冷却原子気体で探る中性子星物質

— その可能性と課題

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(APCTP, POSTECH, RIKEN)

Contents

I. Introduction

II. Neutron matter in neutron star crusts

III. BCS-BEC crossover & unitary Fermi gases

IV. Simulating neutron star matter using cold atoms

References

- S. Shapiro and S. L. Teukolsky:
  "Black Holes, White Dwarfs, and Neutron Stars"
  Wiley (1983).

  Neutron stars and neutron star matter in general.

- C. J. Pethick and D. G. Ravenhall:
  "Matter at large neutron excess and the physics of neutron-star crust"
  Annu. Rev. Nucl. Part. Sci. **45**, 429 (1995).

  Matter in neutron star crusts
  from nuclear and many-body physics point of view.

- S. Giorgini *et al*., Rev. Mod. Phys. **80**, 1215 (2008).
- I. Bloch *et al*., Rev. Mod. Phys. **80**, 885 (2008).

Introduction

Symmetric nuclear matter @ ρ_0

"Basic" system of nuclear phys. is complicated!

Nuclear matter at $\begin{cases} x \equiv n_p/(n_p + n_n) \simeq 0.5 \\ \rho \simeq \rho_0 = 0.16\ \mathrm{fm}^{-3} \end{cases}$

(mass density: 3×10^{14} g cm^{-3})

nuclear matter: mixture of p & n with complicated int.

- 4 spin-isospin DOFs
- Self bound (clustering at $\rho < \rho_0$)
- 3-body force

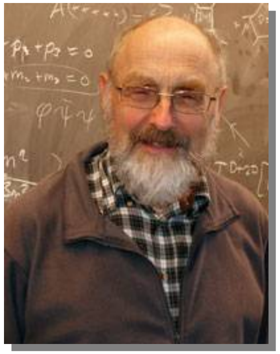
Low density neutron matter

Simpler system

Neutron matter at low densities

$$x = 0 \quad \& \quad \rho \ll \rho_0$$

- Only 2 spin-isospin DOFs
- Non-self bound
- Pauli principle → 3-body force suppressed.



Pandharipande, Pethick, etc.

Current status

Recent progress

- Theory
  - Improved many-body calc.

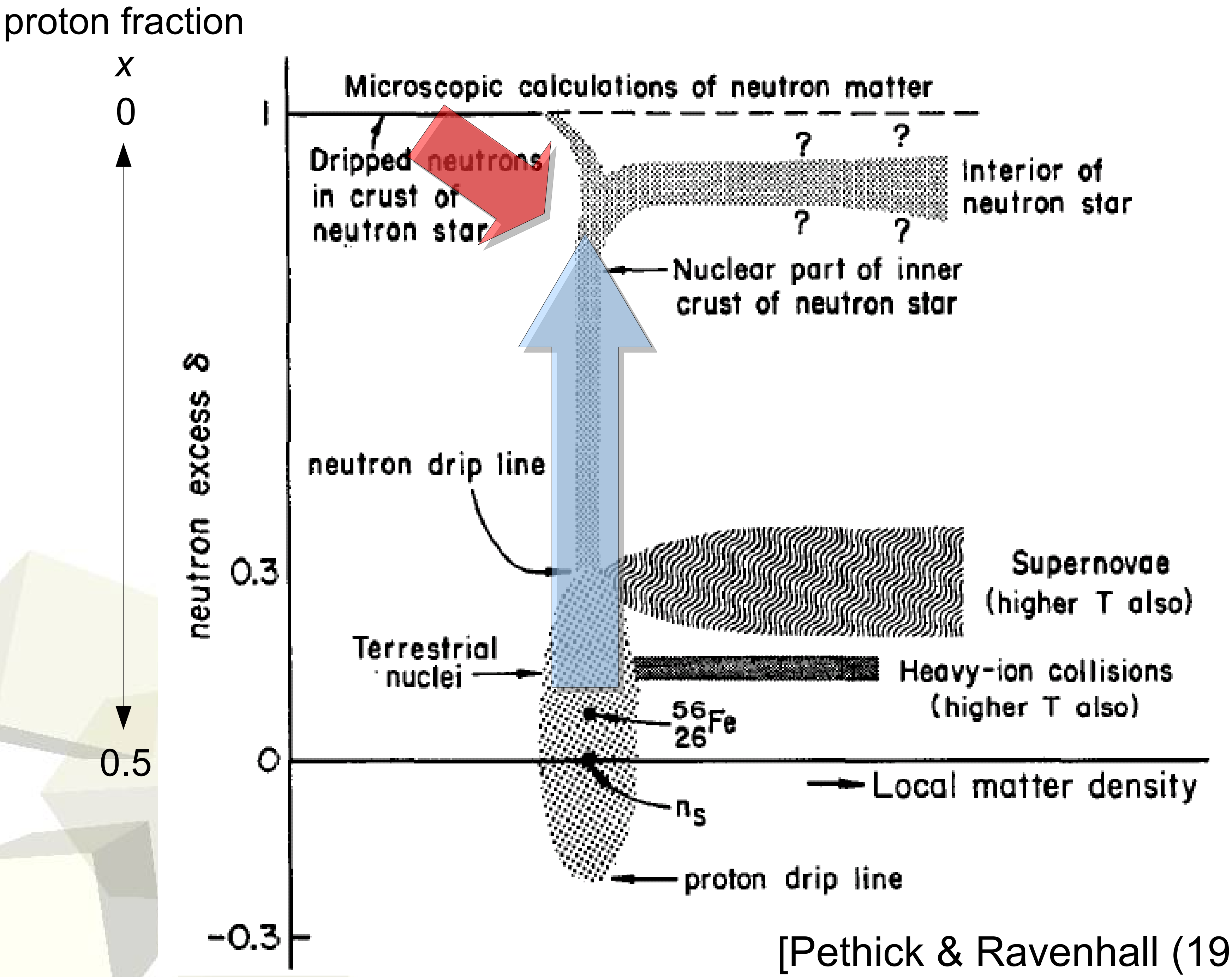
    QMC calc. for neutron matter & cold gases
  - Constraints on EOS

- Experiment
  - Realization of unitary Fermi gases

- Observation
  - Discovery of 1.97M_{solar} NS

[Demorest *et al*. Nature **467**, 1081 (2010)]

Two ways to neutron star matter

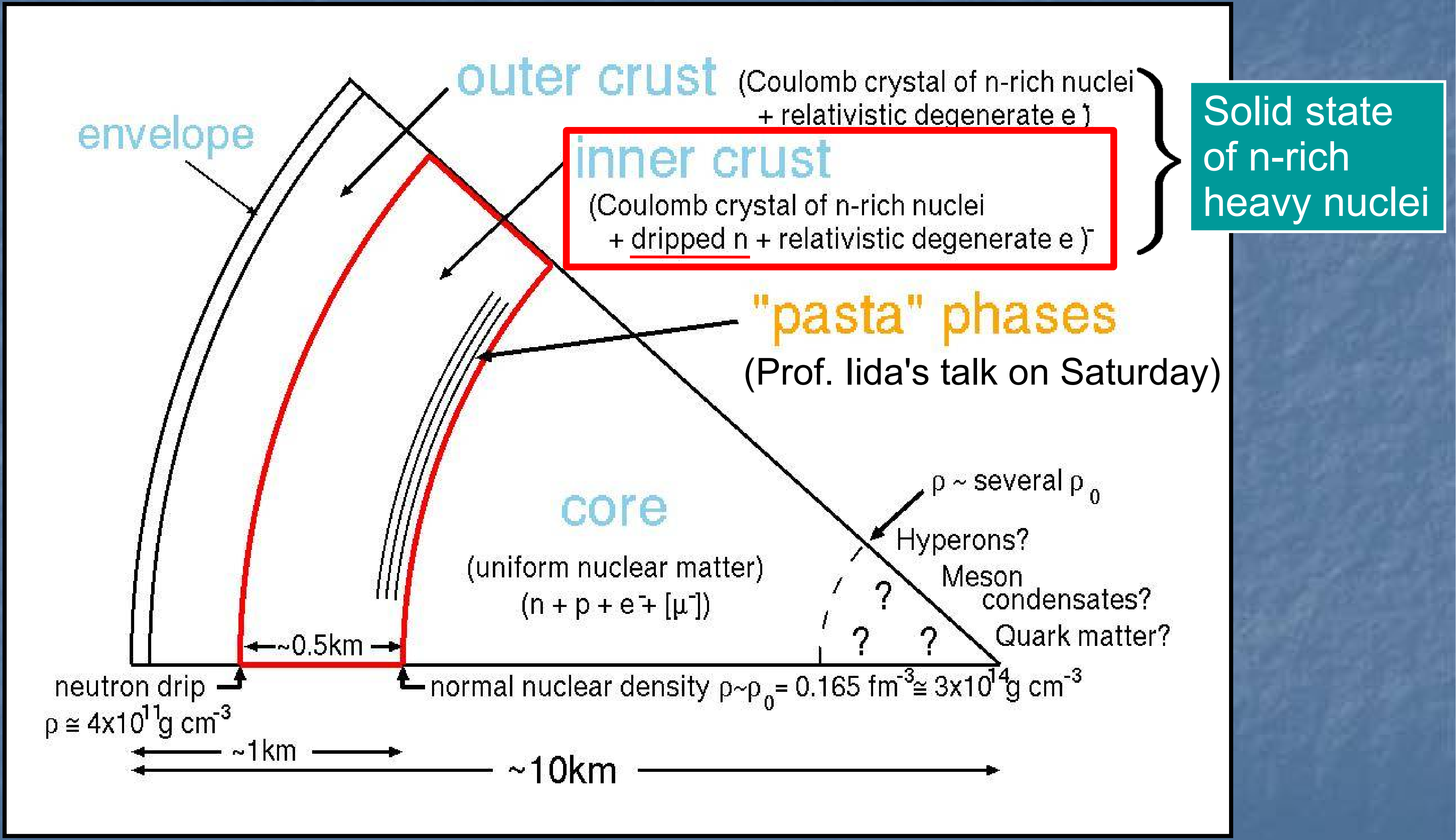


[Pethick & Ravenhall (1995)]

Neutron matter in NS crusts

New Scientist

Structure of neutron stars



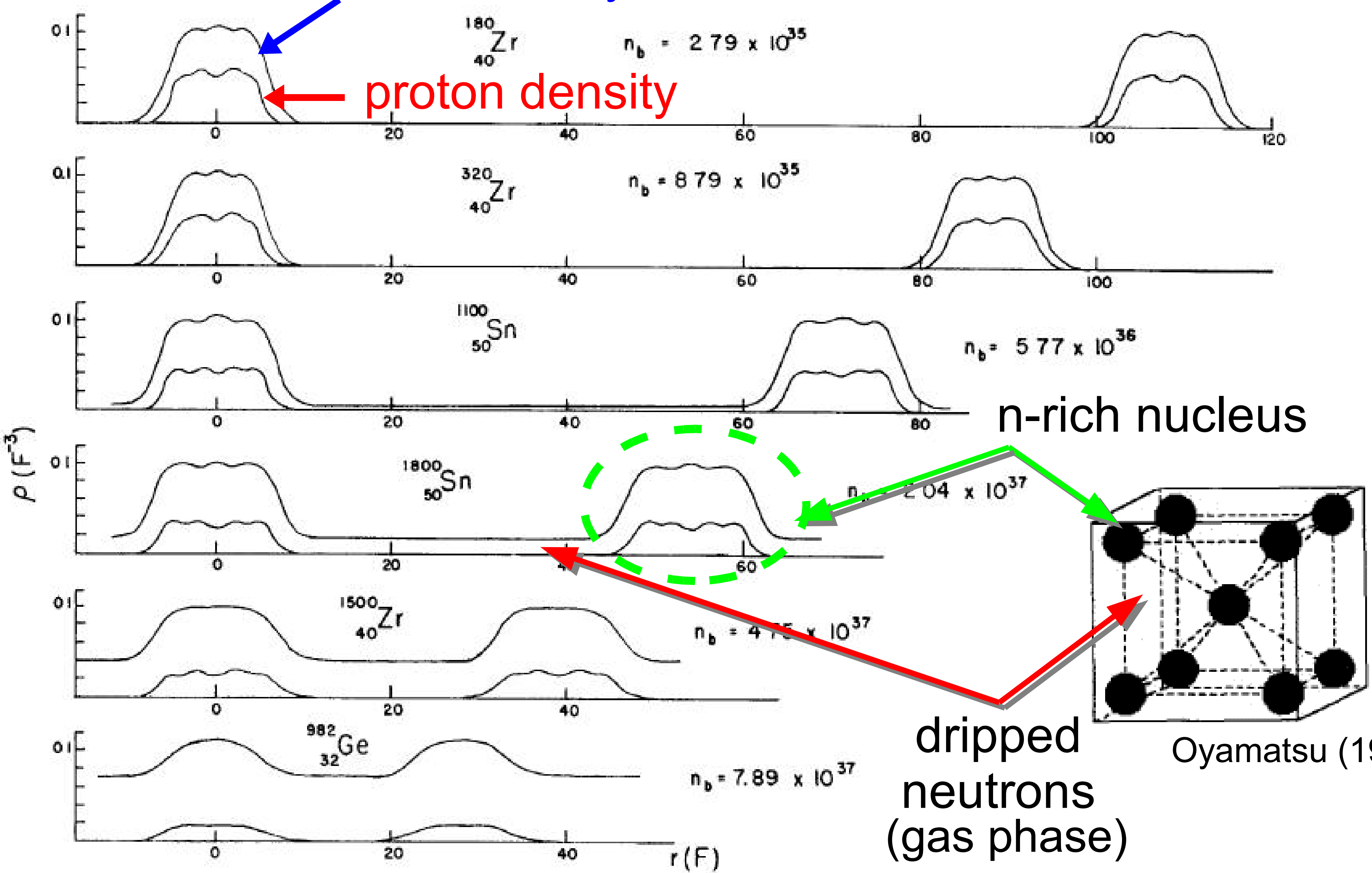
(Prof. Iida's talk on Saturday)

Solid state of n-rich heavy nuclei

Neutronization: $p_N + e^- \rightarrow n_N + \nu_e$

Structure of neutron star matter

low density



neutron density

proton density

n-rich nucleus

dripped neutrons (gas phase)

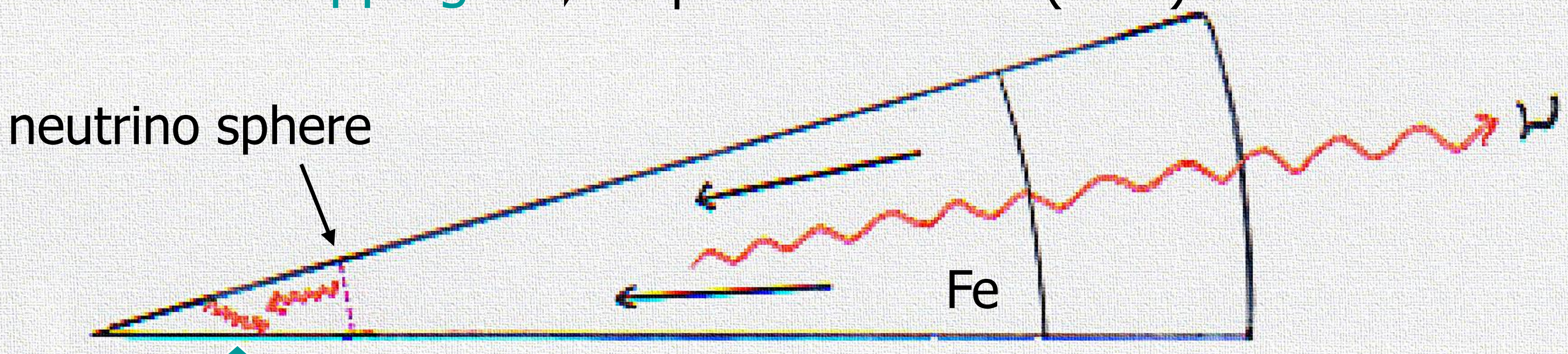
Oyamatsu (1993)

high density

Negele & Vautherin, NPA **207**, 298 (1973)

Matter in supernova inner cores

Neutrino Trapping → supernova matter (SNM)



$Y_L = (\langle n_e \rangle + \langle n_\nu \rangle)/\langle n_b \rangle \sim 0.3$ is conserved

High density: dynamical time scale of the core < diffusion time scale of ν's

→ ν's are trapped

$$p_N + e^- \rightarrow n_N + \nu_e$$

cannot escape

$$\mu_p + \mu_e = \mu_n + \underline{\mu_{\nu_e}}$$

supernova matter (SNM) $x \sim 0.3$

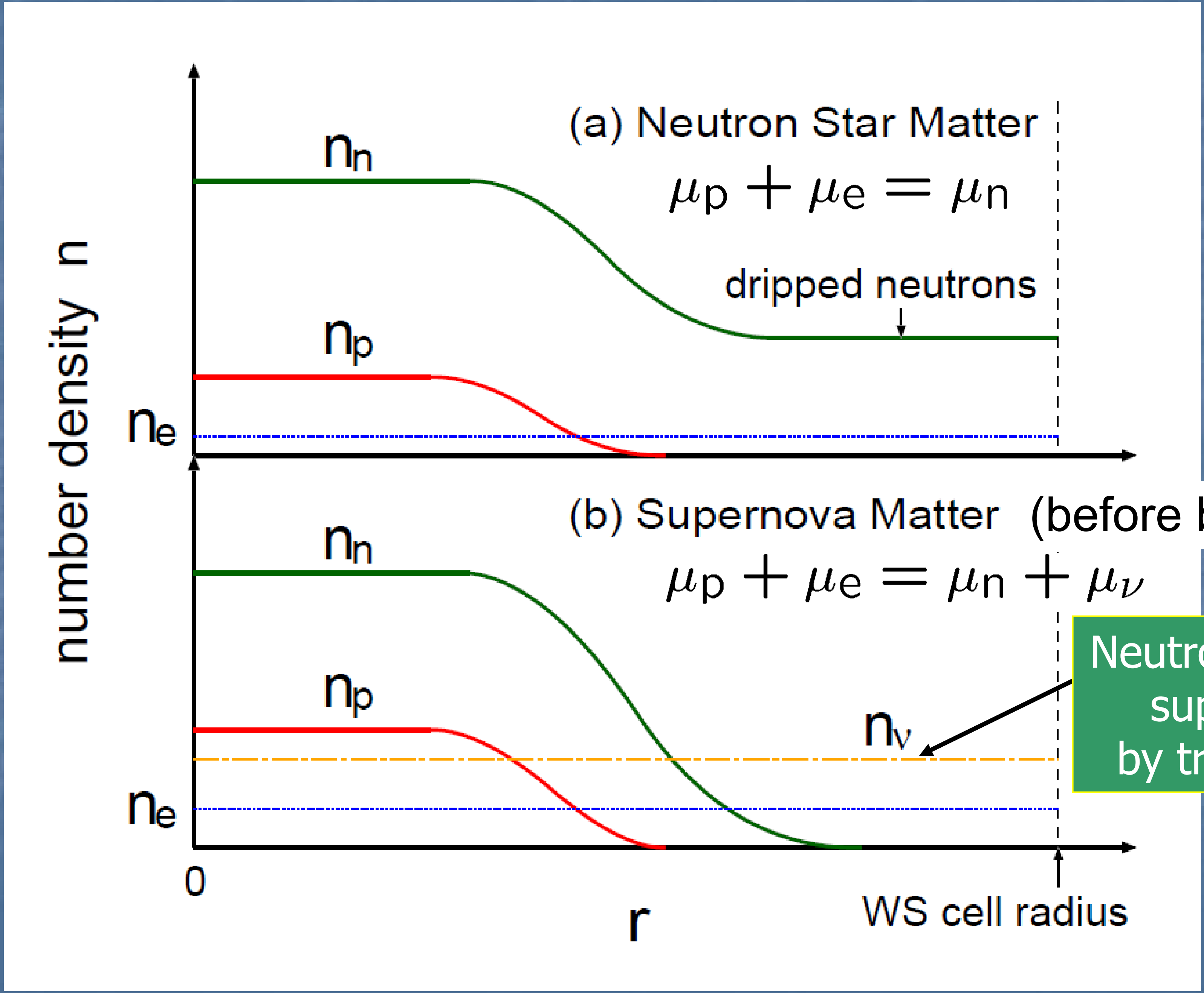
neutron star matter (NSM) $x < 0.1$

x : proton fraction

Central density increases up to $\sim \rho_0$

But, no dripped neutrons.

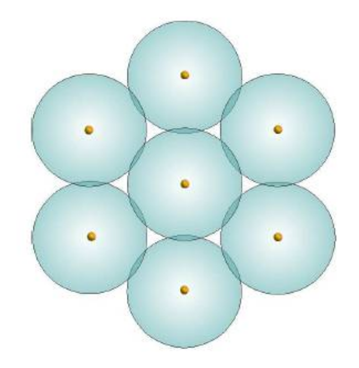
Neutron star matter & supernova matter



(before bounce, low *T*)

Neutronization is suppressed by trapped ν's.

WS cells



Importance of NS crusts

Neutron star crust

Thickness: ~ 1 km (cf. radius of NS: ~10km)

Total mass: ~ O(0.01) M_{solar}

(cf. mass of NS: ~1.4M_{solar})

Negligible? — Depending on phenomena.

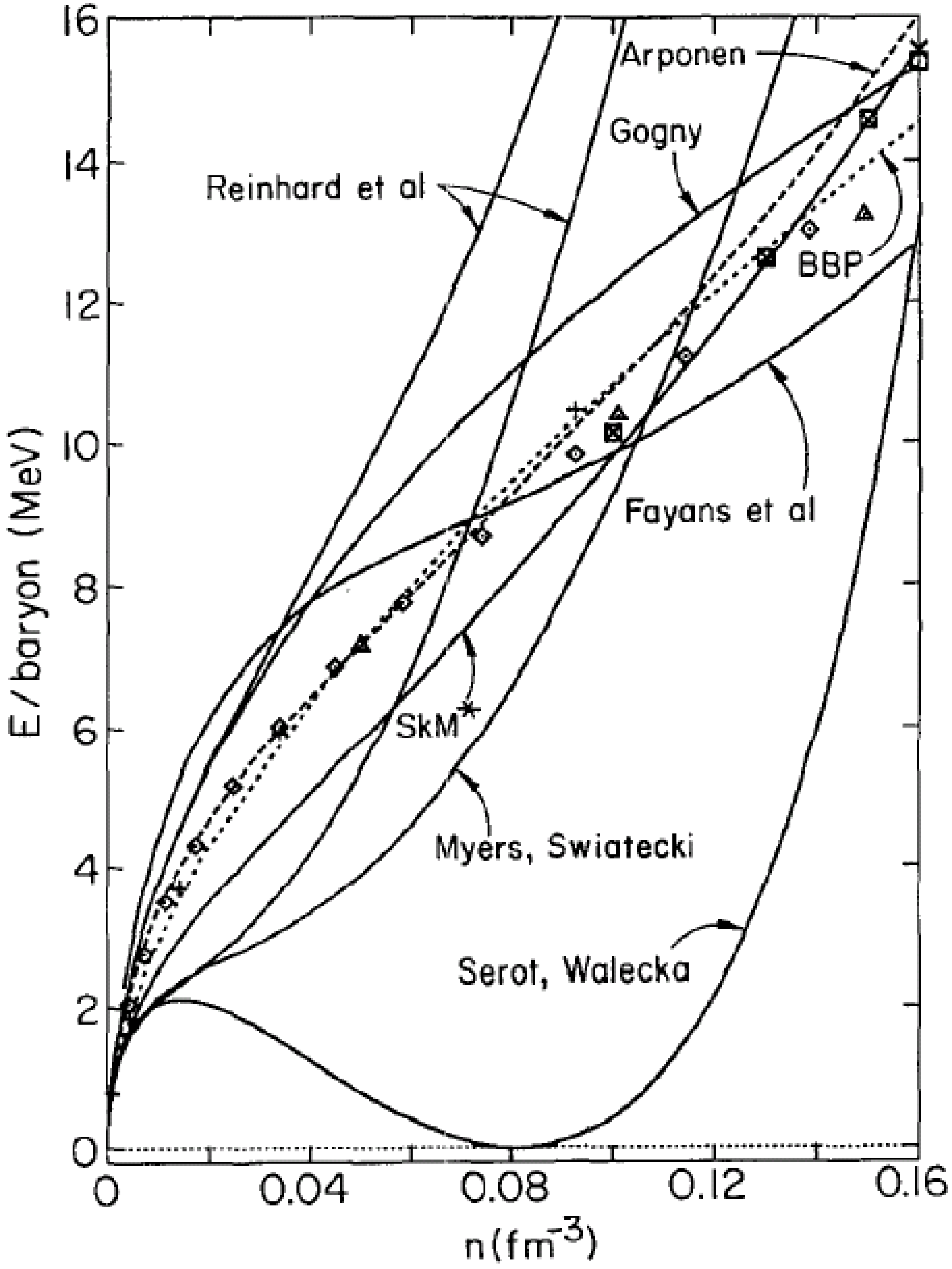
Outer parts have direct consequences of observations.

- NS cooling

  Neutron superfluidity in crusts matters.

- Torsional oscillations of crusts

Uncertainty of neutron matter



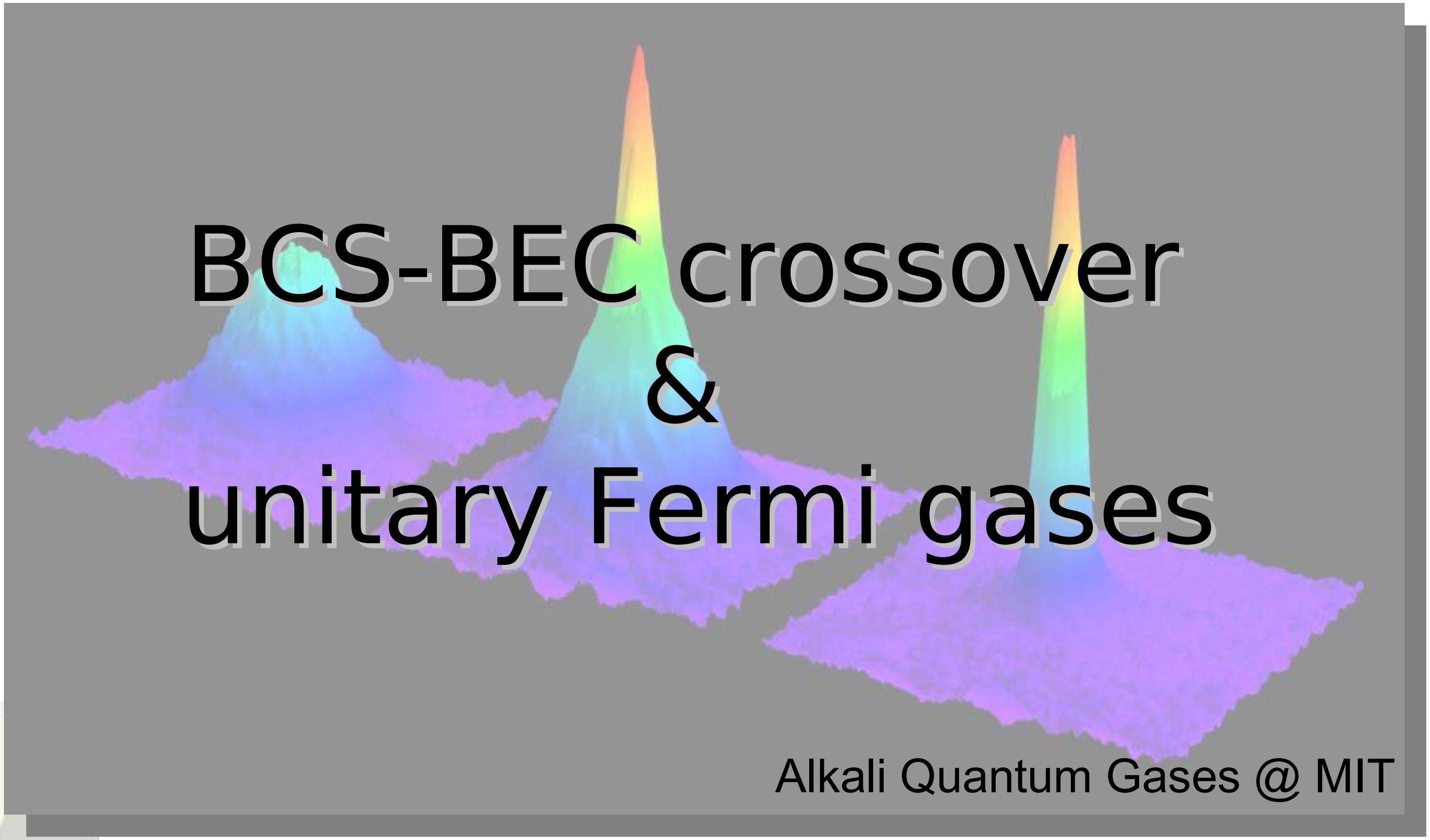
Energy/particle of n-matter

Symbols: microscopic calc. with realistic nucl. int.

Solid lines: phenom. models

Skyrme HF, RMF, etc.

[Pethick & Ravenhall (1995)]



BCS-BEC crossover
&
unitary Fermi gases

Alkali Quantum Gases @ MIT

Unique features of cold atom gases

- Controllability & Flexibility

  Manipulate system parameters

  Density ← Gas phase: low bulk energy

  Strength & sign of the interaction ← Feshbach res.

  etc.

- Observability & Measurability

  Microscopic scales are large

  → "Seeing is believing"

Two directions of research

1. Analogical study

Analogical model of NS matter using cold atoms

Finding new aspects

e.g. Maeda *et al*. PRL **103**, 085301 (2009); arXiv:1205.1086.

2. Quantitative study

(main focus of this talk)

Based on universality of the systems

Attack uncertainties of nuclear systems

Ultracold and ultradilute

Cold atom gases are ultracold and ultradilute

"Ultradilute": (particle separation) >> (range of atomic pot.)

$r_s \sim k_F^{-1} \sim 100\text{nm}$ $R_{\text{vdW}} \sim 10\, a_0$

Density $n \sim 10^{12} – 10^{15}\ \text{cm}^{-3}$

($N \sim 10^6$, $R \sim 10\text{-}100\ \mu\text{m}$)

$$k_B T_F \sim \frac{\hbar^2}{2mr_s^2} \sim 1\mu\text{K}$$

"Ultracold": (thermal de Broglie) $\gtrsim$ (particle separation)

$$\lambda_T \sim \hbar/\sqrt{mk_BT}$$

Coldness & diluteness

➡ Atom-atom int.: Low-energy & two-body scattering

All we need is *s*-wave scattering length a_s.

Typically, $a_s \sim 100\, a_0$, but tunable.

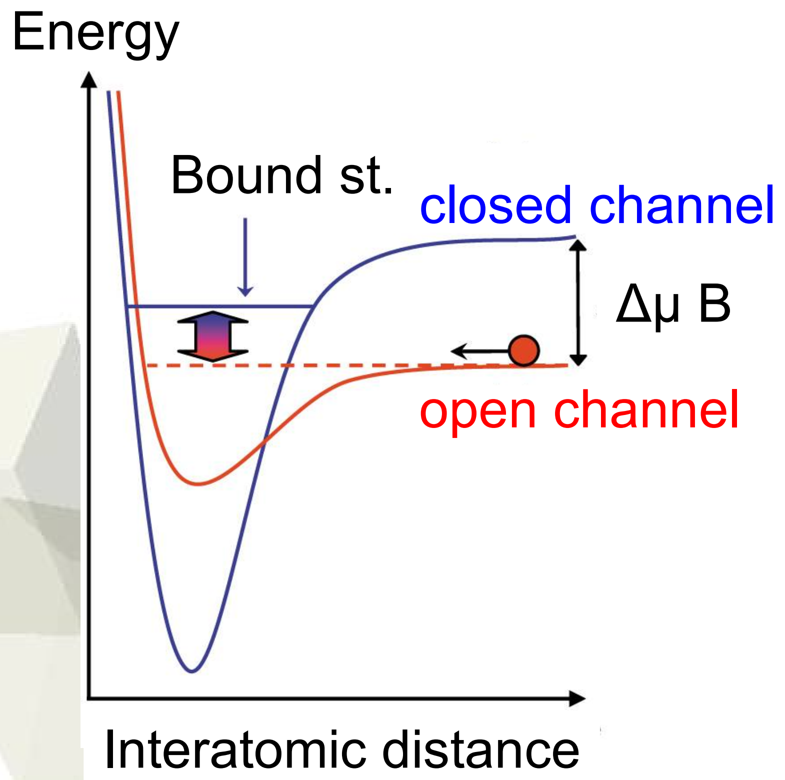
Feshbach resonances

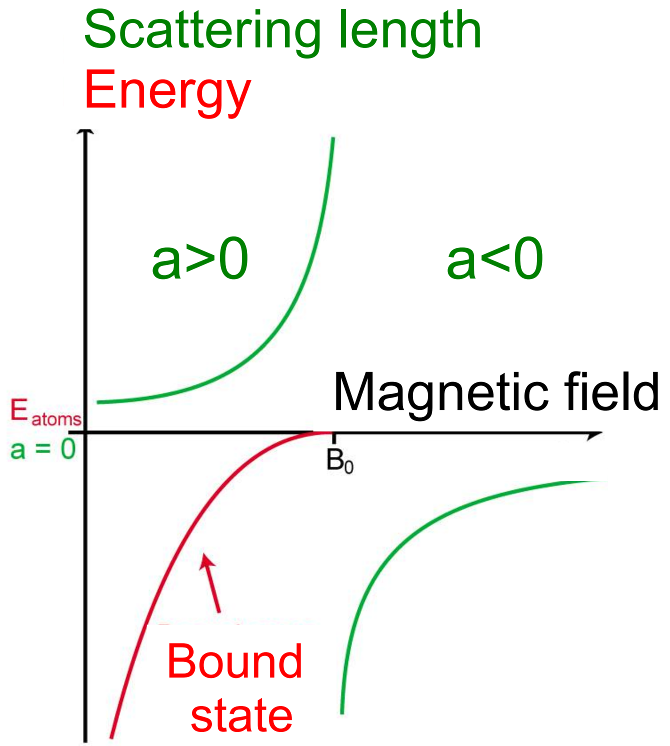
Interaction is determined only by scattering length.

a_s : tunable by magnetic field

zero energy scattering wave func.: $\psi = 1 - \frac{a_s}{r}$

$$(\text{energy of bound st.}) \left\{ \begin{matrix} < \\ = \\ > \end{matrix} \right\} (\text{threshold energy}) \Rightarrow a_s \left\{ \begin{matrix} > 0 \\ = \pm\infty \\ < 0 \end{matrix} \right.$$





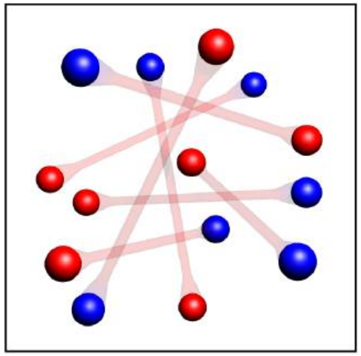
BCS-BEC crossover

BCS-BEC crossover by Feshbach resonance

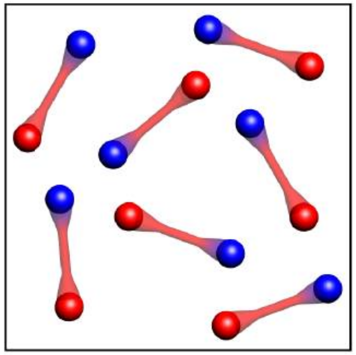
$1/k_\mathrm{F} a_s$

-∞ 0 +∞

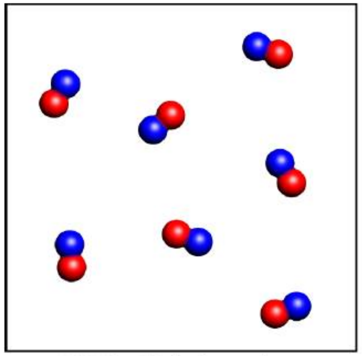
BCS ↔ Unitarity ↔ BEC



BCS state



Crossover Superfluid



BEC of Molecules

Cooper pair size >> interparticle distance

$\xi \equiv \hbar^2 k_F / m\Delta$ k_F^{-1}

Scattering length

$a_s \to \pm\infty$

universality

BEC of bosonic mol.
made of
deeply bound fermions

Universality in unitary Fermi gases

Energy scales: $\hbar^2/ma_s^2,\ E_F,\ k_BT$

$|a_s| \longrightarrow \infty$ ➡ Relevant characteristic scales are

$(k_F a_s \gg 1)$ (k_F or n) & T

Sys. does not depend on details of int.: universality

$$E = \xi\ E_F$$

$$\Delta = \delta\ E_F$$

$$T_c = \alpha\ T_F$$

ξ, δ, α : universal parameter of $\lesssim O(1)$

ξ: Bertsch parameter

momentum distribution

$$n_\sigma(k) \xrightarrow[k \gg k_F]{} C/k^4$$

C: Tan's contact parameter

[Tan, Ann. Phys. **323**, 2952 (2008)]

Art of BCS wave function

BCS wave func.

$$\Psi_{\mathrm{BCS}} \propto \prod_{\mathbf{k}} \left(u_{\mathbf{k}} + v_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow} \right) |0\rangle$$

Amp. of
k↑ & -**k**↓
are empty

Amp. of
k↑ & -**k**↓
are occupied

$$\propto \exp\left(\alpha b_0^{\dagger}\right)|0\rangle$$

$$\alpha\phi_{\mathbf{k}} \equiv v_{\mathbf{k}}/u_{\mathbf{k}} \qquad b_0^{\dagger} \equiv \sum_{\mathbf{k}} \phi_{\mathbf{k}} c^{\dagger}_{\mathbf{k},\uparrow} c^{\dagger}_{-\mathbf{k},\downarrow}$$

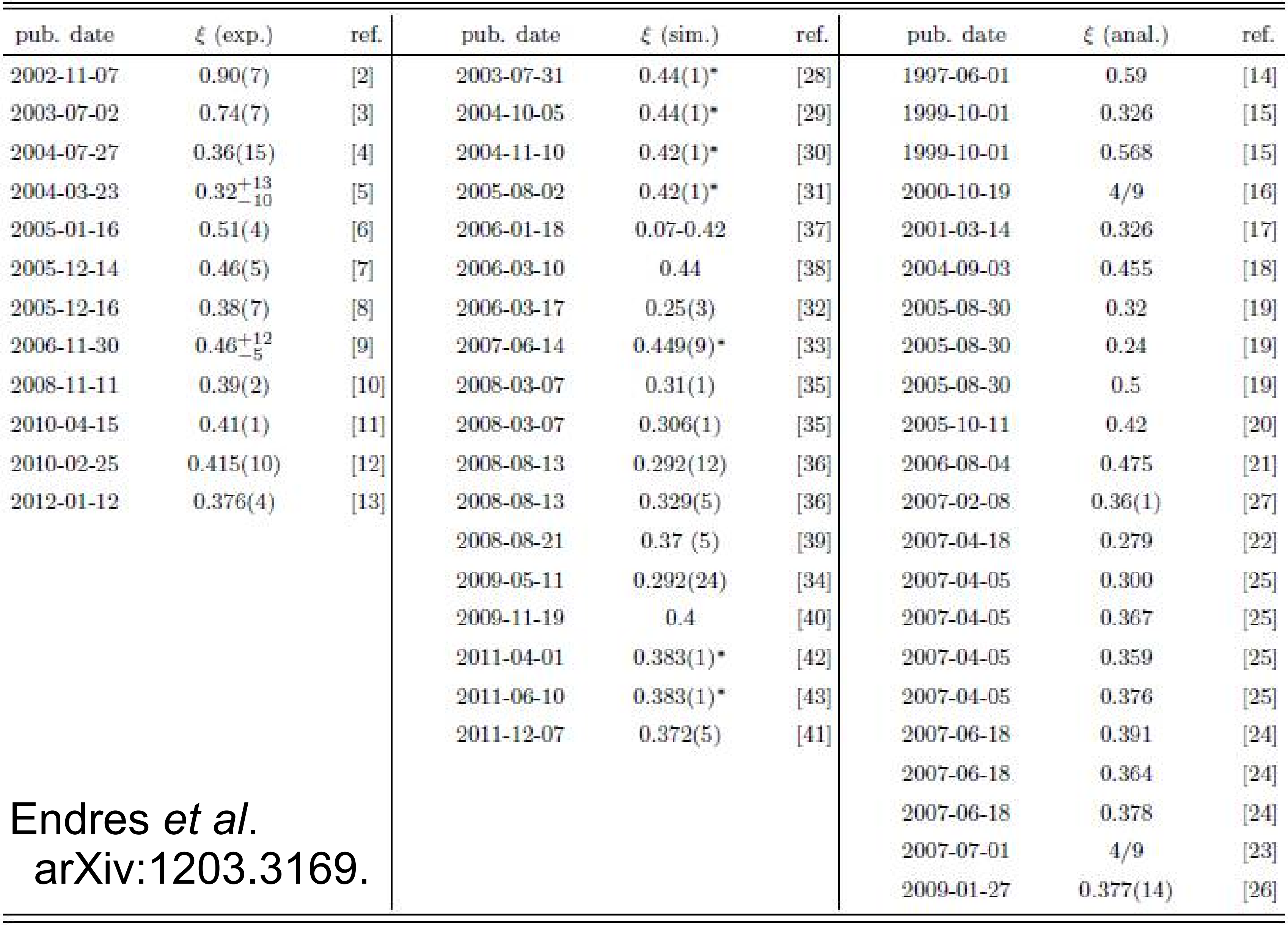
Strong coupling (BEC) limit: $b_0^{\dagger}$ → bosonic op.

$$\Psi_{\mathrm{BCS}} \propto \exp\left(\alpha b_0^{\dagger}\right)|0\rangle \longrightarrow \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$ Coherent st.

Ψ_{BCS} covers BEC of bosonic pairs.

"Time evolution" of Bertsch parameter

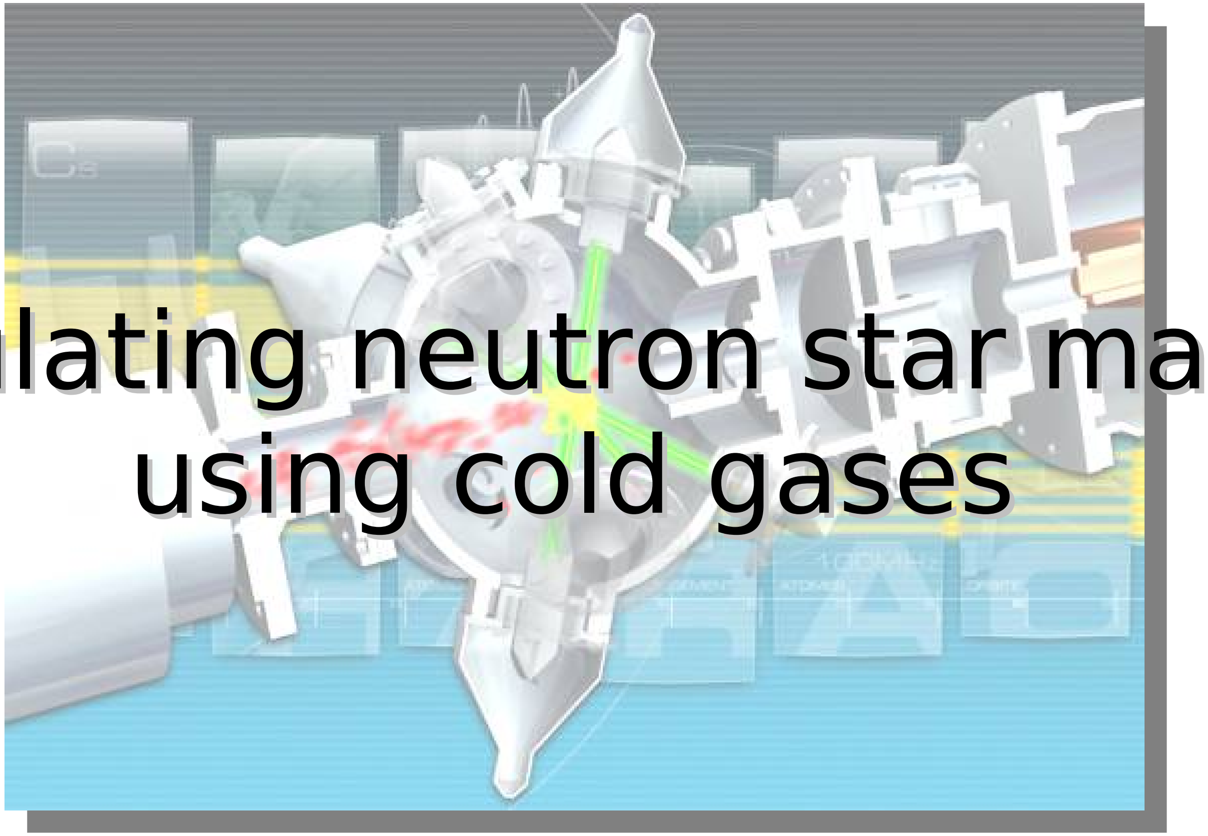
$\xi = 0.3\sim0.45$ Recent QMC: $\xi \approx 0.38$



pub. date	ξ (exp.)	ref.	pub. date	ξ (sim.)	ref.	pub. date	ξ (anal.)	ref.
2002-11-07	0.90(7)	[2]	2003-07-31	$0.44(1)^*$	[28]	1997-06-01	0.59	[14]
2003-07-02	0.74(7)	[3]	2004-10-05	$0.44(1)^*$	[29]	1999-10-01	0.326	[15]
2004-07-27	0.36(15)	[4]	2004-11-10	$0.42(1)^*$	[30]	1999-10-01	0.568	[15]
2004-03-23	0.32^{+13}_{-10}	[5]	2005-08-02	$0.42(1)^*$	[31]	2000-10-19	4/9	[16]
2005-01-16	0.51(4)	[6]	2006-01-18	0.07-0.42	[37]	2001-03-14	0.326	[17]
2005-12-14	0.46(5)	[7]	2006-03-10	0.44	[38]	2004-09-03	0.455	[18]
2005-12-16	0.38(7)	[8]	2006-03-17	0.25(3)	[32]	2005-08-30	0.32	[19]
2006-11-30	0.46^{+12}_{-5}	[9]	2007-06-14	$0.449(9)^*$	[33]	2005-08-30	0.24	[19]
2008-11-11	0.39(2)	[10]	2008-03-07	0.31(1)	[35]	2005-08-30	0.5	[19]
2010-04-15	0.41(1)	[11]	2008-03-07	0.306(1)	[35]	2005-10-11	0.42	[20]
2010-02-25	0.415(10)	[12]	2008-08-13	0.292(12)	[36]	2006-08-04	0.475	[21]
2012-01-12	0.376(4)	[13]	2008-08-13	0.329(5)	[36]	2007-02-08	0.36(1)	[27]
			2008-08-21	0.37 (5)	[39]	2007-04-18	0.279	[22]
			2009-05-11	0.292(24)	[34]	2007-04-05	0.300	[25]
			2009-11-19	0.4	[40]	2007-04-05	0.367	[25]
			2011-04-01	$0.383(1)^*$	[42]	2007-04-05	0.359	[25]
			2011-06-10	$0.383(1)^*$	[43]	2007-04-05	0.376	[25]
			2011-12-07	0.372(5)	[41]	2007-06-18	0.391	[24]
						2007-06-18	0.364	[24]
						2007-06-18	0.378	[24]
						2007-07-01	4/9	[23]
						2009-01-27	0.377(14)	[26]

Endres *et al*.
arXiv:1203.3169.

Simulating neutron star matter using cold gases

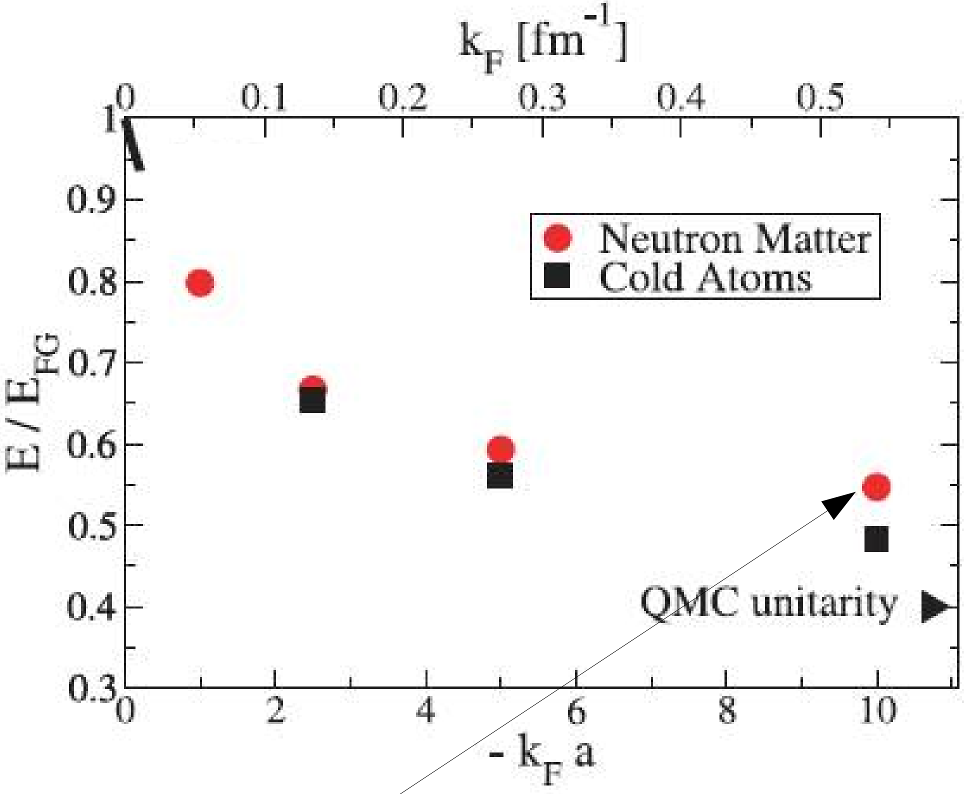


Comparison btwn n-matter & cold atoms

		Neutron matter in NS crust $(\rho \lesssim \rho_0)$	Cold Fermi gas at unitarity
Particle separation	$r_s \sim k_F^{-1}$	~ 1 fm	~ 100 nm
Temperature	T	~ 100 keV	$\gtrsim$ 100 nK
Degeneracy temp.	T_F	~ 100 MeV	~ 1 μK
	T/T_F	~ 10^{-3}	$\gtrsim$ 0.1
Scattering length	a_s	-18.9 fm	∞
	$k_F\vert a_s\vert$	~ 19 >> 1 very large!	∞
Effective range $k\cot\delta = -\frac{1}{a_s} + \frac{1}{2}r_e k^2 + O(k^4)$	r_e	2.75 fm	4.7 nm (^{6}Li @ 834G)
	$k_F\vert r_e\vert$	~ 3 (non-negligible)	~ 10^{-2}

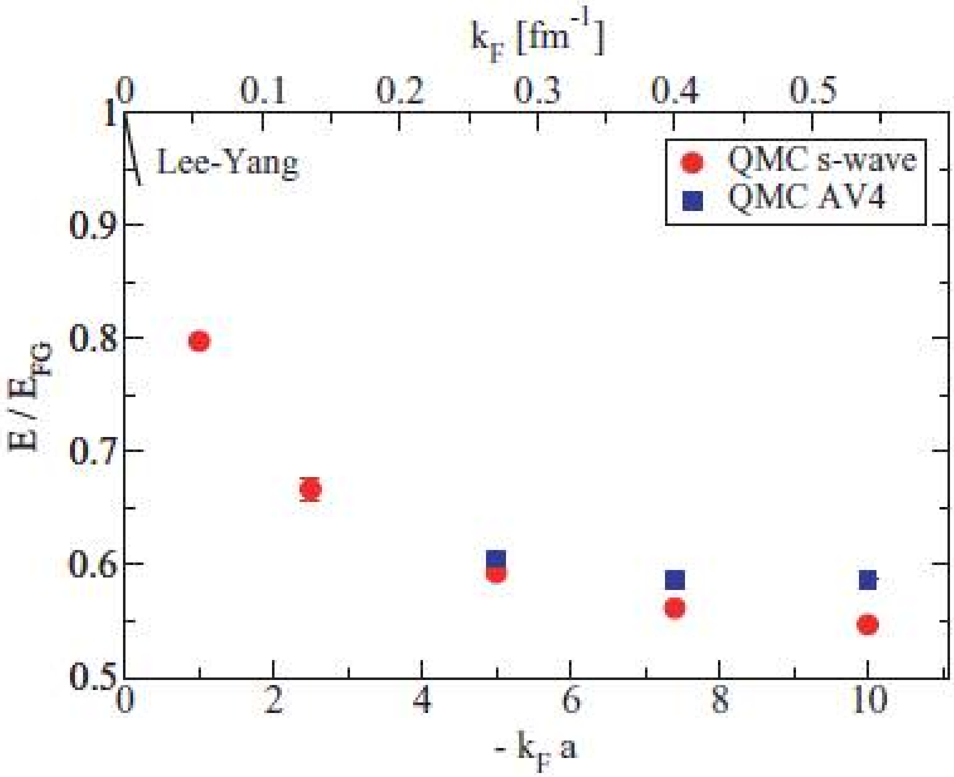
QMC results for energy at T=0

Neutron matter & cold atoms



Gezerlis & Carlson (2008)

s-wave only & with *p*-wave



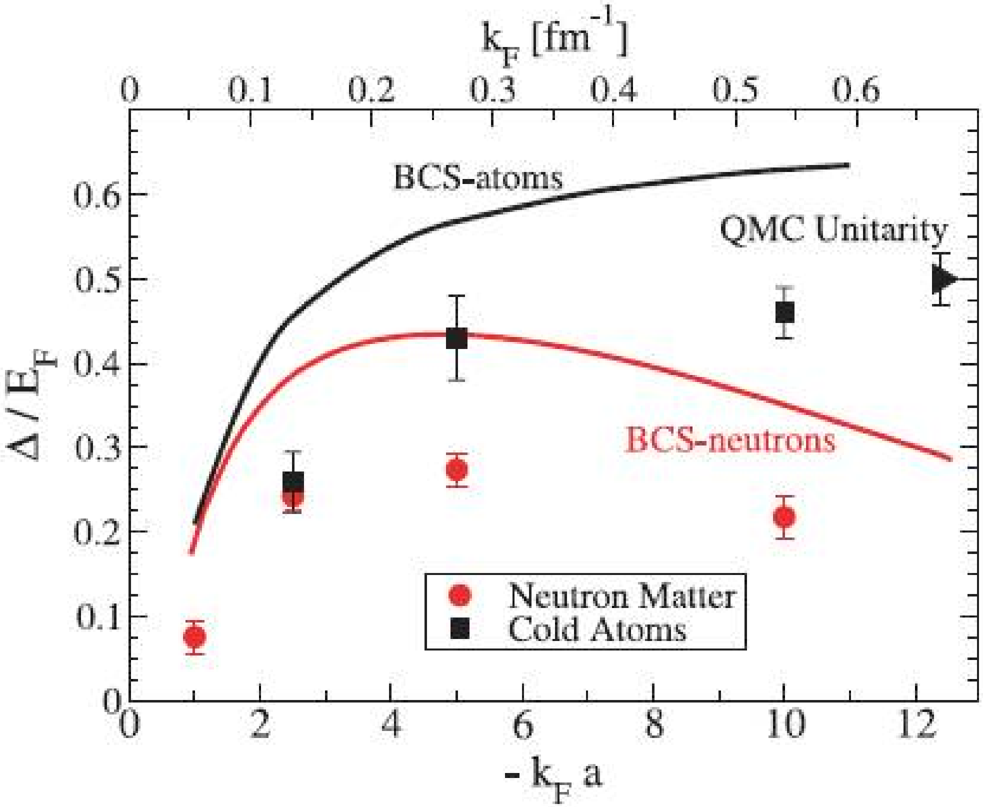
Gezerlis & Carlson (2010)

$k_F r_e \sim 1$ non-negligible r_e

Good agreement for EOS.

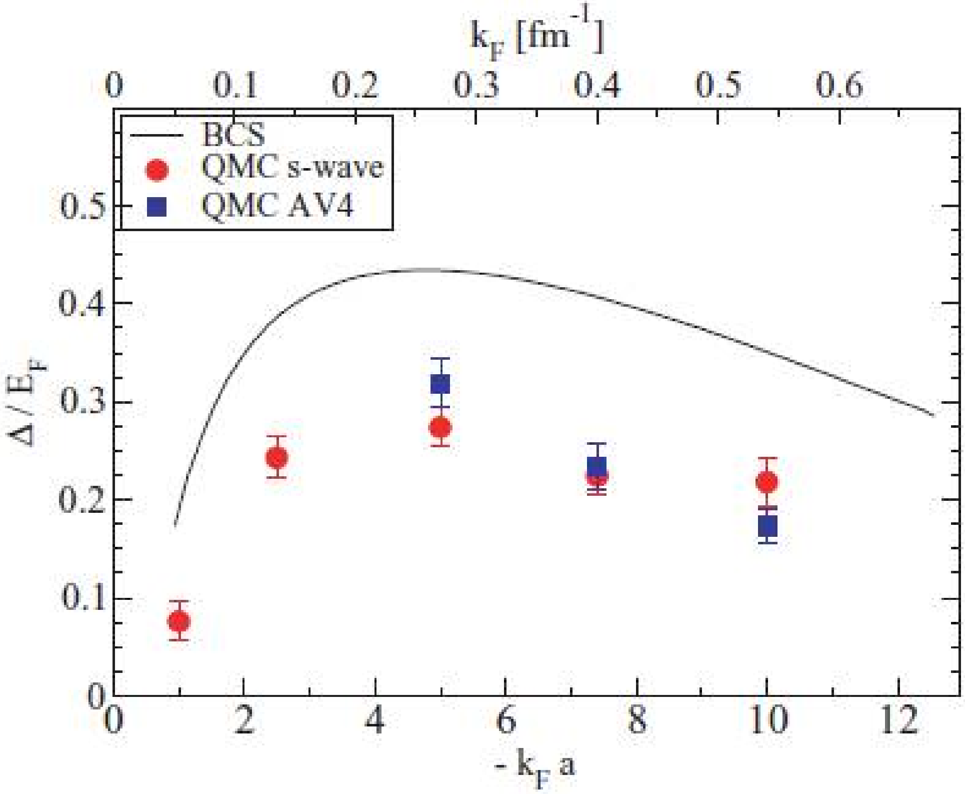
QMC results for Δ at T=0

Neutron matter & cold atoms



Gezerlis & Carlson (2008)

s-wave only & with *p*-wave



Gezerlis & Carlson (2010)

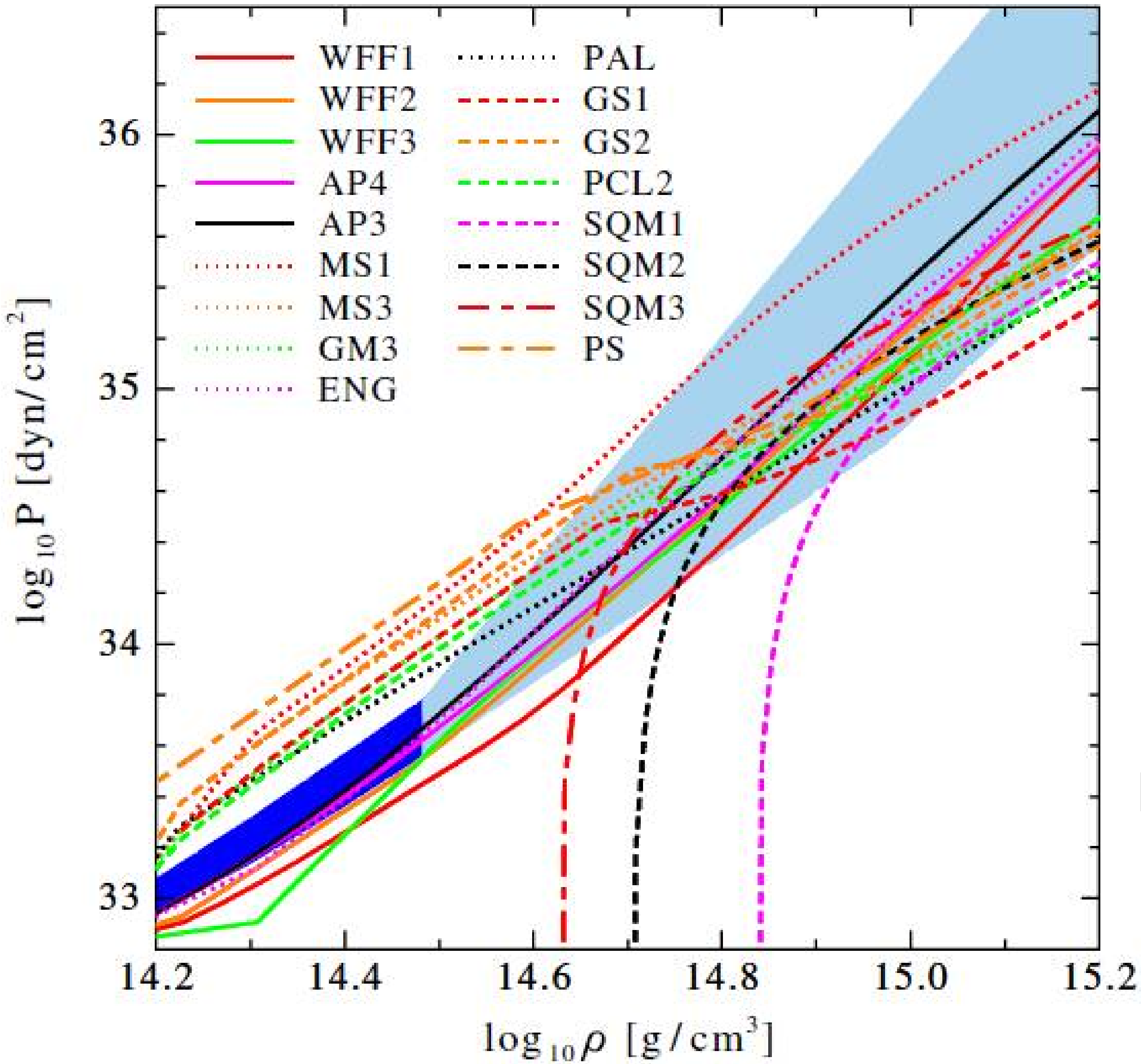
Δ is more sensitive to effect of r_e. ⬅ Exponential dep. on int.

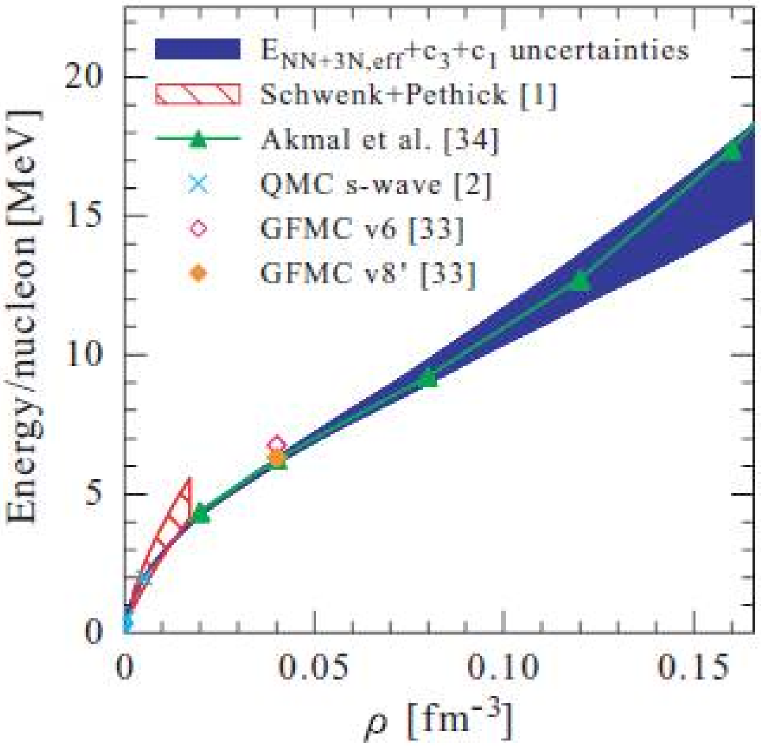
$$\Delta_{\rm BCS} \propto E_F \exp\left(-\pi/2k_F|a|\right)$$

Induced int.
(Gor'kov & Melik-Barkhudarov) ➡

$$\Delta/\Delta_{\rm BCS} = 1/(4e)^{1/3} \simeq 0.45$$

Constraint on EOS





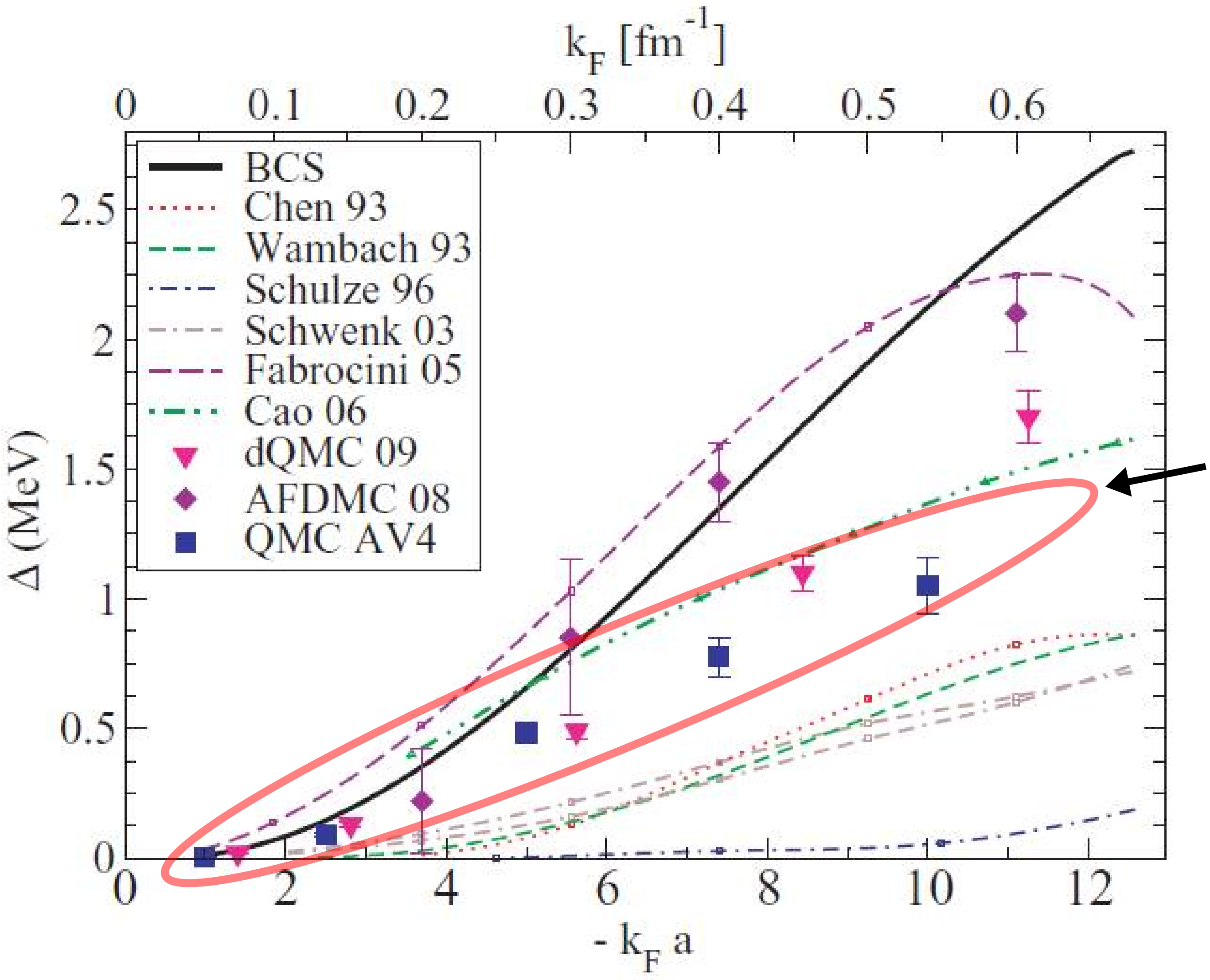
[Hebeler & Schwenk (2010)]

Extrapolation taking account of uncertainty in 3N int. within Ch EFT keeping constraint in the low density regime.

Stronger (factor of 6) constraint on EOS at ρ_0.

Hebeler et al. PRL **105**, 161102 (2010).

$^1S_0$ pairing gap of neutron matter



Recent trend

Gezerlis & Carlson, PRC **81**, 025803 (2010).

More results are awaited!

Toward simulating n-matter by cold atoms

Neutron matter in NS crusts: $k_F r_e \sim 1$ Non-negligible!

$$k \cot \delta = -\frac{1}{a_s} + \frac{1}{2} r_e k^2 + O(k^4)$$

Cold Fermi gases

r_e=2.7nm for ^{6}Li @ 834G $k_F r_e \sim 10^{-2}$ Too small!

r_e=-3700nm for ^{6}Li @ 543G $k_F r_e \sim 10$ Too large!

A method to control r_e

[Marcelis, Verhaar & Kokkelmans, PRL **100**, 153201 (2008)]

Use E-field in addition to B-field. (dipolar int. induced by E-field)

E-field → Potential res. in open channel

B-field → Feshbach res. btwn open & close

} Interplay

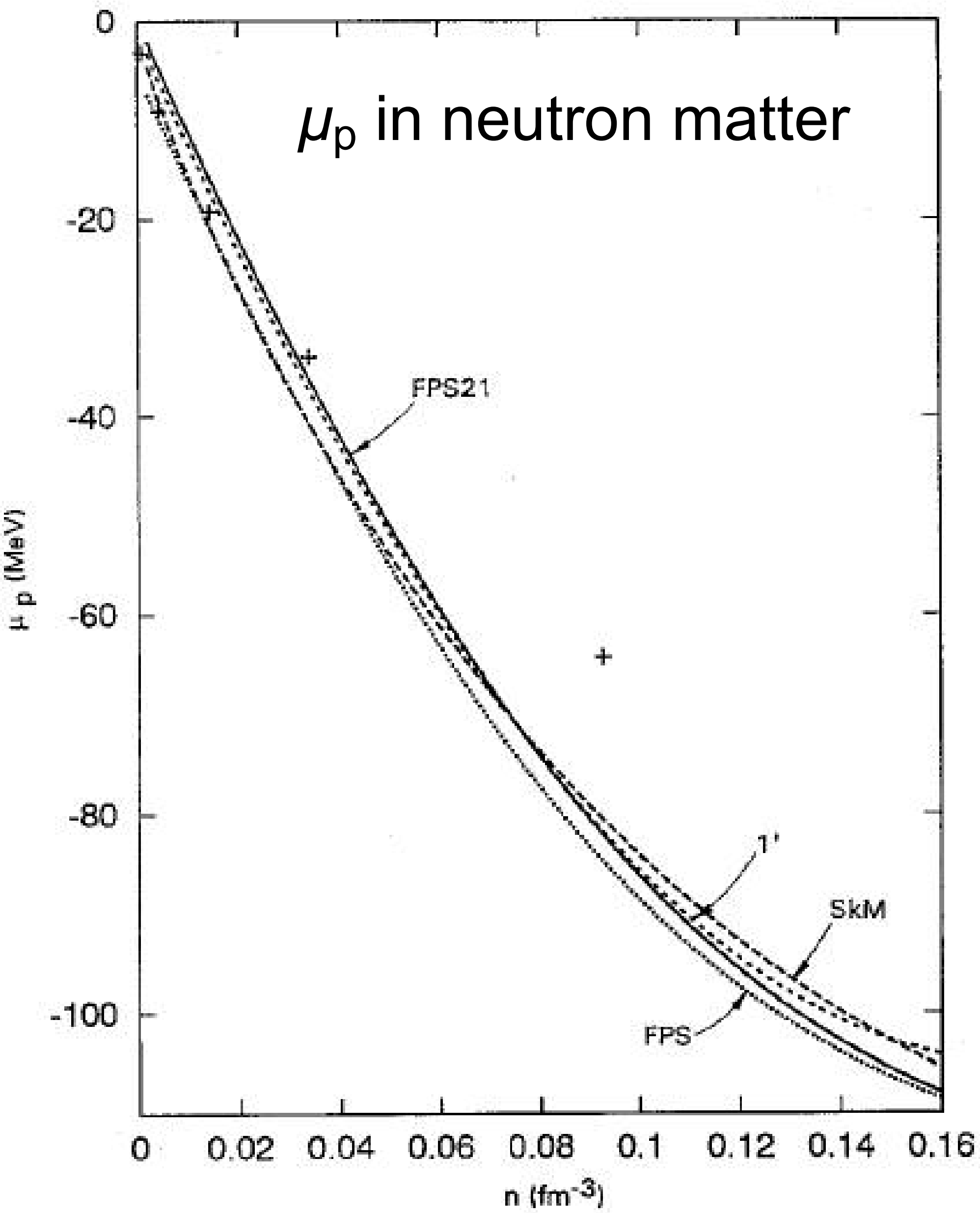
$$a(B) = r_0 + a_P \left(1 - \frac{\Delta B}{B - B_0}\right)$$ with $$a_P = a_P(E)$$

$$r_e(B) \sim \hbar^2 / [m a_P \Delta\mu (B - B_0')]$$

$\Delta\mu$: mag. mom. diff.

Future challenges

Mix a few protons into neutron matter



μ_p in neutron matter

2-body p-n system

Deuteron b. st. due to tensor force in spin triplet channel.

- Proton chem. pot. μ_p in n-matter

  Related to symmetry energy.

- n-rich low density nuclear matter

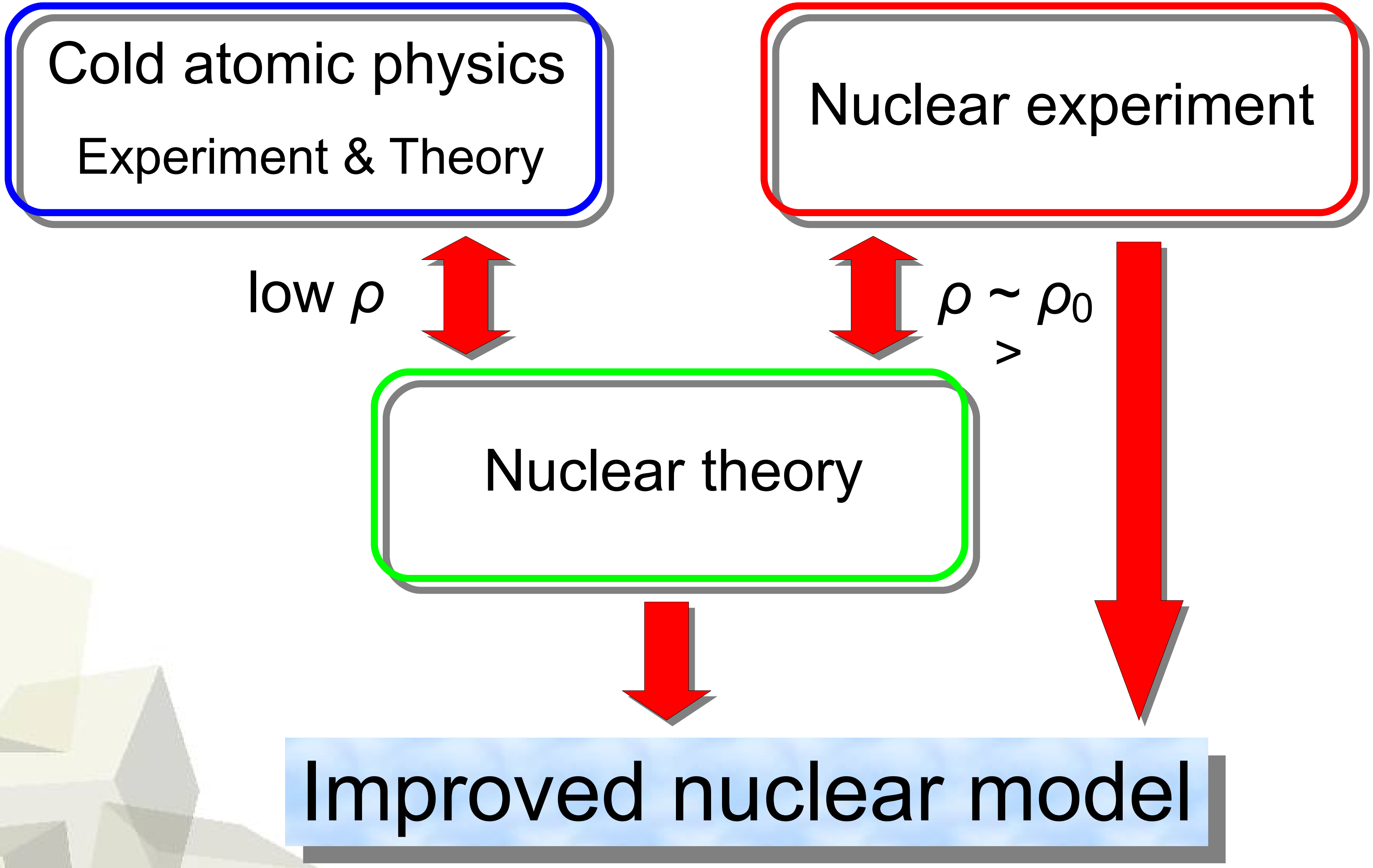
  Nonzero density of protons

- Matter in the crusts

  Existence of nuclei

[Pethick et al. NPA **584**, 675 (1995)]

Towards improved nuclear model



Cold atomic physics

Experiment & Theory

Nuclear experiment

low ρ

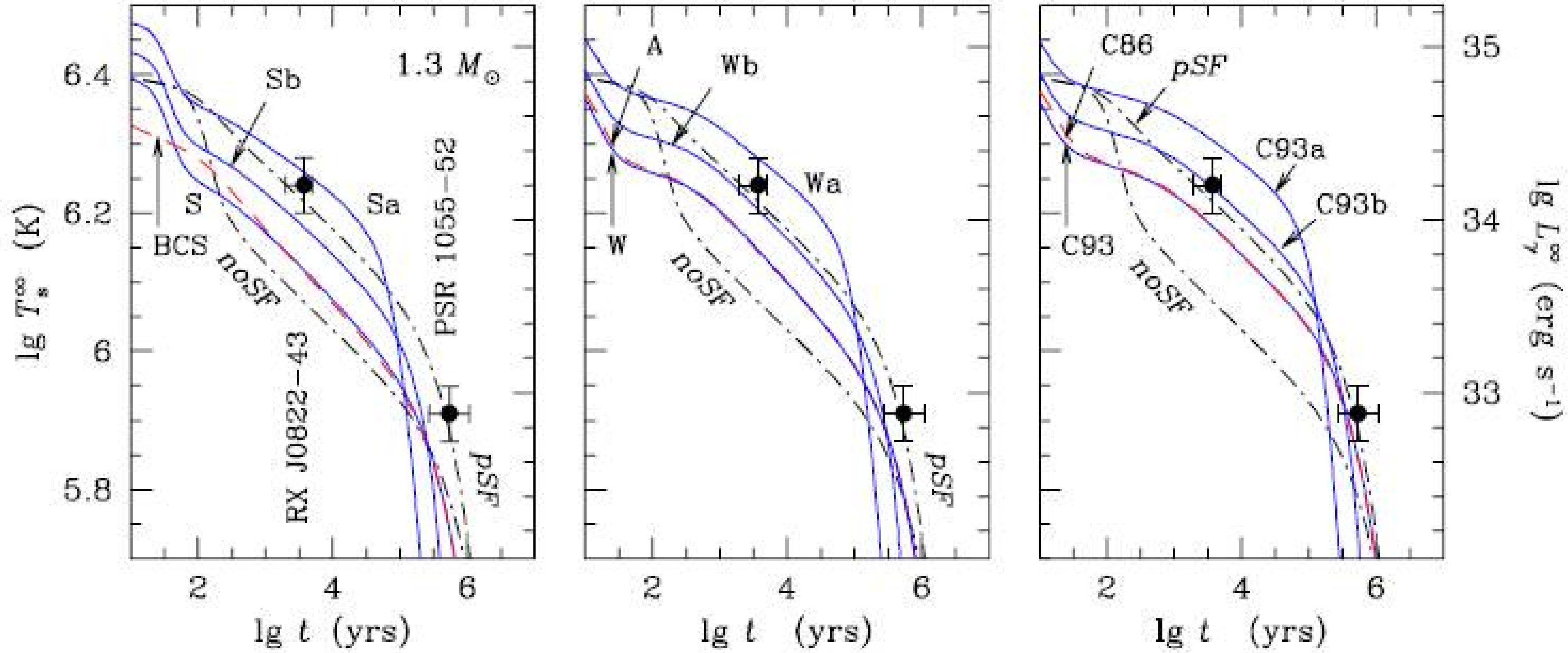
$\rho \gtrsim \rho_0$

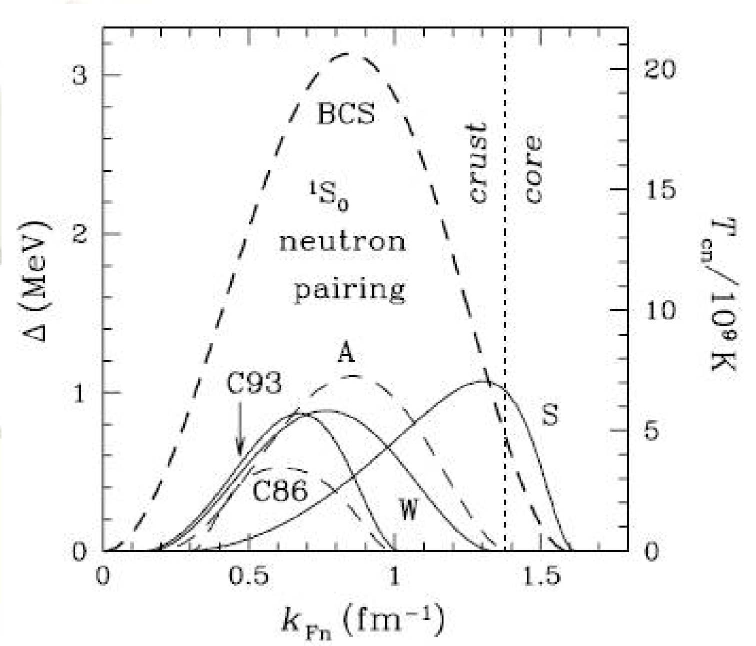
Nuclear theory

Improved nuclear model

We need strong support from nuclear theorists!

Effect of n-superfluid on NS cooling





NS cooling curve depends on pairing gap of neutron matter in crusts!

Yakovlev & Pethick,
Annu. Rev. Astron. Astrophys. **42**, 169 (2004)

QMC calc.

Gezerlis & Carlson, PRC **81**, 025803 (2010)

N = 66-68

Smallest L = 20.8 fm

VMC & GFMC

AV4 (a part of AV18)

$$v_4(r) = v_c(r) + v_\sigma(r)\sigma_1 \cdot \sigma_2$$

Pairing gap:

$$\Delta(N) = E(N) - \frac{1}{2}[E(N+1) + E(N-1)]$$

VMC

VMC (Variational MC)

$$\langle H \rangle = \frac{\int d\mathbf{R}\Psi_{\mathrm{v}}^{*}(\mathbf{R})H\Psi_{\mathrm{v}}(\mathbf{R})}{\int d\mathbf{R}|\Psi_{\mathrm{v}}(\mathbf{R})|^2} \geq E_0$$

Optimize ψ_v to minimize $\langle H \rangle$

$$\Psi_{\mathrm{v}} = \Psi_J(\mathbf{R})\Psi_{\mathrm{BCS}}(\mathbf{R})$$

Jastrow factor: $\Psi_J = \prod_{i<j} f_{\uparrow\uparrow}(r_{i,j}) \prod_{i'<j'} f_{\downarrow\downarrow}(r_{i',j'}) \prod_{i,i'} f_{\uparrow\downarrow}(r_{i,i'})$

BCS wave func.: $\Psi_{\mathrm{BCS}} = \mathcal{A} \prod_{i<j'} \phi(r_{i,j'})$

$$\phi(r) = \sum_{\mathbf{k}} \frac{v_{\mathbf{k}}}{u_{\mathbf{k}}} e^{i\mathbf{k}\cdot\mathbf{r}}$$