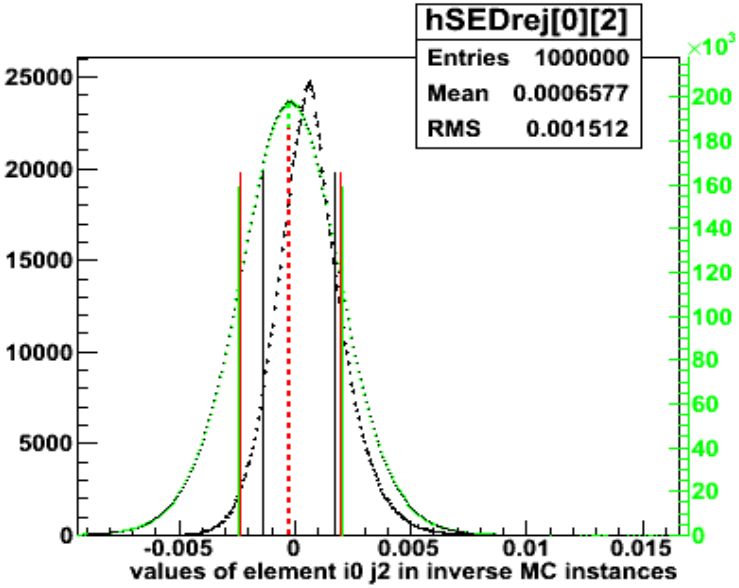
- ACC discrimination power via Cerenkov thresholds

Refractive Index	Cherenkov Threshold Momentum for Particle Species i				
	e [MeV/c]	μ [GeV/c]	π [GeV/c]	K [GeV/c]	p [GeV/c]
1.010	3.6	0.75	0.98	3.48	6.62
1.013	3.2	0.65	0.86	3.05	5.80
1.015	2.9	0.61	0.80	2.84	5.40
1.020	2.54	0.53	0.69	2.46	4.67
1.028	2.14	0.44	0.59	2.07	3.94

Backup: Particle Misidentification Correction-Uncertainty Propagation Procedure

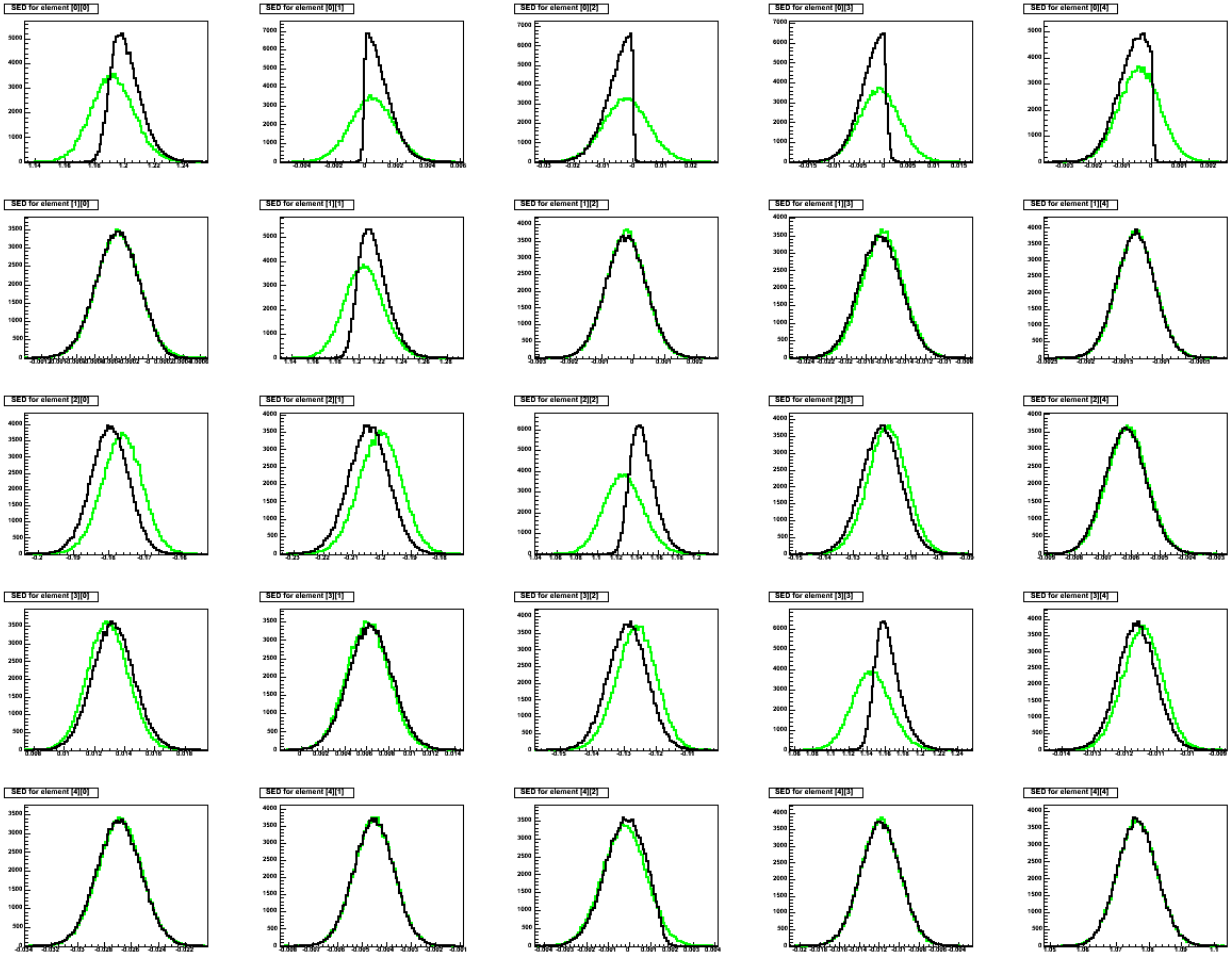
- For inverse values: take analytical inverse.
- For uncertainties: Build MC ‚instances' of original PID matrix by varying each element within its uncertainties, impose probability requirements, sample size **10^6**





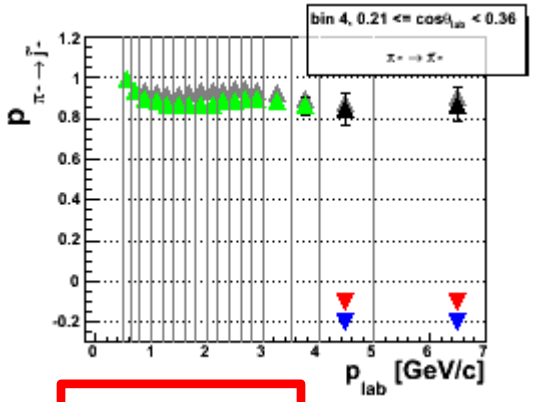
Backup- Accounting for Probability Nature of Matrix Elements in Uncertainty Propagation through Matrix Inversion for Sample PID Matrix

- green: all MC variants of original PID matrix.
- black: rejecting all variant matrices which are not complying with probability interpretation (columnsum > 1.0 || matrix element < 0.0).

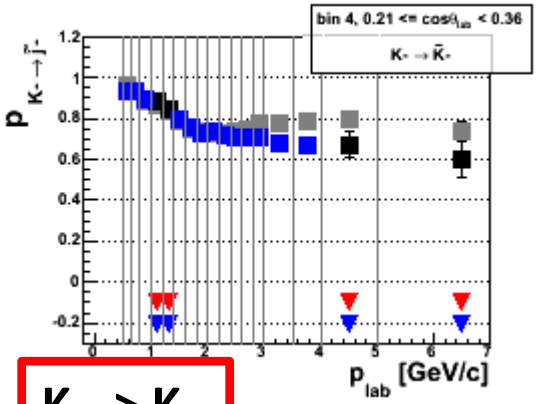


4a. Correction for Particle Misidentification
IV. PID Probability Extrapolation

- PID probabilities not extractable for entire kinematic plane from real data
- To obtain PID information and representative uncertainties also for highest z: extrapolate real data probabilities with MC information.
- To account for sample impurity of real data HadronBJ skim, extract MC information from ‚MC HadronBJ' = uds + charm + 2gamma + QED events.
- Perform PID correction with probabilities from two different extrapolation methods. PID-corrected yields are average of corrections.



pi- -> pi-



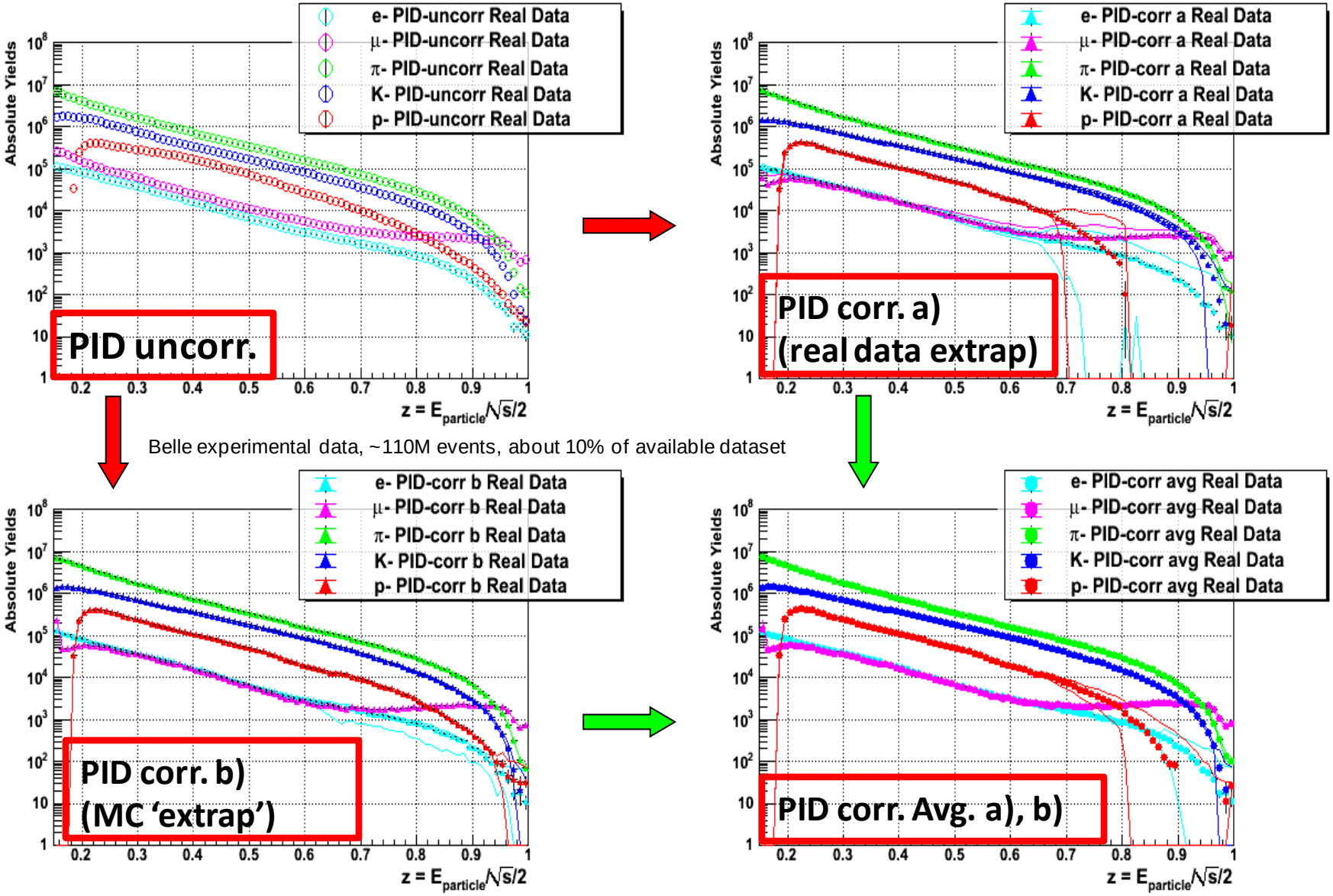
K- -> K-

Color: real data probabilities.

Gray: MC probabilities.

Black: method a) extrapolated probabilities.
(Triangles below 0: internal indicators.)

4a. Correction for Particle Misidentification
V. PID Probability Correction Results

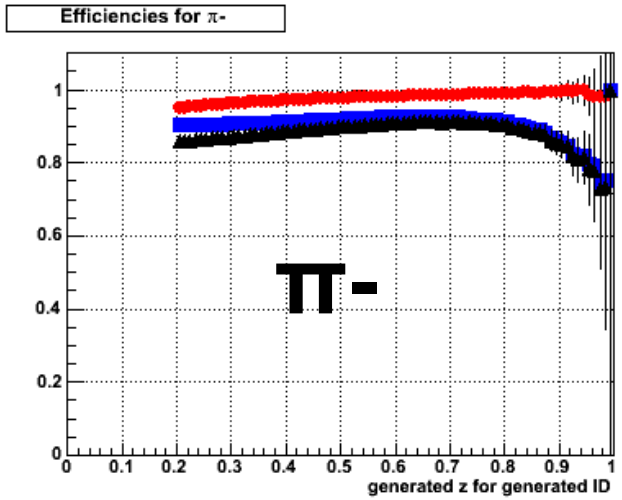


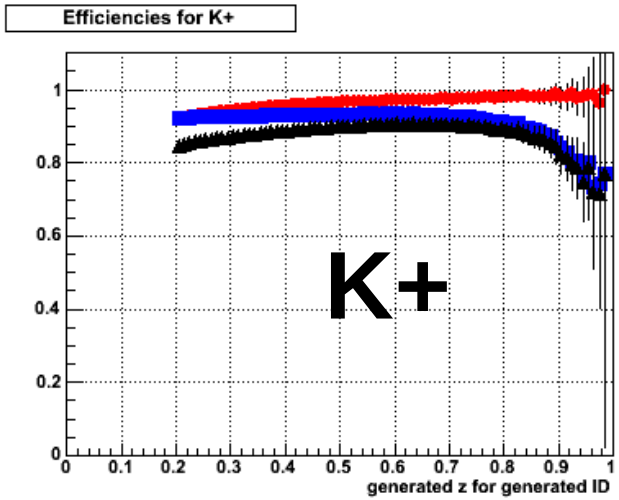
Belle experimental data, ~110M events, about 10% of available dataset

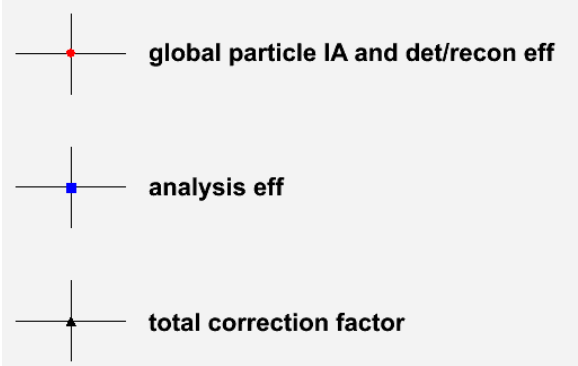
4c. Acceptance Corrections

- MC-based correction for decay-in-flight losses, detector interaction, detector/tracking efficiencies, analysis acceptance.
- Account for dependence of correction factor on MC hadronic interaction package

$$N_z^{h,\,\text{measured}} = N_z^{h,\,\text{all}} \underbrace{\varepsilon_{z,\,\text{global}}^{h\,\text{all}} \varepsilon_{z,\,\text{analysiscuts}}^{h\,\text{all}}}_{\varepsilon_{z,\,\text{total}}^{h}}$$







10^7 MC continuum events

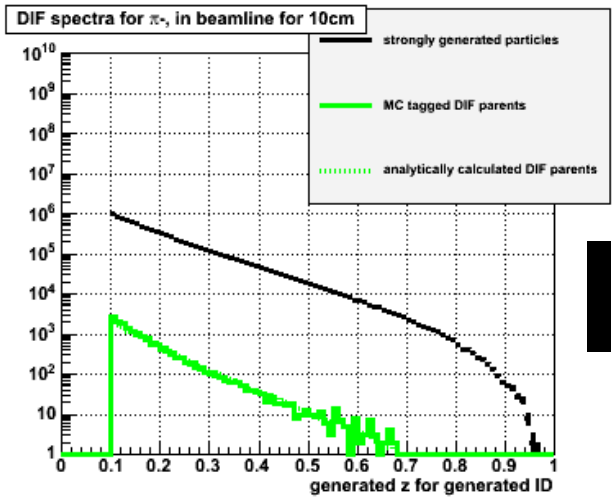
Backup: Pion decay-in-flight study

10cm

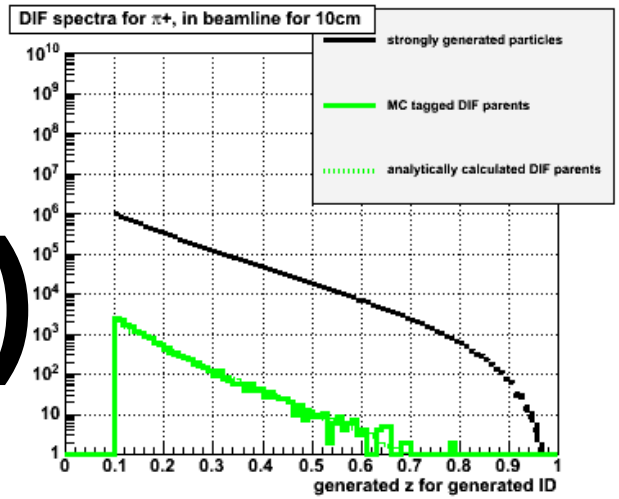
I) Absolute yields of all strongly generated particles, MC-tagged and analytically calculated DIF parents

II) MC-tagged and analytically calculated DIF parents relative to all strongly generated particles

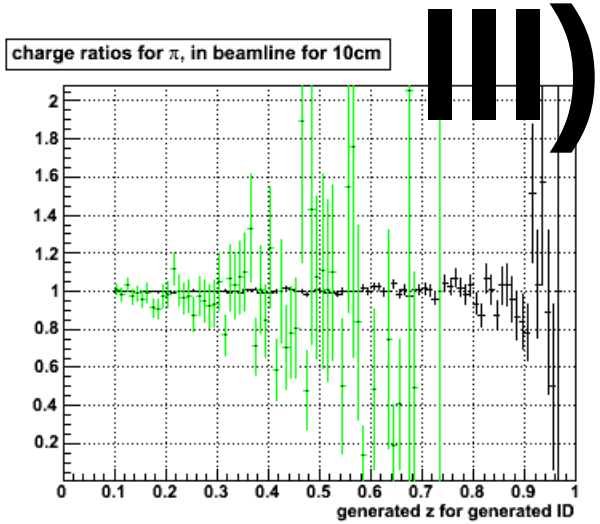
III) charge ratio of all strongly generated particles and MC-tagged DIF parents

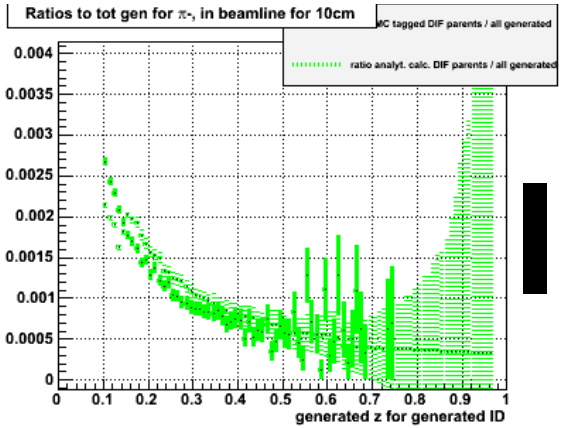


I)

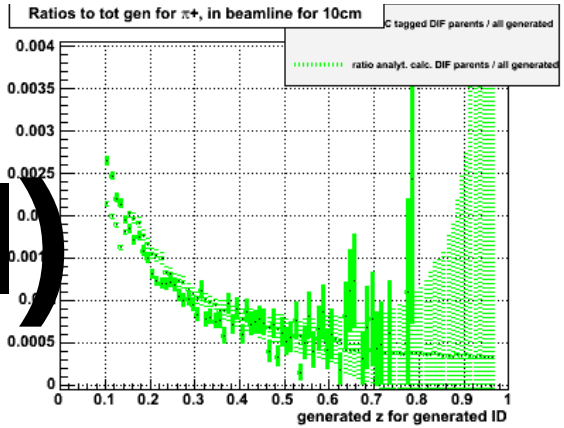


III)





II)



Summary: good consistency between pi+ and pi- MC and with analytical calculation along beampipe

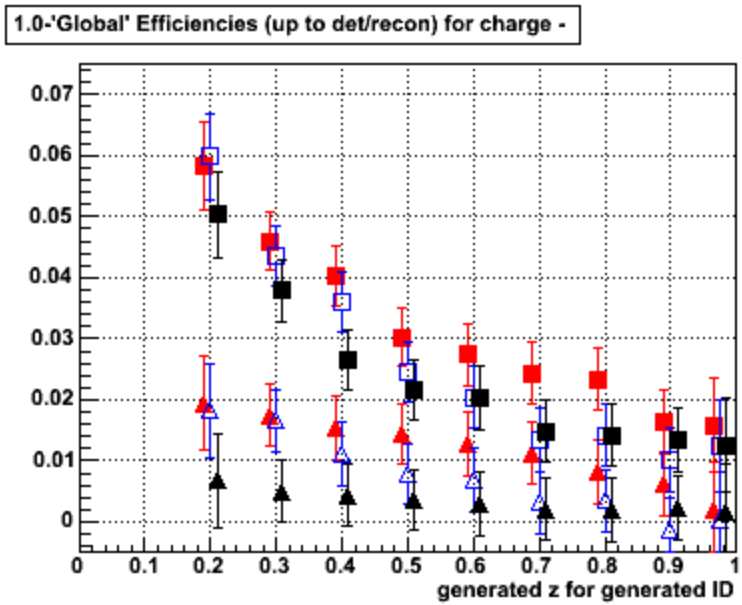
5. Acceptance Correction I

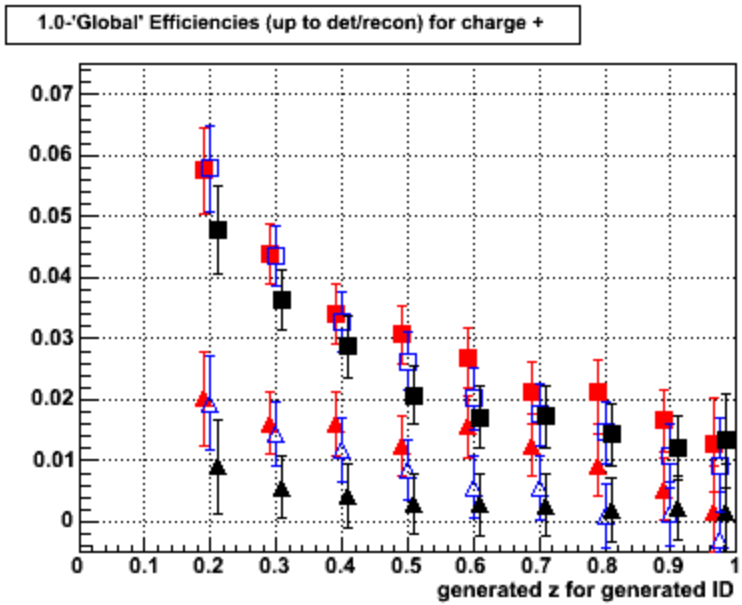
• for systematic uncertainty on ε_{global}: generated 1M particles each for pion/kaon, negative/positive, Fluka/Geisha/no hadronic interaction.

$$N_z^{\pi,\,\mathrm{analysis}} = N_z^{\pi,\,\mathrm{all}}\, \varepsilon_{z,\,\mathrm{global}}^{\pi\,\mathrm{all}}\, \varepsilon_{z,\,\mathrm{analysiscuts}}^{\pi\,\mathrm{all}}$$

single particle MC: 1M per sample, exp55 conditions

- π from FLUKA
- π from GEISHA
- π from no hadronic IA
- K from FLUKA
- K from GEISHA
- K from no hadronic IA



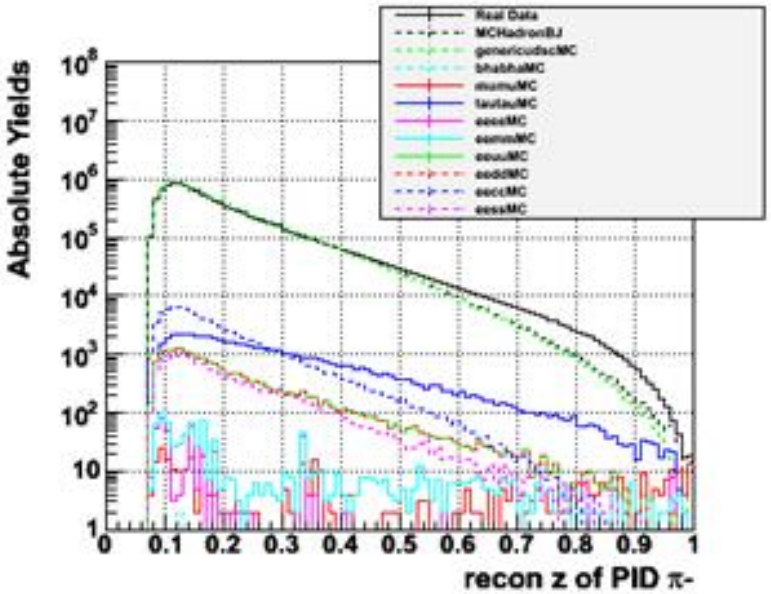


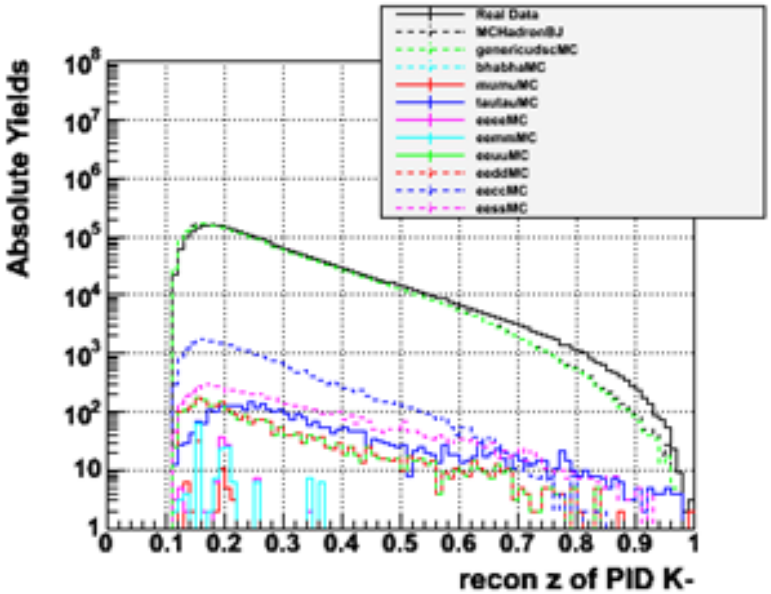
Backup: non-qqbar contributions in MCHadronBJ

Exp 41 stream# 1, 63 runs continuum;

generic uds + charm + 'Nagoya MC' vs real data HadronBJ skim

IPdr < 13mm, IPdz < 4cm; N_SVD(rphi + z) >= 3; Evis > 7 GeV

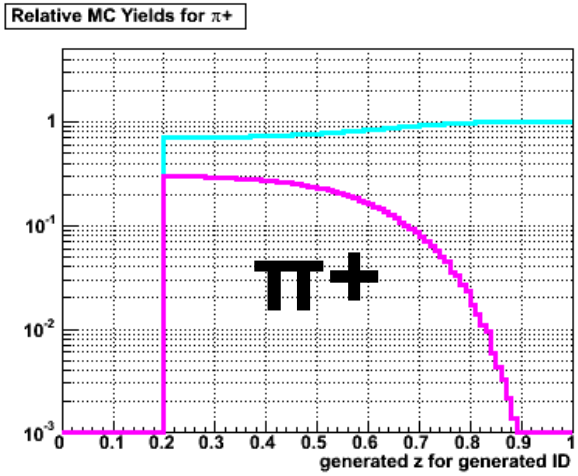


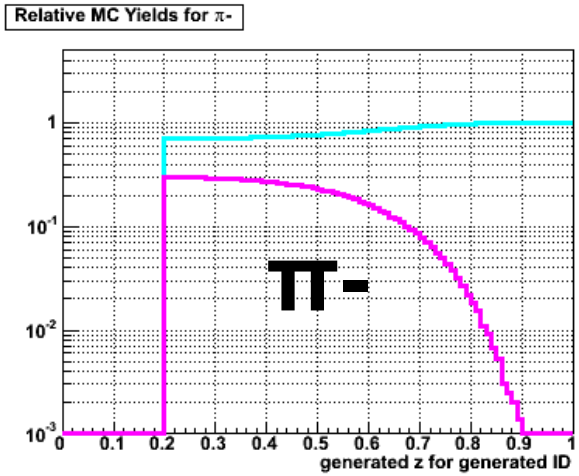


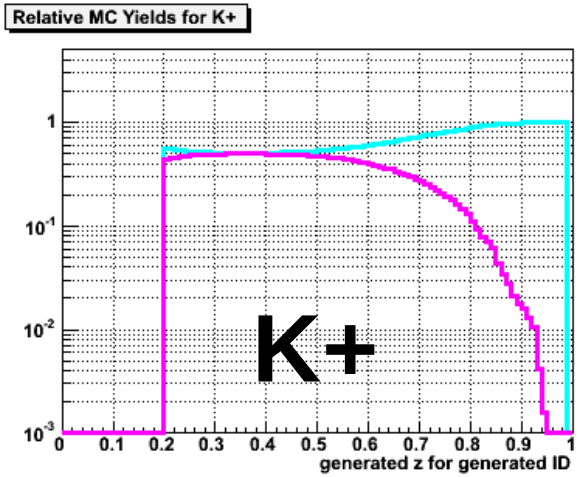
Charged Pion and Kaon Multiplicities from Belle

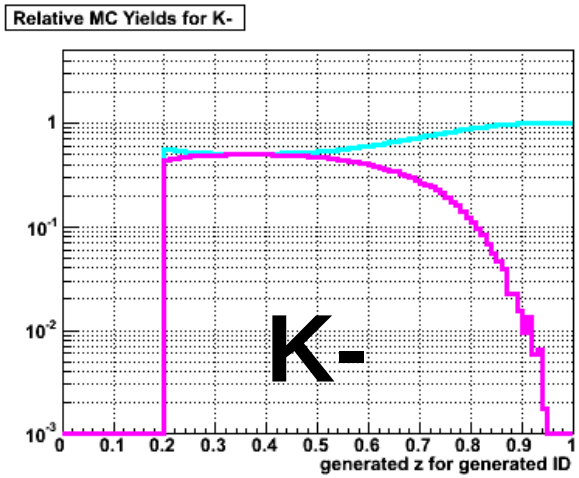
- MC fractions of particles produced in strong or weak decays in e+e- -> qqbar (u,d,s,c) -> X +pi, X + K
- plotted is strong/(strong+weak) and weak/(strong+weak)

strong decay products
weak decay products









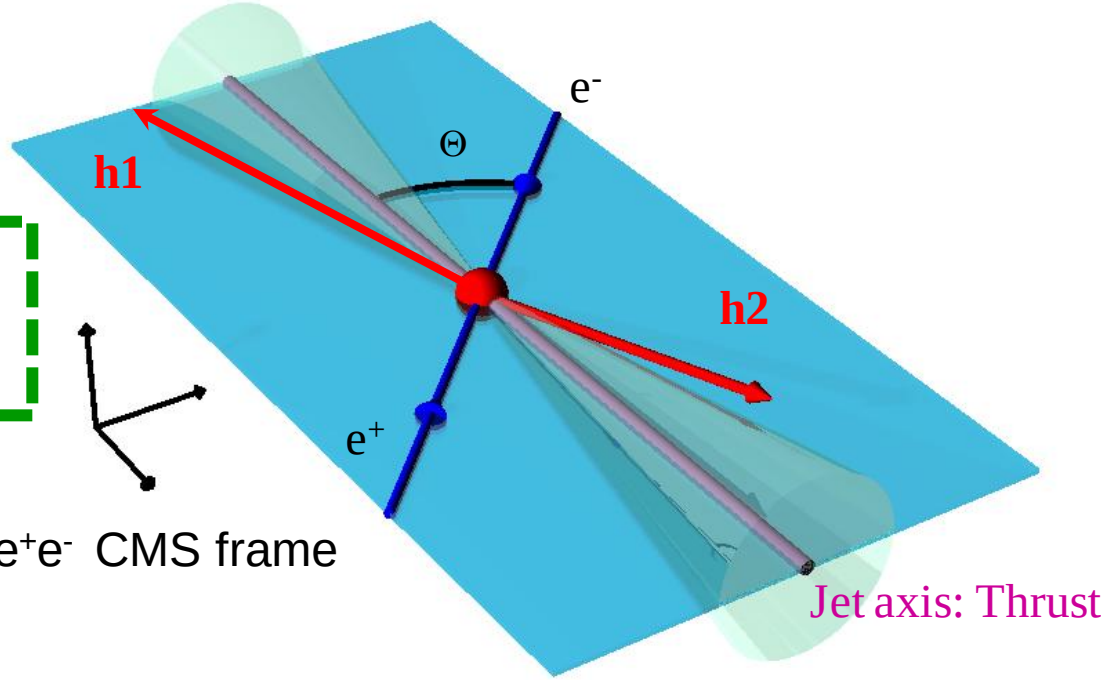
Belle Monte Carlo data, ~500M events

3. Chiral-odd Fragmentation Functions for the extraction of transversity distribution

- To determine transverse quark spin distributions:
  pp, SIDIS measure:
  transversity X **Collins FF:** $\delta q \times H_1^{\perp}$
  or
  transversity X **Interference FF (IFF):** $\delta q \times H_1^{\sphericalangle}$
- **Belle** can measure: **Collins** X **Collins** or **IFF** X **IFF** by identifying two/four hadrons in **two hemispheres** in final state, $e^+e^- \rightarrow$ **(h)(h) X** / $\rightarrow$ **(hh)(hh) X**

Event topology in

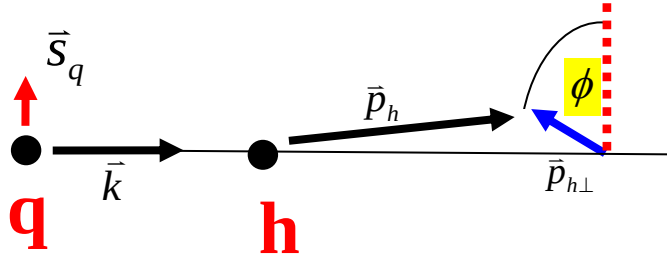
$e^+e^- \rightarrow q\bar{q} \rightarrow$ (h)(h)X



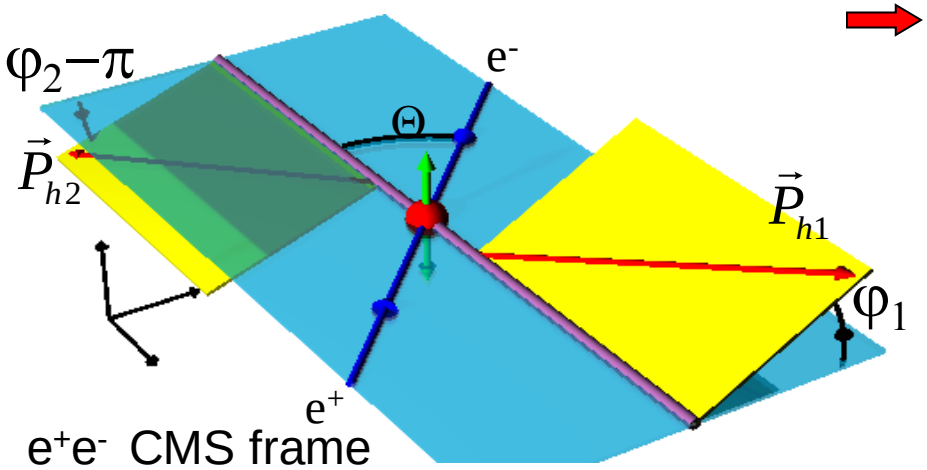
3a. Collins Fragmentation Functions $H_1^{\perp}$ at Belle

- **Collins effect**: J.C. Collins, Nucl. Phys. B396, 161(1993)
  fragmentation of **polarized quark** into **unpolarized hadron**:
  azimuthal dependence of hadron production cross-section $\propto \left(\vec{k} \times \vec{p}_{h\perp}\right) \cdot \vec{s}_q$

$$\propto \sin\phi$$



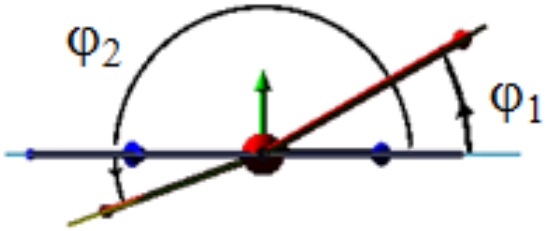
- In **unpolarized e⁺e⁻** quark spin unknown -> Collins effect would average out on single jet -> measure simultaneously in **both hemispheres** (quark spins correlated)



2-hadron inclusive cross section:

$$\frac{d\sigma\left(e^+e^- \to h_1 h_2 X\right)}{d\Omega dz_1 dz_2}$$

$$\propto \sin^2\Theta \, \cos\left(\varphi_1 + \varphi_2\right) H_1^{\perp[1]}(z_1) \bar{H}_1^{\perp[1]}(z_2)$$

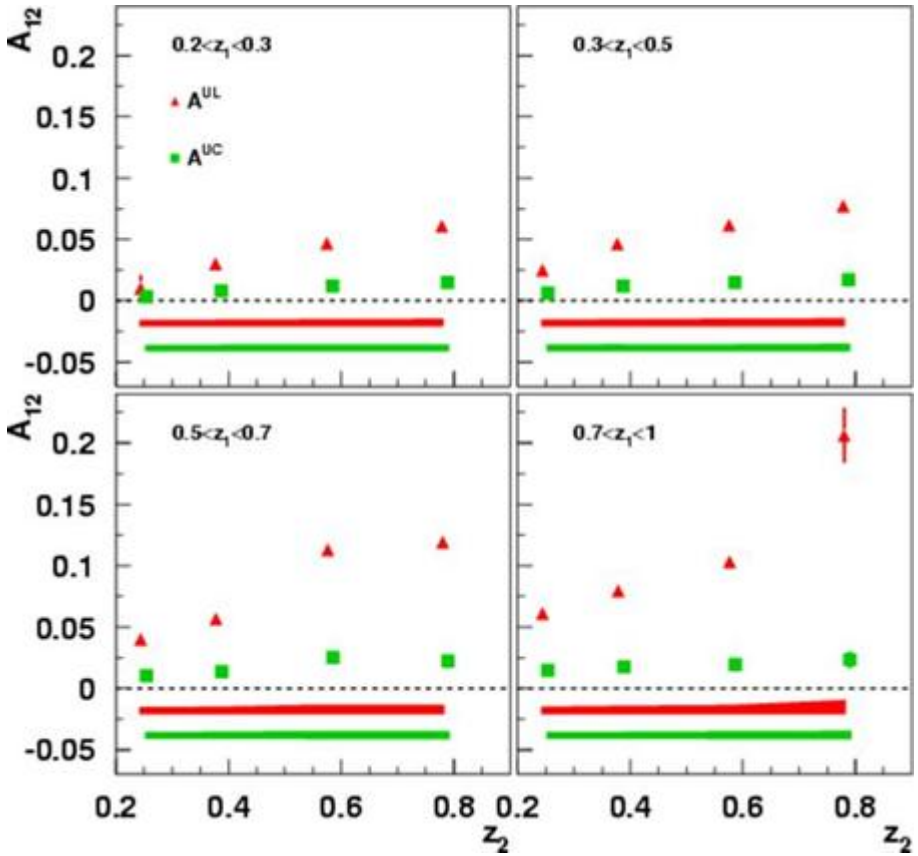


3a. Collins Fragmentation Functions $H_1^{\perp}$ at Belle

- Measure 'double ratios' of **normalized two-pion yields** -> access to favored-disfavored **Collins FF** (transverse spin asymmetries)

First published: Phys. Rev. Lett. 96, 232002 (2006)

Larger data sample: Phys. Rev. D 78, 032011 (2008)



- **first direct measurement** of Collins effect

- **significant, nonzero asymmetries** implying significant spin-dependent effects in

$$e^+e^- \to q\bar{q}\ (q = u, d, s)$$

fragmentation

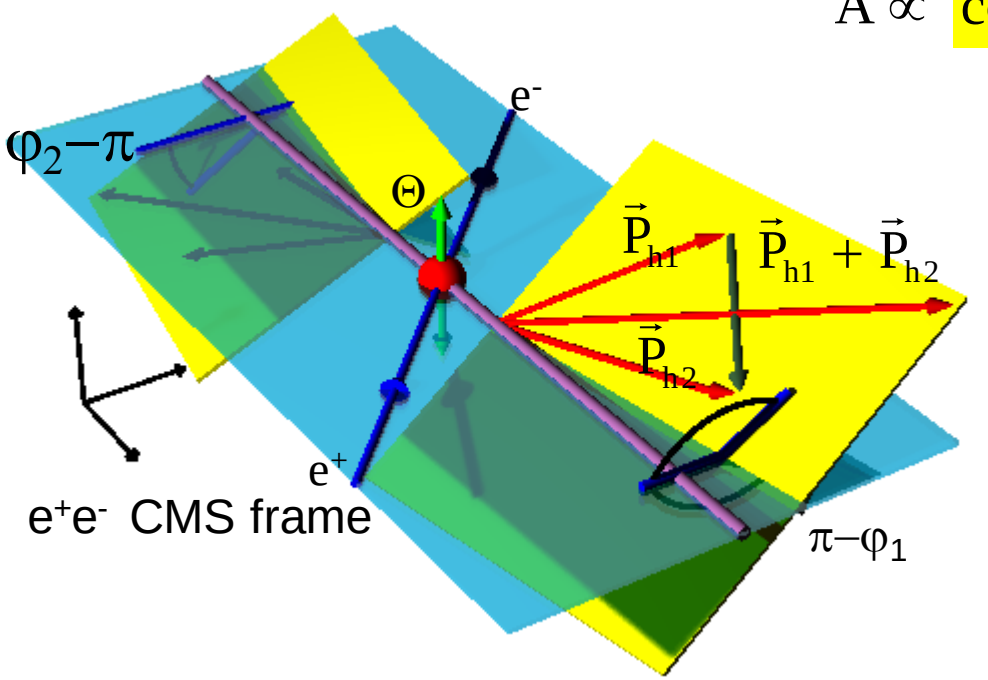
- this measurement used in first **extraction of quark transversity** distributions :

Phys. Rev. D 75, 054032 (2007), Update in Nucl. Phys. Proc. Suppl. 1 91, 98-107 (2009)

3b. Interference Fragmentation Functions $H_1^{\sphericalangle}$ at Belle

- Again asymmetry would **average out** on single jet (**unpolarized initial state** at Belle, quark spin state not known)
- -> measure simultaneous **azimuthal modulation** of **two pion pair yields** in **opposing hemispheres**

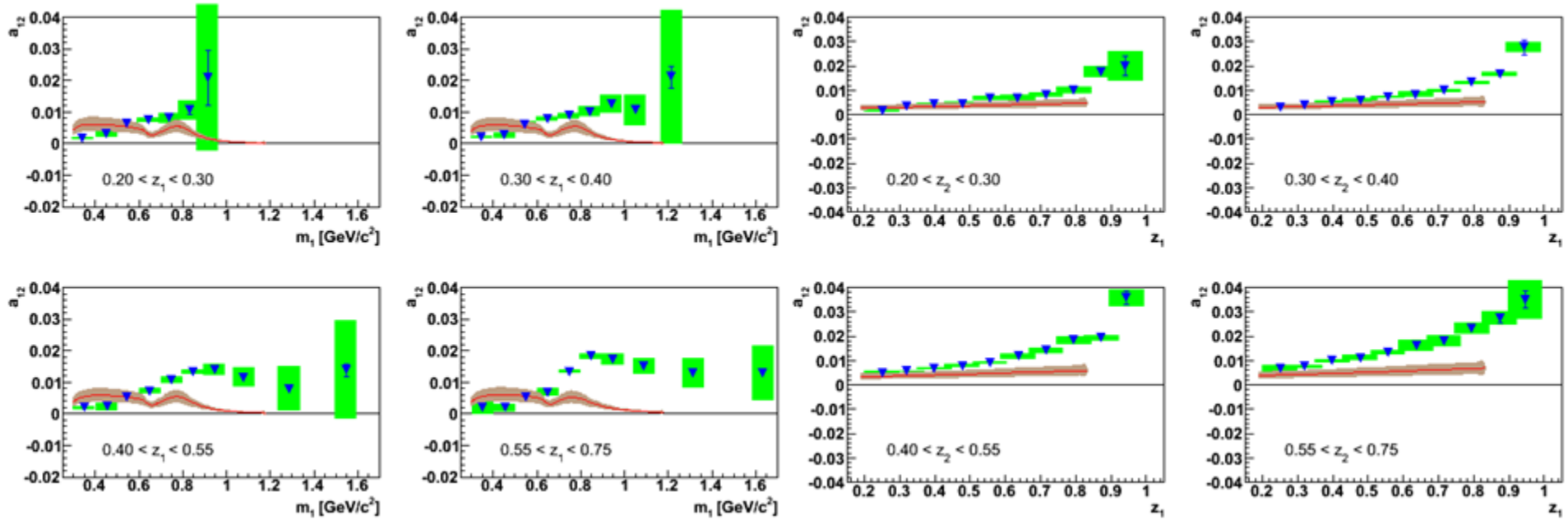
$$A \propto \cos(\varphi_1 + \varphi_2)\, H_1^{\sphericalangle}(z_1, m_1)\, \overline{H}_1^{\sphericalangle}(z_2, m_2)$$



Model predictions for **IFF** from:

Jaffe et al PRL **80,** (1998)
Bacchetta et al. PRD **79, (2009)**

3b. Interference Fragmentation Functions $H_1^{\sphericalangle}$ at Belle- Model predictions



Red line: theory prediction + uncertainties, Bacchetta et al, PRD 79 (2009)

Blue points: data (after sign change and cos moment, decay angular moments adjustment)

- Model prediction: **IFF** larger around invariant mass of correlating resonance, then fall-off
- Measured: ~ **constant IFF at high invariant masses**, **raising IFF** for **increasing fractional energy**