Introduction on spin-dependent fragmentation and evolution of TMDs

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Workshop on Fragmentation Functions and QCD 2012
RIKEN, Wako, Japan, November 9–11, 2012

Outline:

- Introduction
  - Spin-dependent fragmentation functions
  - Transverse momentum dependent distributions
- TMD factorization
  - Evolution of TMDs: basic idea
  - Evolution of Sivers function
    - Difference between SIDIS and DY regarding sign change
  - Evolution of Collins function
- Comments on phenomenology
- Summary

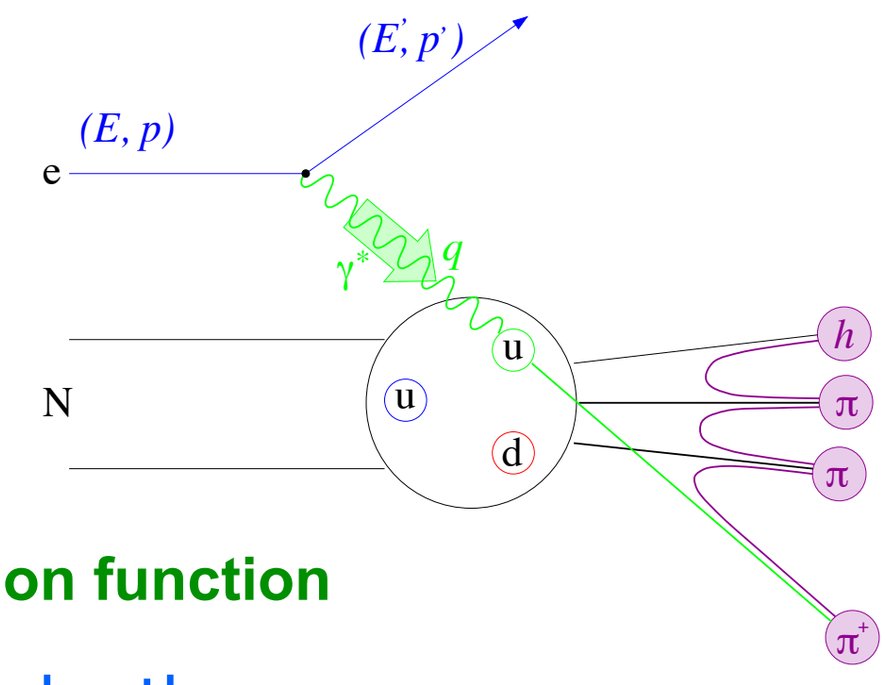
Parton distribution and fragmentation function

- One of the major goal of hadron physics is to understand hadron structure:
  - how quarks and gluons distributed inside the hadron
  - how quarks and gluons are formed into hadron
- One of the major tool to extract these information is through the high energy scattering experiments, by relying on QCD factorization

PDF: parton distribution function

$$d\sigma \sim \phi(x) \otimes \hat{\sigma} \otimes D(z)$$

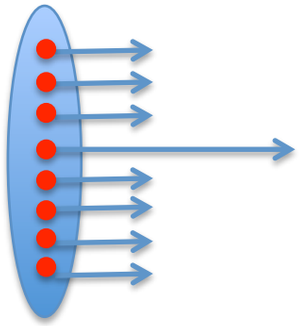
FF: fragmentation function



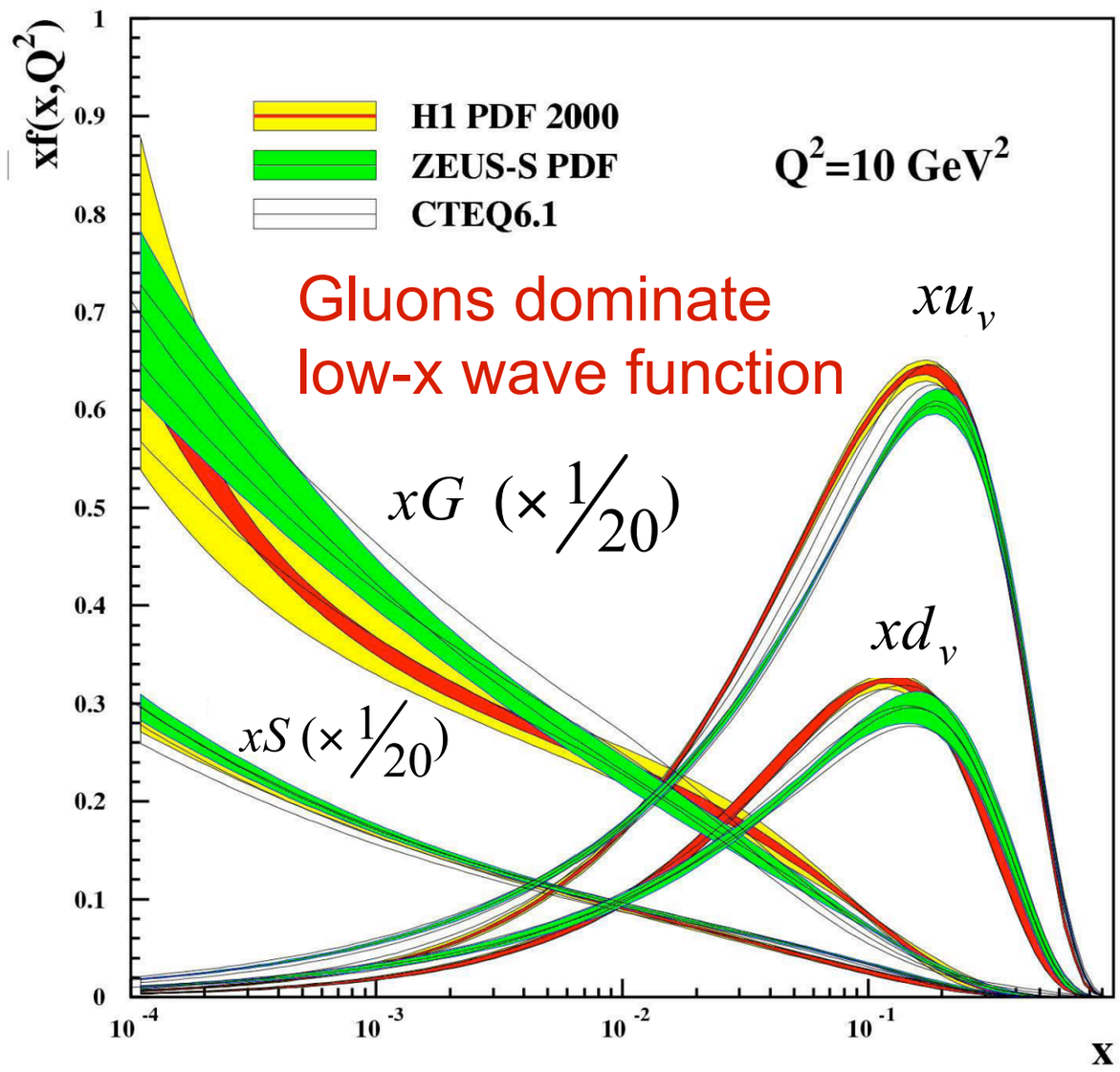
- PDFs and FFs are closely related to each other
  - The better extraction of one could lead to better understanding of the other

The status of collinear PDFs and FFs

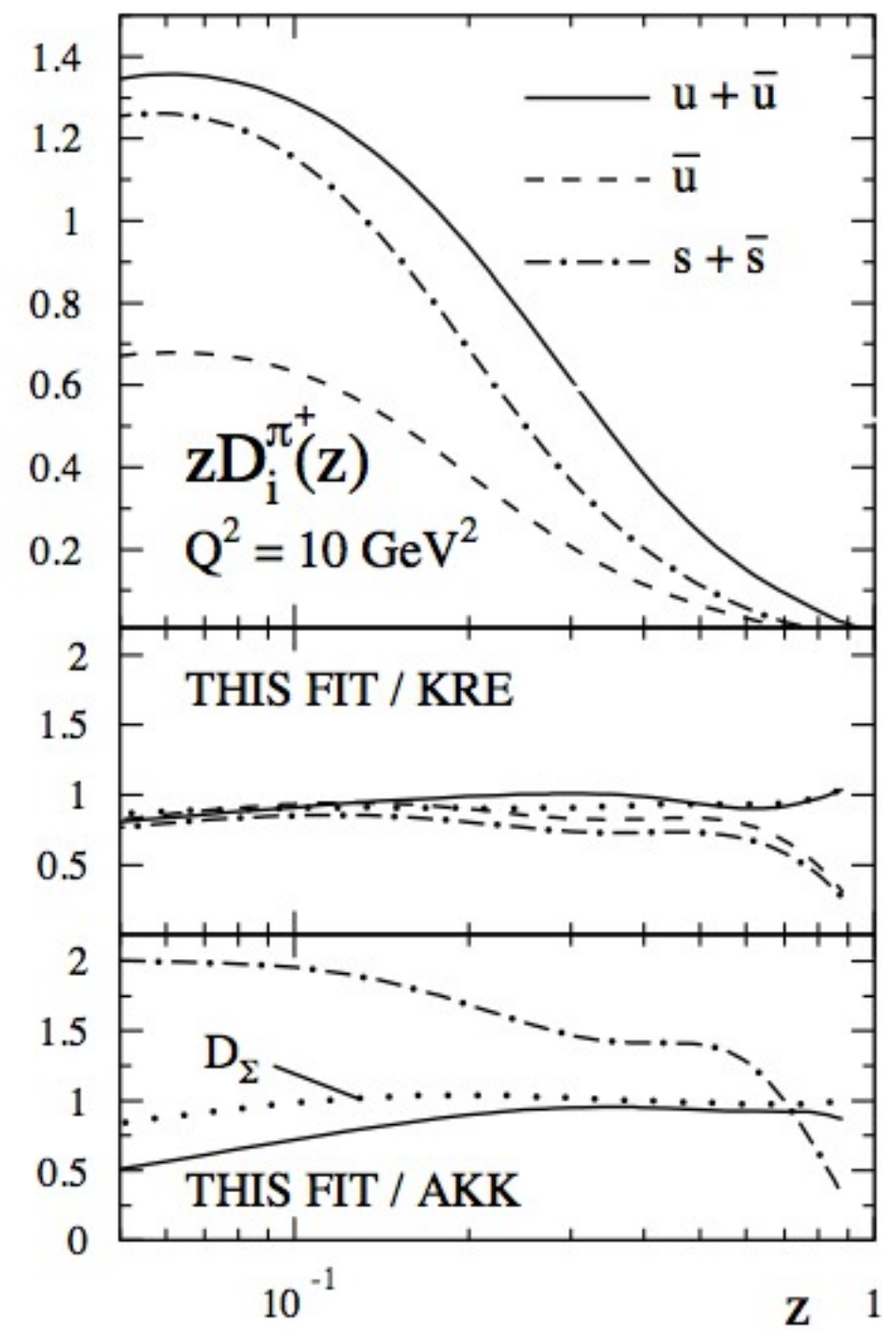
- The collinear PDFs and FFs are widely studied



$$p_a = x_a P_A$$

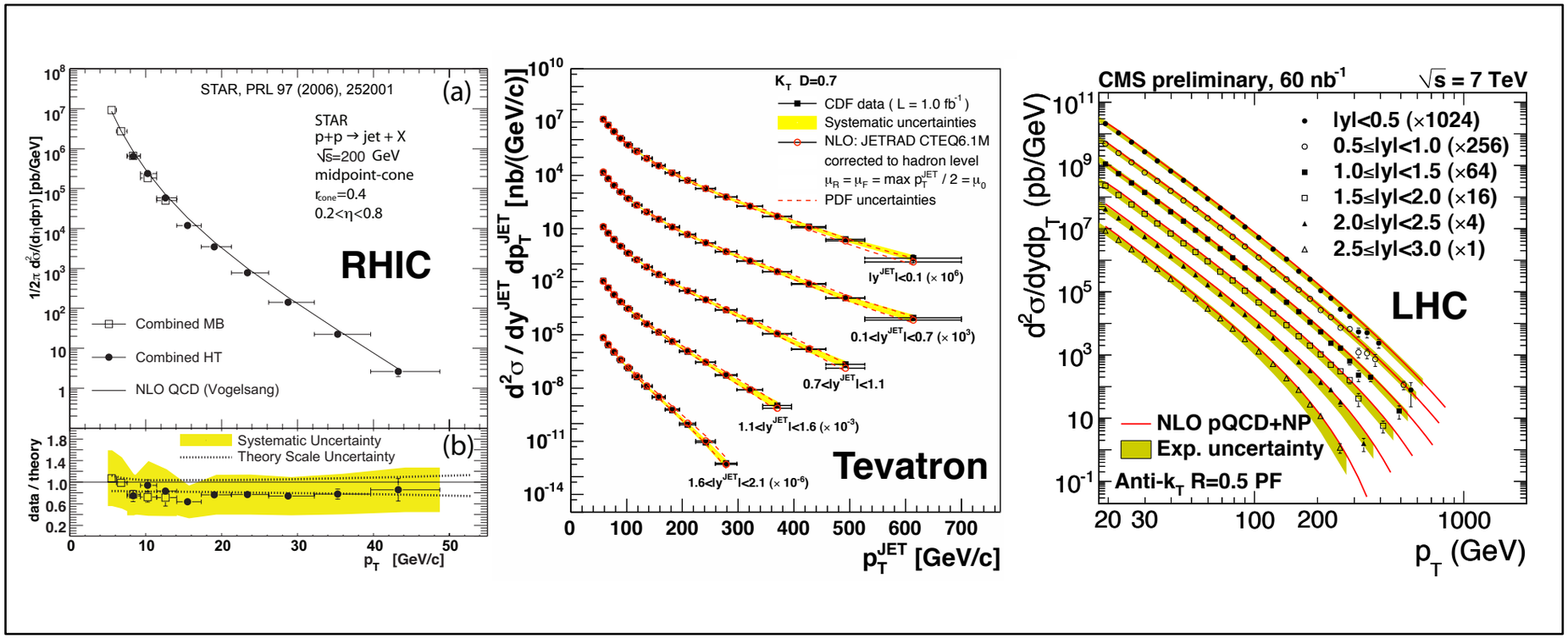


DSS parametrization

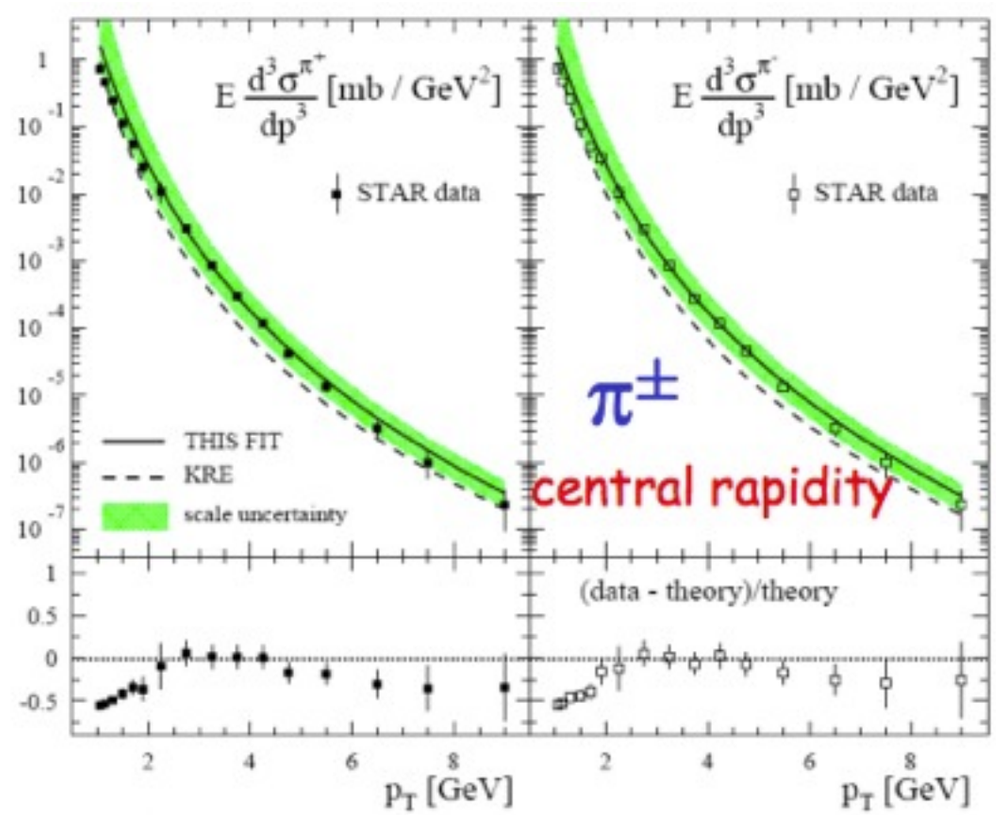


pQCD with PDFs and FFs: they generally work very well

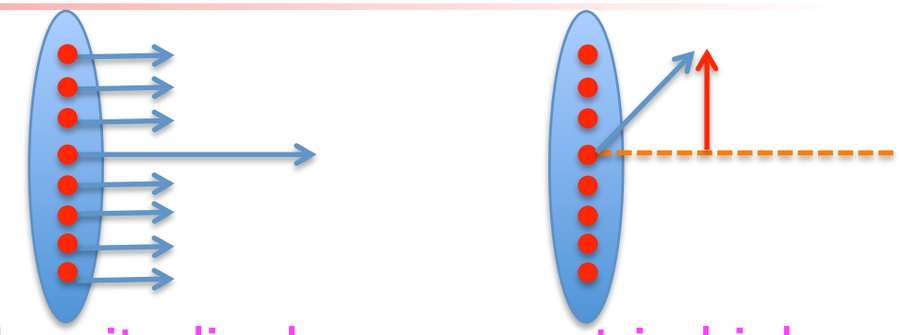
- Jet cross section (p+p→jet+X): only PDFs



- Involving hadron
  - both PDFs and FFs



Going beyond the current picture: two ways - I



- Beyond the collinear PDFs and FFs
  - probe also parton the transverse momentum
  - since parton kt is always much smaller than the longitudinal component in high energy experiments, one typically need "transverse spin" to correlate with the parton kt
  - kt-dependent PDFs and FFs are usually involving spin vector, either the hadron spin or the parton spin

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

parton distribution function

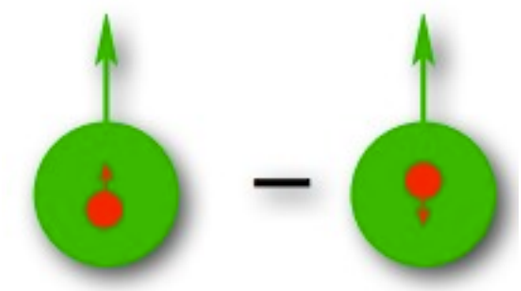
quark pol.

U	L	T
D_1		$H_1^\perp$

Fragmentation function

Three important TMDs will be addressed

- Transversity: survive kt-integration
  - a distribution of transversely polarized quark in a transversly polarized hadron

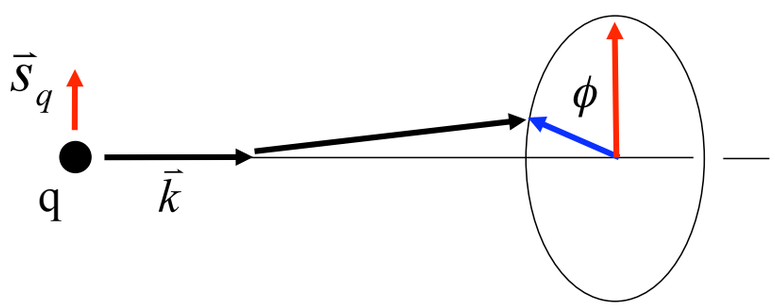


- Sivers function: change sign from SIDIS to DY

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \equiv f_{q/h}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/h^\uparrow}(x,k_\perp)\,\vec{S}\cdot\hat{p}\times\hat{\mathbf{k}}_\perp$$

- Collins function: universal function

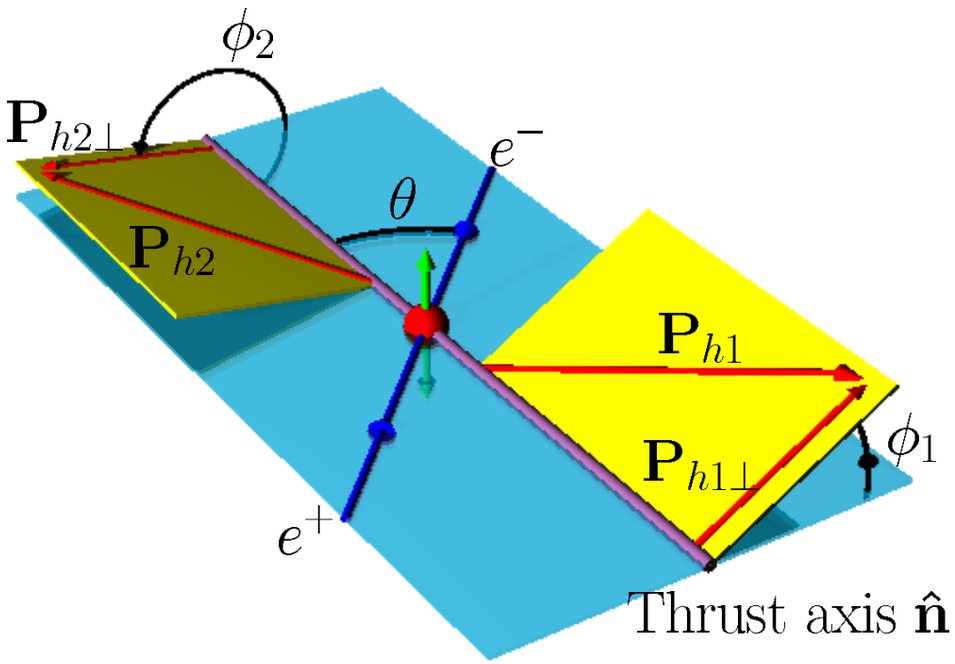
$$\Delta(z,p_\perp) \quad = \quad \frac{1}{2}\left[D(z,p_\perp^2)\not{n} + H_1^\perp(z,p_\perp^2)\sigma^{\mu\nu}\frac{p_{\perp\mu}n_\nu}{M}\right]$$

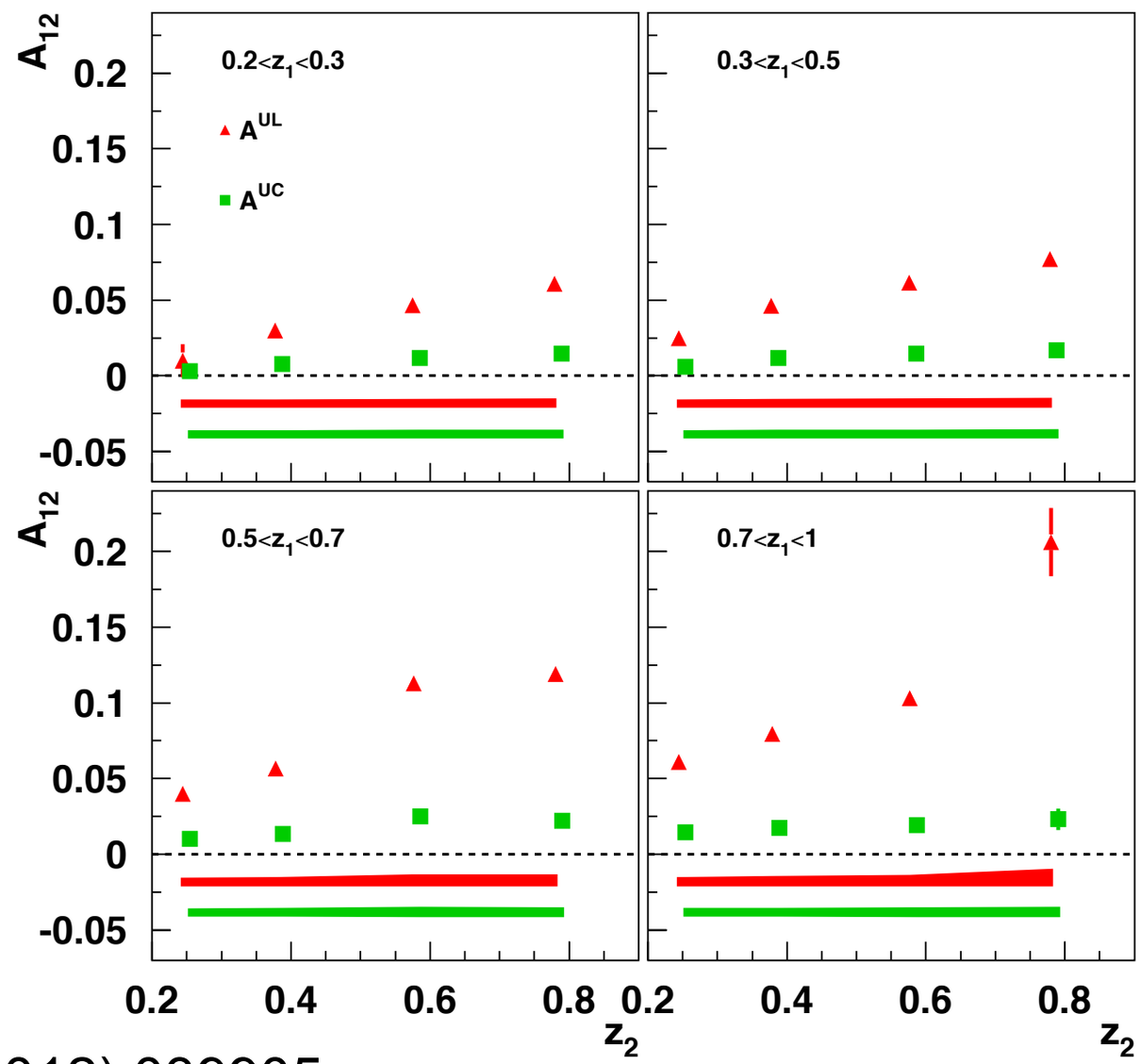


Current status for Collins fragmentation function

- Collins effect observed by BELLE Collaboration: $\cos(\Phi_1+\Phi_2)$

$$A(z_1,z_2,\theta,\varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1\,dz_2\,d\cos\theta\,d(\varphi_1+\varphi_2)}$$
$$= 1+\frac{1}{8}\frac{\sin^2\theta}{1+\cos^2\theta}\cos(\varphi_1+\varphi_2)\frac{\sum_q e_q^2\,\Delta^N D_{h_1/q^\uparrow}(z_1)\,\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1)\,D_{h_2/\bar{q}}(z_2)}$$



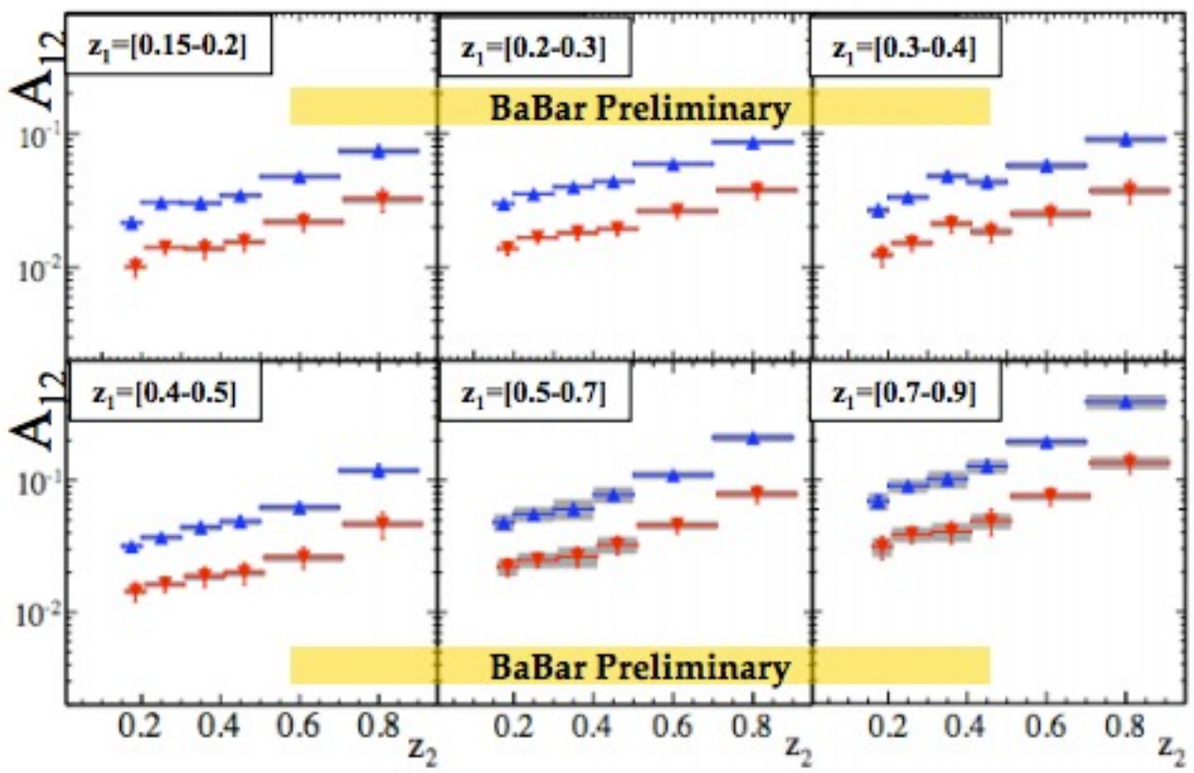


See also update: R. Seidl, PRD 86 (2012) 039905

PRL 96, 232002 (2006)
PRD 78, 032011 (2008)

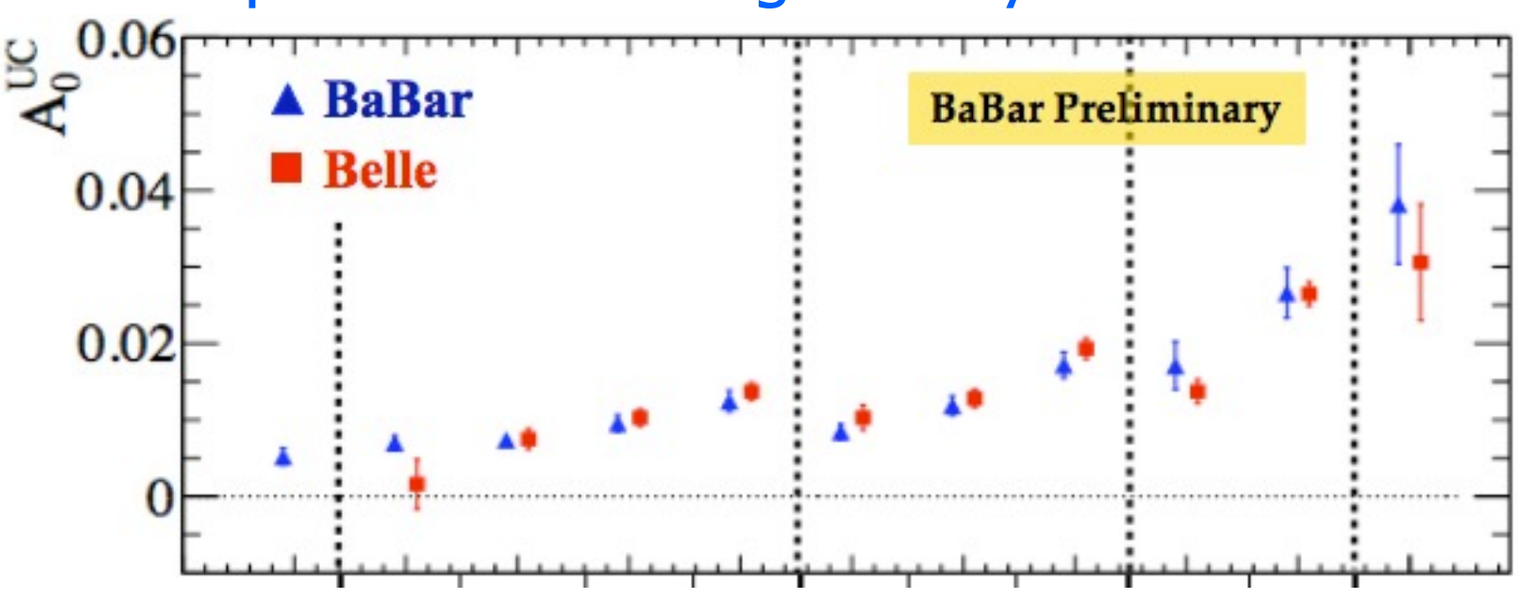
Also observed by BarBar

- Collins effect from BarBar



Garzia, talk at QCD-N'12
see more on Muller's talk

- Compare with Belle: generally consistent

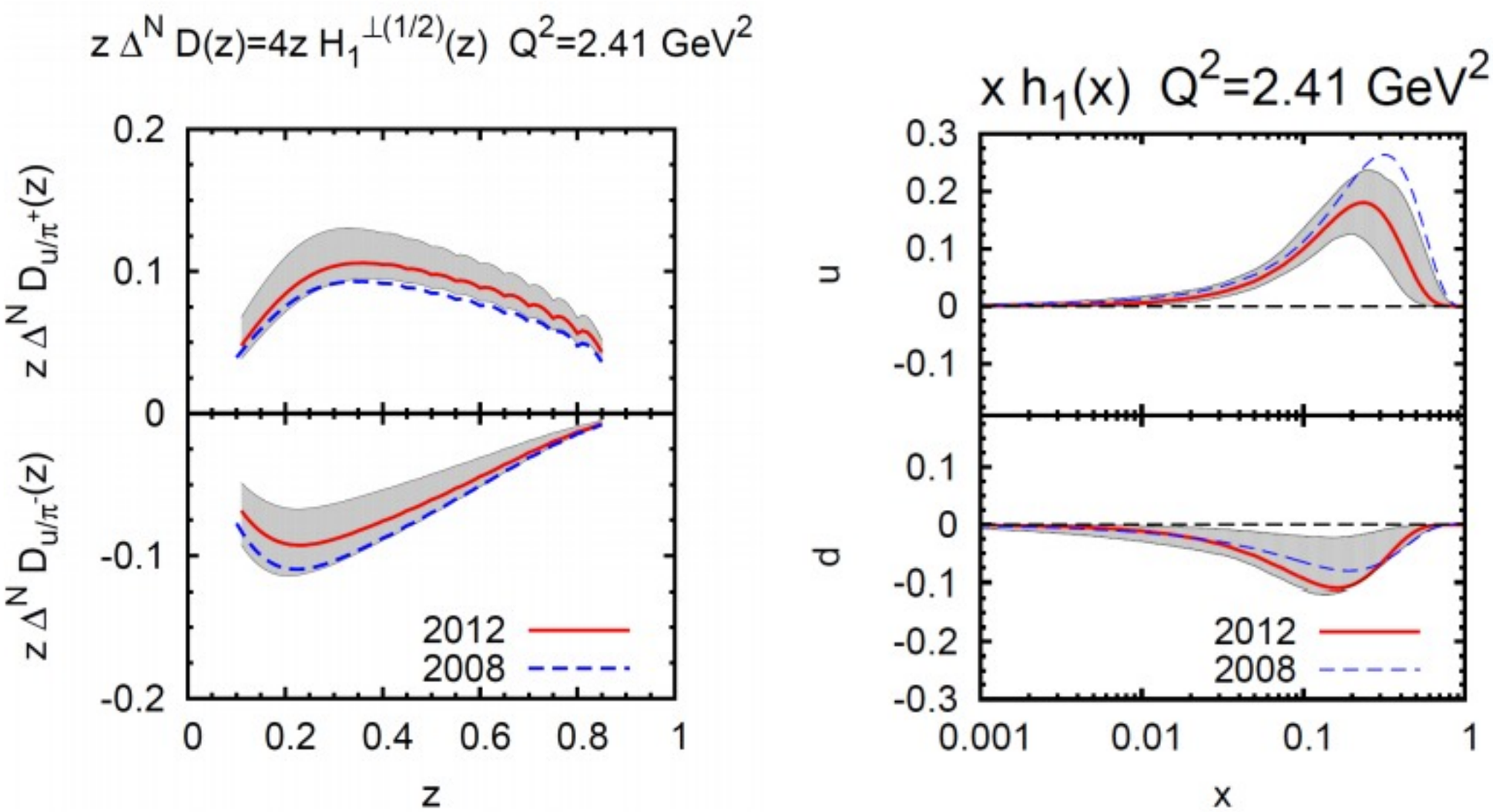


some discrepancy for A^{UC}
should be now resolved

R. Seidl, PRD86 (2012) 039905

Extraction of Collins in e+e- could help extract transversity

- Collins extraction from e+e-

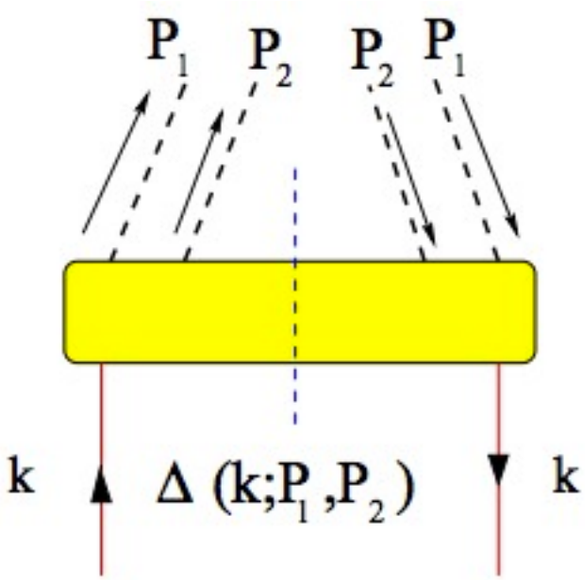


S. Melis, 2012

- Combine this with SIDIS data

Going beyond the current picture: two ways - II

- Dihadron fragmentation function



Dihadron

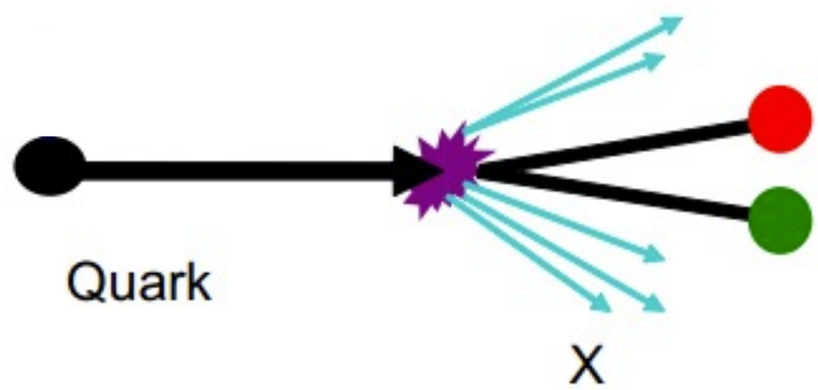
quark pol.

U	L	T
D_1		$H_1^{\sphericalangle}$

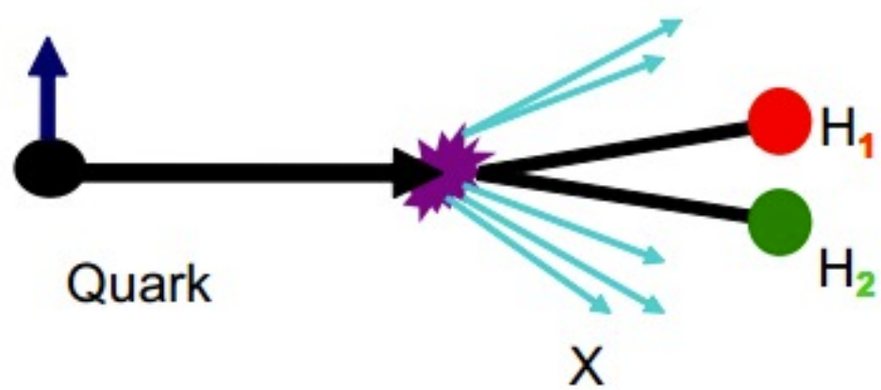
Bianconi-Boffi-Jakob-Radici, 99

$$\Delta \sim \gamma^- D_1(z_1, z_2, M_h^2) + \frac{\sigma^{-\alpha} R_{T\alpha}}{2|R_T|} H_1^{\sphericalangle}(z_1, z_2, M_h^2)$$

- DFF $D_1(z_1, z_2, M_h^2)$



- IFF $H_1^{\sphericalangle}(z_1, z_2, M_h^2) \sim S_q \times (P_{1\perp} - P_{2\perp})$

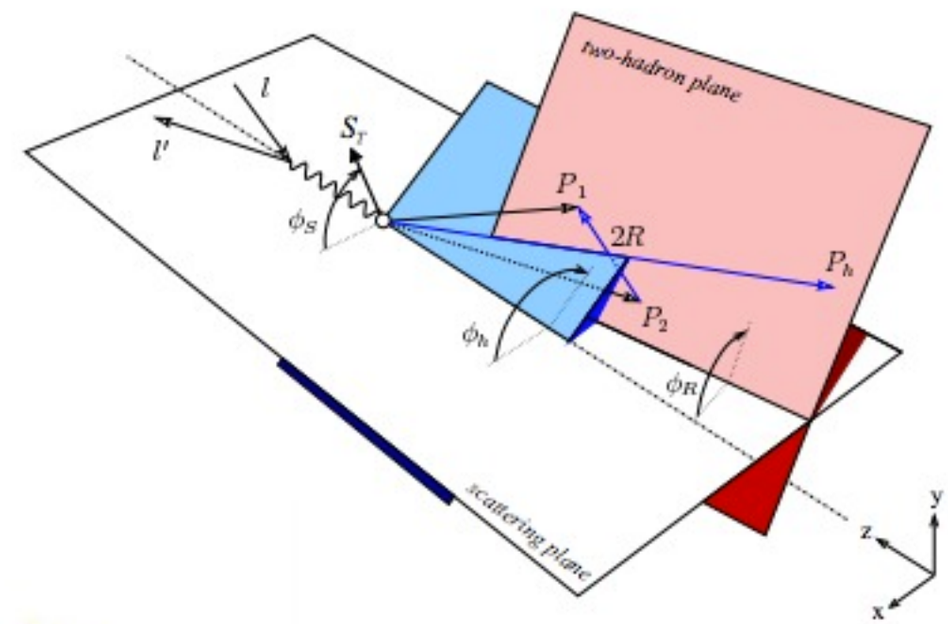


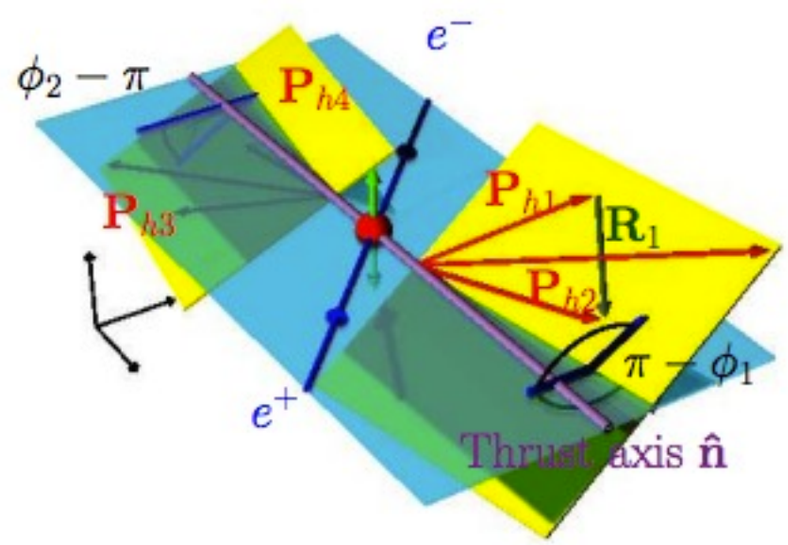
They are better theoretical objects

- They follow a simple collinear DGLAP type evolution equation
  - $D_1(z_1, z_2, M_h^2)$ follow the DGLAP evolution for spin-averaged collinear fragmentation function
  - $H_1^{\sphericalangle}(z_1, z_2, M_h^2)$ follow the DGLAP evolution for the quark transversity

    Ceccopieri-Radici-Bacchetta, 07

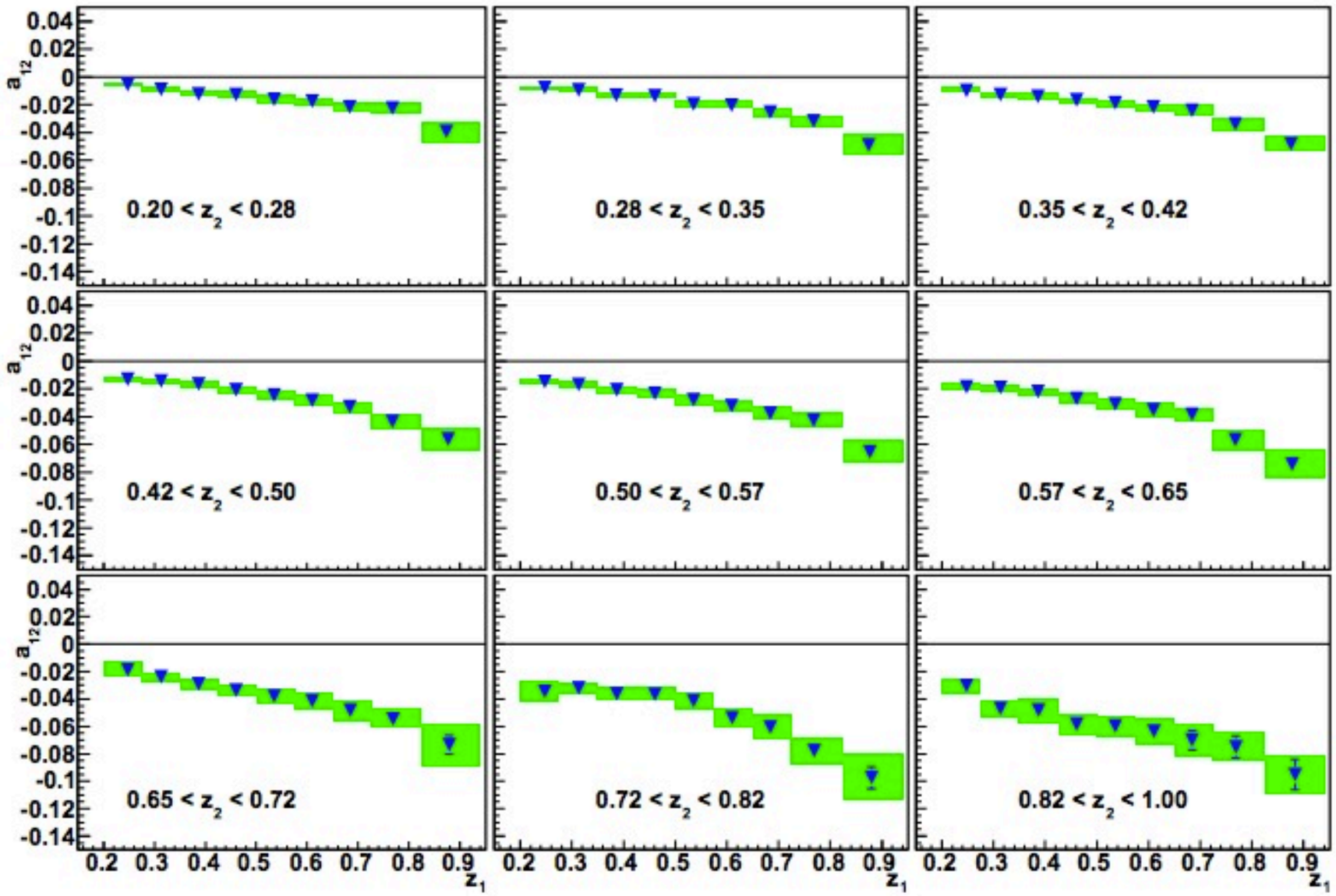
- Usual collinear factorization could be used to describe the experimental data
  - SIDIS: hadron pair, transversity × IFF
  - e+e-: two hadron pair, IFF × IFF
  - pp:
    - hadron pair, transversity × IFF
    - two hadron pair, IFF × IFF





Data on IFF

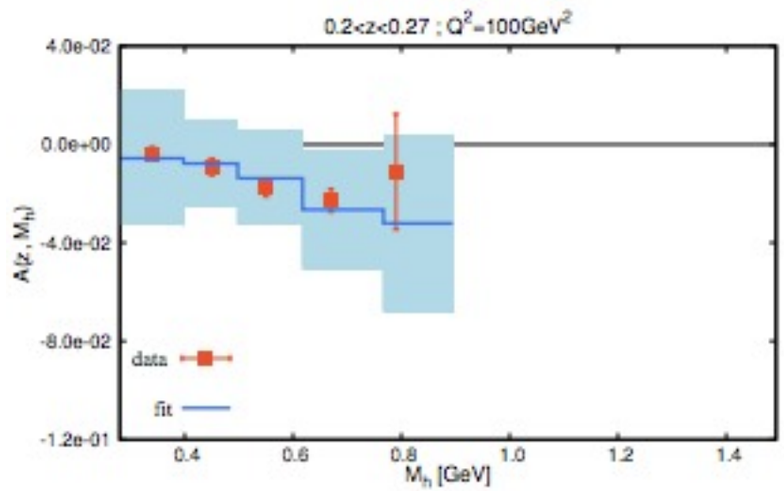
- Belle PRL, arXiv: 1104.2425

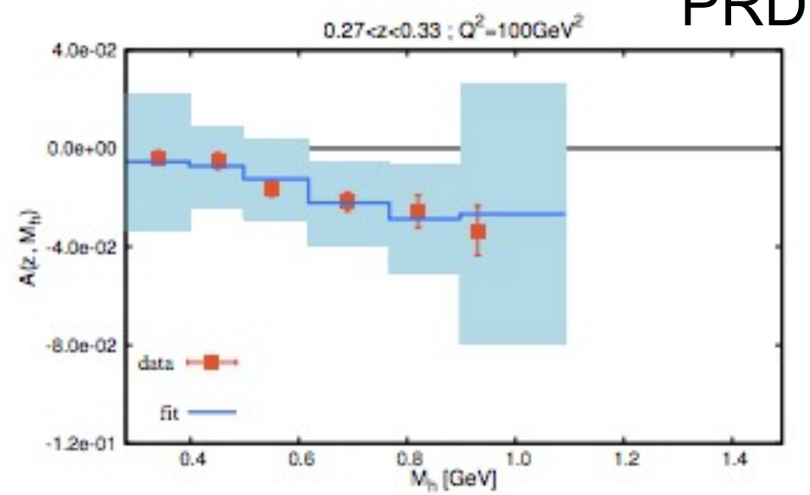


Extract IFF from these data

- First extraction has been made by Courtoy-Bacchetta-Radici-Bianconi

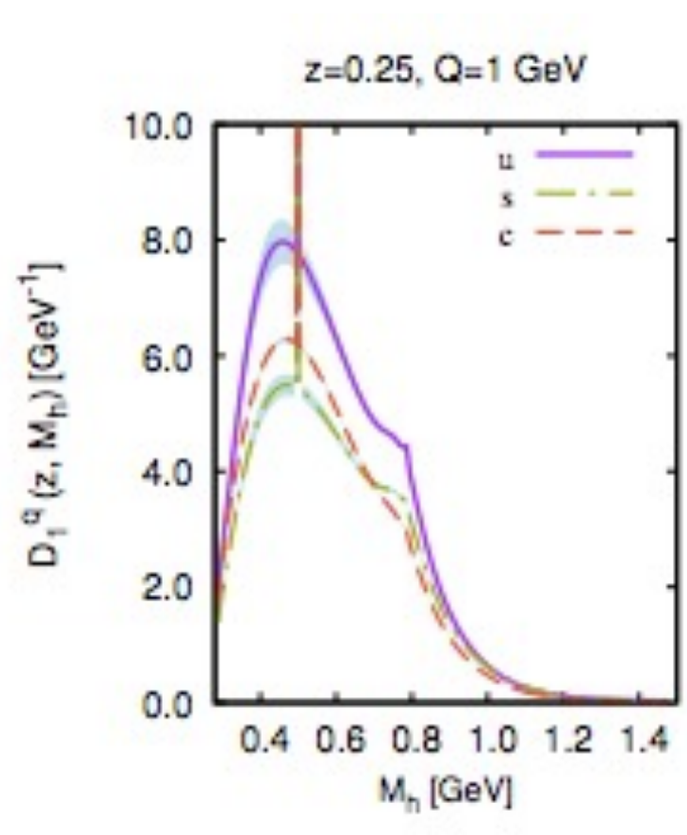
PRD, arXiv: 1202.0323

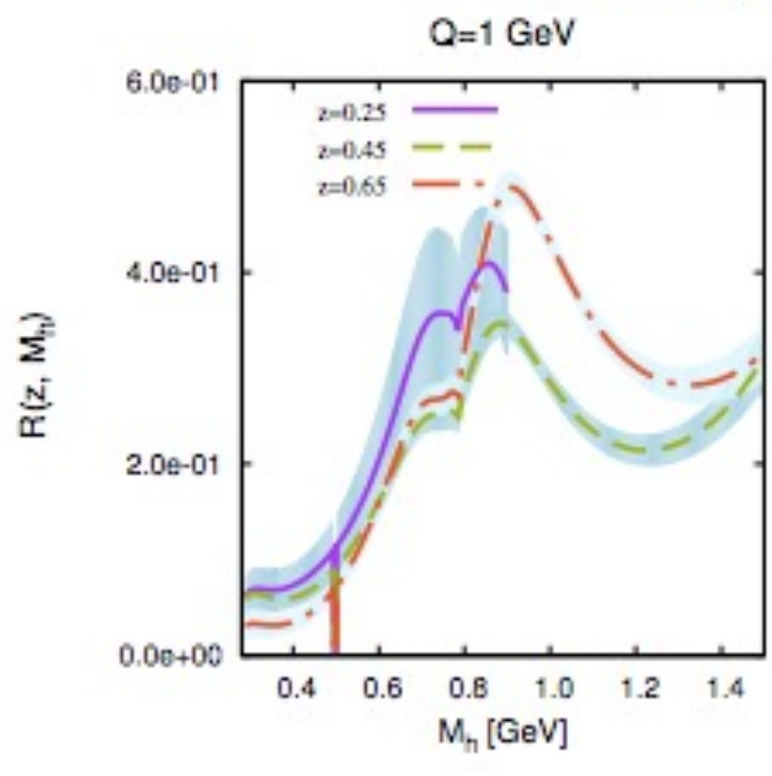




- DFF and IFF

$$R(z, M_h) = \frac{|\boldsymbol{R}|}{M_h} \frac{H_{1,sp}^{\sphericalangle u}(z, M_h; Q_0^2)}{D_1^u(z, M_h; Q_0^2)}$$

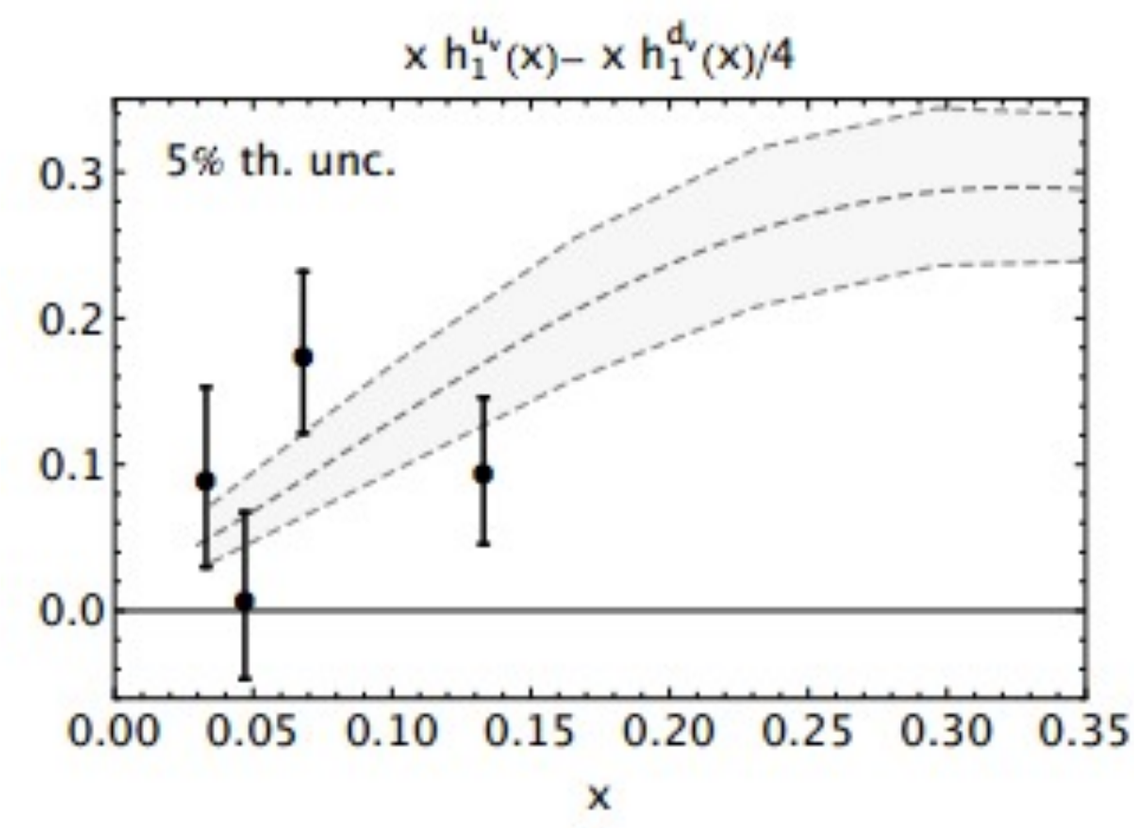




Extract transversity from dihadron fragmentation function

- A first extraction of transversity (a combination) from dihadron fragmentation function (based on SIDIS data) has been made by Bacchetta-Courty-Radicci
  - It is in a reasonable agreement with the transversity extracted from Collins effect

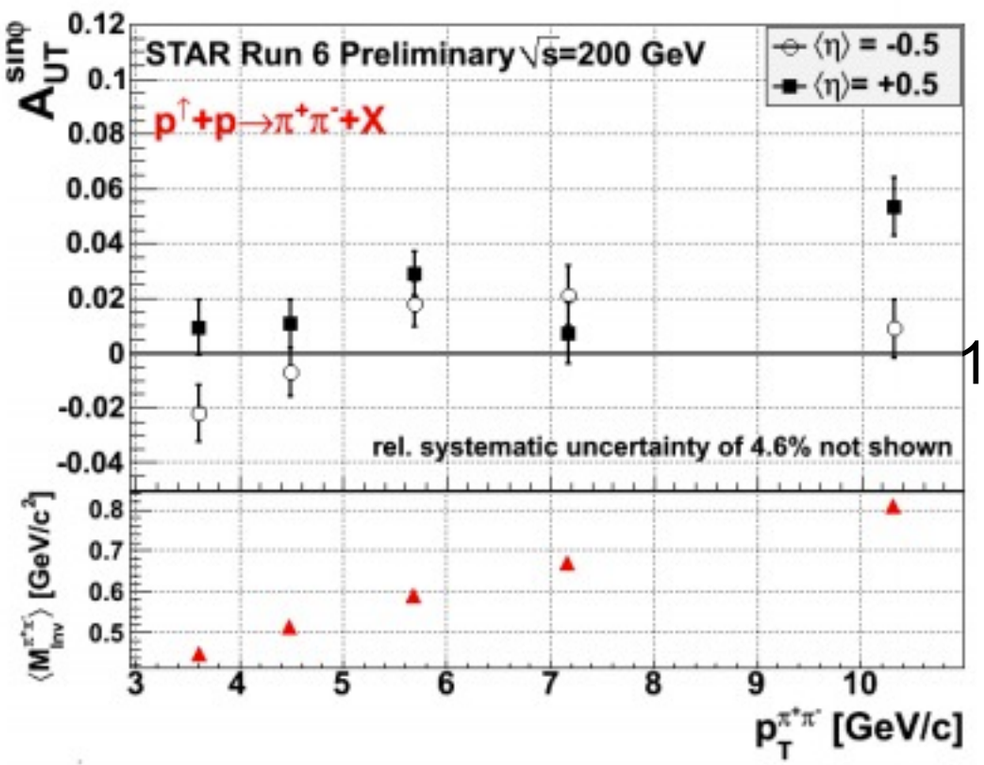
PRL, arXiv: 1104.3855



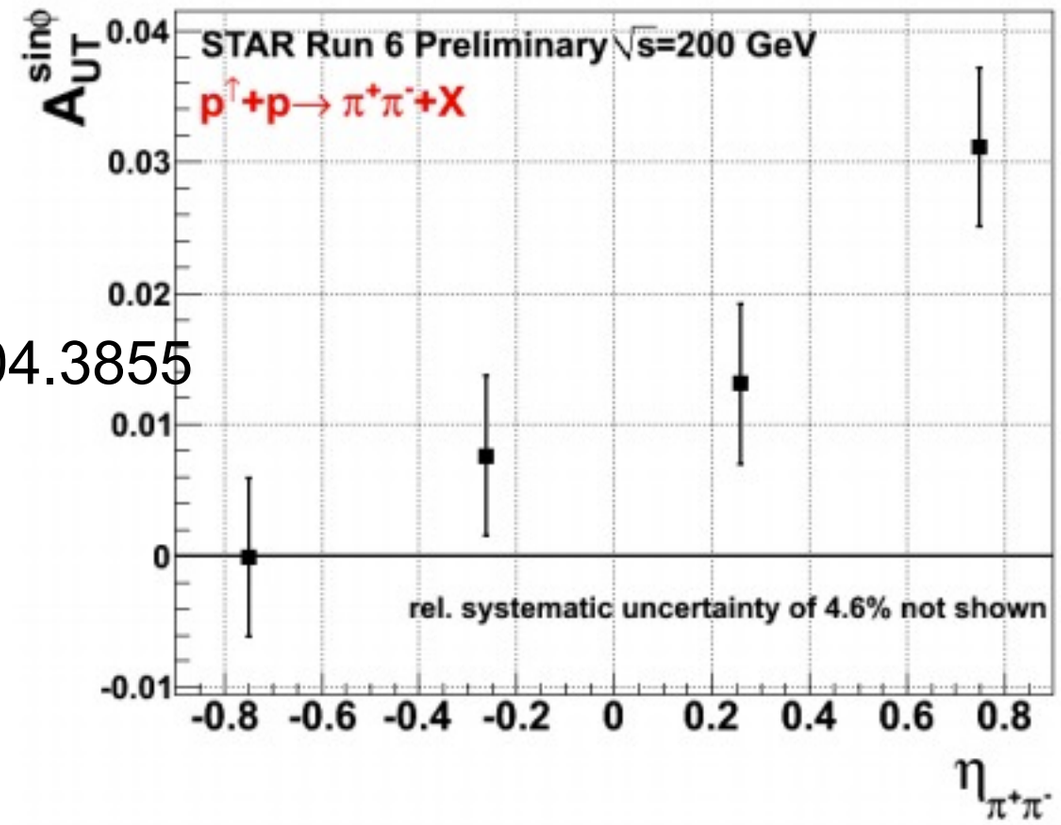
IFF in proton+proton

- STAR also has measurements now, it will be good to use these data to extract transversity

Vossen, RHIC/AGS User's meeting, 2012



1104.3855



Evolution effect

- Our experimental data span very different energies:
  - SIDIS is performed at Q^2~1-3 GeV^2, Belle and BarBar at Q^2~100 GeV^2
  - Future RHIC experiment can also access very high Q^2~16-81 GeV^2 for DY
  - Our kt-extended parton model has made great success in recent years
  - It is the time now to go beyond these parton model result, and understand the evolution of the relevant TMDs
- So far we talked about evolution of DFF and IFF, saying that they are following the usual DGLAP evolution equation (with splitting kernel either same as unpolarized PDF or transversity)
- What does evolution means?
  - How they are connected to higher order corrections?

Cross section depends on where you factorize?

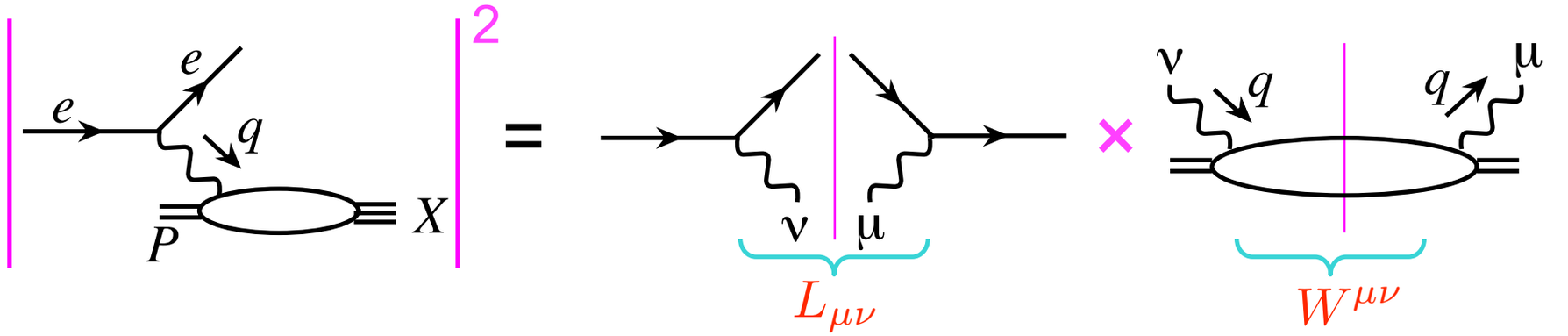
- Start with collinear factorization, cross section can be written as follows

$$\sigma(x, Q^2) = \hat{\sigma}(x, Q^2/\mu^2) \otimes \phi_{a/p}(x, \mu^2)$$

- In the QCD formalism, there is a factorization scale dependence μ^2, where does it come from? How does the PDFs change (evolve) with respect to this scale?
  - If in the hard part, one choose $\mu \sim Q$, then we need PDFs at all different scales as experimental measurements are done at different Qs

Recall: DIS again

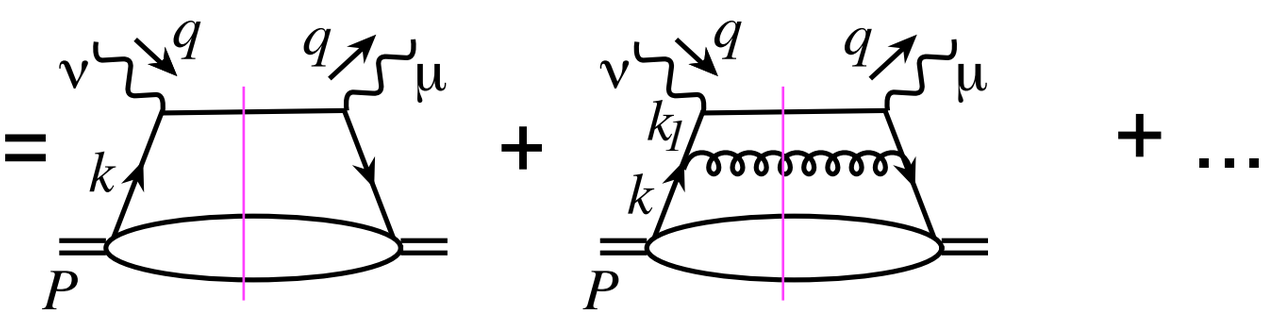
- Deep Inelastic Scattering (DIS)

$\sigma =$ 

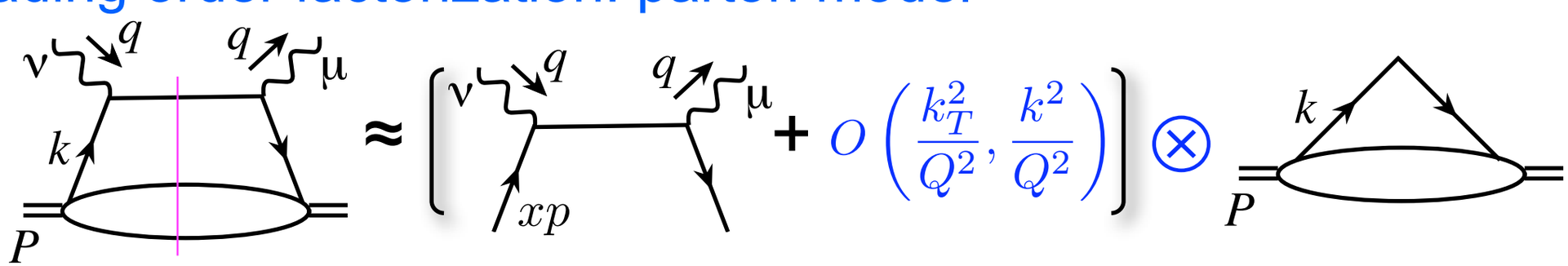
$L_{\mu\nu}$ (leptonic tensor) $W^{\mu\nu}$ (hadronic tensor)

All the interesting physics (QCD dynamics) is contained in $W^{\mu\nu}$

- Hadronic tensor in perturbative expansion

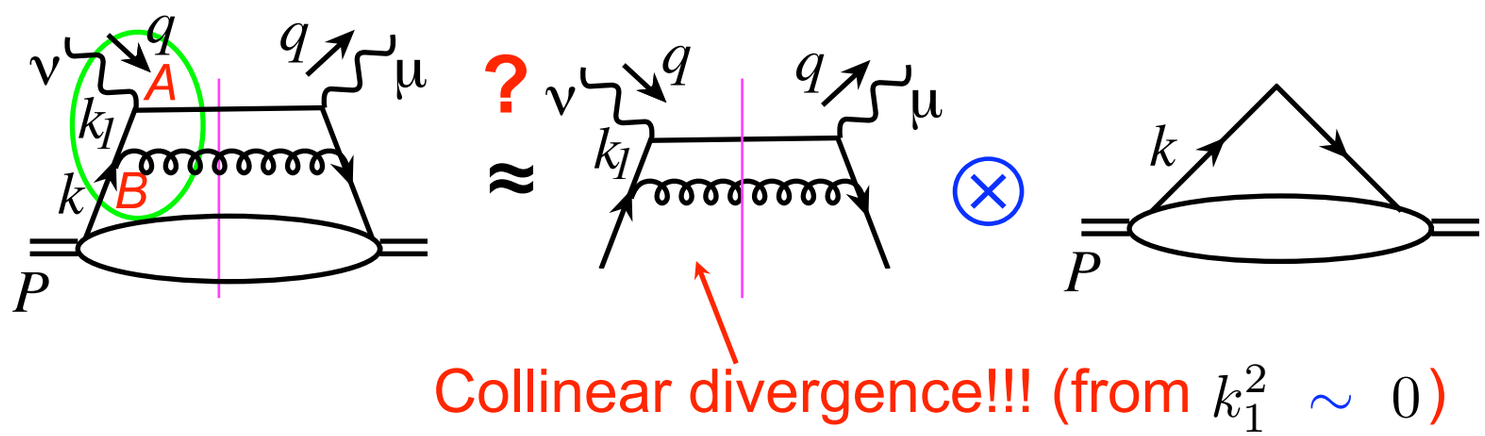
$W^{\mu\nu} =$ 

- Leading order factorization: parton model



QCD dynamics beyond leading order

- Radiative corrections



$$\Rightarrow \int d^4k_1 \frac{i}{k_1^2+i\epsilon}\frac{-i}{k_1^2-i\epsilon} \Rightarrow \infty$$

- ❖ $k_1^2 \sim 0$ intermediate quark is on-shell

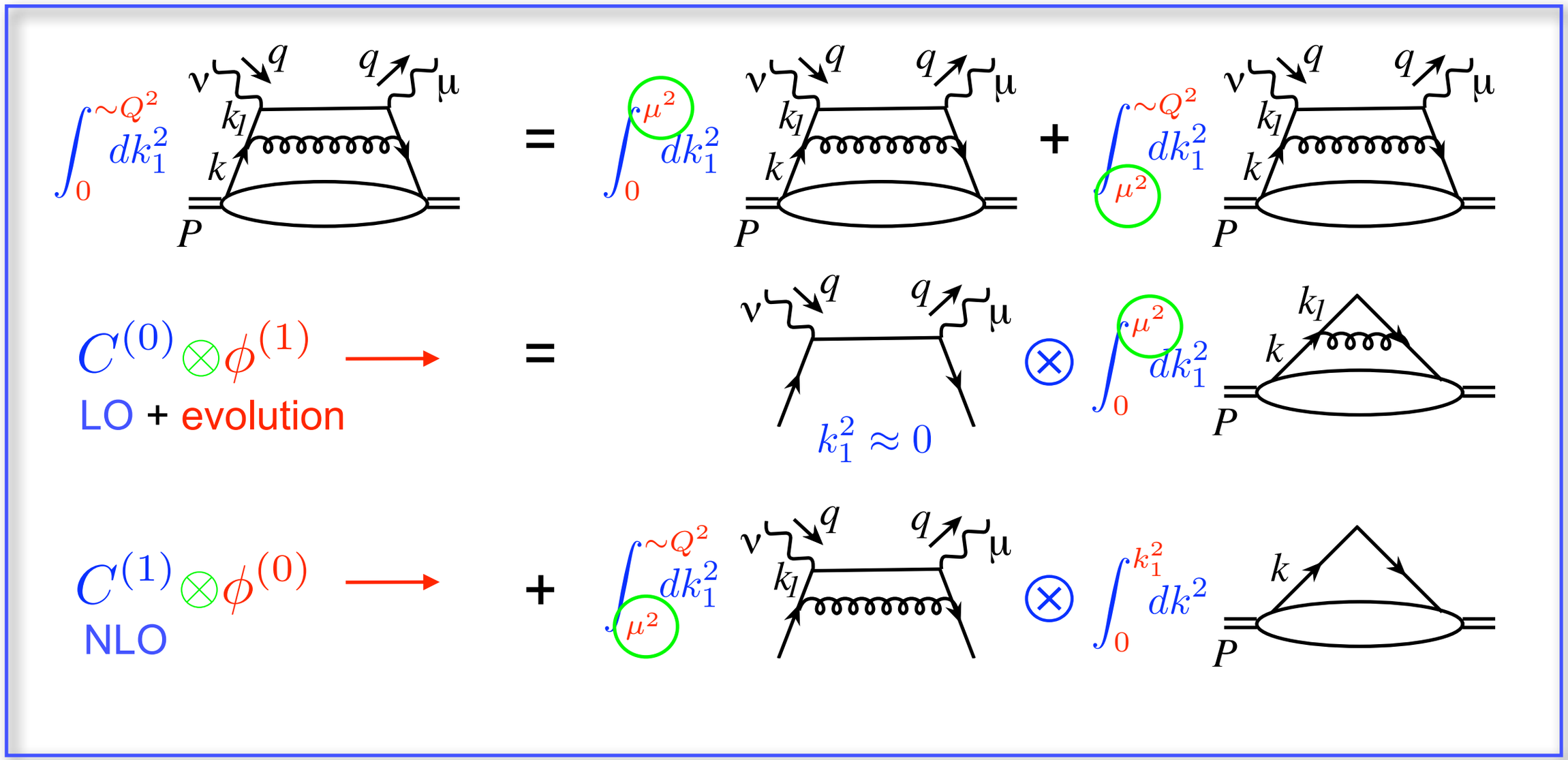
  $t_{AB} \rightarrow \infty$
- ❖ gluon radiation takes place long before the photon-quark interaction

  $\Longrightarrow$ a part of PDF

Partonic diagram has both long- and short-distance physics

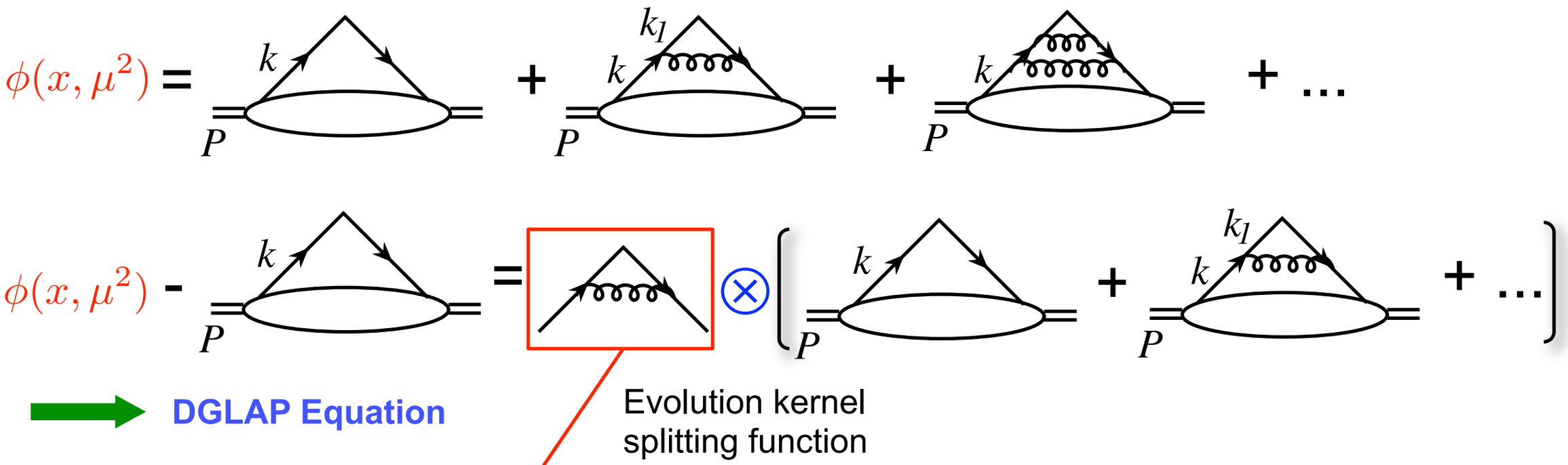
Factorization: separation of short- from long-distance

- Systematically remove all the long-distance physics into PDFs



DGLAP evolution = resummation of single logs

- Evolution = Resum all the gluon radiation



DGLAP Equation

$$\frac{\partial}{\partial \ln \mu^2}\phi_i(x,\mu^2) = \sum_j \boxed{P_{ij}\left(\frac{x}{x'}\right)} \otimes \phi_j(x',\mu^2)$$

- By solving the evolution equation, one resums all the single logarithms of $\left(\alpha_s \ln \frac{\mu^2}{\Lambda^2}\right)^n$

- Same idea for DFF and IFF

Go from Collinear PDFs to TMDs

- Evolution of collinear PDFs follow the usual DGLAP-type evolution equation, which is equivalent to resum the single-logarithmic contributions to all order

$$\left(\alpha_s \ln \frac{Q^2}{\mu^2}\right)^n$$

- Evolution of TMDs follow Collins-Soper-type evolution equation, which is equivalent to resum the double-logarithmic contributions to all order, which is usually more difficult

$$\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$$

Evolution of TMDs: basic idea

- These double logarithms in the cross section level come from the double logarithm in the kt-dependent distribution and fragmentation functions
  - A generic idea: parton distribution/fragmentation function $\phi(x, k_\perp^2)$ is associated with a hadron which has a large longitudinal component P^+, with these two scales in the TMDs (kt, P^+), its perturbative tail will have double logarithms $\left(\alpha_s \ln^2 \frac{P^{+2}}{k_\perp^2}\right)^n$
  - It is these logarithms which will translate into the cross section level $\left(\alpha_s \ln^2 \frac{Q^2}{q_T^2}\right)^n$
- In the true calculation, these P^+ comes in as a Lorentz invariant

$$\zeta = 4(P \cdot v)/v^2$$

- Thus one studies the ζ variable dependence of the TMDs, this is the evolution of TMDs
  - technique part: ζ is introduced to regularize the rapidity divergence

Evolution is simpler in b-space: SIDIS cross section

- in momentum space:

$$F(x_B, z_h, P_{h\perp}, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 \int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\ell}_\perp$$
$$\times q\left(x_B, k_\perp, \mu^2, x_B\zeta, \rho\right) \hat{q}_h\left(z_h, p_\perp, \mu^2, \hat{\zeta}/z_h, \rho\right) S(\vec{\ell}_\perp, \mu^2, \rho)$$
$$\times H\left(Q^2, \mu^2, \rho\right) \delta^2(z_h\vec{k}_\perp + \vec{p}_\perp + \vec{\ell}_\perp - \vec{P}_{h\perp}) \ ,$$

- in b-space:

$$F(x_B, z_h, b, Q^2) = \sum_{q=u,d,s,\ldots} e_q^2 q\left(x_B, z_h b, \mu^2, x_B\zeta, \rho\right) \hat{q}\left(z_h, b, \mu^2, \hat{\zeta}/z_h, \rho\right)$$
$$\times S(b, \mu^2, \rho) H\left(Q^2, \mu^2, \rho\right) \ ,$$

  - b-space the cross section is a simple product of TMDs and hard-coefficient functions, thus the evolution is simpler in b-space

Evolution of TMDs

- Since one now needs to resum double logarithms, typically it involves two steps: Idilbi-Ji-Ma-Yuan, 2004
  - Energy evolution of the unpolarized PDFs

$$\zeta \frac{\partial}{\partial \zeta} q(x, b, \mu, \zeta) = (K(\mu, b) + G(\mu, \zeta))\, q(x, b, \mu, \zeta)$$

  - Since it contains double logarithms, the kernel still contains single logarithms

$$\mu \frac{d}{d\mu} K(\mu, b) = -\gamma_K = -\mu \frac{d}{d\mu} G(\mu, \zeta)$$

- Solving these two equations, equivalently one resums the double logs
  - First for the evolution equation of K and G

$$K(b, \mu) + G(x\zeta, \mu) = K(b, \mu_L) + G(x\zeta, \mu_H) - \int_{\mu_L}^{\mu_H} \frac{d\tilde{\mu}}{\tilde{\mu}} \gamma_K(\alpha(\tilde{\mu}))$$

  - Then feed the solution back to the energy evolution equation

$$q(x, b, \mu, x\zeta, \rho) = \exp\left\{ -\int_{\mu_L}^{C_2 x\zeta} \frac{d\mu}{\mu} \left[\ln\left(\frac{C_2 x\zeta}{\mu}\right) \gamma_K(\alpha(\mu)) - K(b, \mu_L) - G(\mu/C_2, \mu) \right] \right\}$$
$$\times q(x, b, \mu, x\zeta_0 = \mu_L/C_2, \rho)\ ,$$

Solve evolution equations

- Once all the logs are resummed, the rest of b-dependent PDFs and FFs can be expanded as collinear PDFs and FFs

$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0 F_{UU}$$

$$F_{UU} = \int \frac{d^2 b}{(2\pi)^2} e^{i\vec{q_\perp}\cdot\vec{b}} W_{UU}(b, Q, x_B, z_h)$$

$$W_{UU}(b, Q, x_B, z_h) = e^{-S(b,Q)} \sum_q e_q^2 \left(C_{q/i} \otimes f_{i/A}\right)(x_B, \mu = \frac{c}{b}) \times \left(D_{B/j} \otimes \tilde{C}_{j/q}\right)(z_h, \mu = \frac{c}{b})$$

- All the large logarithms are resummed to the Sudakov exponential term

$$S(b, Q) = \int_{c^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln(Q^2/\mu^2) + B\right] \qquad A = \sum_{n=1} A^{(n)} \left(\frac{\alpha_s}{\pi}\right)^n$$

$$A^{(1)} = C_F \qquad B^{(1)} = -\frac{3}{2} C_F$$

Evolution of Sivers function

- The Collins-Soper energy evolution is really for the whole correlator

$$\mathcal{Q}(x,k_\perp,\mu,x\zeta)=\frac{1}{2}\int\frac{d\xi^- d^2\vec{b}_\perp}{(2\pi)^3}e^{-ix\xi^- P^+ + i\vec{b}_\perp\cdot\vec{k}_\perp}\langle P|\overline{\psi}_q(\xi^-,\vec{b}_\perp)\mathcal{L}_v^\dagger\gamma^+\mathcal{L}_v\psi_q(0)|P\rangle.$$

- So for Sivers function, it really is $k_\perp^\alpha f_{1T}^\perp(x,k_\perp^2)$ that evolves as a whole
  - in b-space, it is (also the b-derivative of Sivers function)

$$\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)=\frac{1}{M}\int d^2k_\perp e^{-i\vec{k}_\perp\cdot\vec{b}_\perp}k_\perp^\alpha\, f_{1T}^\perp(x,k_\perp,\mu,\zeta)$$

  - it follows the same energy evolution equation

Kang-Xiao-Yuan, PRL, 2011

$$\zeta\frac{\partial}{\partial\zeta}\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)=[K(\mu,b)+G(\mu,\zeta)]\,\tilde{f}_{1T}^{(\perp\alpha)}(x,b,\mu,\zeta)$$

  - Thus one should get a very similar resummation formalism

The Sivers effect for SIDIS

- The resummation formalism (consistent with experimental convention)

$$\frac{d\sigma}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0 \left[F_{UU} + |s_\perp| \sin(\phi_h - \phi_s) F_{UT}^{\sin(\phi_h - \phi_s)} \right]$$

  - spin-dependent structure function

$$F_{UT}^{\sin(\phi_h - \phi_s)} = -\frac{1}{4\pi} \int_0^\infty db\, b^2 J_1(q_\perp b) W_{UT}(b, Q, x_B, z_h)$$

Qiu-Sterman function

$$W_{UT}(b, Q, x_B, z_h) = e^{-S(b,Q)} \sum_q (\Delta C_{q/i}^T \otimes T_{i,F})(x_B, \mu = \frac{c}{b})$$

$$\times (D_{B/j} \otimes \tilde{C}_{j/q})(z_h, \mu = \frac{c}{b})$$

  - only soft-gluonic pole Qiu-Sterman function appears in this part

$$(\Delta C_{q/i}^T \otimes T_{i,F})(x_B, \mu) = \int_{x_B}^1 \frac{dx}{x} \Delta C_{q/i}^T(\frac{x_B}{x}, \mu) T_{i,F}(x, x, \mu)$$

The evolution of Collins function

- Recall: the evolution equation is really for the correlator, immediately, you know the evolution for Collins function
  - Again it is $p_\perp^\alpha H_1^\perp(z, p_\perp^2)$ which should be evolving as a whole
  - In the b-space, define

$$\tilde{H}_1^{(\perp\alpha)}(z, b, \mu, \zeta) = \frac{1}{M} \int d^2 p_\perp e^{-i\vec{p}_\perp \cdot \vec{b}_\perp} p_\perp^\alpha \, H_1^\perp(z, p_\perp, \mu, \zeta)$$

  - It follows the same evolution just like unpolarized fragmentation function
  - quark transversity follows the same evolution for the unpolarized PDFs f1
- One thus also has the same type of resummation formalism for the Collins effect

Comments on SIDIS and DY

- The only difference comes from so-called coefficient function
  - leading order

$$\Delta C^{T(0)}_{i/j}(z,\mu=\frac{c}{b}) = \delta_{ij}\delta(1-z) \qquad \text{DY}$$

$$\Delta C^{T(0)}_{i/j}(z,\mu=\frac{c}{b}) = -\delta_{ij}\delta(1-z) \qquad \text{SIDIS}$$

  - at next-leading-order: well-known difference due to $Q^2>0$ (<0)

$$\Delta C^{T(1)}_{i/j}(z,\mu=\frac{c}{b}) = \delta_{ij}\left[-\frac{1}{4N_c}+\frac{C_F}{2}(\frac{\pi^2}{2}-4)\delta(1-z)\right] \qquad \text{DY}$$

$$\Delta C^{T(1)}_{i/j}(z,\mu=\frac{c}{b}) = -\delta_{ij}\left[-\frac{1}{4N_c}+\frac{C_F}{2}(-4)\delta(1-z)\right] \qquad \text{SIDIS}$$

  - Thus in the full perturbative QCD region, Sivers between SIDIS and DY is not just a sign: it is interesting to study the consequence

Phenomenological study: what's the difficulty?

- For small qt region, we could use the resumed formalism. Don't need to worry about the perturbative tail from collinear twist-3 contribution

- Only at small b-region (corresponds to large momentum), one can calculate the relevant coefficients perturbatively.

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi}\int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b}) \times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- However, in order to Fourier transform back to qt-space, we need the whole b-region. Since large b-region will be non-perturbative, we need a non-perturbative input. This part should be universal if QCD factorization holds for the process.

The parametrization for the non-perturbative function

- Different approaches for the non-perturbative functions in "DY"

$$\frac{d\sigma}{dQ^2 dy d^2 q_\perp} = \frac{\sigma_0}{2\pi}\int_0^\infty db\, b J_0(q_\perp b) W_{UU}(b, Q, x_A, x_B)$$

$$W_{UU}^{pert}(b, Q, x_A, x_B) = e^{-S(b,Q)} \sum_q e_q^2 (C_{q/i} \otimes f_{i/A})(x_A, \mu = \frac{c}{b}) \times (C_{\bar{q}/j} \otimes f_{j/B})(x_B, \mu = \frac{c}{b})$$

- Parametrize the full b-space function

$$W_{UU}(b, Q, x_A, x_B) = W_{UU}^{pert}(b, Q, x_A, x_B) F^{NP}(b, Q, x_A, x_B)$$

  - function form (through extrapolation): Qiu-Zhang, 2001
  - fitted form directly from experiments: Brock-Landry-Nadolsky-Yuan, 2003

$$W_{UU}(b, Q, x_A, x_B) = W_{UU}^{pert}(b_*, Q, x_A, x_B) F^{NP}(b, Q, x_A, x_B) \quad b_* = \frac{b}{\sqrt{1 + (b/b_{max})^2}}$$

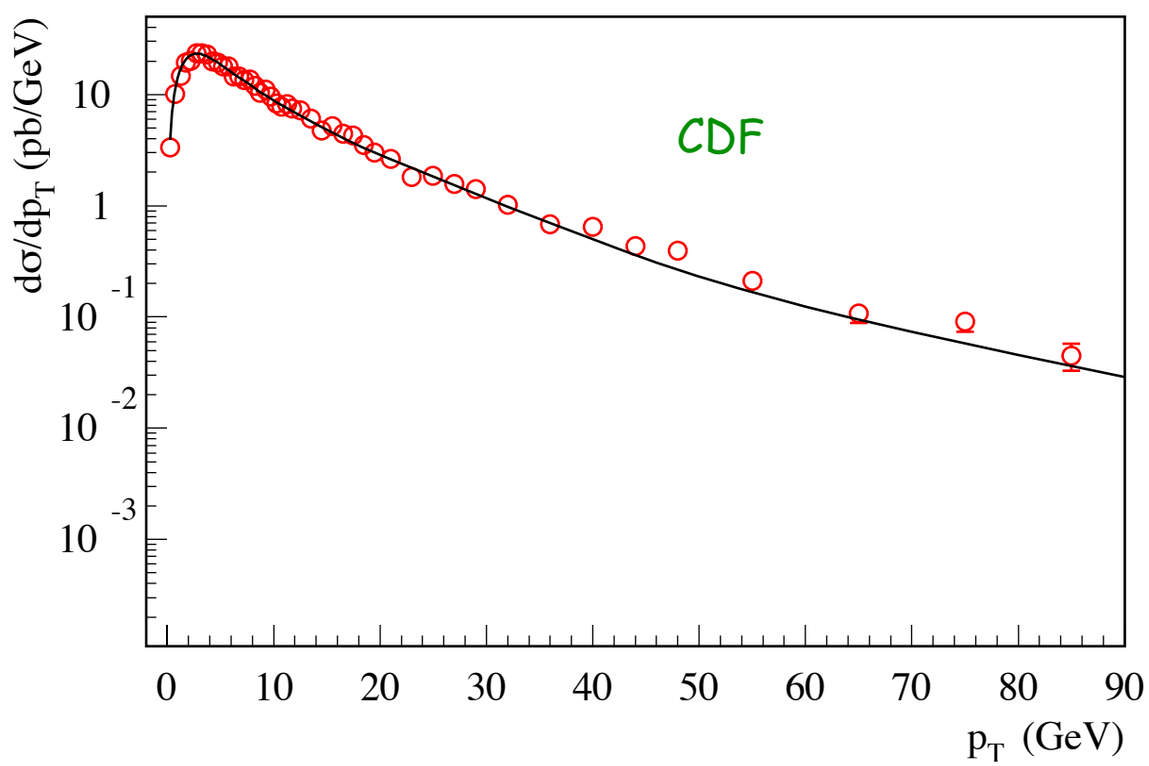
$$F^{NP}(b, Q, x_A, x_B) = \exp\left\{ -\left[g_1 (1 + g_3 \ln(100 x_A x_B)) + g_2 \ln\left(\frac{Q}{2Q_0}\right)\right] b^2 \right\}$$

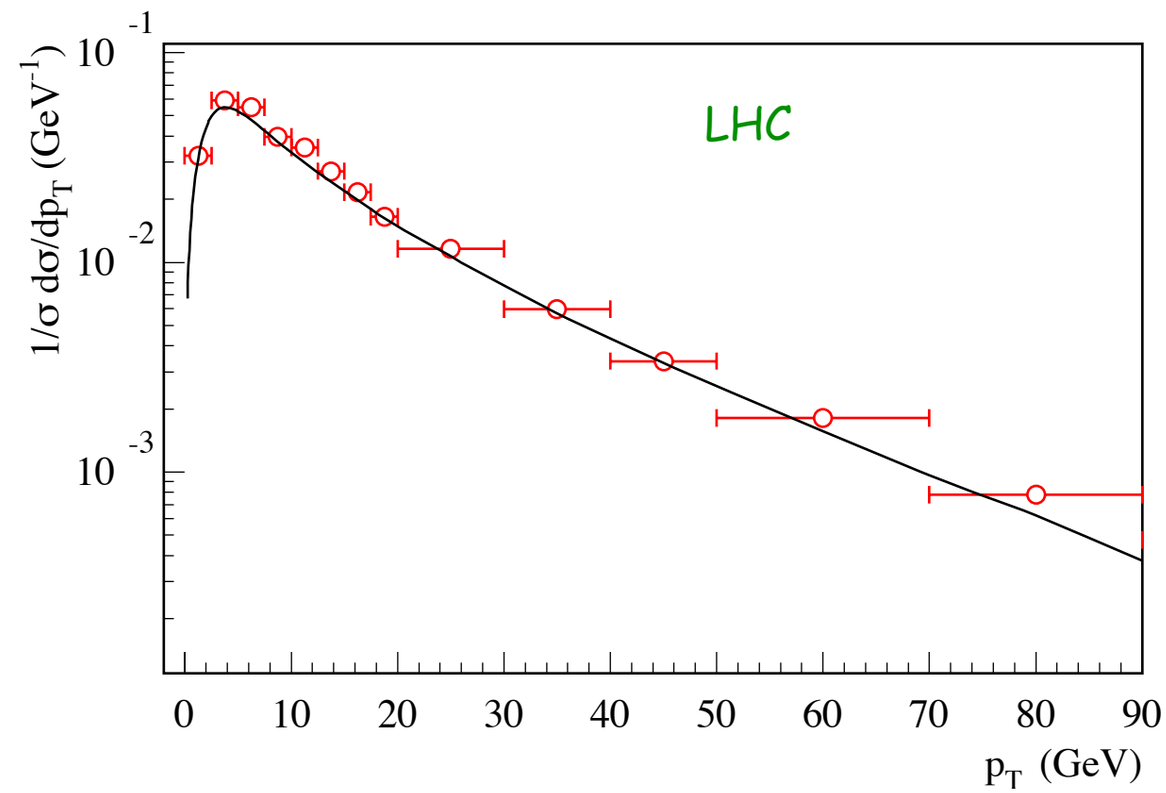
Works perfectly fine for Tevatron and LHC

- Z boson production at Tevatron and LHC

Kang-Qiu, 2012, to appear

  - the non-perturbative function is not suitable for low energy (highly biased by high energy data)



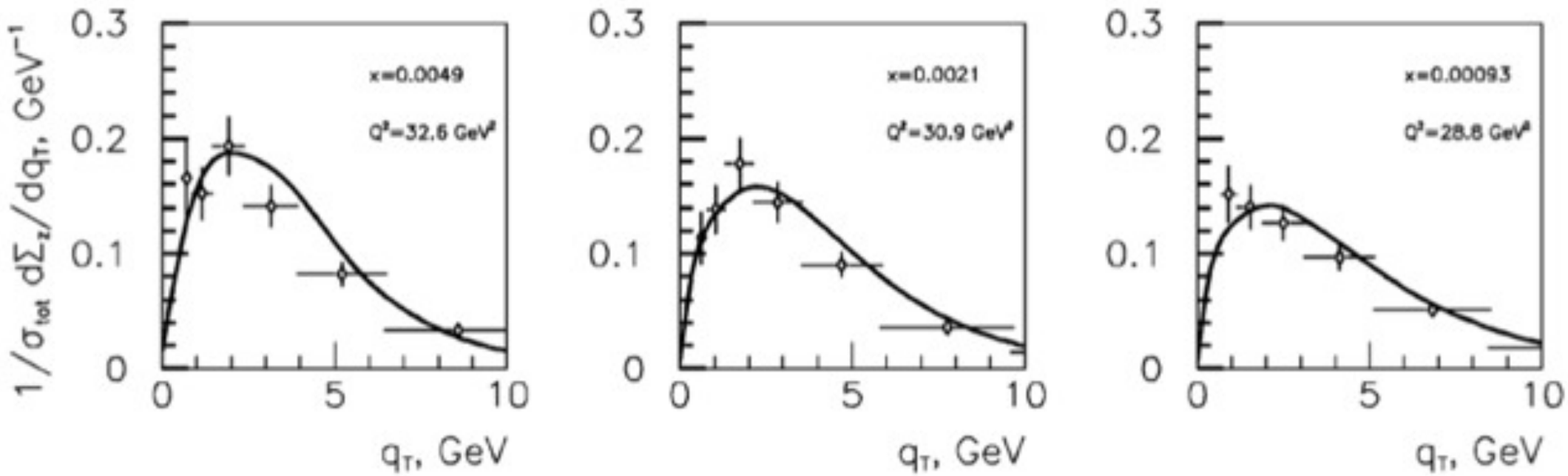


Works very well for HERA data: SIDIS

- z-flow distribution

$$\frac{d\Sigma_z}{dx\,dQ^2\,dq_T^2} = \sum_B \int_{z_{min}}^{1} z \frac{d\sigma(e+A\to e+B+X)}{dx\,dz\,dQ^2\,dq_T^2} dz.$$

Nadolsky, et.al., 1999, 2001

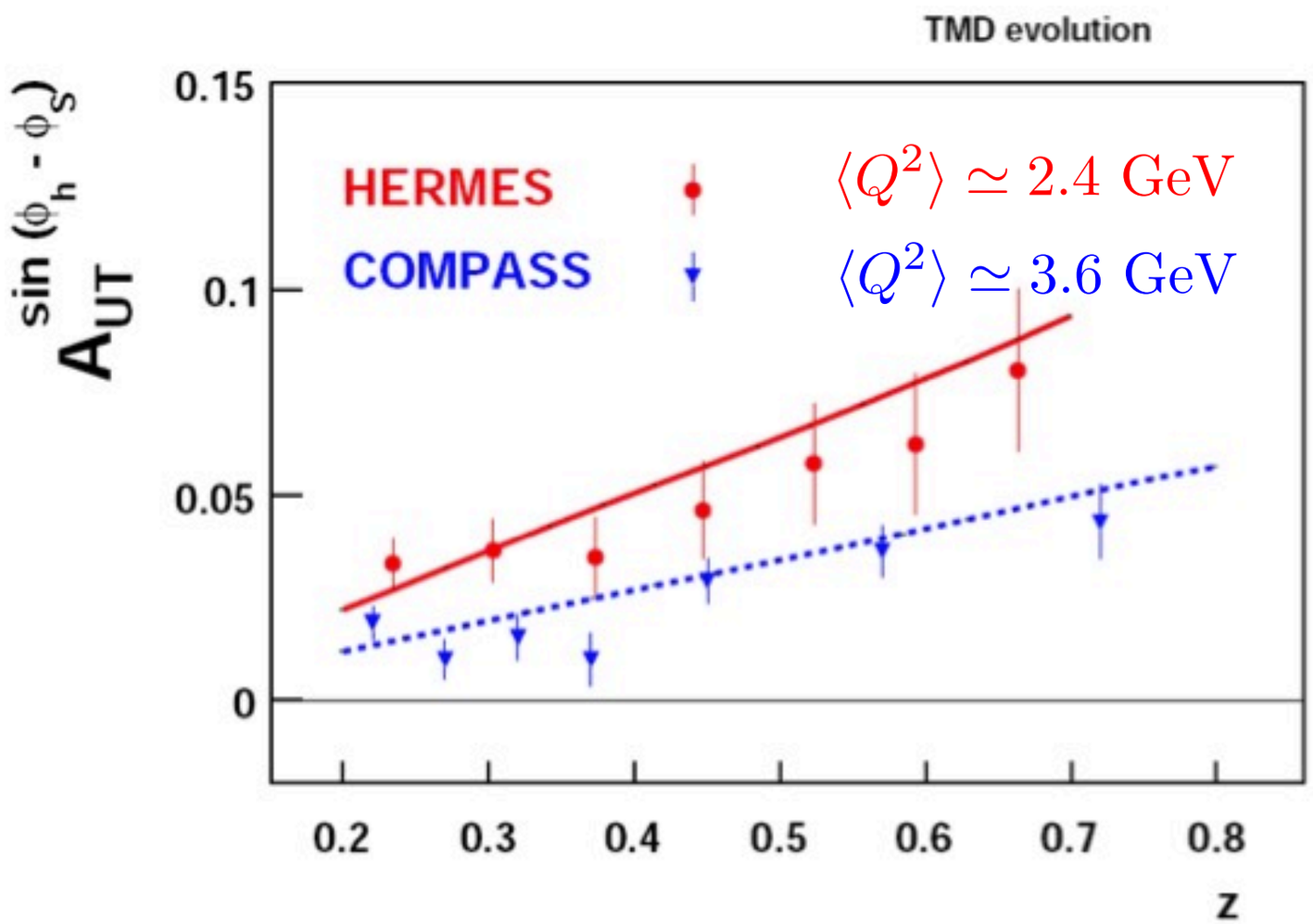


- The non-perturbative function fitted from HERA only works for small-x data, and do not apply for large x (Sivers data)
  - note this function in SIDIS is different from that in DY, as SIDIS non-perturbative function comes from PDF and FF, while DY only comes from PDF
  - In the current Sivers analysis, this functional form is not adopted. It will be a very good cross-check to see if the current form can describe HERA data!!!

Works directly with TMDs

- Seem to work fine

Aybat-Prokudin-Rogers, 2011



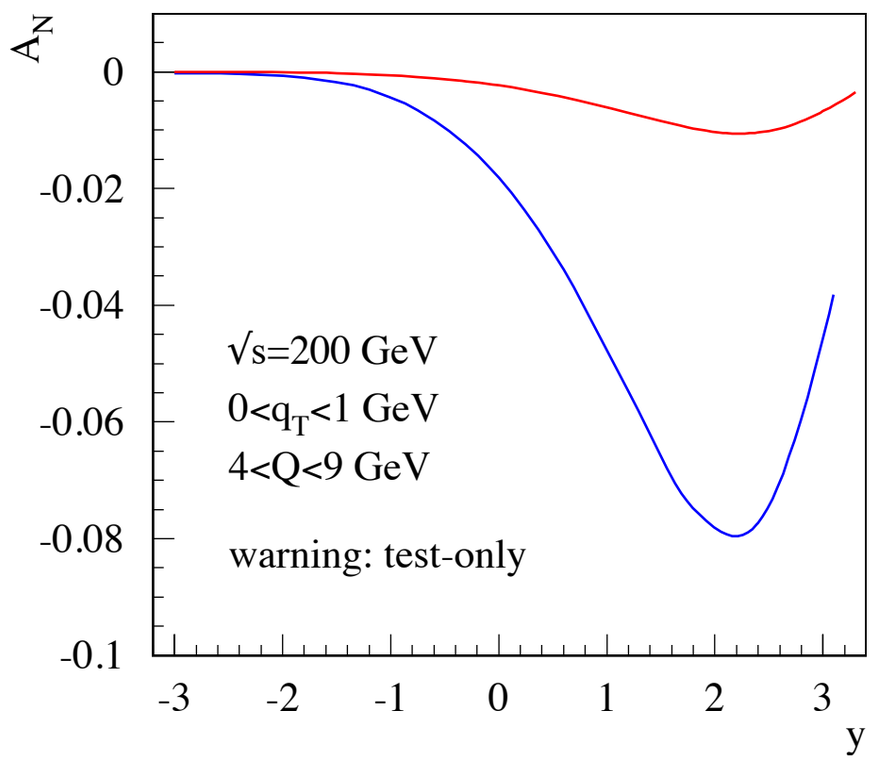
- See also work by Anselmino, et.al.

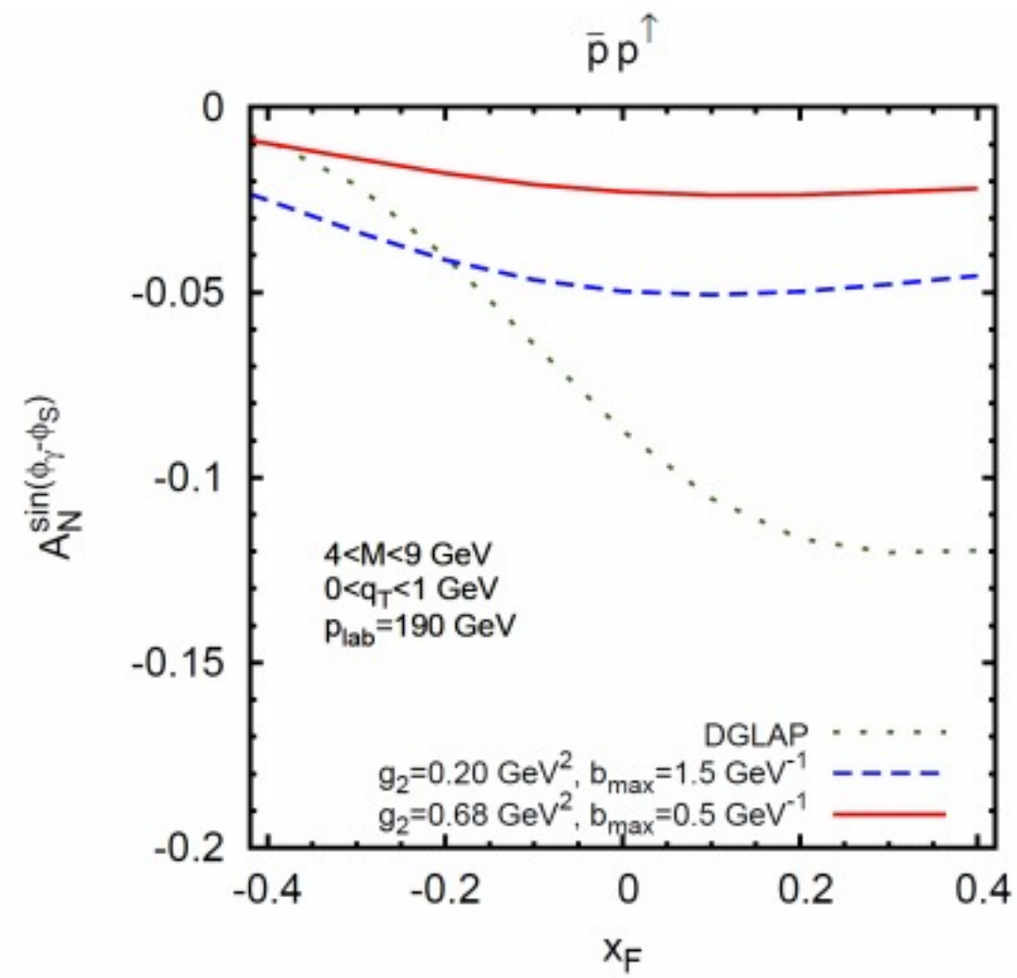
Anselmino-Boglione-Melis, 2012

  - With TMD evolution, it seems to describe the data slightly better

Naively apply this functional form ...

- It leads to dramatic effect, which might not be true





S. Melis, 2012

- One needs a refit, which concentrates on the low energy data

Summary

- There are tremendous progress recently in measuring/extracting spin-dependent fragmentation function, in turn to better extract other TMD parton distribution functions, particularly quark transversity

- QCD evolution is very important in the future for better understanding of the spin asymmetries

- QCD evolution for all TMDs in principle has already become available

- We just need more time/better data to pin down our theoretical parameters, in order to apply them more accurately in the phenomenology

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Thank you