Global Analysis of Fragmentation Functions

Hiroyuki Kawamura (KEK)

Workshop on Fragmentation Functions and QCD
RIKEN, Nov.9-11, 2012

Work in collaboration with Masanori Hirai (Tokyo Univ. of Science)
Shunzo Kumano (KEK)
Koichi Saito (Tokyo Univ. of Science)

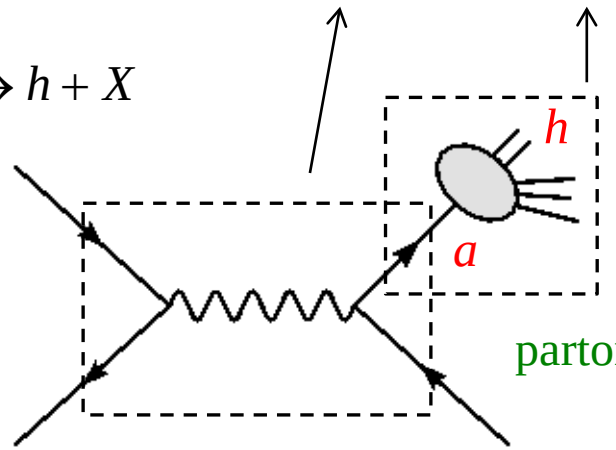
Introduction

- Fragmentation Functions describe the transition of a parton into hadrons.
  - key ingredients for calculating the inclusive production of hadrons.

cross section $$d\sigma^h = d\hat{\sigma}^a \otimes D_a^h(z)$$

z : mom. fraction of outgoing hadron

ex. $e^+ + e^- \to h + X$



fragmentation function (FF)

partonic cross section

- FFs are non-perturbative quantities and need to be extracted from experimental data. "Global analysis"
  - In this talk, we report on some preliminary results of our new analysis as an update of the previous one (HKNS07) with the new Belle data.

Introduction

- Precise knowledge of FFs is useful for various phenomenology including:
  - extraction of pol. PDFs via SIDIS at Hermes/Compass/Jlab, and high-Pt meson production in pol. pp collision at RHIC.
  - investigation of QGP states via high-Pt meson production at RHIC/LHC.
  - study of semi-hard QCD dynamics (hadronization etc.) .

- Global analysis of FFs have been performed by several collaborations.

AESSS, C. Aidala, F. Ellinghaus, R. Sassot, J. Seele, M. Stratmann, arXiv:1009.6145.
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DSS, D. de Florian, R. Sassot and M. Stratmann, PRD75 (2007) 114010; PRD76 (2007) 074033.
DSV, D. de Florian, M. Stratmann and W. Vogelsang, PRD57 (1998) 5811.
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KKP, B. A. Kniehl, G. Kramer and B. Pötter, NPB582 (2000) 514.
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S. Kretzer, PRD62 (2000) 054001.

ref. FF database : http://www2.pv.infn.it/~radici/FFdatabase/

Global χ^2 analysis of FFs

- How to determine FFs from experimental data (e^+e^- collision).

FFs at the initial scale : $D_i^h\left(\xi, Q_0^2\right) = N_i^h x^{\alpha_i^h}(1-x)^{\beta_i^h}(1+\gamma_i^h(1-x)^{\delta_i^h})$ $\quad Q_0 \sim 1\text{GeV}$

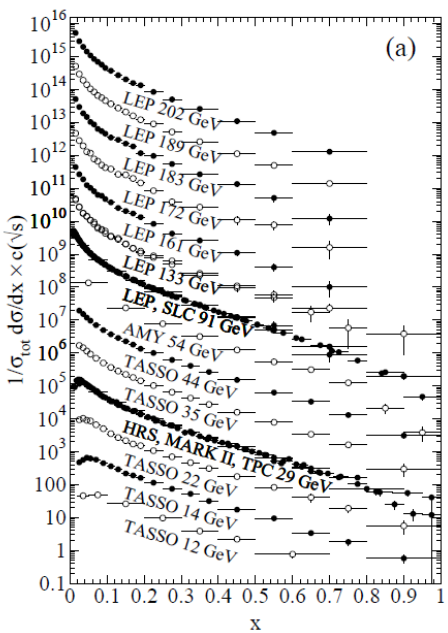
DGLAP evolution eq.: $$\frac{\partial}{\partial \ln Q^2} D_j^h(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi}\sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{z}{\xi}, \alpha_s\right) D_i^h\left(\xi, Q^2\right)$$

observable

$$\frac{1}{\sigma_{tot}}\frac{d\sigma^h}{dz} = \sum_i \int_z^1 \frac{d\xi}{\xi} C_i\left(\frac{z}{\xi}, Q^2\right) D_i^h\left(\xi, Q^2\right)$$

$z = 2E_h / Q$

$Q \sim \sqrt{s}$



exp. data

$$\chi^2 = \frac{(d\sigma^{data} - d\sigma^{theo})^2}{(\delta d\sigma^{data})^2}$$

$\{N_i^h, \alpha_i^h, \beta_i^h, \gamma_i^h, \delta_i^h\}$

Global χ^2 analysis of FFs

- Experimental data

(1) $e^+ + e^- \to h + X$

☺ high precision data (LEP), free from PDFs, easy to handle.

☻ poor constraints on $D^h_{q-\bar{q}}$ & D^h_g . LEP data only sensitive to the singlet part.

(2) $e^\pm + P \to e^\pm + h + X$

☺ sensitive to flavour decomposition & $D^h_{q-\bar{q}}$.

☻ poor constraints on D^h_g , PDF dependence, 1 extra convolution integral.

(3) $P + P(\bar{P}) \to h(P_T) + X$

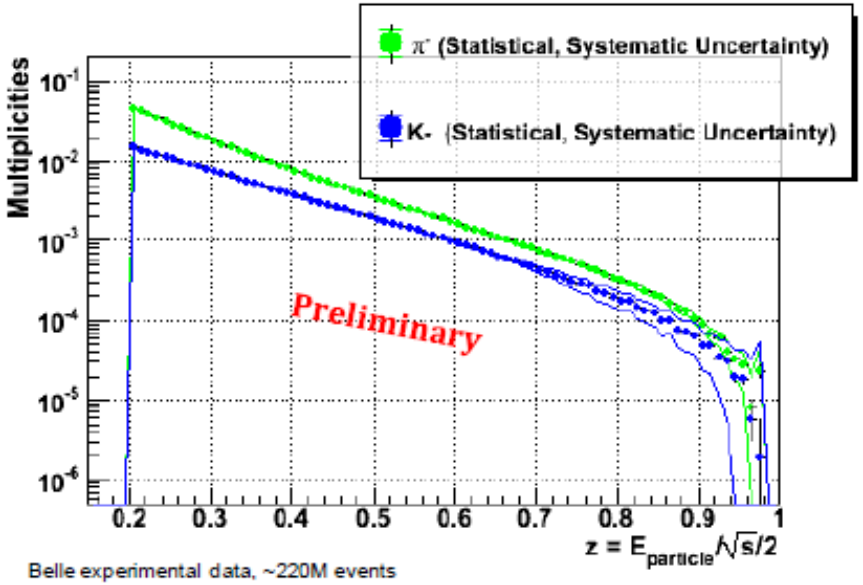
☺ sensitive to $D^h_g, D^h_{q-\bar{q}}$

☻ PDF dependence, 2 extra convolution integrals.

- HKNS07 uses (1) only. Here, we again take the same simple but straightforward way, expecting the new Belle data to be helpful to overcome the cons.

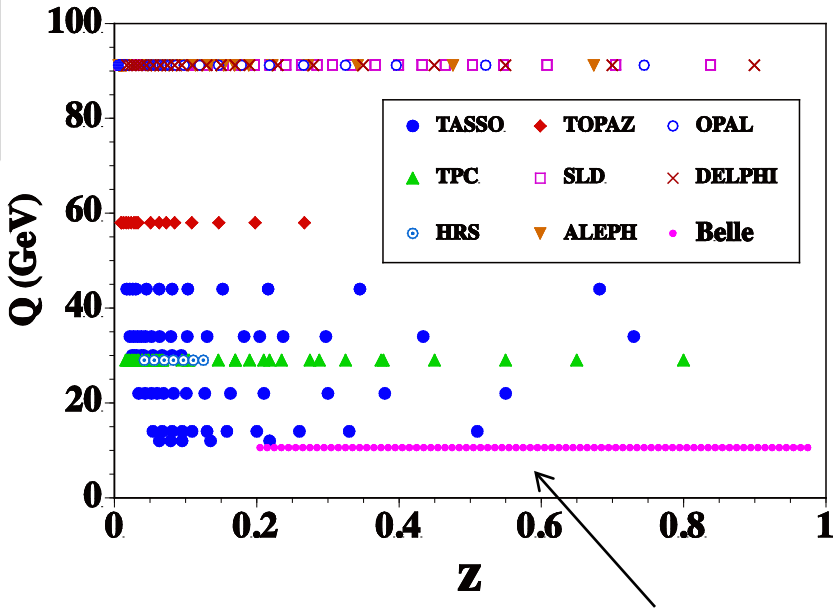
cf. DSS07 uses (1) & (2) & (3), while AKK08 uses (1) & (3), employing more sophisticated convolution techniques.

New data from Belle



Leitgab DIS2012

Kinematical coverage (pion)



Belle data

Belle data covers the large-z, small-Q region with high precision.

→ Constraints on FFs at large z & gluon FF (& q-qbar decomposition?).

Analysis Method

- Parametrization
- Momentum sum rule
- DGLAP evolution
- Kinematical cut
- Error estimation, etc.

Analysis Method (1)

- Parametrization

π^+ 5 independent FFs. fixed to meet the mom. sum rule. (see below)

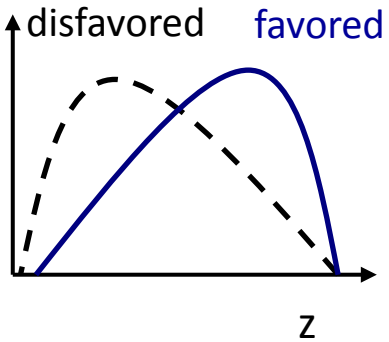
u & $\bar{d}$ favored FFs: $D_u^{\pi^+}(z,Q_0^2) = D_{\bar{d}}^{\pi^+}(z,Q_0^2) = N_u^{\pi^+} z^{\alpha_u^{\pi^+}} (1-z)^{\beta_u^{\pi^+}}$

$\bar{u}$ & d & s & $\bar{s}$ disfavored FFs: $D_{\bar{u}}^{\pi^+}(z,Q_0^2) = D_d^{\pi^+}(z,Q_0^2) = D_s^{\pi^+}(z,Q_0^2) = D_{\bar{s}}^{\pi^+}(z,Q_0^2) = N_{\bar{u}}^{\pi^+} z^{\alpha_{\bar{u}}^{\pi^+}} (1-z)^{\beta_{\bar{u}}^{\pi^+}}$

gluon FF: $D_g^{\pi^+}(z,Q_0^2) = N_g^{\pi^+} z^{\alpha_g^{\pi^+}} (1-z)^{\beta_g^{\pi^+}}$

charm FF: $D_c^{\pi^+}(z,m_c^2) = D_{\bar{c}}^{\pi^+}(z,m_c^2) = N_c^{\pi^+} z^{\alpha_c^{\pi^+}} (1-z)^{\beta_c^{\pi^+}}$

bottom FF: $D_b^{\pi^+}(z,m_b^2) = D_{\bar{b}}^{\pi^+}(z,m_b^2) = N_b^{\pi^+} z^{\alpha_b^{\pi^+}} (1-z)^{\beta_b^{\pi^+}}$



Initial scale(s):

$$n_f = \begin{cases} 3, & Q_0^2 < Q^2 < m_c^2 \\ 4, & m_c^2 < Q^2 < m_b^2 \\ 5, & m_b^2 < Q^2 < m_t^2 \\ 6, & m_t^2 < Q^2 \end{cases}, \quad \begin{pmatrix} Q_0^2 = 1\ \mathbf{GeV}^2 \\ m_c = 1.43\ \mathbf{GeV} \\ m_b = 4.3\ \mathbf{GeV} \end{pmatrix}$$

Analysis Method (2)

K^+ 6 independent FFs.

u favored FFs: $D_u^{K^+}(z,Q_0^2) = N_u^{K^+} z^{\alpha_u^{K^+}} (1-z)^{\beta_u^{K^+}}$

$\bar{s}$ favored FFs: $D_{\bar{s}}^{K^+}(z,Q_0^2) = N_{\bar{s}}^{K^+} z^{\alpha_{\bar{s}}^{K^+}} (1-z)^{\beta_{\bar{s}}^{K^+}}$

$m_s \neq m_u$

$\bar{u}$ & d & $\bar{d}$ & s disfavored FFs: $D_{\bar{u}}^{K^+}(z,Q_0^2) = D_d^{K^+}(z,Q_0^2) = D_{\bar{d}}^{K^+}(z,Q_0^2) = D_s^{K^+}(z,Q_0^2) = N_{\bar{u}}^{K^+} z^{\alpha_{\bar{u}}^{K^+}} (1-z)^{\beta_{\bar{u}}^{K^+}}$

gluon FF: $D_g^{K^+}(z,Q_0^2) = N_g^{K^+} z^{\alpha_g^{K^+}} (1-z)^{\beta_g^{K^+}}$

charm FF: $D_c^{K^+}(z,m_c^2) = D_{\bar{c}}^{K^+}(z,m_c^2) = N_c^{K^+} z^{\alpha_c^{K^+}} (1-z)^{\beta_c^{K^+}}$

bottom FF: $D_b^{K^+}(z,m_b^2) = D_{\bar{b}}^{K^+}(z,m_b^2) = N_b^{K^+} z^{\alpha_b^{K^+}} (1-z)^{\beta_c^{K^+}}$

- Momentum sum rule

$$\sum_h M_i^h \equiv \sum_h \int_0^1 z D_i^h(z,Q_0^2)dz = \sum_h N_i^h B(\alpha_i^h + 2, \beta_i^h + 1) \Rightarrow \boxed{0 < M_i^h < 1}$$

Actually, we take M_i^h instead of N_i^h as the fitting parameters.

Analysis Method (3)

- DGLAP evolution

$$\frac{\partial}{\partial \ln Q^2} D_j^h(z,Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \sum_i \int_z^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{z}{\xi}, \alpha_s\right) D_i^h\left(\xi, Q^2\right)$$

— Scale evolution is carried out in z-space using the Gauss-Legendre quadrature.

ex. Flavour NS FF $\qquad t \equiv \log Q^2$

$$D^h_{q_i^-}(x_m, t_{\ell+1}) = D^h_{q_i^-}(x_m, t_\ell) + \Delta t \frac{\alpha_s(t_\ell)}{2\pi} \frac{1-x_m}{2} \sum_j \sum_{k=1}^{N_{GL}} w_k \frac{1}{x_k} P_{q_j q_i}(x_k) D^h_{q_j^-}\left(\frac{x_m}{x_k}, t_\ell\right).$$

Hirai, Kumano, Comput.Phys.Commun.183(2012)1002

- Heavy quarks threshold

— We change the flavor number in the beta function and DGLAP equation at $Q^2 = m_c^2, m_b^2$, while their pair creation is taken into account at $Q^2 = 4m_c^2, 4m_b^2$.

Analysis Method (4)

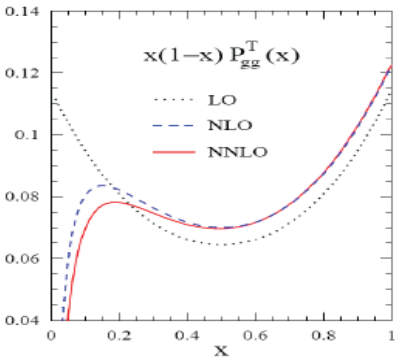
Stratmann, Vogelsang (98)
Albino, Kniehl, Kramer (08)

- Small-z cut for experimental data

$$z > 0.1\ (Q < M_Z)\ ,\ \ z > 0.05\ (Q = M_Z)$$

to avoid (1) finite hadron mass effects $\sim 4m_h^2/(sz^2)$
(2) large negative NLO splitting functions.

$$P^{T,(1)}(z) \sim 4C_A C_F \log^2 z / z$$



Moch, Vogt (06)

- Error estimation by the Hessian method.

$$\Delta\chi^2(\xi) = \chi^2(\hat{\xi} + \delta\xi) - \chi^2(\hat{\xi}) = \sum_{i,j} H_{ij}\delta\xi_i\delta\xi_j,$$

$$\Delta\chi^2 \sim N_{\text{par.}} \leftrightarrow 1\sigma$$

$$[\delta D_i^h(z)]^2 = \Delta\chi^2 \sum_{j,k} \left(\frac{\partial D_i^h(z,\xi)}{\partial \xi_j}\right)_{\hat{\xi}} H^{-1}_{jk} \left(\frac{\partial D_i^h(z,\xi)}{\partial \xi_k}\right)_{\hat{\xi}}$$

– Hessian method is O.K. if $\chi 2$ function is a quadratic function of the fit parameters around the minimum.
– More robust error estimation can be done by the Langarnge multiplier method.

Epele, Llubarof f, Sassot, Stratmann, PRD86(2012)074028

Preliminary Result

(1) Pion Fragmentation Functions

Results for π^+ fit

- Total number of data = 342 (Belle 78)

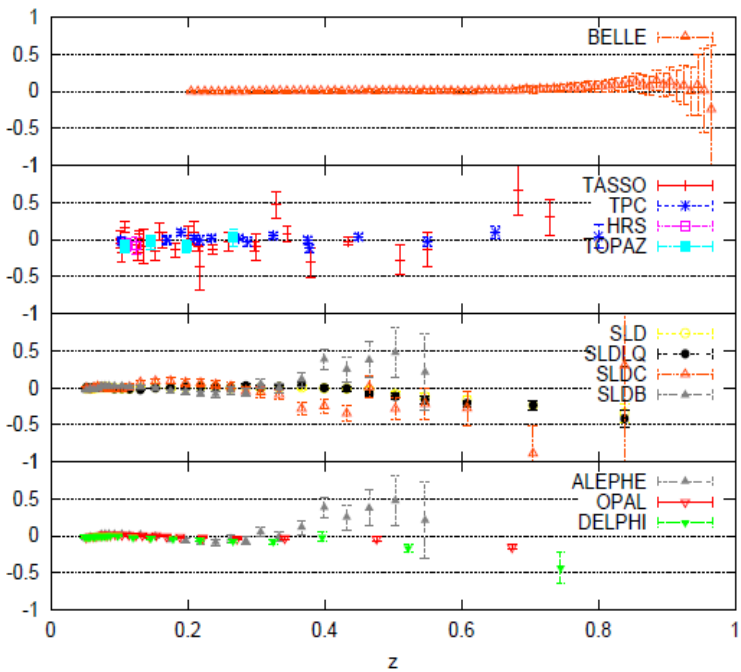
$\Lambda_{QCD}^{n_f=4} = \mathbf{0.220}$ (LO), $\mathbf{0.323}$ (NLO)

— To meet the momentum sum rule, we fixed a parameter for the gluon FF $\beta_g^{\pi^+} = 8$.

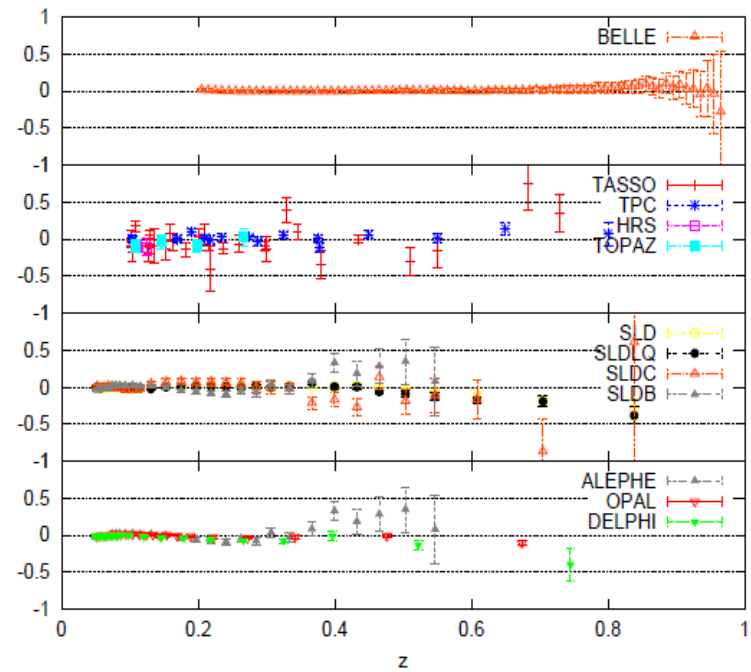
— Good convergences for both of the LO & NLO FFs.

$[D(z,Q_0^2) = Nz^{\alpha}(1-z)^{\beta}]$

LO $\chi^2 = 612.9, \quad \chi^2/(d.o.f) = 1.87$



NLO $\chi^2 = 498.2, \quad \chi^2/(d.o.f) = 1.51$

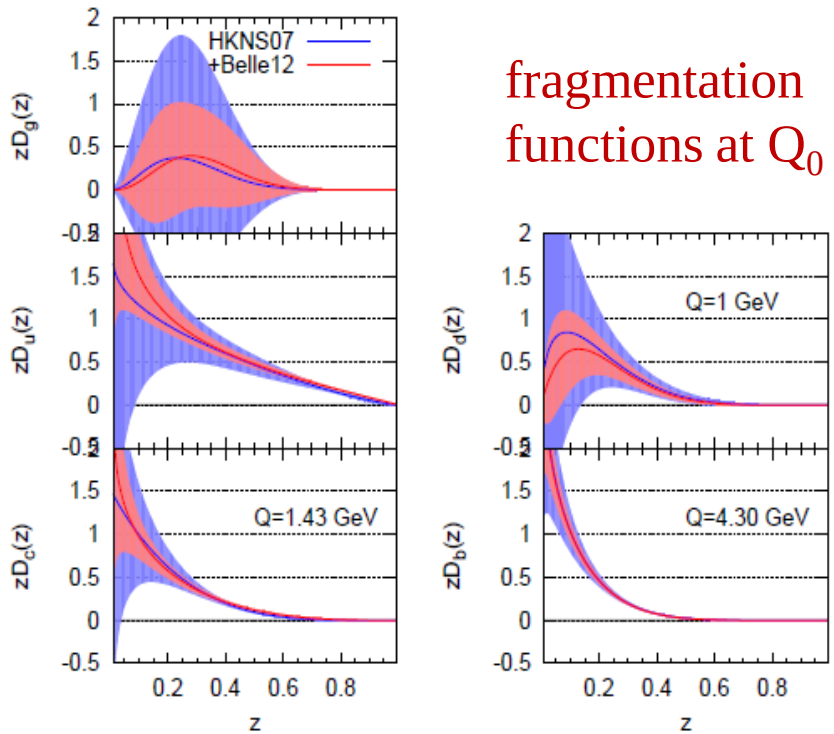


(Data-Theory)/Theory

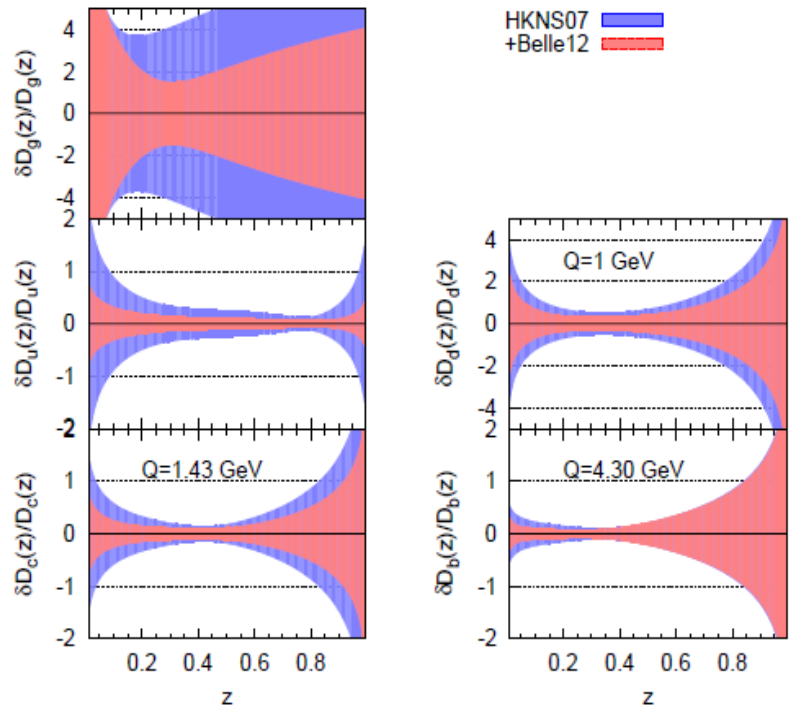
Results for π^+ FFs (LO)

- LO pion FFs with uncertainties

fragmentation functions at Q_0



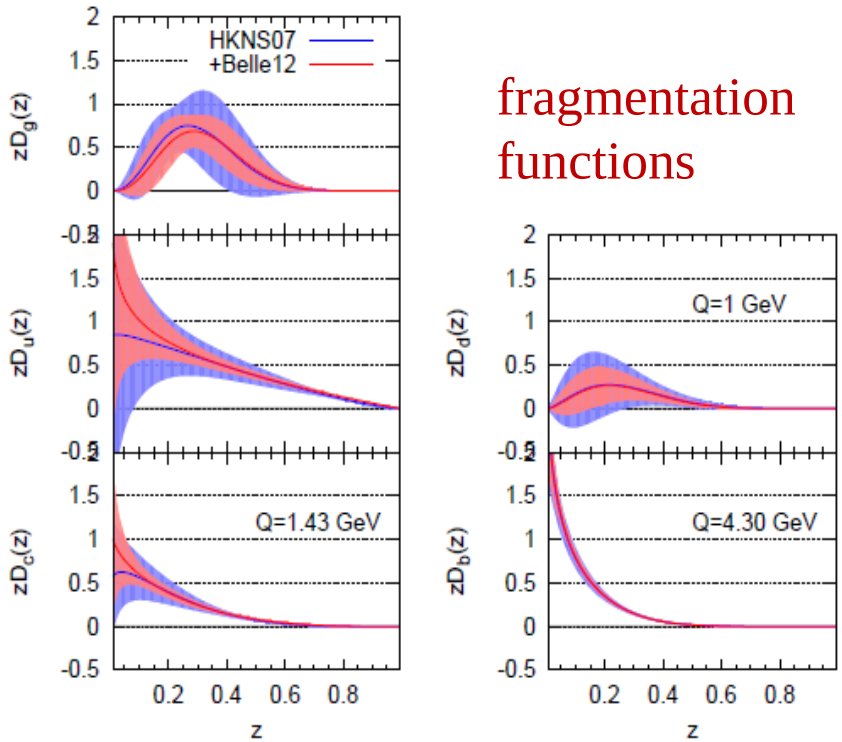
relative uncertainties



- Uncertainty of the gluon FF significantly reduced.
- Not only that, errors of other FFs are also reduced.
- Best fit results of the u-quark, d-quark FFs changed, while the sum of them does not seem to change so much.

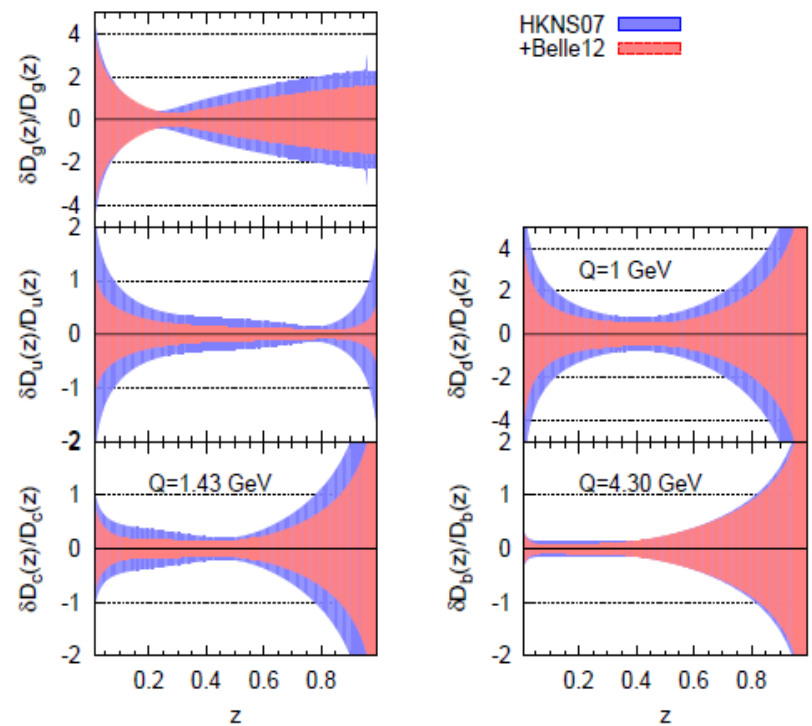
Results for π^+ FFs (NLO)

- NLO pion FFs with uncertainties



fragmentation functions

relative uncertainties



- Uncertainty of the gluon FF reduced.
- Uncertainty of other FFs are also reduced except the b-quark FF.
- Best fit results of u-quark FF enhanced in the small-z region.

Preliminary Result

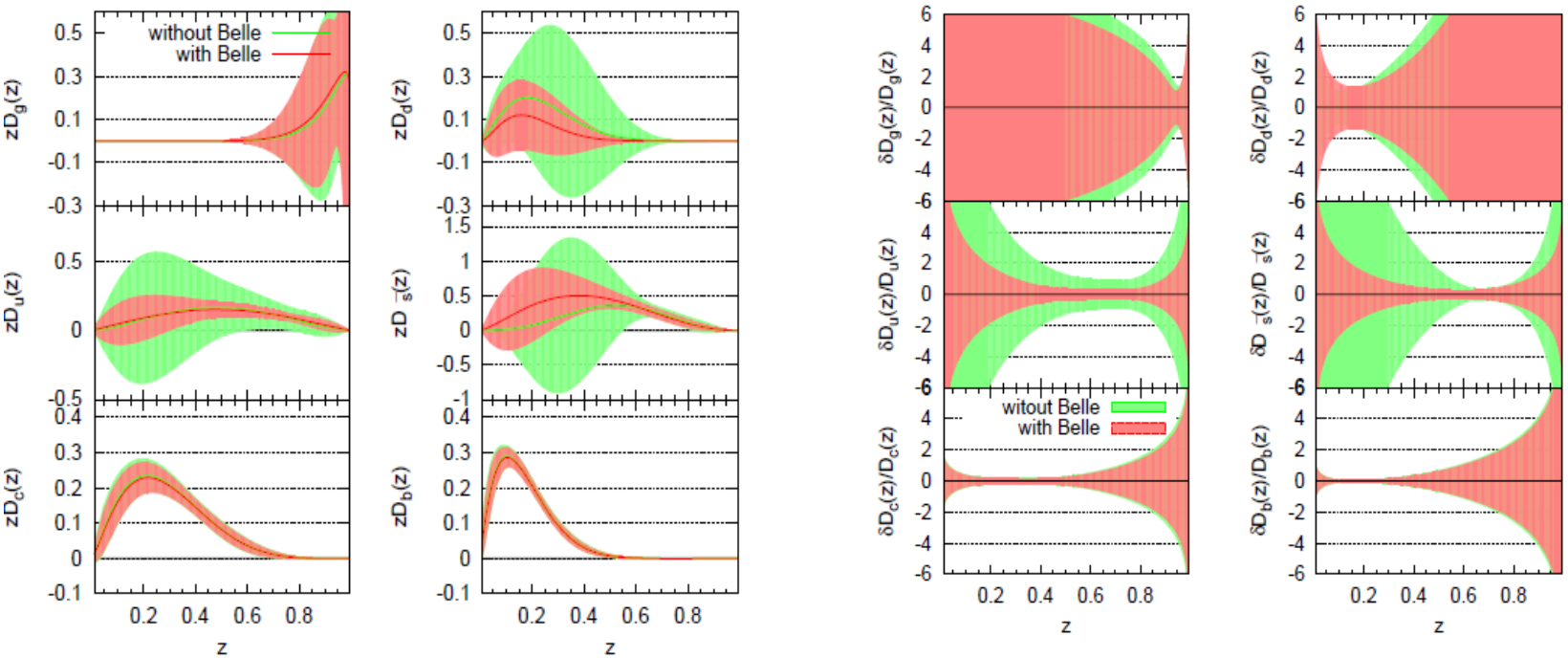
(2) Kaon Fragmentation Functions

Results of K^+ FFs (LO)

- Total number of data = 322 (Belle 77)

$[D(z,Q_0^2) = Nz^{\alpha}(1-z)^{\beta}]$

  - As in the pion case, we take a fixed value for the gluon parameter: $\alpha_g^{\pi^+} = 10$ to ensure $\beta_g^{\pi^+} > 0$.



- Even after including the Belle data, the gluon FF was poorly determined.
- sbar (favoured) & d (disfavoured) changes, while others are almost same.
- Uncertainties of u, sbar (both favoured) are reduced.
- The result does not change if we change $\alpha_g^{\pi^+}$.

Summary

- The new data from Belle cover a wide range of z (0.8>z>0.2) with a very high precision at a low scale Q = 10.58GeV.

  → They would be helpful to constrain the gluon FF as well as the quark FFs through mixing by the scale evolution.

- We studied the impact of the new Belle data on the determination of the fragmentation functions.

π^+

- Uncertainty of the gluon FF significantly reduced (LO & NLO)
- Some of the quark FFs change within the previous uncertainties (LO & NLO).

K^+

- Almost no impact on the gluon FF (LO & NLO)
- Some of the quark FFs changed within the previous uncertainties (LO & NLO).

- An update of HKNS07 with the final data is coming soon.