Deformed Halo Nuclei
~ theoretical aspects ~

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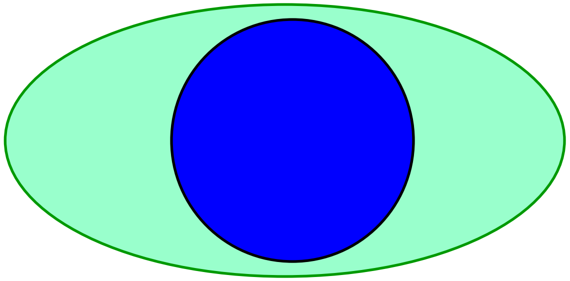
1. Deformed halo nucleus: what is it?
2. Single-particle motion in a deformed potential
3. Particle-rotor model and its appliation to ^{31}Ne
4. 2n halo nuclei: odd-even staggering of σ_R and pairing correlation
5. Summary

What is “deformed halo”? : definition

halo: core-valence decoupling

halo + deformation:



spherical core
+ deformed valence
orbit

cf. ^{17}O : slightly oblate

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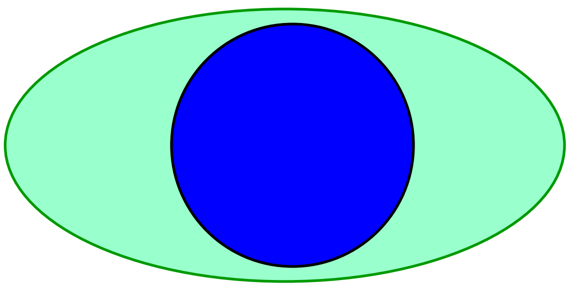

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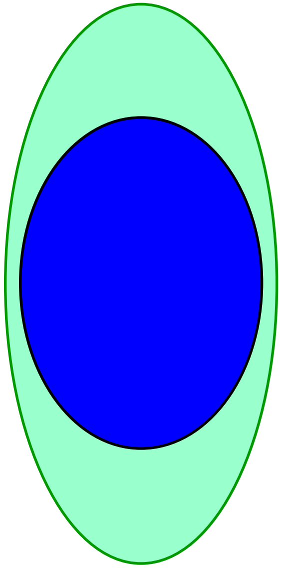
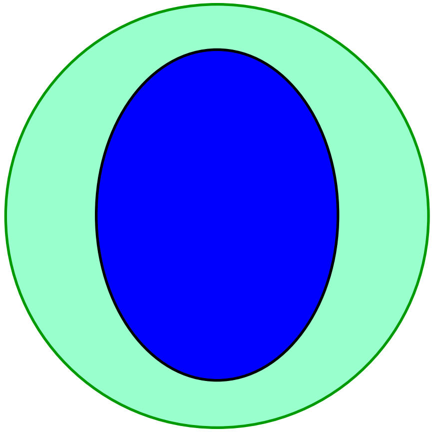


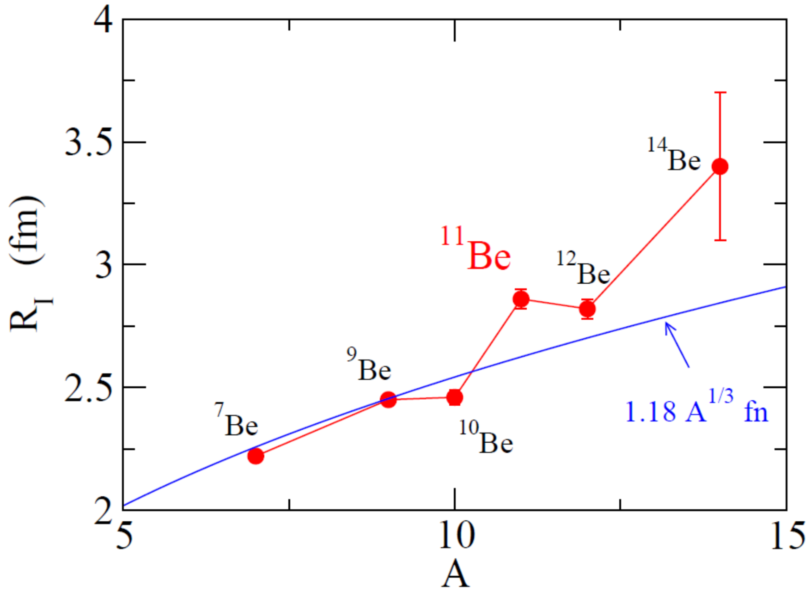
deformed core
+ def. orbit



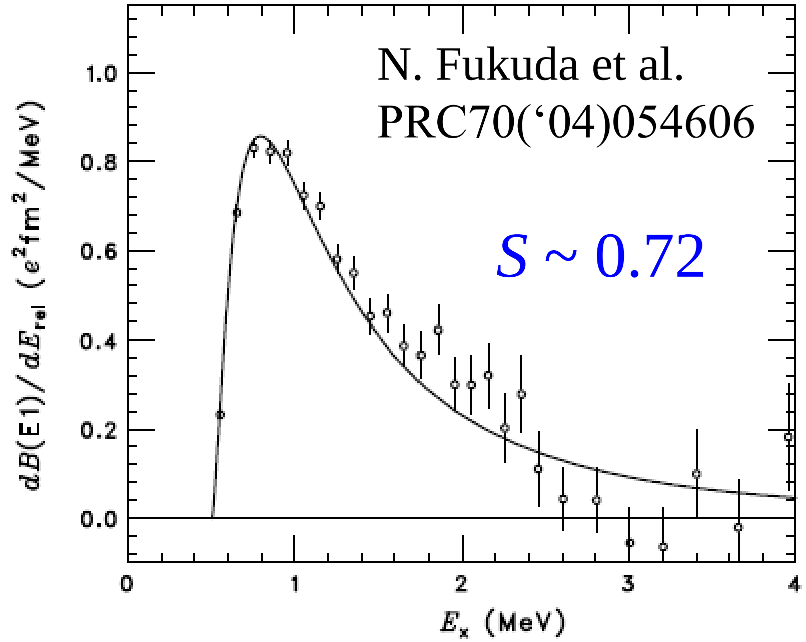
deformed core
+ spherical orbit

deformed halo nucleus

Well-known example: ^{11}Be   (S_n = 504 +/- 6 keV)



I. Tanihata et al.,
PRL55('85)2676; PLB206('88)592



0.32 MeV

1/2$^-$

1/2$^+$

^{11}Be

?

$1p_{1/2}$

$2s_{1/2}$

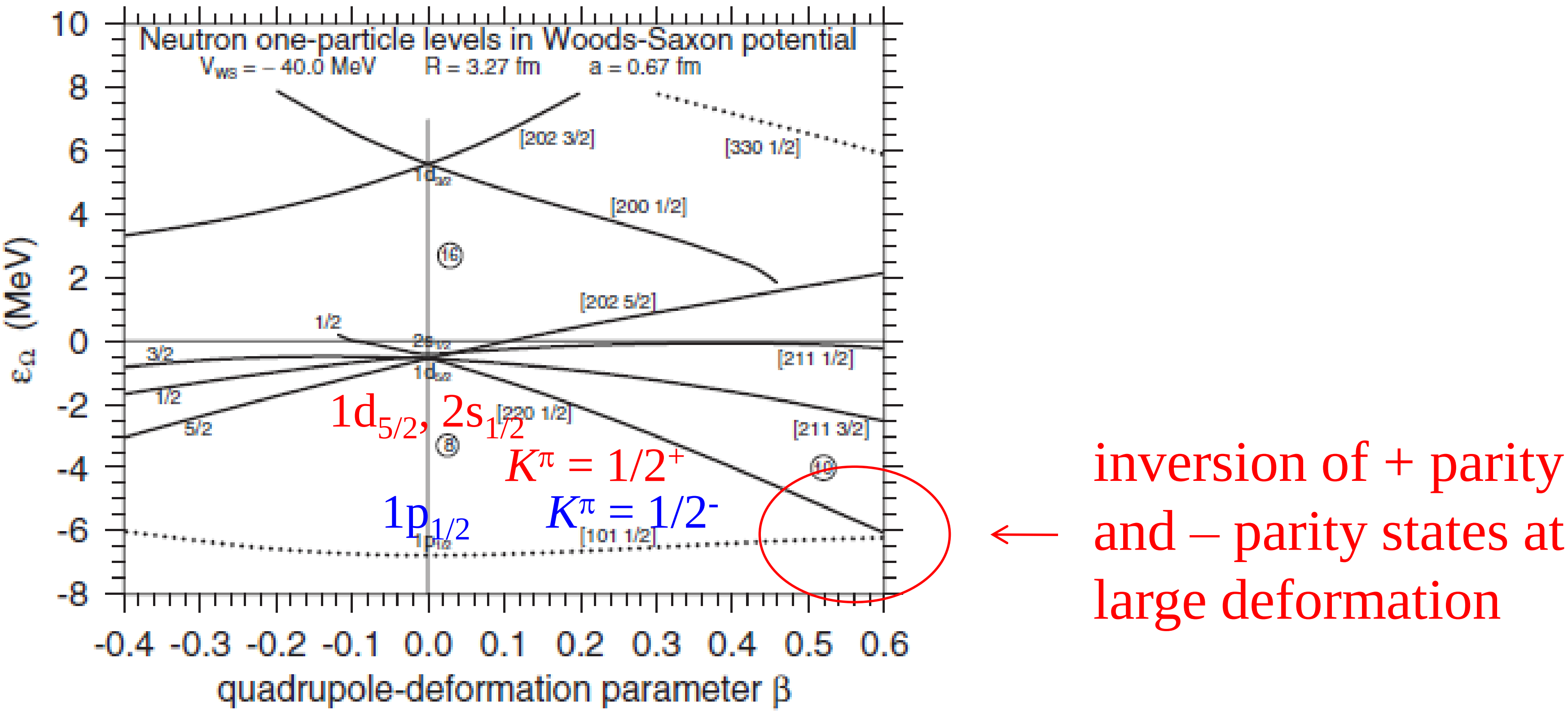
$1p_{3/2}$

$1s_{1/2}$

"parity inversion"

deformed ^{11}Be ? ⟶ single-particle motion in a deformed potential

Can deformation effect explain the level scheme of ^{11}Be ?



I. Hamamoto, J. Phys. G37('10)055102

cf. <u>coupled-channels calculation with finite core excitation energies</u>:

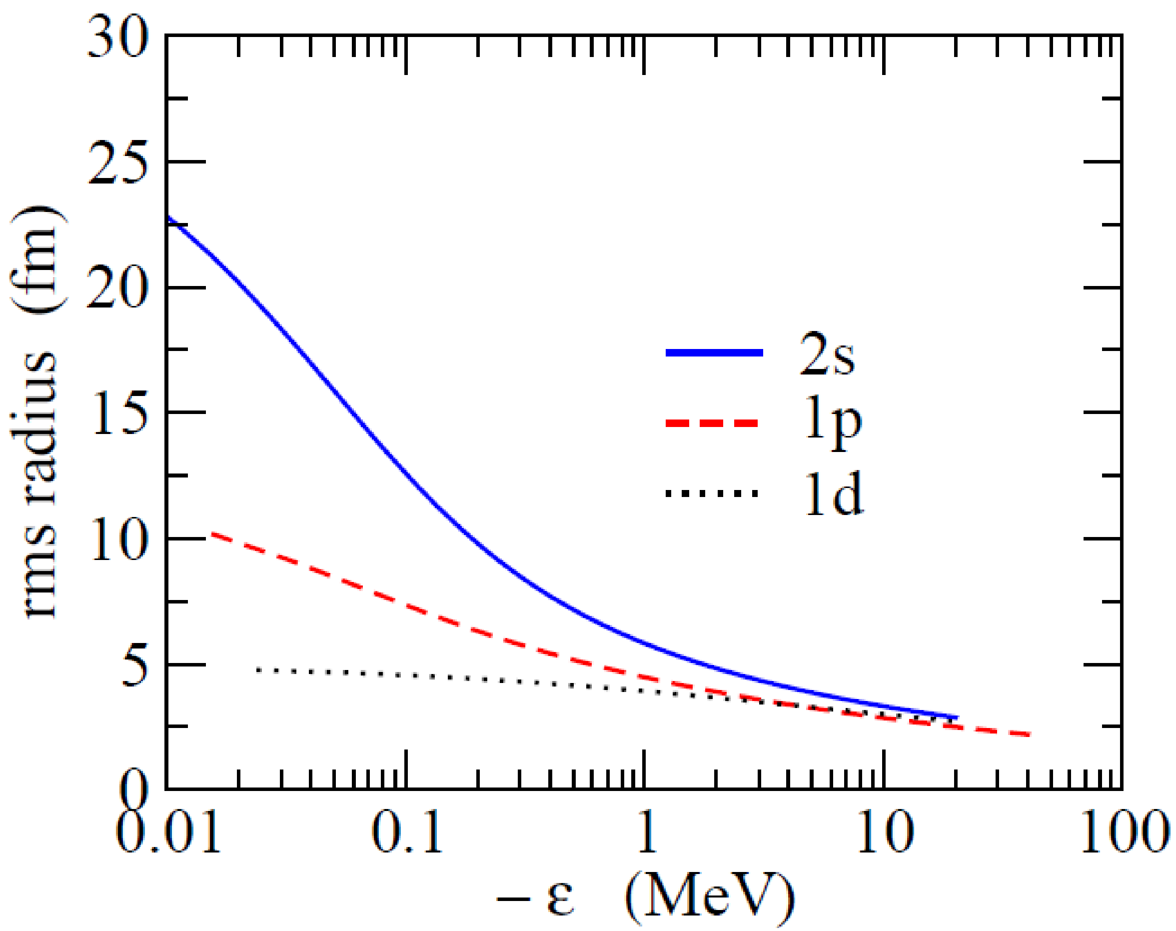
H. Esbensen, B.A. Brown, H. Sagawa, PRC51('95)1274

F.M. Nunes, I.J. Thompson, R.C. Johnson, NPA596('96)171

Role of s.p. angular momentum in halo formation

$$\langle r^2 \rangle \propto \begin{cases} 1/|\epsilon_0| & (l=0) \\ 1/\sqrt{|\epsilon_1|} & (l=1) \\ const. & (l=2) \end{cases}$$

K. Riisager,
A.S. Jensen, and
P. Moller, NPA548('92)393



radius: diverges for l = 0,1
in the zero binding limit

halo (anomalously large
radius): l = 0 or 1

s.p. motion in a deformed potential

halo : only for $l = 0$ or 1

⇨ however, a possibility is enlarged for a deformed nucleus

deformed potential $V(r,\theta)$ $\longrightarrow$ mixture of angular momenta

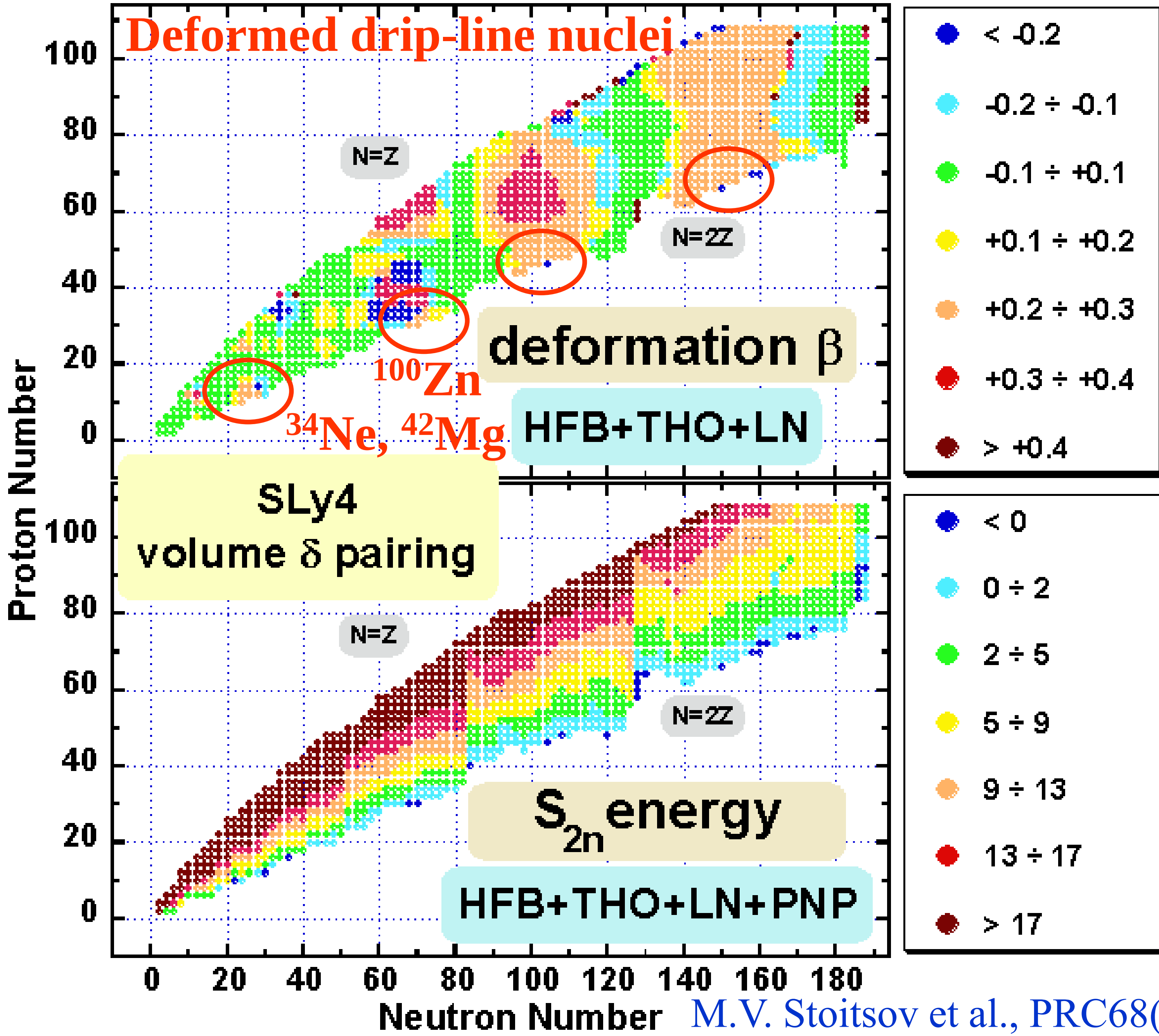
e.g.,

$$|d_{5/2}\rangle \to |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots$$

$$|f_{7/2}\rangle \to |f_{7/2}\rangle + |p_{3/2}\rangle + |p_{1/2}\rangle + \cdots$$

(note) $s_{1/2}$: $\Omega^\pi = 1/2^+$ only
$p_{1/2}$: $\Omega^\pi = 1/2^-$ only
$p_{3/2}$: $\Omega^\pi = 3/2^-$ and $1/2^-$ only

$\longrightarrow$ possibility of halo only for s.p. states with $\Omega^\pi = 1/2^+, 1/2^-, 3/2^-$



^{32}Ne:
β=0.151
^{34}Ne:
β=0.277
^{42}Mg:
β=-0.18
^{100}Zn:
β=0.244

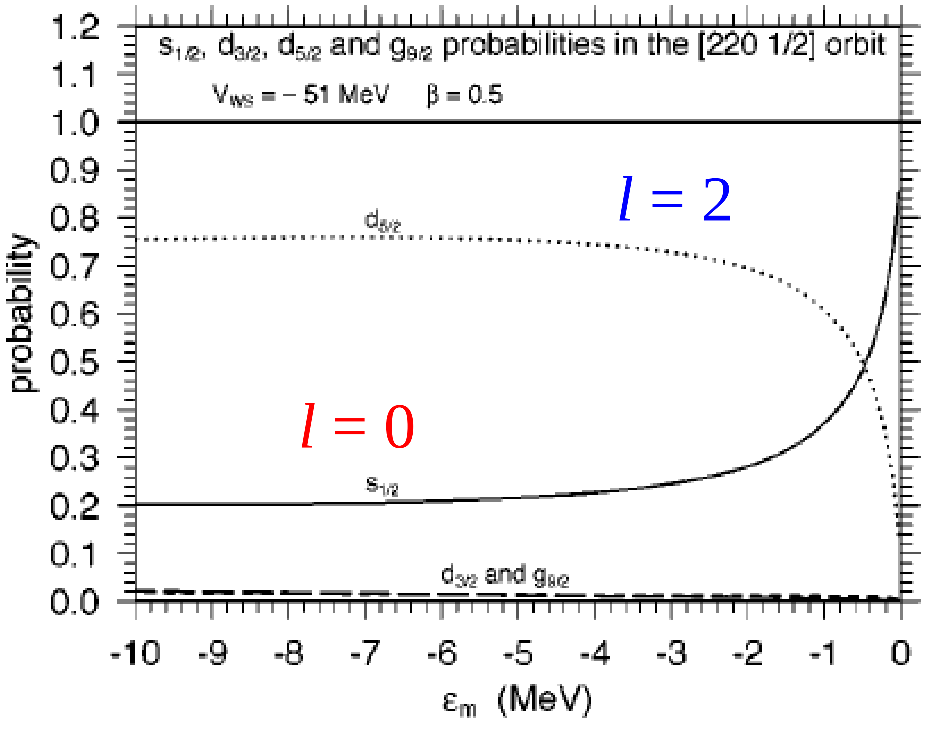
M.V. Stoitsov et al., PRC68('03)054312

s.p. motion in a deformed potential

$$|d_{5/2}\rangle \;\rightarrow\; |d_{5/2}\rangle + |s_{1/2}\rangle + |g_{7/2}\rangle + \cdots$$

$$\rightarrow\; |s_{1/2}\rangle \qquad (|\epsilon| \rightarrow 0)$$

T. Misu, W. Nazarewicz,
and S. Aberg, NPA614('97)44
(deformed square well)



I. Hamamoto, PRC69('04)041306(R)
(deformed Woods-Saxon)

reason for *s*-wave dominance

$$\Psi(\boldsymbol{r})=\sum_l R_l(r)Y_{lK}(\hat{\boldsymbol{r}})\equiv\sum_l \psi_{lK}(\boldsymbol{r})$$

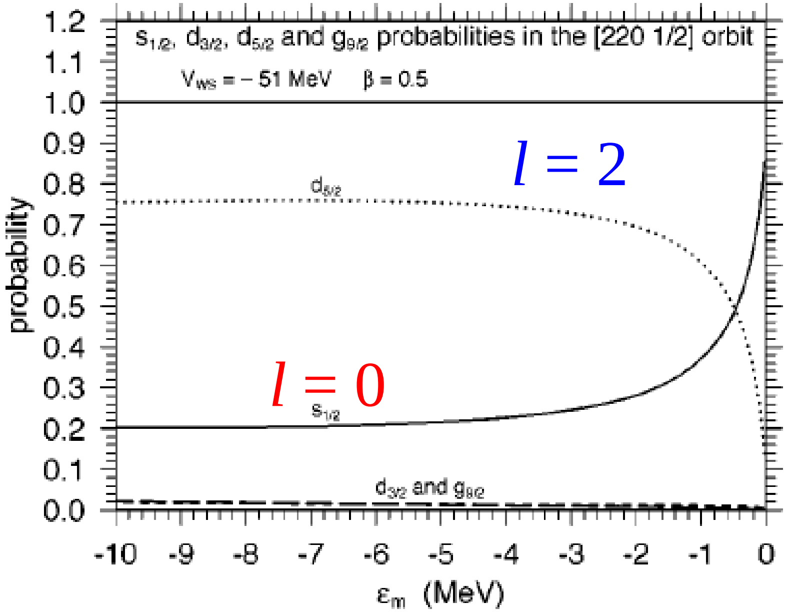
$$P_l=\frac{\langle\psi_{lK}|\psi_{lK}\rangle}{\langle\Psi|\Psi\rangle}=\frac{\langle\psi_{lK}|\psi_{lK}\rangle}{\sum_{l'}\langle\psi_{l'K}|\psi_{l'K}\rangle}$$

(note) $\langle\psi_{lK}|\psi_{lK}\rangle$ diverges for $l = 0$ ($\varepsilon \rightarrow 0$)
finite for $l > 0$

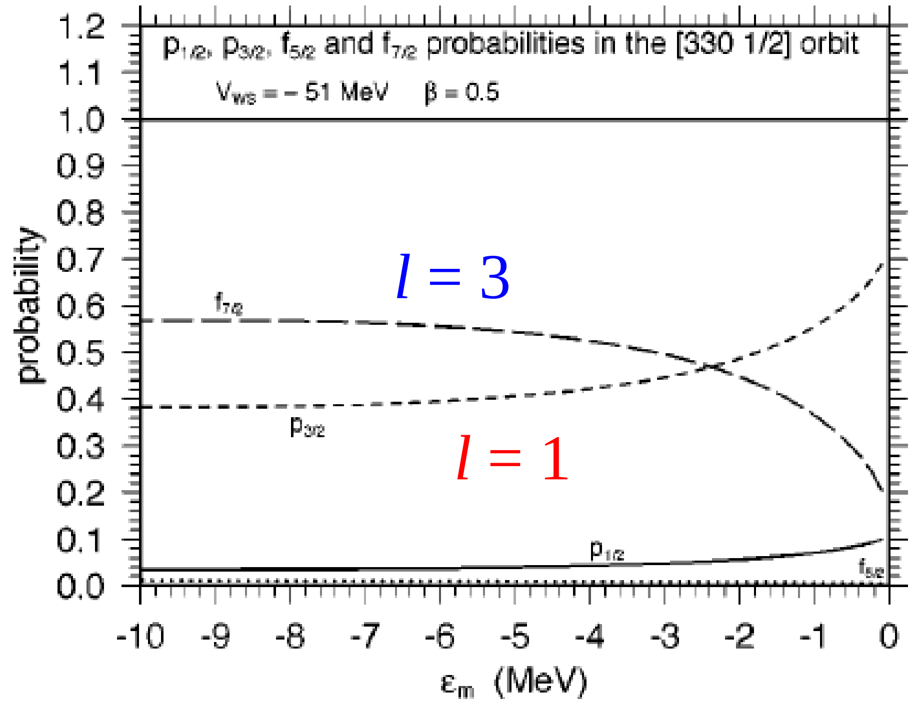
$$P_l\sim\frac{\langle\psi_{lK}|\psi_{lK}\rangle}{\langle\psi_{0K}|\psi_{0K}\rangle}=1\qquad(l=0)$$

(note) $$\boxed{\beta_2\propto\frac{\langle r^2Y_{20}\rangle}{\langle r^2\rangle}\rightarrow 0\qquad(\epsilon\rightarrow 0)}$$

similar dominance phenomenon for *p*-wave



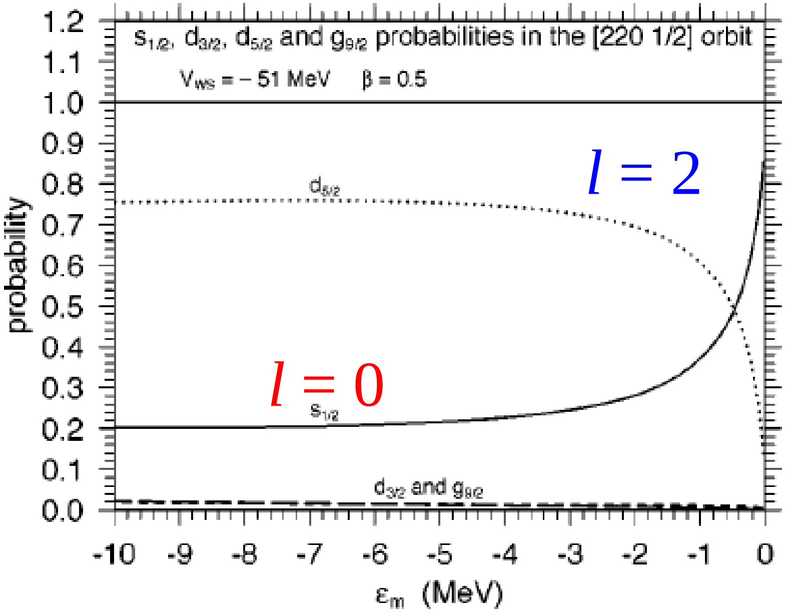
I. Hamamoto, PRC69('04)041306(R)



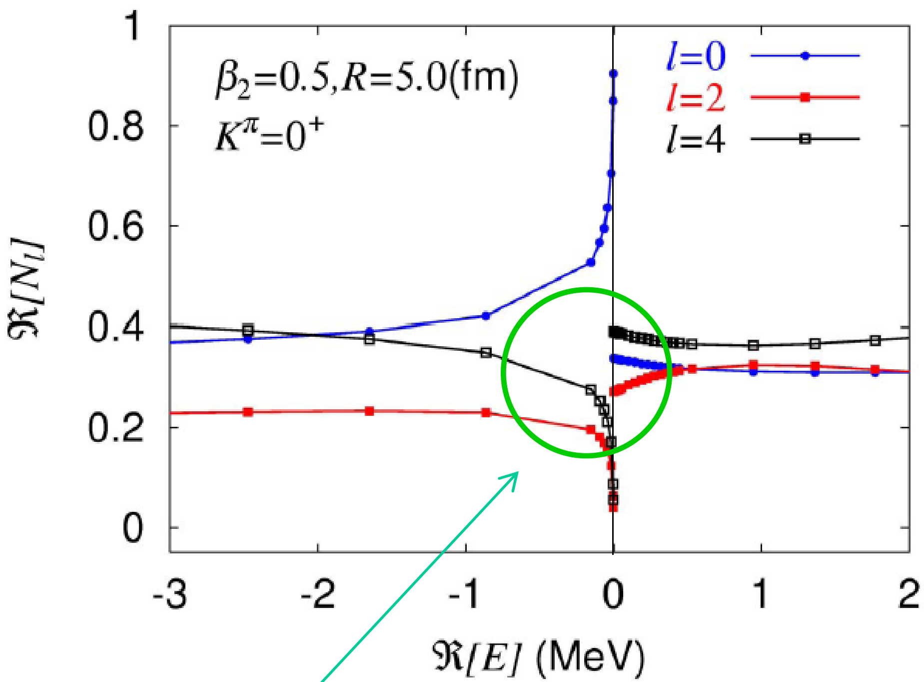
(enhancement of p-wave component, although not 100% in the zero binding limit)

c.f. s-wave dominance and s.p. resonance:
K. Yoshida and K.H., PRC72('05) 064311

c.f. s-wave dominance and s.p. resonance:



I. Hamamoto, PRC69(‘04)041306(R)



The s-wave dominance phenomenon does not continue to scattering states
→ existence of a $K^\pi = 0^+$ resonance

K. Yoshida and K. Hagino, PRC72(‘05)064311

particle-rotor model

Nilsson model: intrinsic (body-fixed) frame formalism

$\longrightarrow$ transformation to the lab. frame

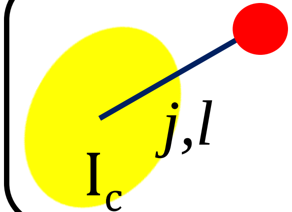
✓angular momentum projection
✓particle-rotor model

For an axially symmetric rotor,
Nilsson: adiabatic (strong coupling) limit of particle-rotor model

$\downarrow$

core + neutron two-body model
with core excitations

$$\Psi_{IM} = \sum_{I_c,j,l} \Big(\quad \Big)^{(IM)}$$



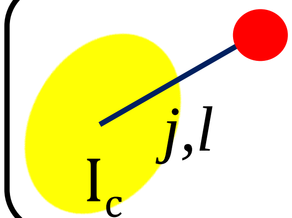
$$= |0^+ \otimes p_{3/2}\rangle + |2^+ \otimes f_{7/2}\rangle + \cdots$$

e.g.,

particle-rotor model

Nilsson: adiabatic (strong coupling) limit of particle-rotor model

$$\Psi_{IM} = \sum_{I_c,j,l} \Big[$$



$$\Big]^{(IM)} \underset{\text{e.g.,}}{=} |0^+ \otimes p_{3/2}\rangle + |2^+ \otimes f_{7/2}\rangle + \cdots$$

all the members of the ground rotational band are degenerate in energy

$\longrightarrow$ K : a good quantum number (no Coriolis coupling)

$$\underbrace{R^{(I)}_{I_c jl}(r)}_{\text{particle-rotor}} = A^{IK}_{jI_c} \cdot \underbrace{\phi_{jlK}(r)}_{\text{Nilsson}}$$

$$A^{IK}_{jI_c} = \sqrt{\frac{2I_c+1}{2I+1}} \cdot \sqrt{2}\, \langle j K I_c 0 | I K \rangle$$

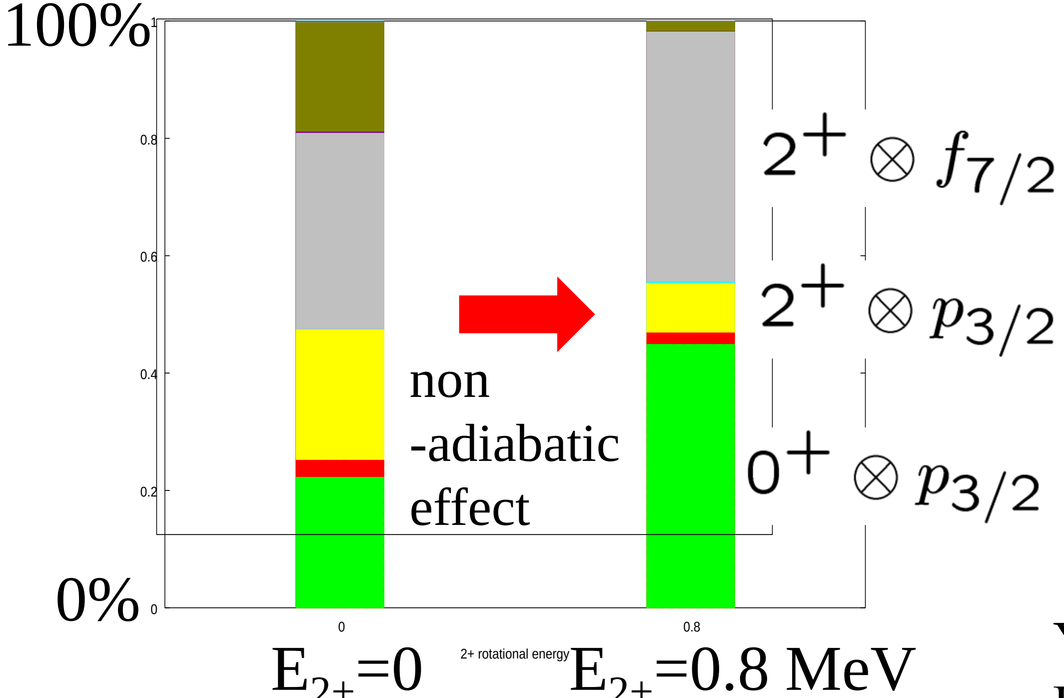
H. Esbensen and C.N. Davids, PRC63('00)014315

particle-rotor model with finite excitation energy

coupled-channels equations

non-adiabatic effect

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2mr^2}+V_0(r)+E_{I_c}-\epsilon\right)R^{(I)}_{I_cjl}(r)$$
$$=-\sum_{I'_c,j',l'}\langle[(jl)I_c]^{(IM)}|V_{\rm def}|[(j'l')I'_c]^{(IM)}\rangle R^{(I)}_{I'_cj'l'}(r)$$



example:

[330 1/2] level at β=0.2

↑

21st neutron

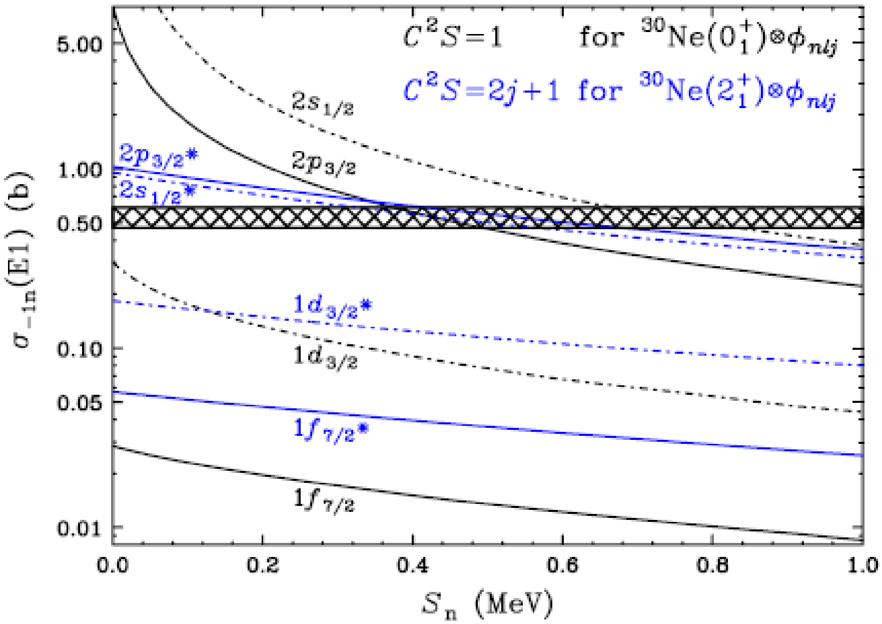
ε = -0.3 MeV

spherical basis with R_{box}=60 fm

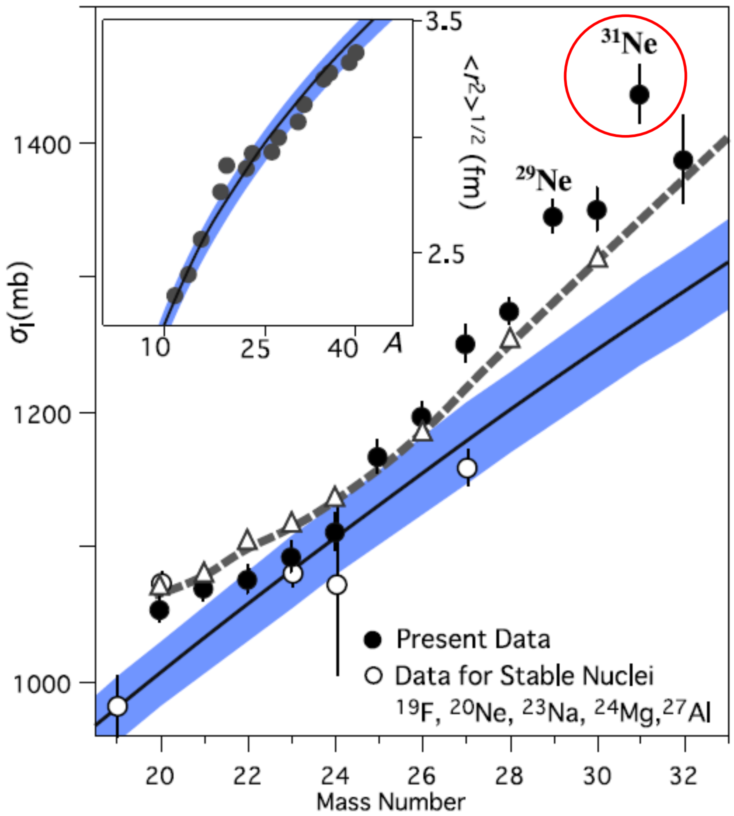
Y. Urata, K.H., and H. Sagawa, PRC83(‘11)041303(R)

Application to ^{31}Ne

large Coulomb breakup and interaction cross sections



T. Nakamura et al., PRL103('09)262501

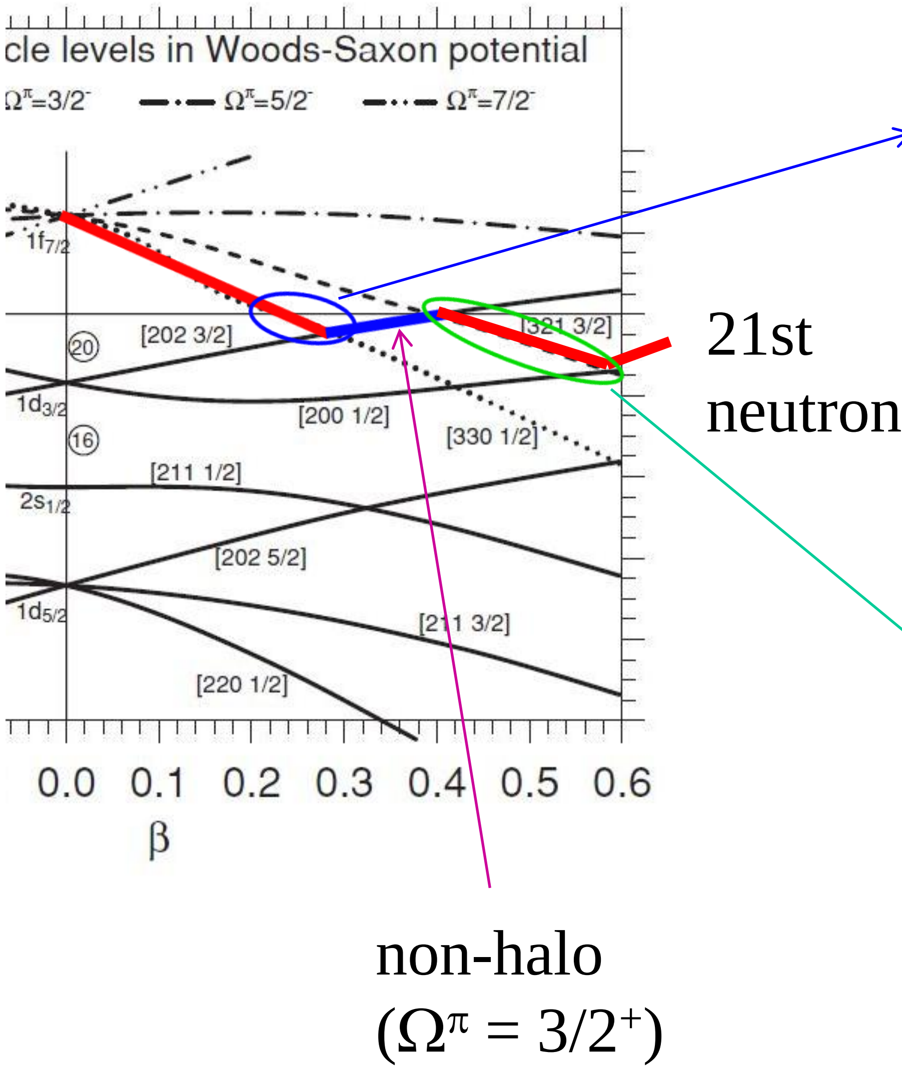


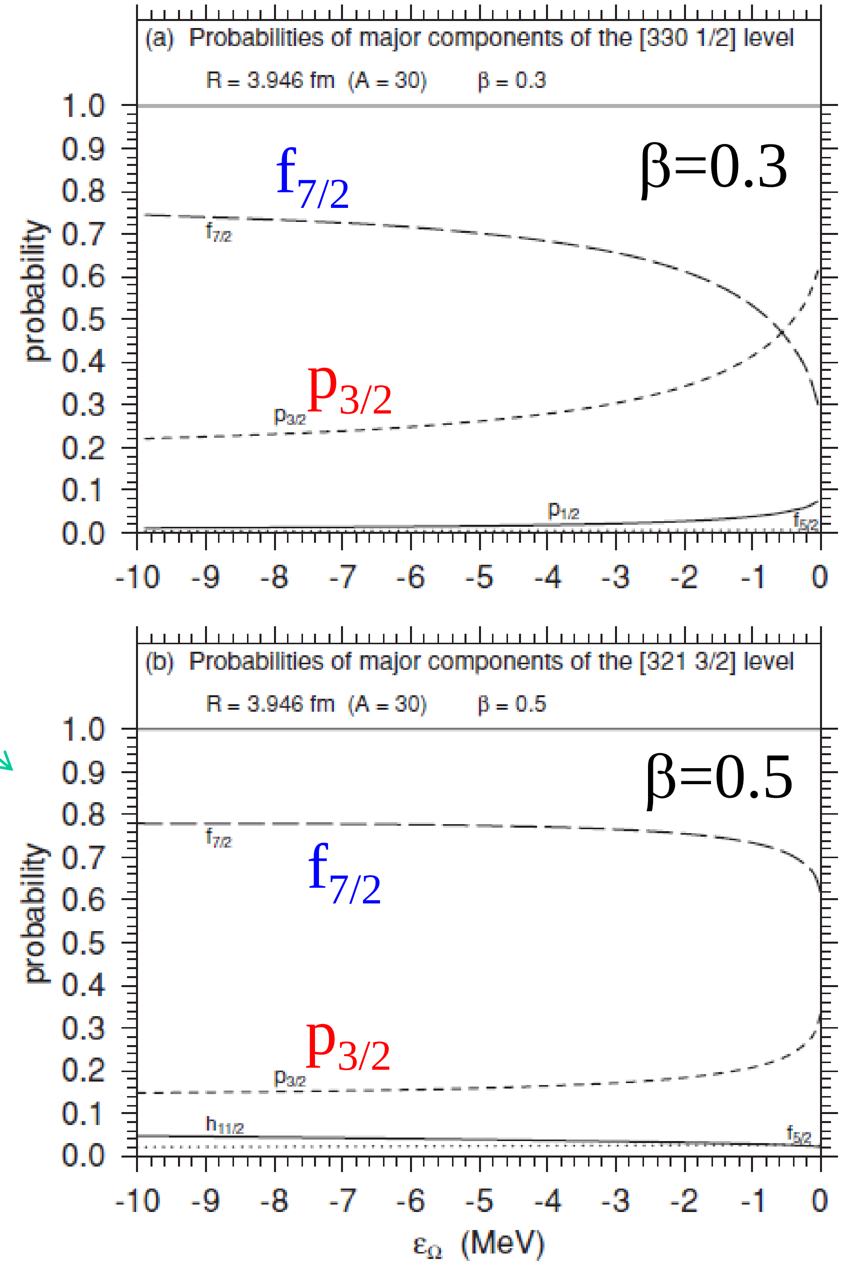
M.Takechi et al., PLB 707('12)357

theoretical studies:

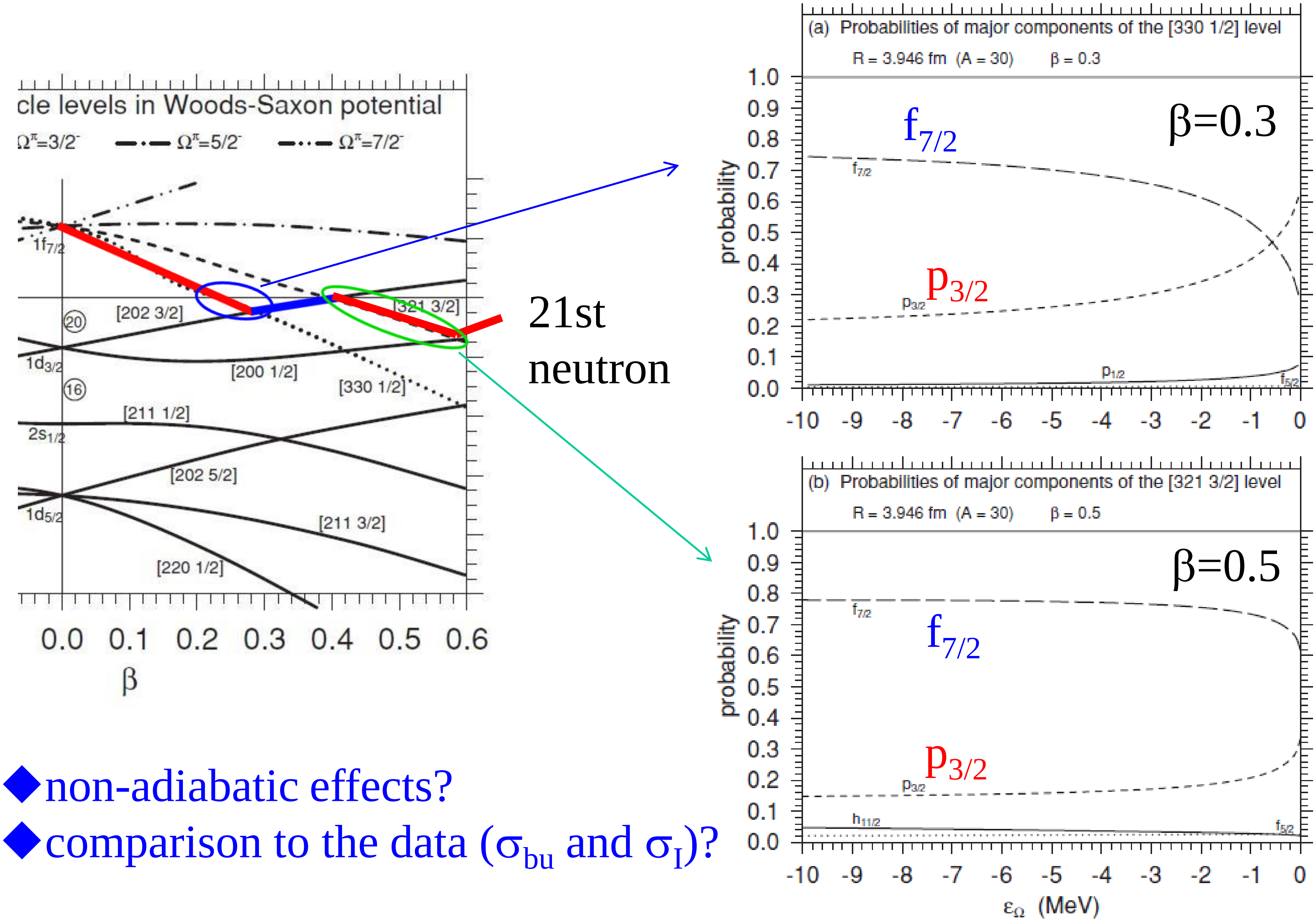
- W. Horiuchi et al., PRC81('10)024606
- I. Hamamoto, PRC81('10)021304(R)
- Y. Urata, K.H., H. Sagawa, PRC83('11)041303(R)
- K. Minomo et al., PRL108('12)052503; PRC84('11)034602; PRC85('12)064613

Nilsson model analysis [I. Hamamoto, PRC81('10)021304(R)]



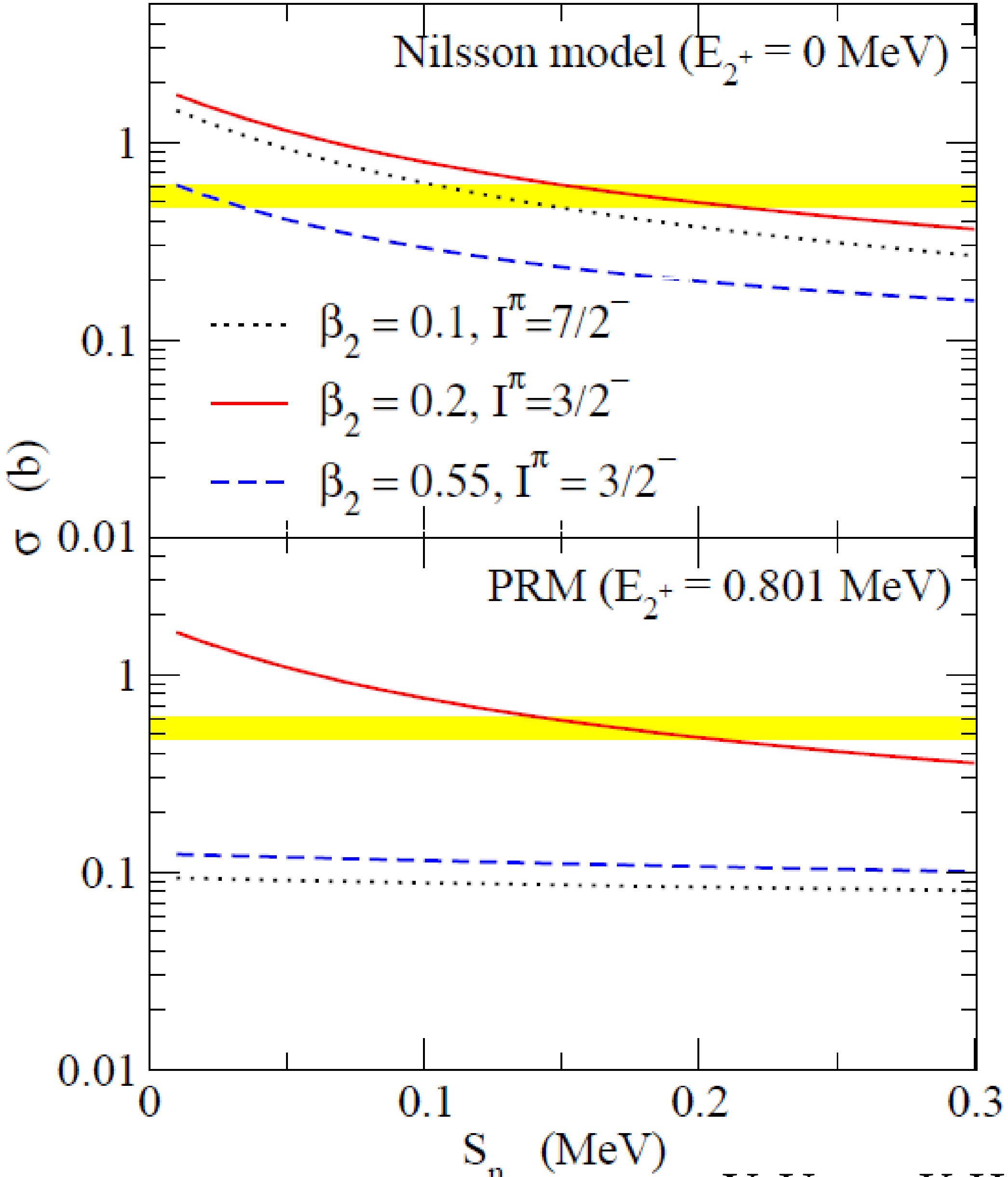


Nilsson model analysis [I. Hamamoto, PRC81('10)021304(R)]



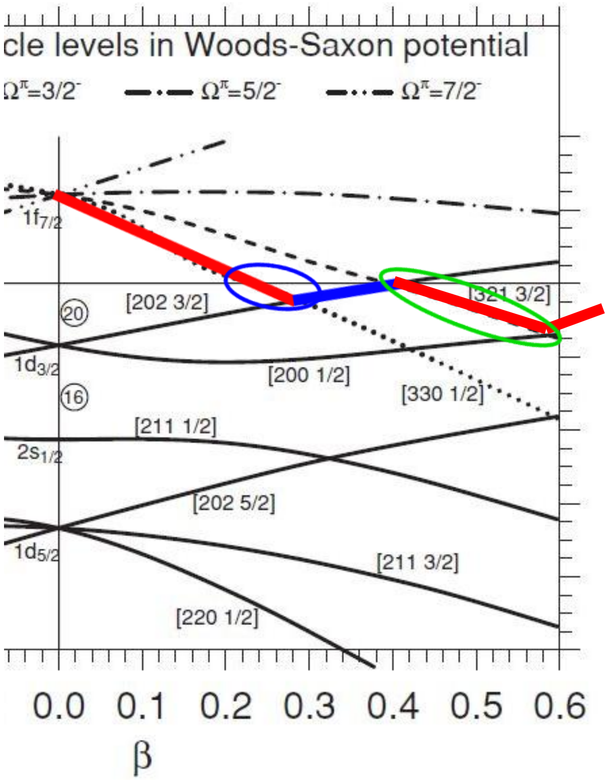
- non-adiabatic effects?
- comparison to the data (σ_{bu} and σ_I)?

Coulomb breakup cross sections



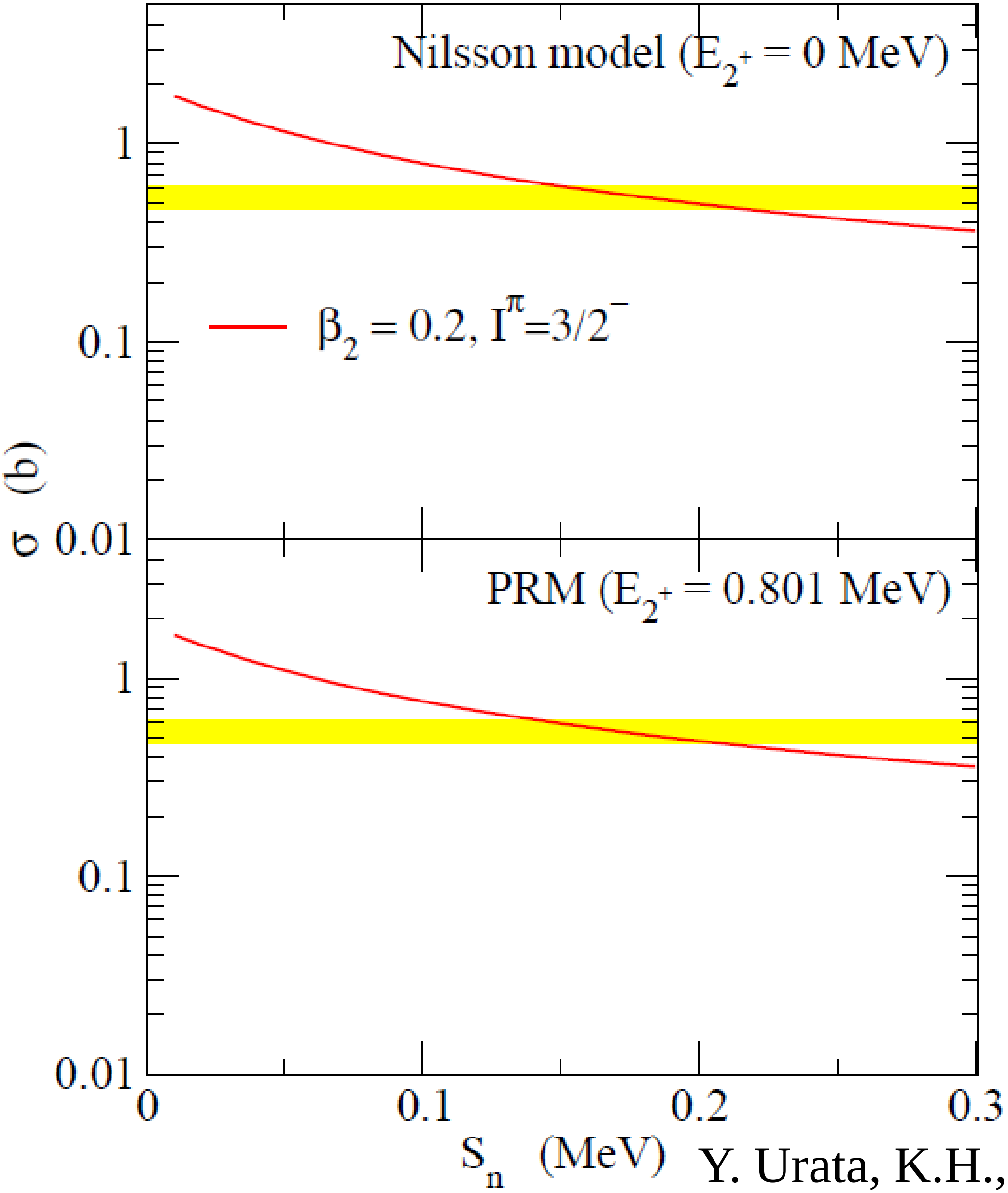
E_{2+} (^{30}Ne) = 0.801(7) MeV
P. Doornenbal et al.,
PRL103('09)032501

S_n (^{31}Ne) = 0.29 +/- 1.64 MeV

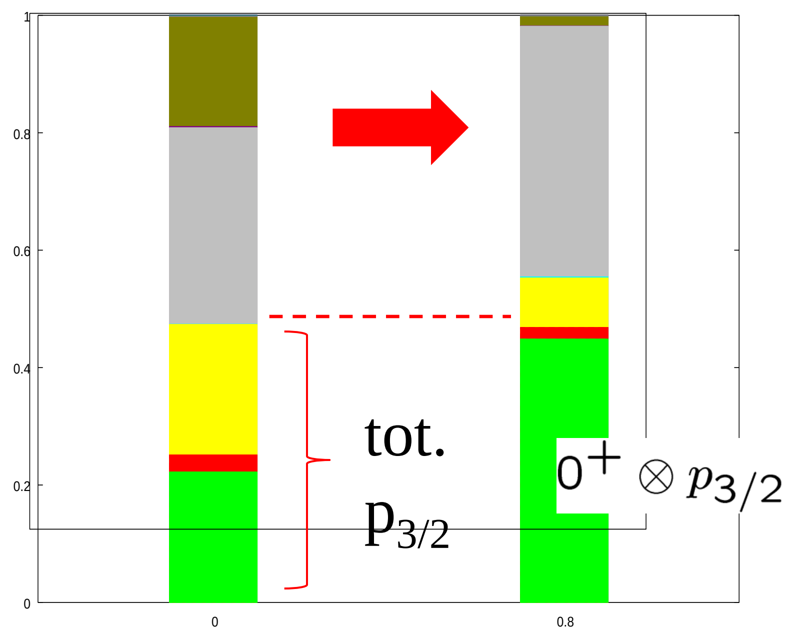


Y. Urata, K.H., and H. Sagawa,
PRC83('11)041303(R)

Coulomb breakup cross sections

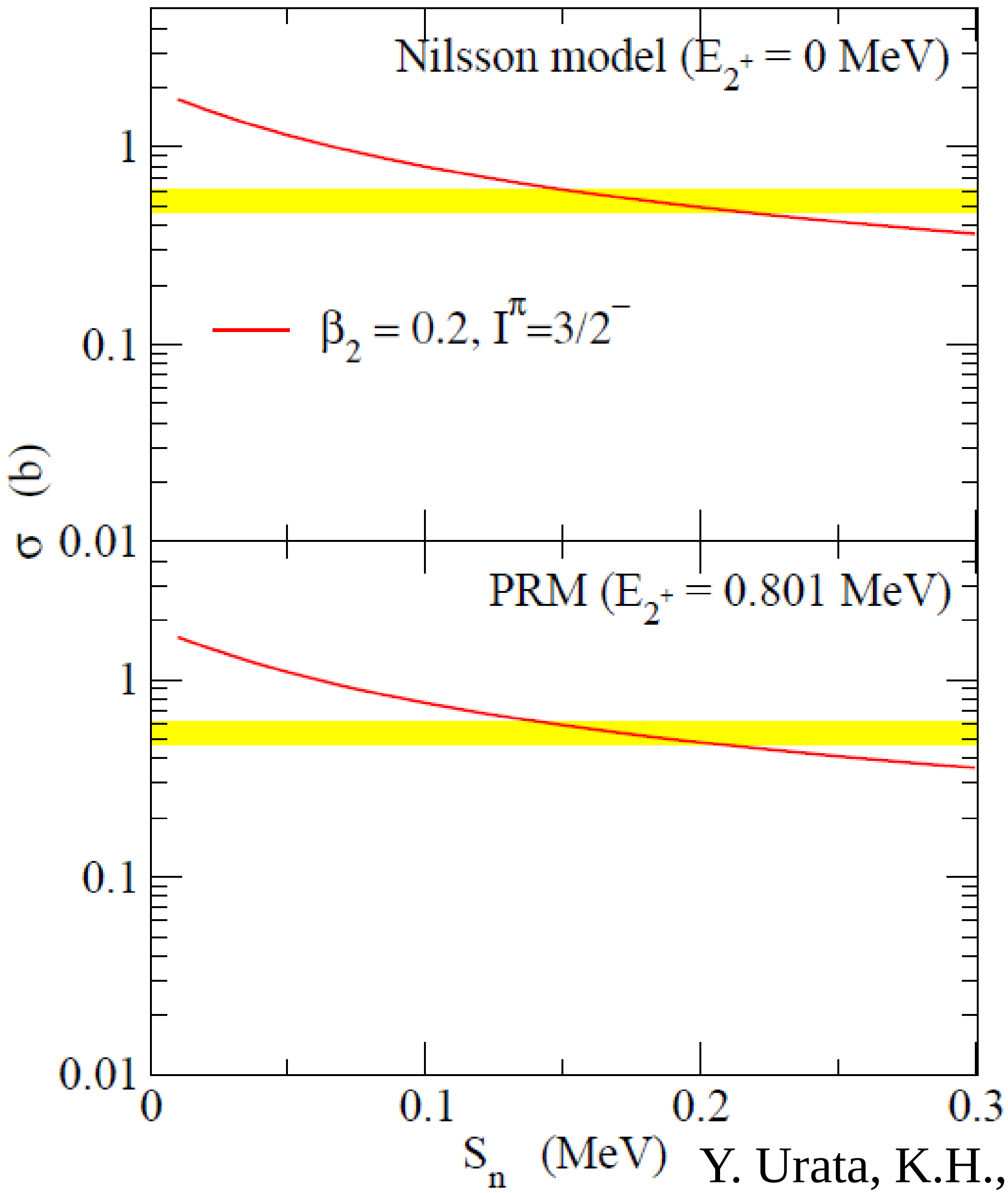


β ~ 0.2 : small non-adiabatic effects



Y. Urata, K.H., and H. Sagawa, PRC83('11)041303(R)

Coulomb breakup cross sections



^{31}Ne → ^{30}Ne (0^+) + n

Coul.BU

$\sigma_{bu}(0+) = 0.45(11)$ b

T. Nakamura et al., preliminary data

RPM for S_n = 0.2 MeV

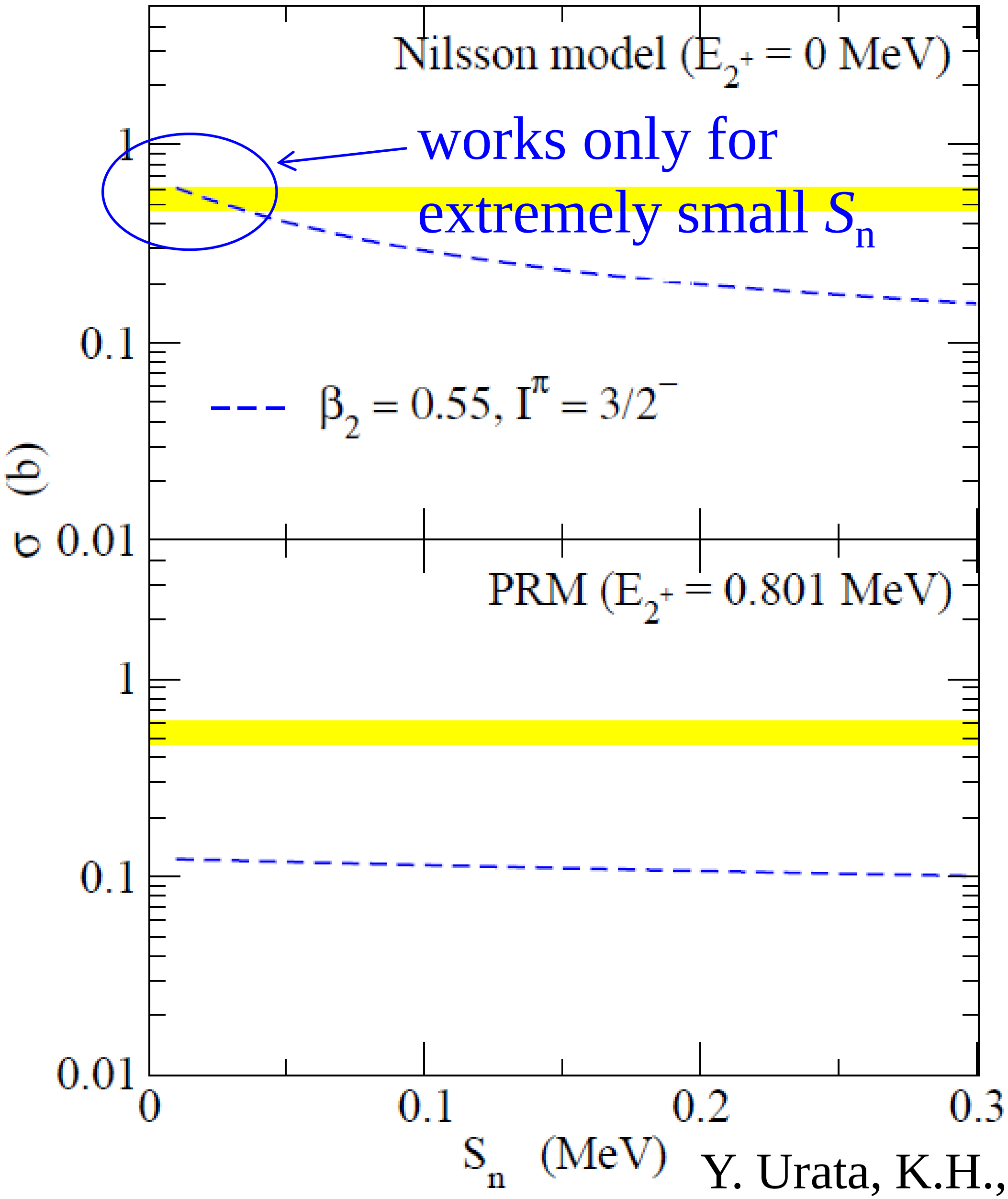
$\sigma_{bu}(0^+) = 0.443$ b

good agreement with the data

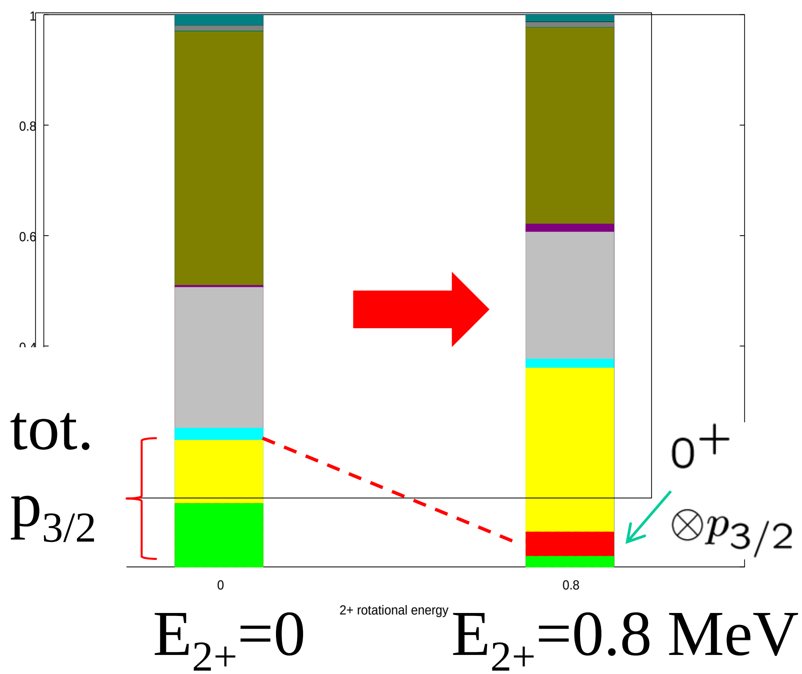
cf. Nilsson: $\sigma_{bu}(0^+) = 0.216$ b

Y. Urata, K.H., and H. Sagawa, PRC83('11)041303(R)

Coulomb breakup cross sections



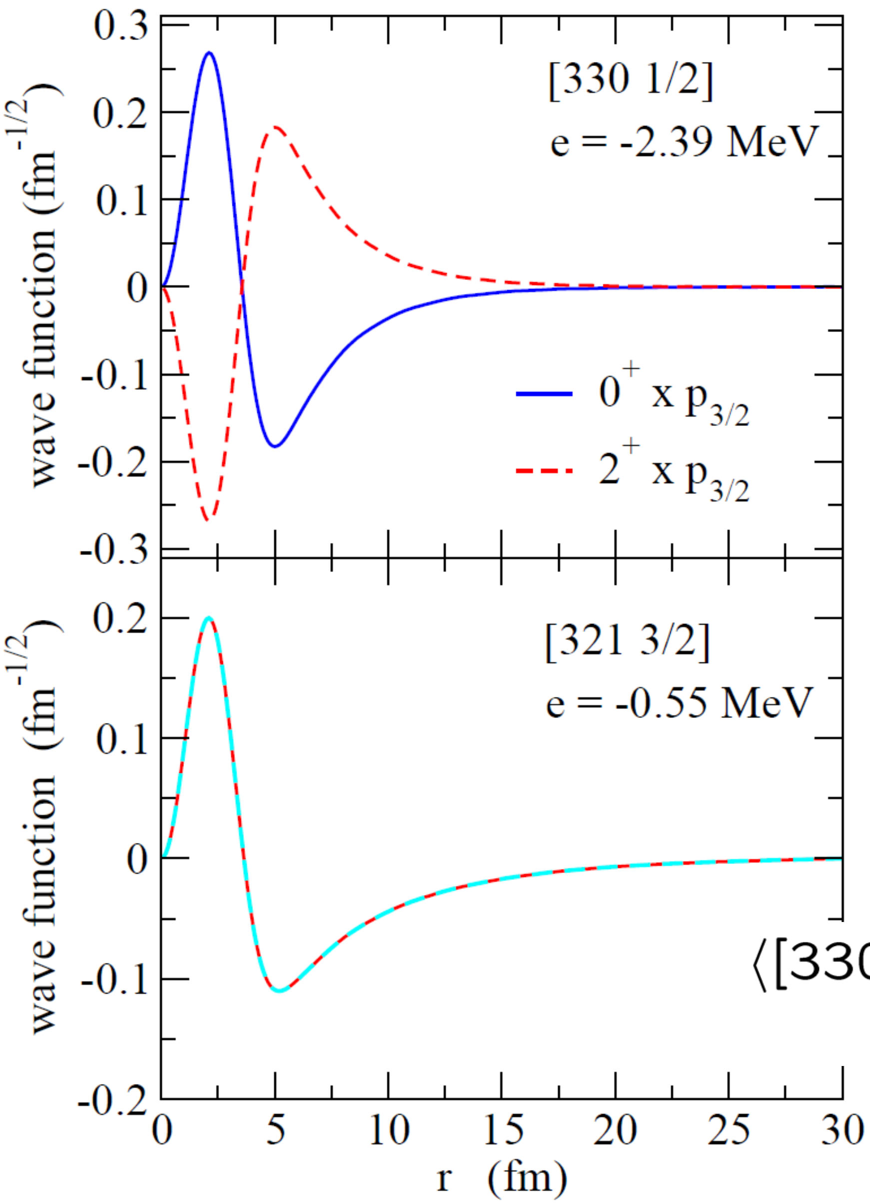
β ~ 0.55 : large non-adiabatic effects

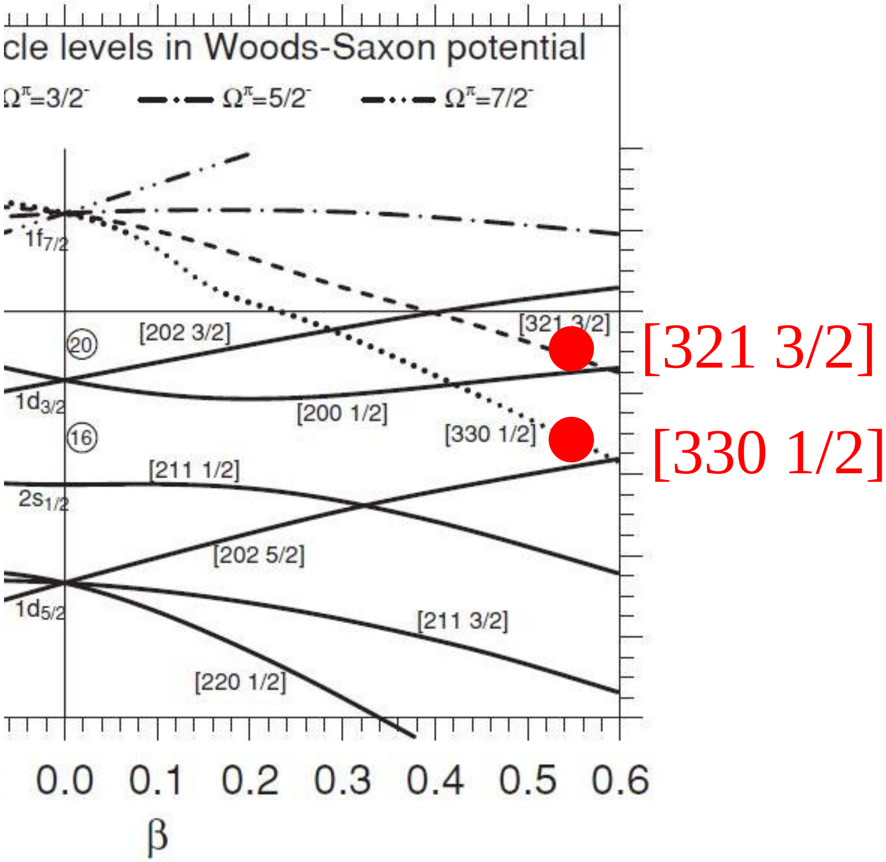


Y. Urata, K.H., and H. Sagawa, PRC83(‘11)041303(R)

two-level mixing between [330 1/2] and [321 3/2]

$I^\pi = 3/2^-$ at β=0.55



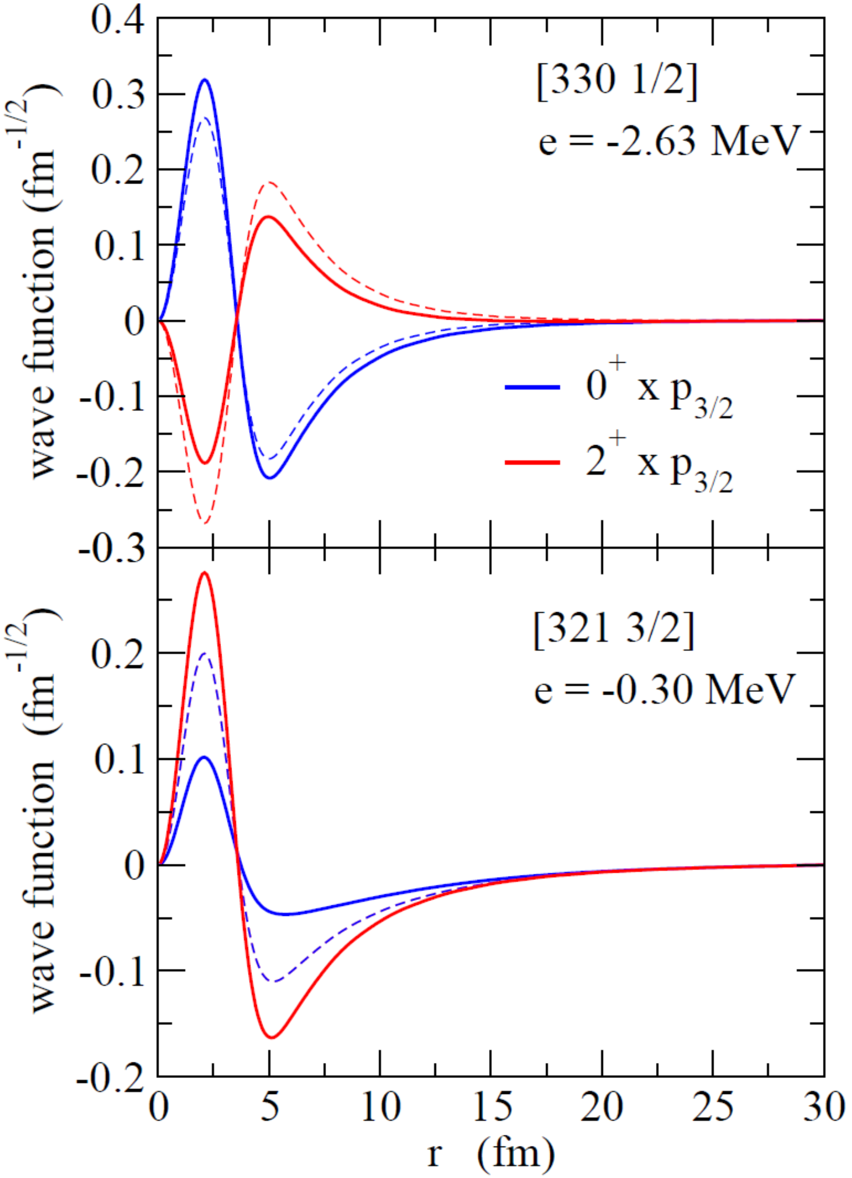


$$\langle [330\ 1/2]^{(I=3/2)} | H_{\rm rot} | [321\ 3/2]^{(I=3/2)} \rangle = -0.718\ \mathrm{MeV}$$

for $E(2^+) = 0.8$ MeV

$\langle[330\ 1/2]|H_{\rm rot}|[321\ 3/2]\rangle$
$= -0.718$ MeV

$\longrightarrow$ diagonalize $\begin{pmatrix} -2.38 & -0.718 \\ -0.718 & -0.547 \end{pmatrix}$



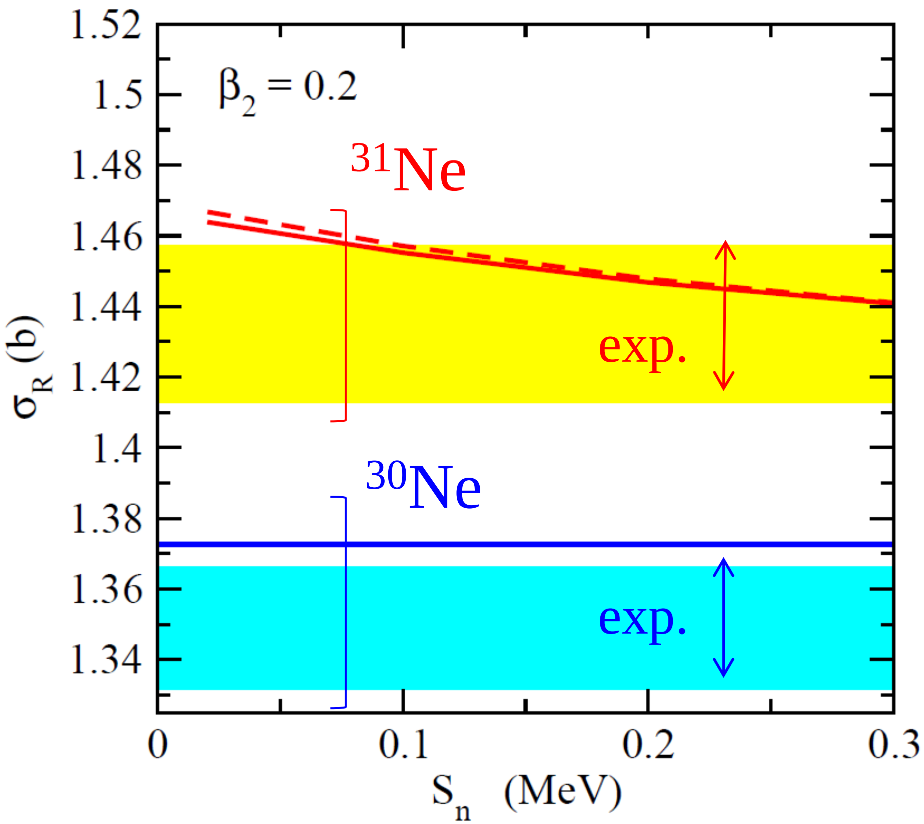
$0.95|[330\ 1/2]\rangle + 0.33|[321\ 3/2]\rangle$

$0^+ \times p_{3/2}$: 19.4% → 27.1%
$2^+ \times p_{3/2}$: 19.4% → 9.85 %

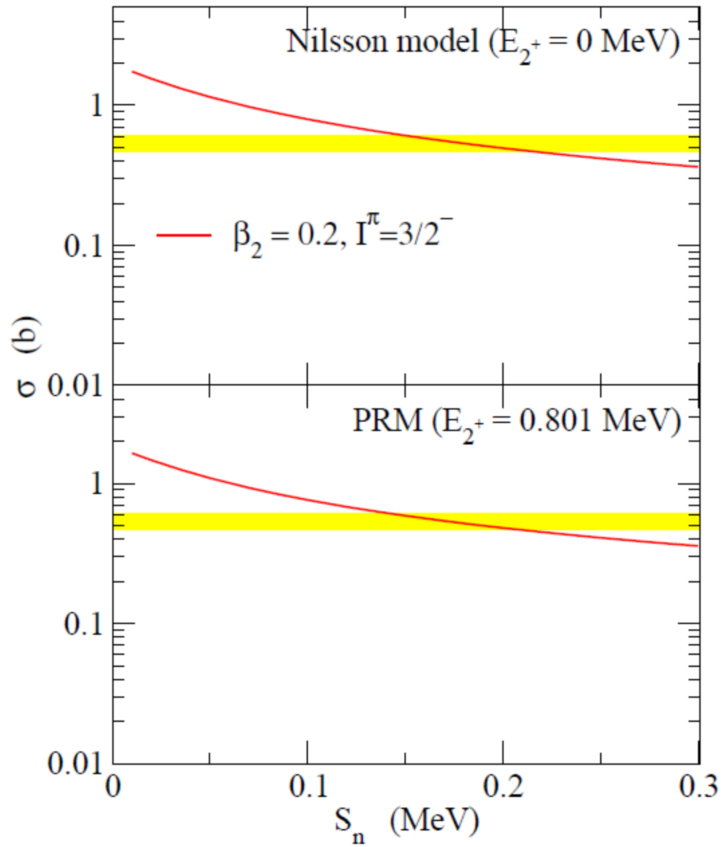
$0.95|[321\ 3/2]\rangle - 0.33|[330\ 1/2]\rangle$

$0^+ \times p_{3/2}$: 10.5% → 2.82%
$2^+ \times p_{3/2}$: 10.5% → 20.1 %

Reaction cross section



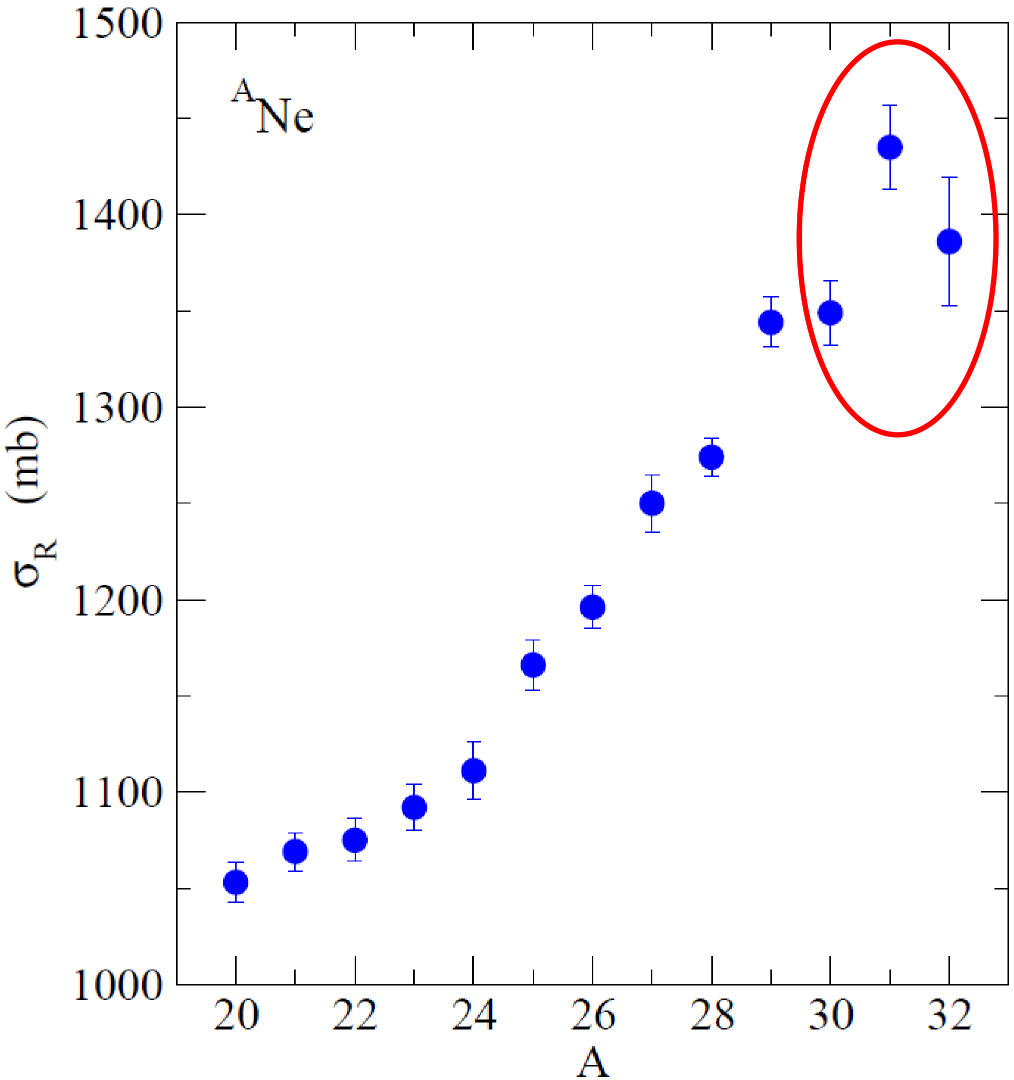
$I^\pi = 3/2^-$ at $\beta \sim 0.2$:
consistent both with σ_{bu} and σ_R



Y. Urata, K.H., and H. Sagawa,
arXiv:1205.2962 [nucl-th]

Odd-even staggering of interaction cross sections

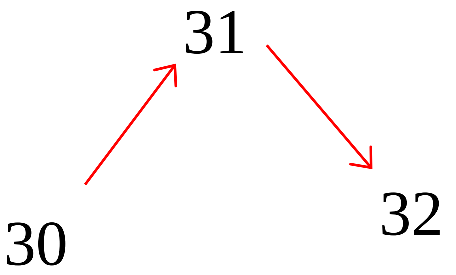
σ_I of unstable nuclei: often show a large odd-even staggering



Typical example:

Recent experimental data on Ne isotopes

M. Takechi et al., Phys. Lett. B707 ('12) 357



clear odd-even effect

- deformation effect?
- pairing effect?

➢pairing anti-halo effect

K. Bennaceur, J. Dobaczewski, and M. Ploszajczak, PLB496('00)154

pairing

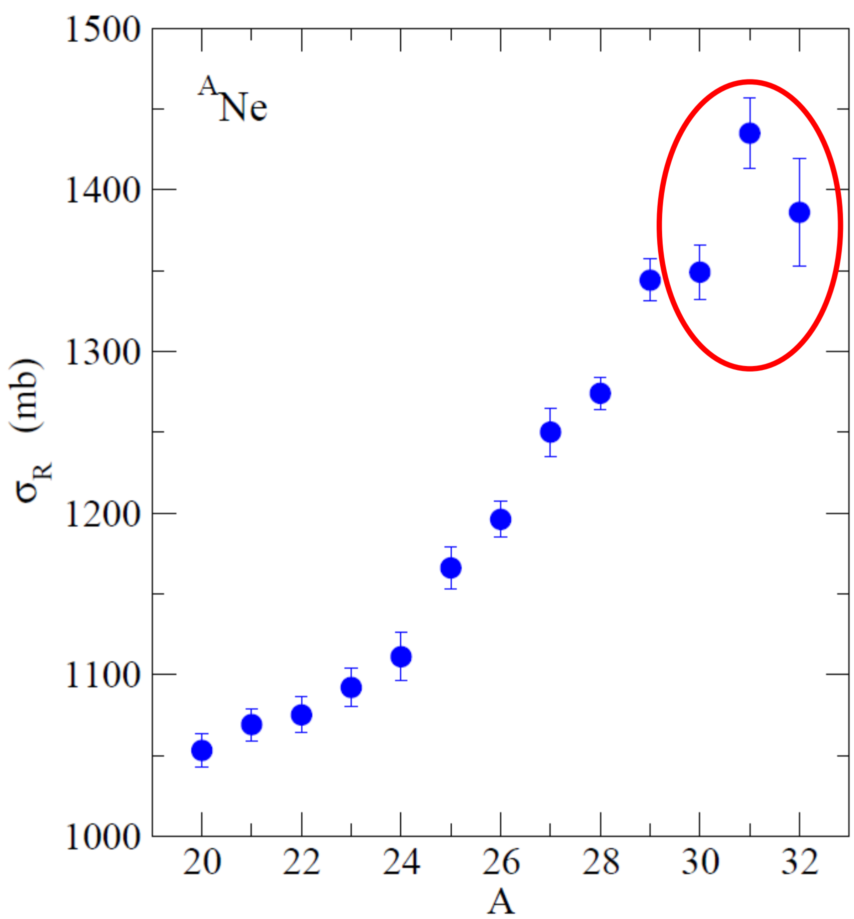
⬇

asymptotic behavior of s.p. wave functions

⬇

suppression of density distribution

➢odd-even staggering of σ_R



Our motivation:

Relation between the odd-mass staggering (OES) of σ_R and pairing (anti-halo) effect?

First experimental evidence for the anti-halo effect?

Model: HFB with a Woods-Saxon mean-field potential

$$\begin{pmatrix} \hat{h}-\lambda & \Delta(r) \\ \Delta(r) & -\hat{h}+\lambda \end{pmatrix}\begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix} = E_k \begin{pmatrix} U_k(r) \\ V_k(r) \end{pmatrix}$$

$$\hat{h} = -\frac{\hbar^2}{2m}\nabla^2 + V_{\mathrm{WS}}(r)$$

${}^{32}_{10}\mathrm{Ne}_{22}$

-0.066 MeV $1f_{7/2}$

-0.321 MeV $2p_{3/2}$

20

^{31}Ne (a =0.75 fm)

$$\Delta(r) = \frac{V_{\mathrm{pair}}}{2}\left(1-\frac{\rho(r)}{\rho_0}\right)\tilde{\rho}_n(r)$$

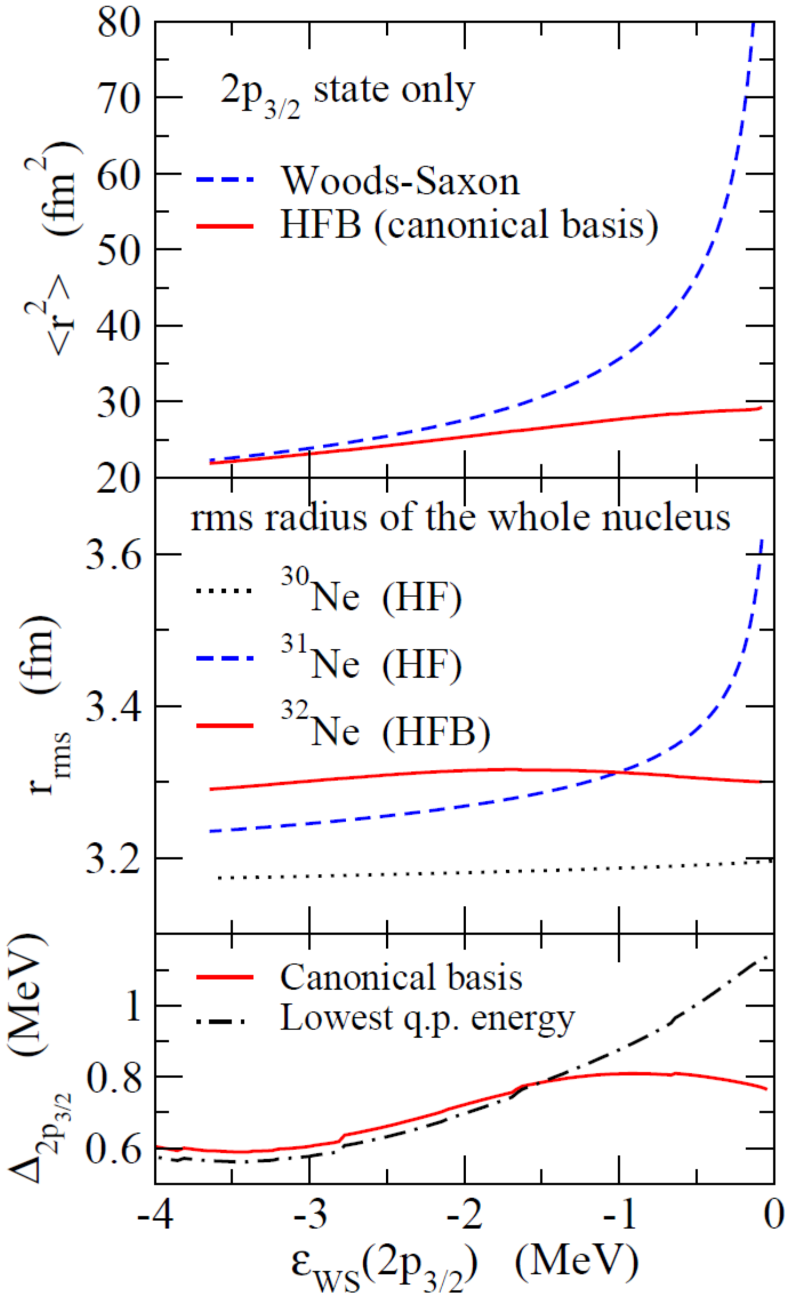
$$\tilde{\rho}_n(r) = -\sum_{k=n} U_k^*(\boldsymbol{r})V_k(\boldsymbol{r})$$

✓λ: self-consistently determined so that N=22

✓E_{cut} = 30 MeV above λ

✓R_{box}=60 fm

rms radius and pairing gap



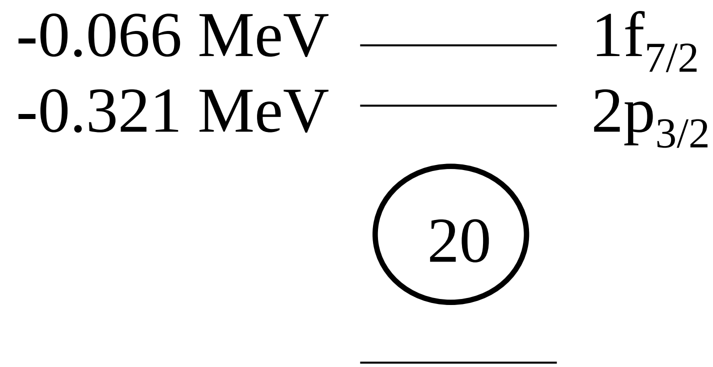
suppression of the radius

the effective pairing gap persists for both the definitions

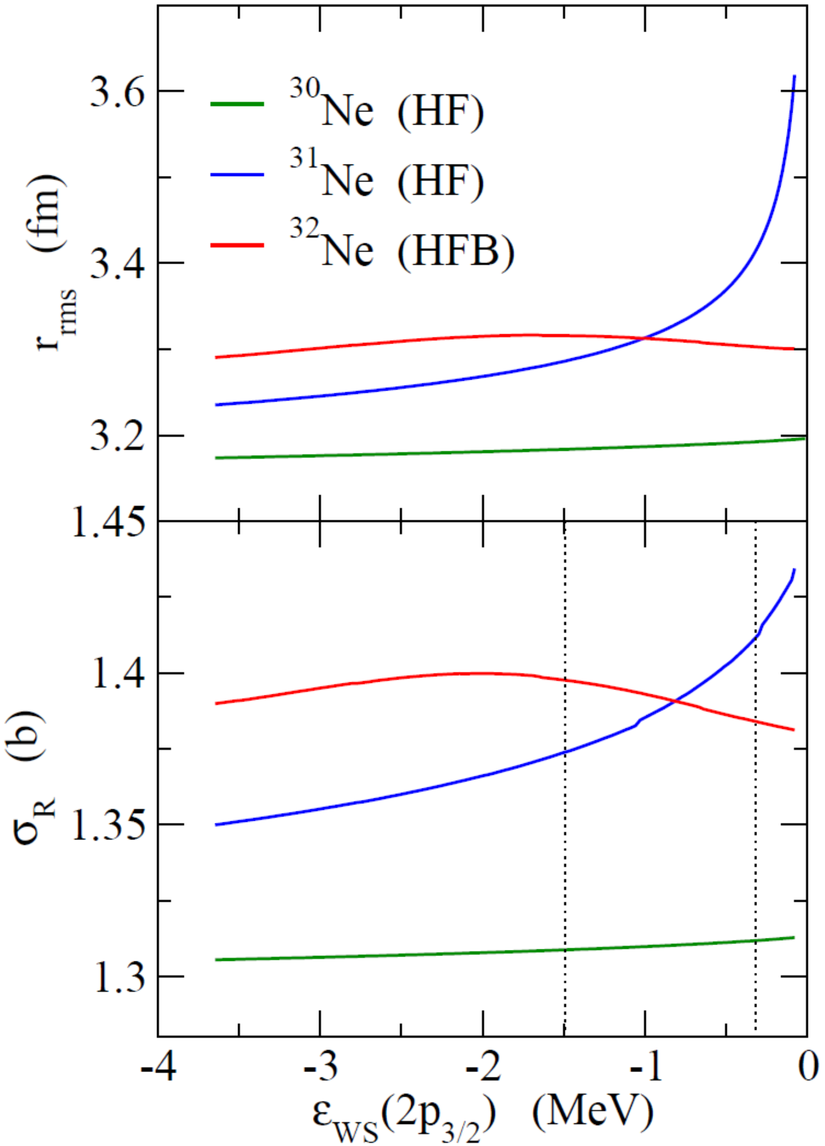
K. H. and H. Sagawa, PRC84('11)011303(R)

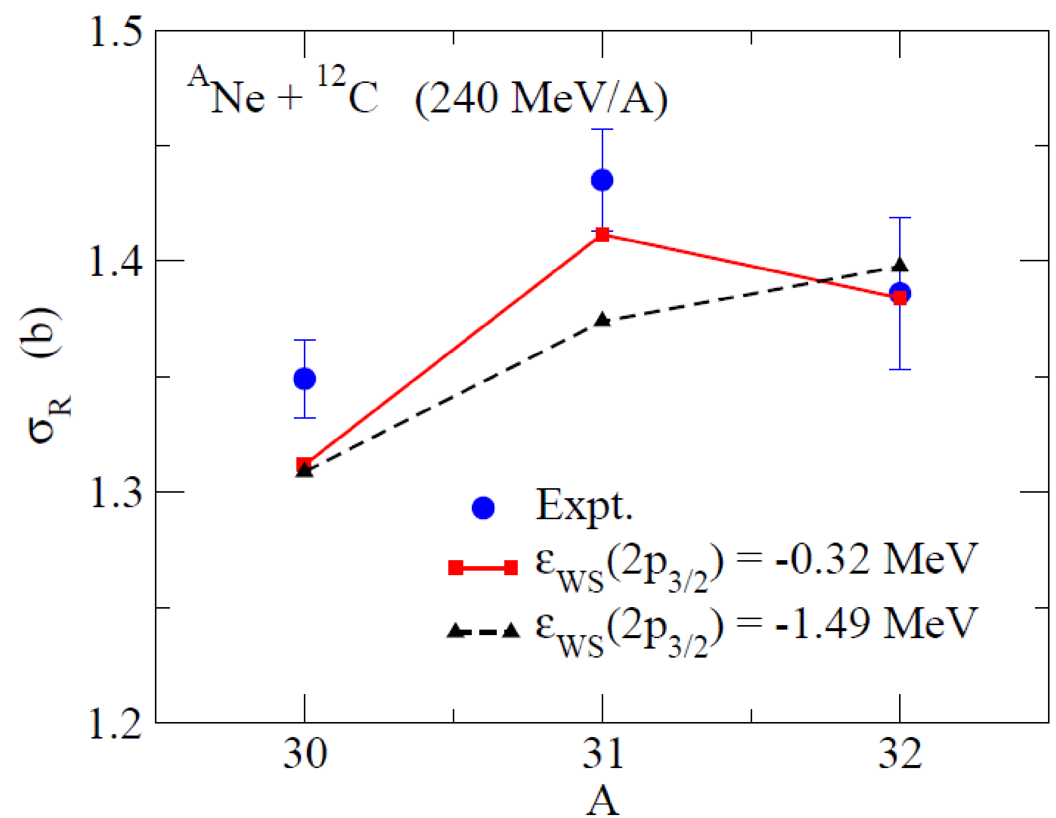
rms radius and reaction cross section

HFB with a spherical Woods-Saxon



^{31}Ne (a =0.75 fm)



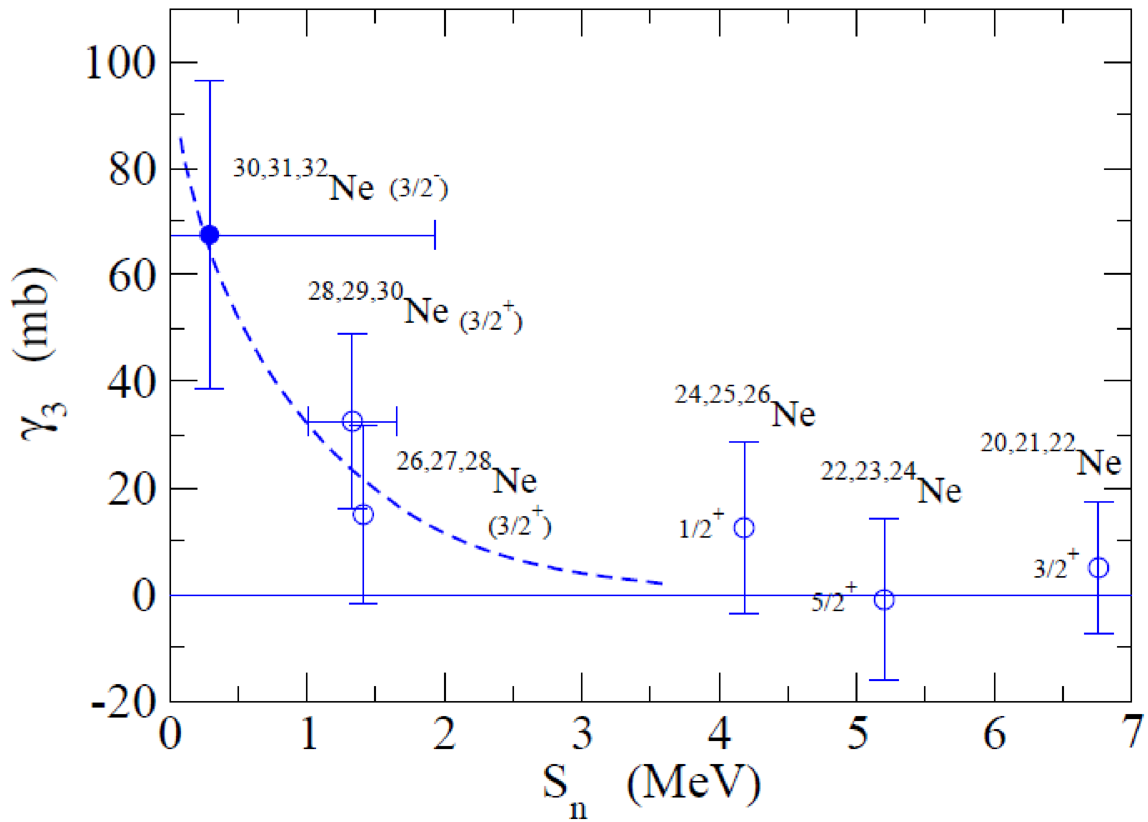


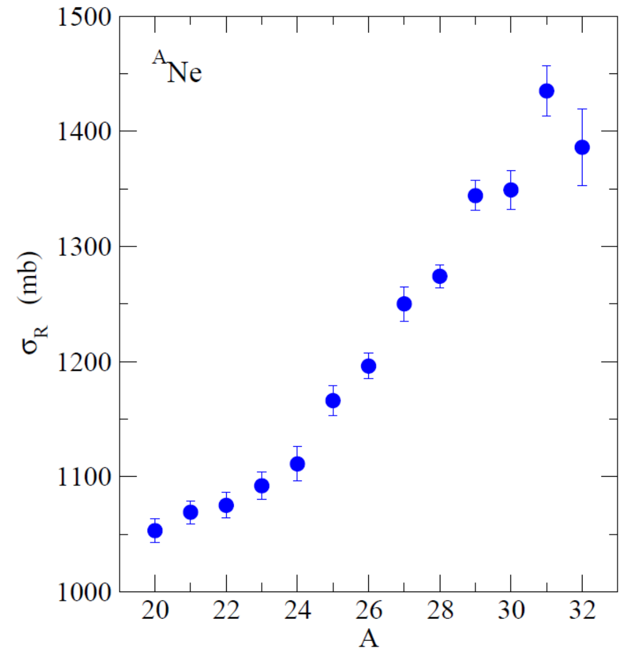
K. H. and H. Sagawa, PRC84('11)011303(R)

Systematics

OES parameter

$$\gamma_3 \equiv -\frac{1}{2}[\sigma_R(A+2) - 2\sigma_R(A+1) + \sigma_R(A)]$$





K. H. and H. Sagawa, PRC85('12)014303

Summary and Discussions

deformation $\longrightarrow$ mixture of angular momenta

$\longrightarrow$ enlarges a possibility of halo formation

- good example: ^{31}Ne

$0^+ \times p_{3/2}$: 44.9 %
$2^+ \times p_{3/2}$: 8.4 %
$2^+ \times f_{7/2}$: 42.7 %

$\longleftarrow$ non-adiabatic particle-rotor model with $\beta \sim 0.2$

$\Rightarrow$ well accounts for $\sigma_{C\text{-bu}}$ (tot), $\sigma_{C\text{-bu}}(0^+)$, and σ_R simultaneously

- Odd-even staggering of σ_R

✓an important role of pairing correlation
✓OES parameter: a good tool to investigate the pairing correlation
✓role of deformation? $\longleftarrow$ deformed HFB (a work in progress)

Summary and Discussions

deformation $\longrightarrow$ mixture of angular momenta

$\longrightarrow$ enlarges a possibility of halo formation

- good example: ^{31}Ne

Other candidates?

^{19}C

^{33,35,37}Mg

^{43,45}S

^{127}Ru

I. Hamamoto, PRC76('07)054319

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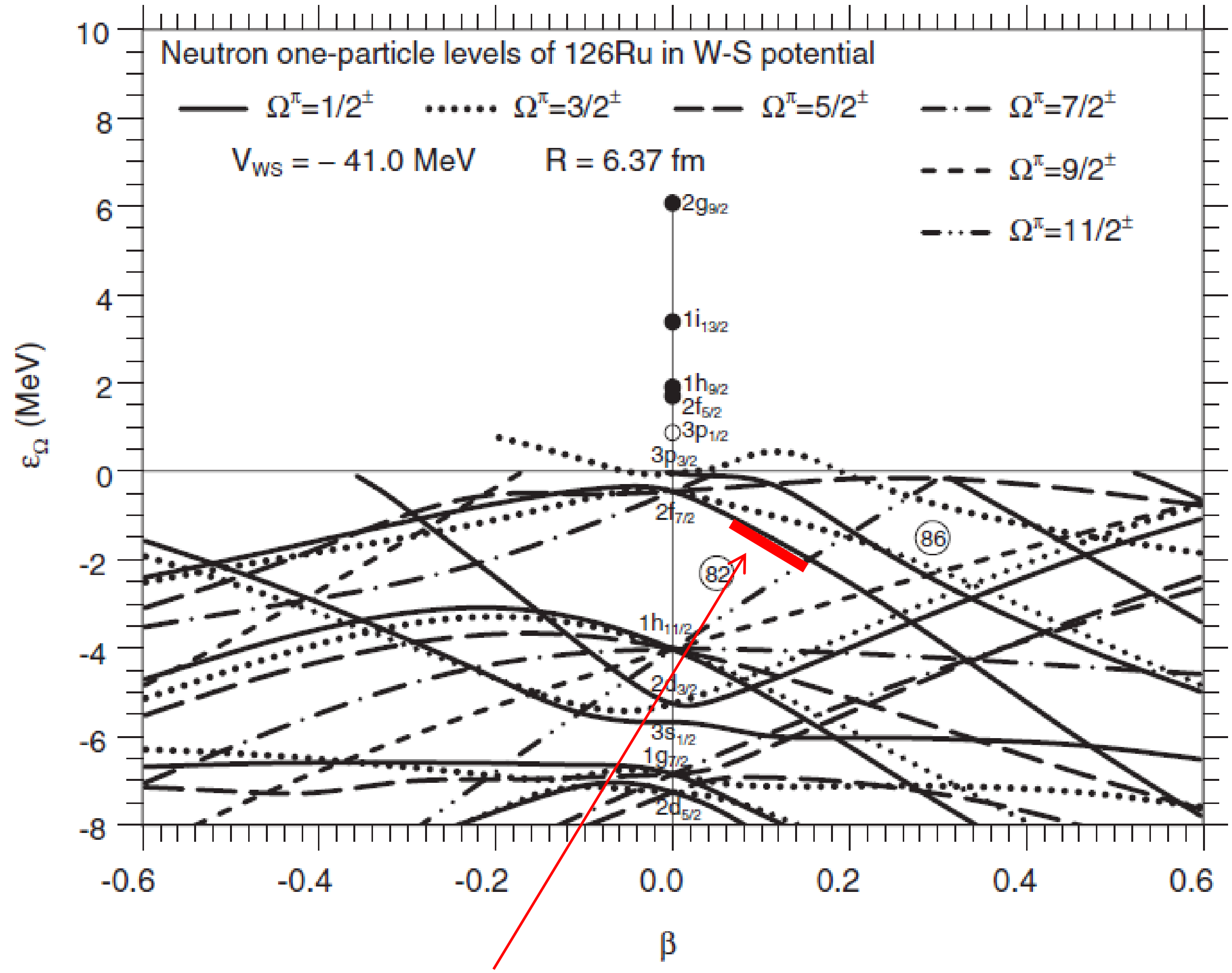
I. Hamamoto, PRC85('12)064329



$^{126}_{44}Ru_{83}$

$\Omega^\pi=1/2^-$

cf. Skyrme-HFB: $\beta = 0$

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Perspectives: deformed halo nuclei

✓Possibility of a heavy halo nucleus

what is the heaviest halo nuclues?

✓“Fine structure” in breakup/transfer reactions

direct population of the 2^+ state after breakup/transfer

$$\left[|j'_p l'_p\rangle \otimes |2^+\rangle\right]^{(IM)}$$

2^+

0^+

core + n

$$\left[|j_p l_p\rangle \otimes |0^+\rangle\right]^{(IM)}$$

cf. proton decay

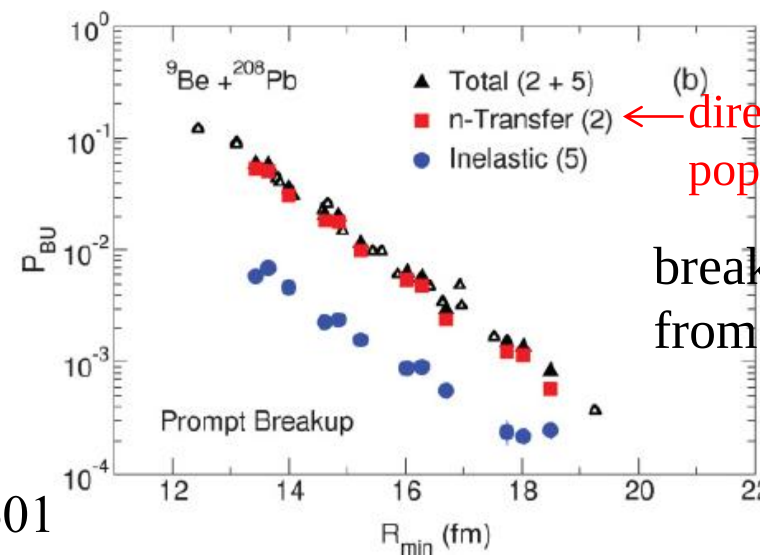
cf. Nakamura-san’s expt.

✓Influence on low-energy heavy-ion reactions (e.g., subbarrier fusion)

interplay between

breakup/ transfer/ rotational

couplings



direct population

breakup from ^{8}Be*

R. Rafiei et al., PRC81(‘10)024601