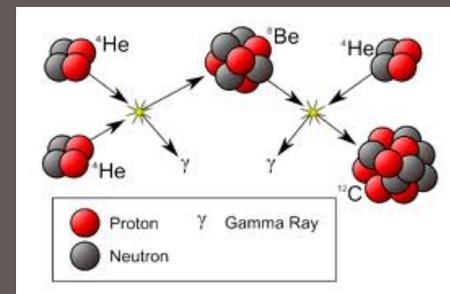
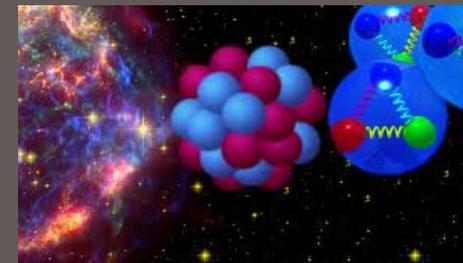
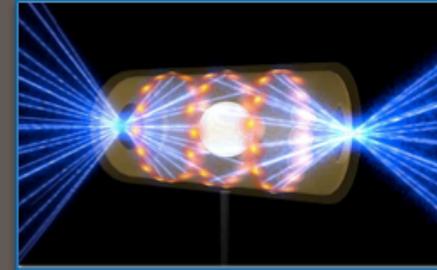


# Calculations with chiral forces in no-core shell model with continuum

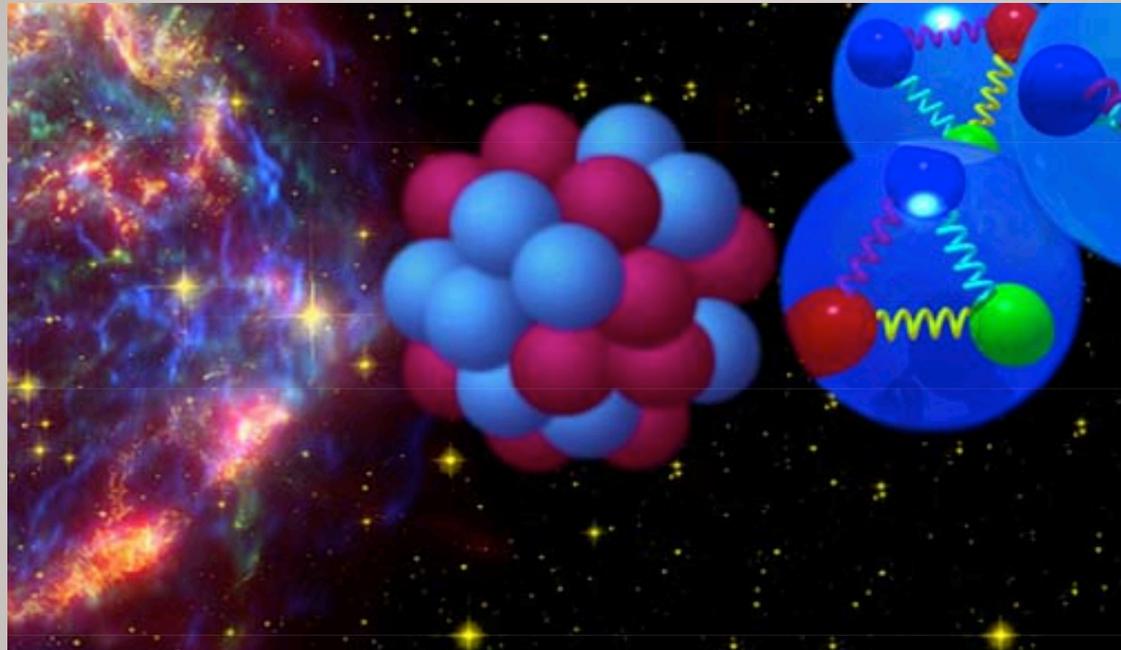
RIBF ULIC Symposium on Perspective in Isospin Physics  
 ~ Role of non-central interactions in structure and dynamics of  
 unstable nuclei ~  
 27-28th August 2012, RIKEN

**Petr Navratil | TRIUMF**

**Collaborators:** Sofia Quaglioni (LLNL),  
 Robert Roth (TU Darmstadt), S. Baroni (ULB),  
 C. Romero-Redondo (TRIUMF), M. Kruse (UA), J. Langhammer (TU  
 Darmstadt), G. Hupin (LLNL), W. Horiuchi (Hokkaido)



## *Ab initio* calculations of nuclear structure and reactions



Connection  
to  
Astrophysics

Connection  
to  
QCD

- Chiral forces
- No-core shell model, NCSM/RGM
  - Neutron rich He isotopes,  $N$ - $^4\text{He}$  scattering,  $^{12}\text{C}$  structure
- No-core shell model with continuum (NCSMC):
  - Unbound  $^7\text{He}$

# Chiral Effective Field Theory

- **First principles for Nuclear Physics:**

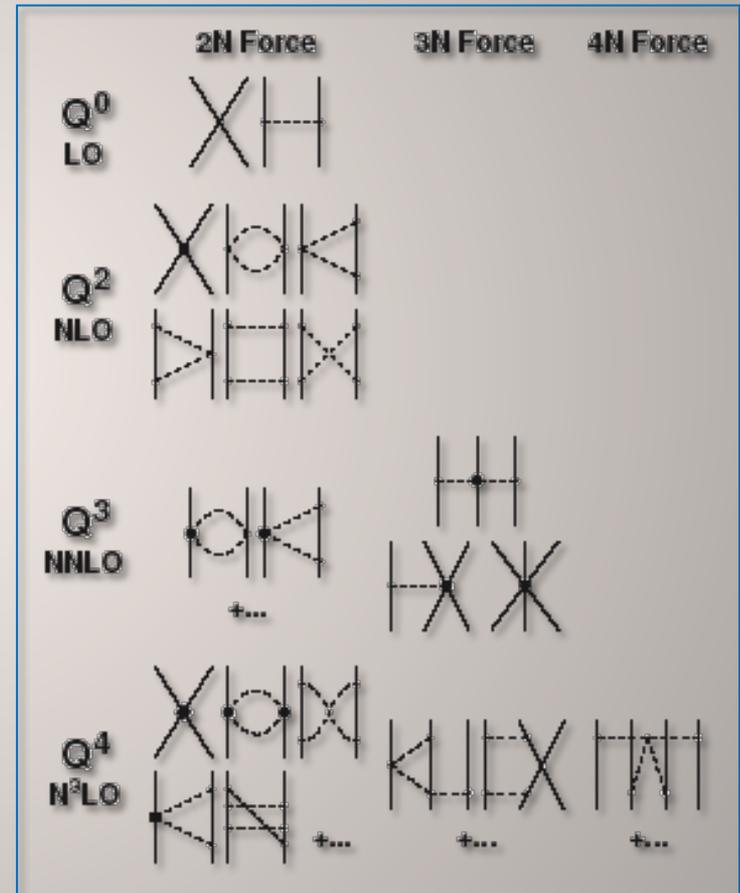
## QCD

- Non-perturbative at low energies
- Lattice QCD in the future

- ***For now a good place to start:***

- **Inter-nucleon forces from chiral effective field theory**

- Based on the symmetries of QCD
  - Chiral symmetry of QCD ( $m_u \approx m_d \approx 0$ ), spontaneously broken with pion as the Goldstone boson
  - Degrees of freedom: nucleons + pions
- Systematic low-momentum expansion to a given order ( $Q/\Lambda_\chi$ )
- Hierarchy
- Consistency
- Low energy constants (LEC)
  - Fitted to data
  - Can be calculated by lattice QCD



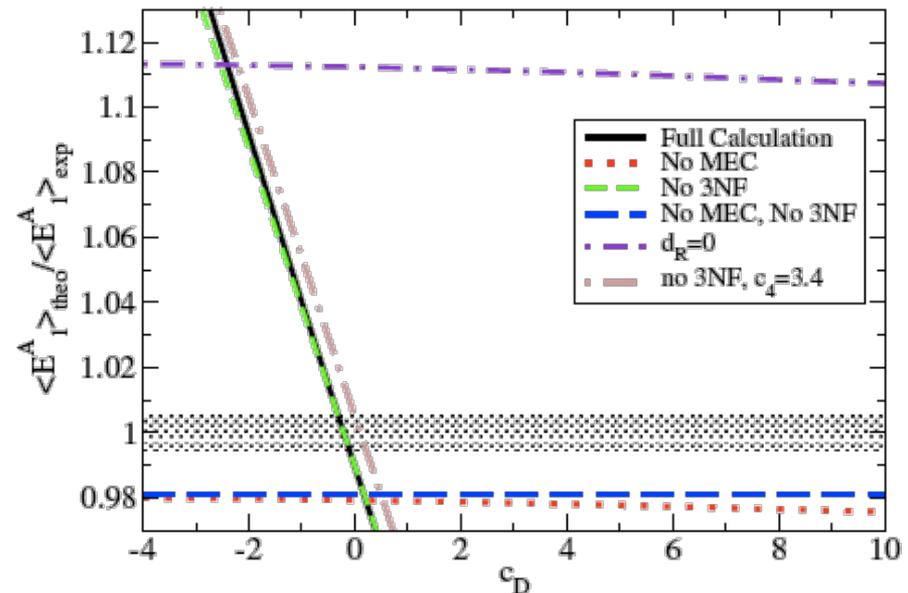
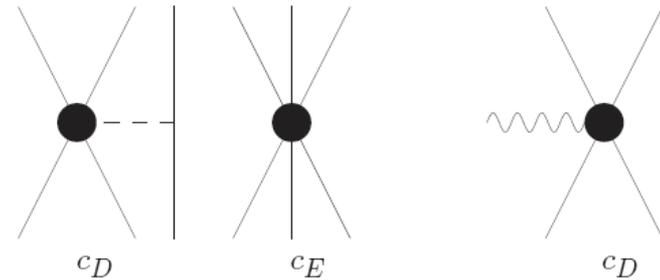
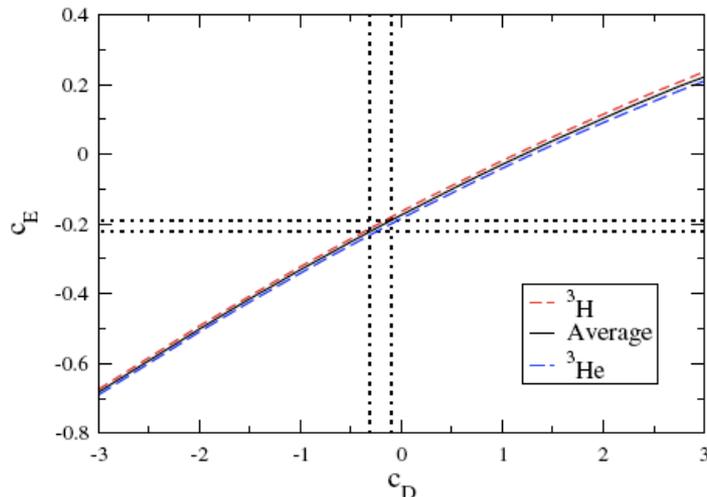
$\Lambda_\chi \sim 1 \text{ GeV}$  :  
Chiral symmetry breaking scale

# Determination of NNN constants $c_D$ and $c_E$ from the triton binding energy and the half life

- **Chiral EFT:**  $c_D$  also in the two-nucleon contact vertex with an external probe
- Calculate  $\langle E_1^A \rangle = |\langle {}^3\text{He} || E_1^A || {}^3\text{H} \rangle|$ 
  - Leading order GT
  - N<sup>2</sup>LO: one-pion exchange plus contact

- **A=3 binding energy constraint:**

$$c_D = -0.2 \pm 0.1 \quad c_E = -0.205 \pm 0.015$$



PRL 103, 102502 (2009) PHYSICAL REVIEW LETTERS week ending 4 SEPTEMBER 2009

Three-Nucleon Low-Energy Constants from the Consistency of Interactions and Currents in Chiral Effective Field Theory

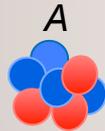
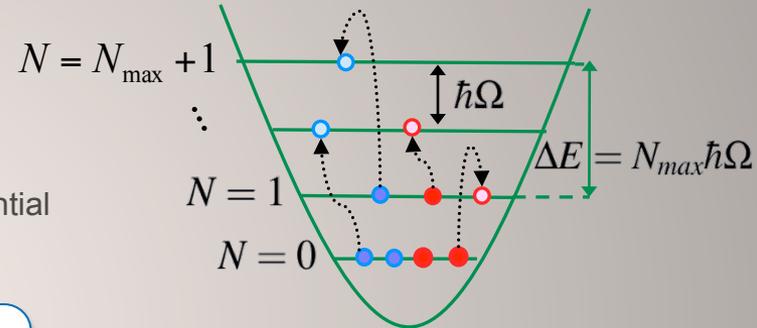
Doron Gazit  
Institute for Nuclear Theory, University of Washington, Box 351550, Seattle, Washington 98195, USA

Sofia Quaglioni and Petr Navrátil  
Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA

# No-core shell model combined with the resonating group method (NCSM/RGM)

- **The NCSM:** An approach to the solution of the  $A$ -nucleon bound-state problem

- Accurate nuclear Hamiltonian
- Finite harmonic oscillator (HO) basis
  - Complete  $N_{max} \hbar\Omega$  model space
- Effective interaction due to the model space truncation
  - Similarity-Renormalization-Group evolved NN(+NNN) potential
- Short & medium range correlations
- No continuum

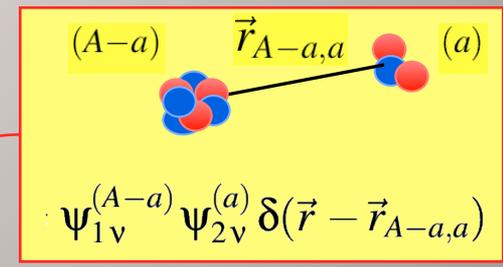


$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni} \Phi_{Ni}^A$$

- **The RGM:** A microscopic approach to the  $A$ -nucleon scattering of clusters

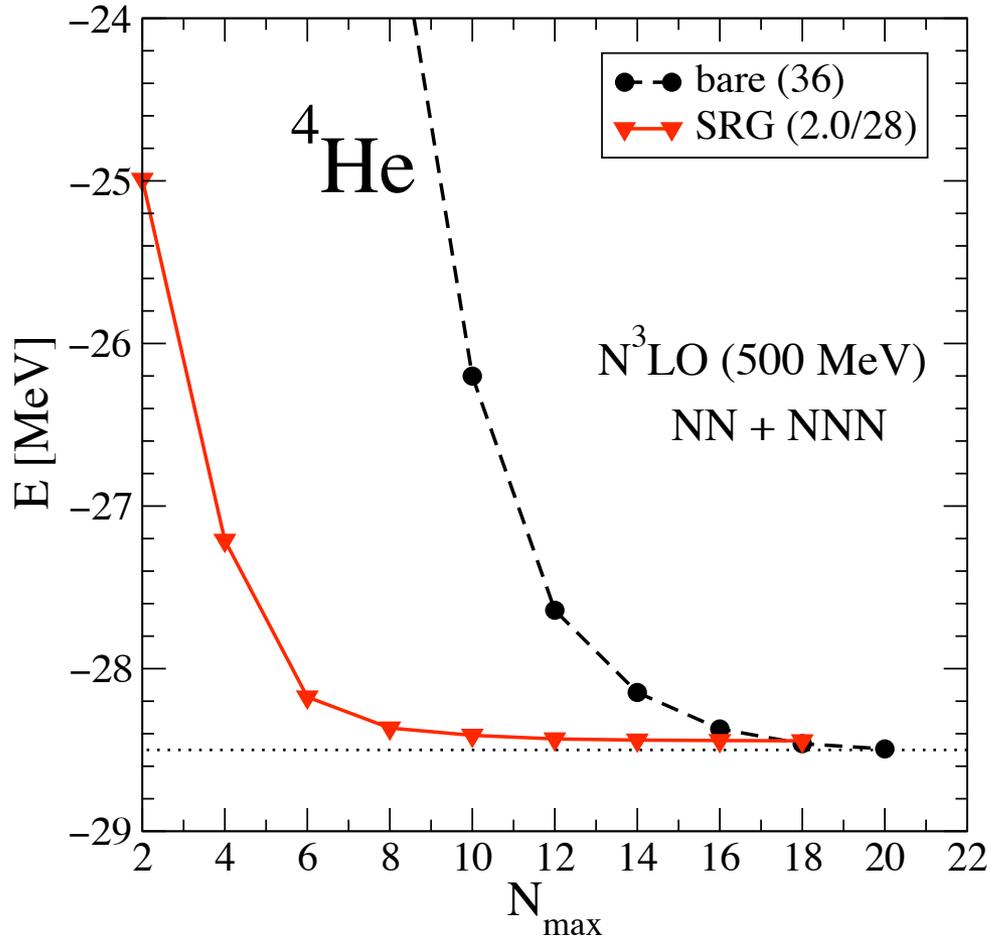
- Long range correlations, relative motion of clusters

$$\Psi^{(A)} = \sum_v \int d\vec{r} \phi_v(\vec{r}) \hat{\mathcal{A}} \Phi_{v\vec{r}}^{(A-a,a)}$$



**Ab initio NCSM/RGM:** Combines the best of both approaches  
 Accurate nuclear Hamiltonian, consistent cluster wave functions  
 Correct asymptotic expansion, Pauli principle and translational invariance

# $^4\text{He}$ from chiral EFT interactions: g.s. energy convergence



## Chiral $N^3\text{LO}$ NN plus $N^2\text{LO}$ NNN potential

- Bare interaction (black line)
  - Strong short-range correlations
    - Large basis needed
- SRG evolved effective interaction (red line)
  - Unitary transformation

$$H_\alpha = U_\alpha H U_\alpha^\dagger \Rightarrow \frac{dH_\alpha}{d\alpha} = [[T, H_\alpha], H_\alpha] \quad (\alpha = 1/\lambda^4)$$

- Two- plus *three*-body components, *four*-body omitted
- Softens the interaction
  - Smaller basis sufficient

PRL 103, 082501 (2009) PHYSICAL REVIEW LETTERS week ending 21 AUGUST 2009

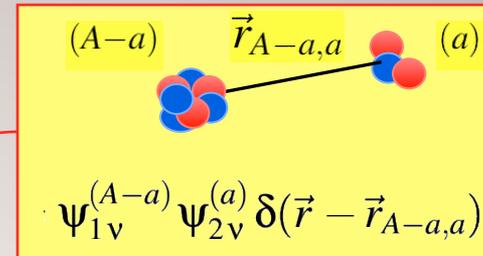
Evolution of Nuclear Many-Body Forces with the Similarity Renormalization Group

E. D. Jurgenson,<sup>1</sup> P. Navrátil,<sup>2</sup> and R. J. Furnstahl<sup>1</sup>

$A=3$  binding energy and half life constraint  
 $c_D = -0.2$ ,  $c_E = -0.205$ ,  $\Lambda = 500$  MeV

# The *ab initio* NCSM/RGM in a snapshot

- Ansatz:  $\Psi^{(A)} = \sum_{\mathbf{v}} \int d\vec{r} \phi_{\mathbf{v}}(\vec{r}) \hat{\mathcal{A}} \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)}$



eigenstates of  $H_{(A-a)}$  and  $H_{(a)}$  in the *ab initio* NCSM basis

- Many-body Schrödinger equation:

$$H\Psi^{(A)} = E\Psi^{(A)}$$

$$T_{\text{rel}}(r) + \mathcal{V}_{\text{rel}} + \bar{V}_{\text{Coul}}(r) + H_{(A-a)} + H_{(a)}$$

$$\sum_{\mathbf{v}} \int d\vec{r} \left[ \mathcal{H}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) - E\mathcal{N}_{\mu\nu}^{(A-a,a)}(\vec{r}', \vec{r}) \right] \phi_{\mathbf{v}}(\vec{r}) = 0$$

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}} H \hat{\mathcal{A}} | \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)} \rangle$$

**Hamiltonian kernel**

$$\langle \Phi_{\mu\vec{r}'}^{(A-a,a)} | \hat{\mathcal{A}}^2 | \Phi_{\mathbf{v}\vec{r}}^{(A-a,a)} \rangle$$

**Norm kernel**

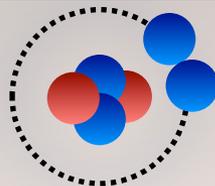
realistic nuclear Hamiltonian

# NNN interaction effects in neutron rich nuclei: He isotopes

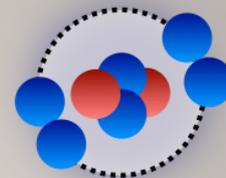
$^4\text{He}$



$^6\text{He}$



$^8\text{He}$

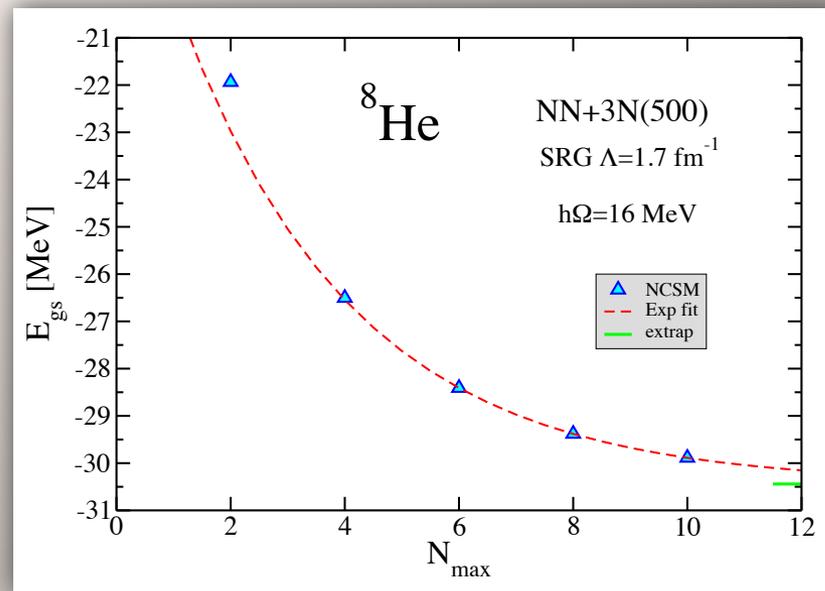
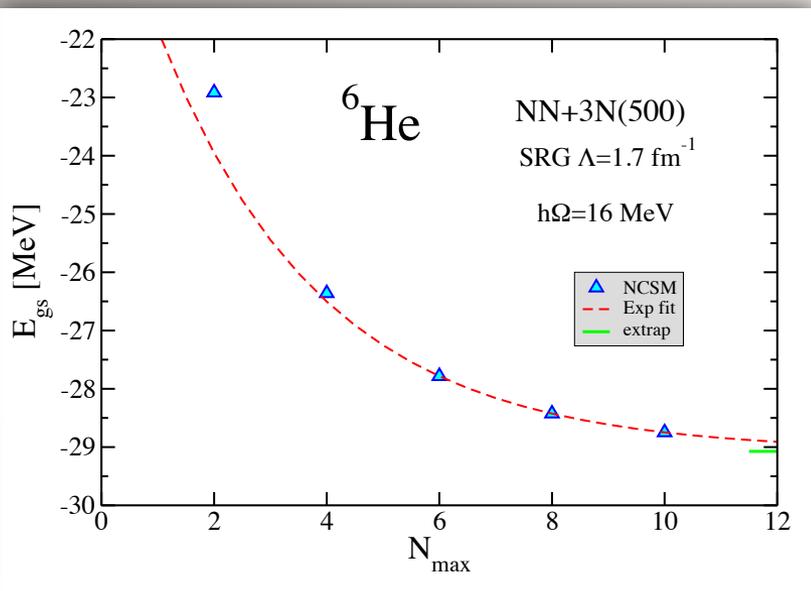


$^6\text{He}$  and  $^8\text{He}$  with SRG-evolved chiral  $N^3\text{LO}$  NN +  $N^2\text{LO}$  NNN

– 3N matrix elements in coupled- $J$  single-particle basis:

- Introduced and implemented by Robert Roth *et al.*
- Now also in my codes: Jacobi-Slater-Determinant transformation & NCSD code
- Example:  $^6\text{He}$ ,  $^8\text{He}$  NCSM calculations up to  $N_{\text{max}}=10$  done with moderate resources

$A=3$  binding energy & half life constraint  
 $c_D=-0.2$ ,  $c_E=-0.205$ ,  $\Lambda=500$  MeV

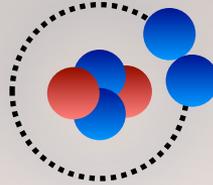


# 3N interaction effects in neutron rich nuclei: He isotopes

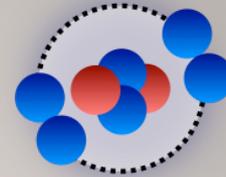
${}^4\text{He}$



${}^6\text{He}$



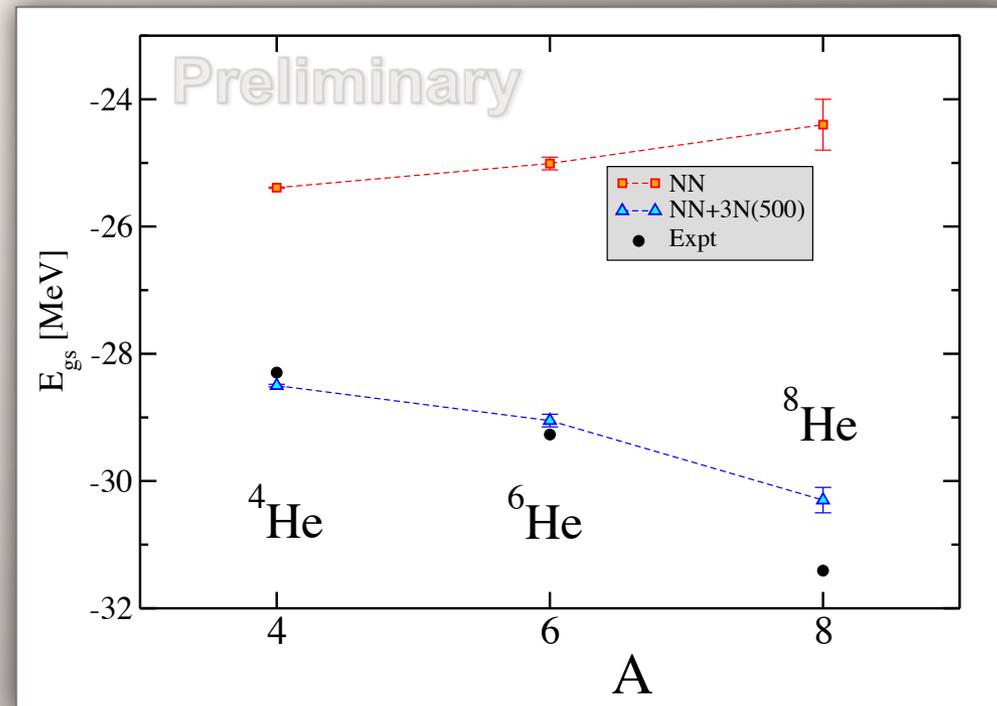
${}^8\text{He}$



- ${}^6\text{He}$  and  ${}^8\text{He}$  with SRG-evolved chiral  $N^3\text{LO NN} + N^2\text{LO 3N}$ 
  - chiral  $N^3\text{LO NN}$ :  ${}^4\text{He}$  underbound,  ${}^6\text{He}$  and  ${}^8\text{He}$  unbound
  - chiral  $N^3\text{LO NN} + N^2\text{LO 3N}(500)$ :  ${}^4\text{He}$  OK, both  ${}^6\text{He}$  and  ${}^8\text{He}$  bound

$A=3$  binding energy & half life constraint  
 $c_D=-0.2$ ,  $c_E=-0.205$ ,  $\Lambda=500$  MeV

**NNN interaction important  
to bind neutron rich nuclei**



# 3N interaction effects in neutron rich nuclei: He isotopes

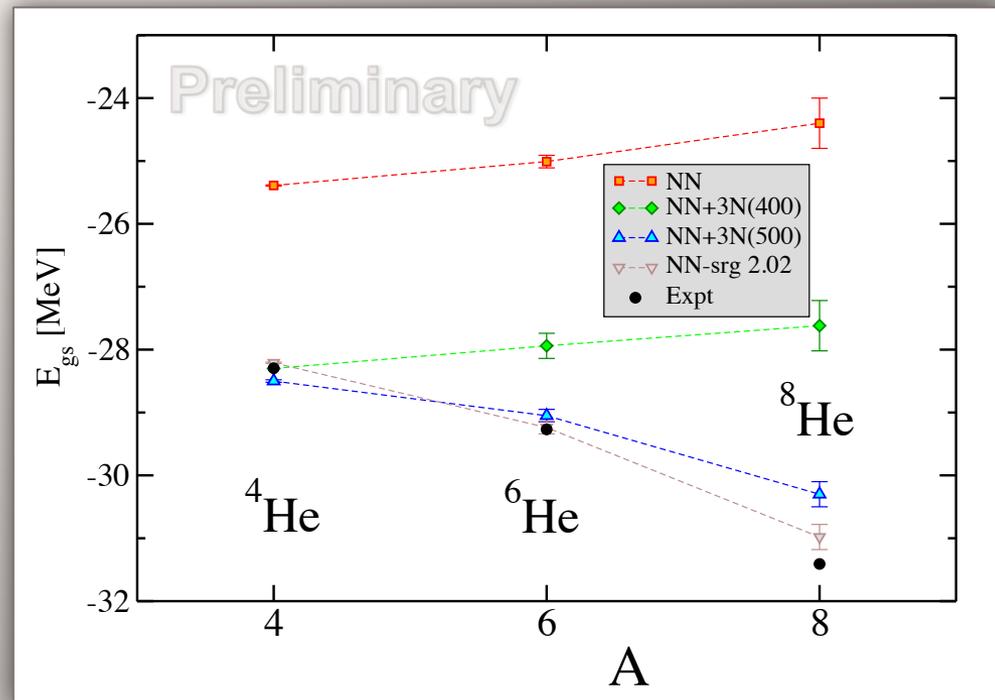
- ${}^6\text{He}$  and  ${}^8\text{He}$  with SRG-evolved chiral  $\text{N}^3\text{LO NN} + \text{N}^2\text{LO 3N}$ 
  - chiral  $\text{N}^3\text{LO NN}$ :  ${}^4\text{He}$  underbound,  ${}^6\text{He}$  and  ${}^8\text{He}$  unbound
  - chiral  $\text{N}^3\text{LO NN} + \text{N}^2\text{LO 3N}(400)$ :  ${}^4\text{He}$  fitted,  ${}^6\text{He}$  barely unbound,  ${}^8\text{He}$  unbound
    - describes quite well binding energies of  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ ,  ${}^{40}\text{Ca}$ ,  ${}^{48}\text{Ca}$
  - chiral  $\text{N}^3\text{LO NN} + \text{N}^2\text{LO 3N}(500)$ :  ${}^4\text{He}$  OK, both  ${}^6\text{He}$  and  ${}^8\text{He}$  bound
    - does well up to  $A=10$ , overbinds  ${}^{12}\text{C}$ ,  ${}^{16}\text{O}$ , Ca isotopes
  - SRG- $\text{N}^3\text{LO NN } \Lambda=2.02 \text{ fm}^{-1}$ :  ${}^4\text{He}$  OK, both  ${}^6\text{He}$  and  ${}^8\text{He}$  bound
    - ${}^{16}\text{O}$ , Ca strongly overbound

${}^4\text{He}$  binding energy &  ${}^3\text{H}$  half life constraint  
 $c_D=-0.2$ ,  $c_E=+0.098$ ,  $\Lambda=400 \text{ MeV}$

$A=3$  binding energy & half life constraint  
 $c_D=-0.2$ ,  $c_E=-0.205$ ,  $\Lambda=500 \text{ MeV}$

**NNN interaction important  
to bind neutron rich nuclei**

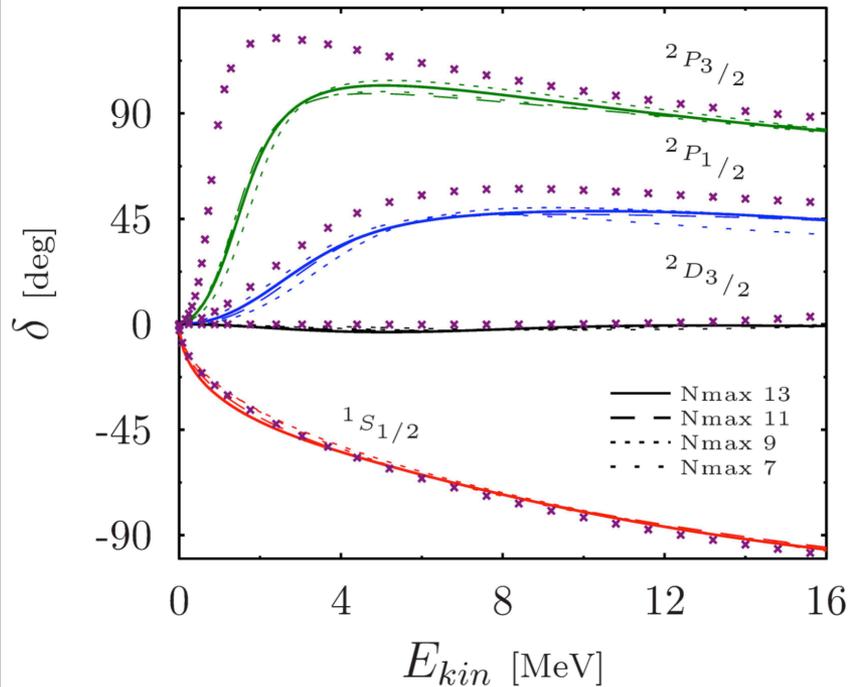
**Our knowledge of the 3N interaction  
is incomplete**



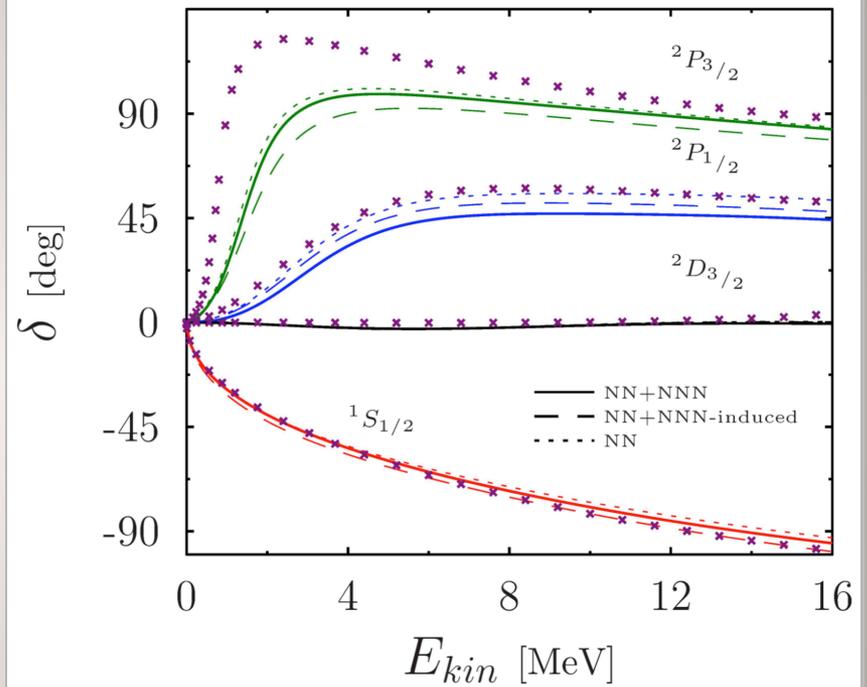
# $n$ - ${}^4\text{He}$ scattering with NN+NNN interactions

G. Hupin, J. Langhammer, S. Quaglioni, P. Navratil, R. Roth, work in progress

## ${}^4\text{He}(n,n){}^4\text{He}$ phase shifts



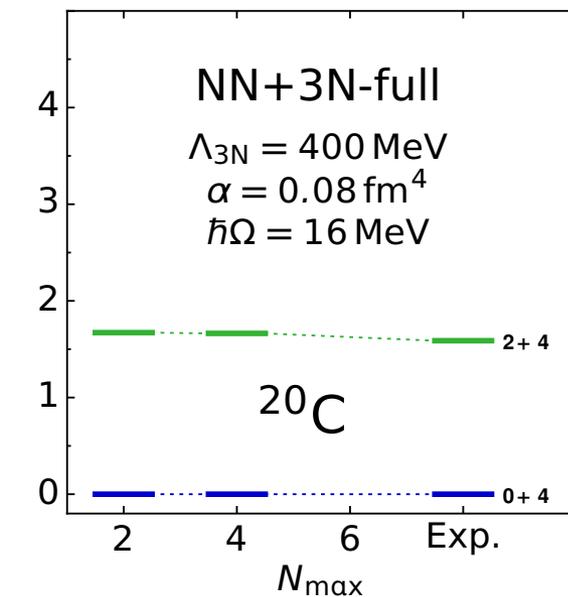
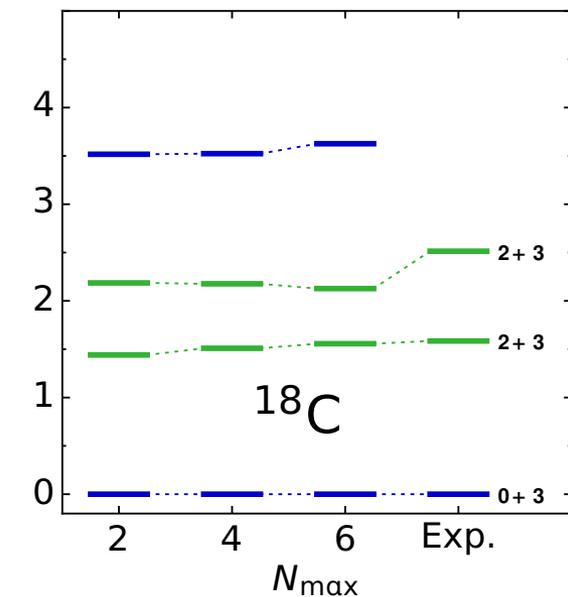
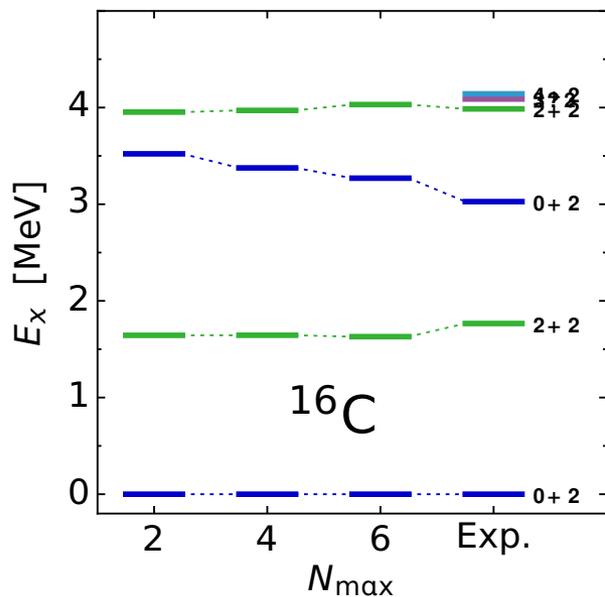
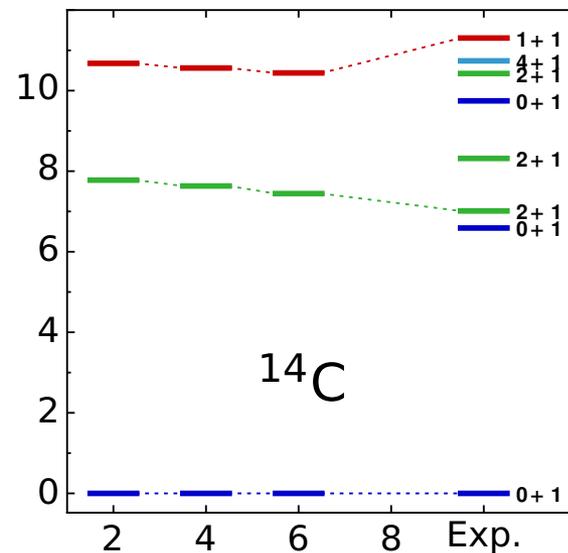
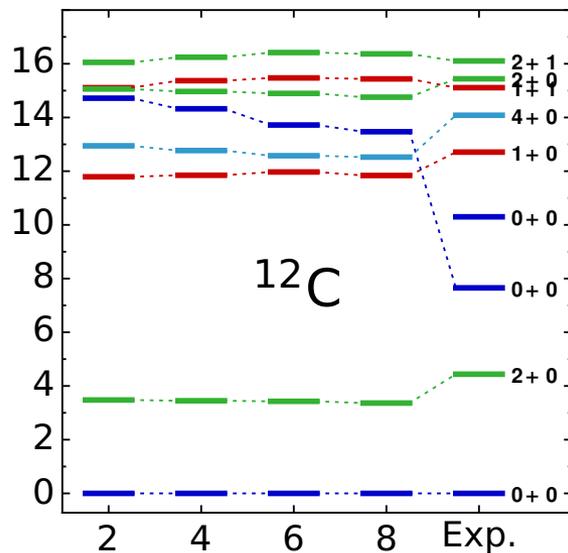
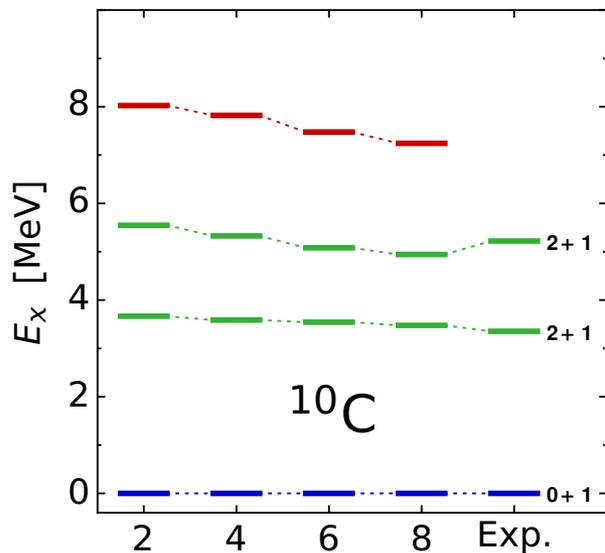
## ${}^4\text{He}(n,n){}^4\text{He}$ phase shifts



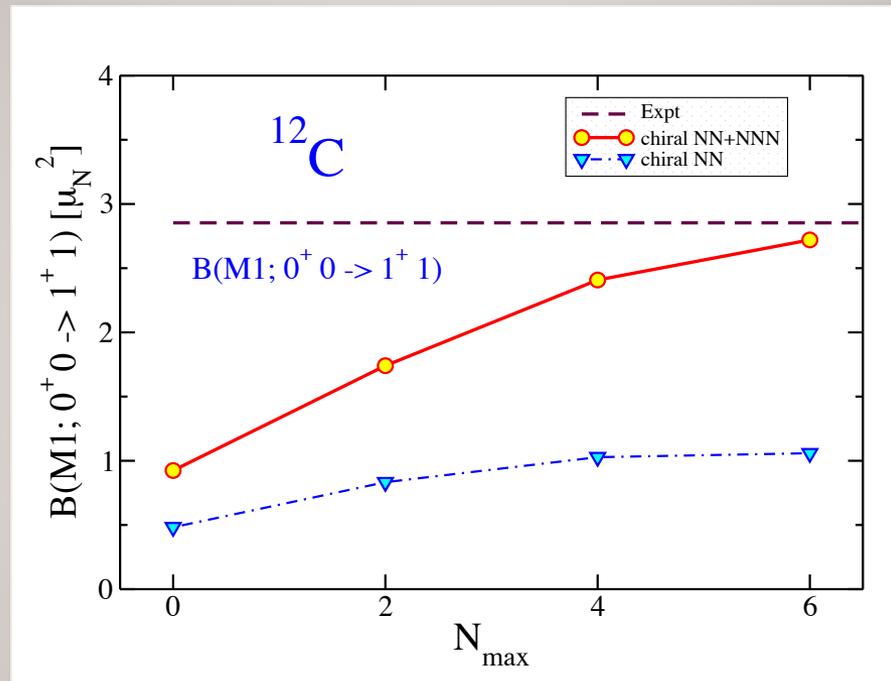
Here:  
 $n + {}^4\text{He}(\text{g.s.})$ , SRG-( $\text{N}^3\text{LO NN} + \text{N}^2\text{LO NNN}$ )  
 potential with ( $\lambda=2 \text{ fm}^{-1}$ ). Convergence with  
 respect to HO basis size ( $N_{\text{max}}$ )

**Largest splitting between  $P$  waves  
 obtained with NN+NNN. Need  ${}^4\text{He}$  excited  
 states and study with respect to SRG  $\lambda$**

# Neutron rich Carbon isotopes from chiral NN+NNN interactions (IT-NCSM, R. Roth *et al.*)



# M1 transitions in $^{12}\text{C}$ sensitive to 3N interaction



Chiral 3N interaction changes occupations of the  $p_{3/2}$  and  $p_{1/2}$  orbits  
 (“increases the gap” between them)

Enhances the M1 transition from the g.s. to  $1^+ 1$  state

Similar increase of the Gamow-Teller transition between g.s. of  $^{12}\text{B}$ ( $^{12}\text{N}$ ) and  $^{12}\text{C}$

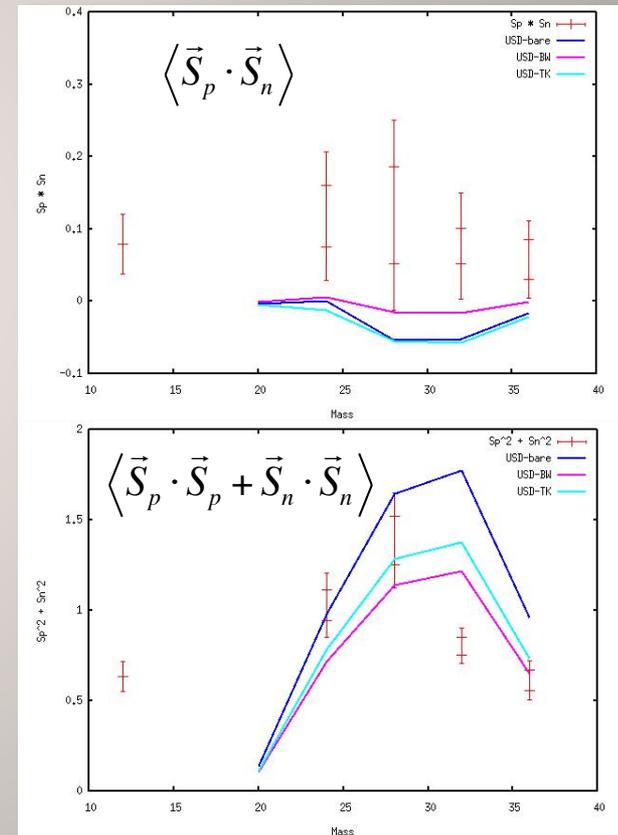
# Tensor correlations and 3N effects in ground states of $^4\text{He}$ and $^{12}\text{C}$

- Tensor correlations related to  $\langle \vec{S}_p \cdot \vec{S}_n \rangle$  and  $\langle \vec{S}_p \cdot \vec{S}_p + \vec{S}_n \cdot \vec{S}_n \rangle$

$$- \vec{S}_p = \frac{1}{2} \sum_{i=1}^A (\frac{1}{2} + t_{z,i}) \vec{\sigma}_i \quad , \quad \vec{S}_n = \frac{1}{2} \sum_{i=1}^A (\frac{1}{2} - t_{z,i}) \vec{\sigma}_i \quad \dots \text{ spin operators}$$

- Experiment: Atsushi Tamii *et al.*
- Ab initio* NCSM:
  - $^{12}\text{C}$   $N_{\text{max}}=6$  only

	$\langle \vec{S}_p \cdot \vec{S}_p + \vec{S}_n \cdot \vec{S}_n \rangle$	$\langle \vec{S}_p \cdot \vec{S}_n \rangle$	$\langle \vec{S}^2 \rangle$
$^4\text{He}$ Minnesota NN	0.04	-0.02	0
$^4\text{He}$ chiral NN	0.19	0.04	0.27
$^4\text{He}$ chiral NN+3N(500)	0.22	0.05	0.32
$^{12}\text{C}$ chiral NN	0.50	0.065	0.63
$^{12}\text{C}$ chiral NN+3N(400)	0.68	0.061	0.80
$^{12}\text{C}$ chiral NN+3N(500)	1.01	0.065	1.14



$^{12}\text{C}$ : chiral NN+3N(400)  
the best agreement with experiment

# New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^{\pi T}}\rangle = \sum_{Ni} c_{Ni} |ANiJ^{\pi T}\rangle$$

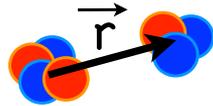
# New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^\pi T\rangle$$

NCSM/RGM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi$$

$$(\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}) \bar{\chi} = E \bar{\chi}$$

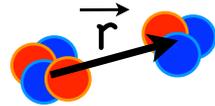
# New developments: NCSM with continuum

NCSM



$$|\Psi_A^{J^\pi T}\rangle = \sum_{Ni} c_{Ni} |ANiJ^\pi T\rangle$$

NCSM/RGM



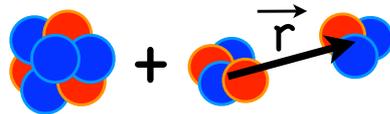
$$|\Psi_A^{J^\pi T}\rangle = \sum_{\nu} \int d\vec{r} \chi_{\nu}(\vec{r}) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\mathcal{H}\chi = E\mathcal{N}\chi$$

$$\bar{\chi} = \mathcal{N}^{+\frac{1}{2}} \chi$$

$$(\mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}}) \bar{\chi} = E \bar{\chi}$$

NCSMC



$$|\Psi_A^{J^\pi T}\rangle = \sum_{\lambda} c_{\lambda} |A\lambda J^\pi T\rangle + \sum_{\nu} \int d\vec{r} \left( \sum_{\nu'} \int d\vec{r}' \mathcal{N}_{\nu\nu'}^{-\frac{1}{2}}(\vec{r}, \vec{r}') \bar{\chi}_{\nu'}(\vec{r}') \right) \hat{A} \Phi_{\nu\vec{r}}^{J^\pi T(A-a,a)}$$

$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \mathcal{N}^{-\frac{1}{2}} \mathcal{H} \mathcal{N}^{-\frac{1}{2}} \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \bar{\chi} \end{pmatrix}$$

# NCSMC formalism

Start from 
$$\begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \bar{\mathcal{H}} \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix} = E \begin{pmatrix} 1 & \bar{g} \\ \bar{g} & 1 \end{pmatrix} \begin{pmatrix} c \\ \chi \end{pmatrix}$$

NCSM sector: 
$$(H_{NCSM})_{\lambda\lambda'} = \langle A\lambda J^\pi T | \hat{H} | A\lambda' J^\pi T \rangle = \varepsilon_\lambda^{J^\pi T} \delta_{\lambda\lambda'}$$

NCSM/RGM sector: 
$$\bar{\mathcal{H}}_{\nu\nu'}(r, r') = \sum_{\mu\mu'} \int \int dy dy' y^2 y'^2 \mathcal{N}_{\nu\mu}^{-\frac{1}{2}}(r, y) \mathcal{H}_{\mu\mu'}(y, y') \mathcal{N}_{\mu'\nu'}^{-\frac{1}{2}}(y', r')$$

Coupling: 
$$\bar{g}_{\lambda\nu}(r) = \sum_{\nu'} \int dr' r'^2 \langle A\lambda J^\pi T | \hat{\mathcal{A}}_{\nu'} | \Phi_{\nu'r'}^{J^\pi T} \rangle \mathcal{N}_{\nu'\nu}^{-\frac{1}{2}}(r', r)$$

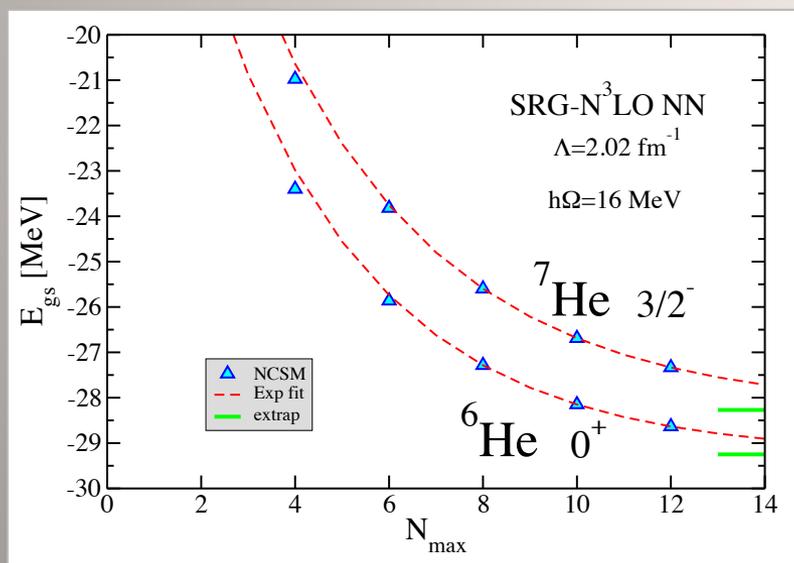
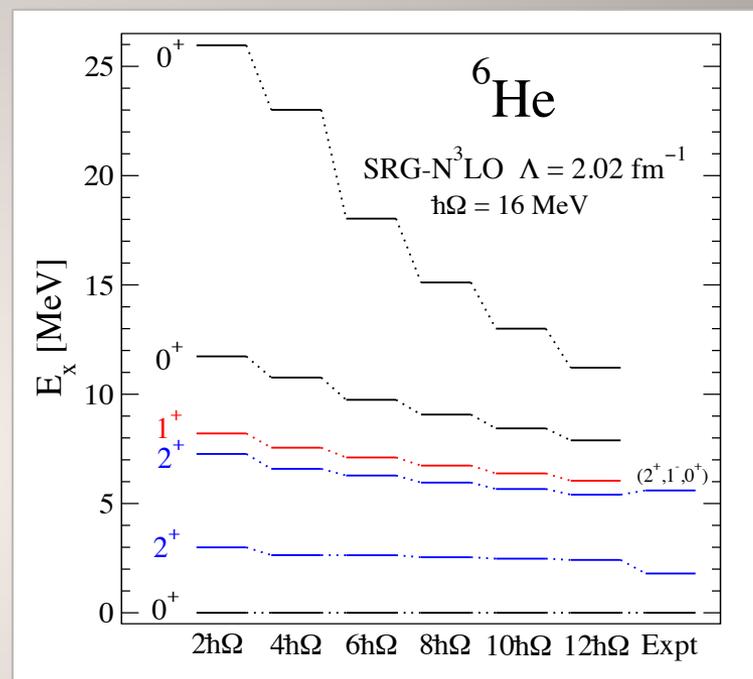
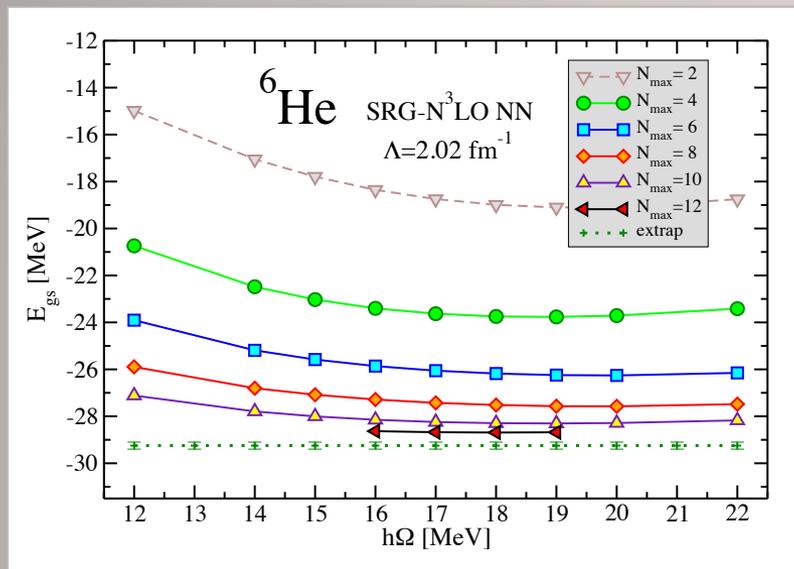
$$\bar{h}_{\lambda\nu}(r) = \sum_{\nu'} \int dr' r'^2 \langle A\lambda J^\pi T | \hat{H} \hat{\mathcal{A}}_{\nu'} | \Phi_{\nu'r'}^{J^\pi T} \rangle \mathcal{N}_{\nu'\nu}^{-\frac{1}{2}}(r', r)$$

Orthogonalization: 
$$N_{\nu r \nu' r'}^{\lambda\lambda'} = \begin{pmatrix} \delta_{\lambda\lambda'} & \bar{g}_{\lambda\nu'}(r') \\ \bar{g}_{\lambda'\nu}(r) & \delta_{\nu\nu'} \frac{\delta(r-r')}{rr'} \end{pmatrix} \quad \bar{H} = N^{-\frac{1}{2}} \begin{pmatrix} H_{NCSM} & \bar{h} \\ \bar{h} & \bar{\mathcal{H}} \end{pmatrix} N^{-\frac{1}{2}} \quad \begin{pmatrix} \bar{c} \\ \bar{\chi} \end{pmatrix} = N^{+\frac{1}{2}} \begin{pmatrix} c \\ \chi \end{pmatrix}$$

Solve with generalized microscopic R-matrix 
$$(\hat{\bar{H}} + \hat{L} - E) \begin{pmatrix} \bar{c} \\ \bar{\chi} \end{pmatrix} = \hat{L} \begin{pmatrix} \bar{c} \\ \bar{\chi} \end{pmatrix}$$

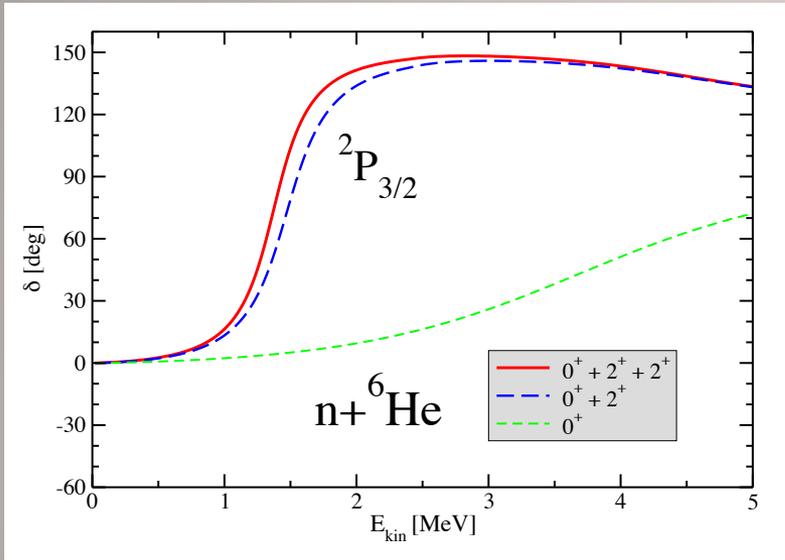
Bloch operator  $\rightarrow \hat{L}_\nu = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{2}\delta(r-a)(\frac{d}{dr} - \frac{B_\nu}{r}) \end{pmatrix}$

# NCSM calculations of ${}^6\text{He}$ and ${}^7\text{He}$ energies

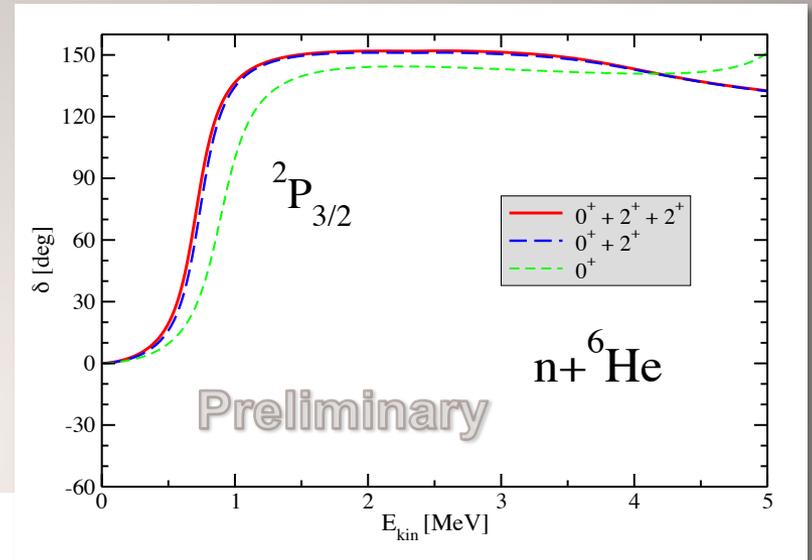


- ✓  $N_{\text{max}}$  convergence OK
- ✓ Extrapolation feasible
  - ${}^6\text{He}$ :  $E_{\text{gs}} = -29.25(15) \text{ MeV}$  (Expt.  $-29.269 \text{ MeV}$ )
  - ${}^7\text{He}$ :  $E_{\text{gs}} = -28.27(25) \text{ MeV}$  (Expt.  $-28.82(30) \text{ MeV}$ )
- ${}^7\text{He}$  unbound ( $+0.44(3) \text{ MeV}$ ), width  $0.16(3) \text{ MeV}$ 
  - NCSM: no information about the width
- All  ${}^6\text{He}$  excited states above  $2^+_1$  broad resonances or states in continuum

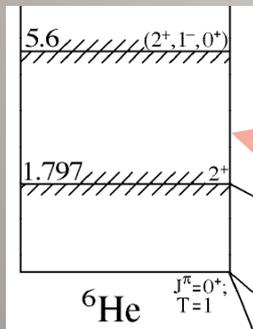
# NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He}+n$



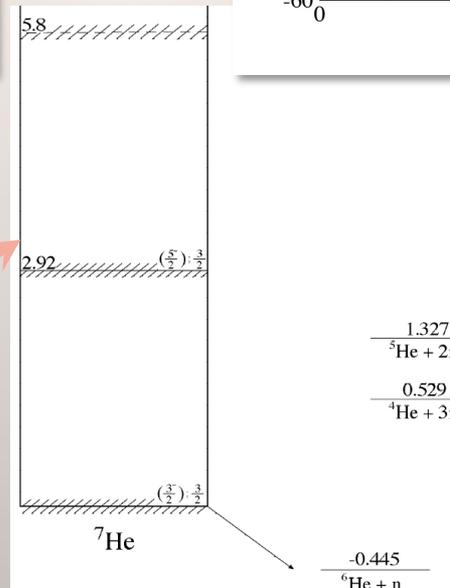
NCSM/RGM  
with up to three  ${}^6\text{He}$  states



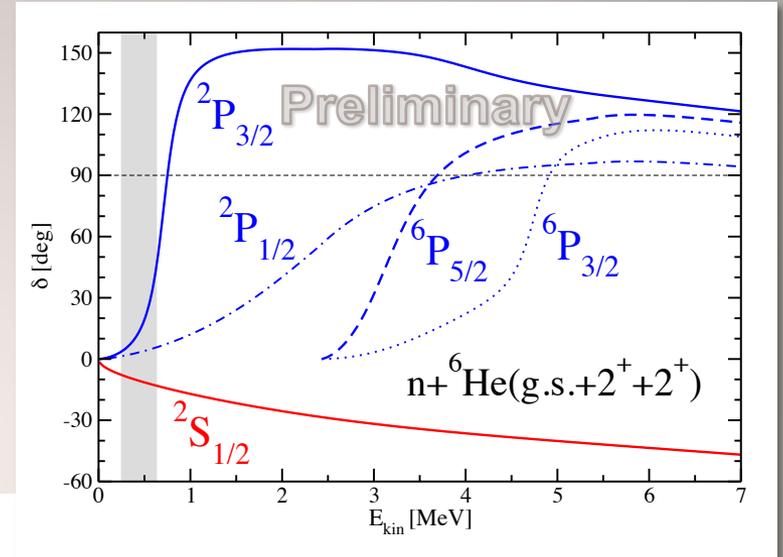
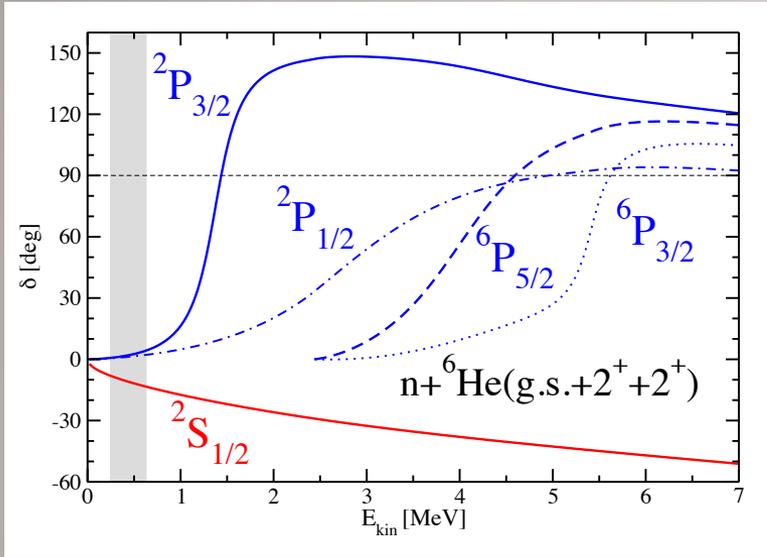
NCSMC  
with up to three  ${}^6\text{He}$  states  
and three  ${}^7\text{He}$  eigenstates  
More 7-nucleon correlations  
Fewer target states needed



Expt.

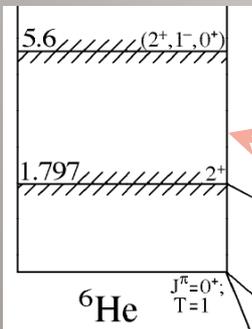


# NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He}+n$

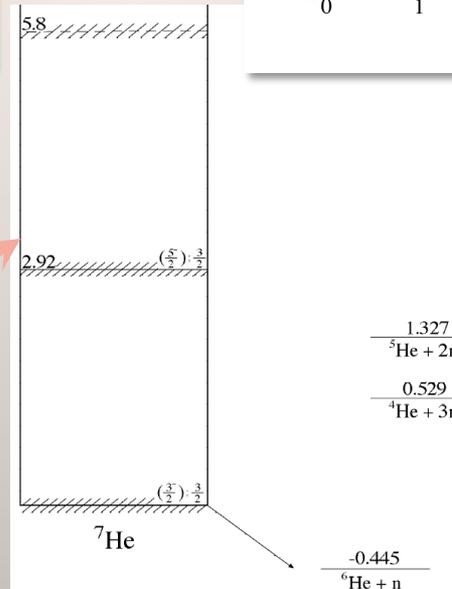


NCSM/RGM  
with three  ${}^6\text{He}$  states

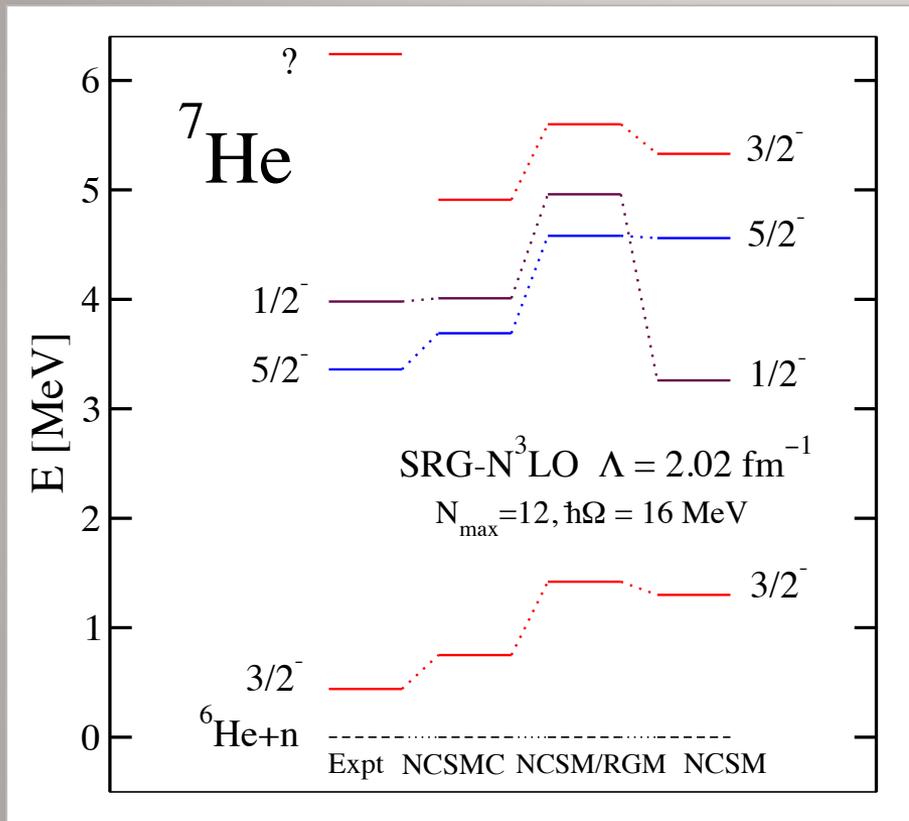
NCSMC  
with three  ${}^6\text{He}$  states  
and three  ${}^7\text{He}$  eigenstates  
More 7-nucleon correlations  
Fewer target states needed



Expt.



# $^7\text{He}$ : NCSMC vs. NCSM/RGM vs. NCSM



Preliminary

$J^\pi$	experiment			NCSMC		NCSM/RGM		NCSM
	E	$\Gamma$	Ref.	E	$\Gamma$	E	$\Gamma$	E
$3/2^-$	0.44(3)	0.16(3)	[29]	0.75	0.31	1.42	0.52	1.30
$5/2^-$	3.36(10)	2.2(3)	[30]	3.69	2.57	4.58	3.06	4.56
$1/2^-$	3.98	10	[42]	4.01	15.15	4.96	14.95	3.26
	3.48	2	[38]					

- NCSMC and NCSM/RGM energies from phase shifts at 90 degrees
- NCSMC and NCSM/RGM widths from the derivatives of phase shifts (preliminary)

$$\Gamma = \frac{2}{\left. \frac{\partial \delta(E_{kin})}{\partial E_{kin}} \right|_{E_{kin}=E_R}}$$

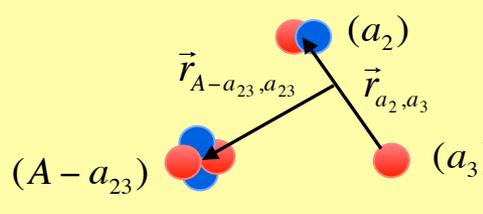
**Experimental controversy:**  
 Existence of low-lying  $1/2^-$  state  
 ... not seen in these calculations

# Three-cluster NCSM/RGM

- The starting point:

$$\Psi_{RGM}^{(A)} = \sum_{a_2 a_3 \nu} \int d\vec{x} d\vec{y} G_{\nu}^{(A-a_{23}, a_2, a_3)}(x, y) \times \hat{A}^{(A-a_{23}, a_2, a_3)} \left| \Phi_{\nu \vec{x} \vec{y}}^{(A-a_{23}, a_2, a_3)} \right\rangle$$

$$\rho^{5/2} \sum_K \chi_{\nu K}^{(A-a_{23}, a_2, a_3)}(\rho) \phi_K^{\ell x \ell y}(\alpha)$$



$$\psi_{\alpha_1}^{(A-a_{23})} \psi_{\alpha_2}^{(a_2)} \psi_{\alpha_3}^{(a_3)} Y^{\ell x, \ell y}(\hat{x}, \hat{y}) \times \delta(\vec{x} - \vec{r}_{a_2, a_3}) \delta(\vec{y} - \vec{r}_{A-a_{23}, a_{23}})$$

- Solves:

$$\sum_{a_2 a_3 \nu K} \int d\rho \rho^5 \left[ H_{a' \nu', a \nu}^{K', K}(\rho', \rho) - E N_{a' \nu', a \nu}^{K', K}(\rho', \rho) \right] \rho^{-5/2} \chi_{\nu K}^{(A-a_{23}, a_2, a_3)}(\rho) = 0$$

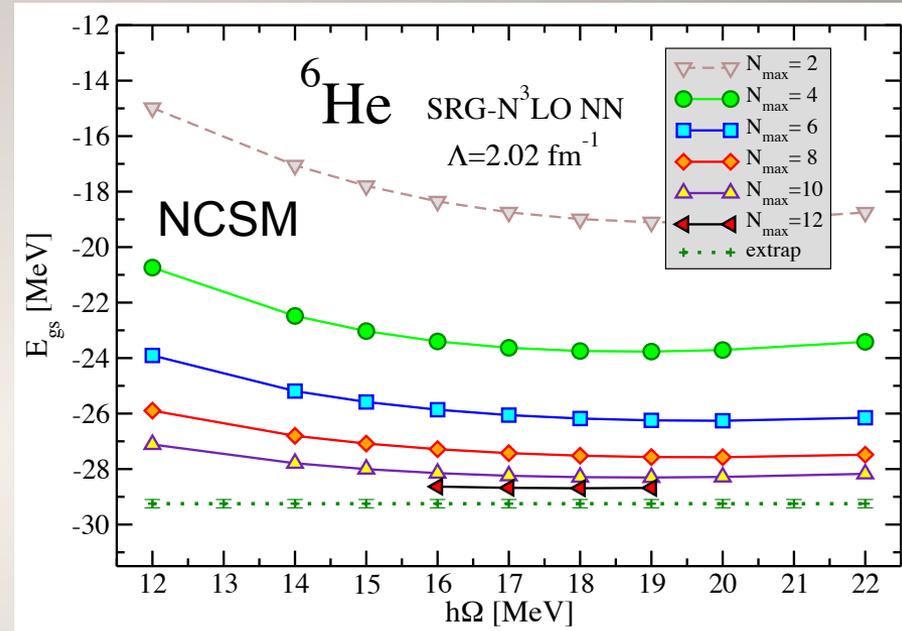
- Where the hyperspherical coordinates are given by:

$$\rho = \sqrt{x^2 + y^2}, \quad \alpha = \arctan\left(\frac{y}{x}\right) \quad \left( x = \rho \cos \alpha, \quad y = \rho \sin \alpha \right)$$

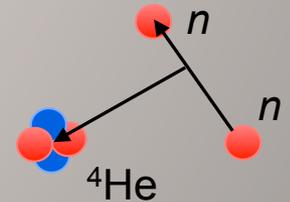
# First results for ${}^6\text{He}$ ground state

S. Quaglioni, C. Romero-Redondo, P. Navratil, work in progress

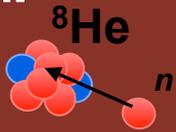
- Preliminary NCSM/RGM results for up to  $N_{\text{max}} = 8$  model space ( $\hbar\Omega = 16$  MeV)
  - $n+n+{}^4\text{He}(\text{g.s.})$
  - SRG-NN chiral with  $\lambda = 2.02 \text{ fm}^{-1}$
- Results compared with NCSM: Gain in binding due to the coupling to continuum
  - At  $N_{\text{max}} = 8$   ${}^6\text{He}$  unbound within the NCSM, bound in the three-cluster NCSM/RGM by  $\sim 1$  MeV
  - Extrapolated NCSM beyond  $N_{\text{max}} = 12$ :  ${}^6\text{He}$  bound by  $\sim 1$  MeV



HO model space	$E_{\text{g.s.}}({}^4\text{He})$ [MeV] (NCSM)	$E_{\text{g.s.}}({}^6\text{He})$ [MeV] (NCSM)	$E_{\text{g.s.}}({}^6\text{He})$ [MeV] (NCSM/RGM)
$N_{\text{max}} = 8$	-27.32	-27.28	-28.38



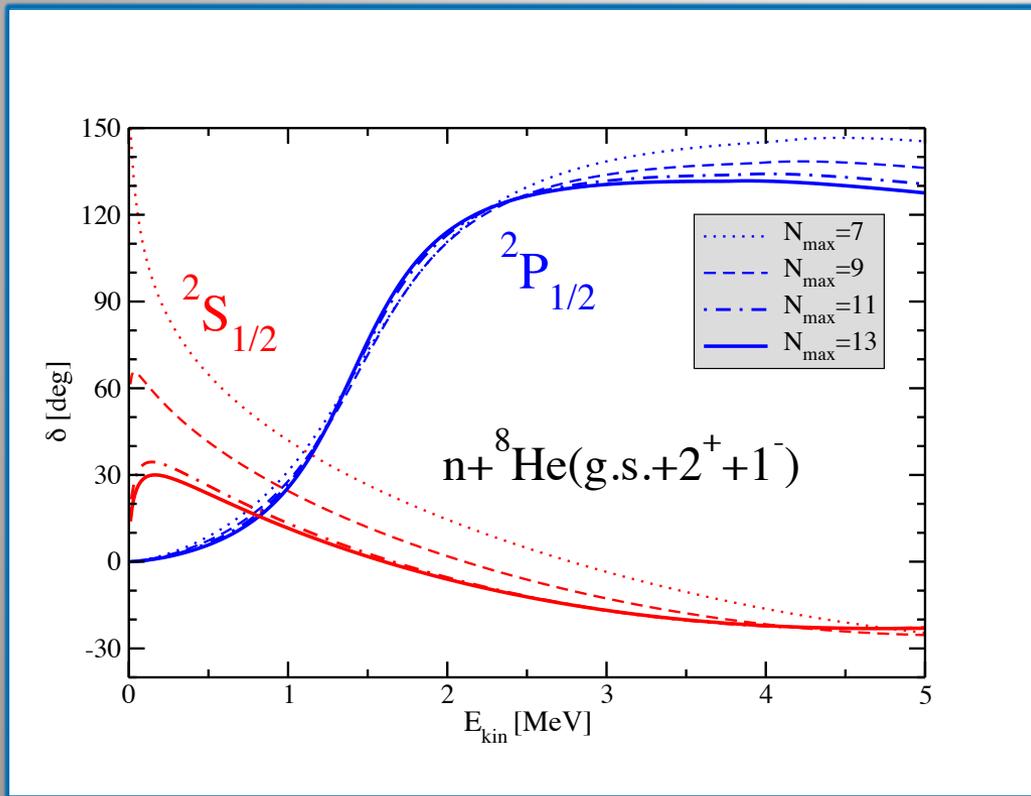
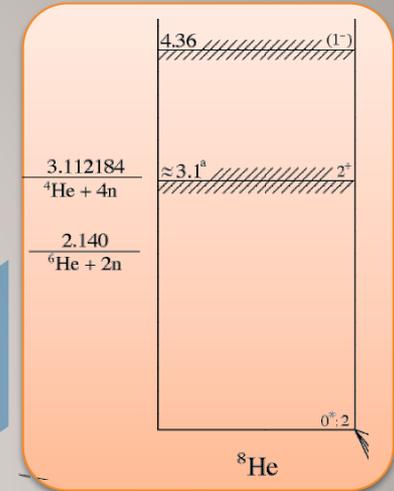
PRELIMINARY



# Structure of the exotic ${}^9\text{He}$ nucleus

- NCSM/RGM calculation of  $n\text{-}{}^8\text{He}$ 
  - SRG- $\text{N}^3\text{LO}$   $NN$  potential with  $\Lambda = 2.02 \text{ fm}^{-1}$
  - ${}^8\text{He}$   $0^+$  g.s. and  $2^+$ ,  $1^-$  excited states included
  - Up to  $N_{\text{max}} = 13$

exotic nuclei



Experiment:

$P$ -wave resonance at

Expt.  $\sim 1.27(10) \text{ MeV}$  (Bohlen *et al.*)

$S$ -wave attraction

Expt.  $a_0 \sim -10 \text{ fm}$  (Chen *et al.*)

$a_0 \sim -3 \text{ fm}$  (Al Falou, *et al.*)

Calculation (preliminary):

**No** bound state

$P$ -wave resonance at  $\sim 1.6 \text{ MeV}$

Attraction in the  $S$ -wave,  $a_0 \sim -13 \text{ fm}$

SRG- $\text{N}^3\text{LO}$   $NN$  too attractive?  $3N$  needed...

# Conclusions and Outlook

- With the NCSM/RGM approach we are extending the *ab initio* effort to describe low-energy reactions and weakly-bound systems
- The first  ${}^7\text{Be}(p,\gamma){}^8\text{B}$  *ab initio* S-factor calculation PLB 704 (2011) 379
- Deuteron-projectile results with SRG- $\text{N}^3\text{LO}$  *NN* potentials:
  - $d$ - ${}^4\text{He}$  scattering PRC 83, 044609 (2011)
  - First *ab initio* study of  ${}^3\text{H}(d,n){}^4\text{He}$  &  ${}^3\text{He}(d,p){}^4\text{He}$  fusion PRL 108, 042503 (2012)
- Under way:
  - $n$ - ${}^8\text{He}$  scattering and  ${}^9\text{He}$  structure
  - ${}^3\text{He}$ - ${}^4\text{He}$  and  ${}^3\text{He}$ - ${}^3\text{He}$  scattering calculations
  - *Ab initio* NCSM with continuum (NCSMC)
  - Three-cluster NCSM/RGM and treatment of three-body continuum
  - Inclusion of **NNN** force
- To do:
  - Alpha clustering:  ${}^4\text{He}$  projectile