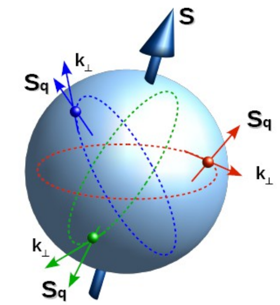


Global Analysis for the extraction of Transversity and the Collins Function

M. Boglione

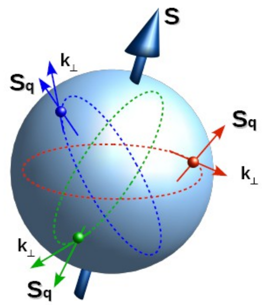
INFN Torino

UNIVERSITÀ DEGLI STUDI DI TORINO
ALMA UNIVERSITAS TAURINENSIS

Fisica Torino

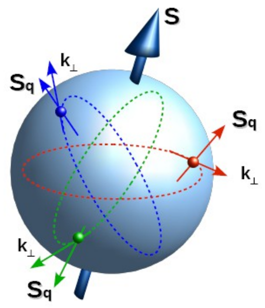
In collaboration with
M. Anselmino, U. D'Alesio, S. Melis, F. Murgia, A. Prokudin

Summary



- **Introduction**: strategy and main ideas
- **Experiments**: old and new data sets
- **Theory**: which functions are we actually extracting ?
- **New data analysis and new results**
- **Open issues and future perspectives**:
  - BaBar data
  - TMD evolution
- **Conclusions**

Strategy and main ideas



- The **transversity distribution function** contains basic information on the spin structure of the nucleons.

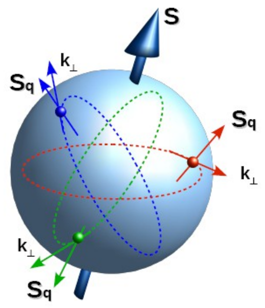
- Being related to the expectation value of a chiral odd operator, it appears in physical processes which require a quark helicity flip; therefore it cannot be measured in usual DIS.

- Drell-Yan → planned experiments in polarized pp at PAX.

- At present, the only chance of gathering information on transversity is **SIDIS**, where it appears associated to the Collins fragmentation function.

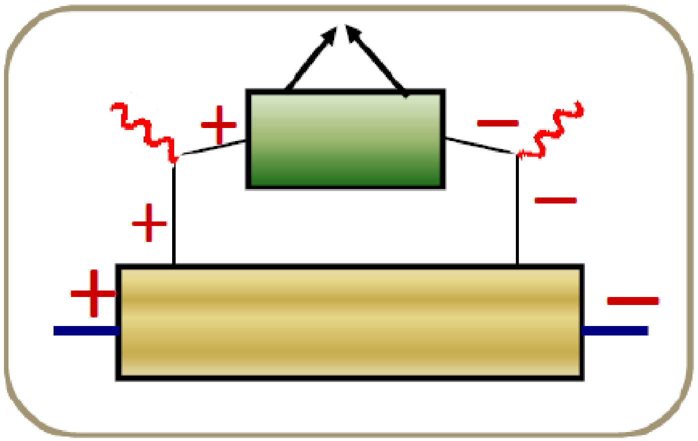
- **DOUBLE PUZZLE**: we cannot determine transversity if we do not know the Collins fragmentation function.

Transversity

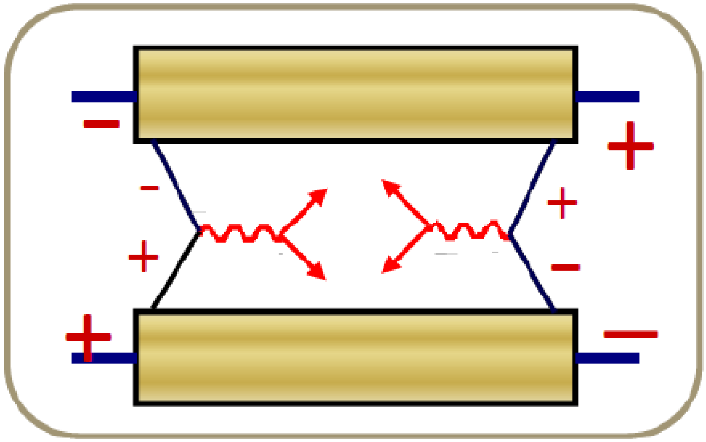


- There is no gluon transversity distribution function
- Transversity cannot be studied in deep inelastic scattering because it is chirally odd
- Transversity can only appear in a cross-section convoluted to another chirally odd function

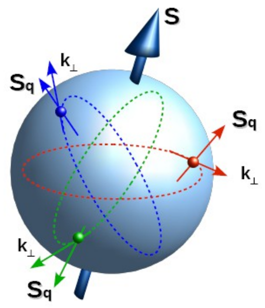
SIDIS



Drell -Yan



Strategy and main ideas



A **global fit** of HERMES and COMPASS **SIDIS** single spin asymmetries combined with **BELLE** data on $e^+e^- \to h_1h_2X$ will allow us to determine both the **transversity** distribution and the **Collins** fragmentation functions of u and d quarks.

Experiments: old and new data

Old data:

BELLE : 2006 first release
2008 new analysis Phys. Rev. D 78, 032011 (2008)

HERMES : 2005 data, A. Airapetian et al., Phys. Rev. Lett. 94 (2005) 012002.
2007 release, M. Diefenthaler, (2007), arXiv:0706.2242 [hep-ex].

COMPASS – d : 2007 data, E.S. Ageev et al., Nucl. Phys. B765 (2007) 31.
2008 release, M. Alekseev et al., (2008), arXiv:0802.2160 [hep-ex].

New data:

BELLE : 2012 erratum, R. Seidl et al., Phys. Rev. D 86, 039905(E) (2012)

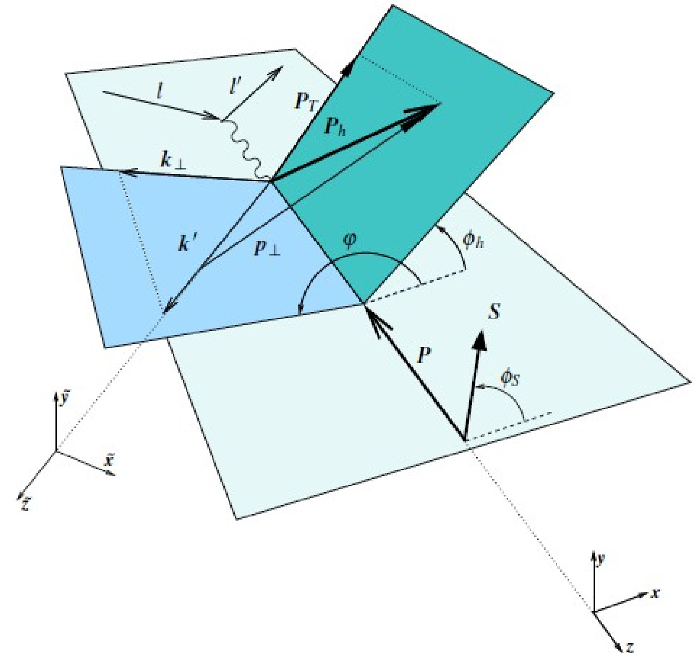
HERMES : 2010 data, Phys. Lett. B 693, (2010)

COMPASS – p : 2010-2011 new data on proton target, charged hadron production presented at Transversity 2011
Hadron-separated (pions and kaons) presented at SPIN 2012.

Theory perspectives

Theory: transversity and Collins function in SIDIS

$$A_{UT}^{\sin(\phi+\phi_S)} \equiv 2\,\frac{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow}]\,\sin(\phi+\phi_S)}{\int \mathrm{d}\phi\,\mathrm{d}\phi_S\,[\mathrm{d}\sigma^{\uparrow}+\mathrm{d}\sigma^{\downarrow}]}$$

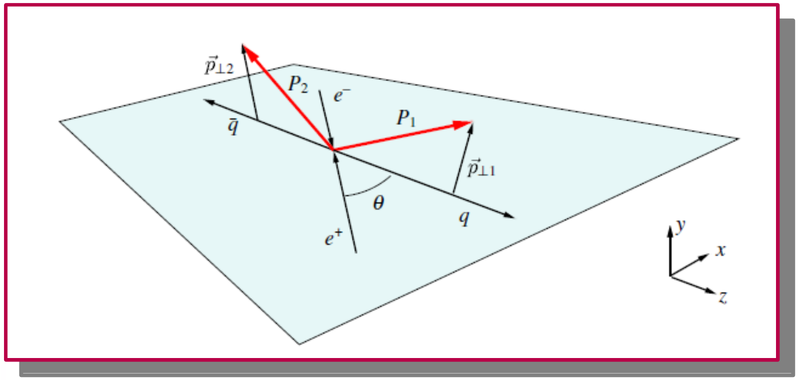


$$\mathrm{d}\sigma^{\uparrow}-\mathrm{d}\sigma^{\downarrow} = \sum_q h_{1q}(x,k_\perp)\otimes \mathrm{d}\Delta\hat{\sigma}(y,\boldsymbol{k}_\perp)\otimes \Delta^N D_{h/q^\uparrow}(z,\boldsymbol{p}_\perp)$$

Transversity

Collins function

BELLE A_{12} – thrust axis method



$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1 dz_2 d\cos\theta d(\varphi_1+\varphi_2)} = \frac{3\alpha^2}{4s}\sum_q e_q^2\Big\{(1+\cos^2\theta)D_{h_1/q}(z_1)D_{h_2/\bar{q}}(z_2) + \frac{1}{4}\sin^2\theta\cos(\varphi_1+\varphi_2)\Delta^N D_{h_1/q^\uparrow}(z_1)\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)\Big\},$$

Having defined

$$\int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z, p_\perp) \equiv \Delta^N D_{h/q^\uparrow}(z).$$

By normalizing to the azimuthal averaged cross section, we have

$$A(z_1, z_2, \theta, \varphi_1+\varphi_2) \equiv \frac{1}{\langle d\sigma\rangle}\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1 dz_2 d\cos\theta d(\varphi_1+\varphi_2)} = 1 + \frac{1}{4}\frac{\sin^2\theta}{1+\cos^2\theta}\cos(\varphi_1+\varphi_2) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1)\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}.$$

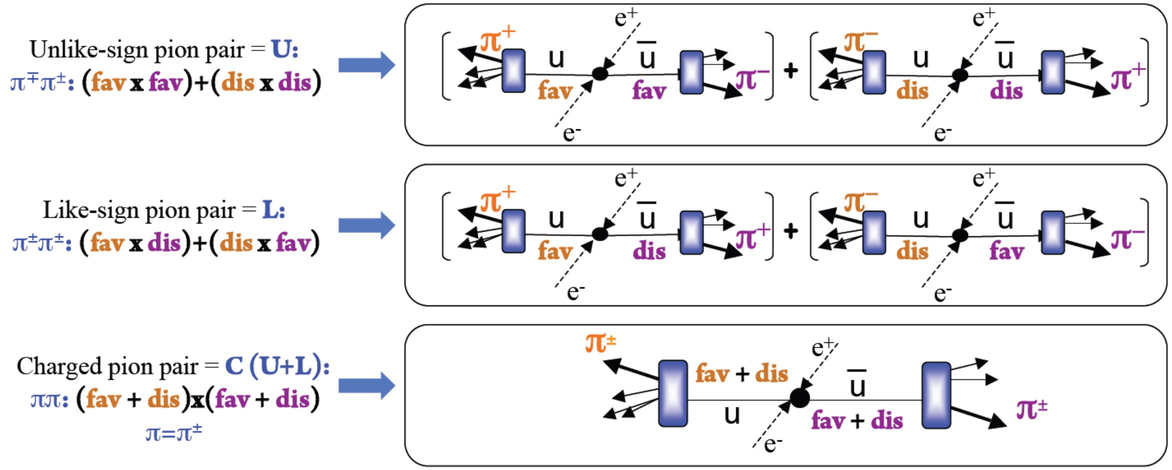
What we are extracting from experimental data Is actually the product of two Collins function azimuthal moments

$$\Delta^N D_{h/q^\uparrow}(z) = \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z, p_\perp) = \int d^2\boldsymbol{p}_\perp \frac{2p_\perp}{z m_h} H_1^{\perp q}(z, p_\perp) = 4H_1^{\perp(1/2)q}(z).$$

BELLE A_{12} – thrust axis method

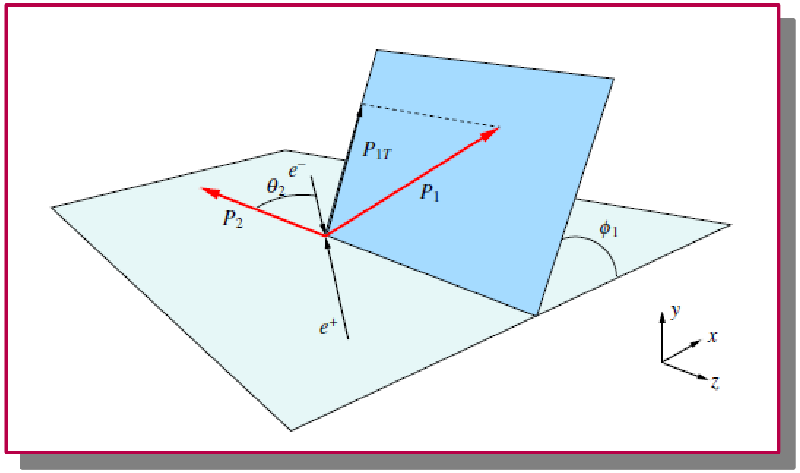
To eliminate false asymmetries, the BELLE (and BaBar) Collaborations measure the ratio of unlike to like sign hadron pair production, and of unlike and like pairs

$$
\begin{aligned}
R \equiv \frac{A_U}{A_L} &= \frac{1 + \frac{1}{4}\cos(\varphi_1 + \varphi_2)\frac{\langle \sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}P_U}{1 + \frac{1}{4}\cos(\varphi_1 + \varphi_2)\frac{\langle \sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}P_L} \\
&\simeq 1 + \frac{1}{4}\cos(\varphi_1 + \varphi_2)\frac{\langle \sin^2\theta\rangle}{\langle 1 + \cos^2\theta\rangle}(P_U - P_L) \\
&\equiv 1 + \cos(\varphi_1 + \varphi_2)A_{12}(z_1, z_2),
\end{aligned}
$$



As Anselm Vossen and David Muller nicely explained in their talks

BELLE A_0 – hadron plane method



Kinematically much more complex than the previous method

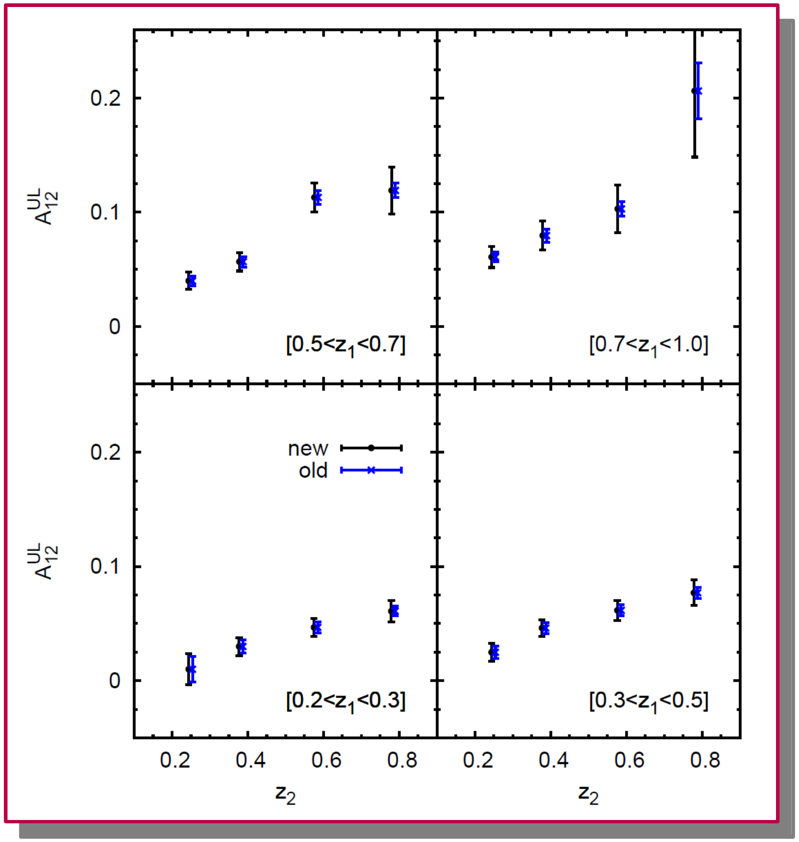
$$R \simeq 1 + \cos(2\phi_1) A_0(z_1, z_2),$$

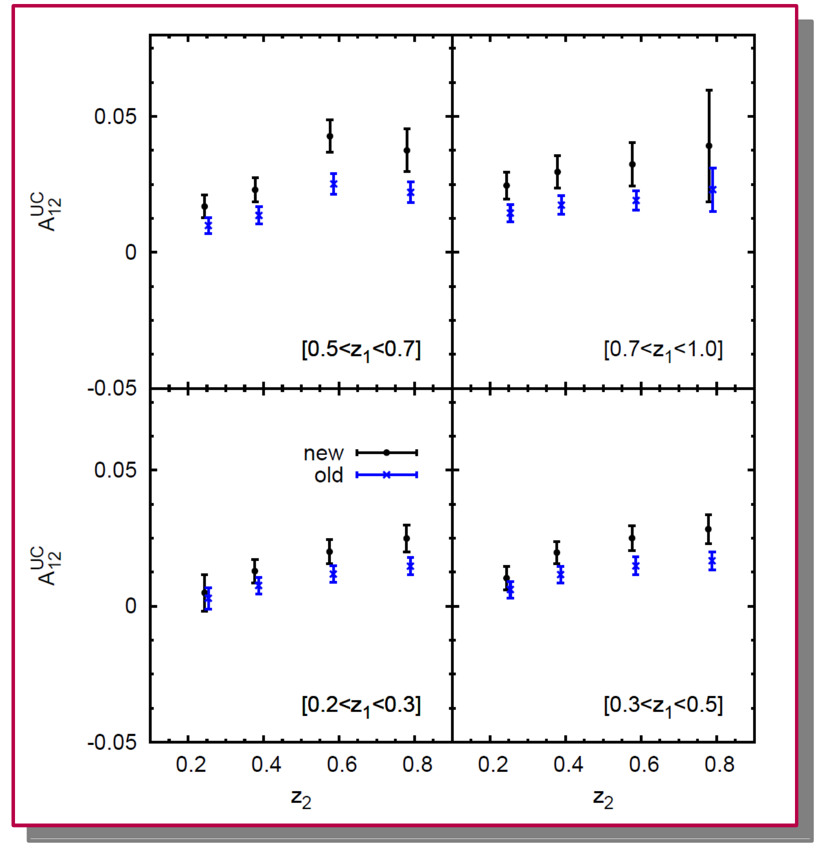
$$A_0(z_1, z_2) = \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} (P_U - P_L),$$

$$A(z_1, z_2, \theta_2, \phi_1) = 1 + \frac{1}{\pi} \frac{z_1 z_2}{z_1^2 + z_2^2} \frac{\sin^2\theta_2}{1 + \cos^2\theta_2} \cos(2\phi_1) \times \frac{\sum_q e_q^2 \Delta^N D_{h_1/q^\uparrow}(z_1) \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2 D_{h_1/q}(z_1) D_{h_2/\bar{q}}(z_2)}$$

The newest data

BELLE A_{12}: old and new data

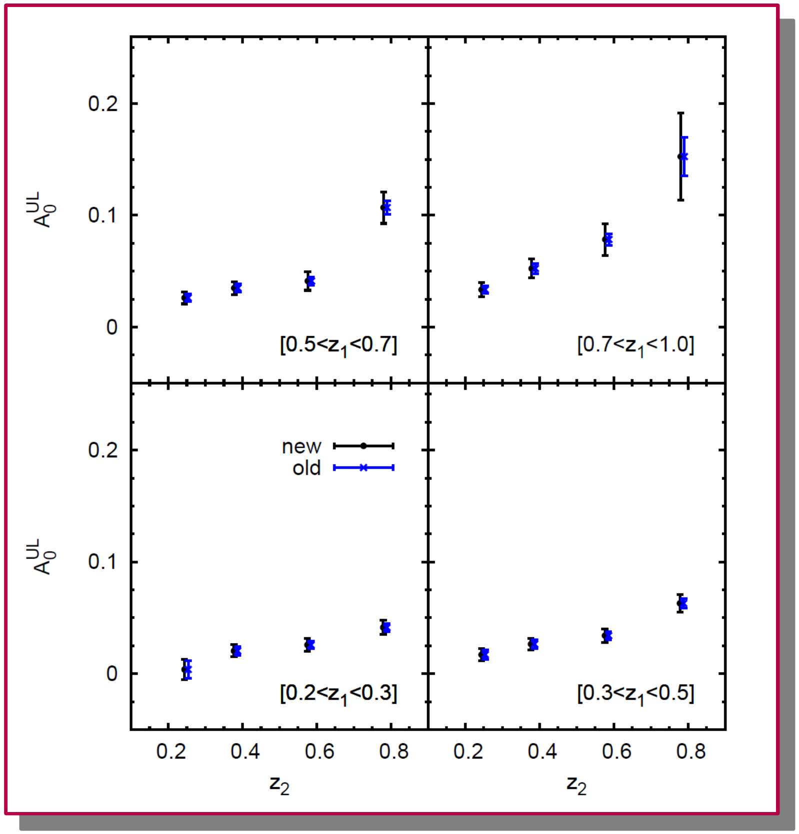


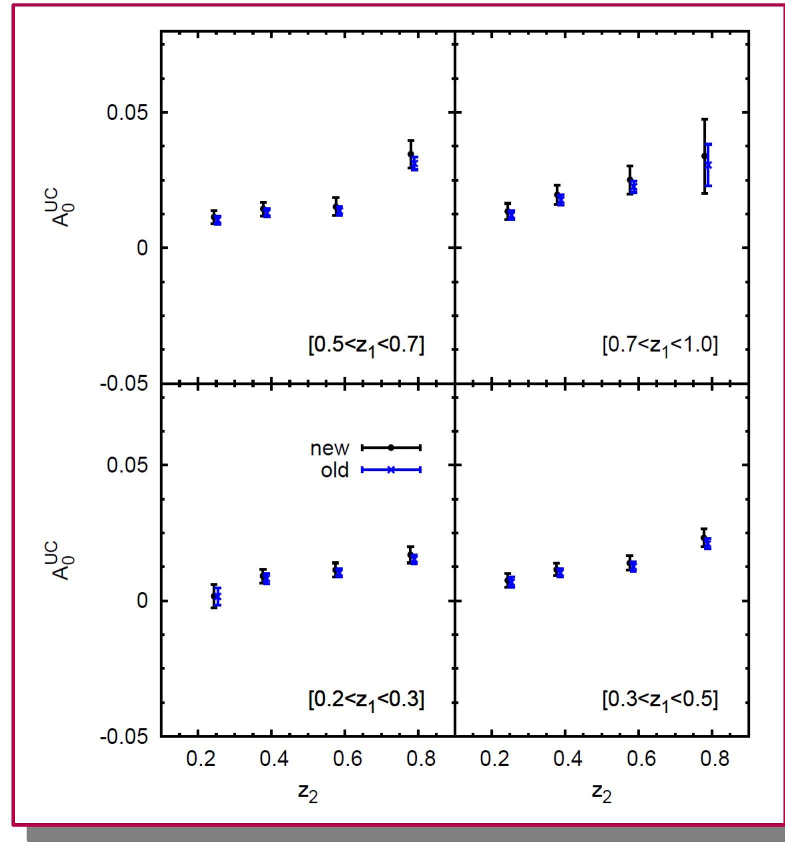


A_{12}^{UL}: same central values
larger error bars

A_{12}^{UC}: larger central values (different normalization)
larger error bars

BELLE A_0: old and new data



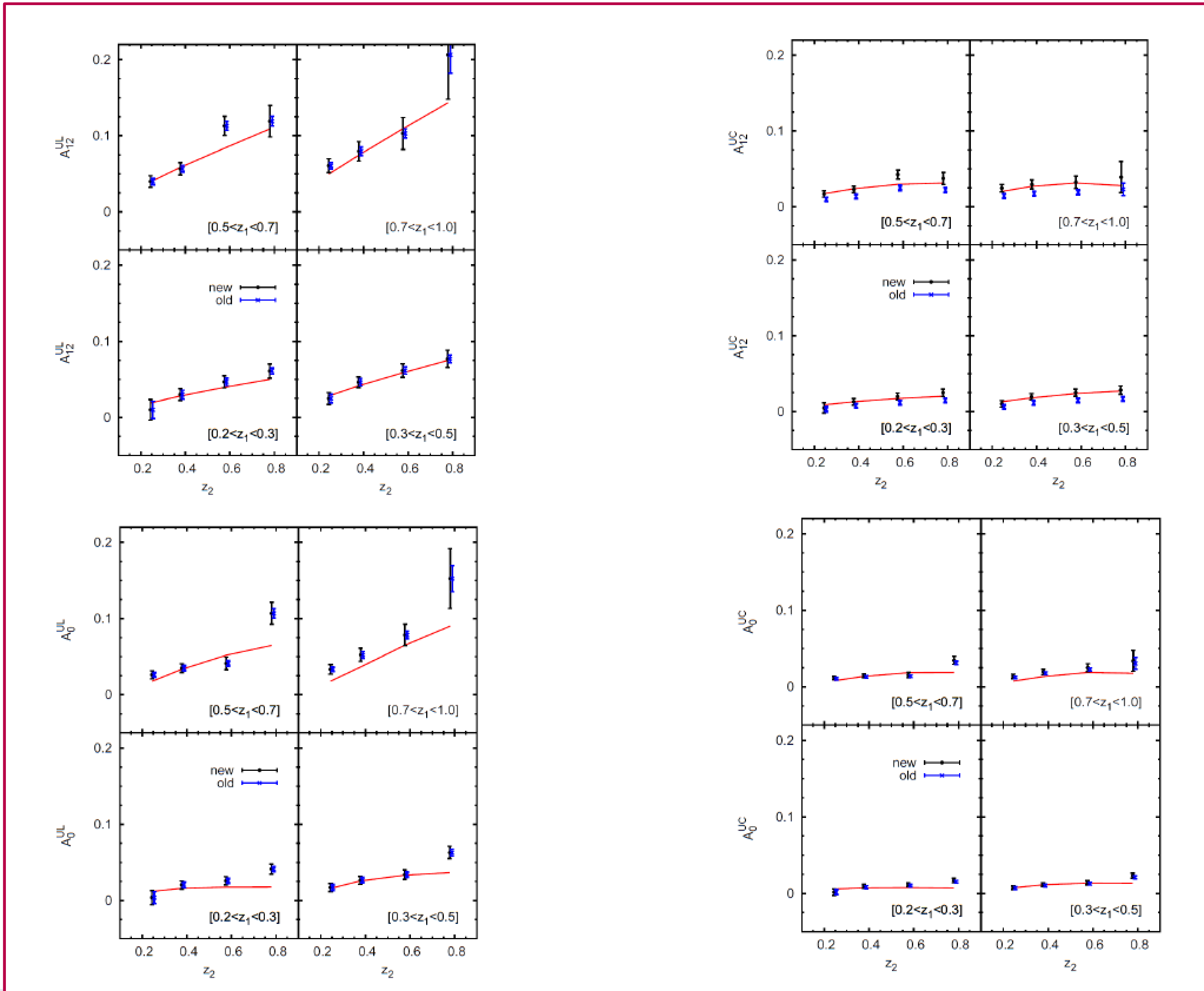


A_{12}^{UL}: same central values
larger error bars

A_{12}^{UC} : larger central values (different mormalization)
larger error bars

Good consistency between UL and UC ratios
Still some tension between thrust axis and hadronic plane methods

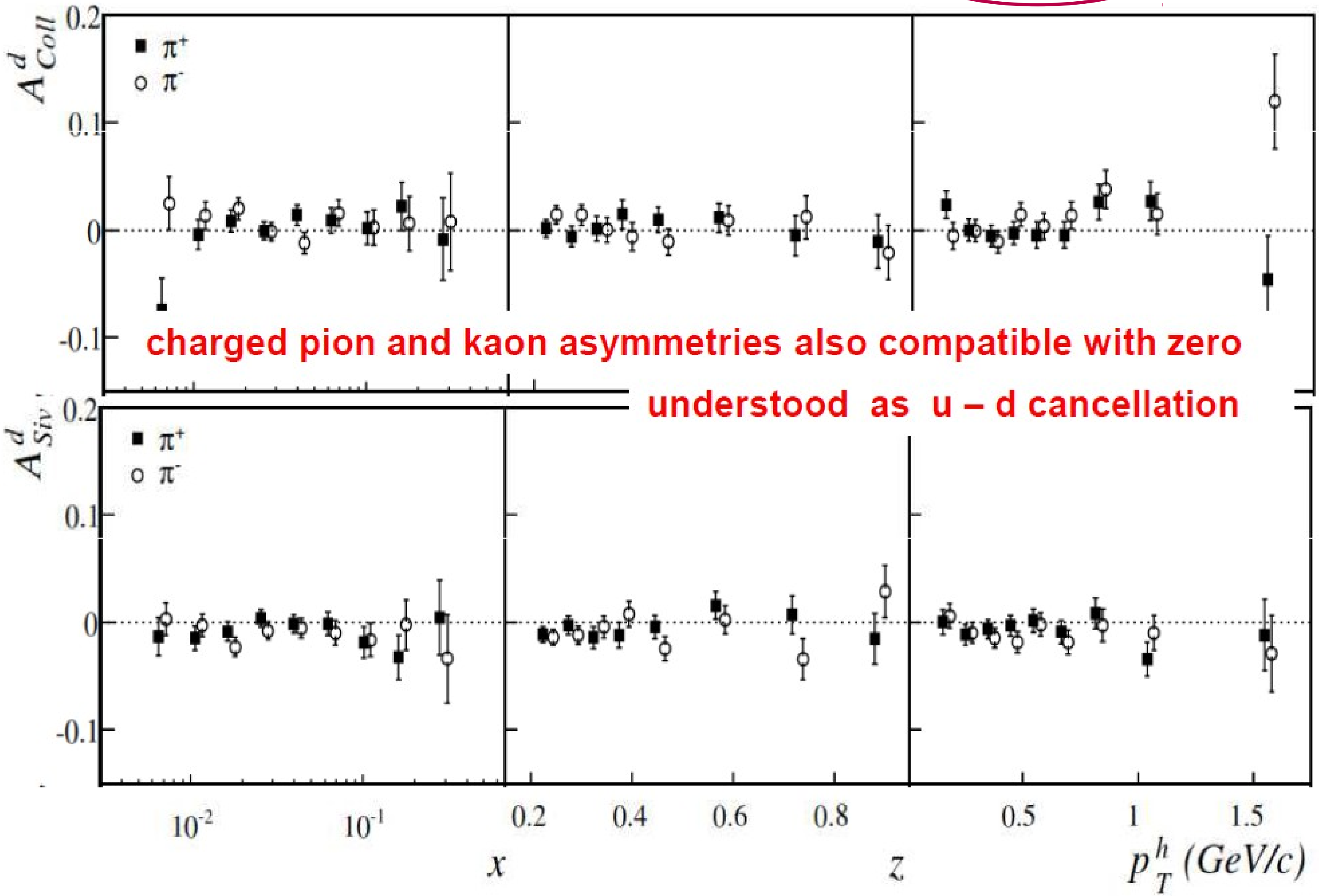
BELLE: old fit, new data



COMPASS deuteron: latest analysis 2009

first results in 2005

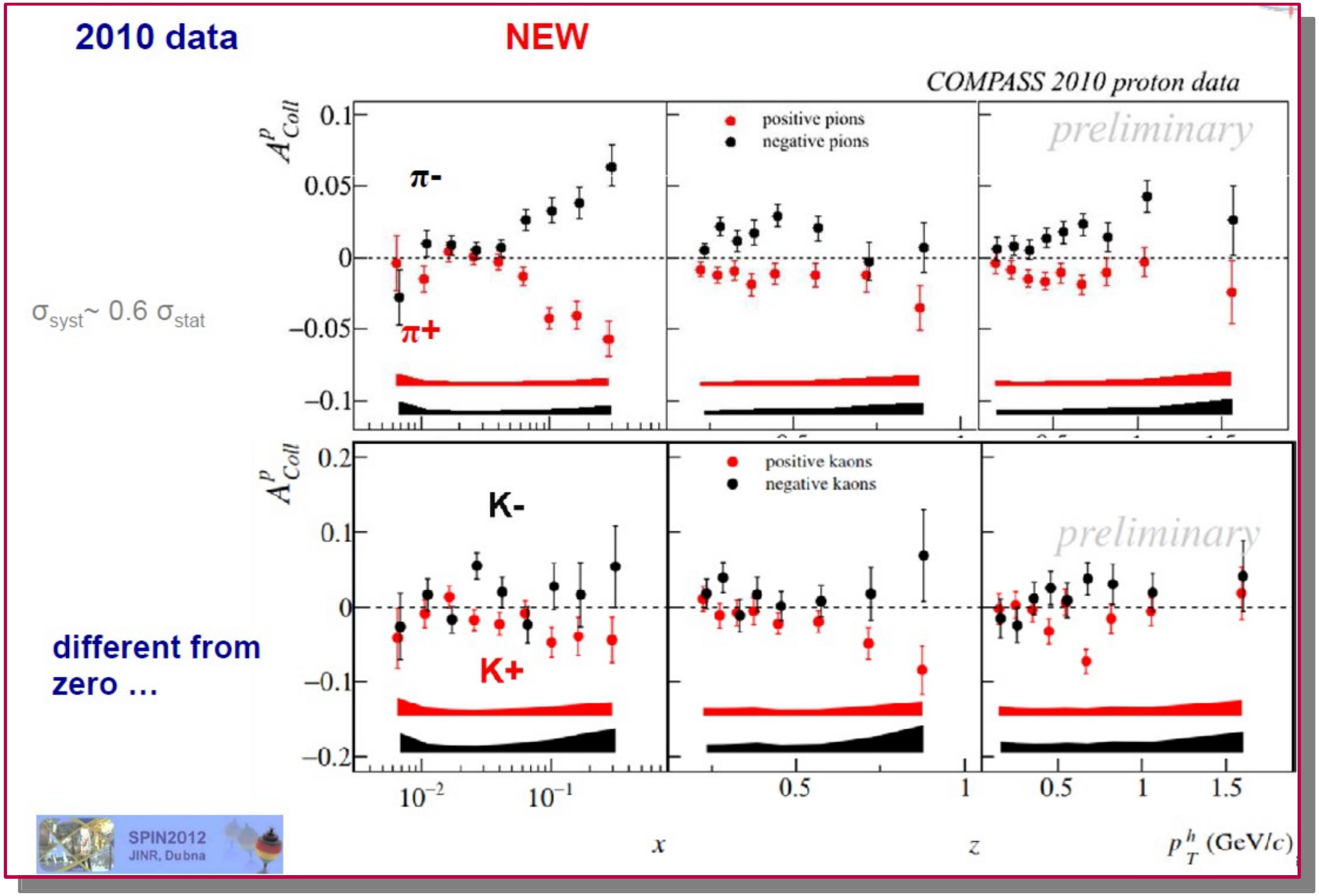
final results on deuteron 2002-2004 data NPB 765 (2007) 31, PLB 673 (2009) 127



charged pion and kaon asymmetries also compatible with zero

understood as u – d cancellation

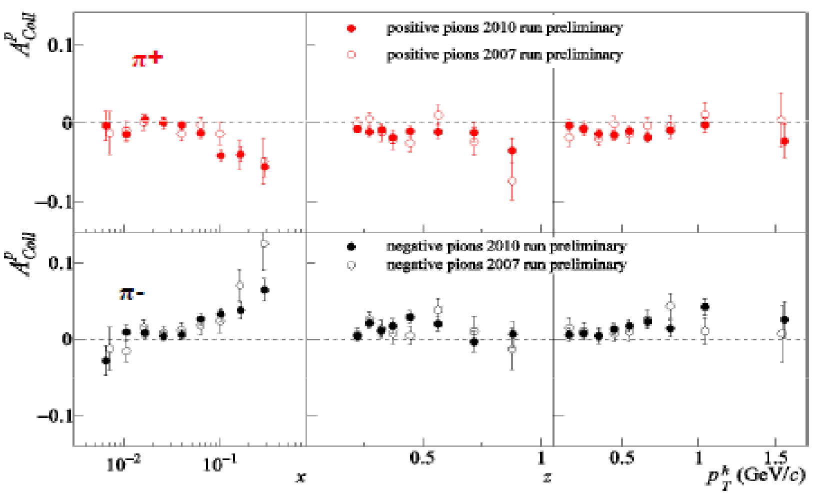
COMPASS proton: newest data 2012



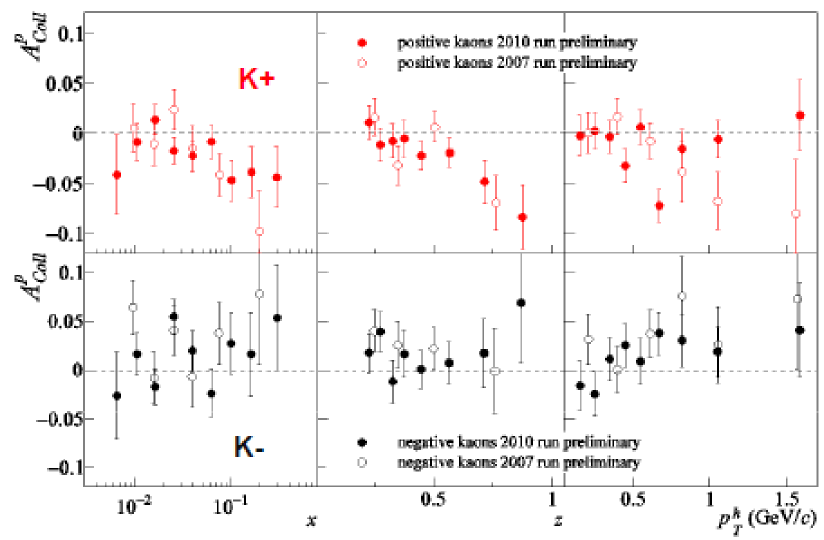
Courtesy of Anna Martin

COMPASS proton:
2007 data versus 2010 data

Collins asymmetry 2010 vs 2007

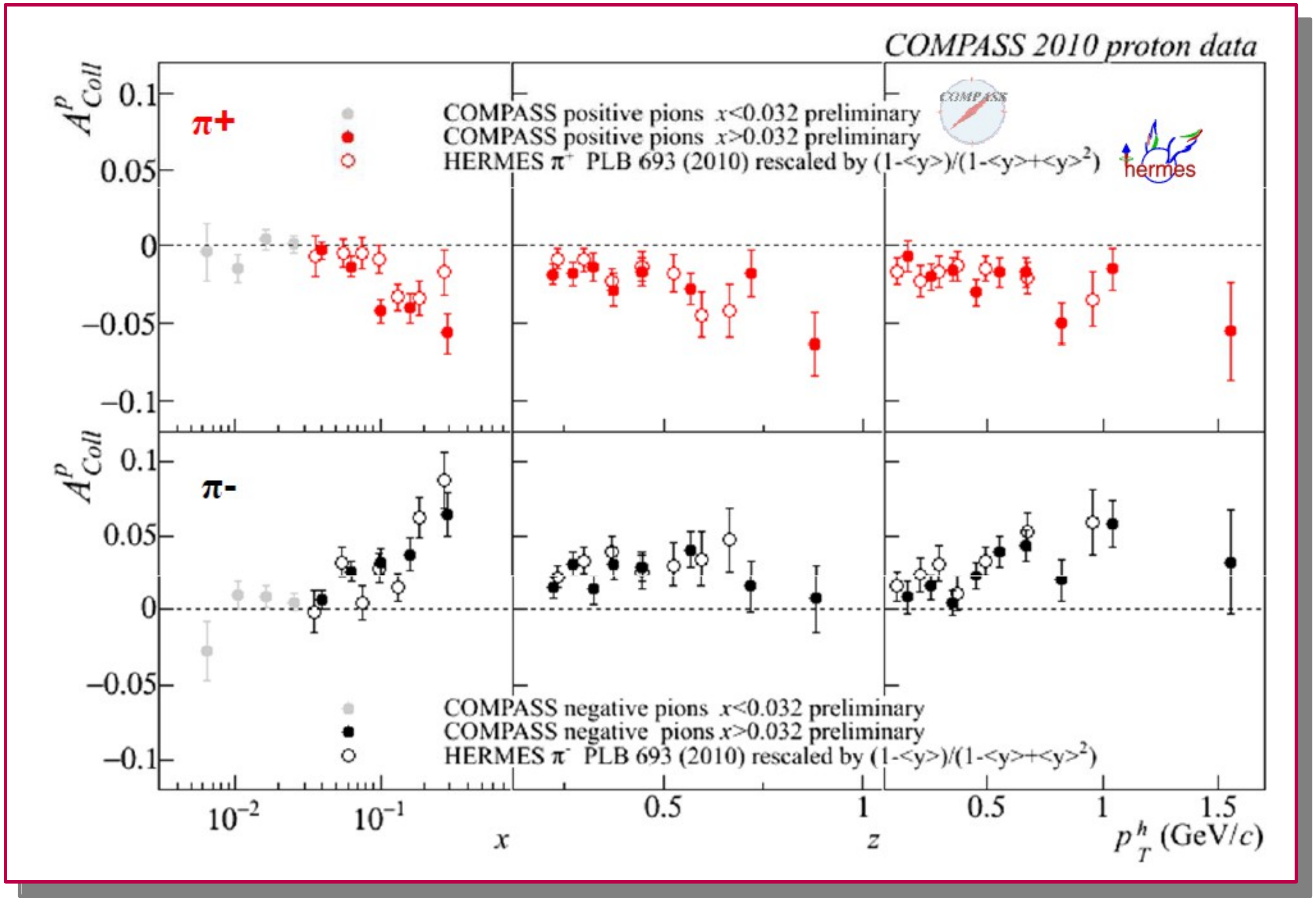


Courtesy of Anna Martin



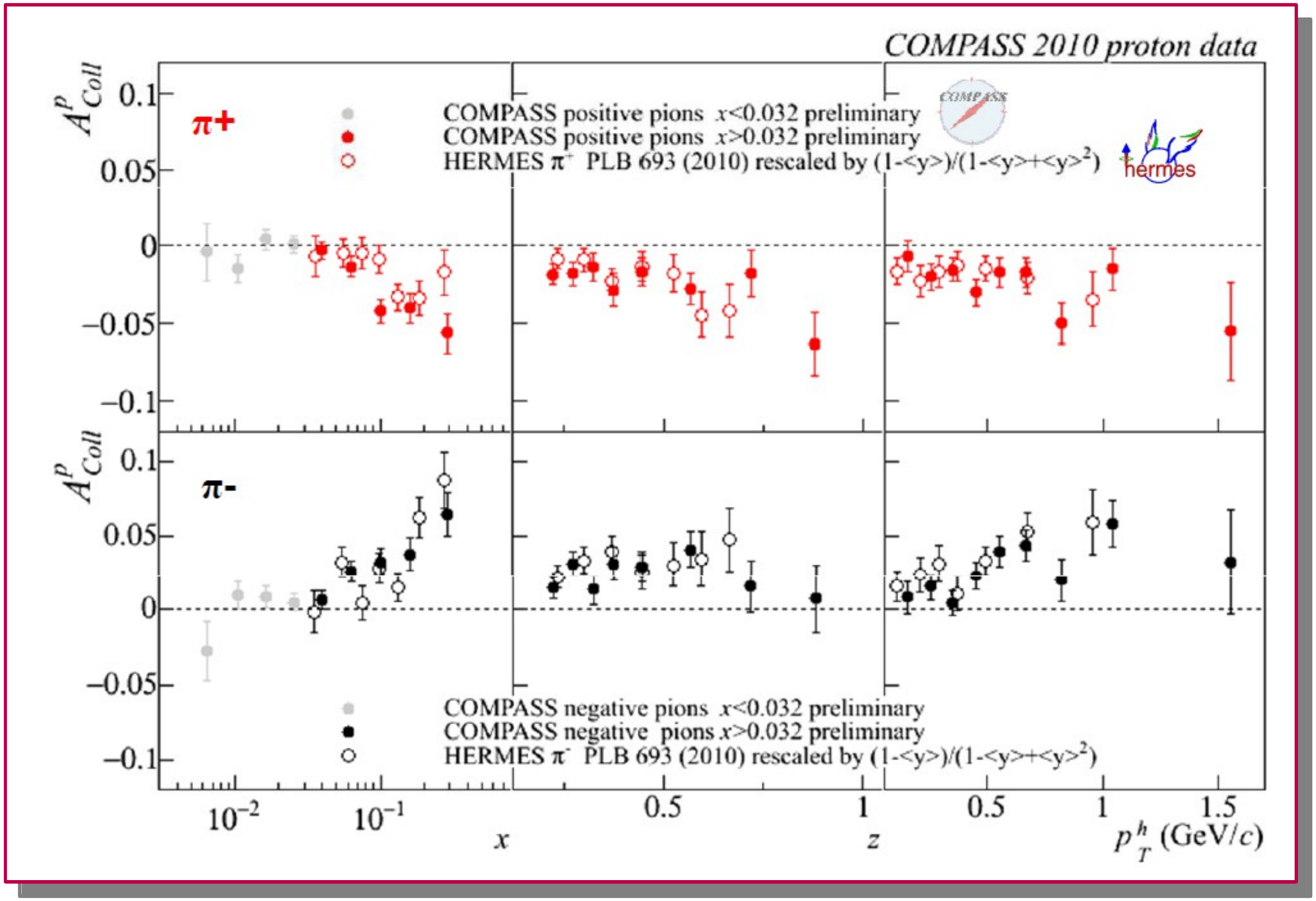
For the moment being only unidentified hadron data are published in PLB 69 (2210) 240

COMPASS and HERMES on proton: comparison



Courtesy of Anna Martin

COMPASS and HERMES on proton: comparison

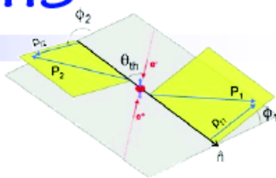


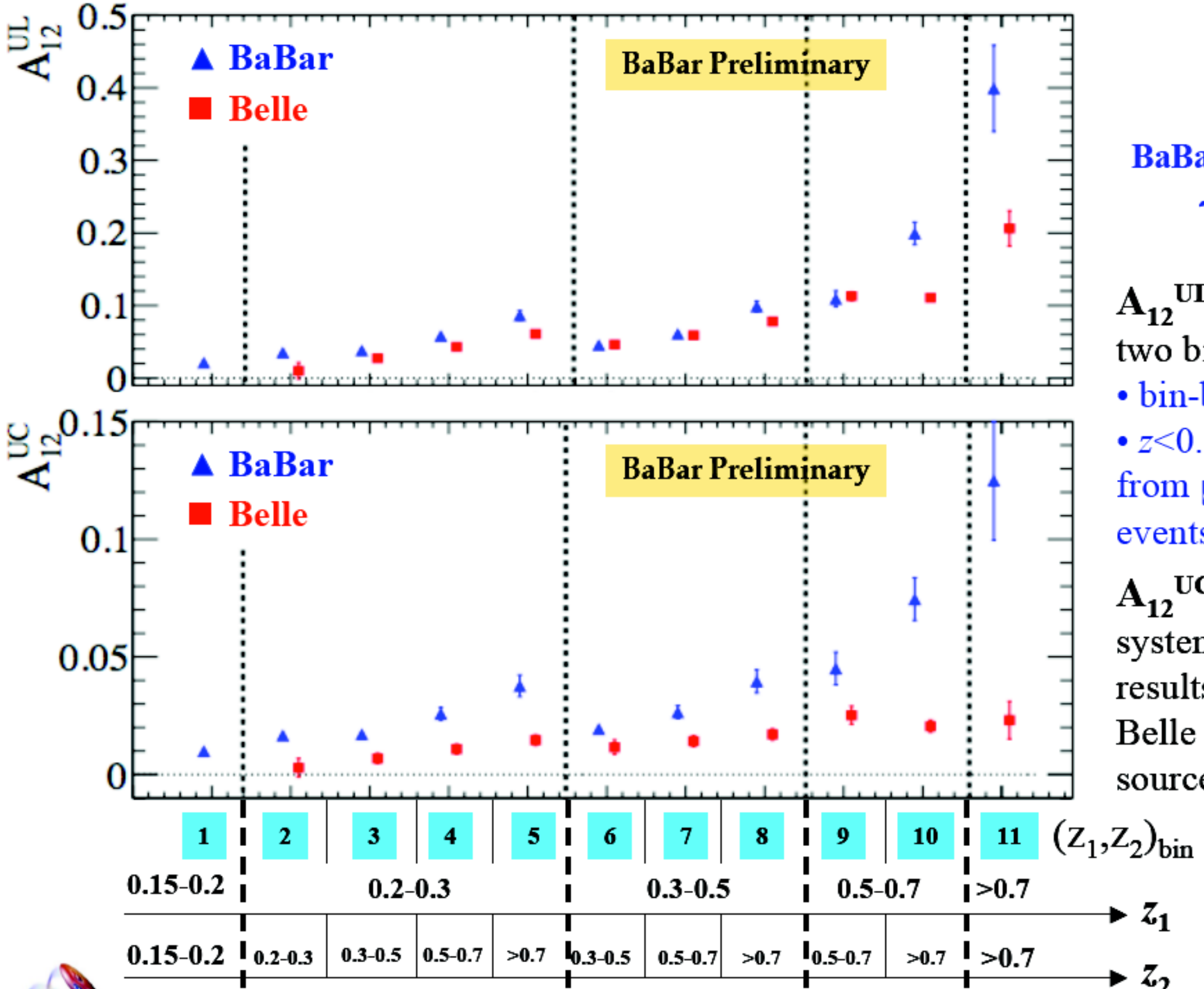
Courtesy of Anna Martin

Future perspectives: BaBar data

BaBar and BELLE: comparison

RF12:BaBar/Belle asymmetries comparisons





BaBar ($0.15<z<0.9$) $\mathcal{L}\sim470$ fb^{-1} **Belle ($0.2<z<1$)** $\mathcal{L}\sim547$ fb^{-1}

PRD 78, 032011 (2008)

A_{12}^{UL}: large discrepancy in the last two bins of z

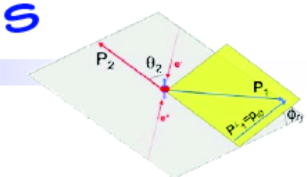
- bin-by-bin correction factors (30%)
- $z<0.9$ to remove the contamination from $\mu\mu\gamma$ background and exclusive events

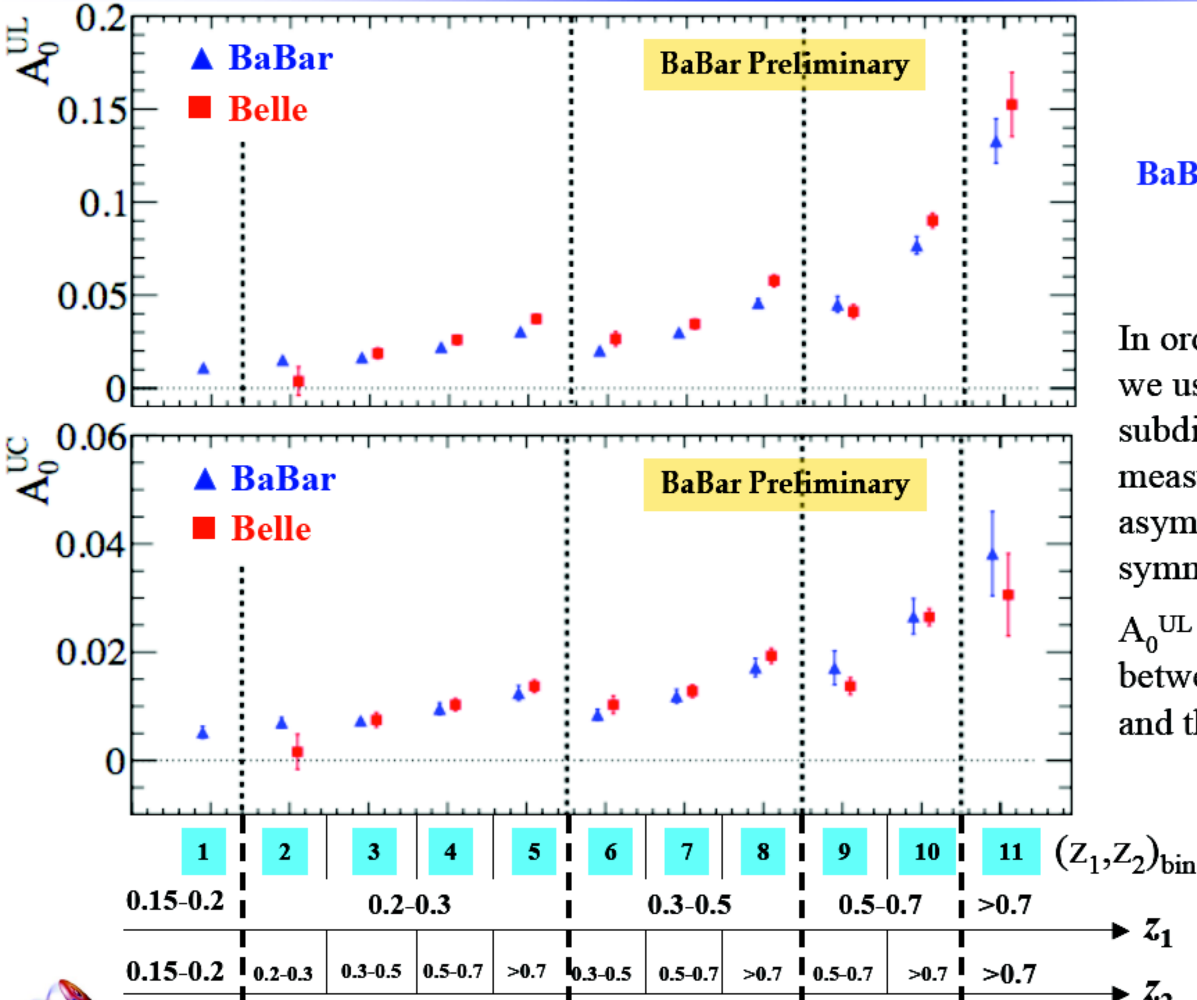
A_{12}^{UC}: BaBar asymmetry systematically above the Belle results for all z.
Belle analysts are investigating the source of discrepancies.

Courtesy of Isabella Garcia

BaBar and BELLE: comparison

RFO:BaBar/Belle asymmetries comparisons





BaBar (0.15<z<0.9) $\mathcal{L}$~470 fb^{-1}

Belle (0.2<z<1) $\mathcal{L}$~547 fb^{-1}

PRD 78, 032011 (2008)

In order to perform this comparison, we used 10 (+1) symmetrized z-bin subdivisions, averaging the measured Belle and BaBar asymmetries which fell in the same symmetric bins

A_0^{UL} and A_0^{UC} : good agreement between the BaBar asymmetries and the Belle results.

Courtesy of Isabella Garcia

The fit:
new global analysis and new results

**(MINUIT ran until a minute ago …
thanks Stefano and Alexei
for your formidable efforts !)**

The fit: parameters and parametrization

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle},$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2.$$

These parameters where extracted in 2005 mostly from EMC data. It is now about time to perform a new extraction of unpolarized TMDs with the outstanding high precision multiplicity data from COMPASS, HERMES, BaBar and hopefully BELLE.

Transversity

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) [f_{q/p}(x) + \Delta q(x)] \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_T}}{\pi \langle k_\perp^2 \rangle_T},$$

$$\mathcal{N}_q^T(x) = N_q^T x^\alpha (1-x)^\beta \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha \beta^\beta},$$

Collins

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle},$$

$$\mathcal{N}_q^C(z) = N_q^C z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^\gamma \delta^\delta},$$

$$h(p_\perp) = \sqrt{2e} \frac{p_\perp}{M} e^{-p_\perp^2/M^2},$$

The fit: parameters and parametrization

Transversity

$$\mathcal{N}_q^T(x) = N_q^T x^{\alpha}(1-x)^{\beta}\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^{\alpha}\beta^{\beta}},$$

Collins function

$$\mathcal{N}_q^C(z) = N_q^C z^{\gamma}(1-z)^{\delta}\frac{(\gamma+\delta)^{(\gamma+\delta)}}{\gamma^{\gamma}\delta^{\delta}},$$

$$h(p_\perp) = \sqrt{2e}\,\frac{p_\perp}{M}\,e^{-p_\perp^2/M^2},$$

Transversity

- N_u ~ 0.5
- N_d ~ -1.0
- α ~ 1.0
- β ~ 3.5 ± large error

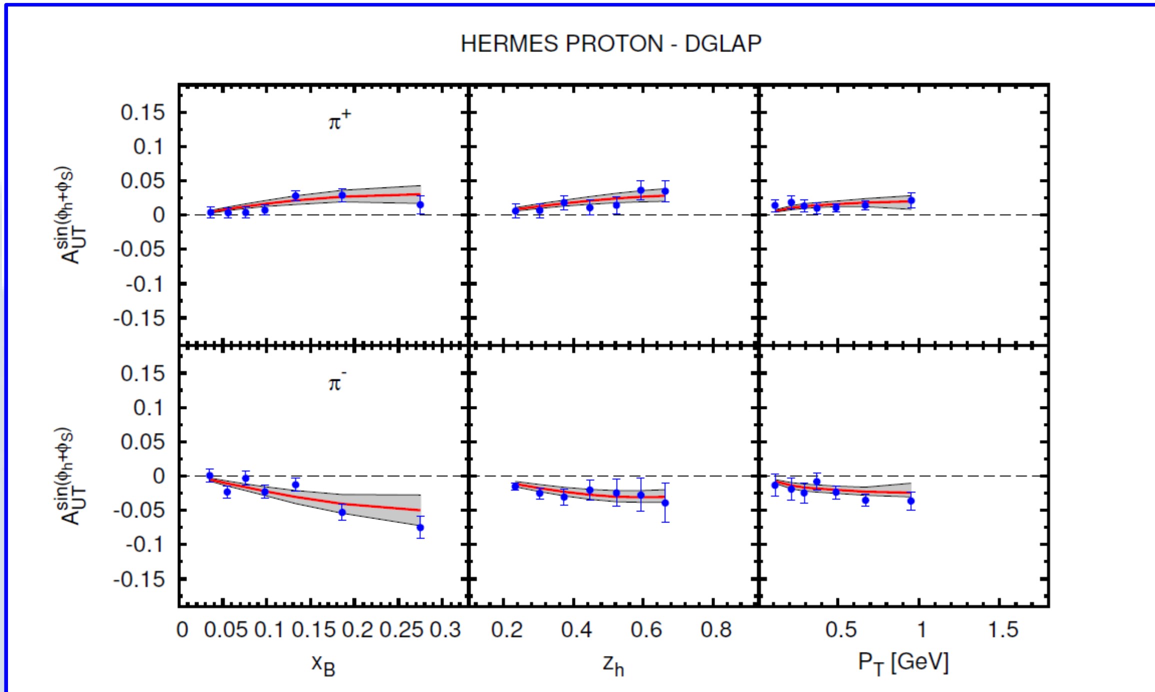
Collins function

- N_{fav} ~ 0.5
- N_{unf} ~ - 1.0
- γ ~ 1.0
- δ ~ 0.0 ± large error
- M^2 ~ 1.5 GeV^2

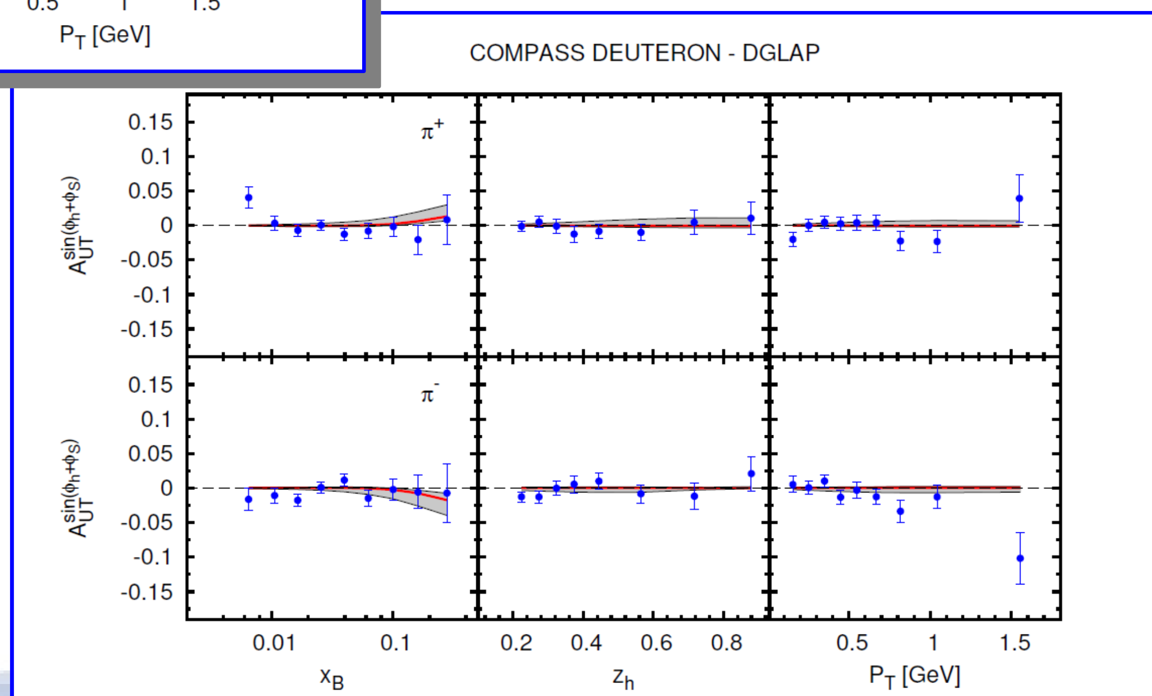
χ^2_{dof} = 0.8

Previous extractions:
Anselmino et al., Phys. Rev. D 75, 054032 (2006)
Nucl.Phys.Proc.Suppl. 191, 98 (2009)

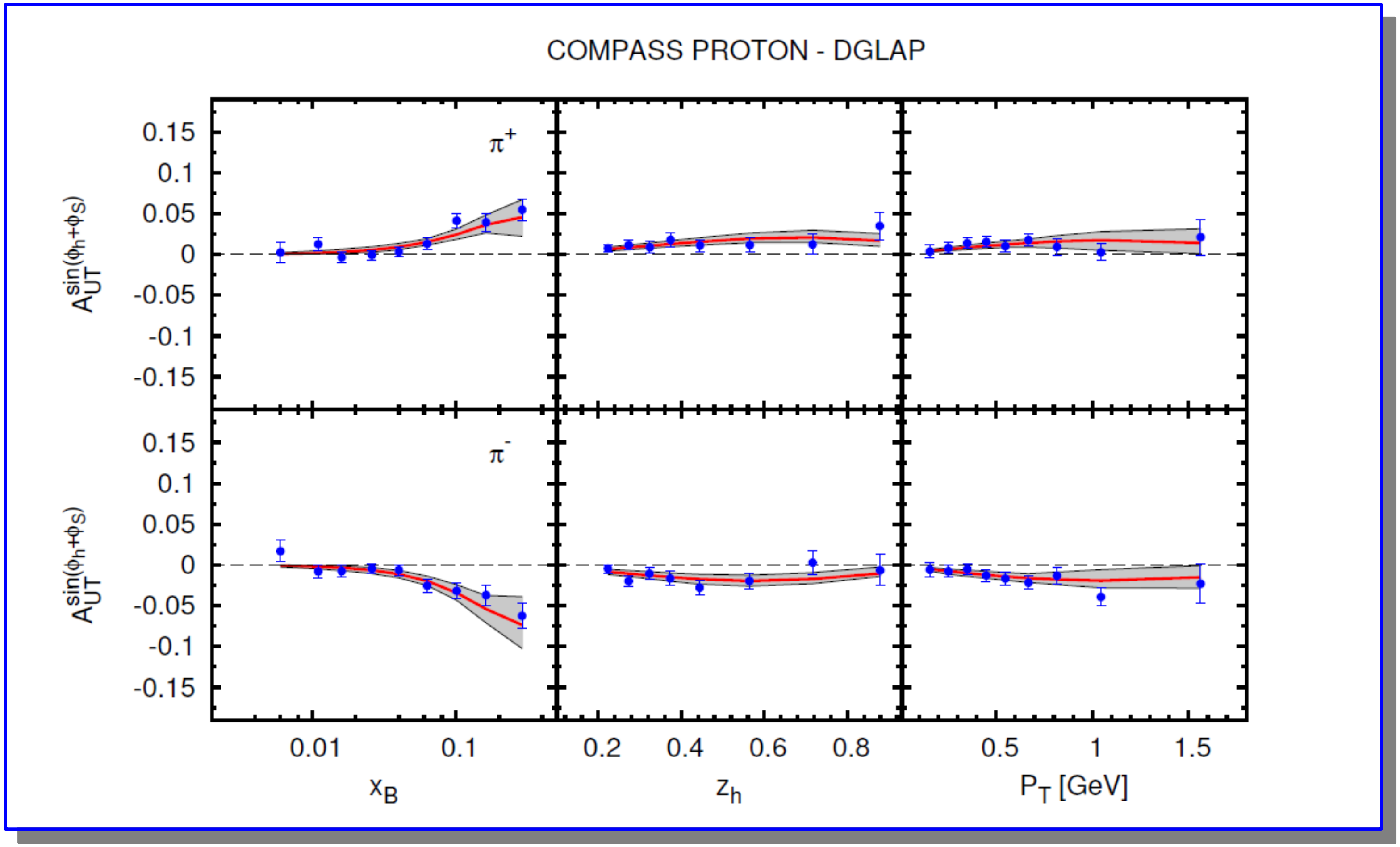
HERMES and COMPASS - deuteron



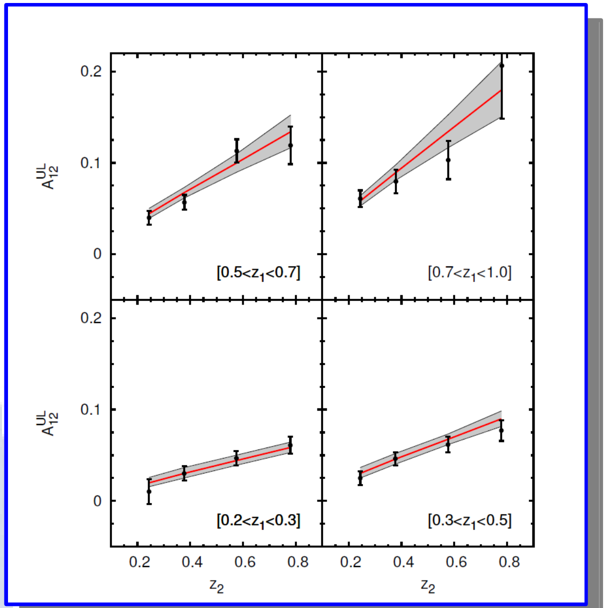
The bands are obtained by exploring the (9) parameter space over the region covered by $\Delta\chi^2 = 17.2$

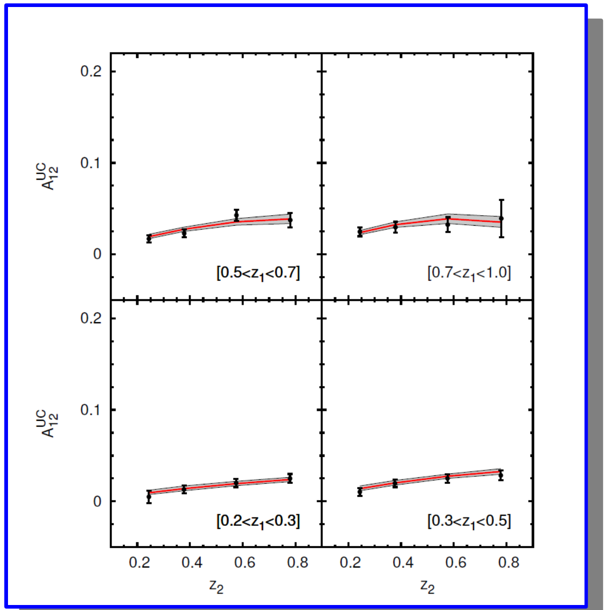


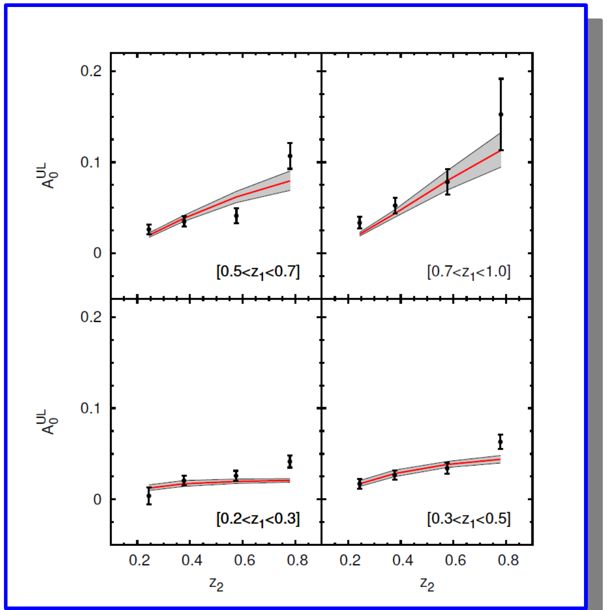
COMPASS - proton

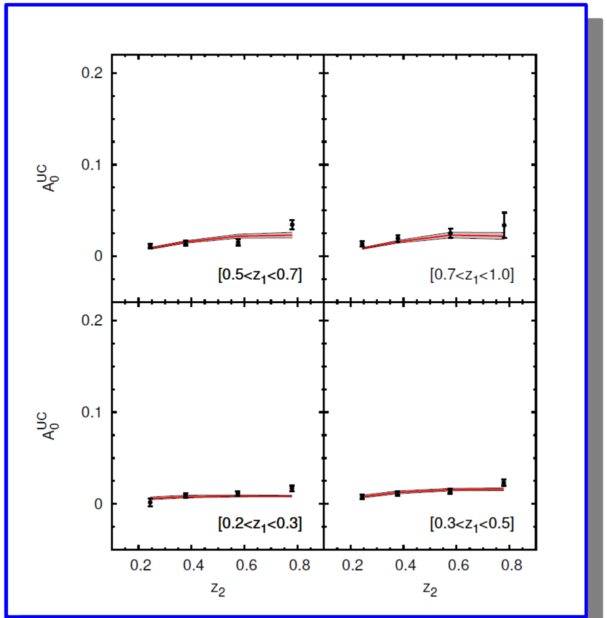


BELLE A_{12} and A_0

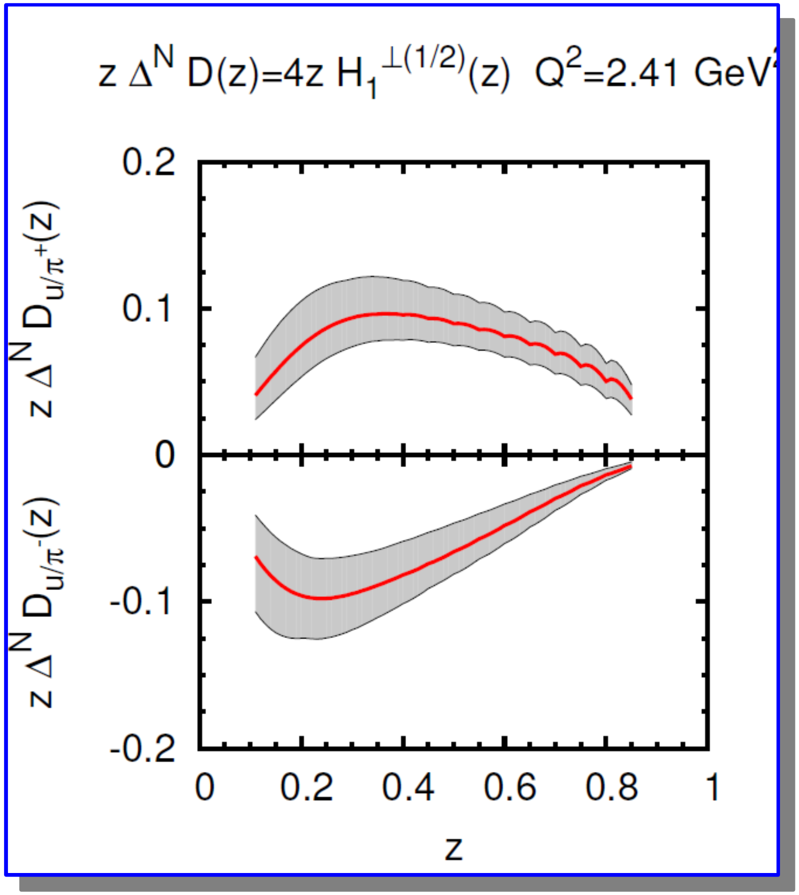


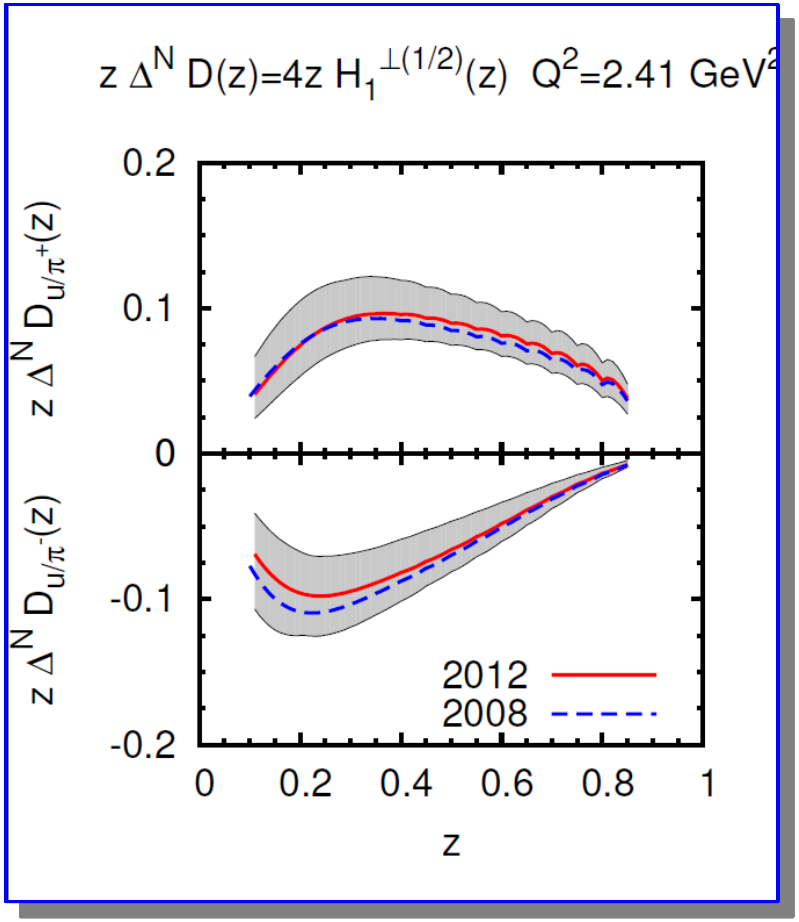




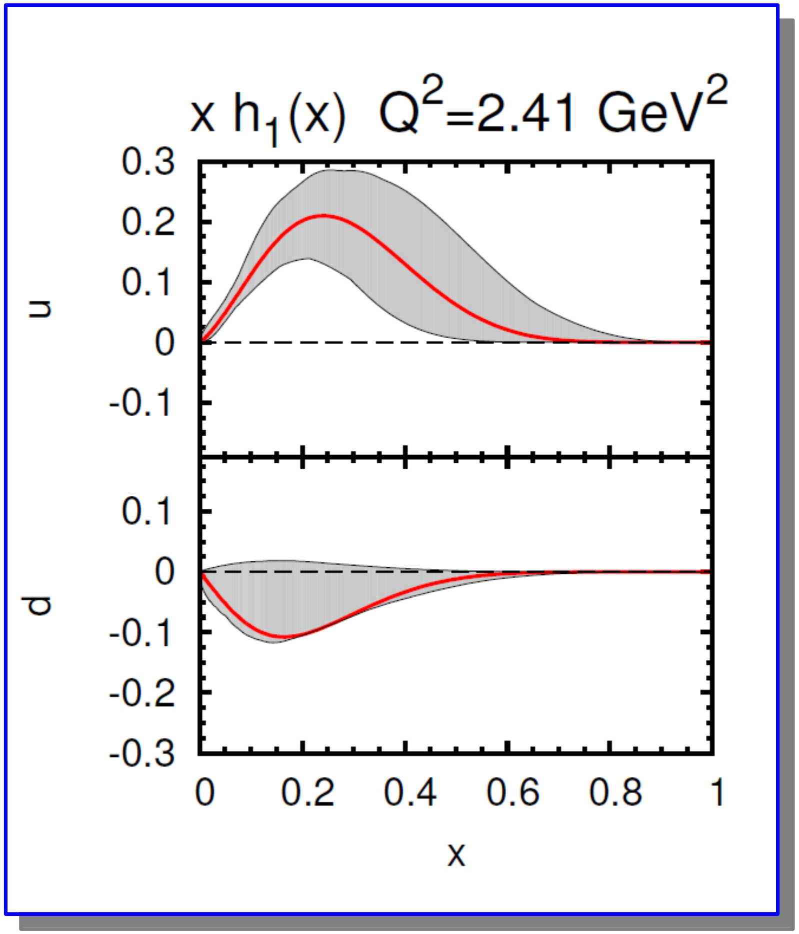


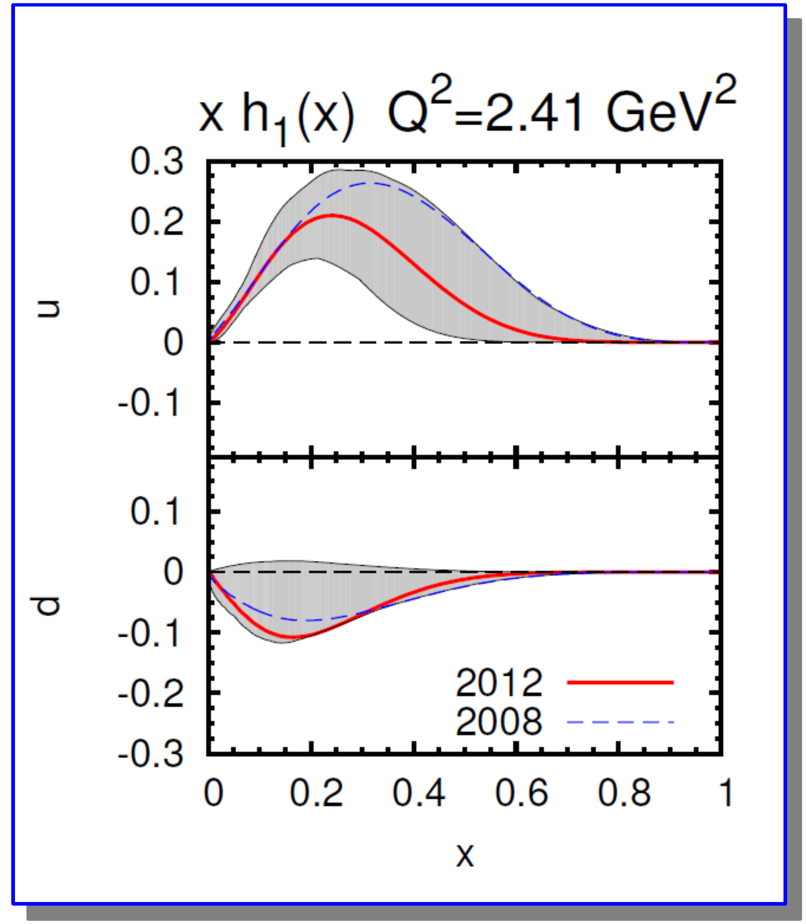
The Collins function



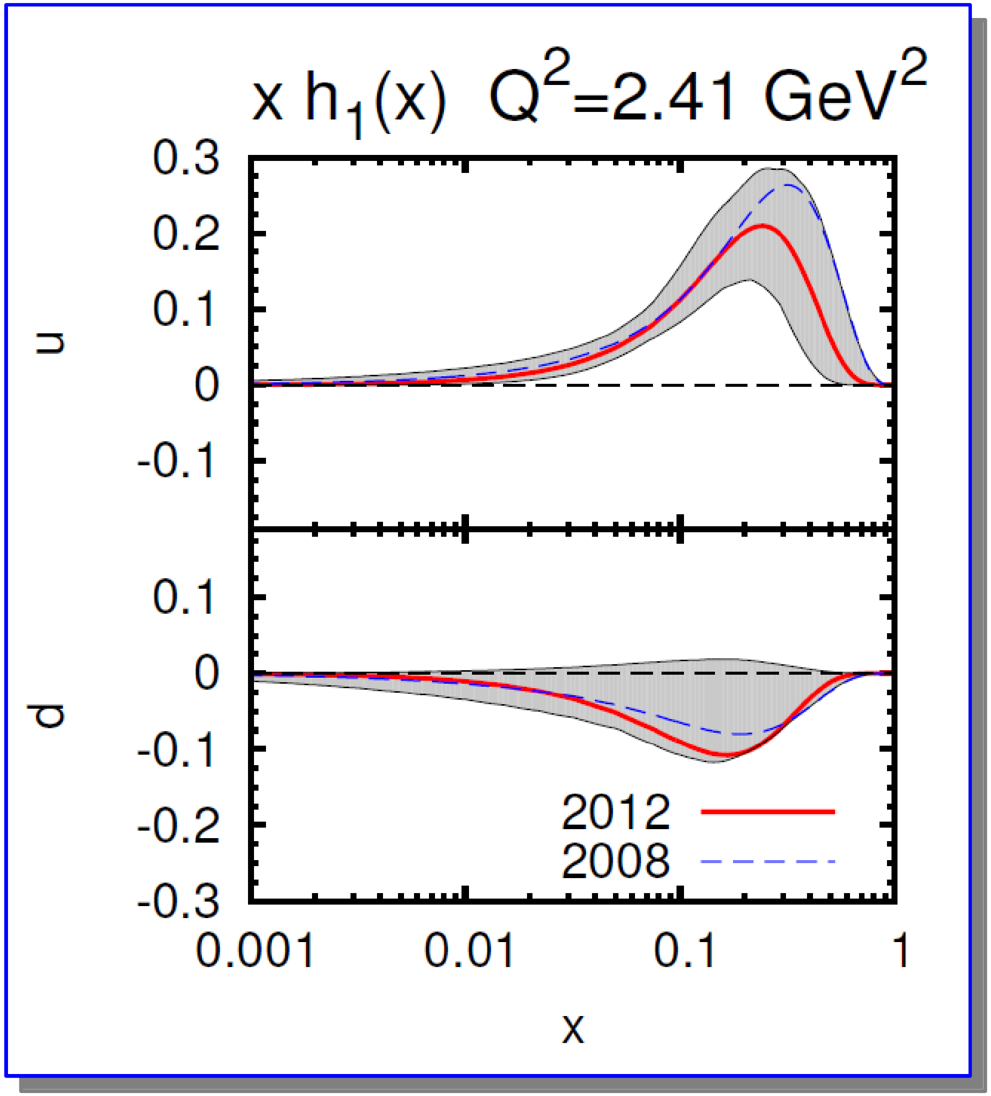


Transversity



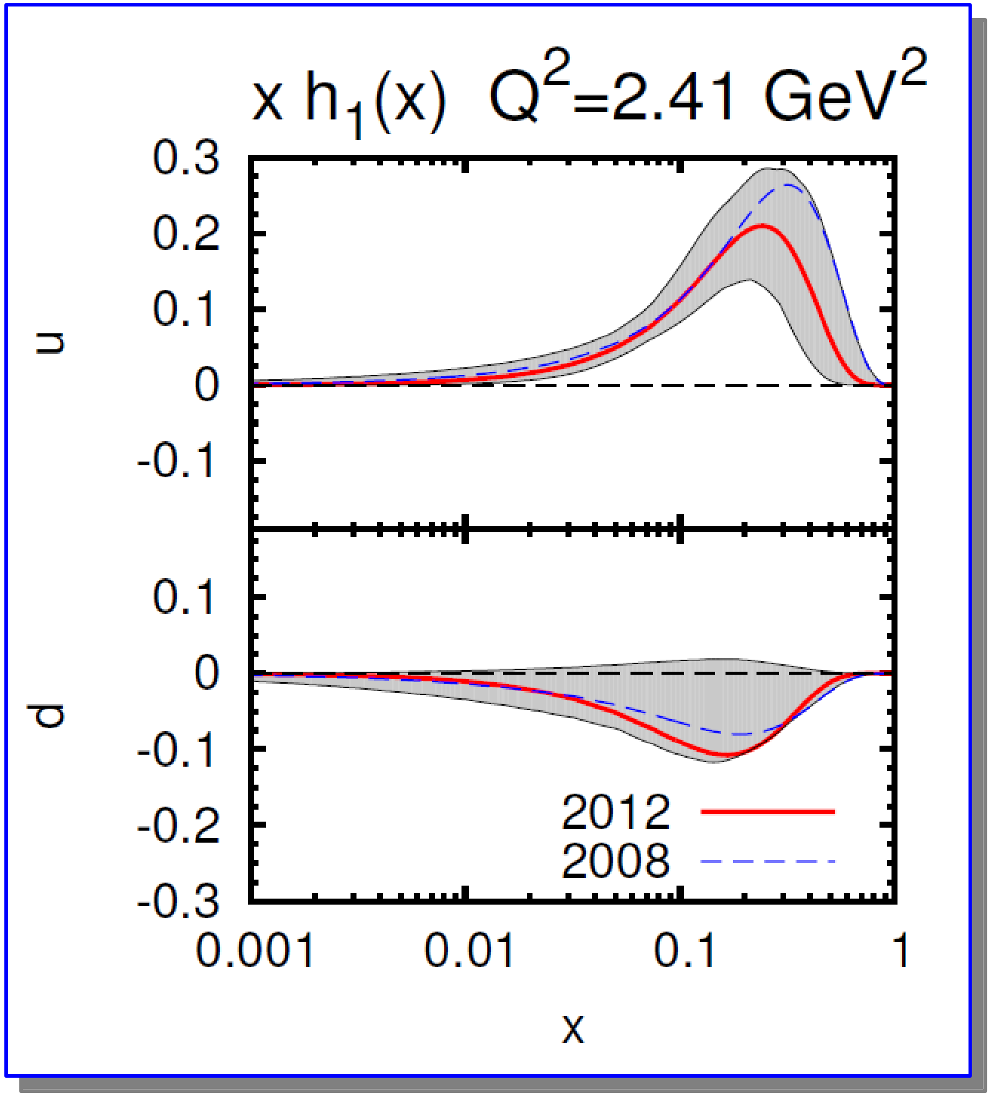


Transversity



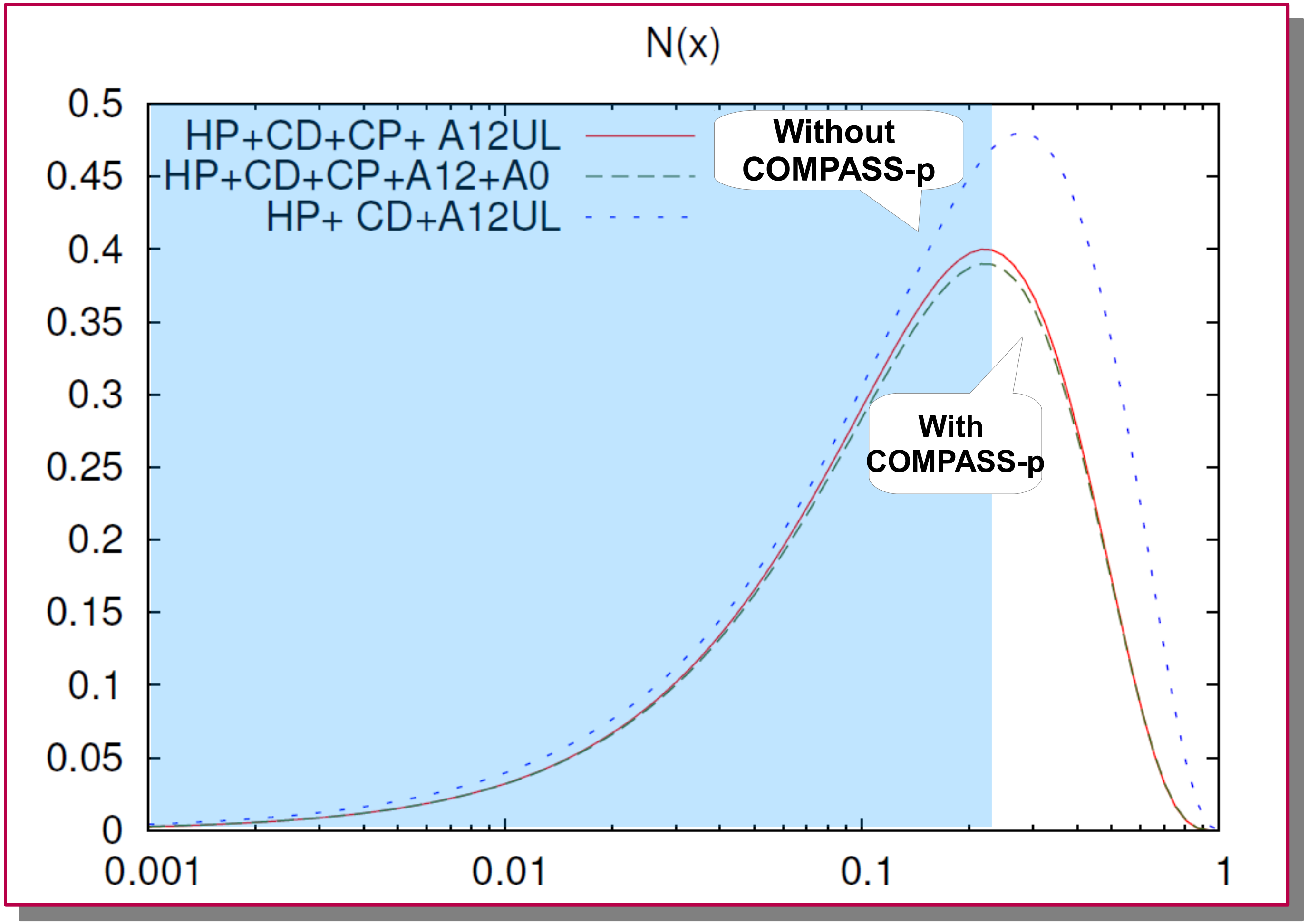
The 2012 extraction is perfectly consistent with the 2008 extraction in the region covered by the experimental data

Transversity



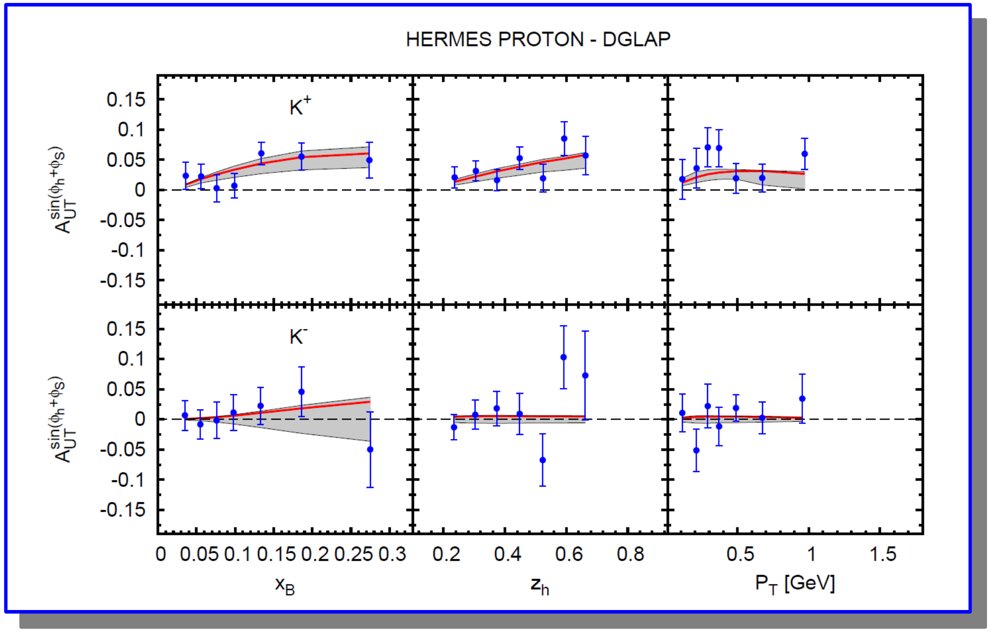
The 2012 extraction is perfectly consistent with the 2008 extraction in the region covered by the experimental data

Impact of new COMPASS-proton data on the fit results



The new COMPASS-proton data show a preference for a smaller transversity functions than those extracted previously

Collins fragmentation function for kaons



$\chi^2_{dof} = 1.0$

Kaon Collins function

$N_{fav} \sim 1.0$

$N_{unf} \sim + 1.0$

$\gamma \sim 1.0$ fixed

$\delta \sim 0.0$ fixed

$M^2 \sim 0.7$ GeV^2

- At present, only SIDIS data for K^+ and K^- production are available for the extraction of the Collins function for kaons.
- In this extraction the transversity functions are those obtained by fitting pion data (only valence contribution) → strong suppression of s and s-bar contributions. As a consequence N_{fav} is stretched to its maximum.
- Favoured and unfavoured K-Collins functions have the same sign,while for pion production they have opposite signs.
- e^+e^- data from BELLE and BaBar will soon be available for kaon pair production (see Francesca Giordano talk) which will allow a proper extraction.

TMD Evolution formalism

TMD Evolution formalism

Let F be either an unpolarized distribution or fragmentation function in the **impact parameter space**.
In general terms, its TMD evolution equation can be written as

$$\widetilde{F}(x, \boldsymbol{b}_T; Q) = \widetilde{F}(x, \boldsymbol{b}_T; Q_0)\, \widetilde{R}(Q, Q_0, b_T)\, \exp\left\{ -g_K(b_T) \ln \frac{Q}{Q_0} \right\}$$

Aybat, Collins, Qiu, Rogers

J.C. Collins, Foundation of Perturbative QCD, Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology, No. 32, Cambridge University Press, 2011.

S. M. Aybat and T. C. Rogers, Phys. Rev. D83, 114042 (2011), arXiv:1101.5057 [hep-ph]

S. M. Aybat, J. C. Collins, J.-W. Qiu and T.C. Rogers, arXiv:1110.6428 [hep-ph]

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Input function

Aybat, Collins, Qiu, Rogers

Unknown, but universal and scale Independent, input function

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}$$

$$\gamma_K(\mu) = \alpha_s(\mu)\,\frac{2\,C_F}{\pi}$$

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2}-\ln\frac{Q^2}{\mu^2}\right)$$

TMD Evolution formalism

Parametrization of the unknown functions

$$\widetilde{f}_{q/p}(x,b_T;Q) = f_{q/p}(x,Q_0)\ \widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-b_T^2\left(\alpha^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\},$$

$$\widetilde{D}_{h/q}(z,b_T;Q) = \frac{1}{z^2}D_{h/q}(z,Q_0)\ \widetilde{R}(Q,Q_0,b_T)\ \exp\left\{-b_T^2\left(\beta^2 + \frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}$$

Under some conditions, it is possible to find an analytical solution for the evolution equations
For the unpolarized TMD parton fragmentation function, we have

$$D_{h/q}(z,p_\perp,Q) = D_{h/q}(z,Q_0)\ R(Q,Q_0)\ \frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2}$$

$$w_F^2 \equiv w_F^2(Q,Q_0) = \langle p_\perp^2\rangle + 2z^2 g_2 \ln\frac{Q}{Q_0}$$

TMD Evolution

TMD evolution Very rapidly broadens and dilutes the functions with growing Q

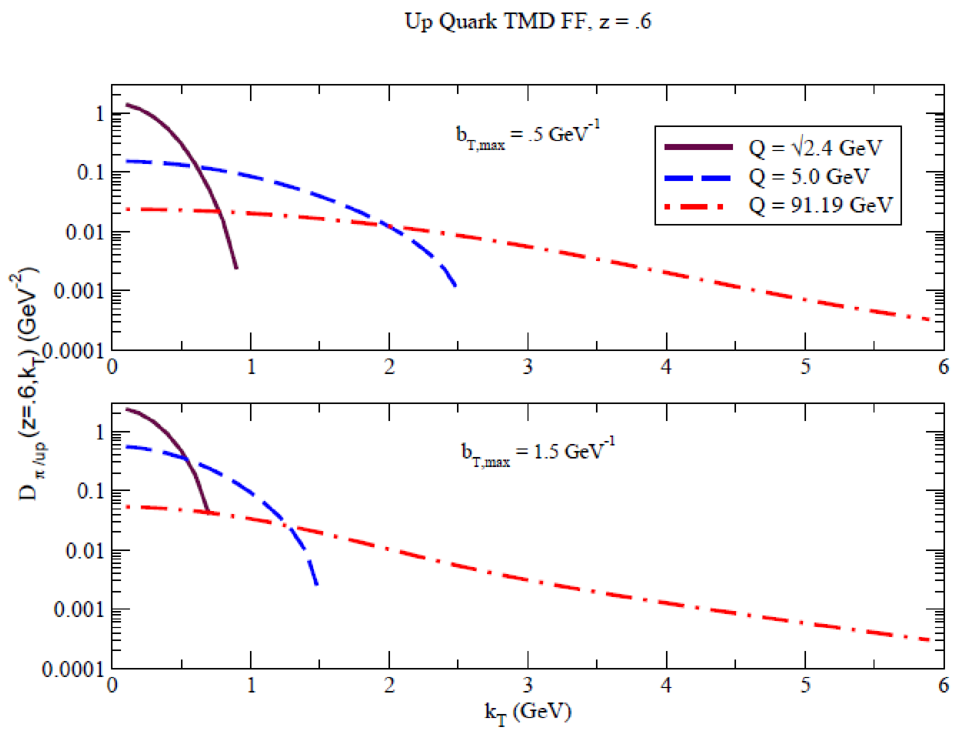
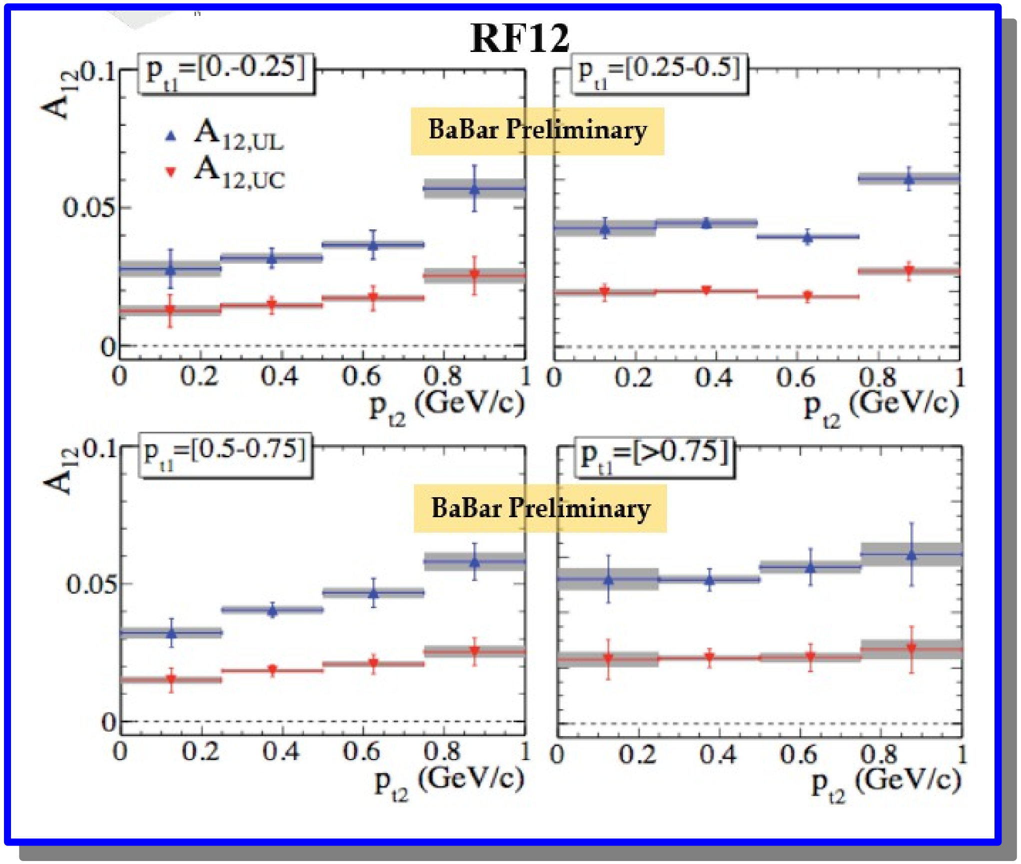


FIG. 4: TMD FF for charged pions from a hadronizing up-quark. The upper plot is for $g_2 = .68$ GeV^2 and the lower plot is for $g_2 = .184$ GeV^2. In each case, TMD FF matches to the STM fit at $Q = \sqrt{2.4}$ GeV. (See online version for color.)

S. Mert Aybat and Ted. C. Roger, Phys.Rev. D83 (2011) 114042

TMD Evolution

TMD evolution very rapidly broadens and dilutes the fragmentation functions (in pt) with growing $Q^2 \rightarrow$ this might explain why BaBar (Q^2 =100 GeV^2) measures a flat asymmetry



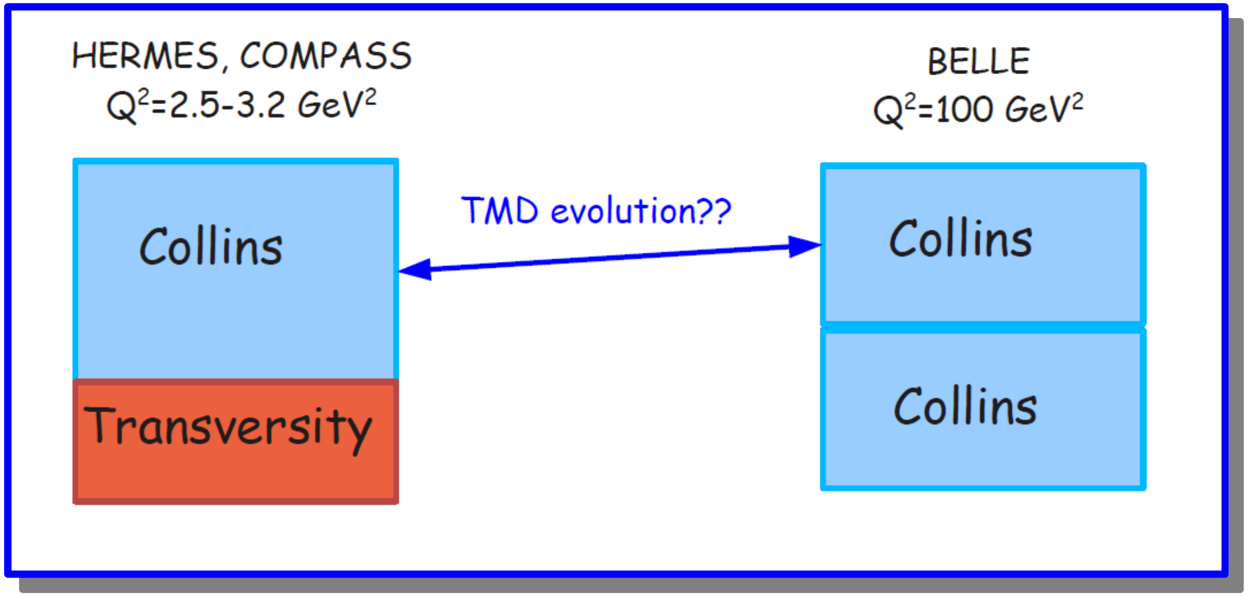
Courtesy of Isabella Garcia

TMD Evolution

- TMD evolution tends to reduce the size of distribution and fragmentation functions as Q^2 grows.

- The Collins TMD evolution is yet (phenomenologically) unknown:
  the perturbative part of the evolution equations is the same as that of the Sivers function,
  but the non perturbative input function is not: we need new modeling and new fits
  (Previous determinations, Nadolsky et al. 1999, 2001, are not appropriate, see talk by Z. Kang).

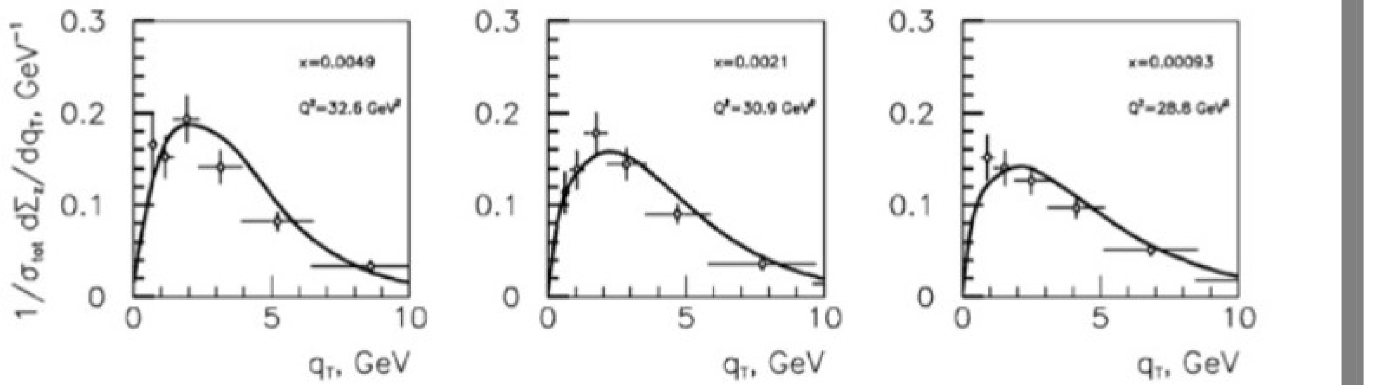
- Evolution tends to reduce the Collins function at larger Q^2
  D. Boer, Nucl. Phys. B603 (2001); Nucl. Phys. B803 (2009),
  BUT the azimuthal moments involved in the Collins asymmetries are probably not smaller !
  Need a more detailed investigation ...



Courtesy of Stefano Melis

TMD Evolution

- z-flow distribution $$\frac{d\Sigma_z}{dx\,dQ^2\,dq_T^2} = \sum_B \int_{z_{min}}^{1} z \frac{d\sigma(e+A \to e+B+X)}{dx\,dz\,dQ^2\,dq_T^2} dz.$$ Nadolsky, et.al., 1999, 200



- The non-perturbative function fitted from HERA only works for small-x data, and do not apply for large x (Sivers data)
  - note this function in SIDIS is different from that in DY, as SIDIS non-perturbative function comes from PDF and FF, while DY only comes from PDF
  - In the current Sivers analysis, this functional form is not adopted. It will be a very good cross-check to see if the current form can describe HERA data!!!

Slide from Zhongbo Kang talk

Conclusions

- First of all, we need a fresh start on the unpolarized TMDs → new extractions of TMDs from high precision data on multiplicities and cross sections from HERMES, COMPASS, BELLE, BaBar ...
- The global analysis to extract transversity and the Collins function is well on its way. We are eagerly waiting for BaBar data to be able to finalize it.
- Extraction of Kaon Collins functions needs more backup from experimental data (BELLE, BaBar)
- TMD evolution needs to be further examined and better understood. As this evolution is very fast, it has serious consequences on many processes (Drell Yan for example).
- We should also explore different evolution schemes, which are now becoming available [for example, Echevarria et al, JHEP 1207 (2012) 002]