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Fine structure of spin-dipole excitations in covariant density functional theory

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Outline

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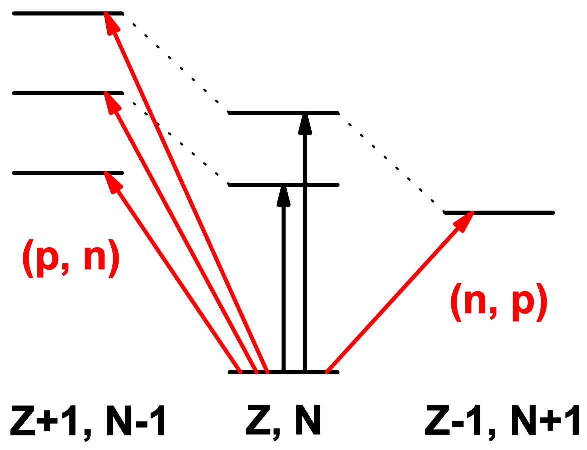
Nuclear spin-isospin resonances



- Nuclear charge-exchange excitations
  - ⋆ β-decay
  - ⋆ charge-exchange reactions

- These excitations play important roles
  - ⋆ spin and isospin properties of the in-medium nuclear interaction
  - ⋆ neutron skin thickness Krasznahorkay:1999, Vretenar:2003, Yako:2006
  - ⋆ β-decay rates of nuclei in r-process path Engel:1999, Borzov:2006
  - ⋆ $\beta\beta$-decay rates Ejiri:2000, Avignone:2008
  - ⋆ inclusive neutrino-nucleus cross sections Kolbe:2003, Vogel:2006
  - ⋆ isospin corrections for superallowed β deacys Sagawa:1996, HL:2009, Towner & Hardy:2010

- Nuclear spin-isospin resonances become one of the key topics in nuclear physics and astrophysics.

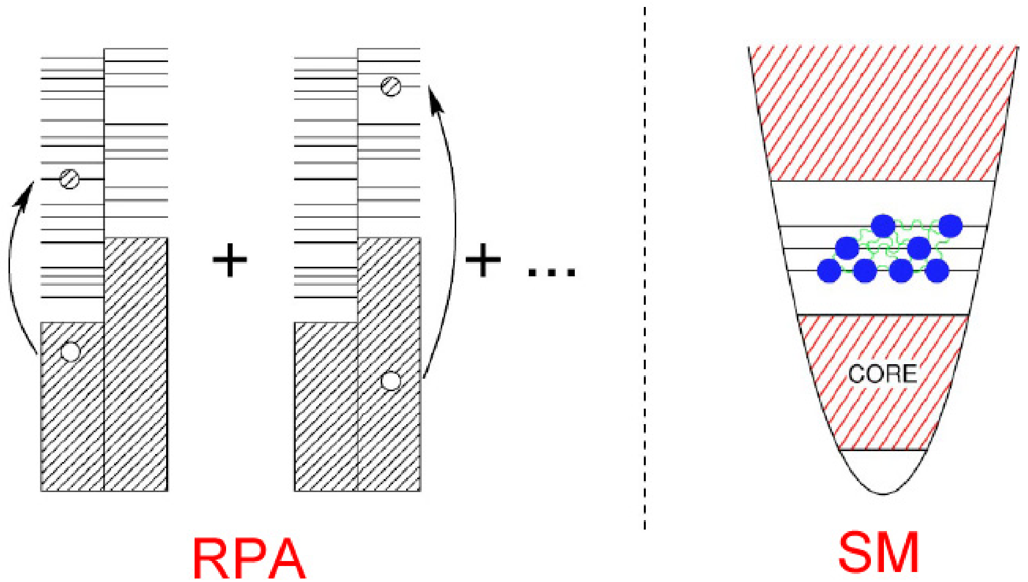
Microscopic theories for spin-isospin resonances

- Shell models ($A \sim 60$)

  Radha:1997, Caurier:1999,2005

- Random Phase Approximation (RPA) based on density functional theories
  - ⋆ traditional (non-relativistic) density functional

    Halbleib:1967, Auerbach:1981, Colò:1994, Engel:1999, Bender:2002, Fracasso:2005, Péru:2008, Bai:2010
  - ⋆ covariant (relativistic) density functional: RH+RPA, RHF+RPA



Covariant density functional theory – RH theory

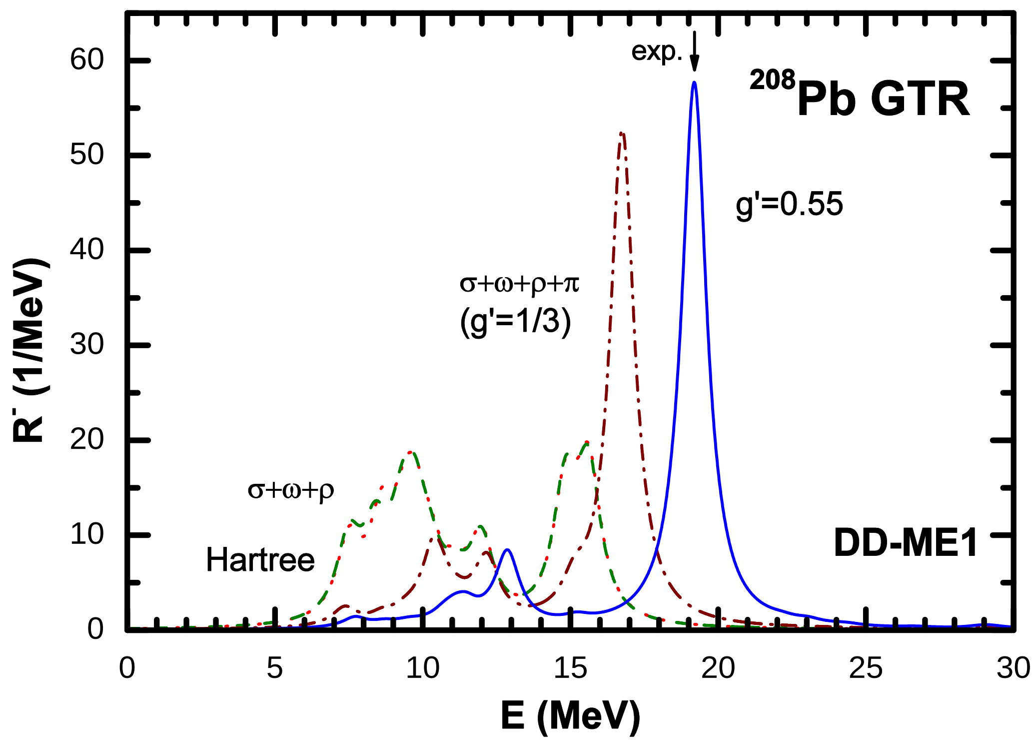
- Covariant density functional theory in Hartree level (RH/RMF theory) has received wide attention due to its successful description of lots of nuclear phenomena.
  Serot:1986, Ring:1996, Vretenar:2005, Meng:2006, Paar:2007, Nikšić:2011
  - ⋆ spin-orbit splittings
  - ⋆ nuclear saturation properties (the Coester line) Brockmann:1990,1992
  - ⋆ binding energy per nucleon E/A Reinhard:1989, Ring:1996
  - ⋆ isotopic shifts in the Pb region Sharma:1993
  - ⋆ halo and giant halo in exotic nuclei Meng:1996,1998,2002
  - ⋆ pseudospin symmetry in nucleon spectrum Ginocchio:1997,2005, HL:2011
  - ⋆ spin symmetry in anti-nucleon spectrum Zhou:2003
  - ⋆ ......

RH+RPA for spin-isospin resonances

- RH+RPA for spin-isospin resonances

  De Conti:1998, 2000, Vretenar: 2003, Ma:2004, Paar:2004, Nikšić:2005

example: Gamow-Teller resonance (GTR) in ^{208}Pb ($\Delta S = 1$, $\Delta L = 0$, $J^{\pi} = 1^{+}$)



a. add π-meson b. fit g' full self-consistency is missing

RHF+RPA for spin-isospin resonances

- Covariant density functional theory in Hartree-Fock level (RHF theory)
  π-meson and nucleon-nucleon tensor interactions are included naturally
  - ⋆ early attempts Bouyssy:1985, 1987, Bernardos:1993, Marcos:2004
  - ⋆ density-dependent RHF (Bogoliubov) theory Long:2006,2007,2010
  - ⋆ proton-neutron effective mass splitting Long:2006
  - ⋆ nuclear shell structures and their evolutions Long:2007,2008,2009, Tarpanov:2008, Moreno-Torres:2010
  - ⋆ spin and pseudospin symmetries in nucleon spectra Long:2006, HL:2010

- A fully self-consistent QRPA approach has been established based on RHFB theory.
- Applications
  - ⋆ nuclear spin-isospin resonances and their fine structures
  - ⋆ β-decay rates of nuclei in r-process path
  - ⋆ inclusive charged-current neutrino-nucleus cross sections
  - ⋆ isospin symmetry-breaking corrections for the superallowed β decays
  - ⋆ ...

Outline

Covariant density functional theory – RHF theory

- Effective Lagrangian density Bouyssy:1987, Long:2006

$$\mathscr{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right) - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi$$
$$+\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu$$
$$+\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \quad (1)$$

- Energy functional of the system

$$E = \langle\Phi_0|\, H\, |\Phi_0\rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_\pi^E + E_A^E \quad (2)$$

Random Phase Approximation

- RPA equations

$$\begin{pmatrix} \mathcal{A} & \mathcal{B} \\ -\mathcal{B} & -\mathcal{A} \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega_\nu \begin{pmatrix} X \\ Y \end{pmatrix} \tag{3}$$

where the matrix elements of particle-hole residual interactions read

$$\mathcal{A} = \begin{pmatrix} (E_A - E_a)\delta_{AB}\delta_{ab} & \\ & (E_\alpha - E_a)\delta_{\alpha\beta}\delta_{ab} \end{pmatrix} + \begin{pmatrix} \langle f_A f_b \,|V|\, f_B f_a - f_a f_B \rangle & \langle f_A f_b \,|V|\, f_\beta f_a - f_a f_\beta \rangle \\ \langle f_\alpha f_b \,|V|\, f_B f_a - f_a f_B \rangle & \langle f_\alpha f_b \,|V|\, f_\beta f_a - f_a f_\beta \rangle \end{pmatrix}, \tag{4a}$$

$$\mathcal{B} = \begin{pmatrix} \langle f_A f_B \,|V|\, f_b f_a - f_a f_b \rangle & \langle f_A f_\beta \,|V|\, f_b f_a - f_a f_b \rangle \\ \langle f_\alpha f_B \,|V|\, f_b f_a - f_a f_b \rangle & \langle f_\alpha f_\beta \,|V|\, f_b f_a - f_a f_b \rangle \end{pmatrix} \tag{4b}$$

- Particle-hole residual interactions in self-consistent RPA
  - ⋆ derived from the second derivative of the energy functional
  - ⋆ with rearrangement terms, if the meson-nucleon couplings are density-dependent

RHF+RPA in charge-exchange channel

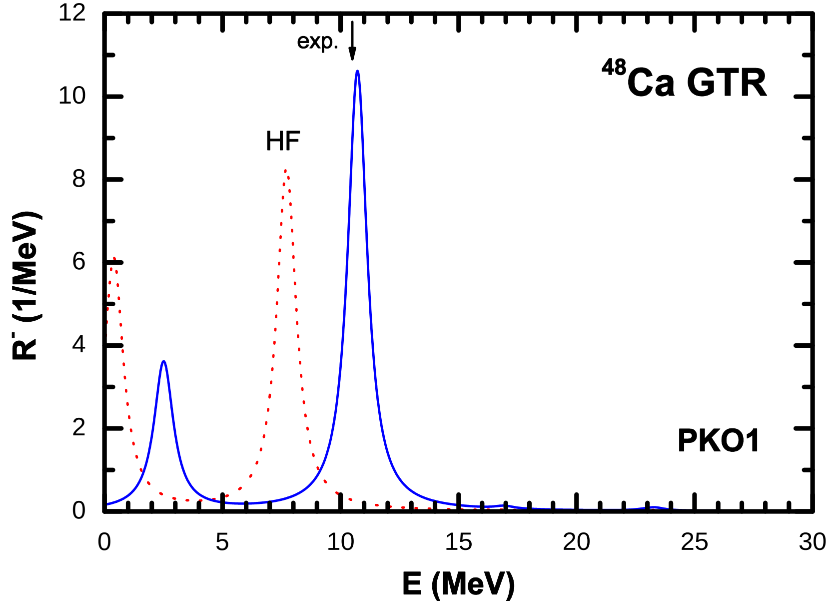
- Particle-hole residual interactions
  - ⋆ σ-meson:
$$V_\sigma(1,2) = -[g_\sigma\gamma_0]_1[g_\sigma\gamma_0]_2 D_\sigma(1,2) \quad (5a)$$
  - ⋆ ω-meson:
$$V_\omega(1,2) = [g_\omega\gamma_0\gamma^\mu]_1[g_\omega\gamma_0\gamma_\mu]_2 D_\omega(1,2) \quad (5b)$$
  - ⋆ ρ-meson:
$$V_\rho(1,2) = [g_\rho\gamma_0\gamma^\mu\vec{\tau}]_1\cdot[g_\rho\gamma_0\gamma_\mu\vec{\tau}]_2 D_\rho(1,2) \quad (5c)$$
  - ⋆ pseudovector π-N coupling:
$$V_\pi(1,2) = -[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^k\partial_k]_1\cdot[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\gamma^l\partial_l]_2 D_\pi(1,2) \quad (5d)$$
  - ⋆ zero-range counter-term of π-meson:
$$V_{\pi\delta}(1,2) = g'[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_1\cdot[\frac{f_\pi}{m_\pi}\vec{\tau}\gamma_0\gamma_5\boldsymbol{\gamma}]_2\delta(\mathbf{r}_1-\mathbf{r}_2), \quad g'=1/3 \quad (5e)$$

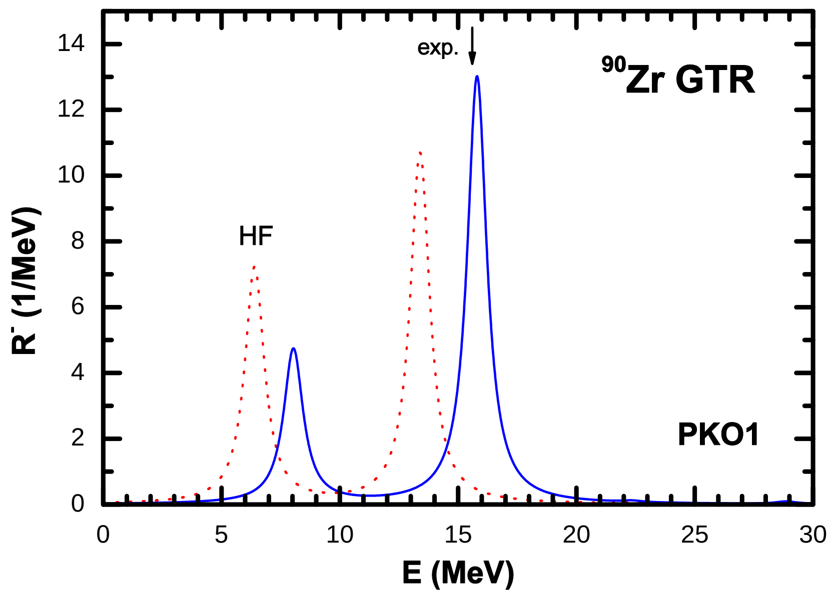
- π-meson is included naturally.
- $g'=1/3$ in the zero-range counter-term of π-meson is maintained for the sake of self-consistency.

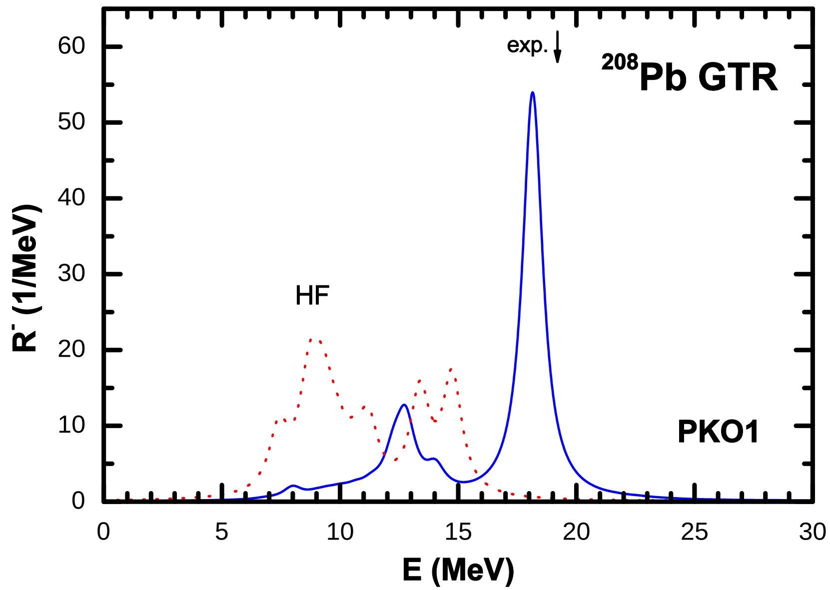
Outline

RHF+RPA for Gamow-Teller resonances

★ Gamow-Teller resonances in ^{48}Ca, ^{90}Zr, and ^{208}Pb







✓ GTR excitation energies can be reproduced in a fully self-consistent way.

HL, Giai, Meng, *PRL* **101**, 122502 (2008)

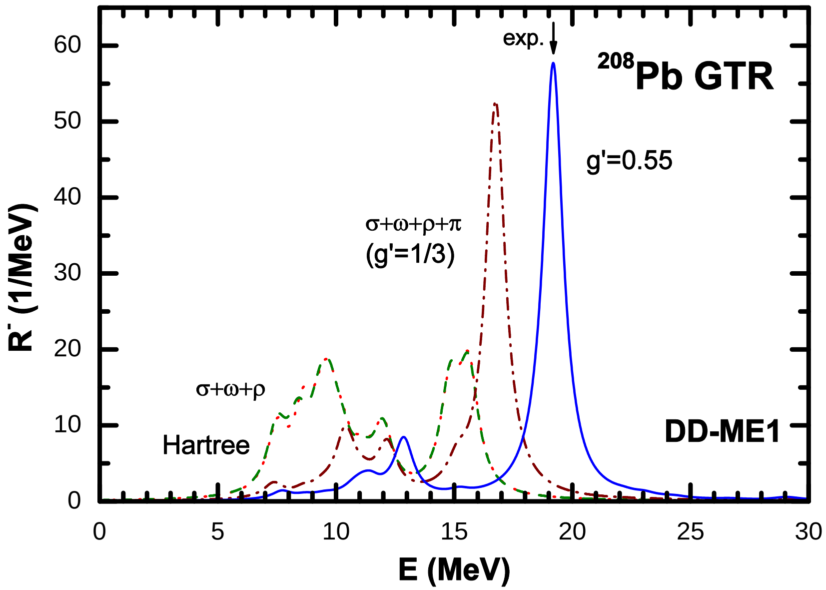
GTR excitation energies and strength

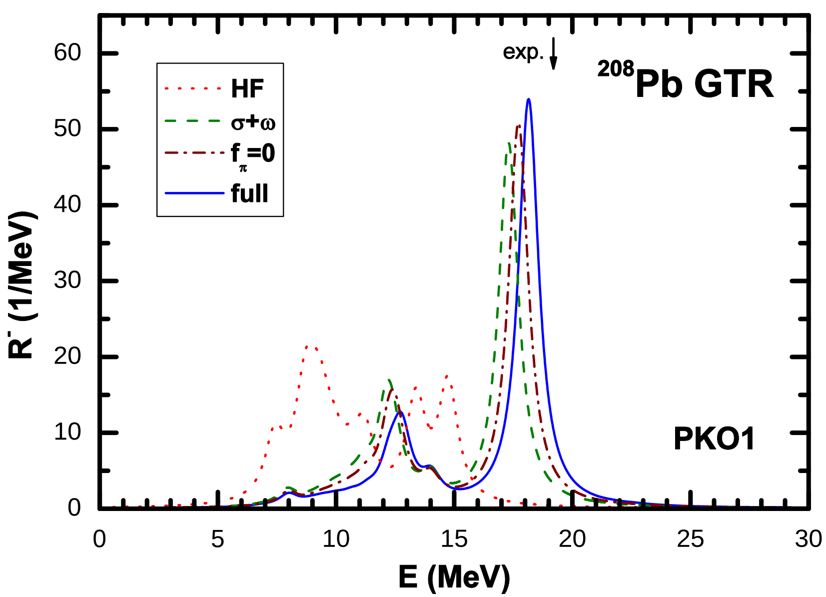
★ GTR excitation energies in MeV and strength in percentage of the $3(N-Z)$ sum rule within the RHF+RPA framework. Experimental and the RH+RPA results are given for comparison.
HL, Giai, Meng, *PRL* **101**, 122502 (2008)

		^{48}Ca		^{90}Zr		^{208}Pb	
		energy	strength	energy	strength	energy	strength
experiment		~ 10.5		15.6 ± 0.3		19.2 ± 0.2	60-70
RHF+RPA	PKO1	10.72	69.4	15.80	68.1	18.15	65.6
	PKO2	10.83	66.7	15.99	66.3	18.20	60.5
	PKO3	10.42	70.7	15.71	68.9	18.14	67.7
RH+RPA	DD-ME1	10.28	72.5	15.81	71.0	19.19	70.6

- The pion is not included in PKO2.

Physical mechanisms of GTR

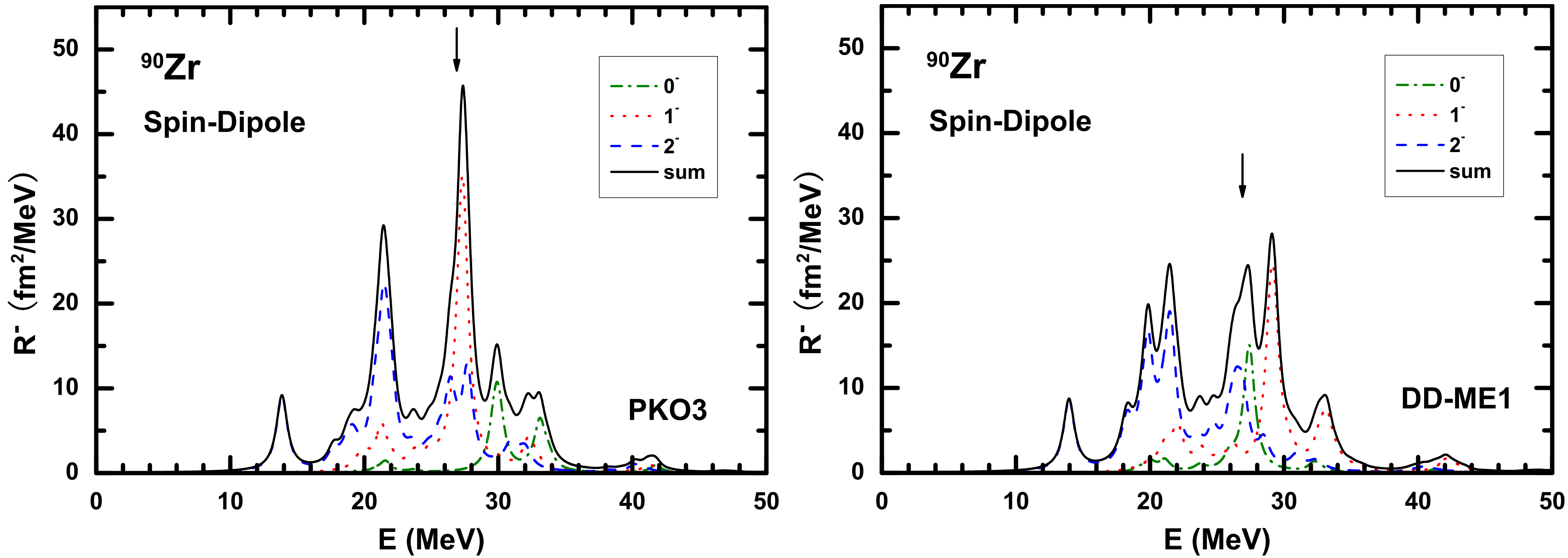




- RH+RPA
  - ⋆ no contribution from isoscalar mesons (σ, ω), because exchange terms are missing.
  - ⋆ π-meson is dominant in this resonance.
  - ⋆ g' has to be refitted to reproduce the experimental data.
- RHF+RPA
  - ⋆ isoscalar mesons (σ, ω) play an essential role via the exchange terms.
  - ⋆ π-meson plays a minor role.
  - ⋆ $g' = 1/3$ is kept for self-consistency.

HL, Giai, Meng, *PRL* **101**, 122502 (2008)

Spin-dipole resonances



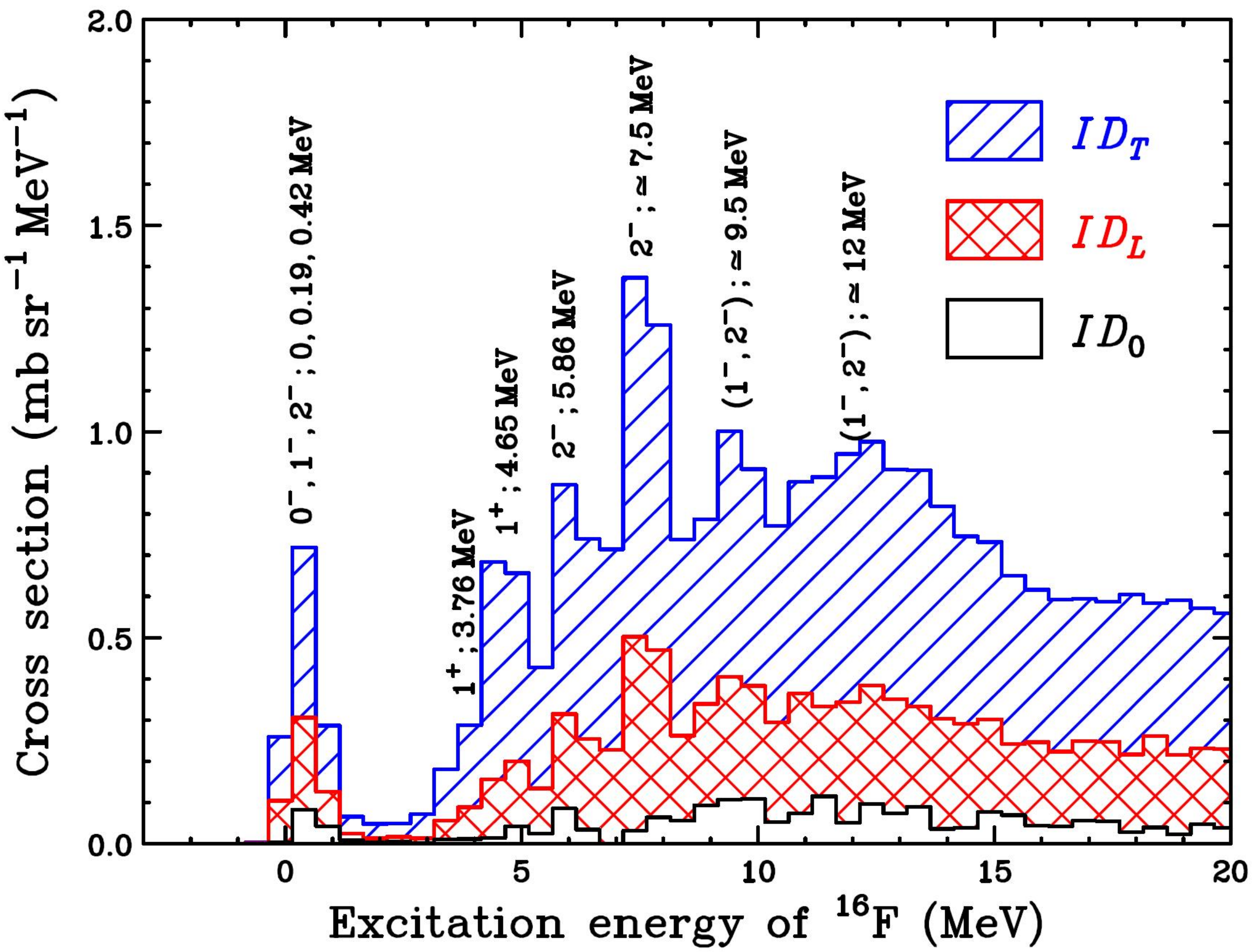
- Main peak can be reproduced by RHF+RPA exp. Yako:2006
- Energy hierarchy
  - ⋆ RHF+RPA: $E(2^-) < E(1^-) < E(0^-)$ agree with SHF+RPA Fracasso:2007
  - ⋆ RH+RPA: $E(2^-) < E(0^-) < E(1^-)$

- Separating experimentally the different components from the total transition strength would be helpful to evaluate the theoretical predictive power, e.g.,
  - ⋆ SDR in ^{16}O Wakasa:2011 and ^{208}Pb Wakasa:2012

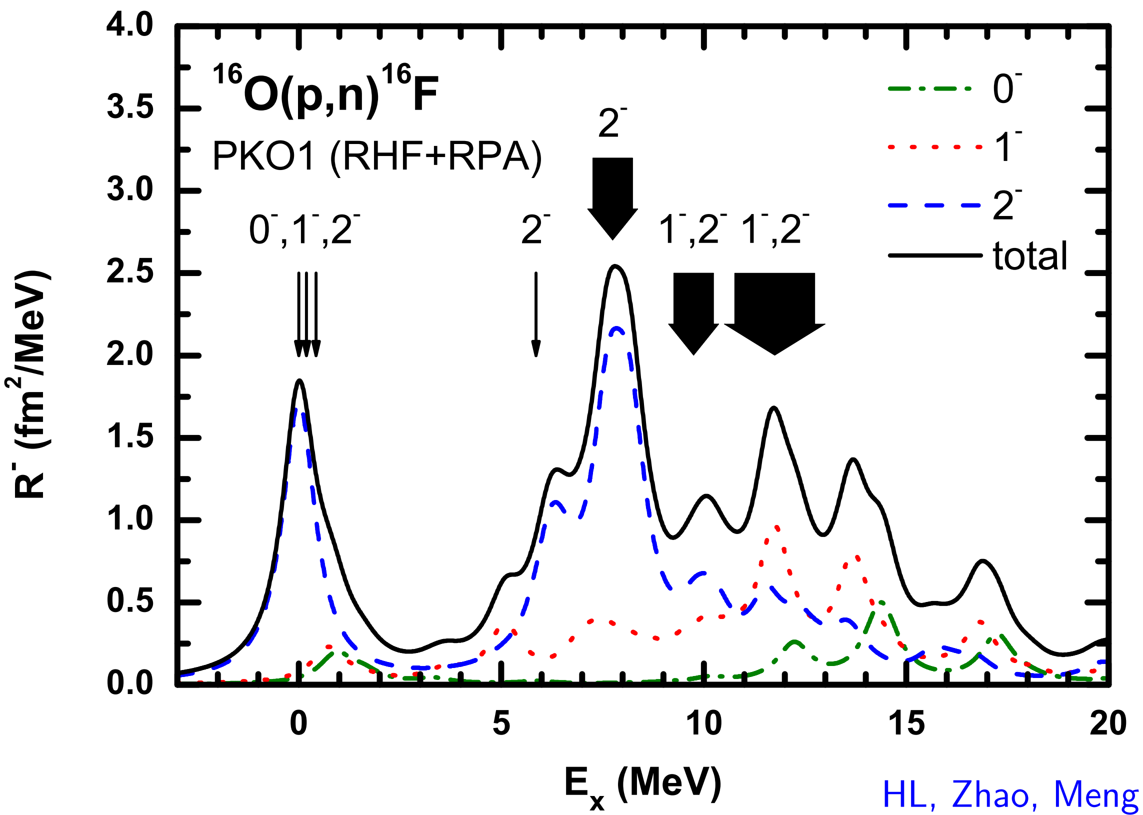
Outline

Fine structure of GT and SD excitations in ^{16}O

- A recent $^{16}O(\vec{p}, \vec{n})^{16}F$ experiment Wakasa *et al.*, *PRC* **84**, 014614 (2011)



Spin-dipole excitations by RHF+RPA

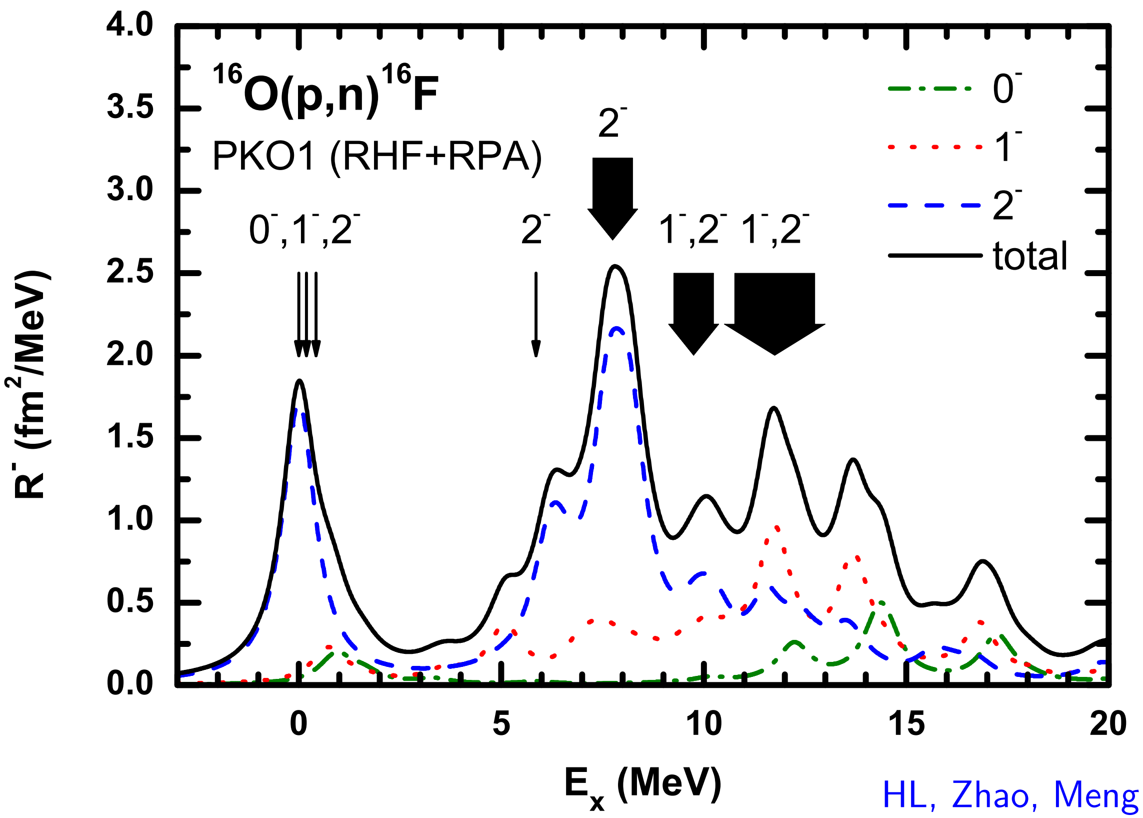


HL, Zhao, Meng, *PRC* **85**, 064302 (2012)

⋆ SDR in T_- channel by RHF+RPA, the lowest RPA state as the reference of E_x. exp. Wasaka:2011

- In general, the 0^-, 1^-, and 2^- excited states are well reproduced within 1 MeV.
- The 0_1^-, 1_1^-, and 2_1^- triplets are found at $E_x \simeq 0$ MeV.
- The shoulder at $E_x = 5.86$ MeV and giant resonance at $E_x \simeq 7.5$ MeV are nicely reproduced. In particular, the shoulder state cannot be described by shell model calculations.

Spin-dipole excitations by RHF+RPA

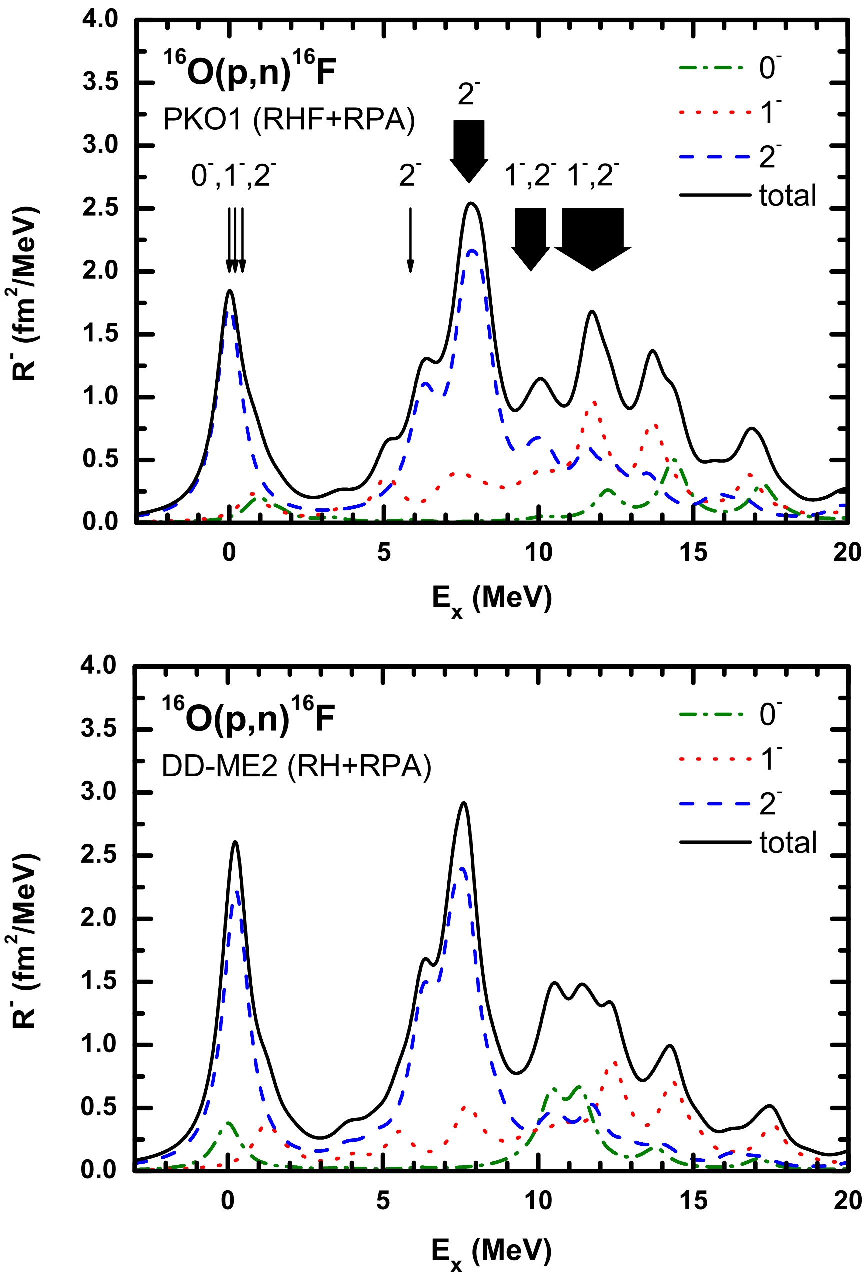


HL, Zhao, Meng, *PRC* **85**, 064302 (2012)

⋆ SDR in T_- channel by RHF+RPA, the lowest RPA state as the reference of E_x. exp. Wasaka:2011

- The mixtures of 1^- and 2^- states at $E_x \simeq 9.5$ MeV and $E_x \simeq 12$ MeV are reproduced, and the former one is dominant by 2^- component, whereas the latter one is dominant by 1^- component.
- The 0^- resonances are predicted to be fragmented at $12 \sim 18$ MeV with the peak at $E_x \simeq 14.5$ MeV.

SD excitations by RHF+RPA and RH+RPA



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RH+RPA results

- The general pattern of 2^- excitations are similar to that of RHF+RPA calculations, except the peak at $E_x \simeq 9.5$ MeV is missing.
- The 1^- resonances are predicted at $12 \sim 15$ MeV, somehow too high in energy by comparing to data.
- The 0^- resonances are predicted to be centralized at $10 \sim 12$ MeV, but not seen in experiments yet.

- By comparing with the experimental date, it is found that the self-consistent RHF+RPA calculations are more favored.

Outline

Density functional theories

- Density functional theories (DFT) Hohenberg & Kohn:1964
  - ⋆ reducing the many-body problems formulated in terms of N-particle wave functions to the one-particle level with the local density distribution $\rho(\mathbf{r})$
  - ⋆ no other method achieves comparable accuracy at the same computational cost

- Kohn-Sham scheme Kohn & Sham:1965
  - ⋆ for any interacting system, there exists a **local** single-particle (Kohn-Sham) potential $v_{\rm KS}(\mathbf{r})$, such that the exact ground-state density of the interacting system can be reproduced by non-interacting particles moving in this local potential:

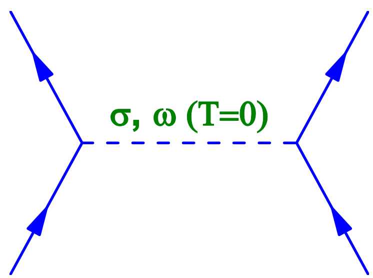
$$\rho(\mathbf{r}) = \rho_{\rm KS}(\mathbf{r}) \equiv \sum_i^{\rm occ} |\phi_i(\mathbf{r})|^2$$

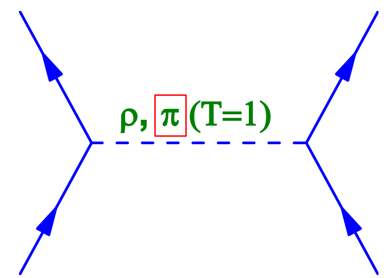
  - ⋆ system energy density functional:

$$E[\rho(\mathbf{r})] = T[\rho(\mathbf{r})] + E_{\rm ext.}[\rho(\mathbf{r})] + E_{\rm H}[\rho(\mathbf{r})] + E_{\rm xc}[\rho(\mathbf{r})]$$

Covariant density functional theory – RHF theory

- Effective Lagrangian density Bouyssy:1987, Long:2006





$$\begin{aligned}\mathscr{L} = \bar{\psi}\left[i\gamma^\mu\partial_\mu - M - g_\sigma\sigma - \gamma^\mu\left(g_\omega\omega_\mu + g_\rho\vec{\tau}\cdot\vec{\rho}_\mu + e\frac{1-\tau_3}{2}A_\mu\right) - \frac{f_\pi}{m_\pi}\gamma_5\gamma^\mu\partial_\mu\vec{\pi}\cdot\vec{\tau}\right]\psi \\ +\frac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\vec{R}_{\mu\nu}\cdot\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}^\mu\cdot\vec{\rho}_\mu \\ +\frac{1}{2}\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{1}{2}m_\pi^2\vec{\pi}\cdot\vec{\pi} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\end{aligned} \tag{6}$$

- Energy functional of the system

$$E = \langle\Phi_0|\,H\,|\Phi_0\rangle = E_k + E_\sigma^D + E_\omega^D + E_\rho^D + E_A^D + E_\sigma^E + E_\omega^E + E_\rho^E + E_\pi^E + E_A^E \tag{7}$$

Zero-range reduction

- Yukawa propagators of the mesons

$$D_i(\mathbf{r},\mathbf{r}') = \frac{1}{4\pi}\frac{e^{-m_i|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}, \qquad D_i(\mathbf{q}) = \frac{1}{m_i^2+\mathbf{q}^2} \tag{8}$$

for $m_i \gg q$,

$$D_i(\mathbf{q}) \approx \frac{1}{m_i^2} - \frac{\mathbf{q}^2}{m_i^4} + \cdots \quad \Rightarrow \quad D_i(\mathbf{r},\mathbf{r}') \approx \frac{1}{m_i^2}\delta(\mathbf{r}-\mathbf{r}') \tag{9}$$

within the zero-order approximation.

- Zero-range reduction of meson-nucleon couplings

$$\alpha_S^{\mathrm{HF}} = -\frac{g_\sigma^2}{m_\sigma^2}, \qquad \alpha_V^{\mathrm{HF}} = \frac{g_\omega^2}{m_\omega^2}, \qquad \alpha_{tV}^{\mathrm{HF}} = \frac{g_\rho^2}{m_\rho^2}, \tag{10}$$

Fierz transformation (I)

- Sixteen Dirac matrices form a complete system

$$O^S = 1, O^V = \gamma^\mu, O^T = \sigma^{\mu\nu}, O^{PS} = \gamma^5, O^{PV} = \gamma^5\gamma^\mu$$

so that any one can be expressed as a linear superposition of variants with a changed sequence of spinors,

$$(\bar{a}O^i d)(\bar{c}O_i b) = \sum_k c_{ik}(\bar{a}O^k b)(\bar{c}O_k d), \tag{11}$$

with the coefficients c_{ik} in the so-called Fierz table Fierz:1937, Okun:1982, Sulaksono:2003

	S	V	T	PS	PV
S	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{4}$
V	1	$-\frac{1}{2}$	0	-1	$-\frac{1}{2}$
T	3	0	$-\frac{1}{2}$	3	0
PS	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{4}$
PV	-1	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$

(12)

- For the isospin coefficients,

$$\delta_{q_a q_d}\delta_{q_c q_b} = \frac{1}{2}\left[\delta_{q_a q_b}\delta_{q_c q_d} + \langle q_a |\vec{\tau}| q_b\rangle \cdot \langle q_c |\vec{\tau}| q_d\rangle\right], \tag{13a}$$

$$\langle q_a |\vec{\tau}| q_d\rangle \cdot \langle q_c |\vec{\tau}| q_b\rangle = \frac{1}{2}\left[3\delta_{q_a q_b}\delta_{q_c q_d} - \langle q_a |\vec{\tau}| q_b\rangle \cdot \langle q_c |\vec{\tau}| q_d\rangle\right]. \tag{13b}$$

Fierz transformation (II)

- Fierz transformation: from $\alpha^{\rm HF}$ to $\alpha^{\rm H}$

$$\alpha^{\rm H}_S = +\frac{7}{8}\alpha^{\rm HF}_S - \frac{4}{8}\alpha^{\rm HF}_V - \frac{12}{8}\alpha^{\rm HF}_{tV} \tag{14a}$$

$$\alpha^{\rm H}_{tS} = -\frac{1}{8}\alpha^{\rm HF}_S - \frac{4}{8}\alpha^{\rm HF}_V + \frac{4}{8}\alpha^{\rm HF}_{tV} \tag{14b}$$

$$\alpha^{\rm H}_V = -\frac{1}{8}\alpha^{\rm HF}_S + \frac{10}{8}\alpha^{\rm HF}_V + \frac{6}{8}\alpha^{\rm HF}_{tV} \tag{14c}$$

$$\alpha^{\rm H}_{tV} = -\frac{1}{8}\alpha^{\rm HF}_S + \frac{2}{8}\alpha^{\rm HF}_V + \frac{6}{8}\alpha^{\rm HF}_{tV} \tag{14d}$$

$$\alpha^{\rm H}_T = -\frac{1}{16}\alpha^{\rm HF}_S \tag{14e}$$

$$\alpha^{\rm H}_{tT} = -\frac{1}{16}\alpha^{\rm HF}_S \tag{14f}$$

$$\alpha^{\rm H}_{PS} = -\frac{1}{8}\alpha^{\rm HF}_S + \frac{4}{8}\alpha^{\rm HF}_V + \frac{12}{8}\alpha^{\rm HF}_{tV} \tag{14g}$$

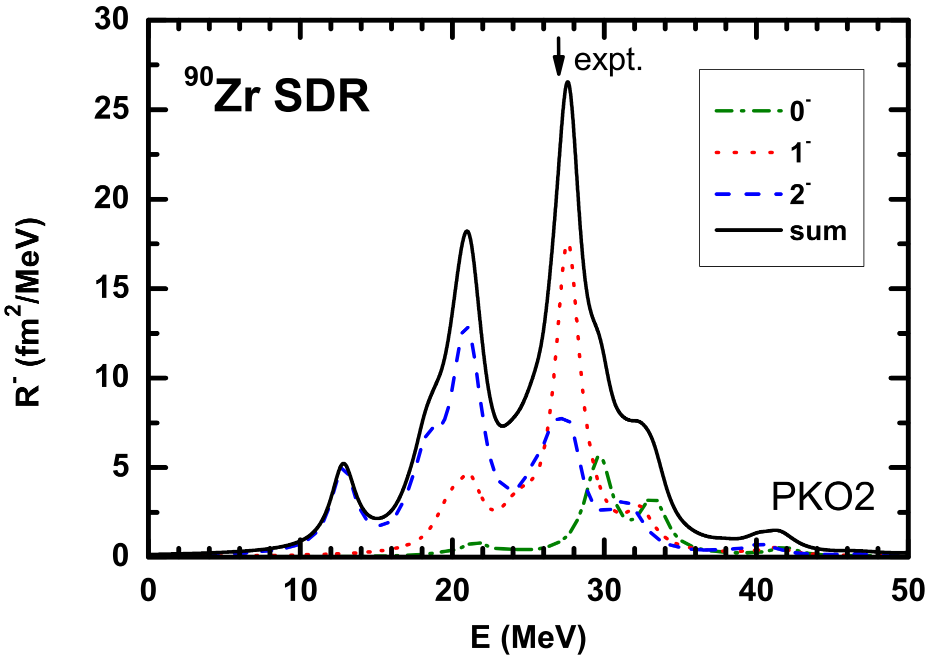
$$\alpha^{\rm H}_{tPS} = -\frac{1}{8}\alpha^{\rm HF}_S + \frac{4}{8}\alpha^{\rm HF}_V - \frac{4}{8}\alpha^{\rm HF}_{tV} \tag{14h}$$

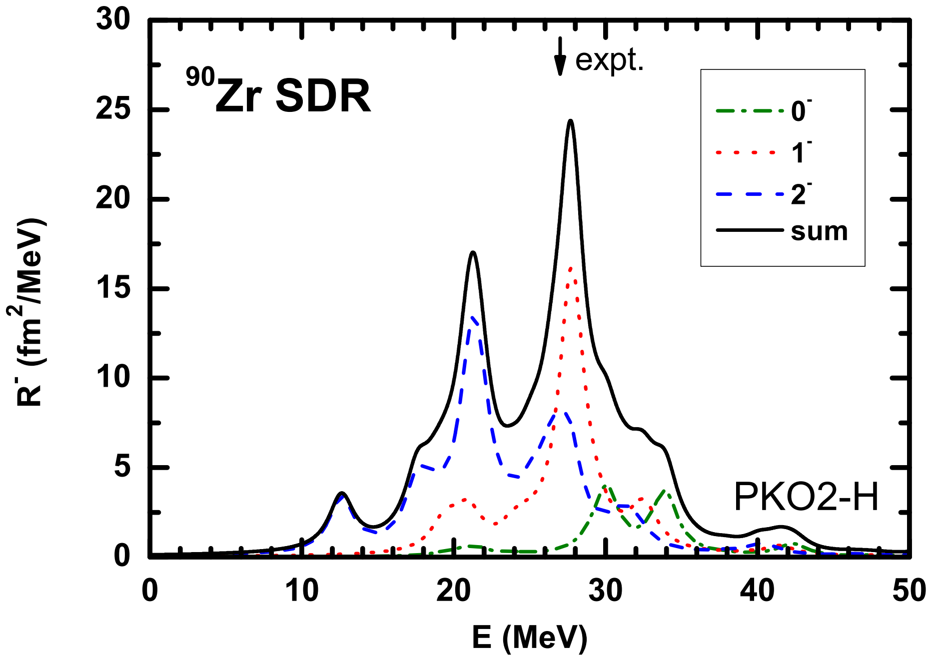
$$\alpha^{\rm H}_{PV} = +\frac{1}{8}\alpha^{\rm HF}_S + \frac{2}{8}\alpha^{\rm HF}_V + \frac{6}{8}\alpha^{\rm HF}_{tV} \tag{14i}$$

$$\alpha^{\rm H}_{tPV} = +\frac{1}{8}\alpha^{\rm HF}_S + \frac{2}{8}\alpha^{\rm HF}_V - \frac{2}{8}\alpha^{\rm HF}_{tV} \tag{14j}$$

HL, Zhao, Ring, Roca-Maza, Meng, *PRC* **86**, 021302(R) (2012)

SDR in localized RHF equivalent RPA





HL, Zhao, Ring, Roca-Maza, Meng, *PRC* **86**, 021302(R) (2012) expt: Yako, Sagawa, Sakai, *PRC* **74**, 051303(R) (2006)

- Not only the total strengths but also the individual contribution from different spin-parity J^π components are almost identical.
- The energy hierarchy $E(2^-) < E(1^-) < E(0^-)$ can be obtained naturally in the present RPA calculations in the local scheme.

- The constraints introduced by the Fock terms of the RHF scheme into the *ph* residual interactions are straight forward and robust.

Outline

Summary and Perspectives

Summary

- ★ A fully self-consistent charge-exchange QRPA approach has been established based on the RHFB theory.
  - ✓ GTR excitation energies can be reproduced in a fully self-consistent way.
  - ✓ Fine structure of SD excitations in ^{16}O can be well reproduced.
- ★ A new method is proposed to construct covariant density functionals based on only local potentials, yet keeping the merits of the exchange terms.

Perspectives

- ?′ To identify the pure tensor forces in the relativistic framework and their effects on the spin-isospin resonances.
- ?′ To develop the deformed relativistic RPA approach with finite amplitude method.
- ?′ ......

Acknowledgments

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- Jie Meng — Peking University, China
- Takashi Nakatsukasa — RIKEN, Japan
- Peter Ring — Technische Universität München, Germany
- Xavier Roca-Maza — INFN-Milano, Italy
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- Pengwei Zhao — Peking University, China

Thank you!