研究計画B03：

冷却原子を用いた中性子過剰な低密度核物質の状態方程式

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理化学研究所（理論）　：　中務孝（連携）　☆冷却原子⇔原子核

岡山大学（理論）　：　水島健（連携）　☆p波相互作用

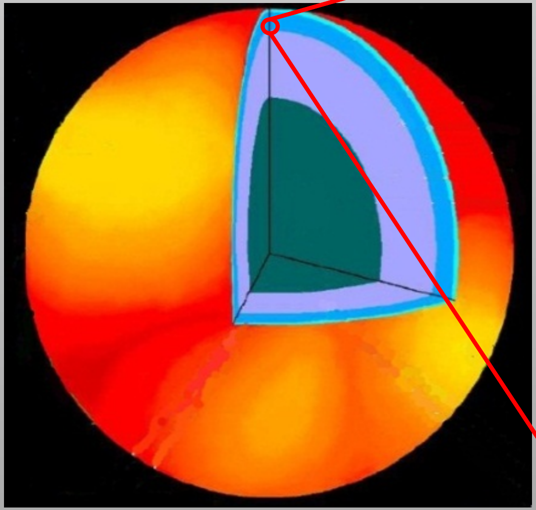
APCTP（理論）　：　渡辺元太郎（協力）　☆冷却原子⇔原子核

Our mission

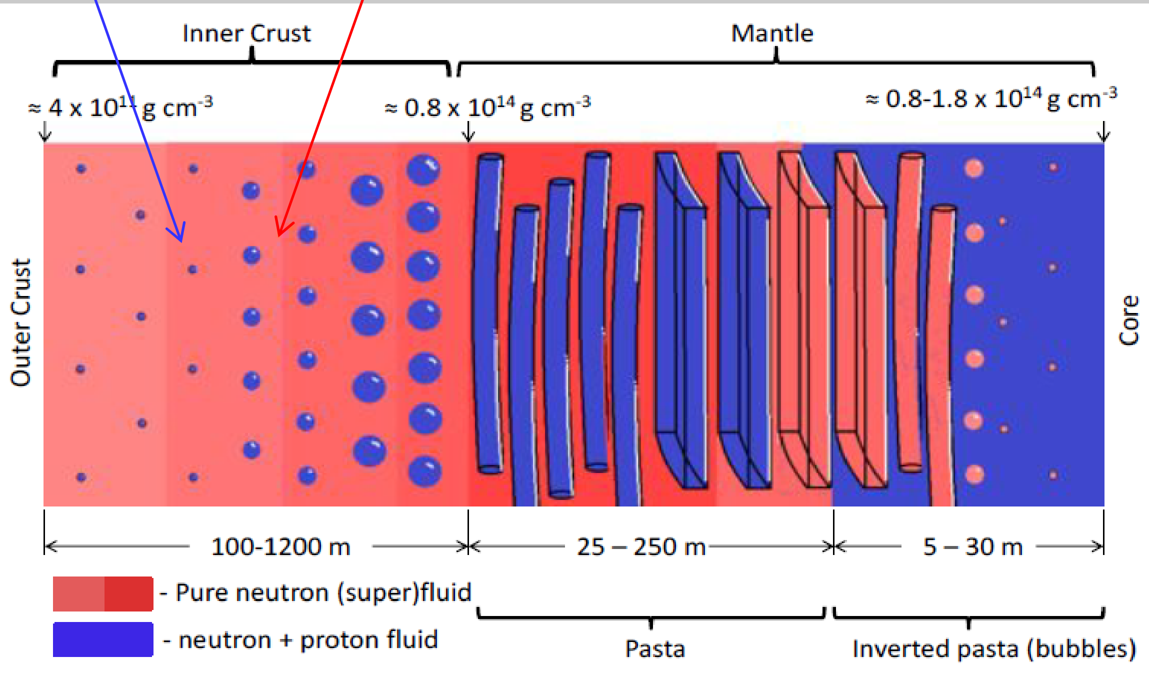
Simulation of neutron matter using cold Fermi gas

bcc crystal lattice of nuclei

Dripped neutrons from nuclei



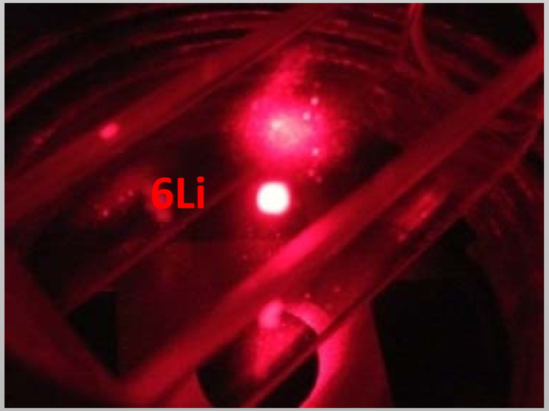
Neutron star

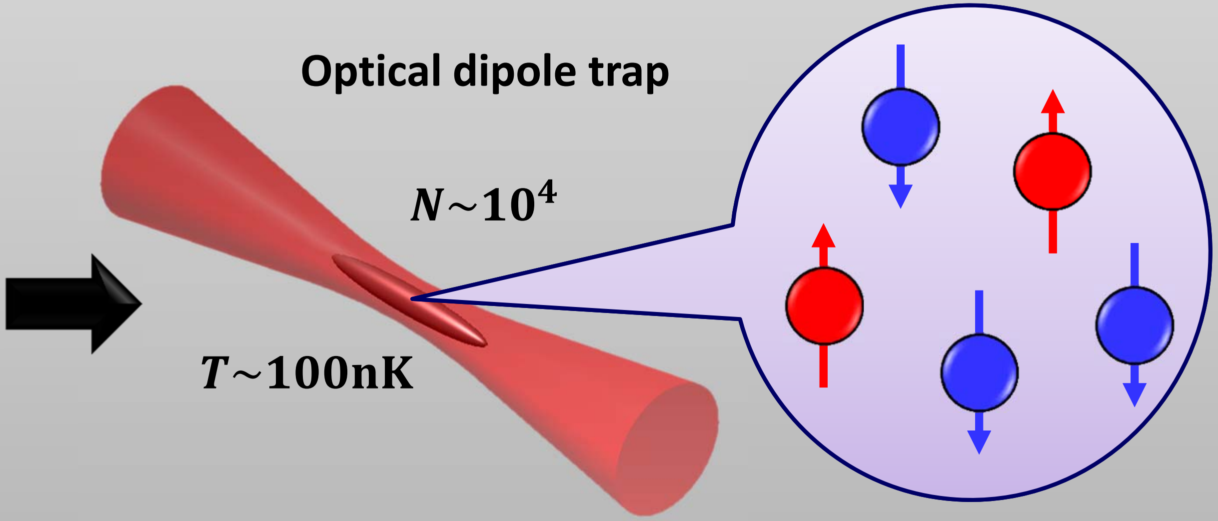


inner crust region [Newton, *et al.*, arXiv:1112.2018]

Cold atom system

Laser cooling
（magneto-optical trap）





Optical dipole trap

$N \sim 10^4$

$T \sim 100\mathrm{nK}$

Two-component Fermi gas

- Approximately **homogeneous** system
- **Dilute** two-components Fermi systems
- **Tunable interactions** by Feshbach resonances
- **Universal** many-body system
- **Precise measurements**

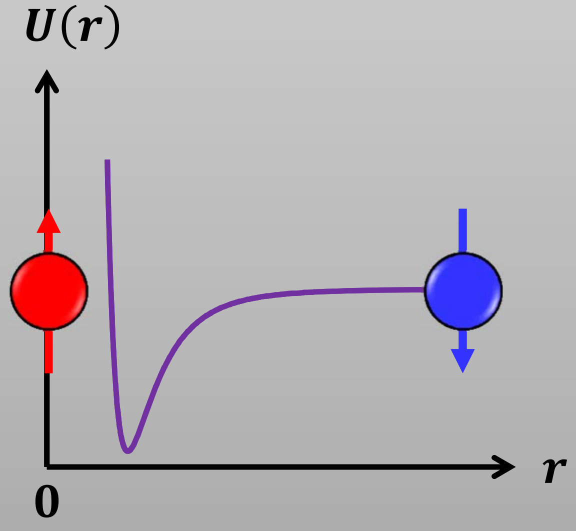
Cold atom vs. Neutron matter

	Cold Fermi atom	Neutron matter
Interaction	s-wave	s-wave
Temperature : T	$\sim 10^{-7}$K	$\sim 10^{9}$K
Fermi temperature : T_{F}	$\sim 10^{-6}$K	$\sim 10^{10}$K
Interparticle distance : $\sim k_F^{-1}$	100nm	$6 \sim 3$fm
Scattering length : a	$-\infty \sim \infty$(Feshbach resonance)	-18.5 ± 0.3fm
Effective range : r_{e}	4.7nm	2.75 ± 0.11fm
↓	↓	↓
Temperature : T/T_F	$10 \sim 0.05$	$0.1 \sim 0$
Interaction : $-1/k_F a$	$-\infty(\text{BEC limit}) \sim +\infty(\text{BCS limit})$	$0.28 \sim 0.04$
Effective range : $k_F r_e$	0.05	$0.53 \sim 3.3$
Phase transition : T_C/T_F	~ 0.2	~ 0.1
Superfluid gap : Δ/E_{F}	~ 0.6	~ 0.2
Lattice potential	Optical lattice, Ion crystal	Nuclei crystal

Universal many-body Fermi system

Why can we simulate neutron matter using cold atoms ?

Because both systems belong to the universal system !



The potential shape depends on
details of the particle

Basic concept

Parameterization of scattering

Ultralow temperature
Diluteness
Two-body elastic collision

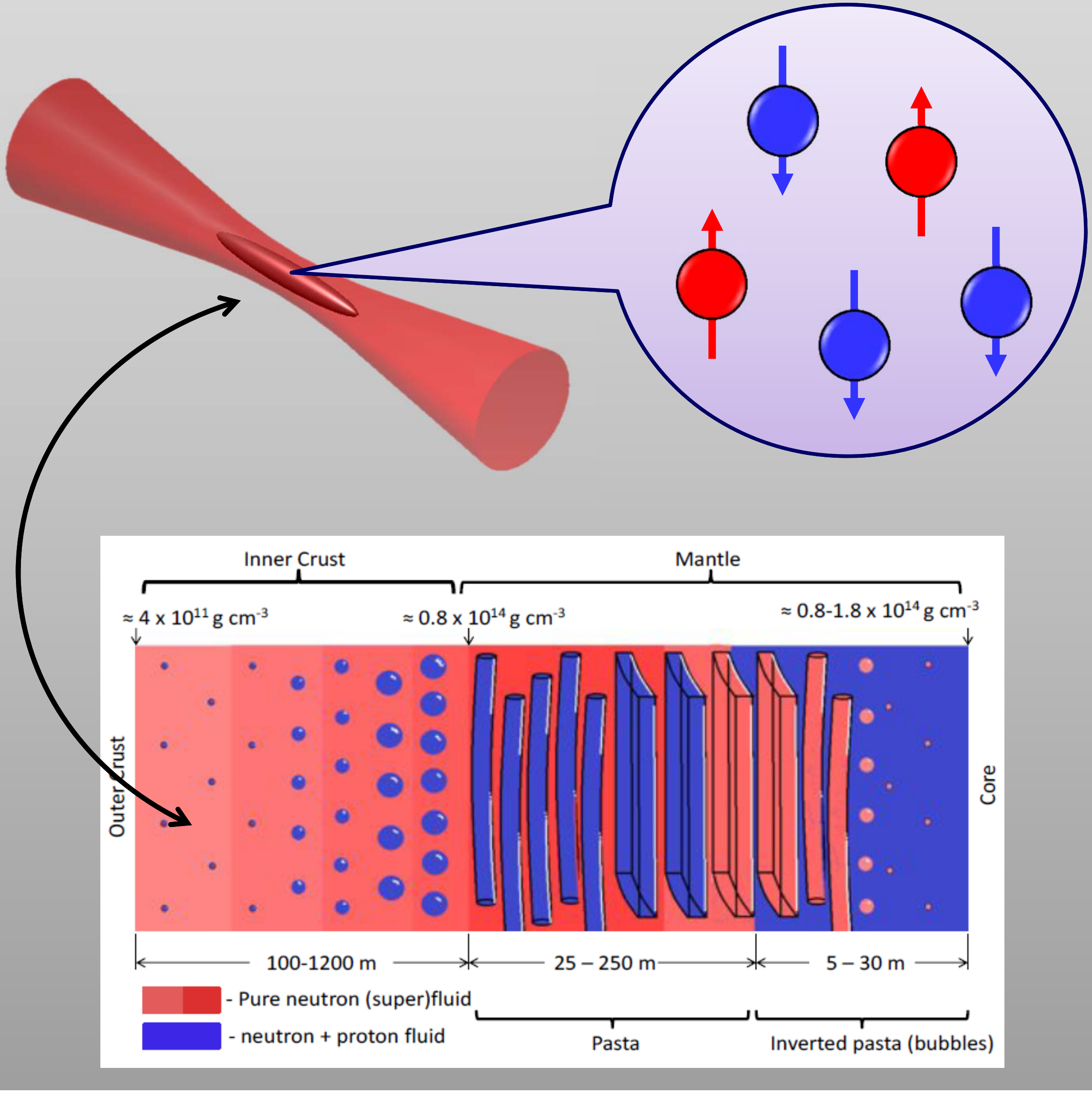
Two-body s-wave phase shifts
No details of the particle

$$k\cot\delta_k = -\frac{1}{a} + \frac{1}{2} r_e k^2 + \mathcal{O}(k^4)$$

Scattering length

Effective range

Simulation of neutron matter using cold atoms



Cold atom

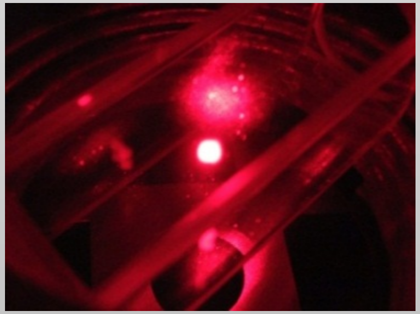
Internal energy :

$$\frac{E}{E_{\mathrm{F}}}\left(\frac{T}{T_{\mathrm{F}}}, -\frac{1}{k_{\mathrm{F}}a}, k_{\mathrm{F}}r_{\mathrm{e}}\right)$$

Inner crust

Our goal

Cold atoms



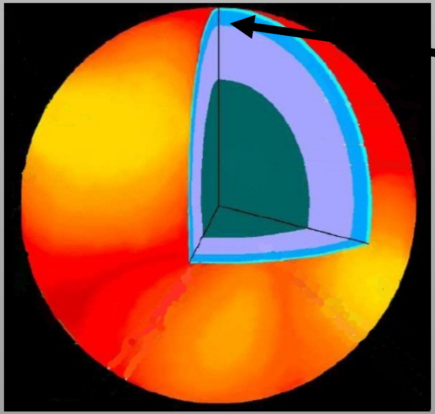
Measurements

- Universal equation of state
- Internal energy
- Specific heat
- Critical temperature
- s,p-wave superfluid
- Superfluid gap
- Superfluid density

⋮

- Benchmark for theories

Inner crust of neutron stars



EOS

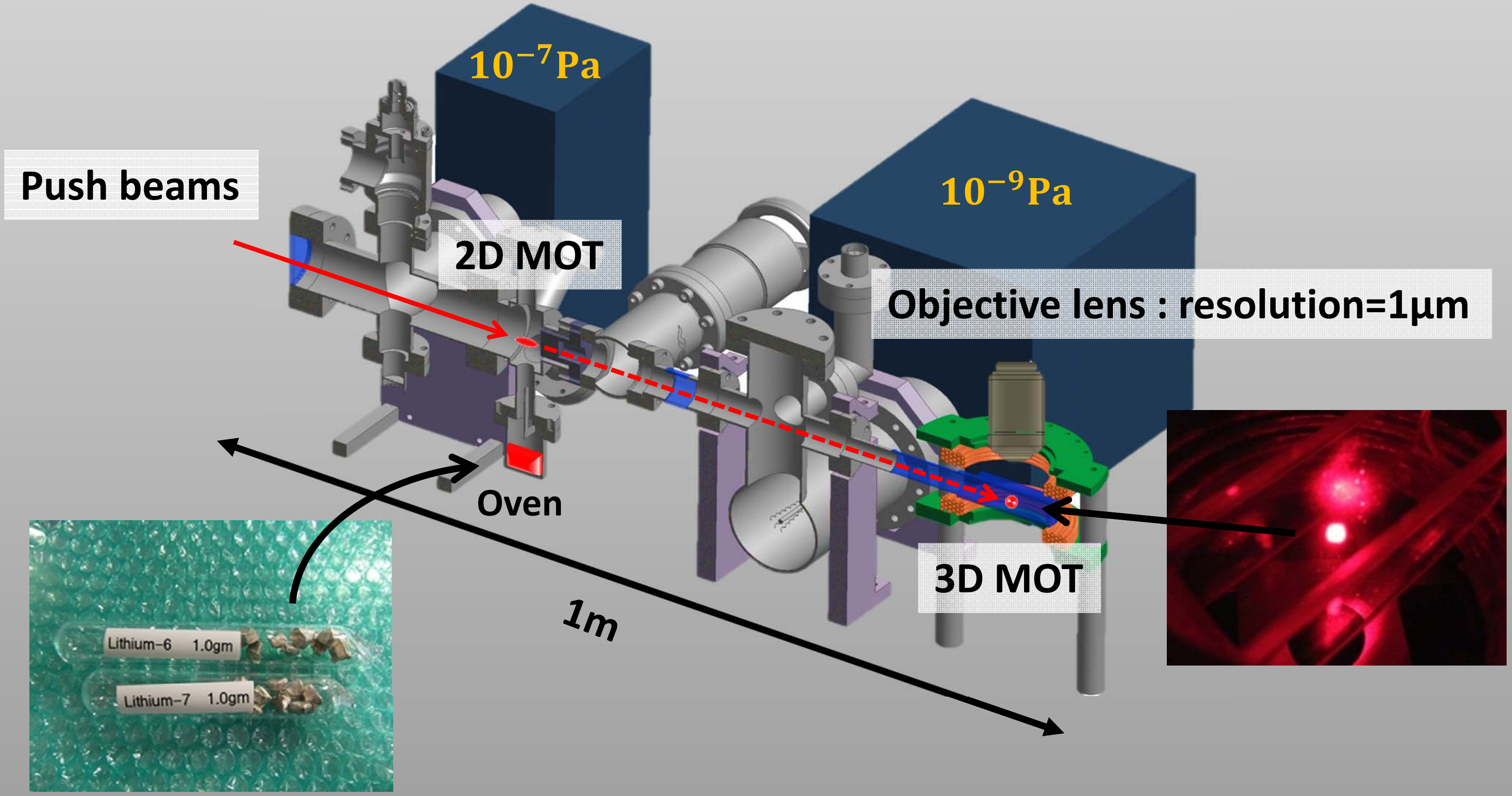
Cooling curve

- ◆ Correction of the effective range
- ◆ Lattice of neutron-rich nuclei
- ◆ Protons

Theories

Cold Atom Lab at University of Tokyo

s-wave interaction, ^{6}Li-^{7}Li (Fermi-Bose) mixture



10^{-7}Pa

Push beams

10^{-9}Pa

2D MOT

Objective lens : resolution=1µm

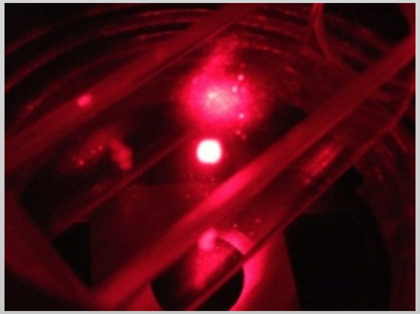
Oven

3D MOT

1m

Present status

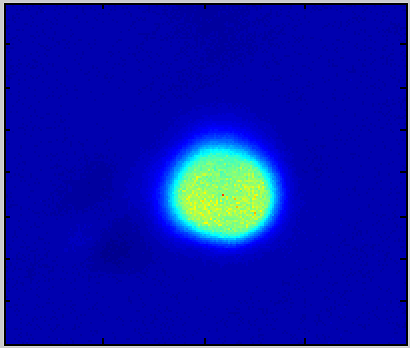
2011/April



Simultaneous MOT of 6Li and 7Li

$N_{6\mathrm{Li}} \sim N_{7\mathrm{Li}} \sim 10^8$

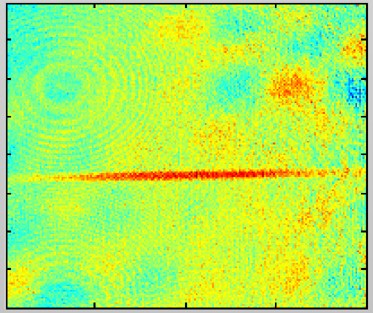
$T \sim 1\mathrm{mK}$



Compression-MOT

$N_{6\mathrm{Li}} \sim N_{7\mathrm{Li}} \sim 10^8$

$T \sim 200\mu\mathrm{K}$



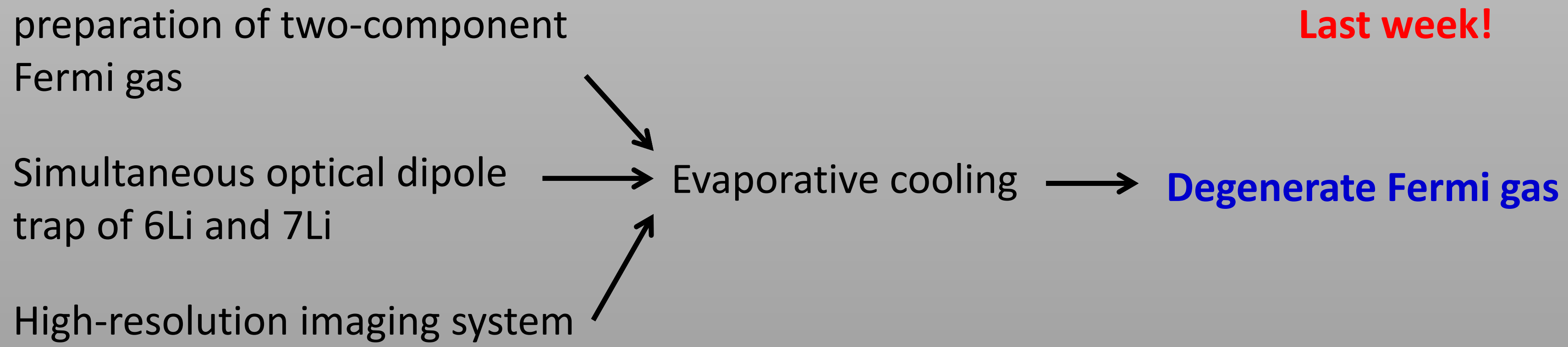
Optical dipole trap

$N_{6\mathrm{Li}} \sim 10^6$

$T \sim 200\mu\mathrm{K}$

Last week!

Next



preparation of two-component Fermi gas

Simultaneous optical dipole trap of 6Li and 7Li

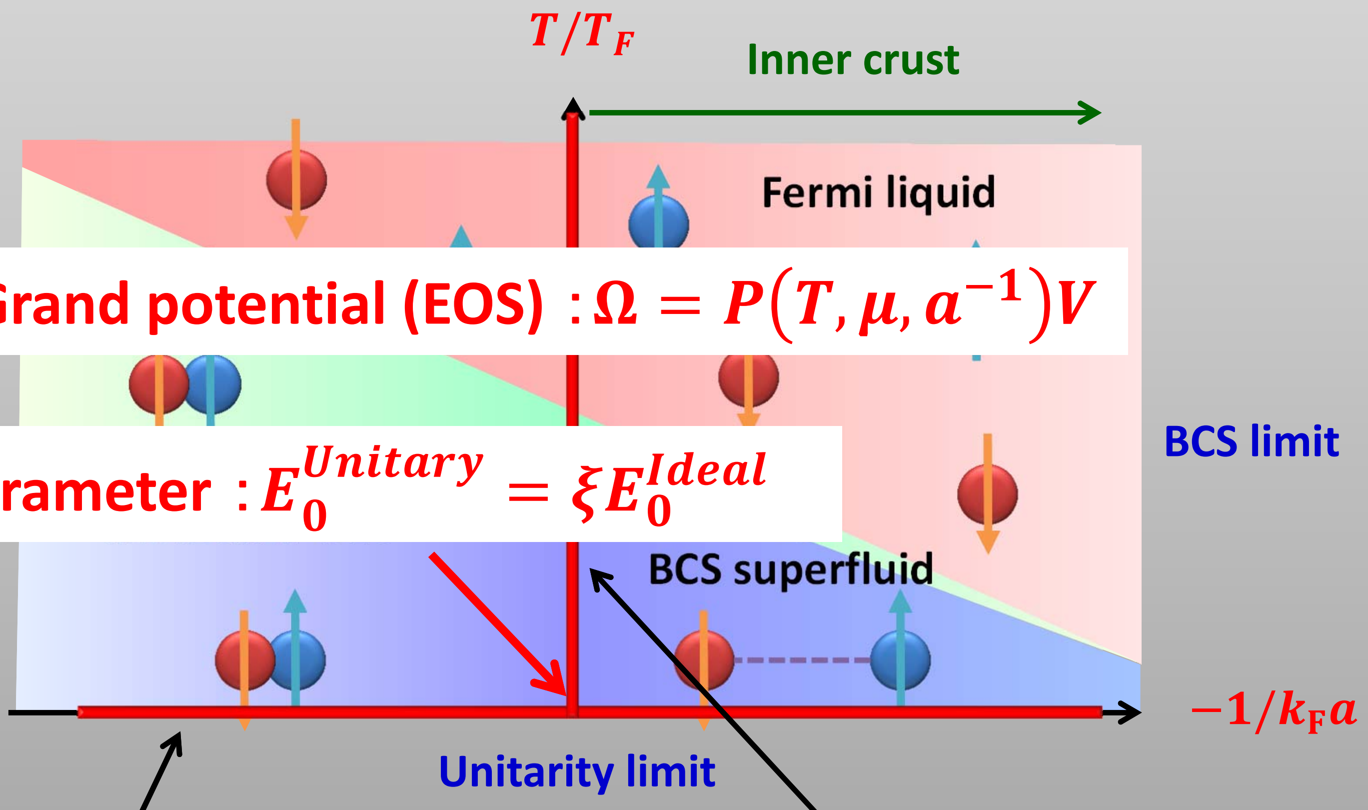
High-resolution imaging system

Evaporative cooling

Degenerate Fermi gas

BCS-BEC crossover in 2-conponent Fermi systems

Dilute limit : $k_F r_e = 0$



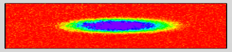
Grand potential (EOS) : $\Omega = P\left(T, \mu, a^{-1}\right)V$

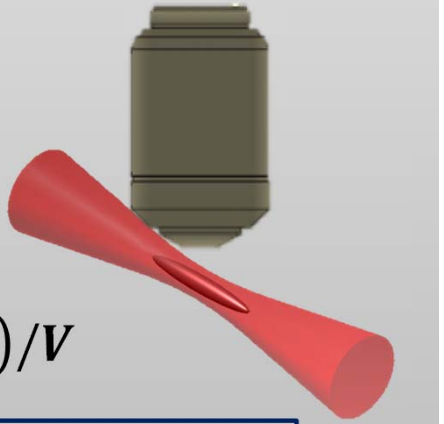
Bertsch parameter : $E_0^{Unitary} = \xi E_0^{Ideal}$

[M. Horikoshi, *et al.*, Science **327**, 442 (2010)]
[S. Nascimbène, *et al.*, Nature **463**, 1057 (2010)]
[M. Ku, *et al.*, Science **335**, *563* (2012)]

[N. Navon, *et al.*, Science **328**, 729 (2010)]

Our method to determine the EOS





EOS for dilute Fermi gases : $\boldsymbol{P\left(T,\mu,a^{-1}\right)}$

↓

But, cold atoms are trapped in a harmonic trap

The gas has **inhomogeneous density distribution**

Position dependent EOS : $\boldsymbol{P(r)=\Omega\left(T,\mu(r),a(B)^{-1}\right)/V}$

- **Local pressure** $\boldsymbol{P}(\boldsymbol{r})$: density distribution is pressure distribution

  ⇒ **Resolution of imaging determine precision of the EOS**

  [Tin-Lun Ho. Nature Physics 6, 131 (2010)]

- **Temperature** $\boldsymbol{T}$: mixing 7Li for thermometry into 6Li

  [S. Nascimbène, *et al.*, Nature 463, 1057 (2010)]

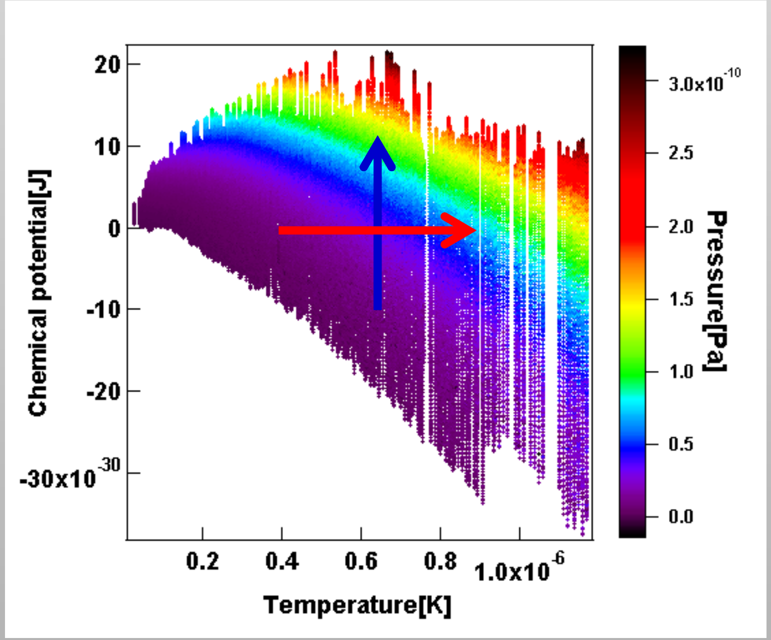
- **Local chemical potential** $\boldsymbol{\mu}(\boldsymbol{r})$: LDA ⇒$\mu(\boldsymbol{r})=\mu(0)-U_{\mathrm{trap}}(\boldsymbol{r})$

  Thermodynamic relation for harmonically trapped system :

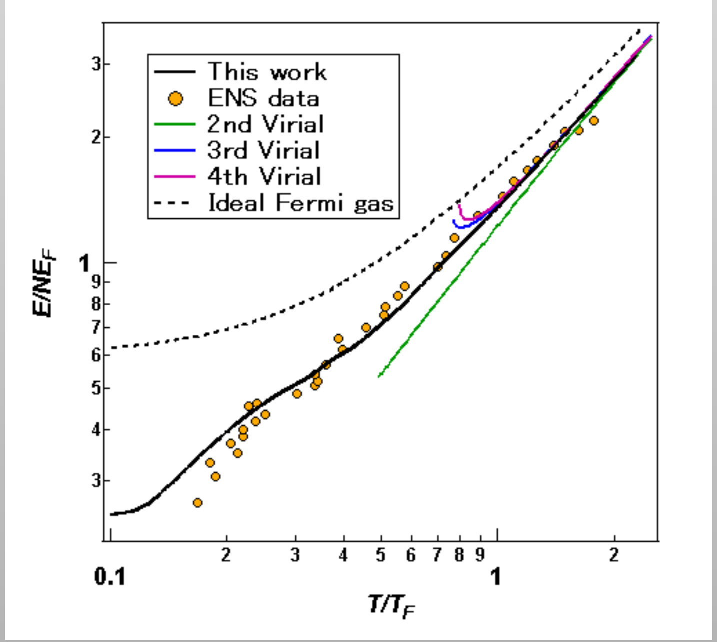
  $$E_{\mathrm{rel}}=TS+\mu(0)N-\frac{5}{3}E_{\mathrm{pot}}$$

EOS of unitary Fermi gases

EOS via ENS's route : $P(\mu, T, a^{-1}=0)$



Internal energy



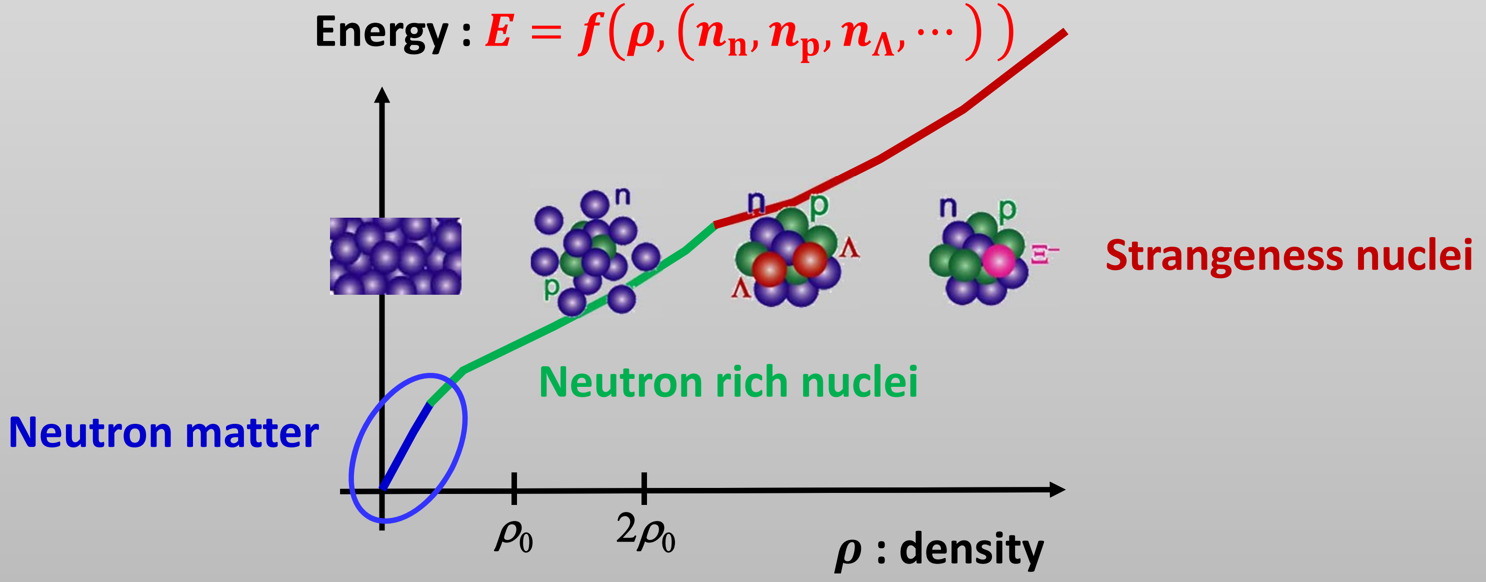
Particle density : $n = (dP/d\mu)_{T,a}$

Entropy density : $s = (dP/dT)_{\mu,a}$

Internal energy density : $\varepsilon = Ts + \mu n - P$

Analyzed data correspond to
[M. Horikoshi, *et al.*,
Science **327**, 442 (2010)]

Determination of EOS for neutron matter



Cold Fermi atoms :

$$\frac{E}{E_{\mathrm{F}}}\left(\frac{T}{T_{\mathrm{F}}}, -\frac{1}{k_{\mathrm{F}}a}\right)$$

Correction

$$\xrightarrow{\qquad\qquad}$$

Neutron matter :

$$\frac{E}{E_{\mathrm{F}}}\left(\frac{T}{T_{\mathrm{F}}}, -\frac{1}{k_{\mathrm{F}}a}, k_{\mathrm{F}}r_{\mathrm{e}}\right) \longrightarrow$$

Neutron stars :

$$\boldsymbol{E = f(\rho, (n_{\mathrm{n}}))}$$

$$\frac{\partial \boldsymbol{E}}{\partial(\boldsymbol{k_F r_e})} = \zeta \quad (\zeta = 0.127(4) \ @ \ |\infty| = 0, T = 0)$$

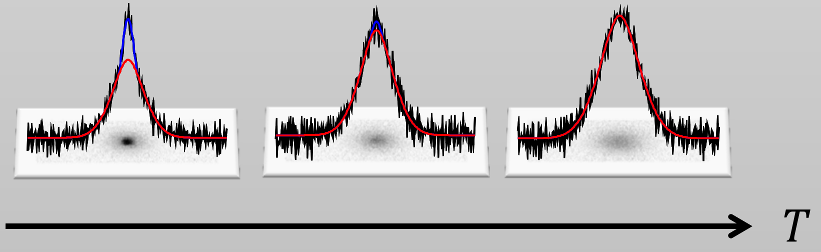
[Forbes, *et al.*, arXiv:1205.4815]

[Werner and Castin, Phys. Rev. A 86, 013626 (2012)]

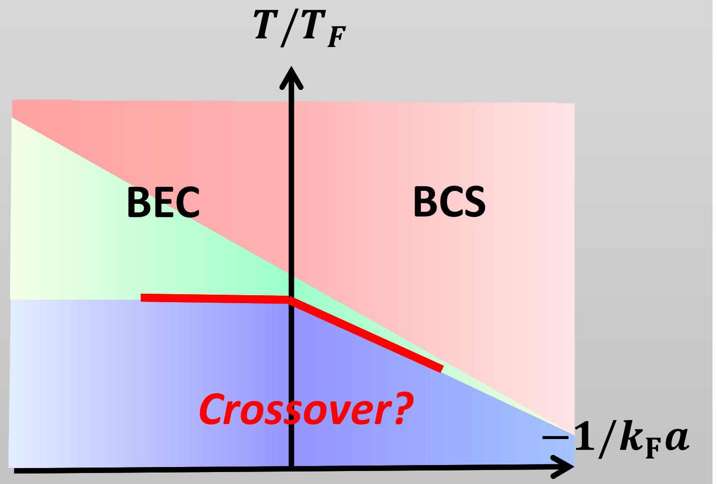
Superfluid phase transition

Emergence of cooper-pair with zero center-of-mass momentum

[M.Greiner *et al.*, *Nature* **426**, 537]



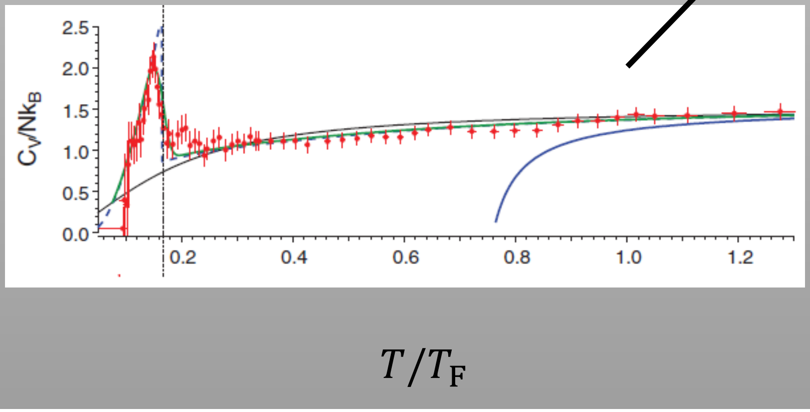
7Li thermometry



Lambda transition of heart capacity

Imaging resolution is 1.4μm

[Mark J. H. Ku, *et al.*, **335**, 563 (2012)]



Challenging issue

Universality class over the crossover

Critical exponent for correlation length :

$\xi \propto \left|\frac{T-T_\mathrm{C}}{T_\mathrm{C}}\right|^{-\nu} (\nu = 0.67 \text{ for 3D-XY model})$

Critical exponent for heat capacity :

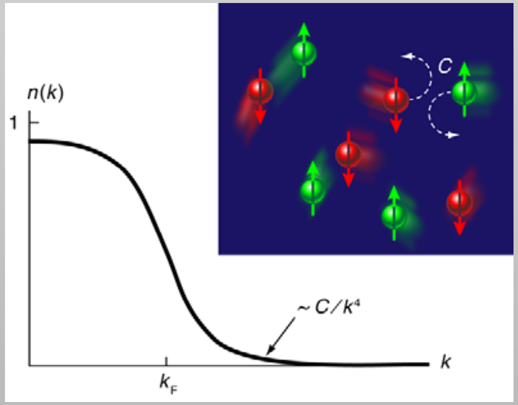
$C_\mathrm{V} \propto \left|\frac{T-T_\mathrm{C}}{T_\mathrm{C}}\right|^{-\alpha} (\alpha = -0.013 \text{ for 3D-XY model})$

$\alpha = 2 - 3\nu$

Universal many-body function, Tan's contact

In 2005, Shina Tan suggested that, the tail of momentum distribution for the universal 2-component Fermi system become

[Shina Tan, cond-mat/0508320]



Physics **3**, 48 (2010)

Tan's Contact

$$n_{\uparrow\text{or}\downarrow}\left(k_\mathrm{F}, |a|^{-1}, \Lambda_\mathrm{T}^{-1} < k < R_\mathrm{vdw}^{-1}\right) = \frac{C(x,\theta)}{k^4}$$

Fourier transform

$$C \equiv 4\pi k_\mathrm{F} N h(x,\theta)$$

The origin is two-particle density matrix at short range :

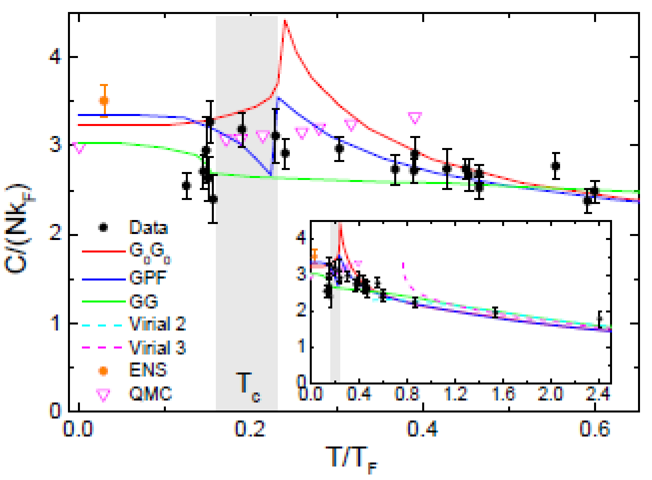
$$\left|\phi_\mathrm{pair}(r)\right|^2 = \langle \psi_\uparrow^\dagger(\boldsymbol{r}_1)\psi_\downarrow^\dagger(\boldsymbol{r}_2)\psi_\downarrow(\boldsymbol{r}_2)\psi_\uparrow(\boldsymbol{r}_1)\rangle$$

$$\xrightarrow{a, k_\mathrm{F}^{-1} \gg r \equiv |r_1 - r_2| \gtrsim R_\mathrm{vdw}} 4\pi k_\mathrm{F} N \cdot h(x,\theta) \cdot \left|\frac{\phi(r)}{4\pi}\right|^2 , \left(\begin{array}{c} \boldsymbol{x \equiv -\frac{1}{k_\mathrm{F}a}, \theta \equiv \frac{T}{T_\mathrm{F}}} \\ \phi(r) = \frac{1}{r} - \frac{1}{a}\end{array}\right)$$

Universal many-body function

Universal many-body function, Tan's contact

Recent measurement from JILA at unitarity limit ($|a| = \infty$)

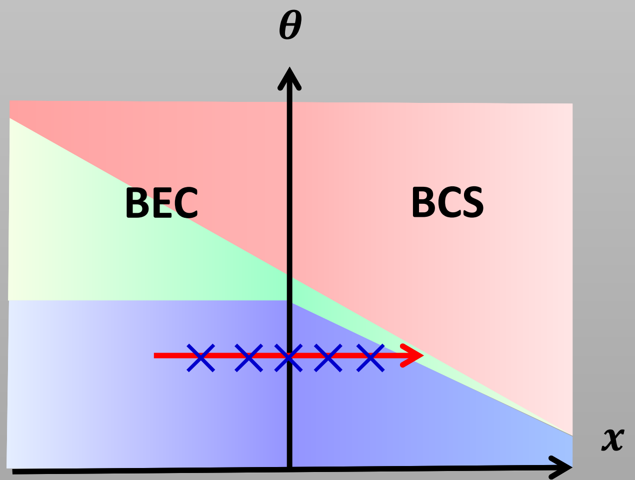


[Yoav Sagi, et al., arXiv:1208.2067]

Peak at the critical temperature?
Contact has some critical exponent?

Some Tan's relation

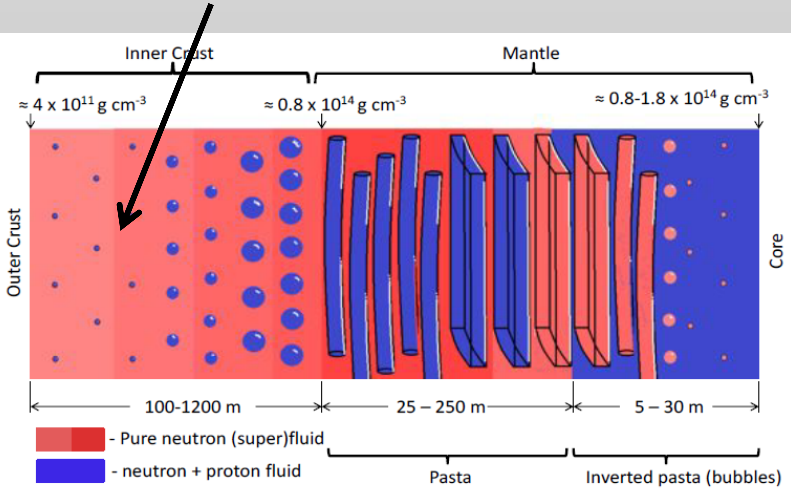
- Adiabatic relation : $\left.\frac{\partial E}{\partial x}\right|_{\theta} = 2\varepsilon_F N h(x,\theta) > 0$
- ***P-E*** **relation** : $PV = \frac{2}{3}\left(E - N\varepsilon_F x h(x,\theta)\right)$



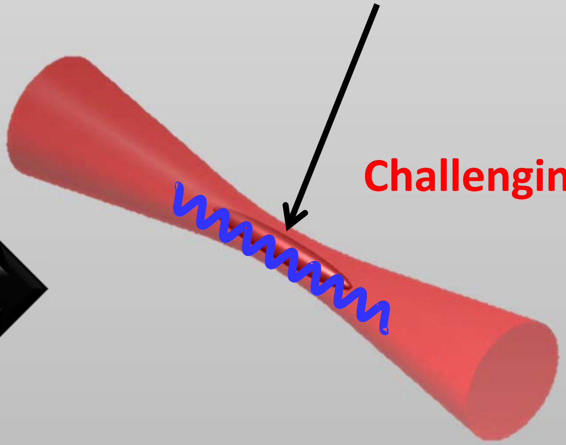
Verification of experimental result

Periodic potential

Dripped neutrons are moving in periodic potential by nuclei crystal



Tunable periodic potential by standing wave of laser beams



Challenging issue

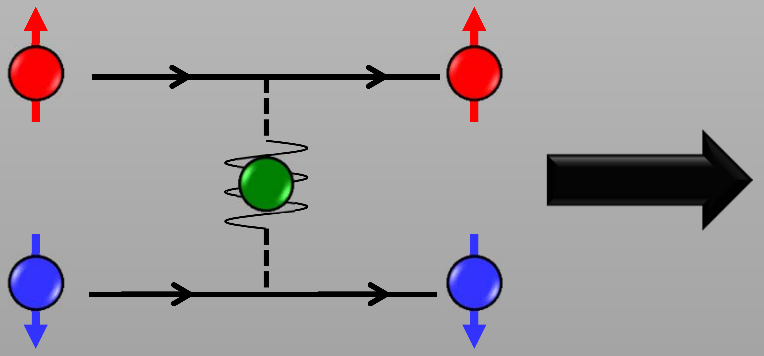
[G. Watanabe, *et al*., Phys. Rev. Lett. **107**, 270404 (2011)]

	Cold atom	Neutron matter
Fermion	6Li, 40K	neutrons
Interparticle distance : $\sim k_F^{-1}$	100nm	6~3fm
Periodic potential ： d	500nm	15~50fm ?
Ratio : $k_F d$	5	5~10
Lattice potential	Optical lattice	Nuclei crystal

Induced interaction : effects of Protons

	Cold atom	Neutron rich matter
Mixture	6Li↑-6Li↓-7Li 6Li↑-6Li↓-6Li→	n↑-n↓-p↑
Type	Femi-Bose Fermi-Fermi	Fermi-Fermi
Scattering length : a_{nn}	$\|\infty\|$ (unitarity limit)	-18.5 ± 0.3fm
Scattering length : a_{np}	2nm (Femi-Bose) -174nm(↑-→), -851nm(↓-→)	5.423fm (triplet) -23.749fm (singlet)
Interparticle distance : $\sim k_F^{-1}$	100nm	6~3fm

Induced interactions :



Challenging issue

Change of T_C
Change of EOS

[H. Heiselberg, et al., Phys. Rev. Lett. **85**, 2418 (2000)]

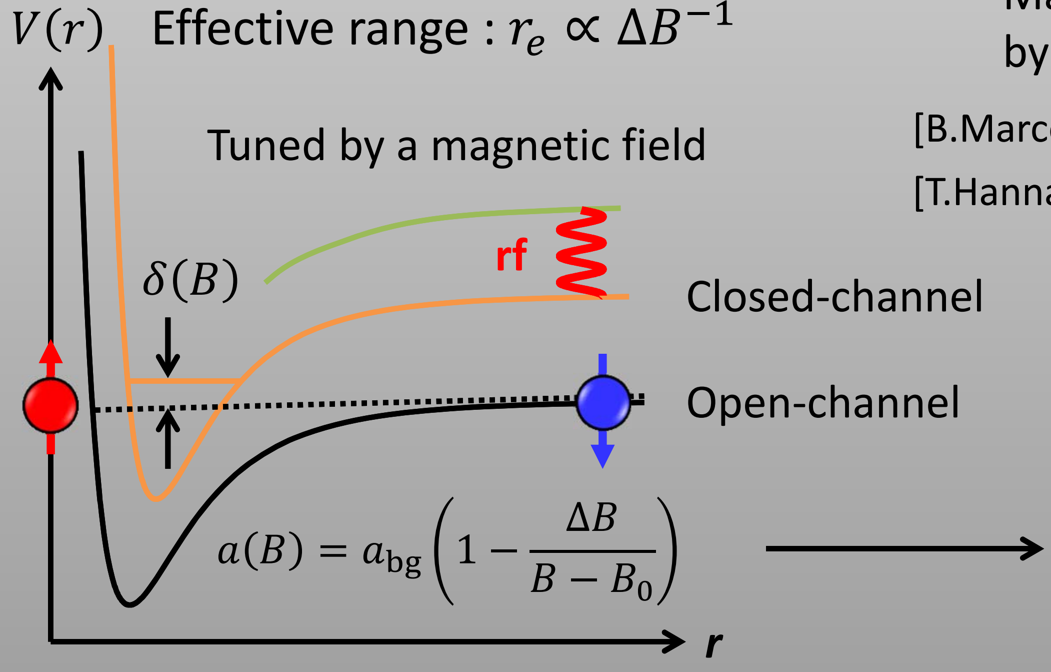
Tuning of effective range

	Cold atom	Neutron matter
Fermion	**6Li**	**neutrons**
Effective range : $k_F r_e$	0.05 →	$0.53 \sim 3.3$

Feshbach resonance

Effective range : $r_e \propto \Delta B^{-1}$

Tuned by a magnetic field



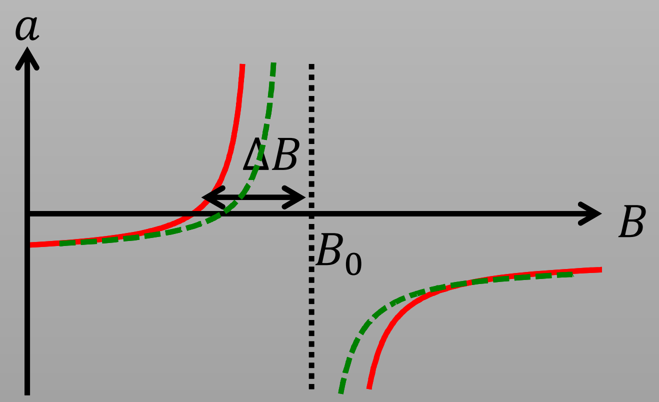
$$a(B) = a_{\mathrm{bg}}\left(1 - \frac{\Delta B}{B - B_0}\right)$$

Challenging issue

Manipulation of Feshbach resonances
by rf field or dc-electric field

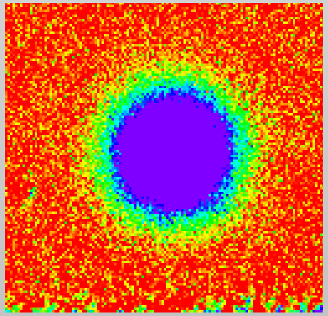
[B.Marcelis, et al., Phys. Rev. Lett. **100**, 153201 (2008)]

[T.Hanna, et al., *New J. Phys.* **12** 083031 (2010)]

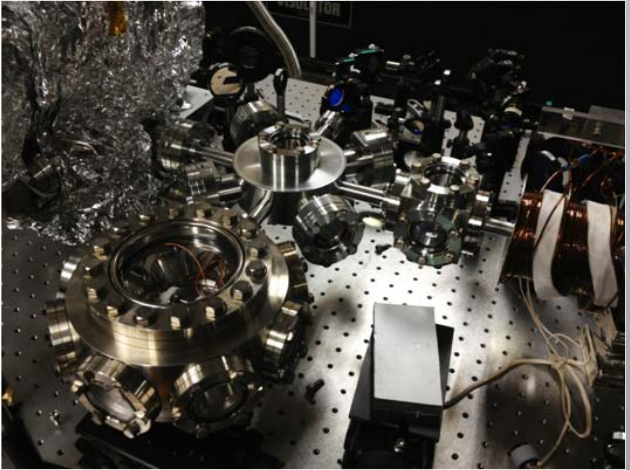


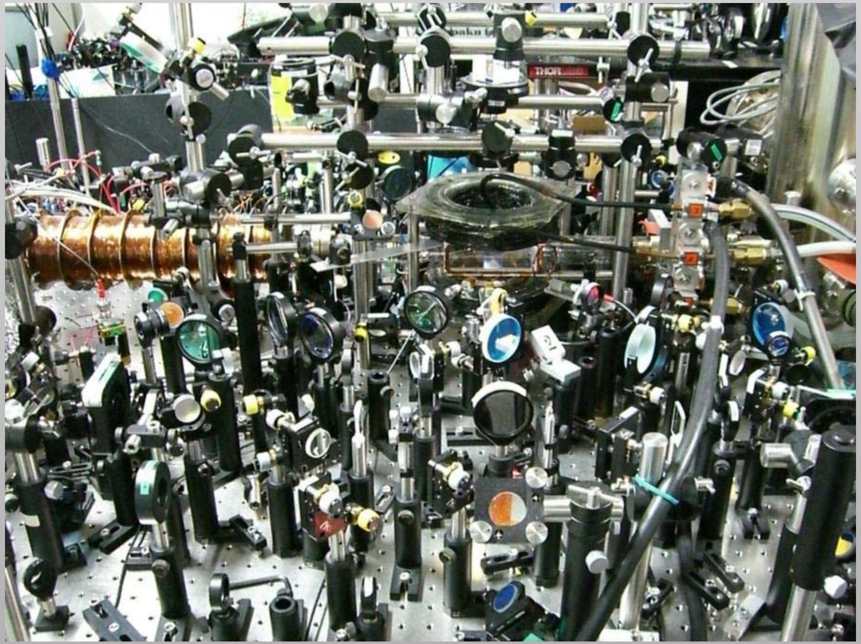
Cold Atom Lab at University of Electro-Communications

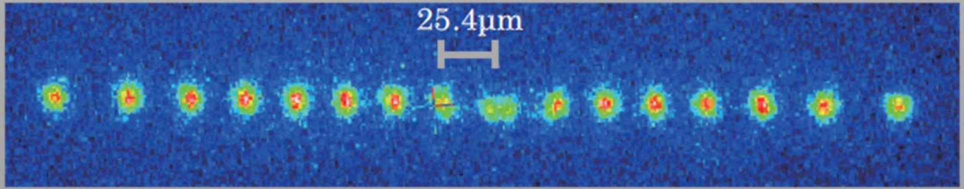
p-wave interaction, ^{6}Li-^{40}Ca^{+} (Fermi-Ion) mixture



Interacting 6Li by p-wave collisions





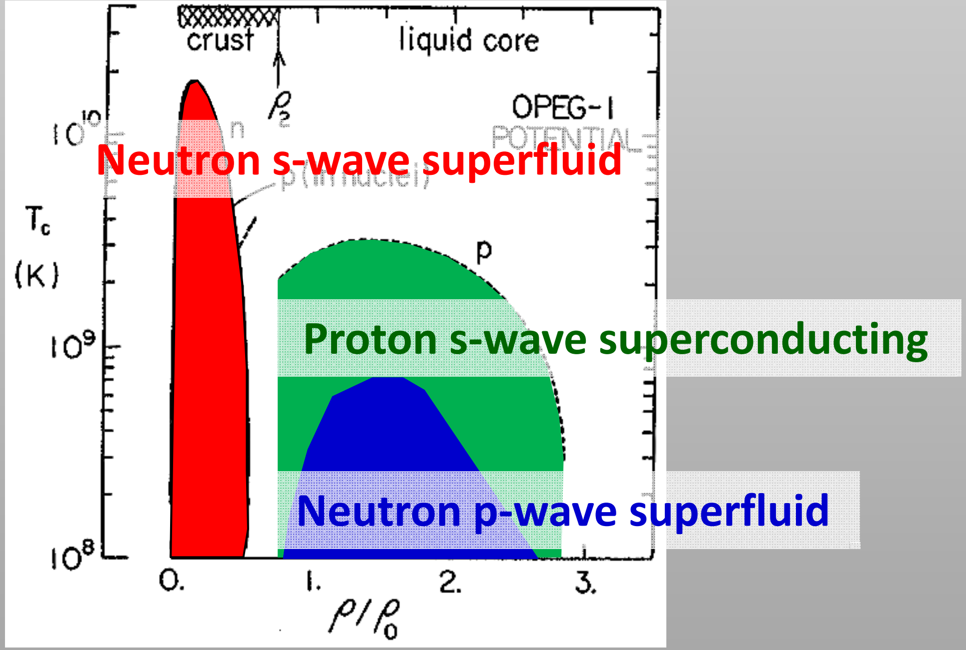


Crystalized cold ions

p-wave superfluidity in neutron stars

Missions : EOS of many-body Fermi system with p-wave interactions
Realization of p-wave superfluid

Superfluid phase diagram in neutron stars



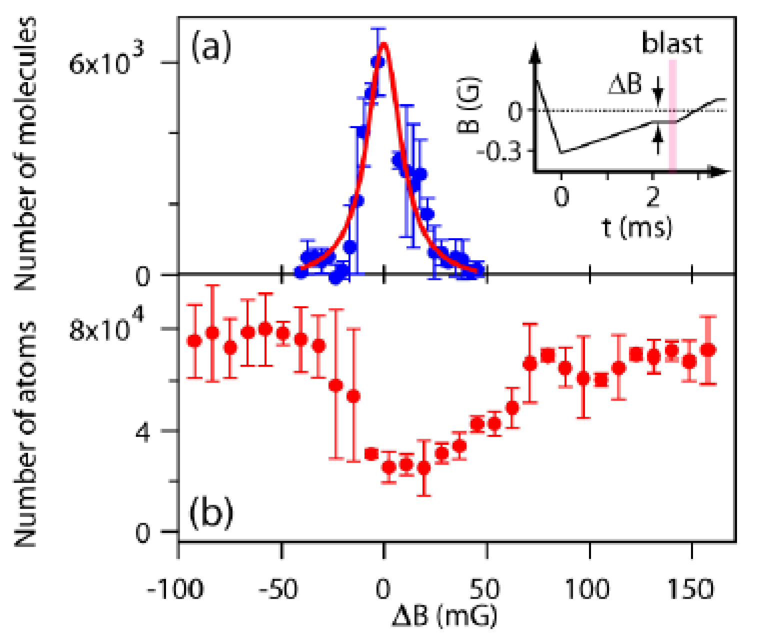
Neutron s-wave superfluid

Proton s-wave superconducting

Neutron p-wave superfluid

[T. Takatsuka and R. Tamagaki, Progress of Theoretical Physics Supplement 112, 27 (1993)]

Cold 6Li atom with p-wave Feshbach resonance



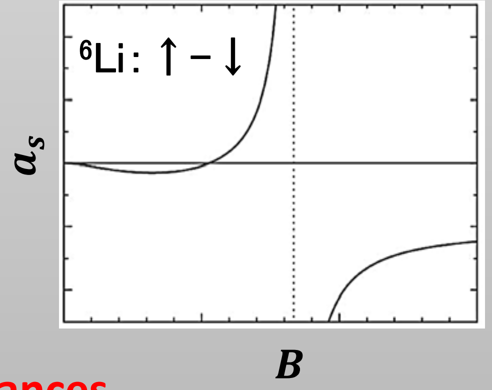
[Y.Inada, *et al.*, Phys. Rev. Lett. **101**, 100401 (2008)]

What is p-wave scattering parameters ?

By knowing parameters of s-wave Feshbach resonances, studies of many-body physics using s-wave interactions have progressed dramatically

s−wave interaction :

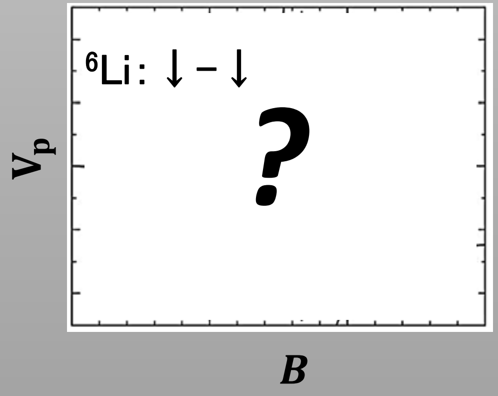
$$f_s(k,B)=\frac{1}{-\frac{1}{a_s(B)}+\frac{1}{2}r_e k^2-ik}\qquad a_s(B)=a_{bg}\left(1-\frac{\Delta B}{B-B_{res}}\right)$$



Today, no one knows parameters of p-wave Feshbach resonances

p−wave interaction :

$$f_p(k,B)=\frac{1}{-\frac{1}{V_p(B)}+\frac{1}{2r_e}k^2-ik^3}\qquad V_p(B)=V_{bg}\left(1-\frac{\Delta B}{B-B_{res}}\right)$$

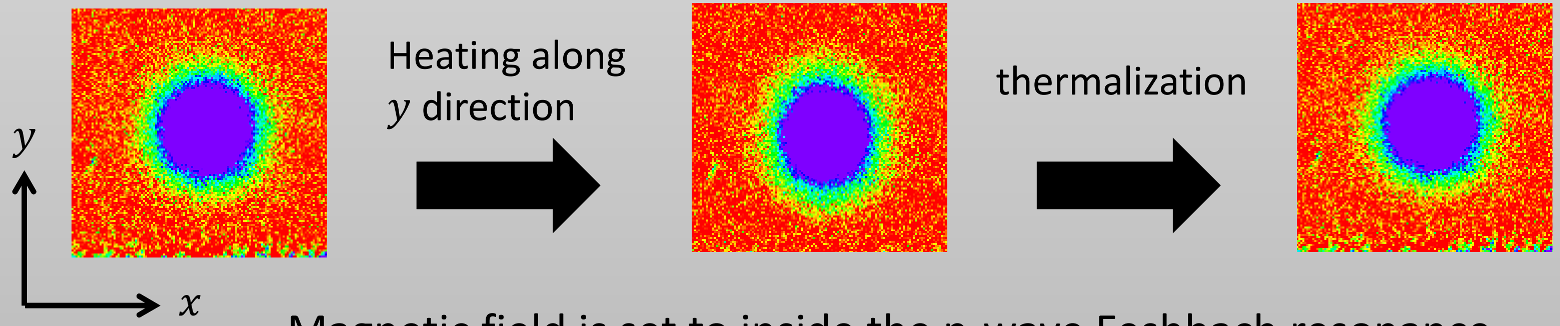


First of all, we must determine them

Determination of p-wave scattering parameters

Method : measurement of thermalization rate

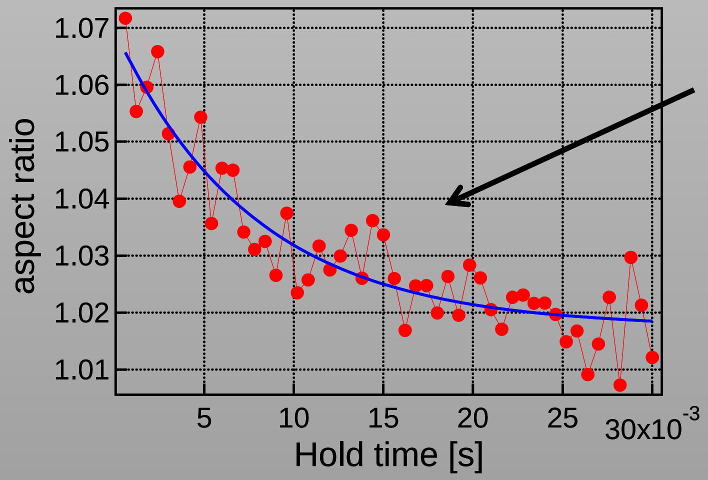
Momentum(temperature) space



Heating along y direction

thermalization

Magnetic field is set to inside the p-wave Feshbach resonance



Exponential decay with a thermalization rate of τ

Collisional cross section averaged over the trap : $\overline{\sigma}(T,B) = \dfrac{2.7\tau}{\bar{n}\bar{v}}$

$$\downarrow$$

Elastic collision cross section : $\sigma = 4\pi\left|f_p(k)\right|$

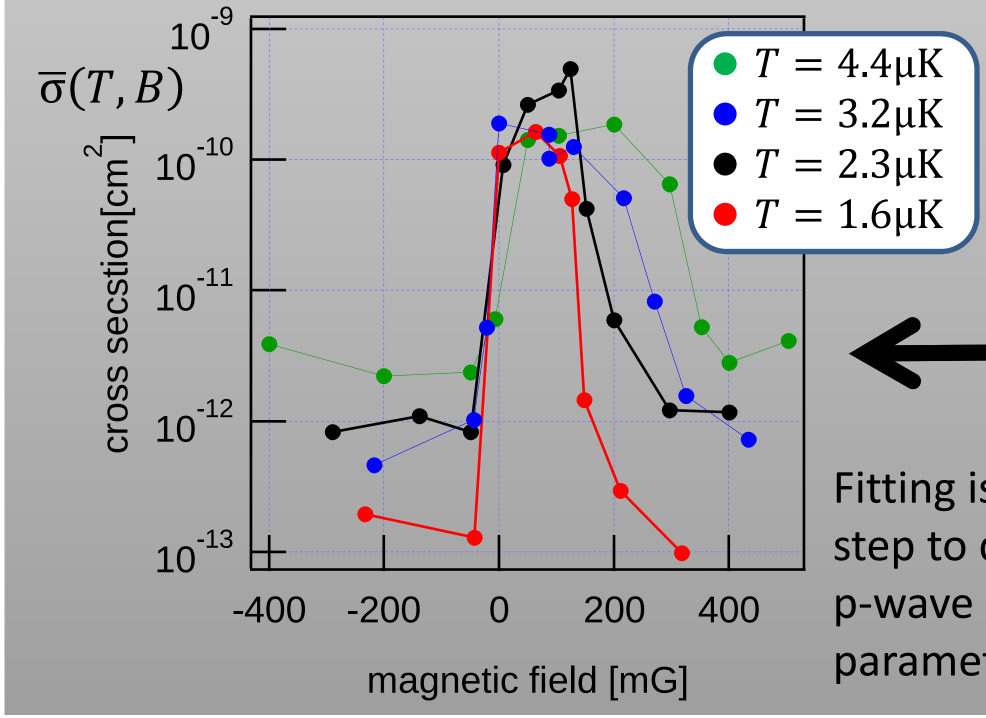
Preliminary data

Elastic collision cross section : $\sigma = 4\pi\left|f_p\left(k, V_p(B).r_e\right)\right|^2$

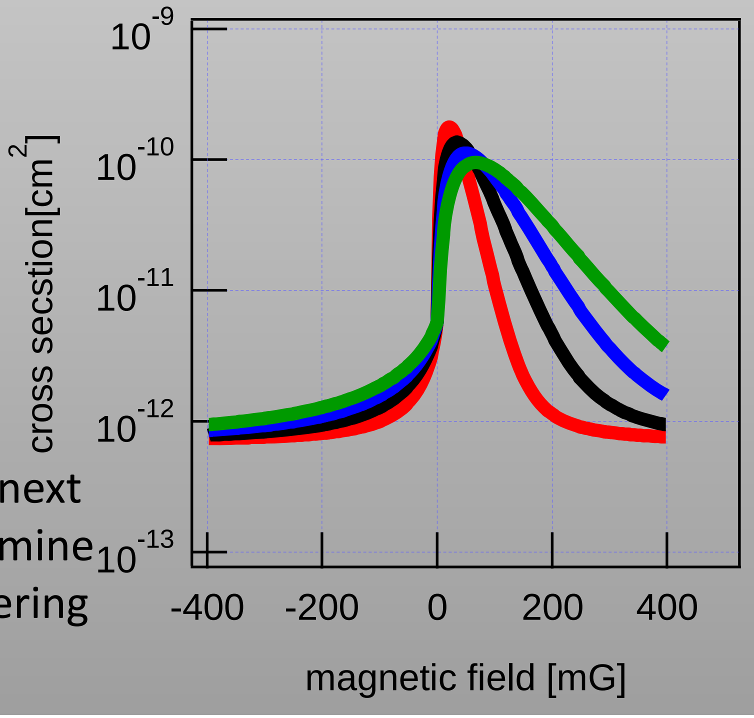
Thermal averaging under the assumption of Maxwell−Boltzman distribution

$$\left\langle\sigma\left(T, V_p(B).r_e\right)\right\rangle = (k_B T)^{-2}\int_0^\infty 4\pi\left|f_p\left(E, V_p(B).r_e\right)\right|^2 E e^{-E/k_B T}\, dE \longleftrightarrow \overline{\sigma}(T,B)$$

Experimental data

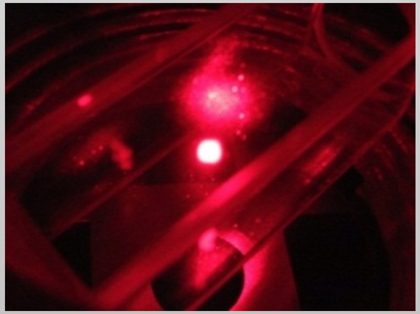


Fitting is the next step to determine p-wave scattering parameters



Summary

Cold atoms



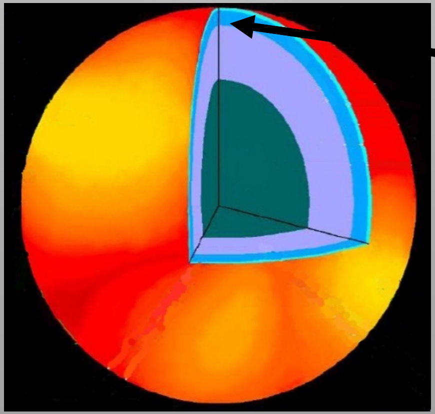
Measurements

- Universal equation of state
- Internal energy
- Specific heat
- Critical temperature
- s,p-wave superfluid
- Superfluid gap
- Superfluid density

⋮

- Benchmark for theories

Inner crust of neutron stars



EOS

Cooling curve

◆ Correction of the effective range
◆ Lattice of neutron-rich nuclei
◆ Protons

Theories