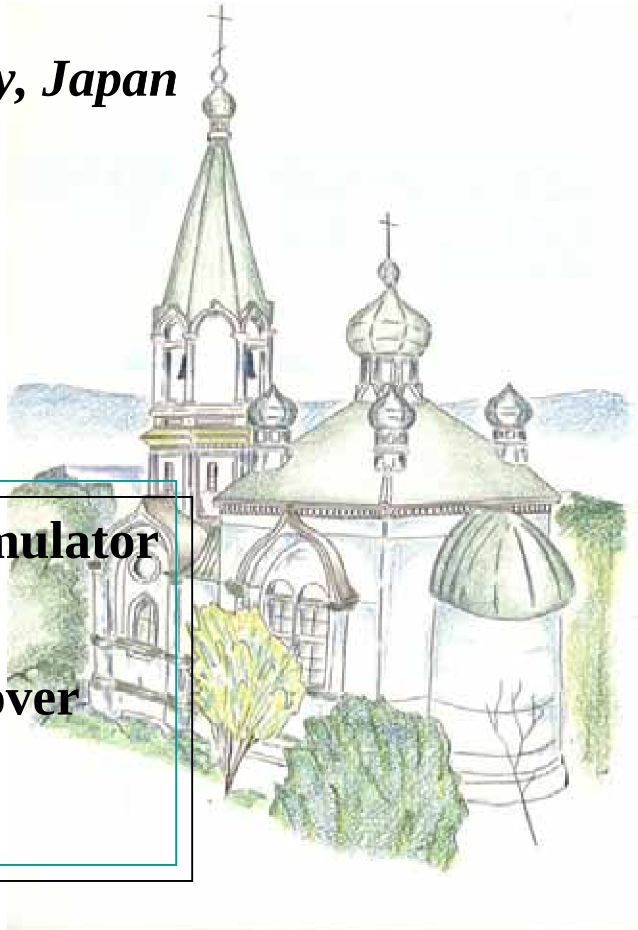
October 26-27 (2012) 新学術領域「中性子星核物質」 RIKEN, Wako, Japan

ユニタリー領域近傍におけるフェルミ原子ガスの強結合効果

Strong-coupling effects in the unitary regime of an ultracold Fermi gas

Yoji Ohashi

Department of Physics, Keio University, Japan



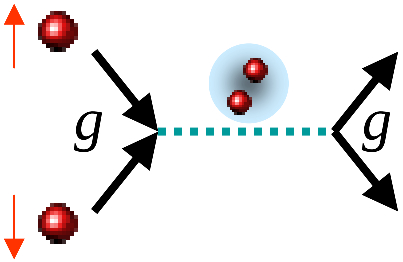
Collaborators:

S. Tsuchiya (SUT)
R. Watanabe (KEIO)
T. Kashimura (KEIO)
D. Inotani (KEIO)

- Introduction: cold Fermi atom gas as a quantum simulator
- current situation in cold Fermi gas physics
- Strong-coupling phenomena in the BCS-BEC crossover
- summary

Cold Fermi atom gas system as a useful Quantum Simulator

ultracold Fermi atom gas



Feshbach resonance

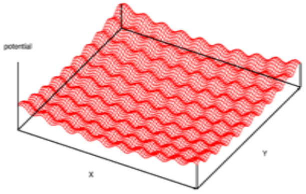
tunable interaction

BCS-BEC crossover

p-wave pairing

hetero pairing

resonance superfluid



optical lattice

$$\frac{(\mathbf{p}-\alpha\mathbf{A})^2}{2m}$$

synthetic gauge field

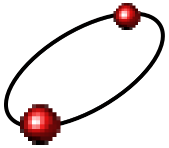
band effect

Hubbard model

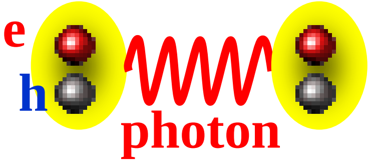
asymmetric LS coupling

non-centro-symmetric state

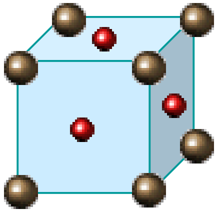
Fermi superfluid physics



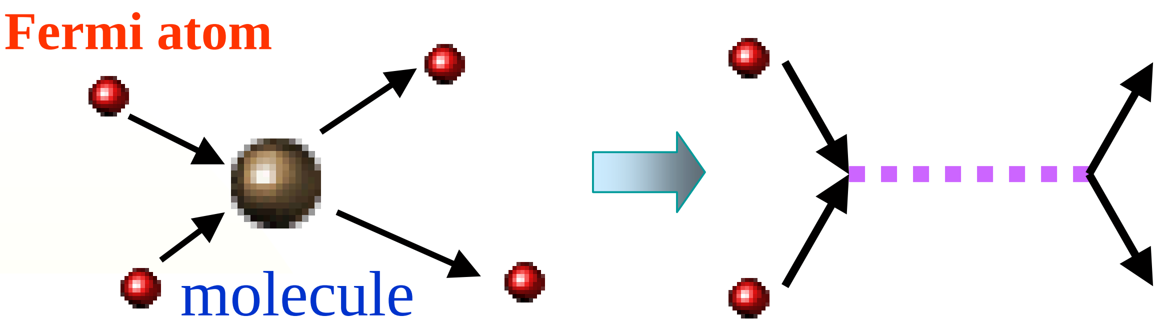
exciton-polariton condensate



material science



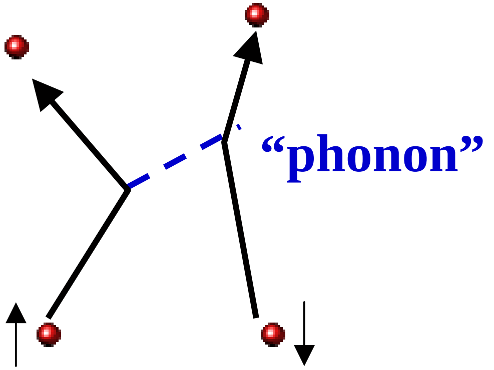
Feshbach resonance



$$V_{eff} = -g^2 \frac{1}{2\nu}$$

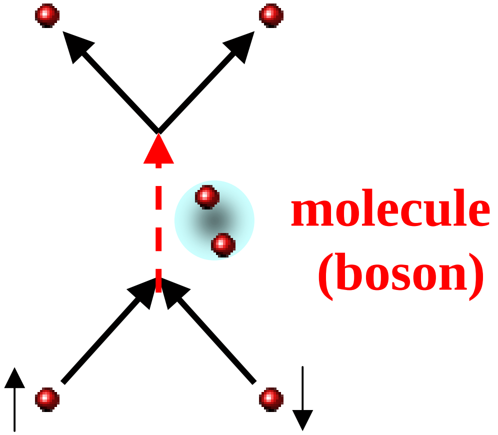
tunable **by magnetic field**

“phonon”-mediated pairing mechanism



- **superconductivity**
  Phonon, AF spin fluctuations
- **superfluid ^{3}He**
  Ferromagnetic spin fluctuations

(Feshbach) resonance pairing machanism

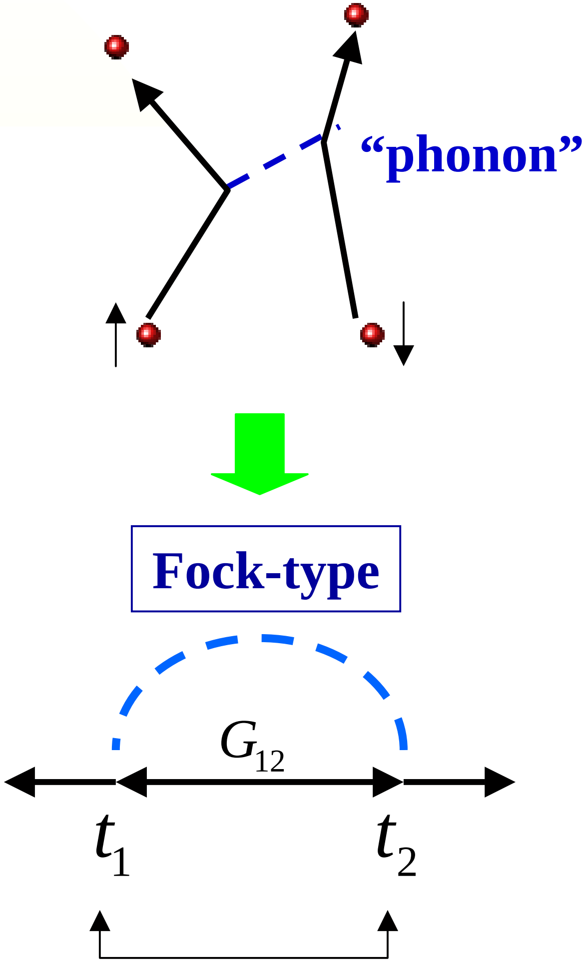


- **superfluid Fermi gas ^{40}K, ^{6}Li**
- **Exciton-polariton**

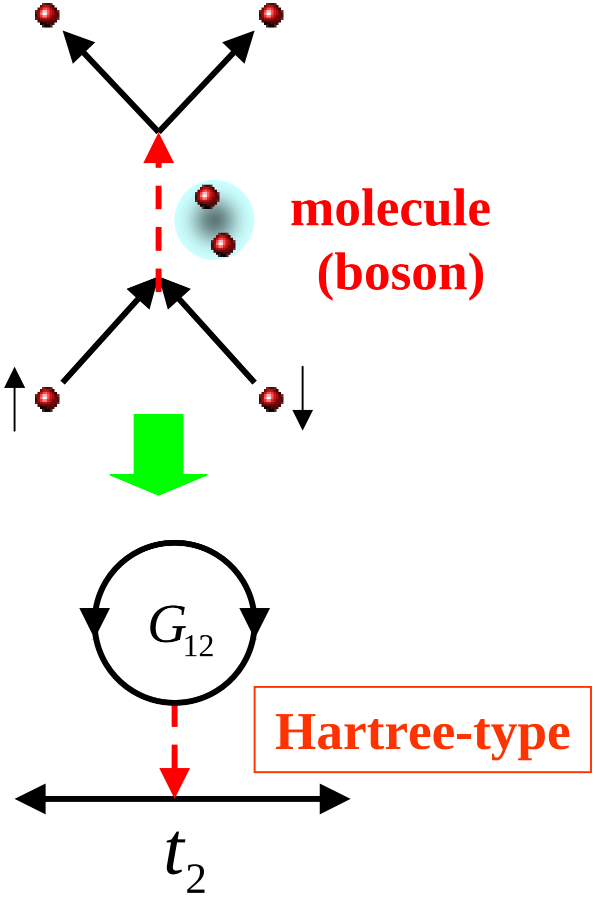
“resonance superfluid”

Feshbach resonance: absence of retardation effect

“phonon”-mediated pairing mechanism



(Feshbach) resonance pairing machanism

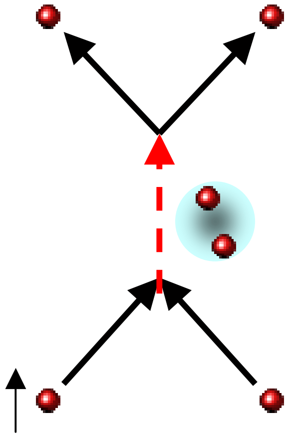


“off-diagonal” self-energy

Σ_{12}

retardation effect: Low-energy thermal Bose excitations cause the pair-breaking effect, which suppresses Tc.

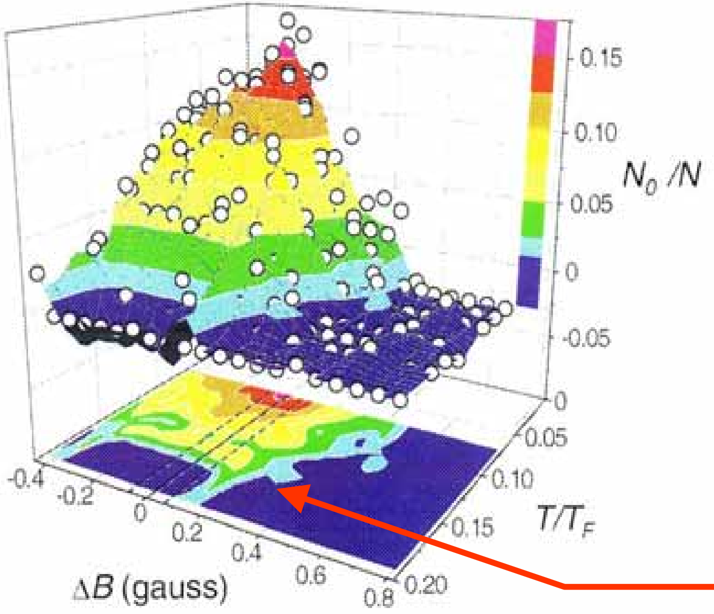
Upper limiut of Tc (?) in the Feshbach resonance mechanism



molecule (boson)

$$V_{eff} = -g^2 \frac{1}{2\nu}$$

^{40}**K Fermi gas** (A similar result has been also obtained in a ^{6}Li gas.)



$$|9/2,-7/2> + |9/2,-9/2>$$

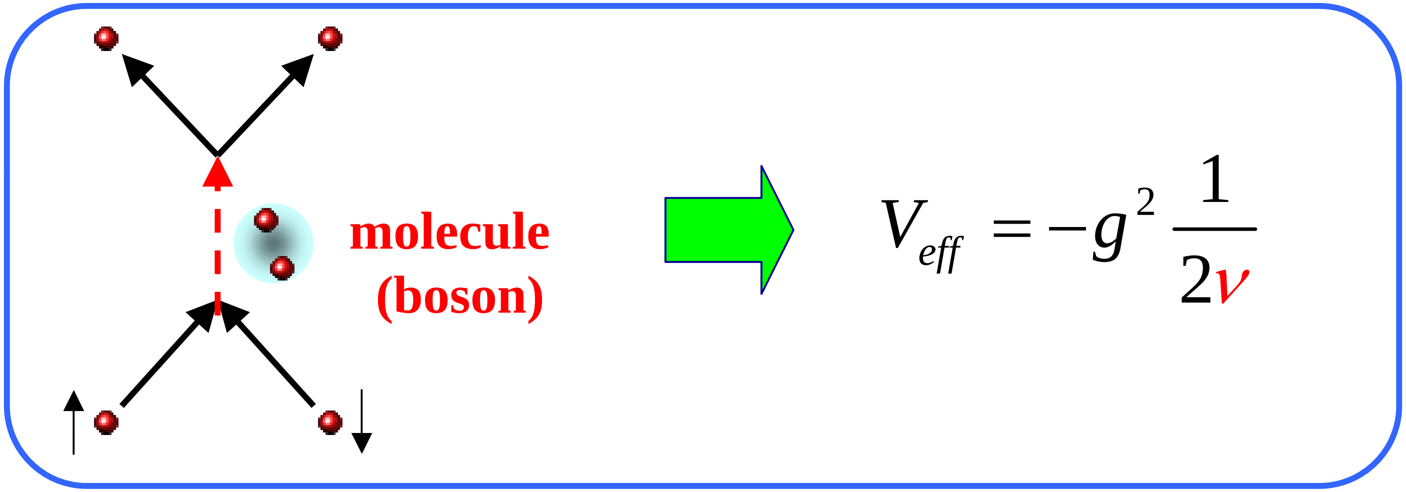
$$T_F = 0.35\mu K$$

$$N = 10^5$$

C. A. Regal, et al. PRL 92 (2004) 040403.

$$T_c^{MAX} / T_F \sim 0.2 >> 10^{-4} - 10^{-2} (metal)$$

Upper limiut of Tc (?) in the Feshbach resonance mechanism

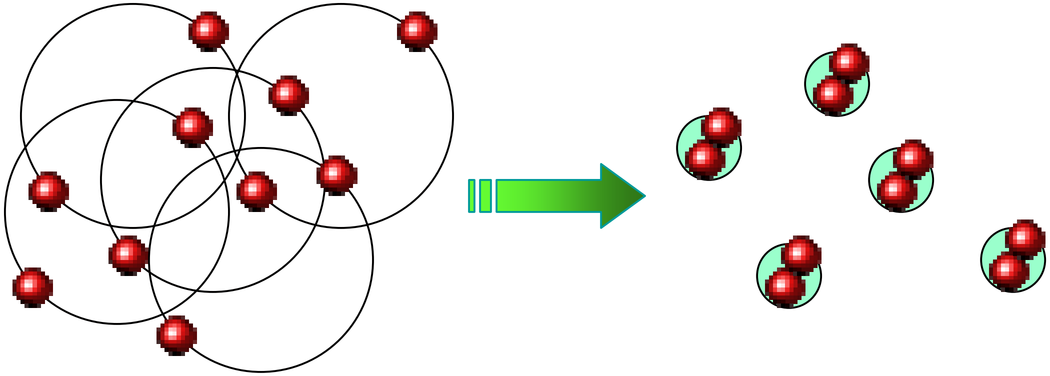


molecule

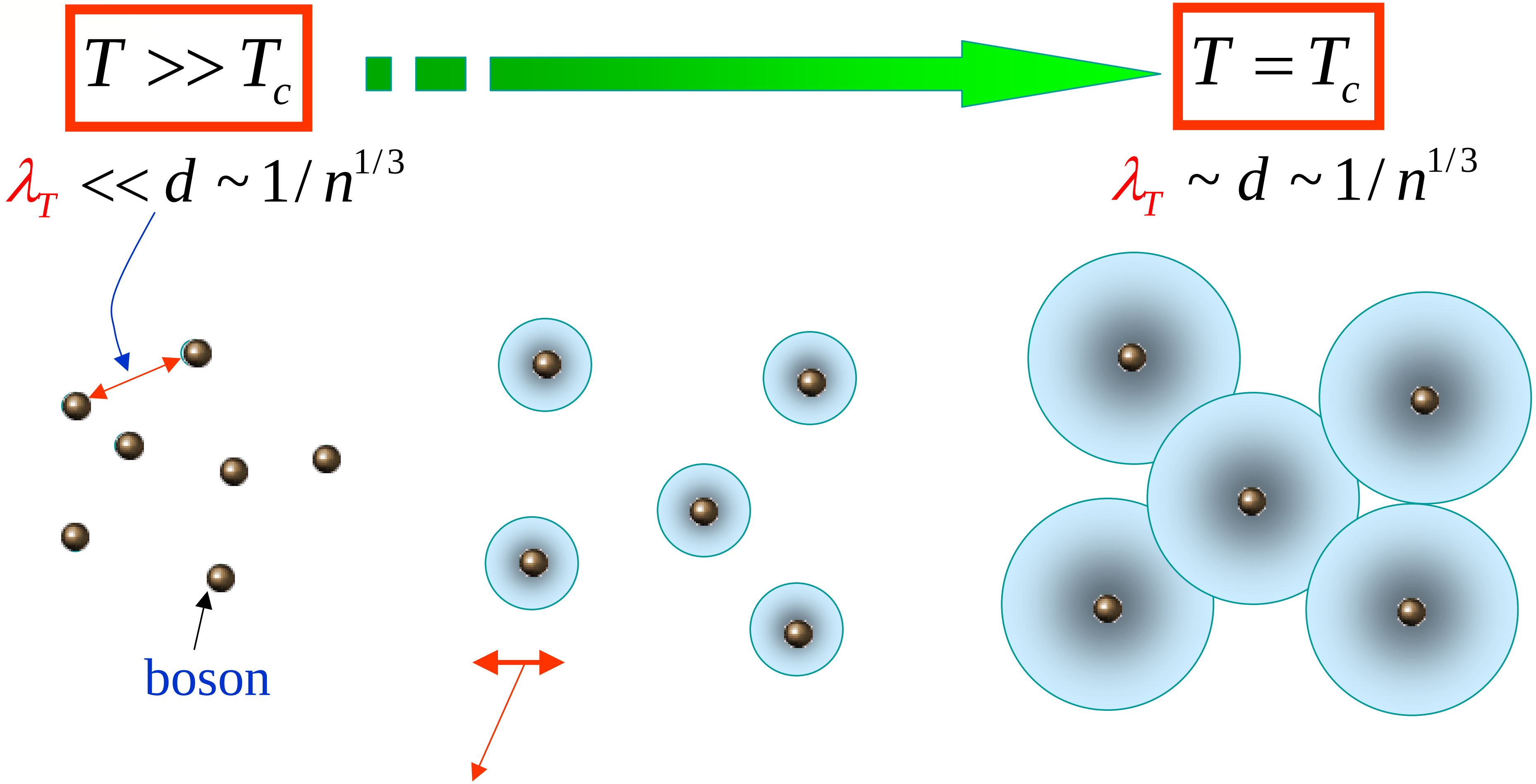
(boson)

$$V_{eff} = -g^2 \frac{1}{2\nu}$$

BCS-BEC crossover adjusted by Feshbach resonance



Bose-Einstein Condensation (BEC)



$T \gg T_c$

$T = T_c$

$\lambda_T \ll d \sim 1/n^{1/3}$

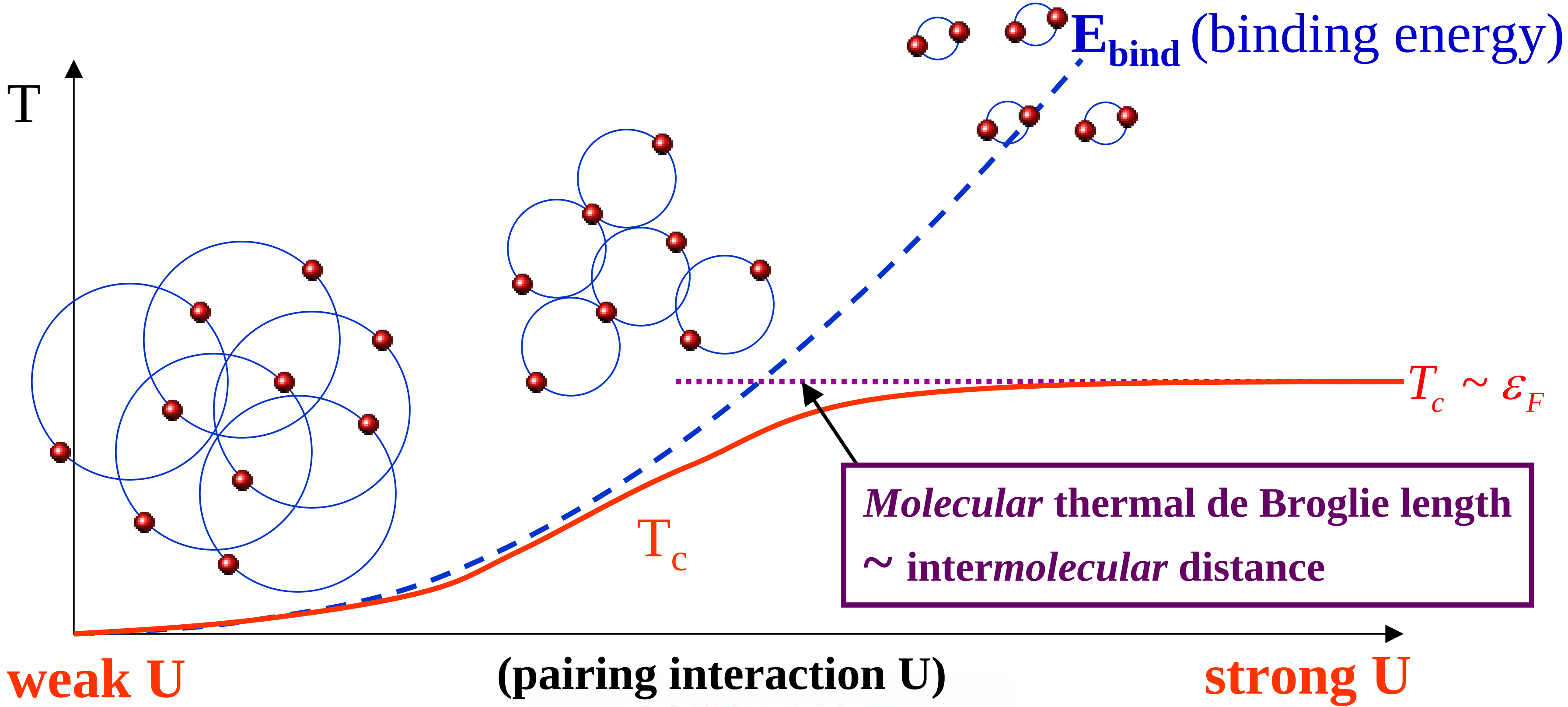
$\lambda_T \sim d \sim 1/n^{1/3}$

boson

thermal de Broglie length

$$\lambda_T = h/\sqrt{2\pi m T}$$

Essence of the BCS-BEC crossover phenomenon



E_{bind} (binding energy)

T

$T_c \sim \varepsilon_F$

T_c

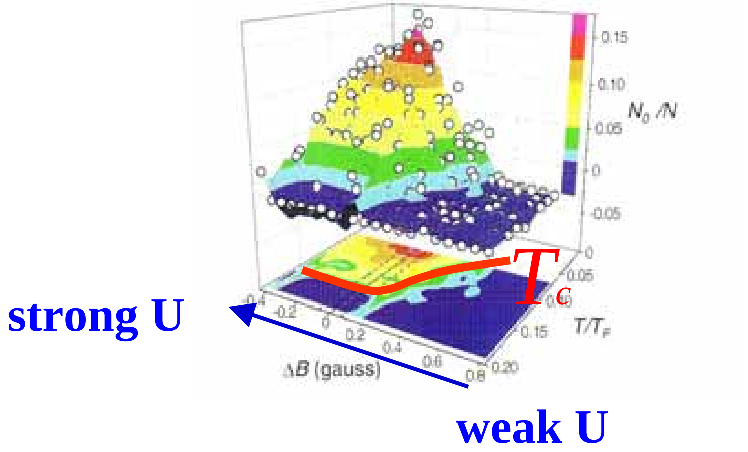
Molecular **thermal de Broglie length**

~ inter*molecular* distance

weak U

(pairing interaction U)

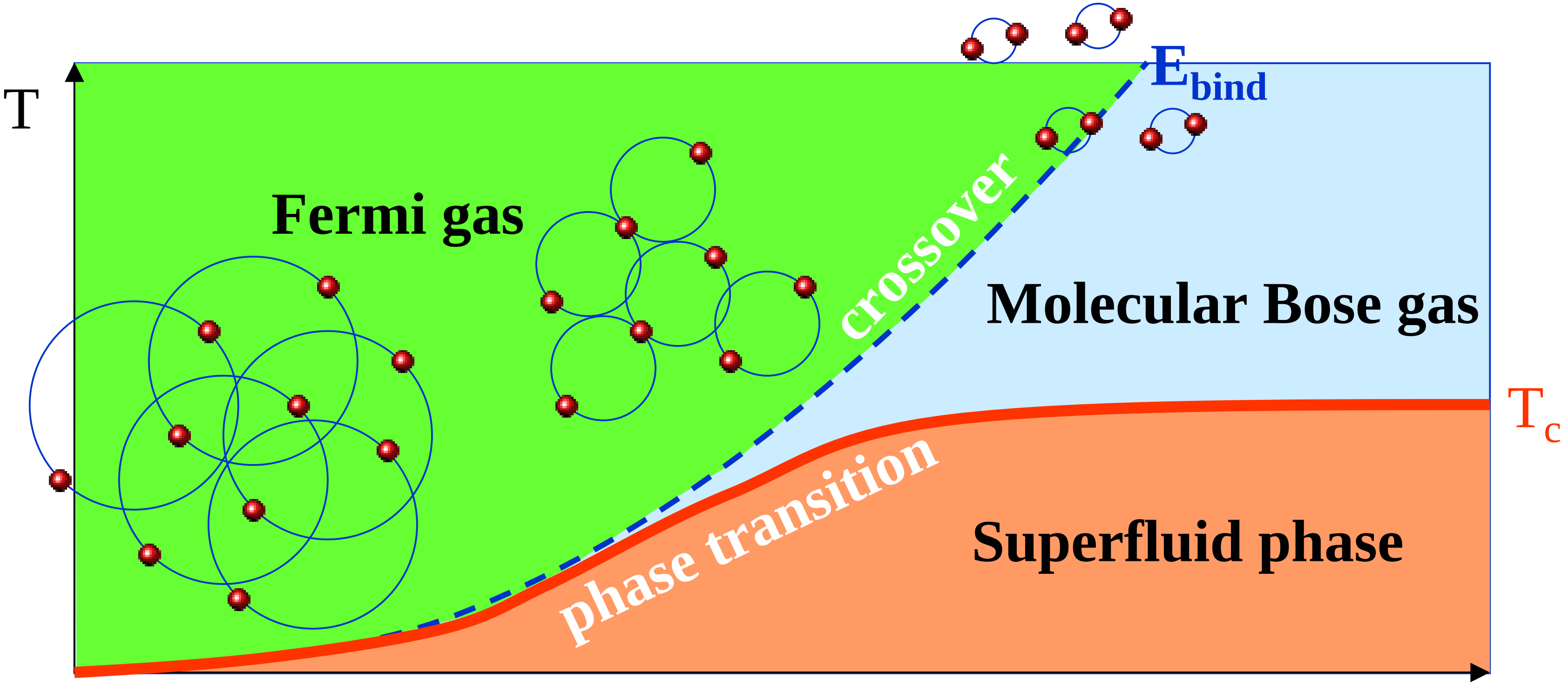
strong U



strong U

weak U

Phase diagram of Fermi superfluids



weak U

crossover region

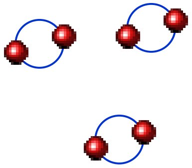
strong U

BCS regime

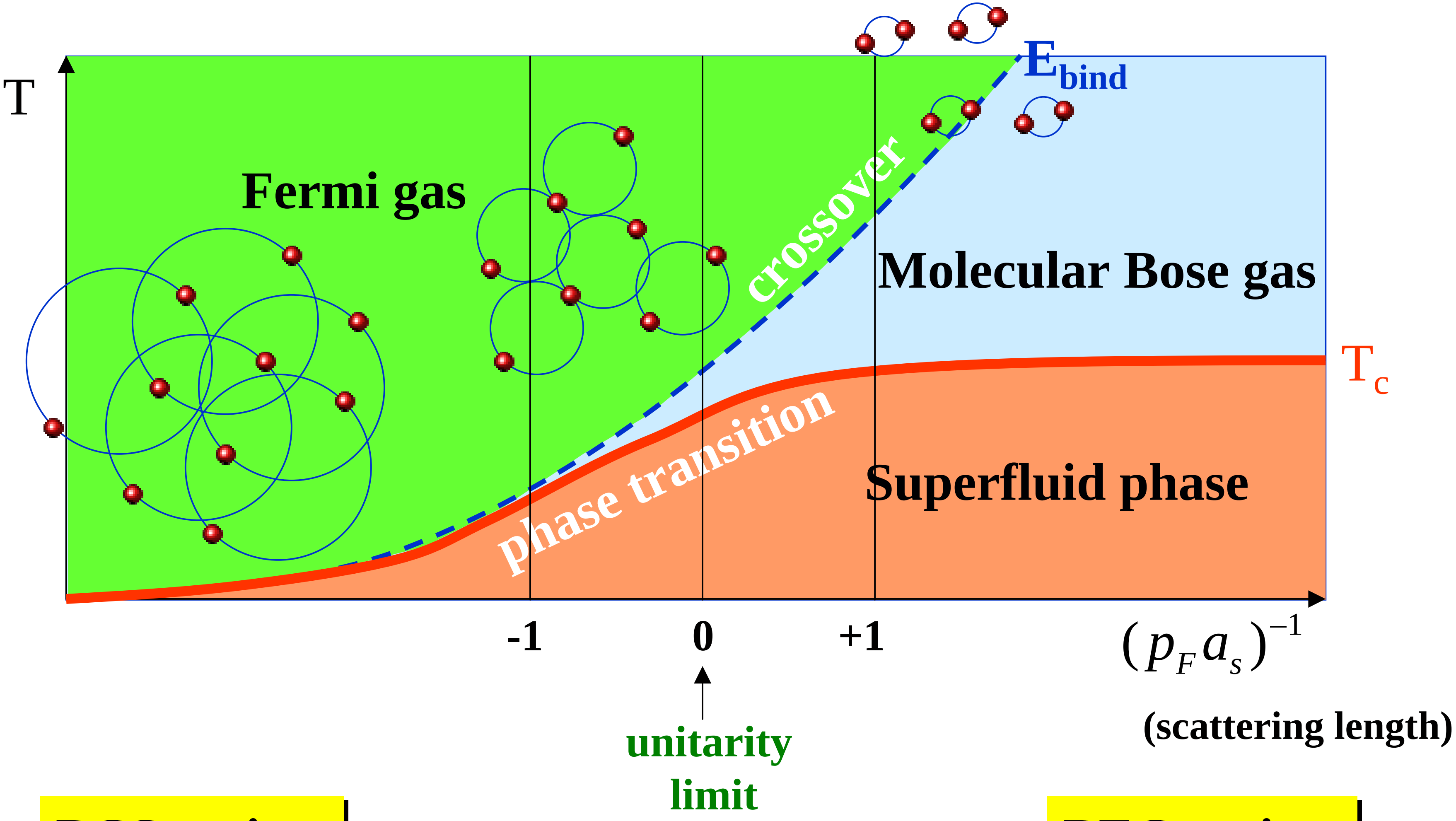
(Fermi superfluid)

BEC regime

(Bose superfluid)



Phase diagram of Fermi superfluids



BCS regime

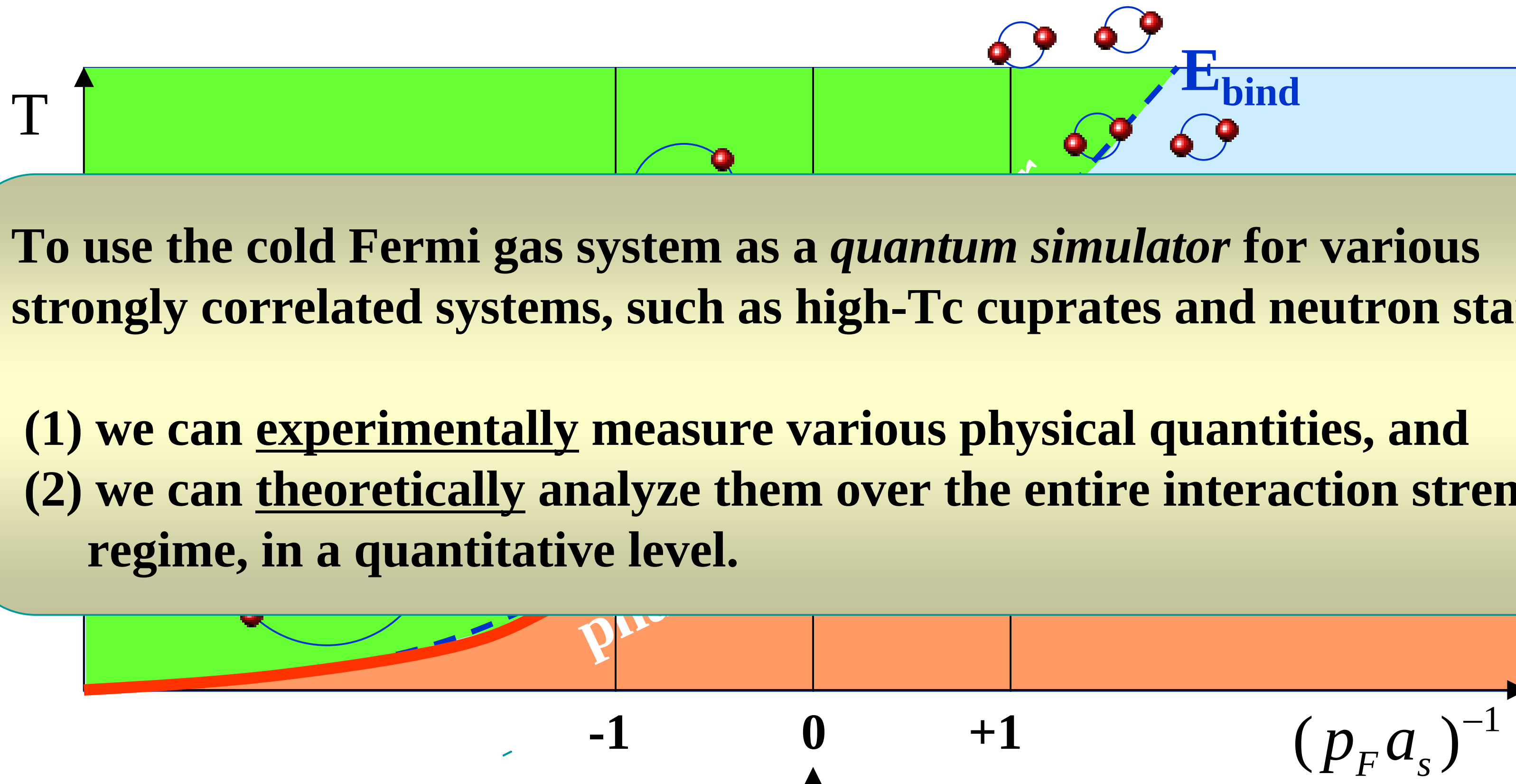
(Fermi superfluid)

crossover regime
unitarity regime

BEC regime

(Bose superfluid)

Can the cold Fermi gas system be used as a quantum simulator?



To use the cold Fermi gas system as a *quantum simulator* for various strongly correlated systems, such as high-Tc cuprates and neutron star,

(1) we can experimentally measure various physical quantities, and
(2) we can theoretically analyze them over the entire interaction strength regime, in a quantitative level.

(scattering length)

unitarity limit

BCS regime

(Fermi superfluid)

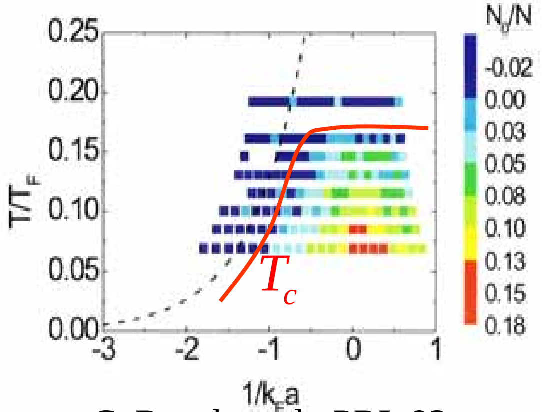
crossover regime
unitarity regime

BEC regime

(Bose superfluid)

Current experimental situation: Very good!

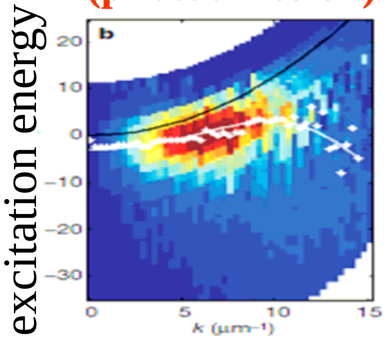
superfluid phase transition temperature Tc



C. Regal, et al. PRL **92** (2004) 040403.

single-particle excitations

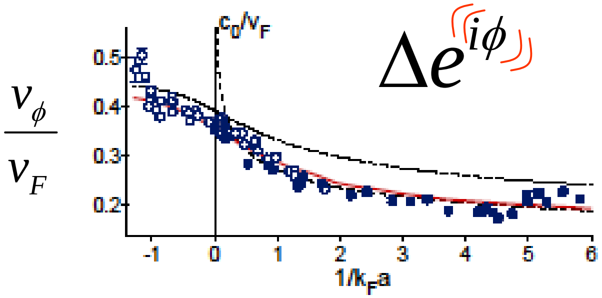
(photoemission)



JILA, Nature **454** (2008) 744

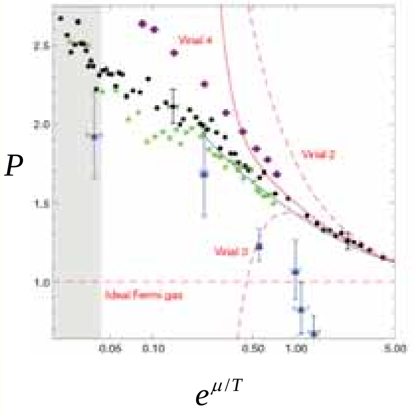
collective excitations (Goldstone mode)

$$\boxed{\omega = v_\phi q}$$



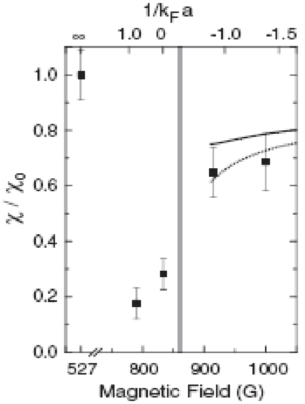
Thomas et al., PRL **98** (2007) 170401

local pressure



Salomon et al., Nature **463** (2010) 1057

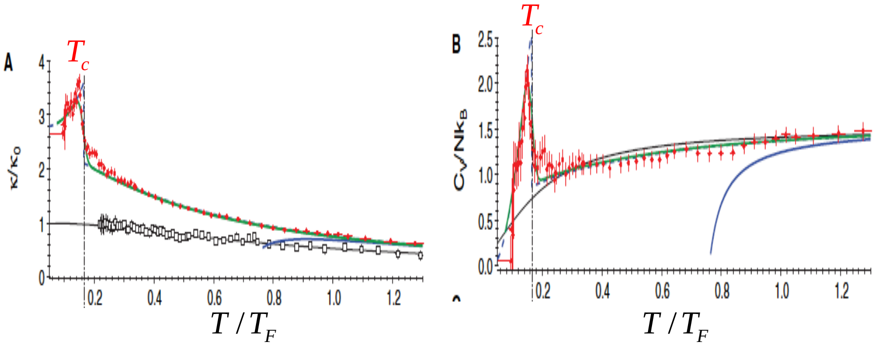
spin susceptibility



Ketterle et al., PRL **106** (2011) 010402

compressibility

specific heat



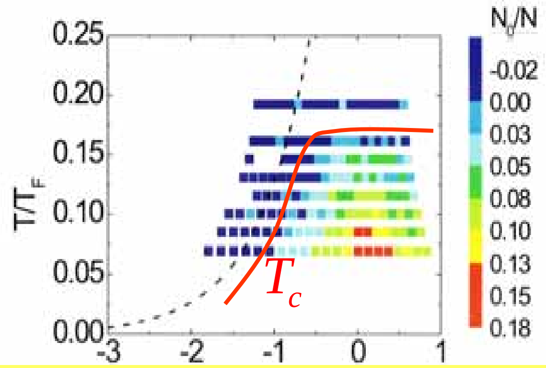
Zwierlein et al., Science **335** (2012) 563

Current experimental situation: Very good!

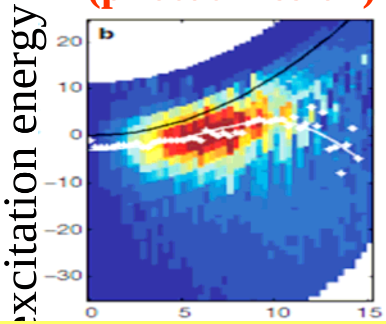
superfluid phase transition temperature Tc

single-particle excitations

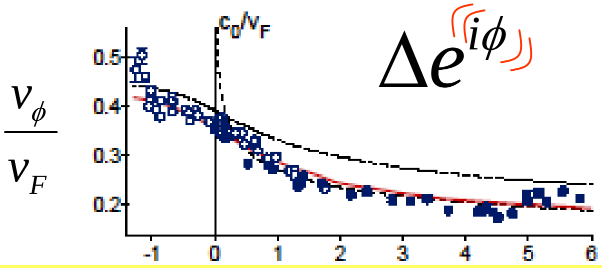
collective excitations (Goldstone mode)



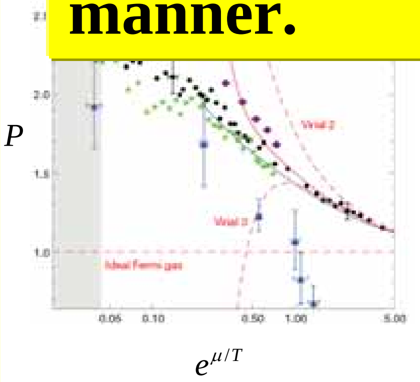
(photoemission)



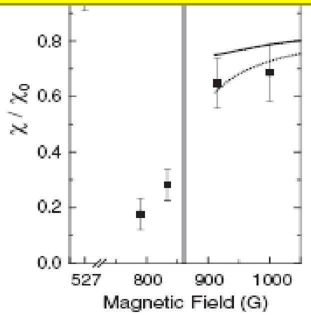
$$\boxed{\omega = v_\phi q}$$



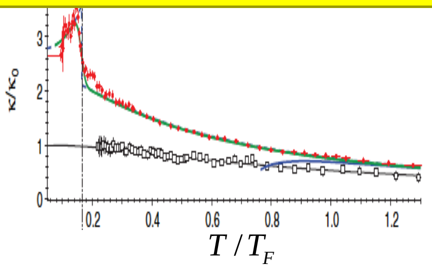
Toward the realization of a Fermi gas quantum simulator, we try to construct a reliable theory to analyze various observable physical quantities in the BCS-BEC crossover region in a unified manner.

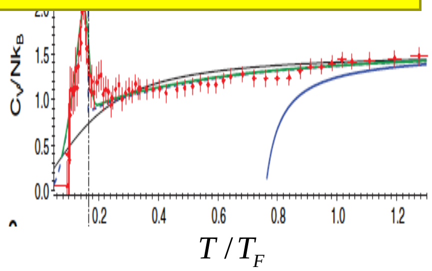


Salomon et al., Nature **463** (2010) 1057



Ketterle et al., PRL **106** (2011) 010402





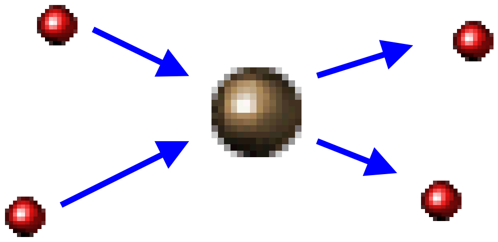
Zwierlein et al., Science **335** (2012) 563

Formulation (*broad* Feshbach resonance: ^{40}K, ^{6}Li)

So far, all the current experiments are using a *broad* Feshbach resonance. In this case, we can ignore details of Feshbach pairing mechanism, and safely consider the BCS-BEC crossover based on the BCS model.

$$H = \sum_{\mathbf{p},\sigma} (\varepsilon_p - \mu) c^{\dagger}_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} - U \sum_{\mathbf{p},\mathbf{q}} c^{\dagger}_{\mathbf{p}+\mathbf{q}\uparrow} c^{\dagger}_{\mathbf{p}'-\mathbf{q}\downarrow} c_{\mathbf{p}'\downarrow} c_{\mathbf{p}\uparrow}$$

- uniform gas is assumed.
- σ : two atomic hyperfine states = ↑ ↓
- U : pairing interaction associated with the F.R.



$\Rightarrow \quad U$

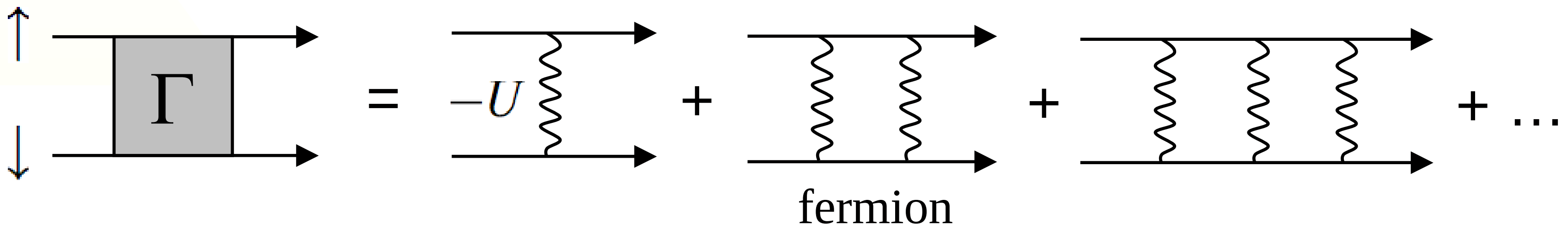
Feshbach resonance

We treat U as a tunable parameter.

Effects of a trap is included within the local density approximation (LDA).

Formulation (*broad* Feshbach resonance: ^{40}K, ^{6}Li)

T_c: Thouless criterion



$$G = \frac{1}{i\omega - (\varepsilon_{\mathbf{p}} - \mu)}$$

- **pole of Γ at $q=\omega=0$** ⟶ T_c

$$1 = U \sum_{\mathbf{p}} \frac{\tanh \frac{\beta}{2}(\varepsilon_p - \mu)}{2(\varepsilon_p - \mu)}$$

μ is know to remarkably deviate from the Fermi energy in the BCS-BEC crossover.

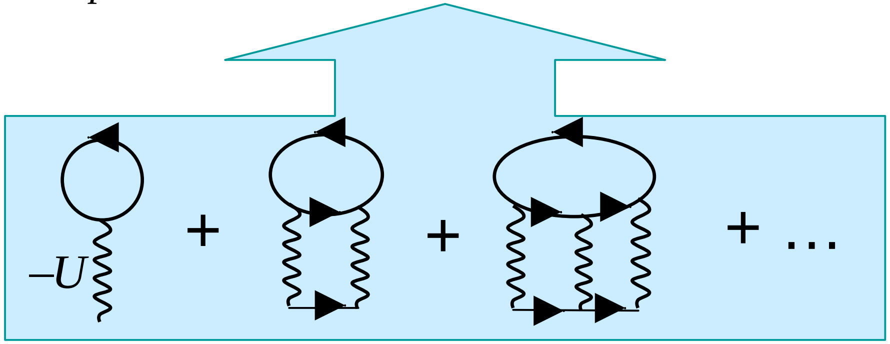
Formulation (*broad* Feshbach resonance: ^{40}K, ^{6}Li)

μ **is determined from the equation for the number of Fermi atoms.**

$$N=\frac{2}{\beta}\sum_{p,i\omega_n}G(p,i\omega_n)e^{i\delta\omega_n}$$

Single-particle Green's function involves self-energy correction, describing effects of fluctuations in the Cooper channel.

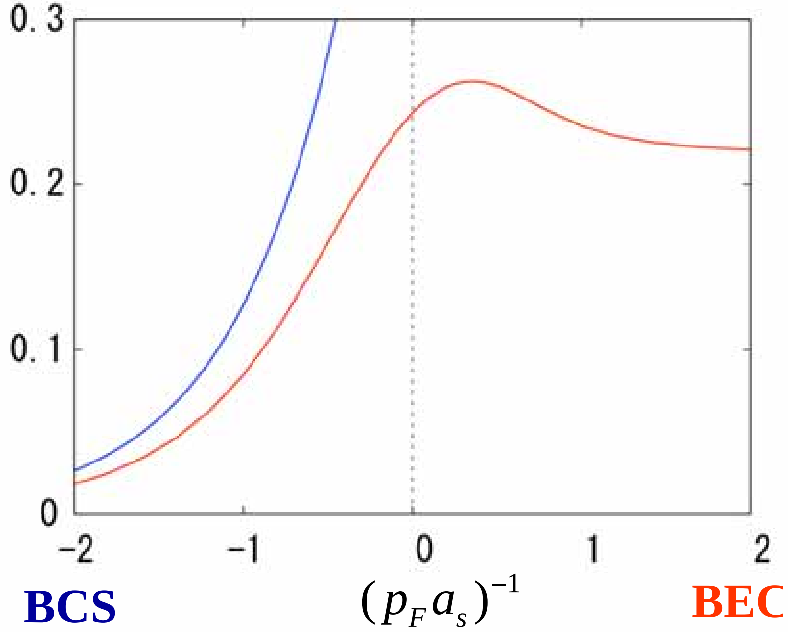
$$G(p,i\omega_n)=\frac{1}{i\omega_n-(\varepsilon_p-\mu)-\Sigma(p,i\omega_n)}$$



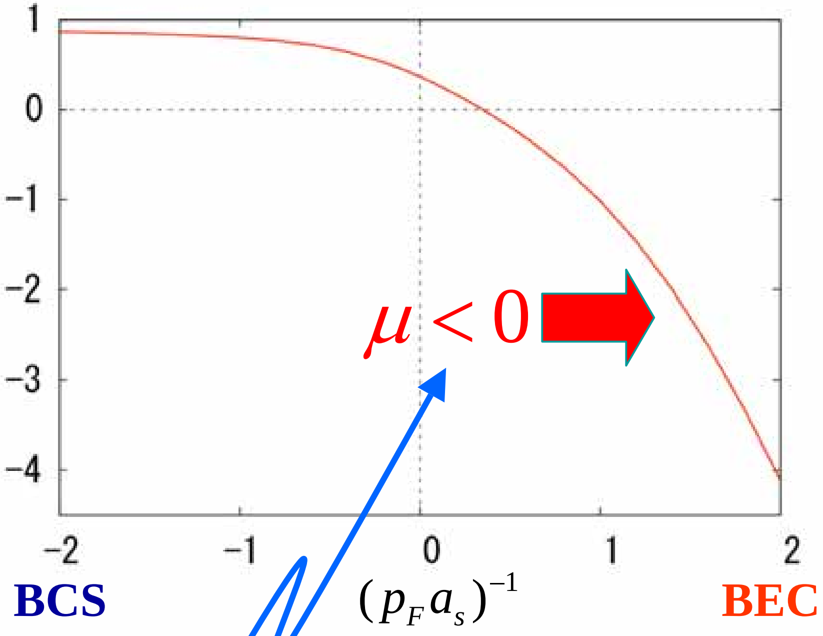
By solving the T_c-equation, together with the number equation, we self-consistently determine T_c and μ.

Self-consistent solutions at T_c in the BCS-BEC crossover

T_c / T_F



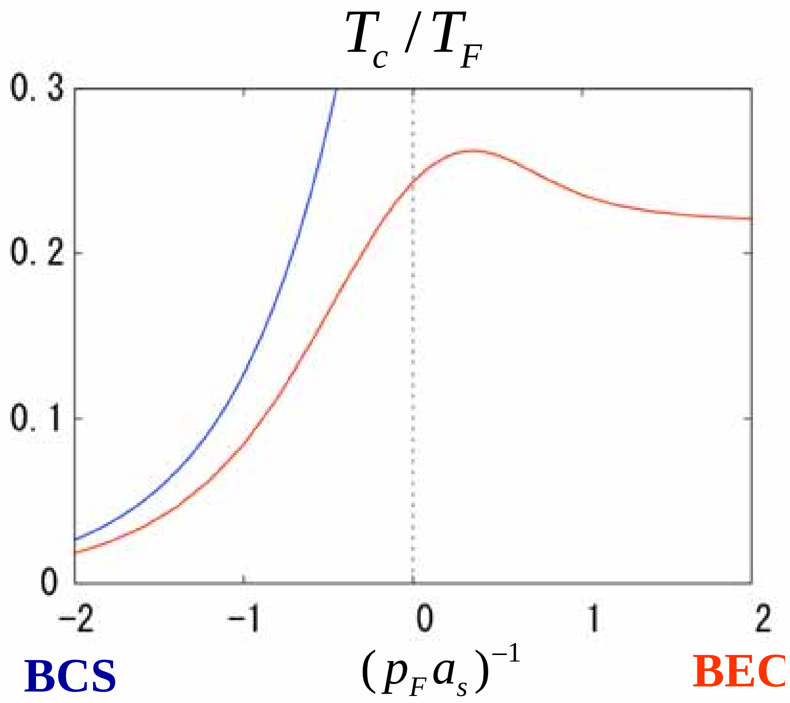
μ / ε_F

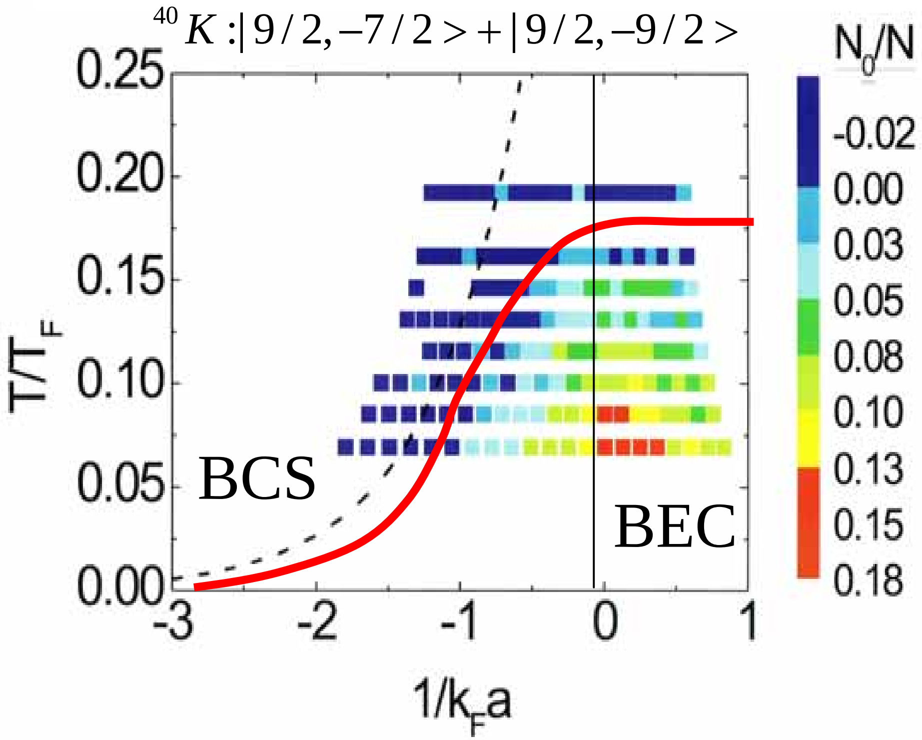


$\mu < 0$

signature of the formation of two-body bound molecues at (and above) Tc.

Self-consistent solutions at T_c in the BCS-BEC crossover



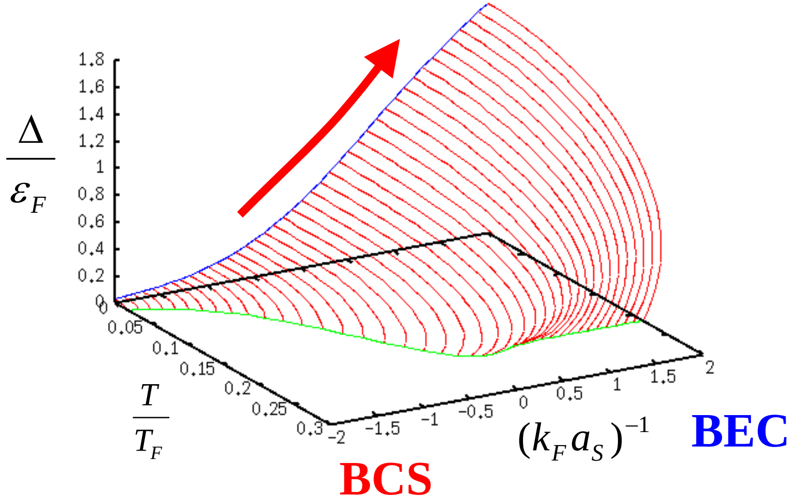


C. A. Regal, et al. PRL 92 (2004) 040403.

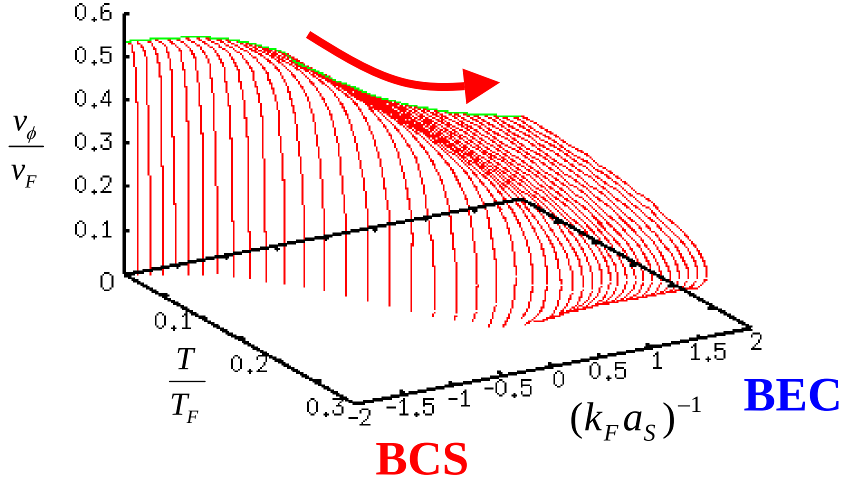
Extension to the superfluid phase below Tc

In the superfluid phase below Tc, we need to treat phase fluctuations and *amplitude fluctuations* of the order parameter Δ in a consistent manner.

superfluid order parameter



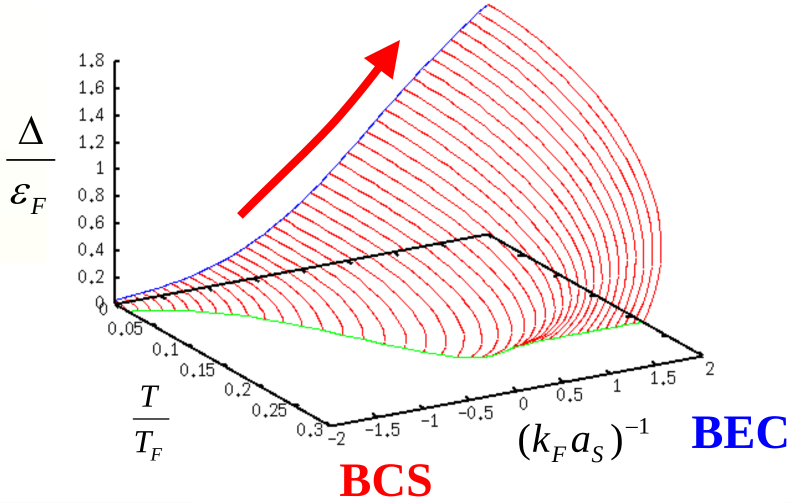
Goldstone sound velocity



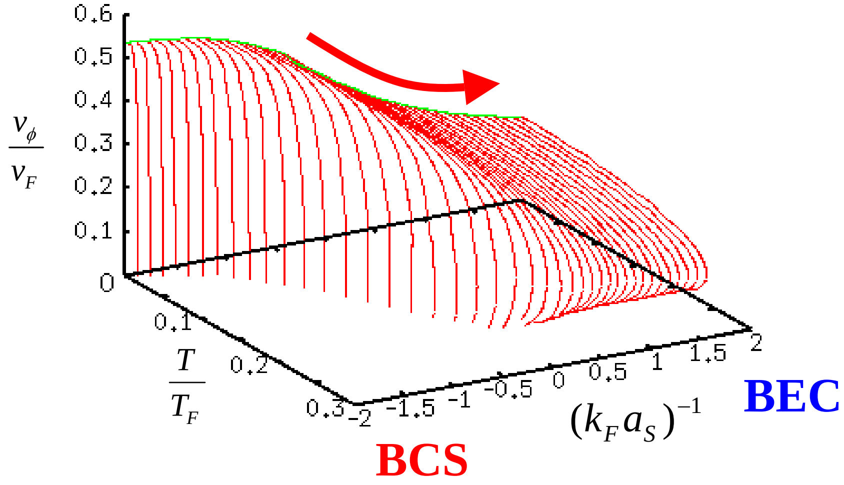
Y. Ohashi et al., PRA **75** (2007) 033609 (Gaussian)

Extension to the superfluid phase below Tc

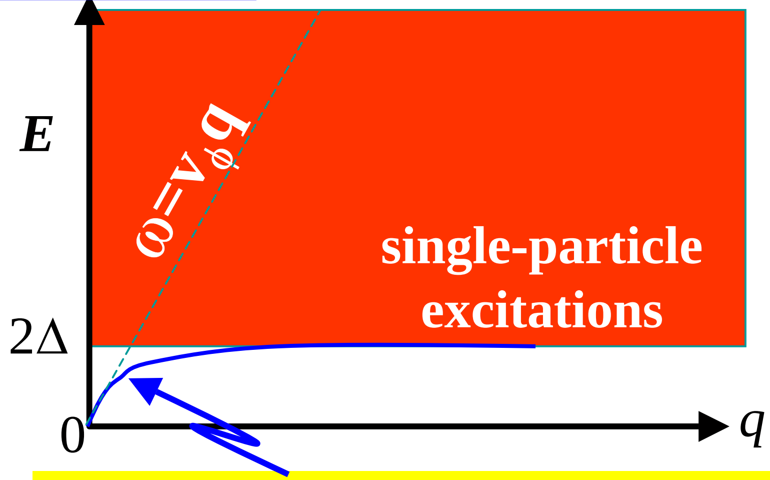
superfluid order parameter



Goldstone sound velocity

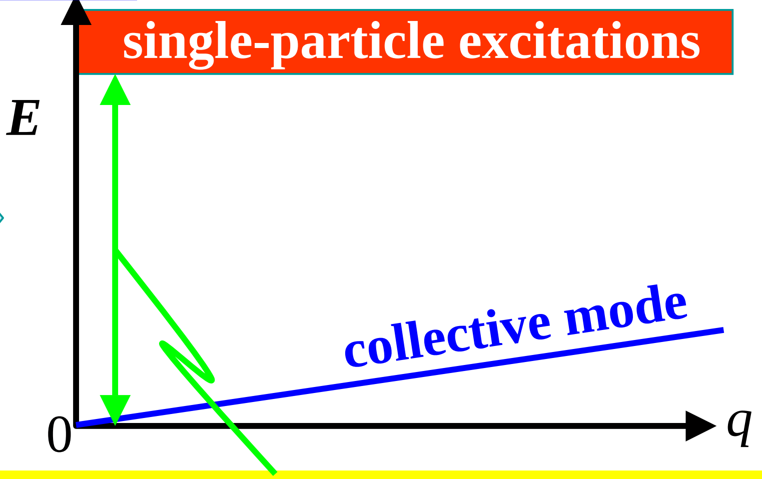


BCS region



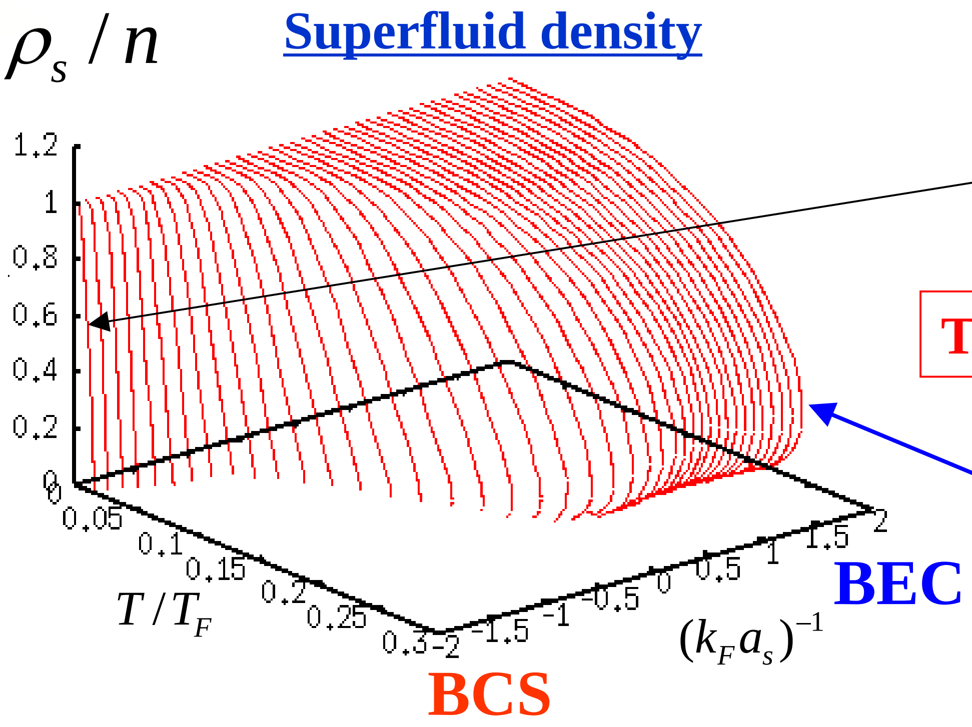
Collective excitations are important only in the small momentum region.

BEC region



Large single-particle excitation gap reflects a strong binding energy.

Superfluid density in the BCS-BEC crossover



Y. Ohashi et al., PRA **75** (2007) 033609 (Gaussian)

$$\rho_s = n - \rho_n^F - \rho_n^B$$

BCS $n - \rho_n^F$

Thermal single-particle excitations

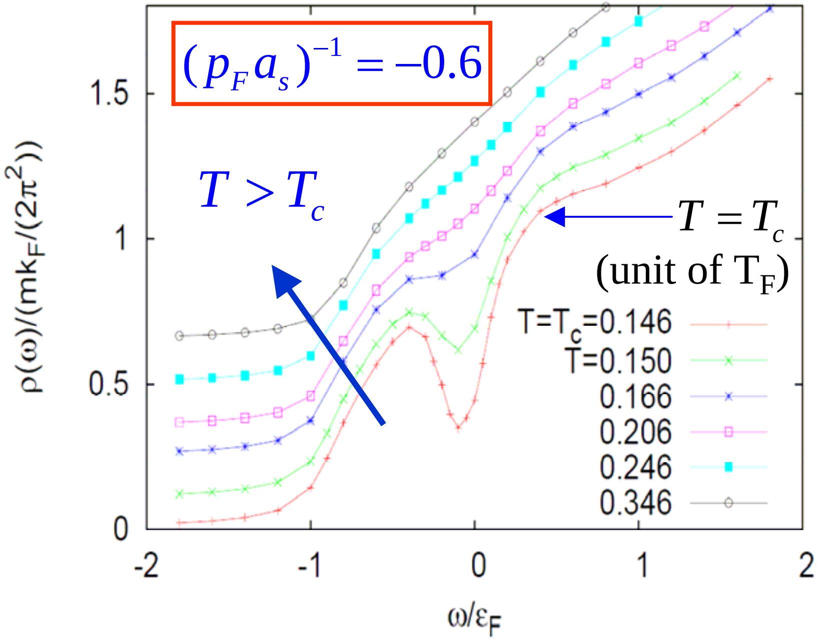
BEC $n - \rho_n^B$

thermally excited collective modes

The origin of the normal fluid density continuously changes from single-particle excitations to collective excitations in the BCS-BEC crossover.

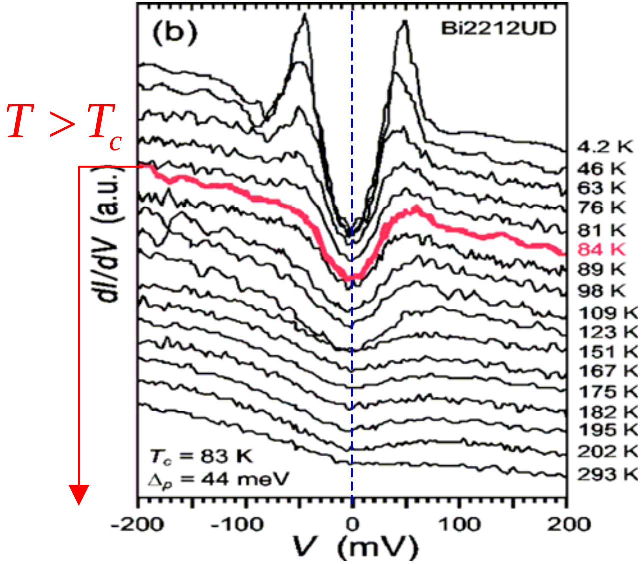
Single-particle properties above Tc (Δ=0!): “pseudogap”

density of states at Tc in the crossover region

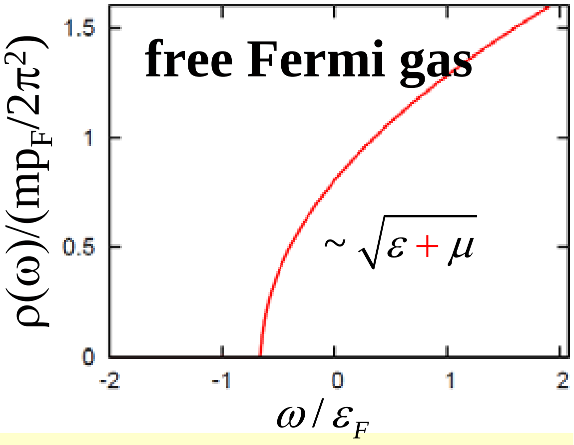


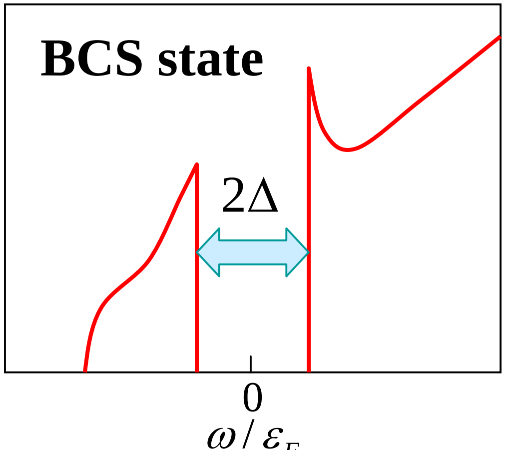
Tsuchiya, YO et at. PRA **80** (2009) 033613

pseudogap in High-Tc cuprates



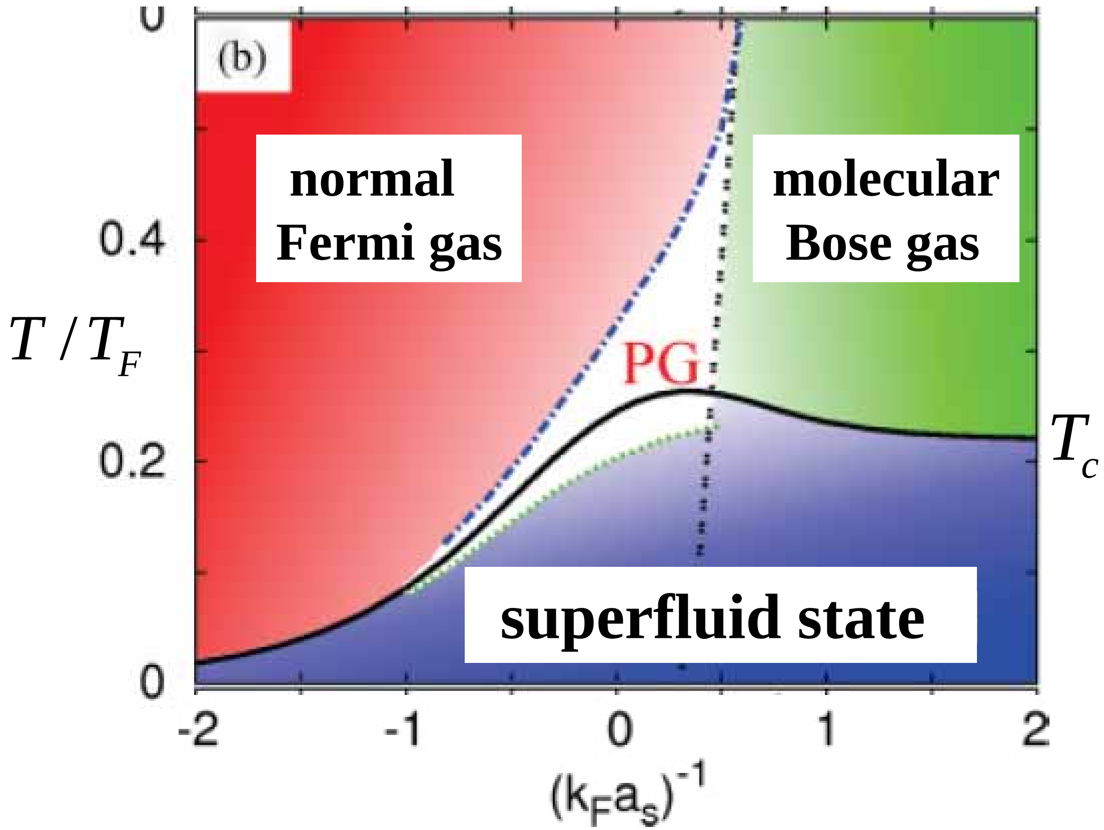
Fisher et at. RMP **79** (2007) 353





Phase diagram of cold Fermi gas in the BCS-BEC crossover

PG: pseudogap regime



Watanabe, YO et at. PRA **82** (2010) 043630, **85** (2012) 039908(E)

Spectral weight at Tc in the BCS-BEC crossover

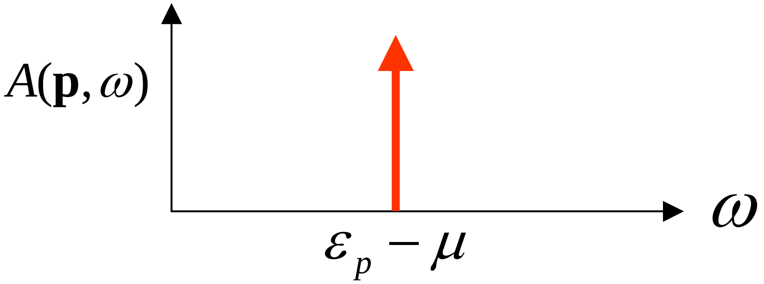
$$\boxed{A(\mathbf{p},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\mathbf{p}, i\omega_n \to \omega_+)}$$

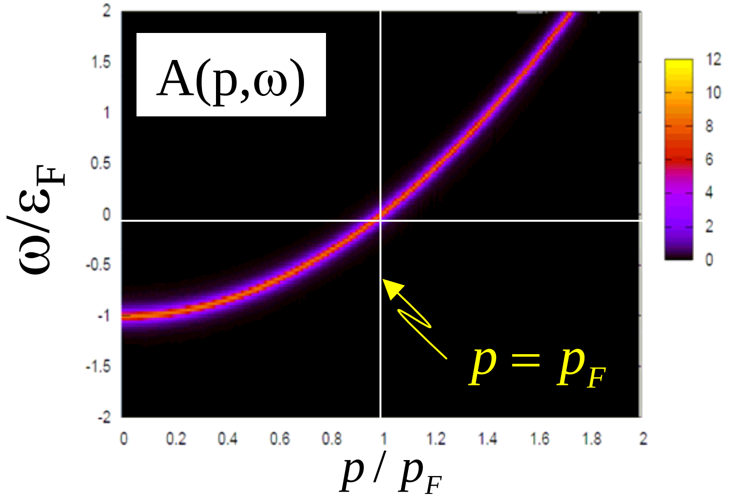
$$DOS = \sum_{\mathbf{p}} A(\mathbf{p},\omega)$$

Free Fermi gas

$$A(\mathbf{p},\omega) = \delta(\omega - (\varepsilon_{\mathbf{p}} - \mu))$$

$$\varepsilon_p = p^2/2m$$

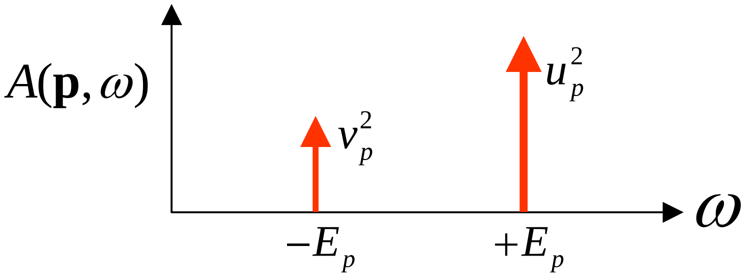


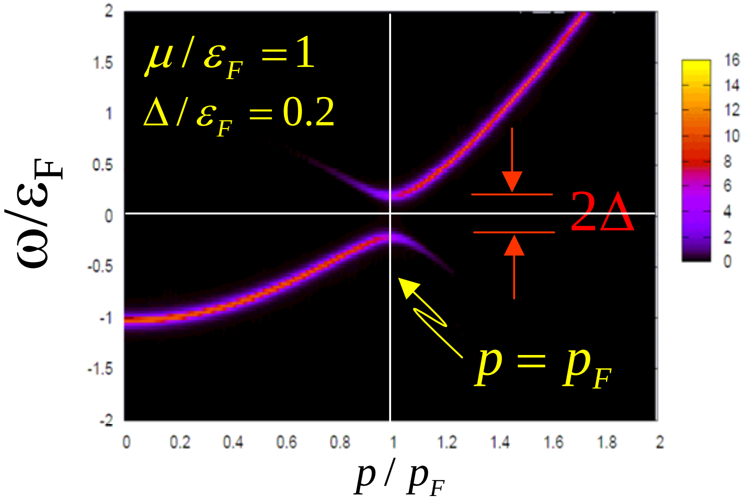


weak-coupling BCS state at T=0

$$A(\mathbf{p},\omega) = u_p^2\,\delta(\omega - E_p) + v_p^2\,\delta(\omega + E_p)$$

$$E_p = \sqrt{(\varepsilon_p - \mu)^2 + \Delta^2}$$





BCS gap = particle-hole coupling by order parameter Δ

$$G_{11}^{BCS}(p,i\omega) = -\frac{i\omega+\xi_{\mathbf{p}}}{\omega^2+\xi_{\mathbf{p}}^2+\Delta^2} = \frac{1}{i\omega-\xi_{\mathbf{p}} - \dfrac{\Delta^2}{i\omega+\xi_{\mathbf{p}}}} \qquad (\xi_{\mathbf{p}} = \varepsilon_{\mathbf{p}} - \mu)$$

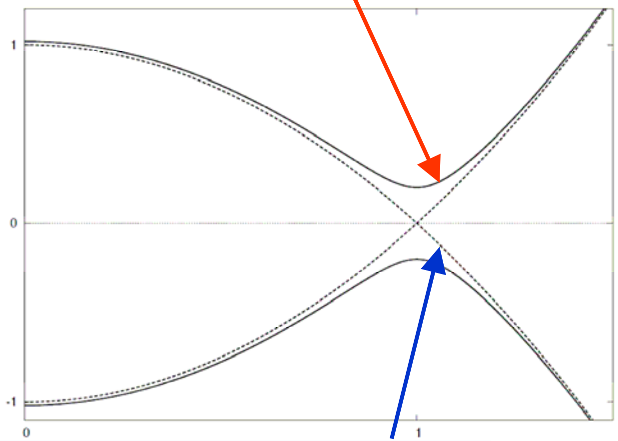
particle Green's function

hole Green's function

$$G(p) = \frac{1}{i\omega-\xi_{\mathbf{p}}}$$

$$G(p) = \frac{1}{i\omega+\xi_{\mathbf{p}}}$$

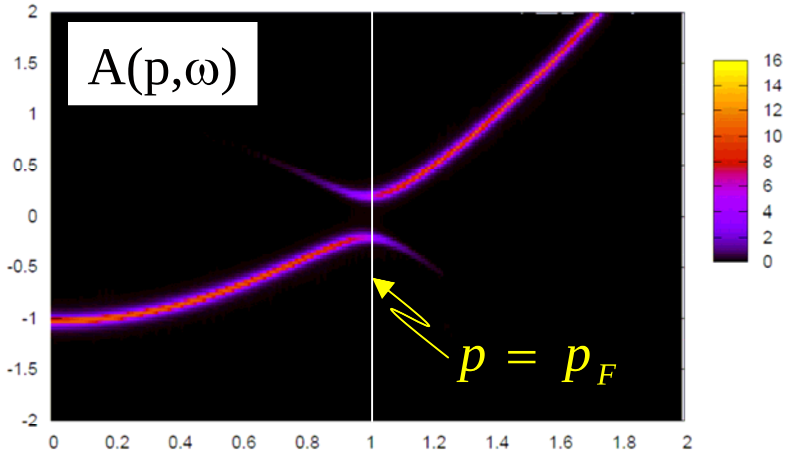
$\varepsilon_p - \mu$: **particle band**



$-(\varepsilon_p - \mu)$: **hole band**

ω / ε_F

p / p_F



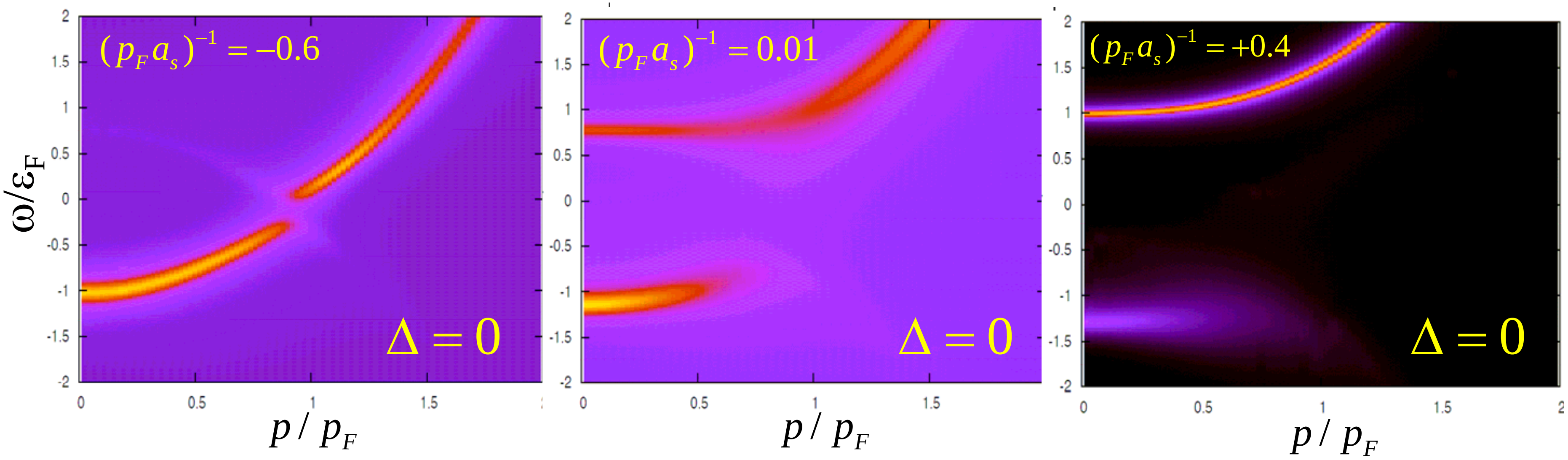
$p = p_F$

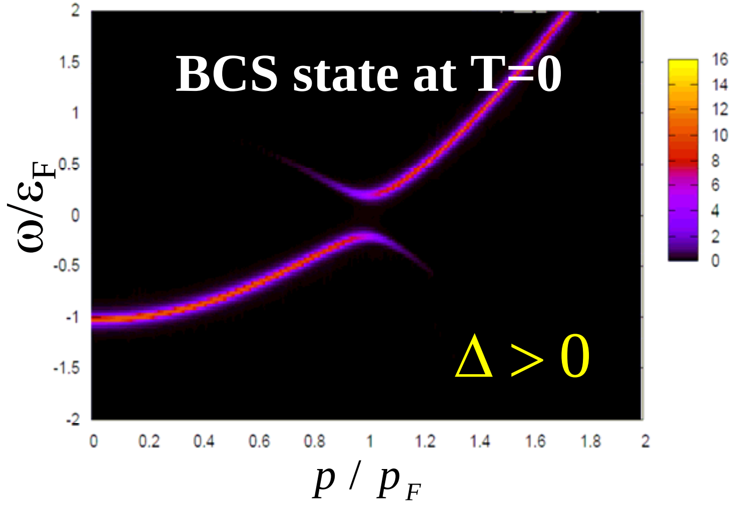
p / p_F

Spectral weight at Tc in the BCS-BEC crossover

$$A(\mathbf{p},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\mathbf{p}, i\omega_n \to \omega_+)$$

BCS → **crossover** → **BEC**



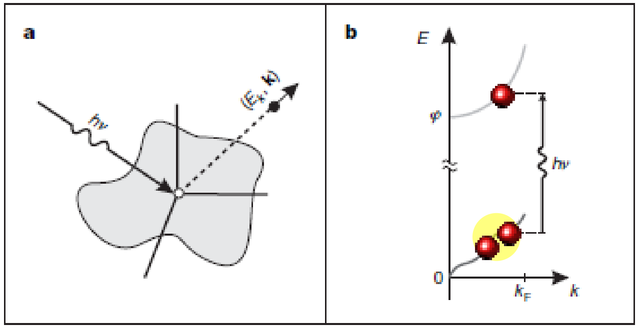


Strong pairing fluctuations induces a particle-hole coupling, leading to the pseudo-gapped single-particle excitations above Tc.

Observation of pseudogap in a cold ^{40}K Fermi gas

"photoemission-type" experimanet (JILA 2008)

Stewart, et al., Nature 454 (2008) 744

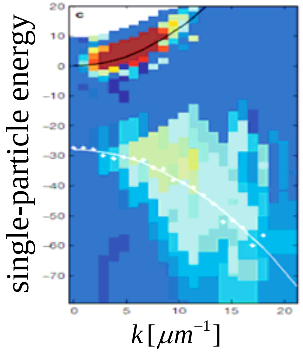


$$I(\mathbf{p},\omega) = A(\mathbf{p},\omega) \times f(\omega)$$

+ harmonic trap

spectral weight

Fermi distribution function

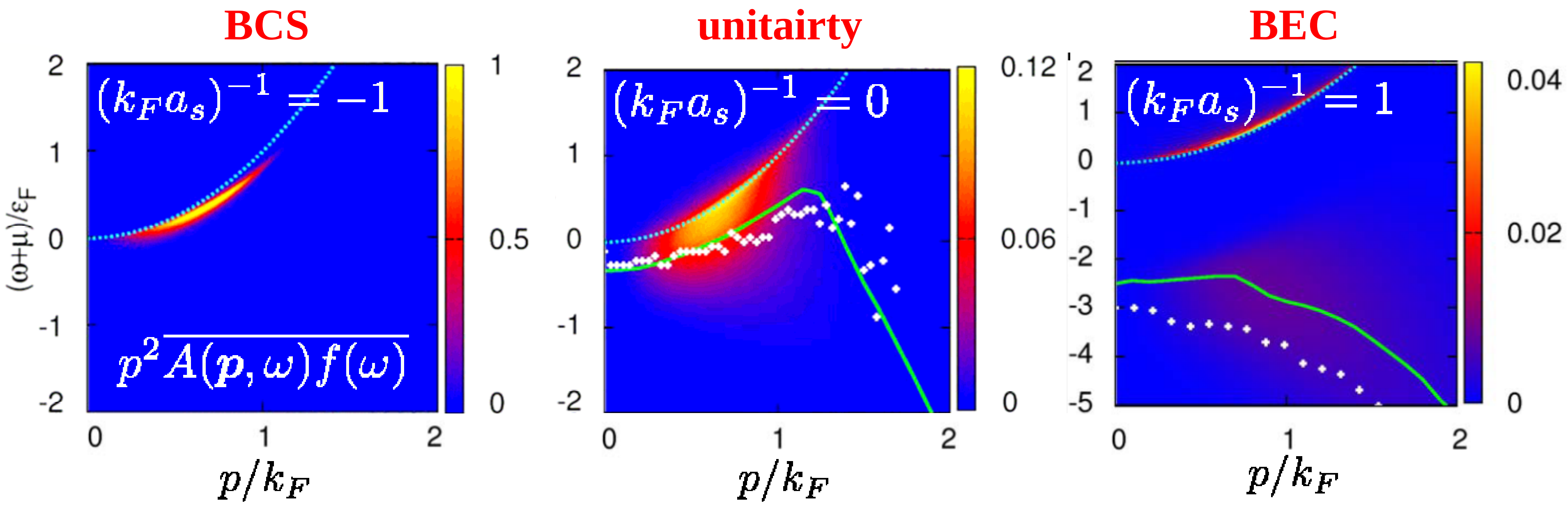


$$\rho(\omega) = -\frac{1}{\pi}\sum_{\mathbf{p}} A(\mathbf{p},\omega)$$

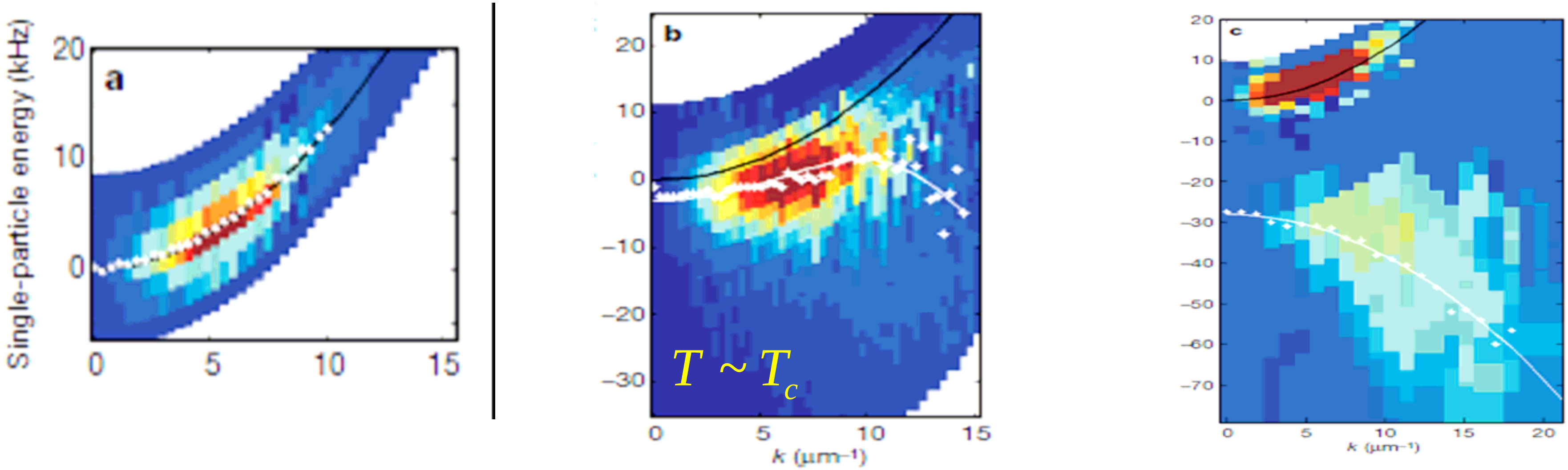
(density of states)

$$A(\mathbf{p},\omega) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\mathbf{p}, i\omega \to \omega + i\delta)$$

Photoemission spectrum at T_c

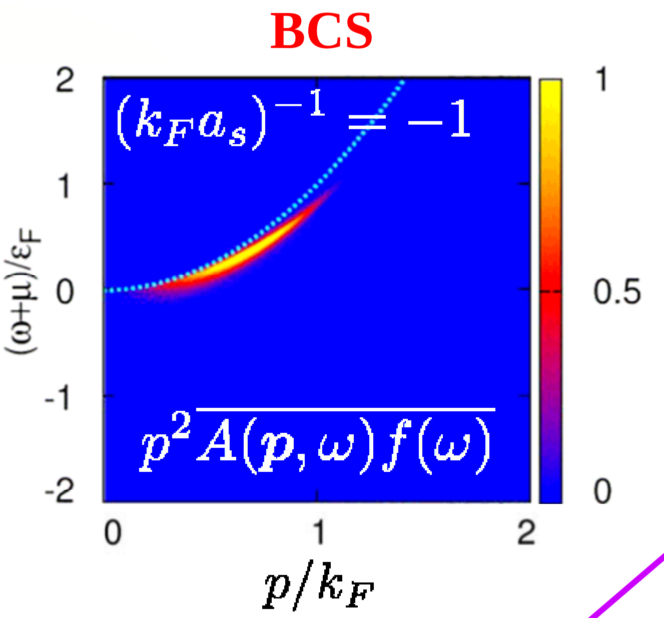


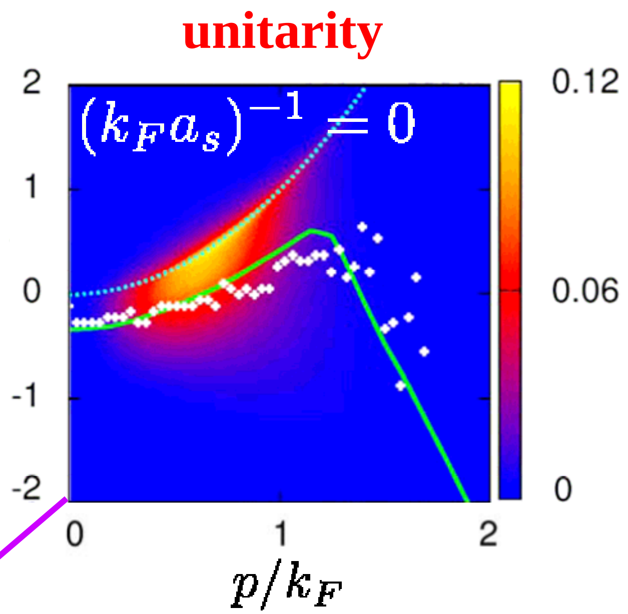
Theory: Tsuchiya, Watanabe, Ohashi, PRA **82** (2010) 033629

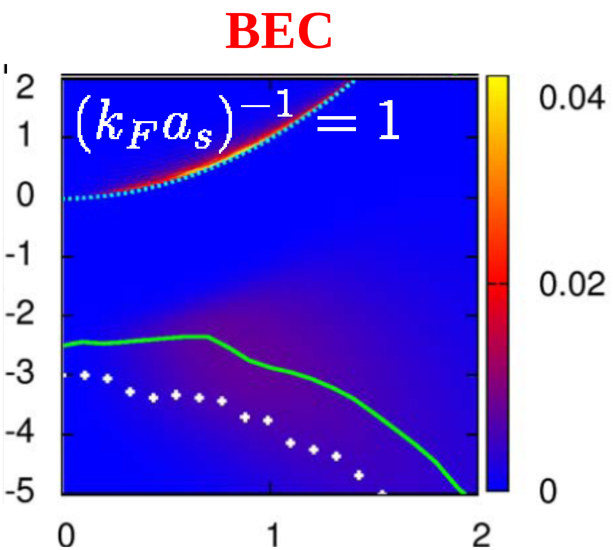


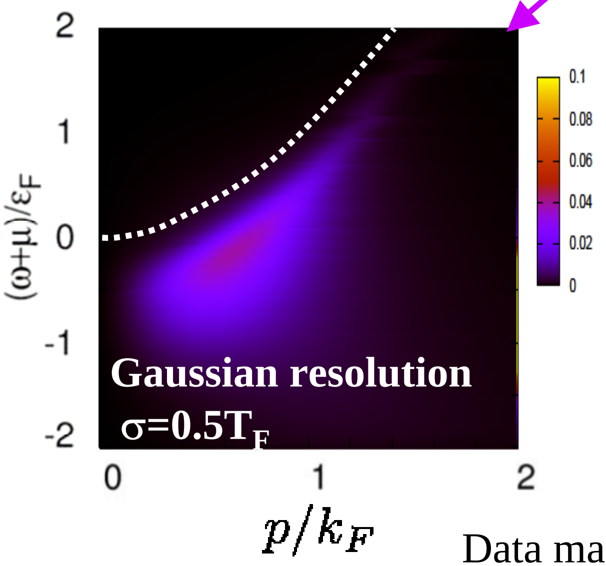
Experiment on ^{40}K: Stewart, Gaebler, Jin, Nature **454** (2008) 744

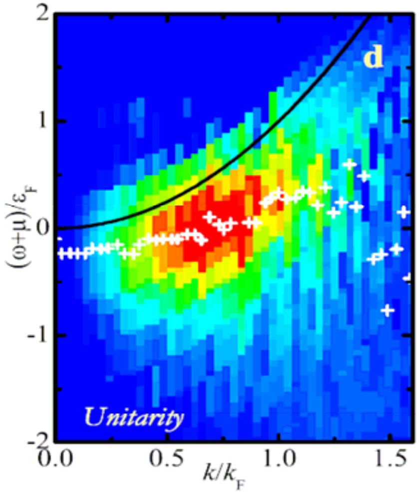
Photoemission spectra in the BCS-BEC crossover at T_c

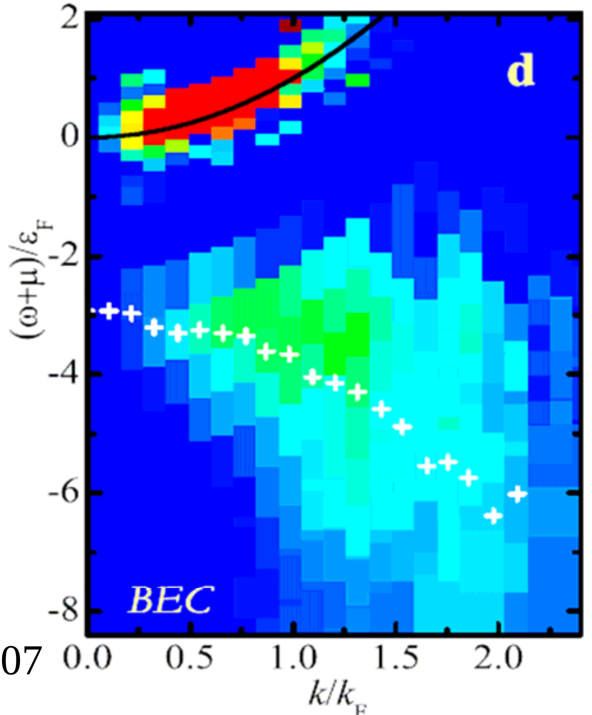






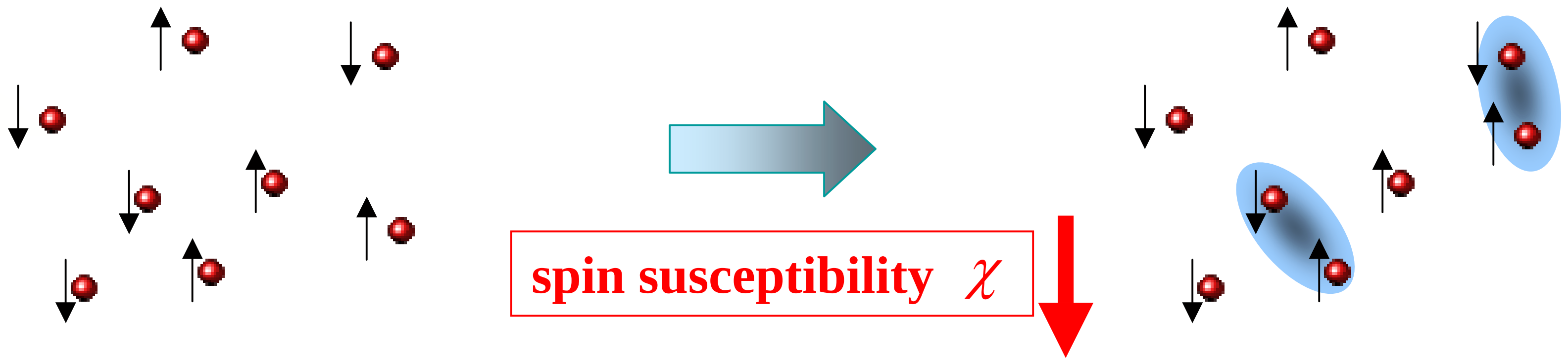






Data mapping: Hu et al., PRL 104 (2010) 240407

Preformed singlet-pairs in the crossover region

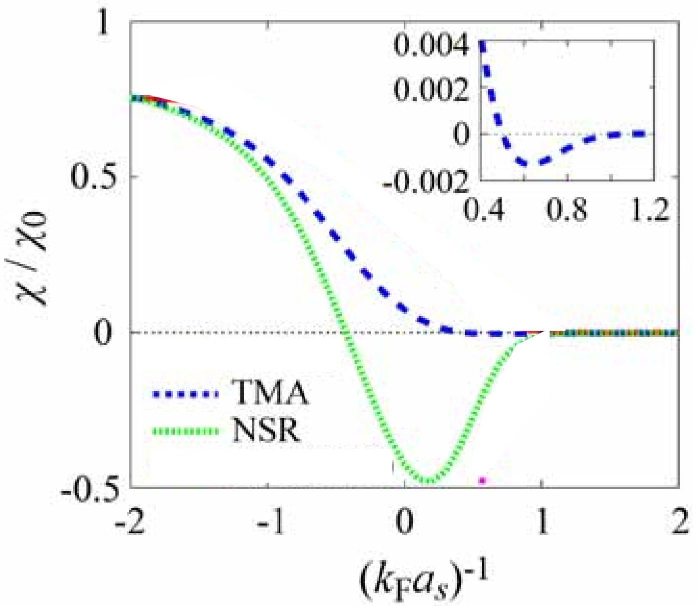


spin susceptibility χ

weak-coupling regime

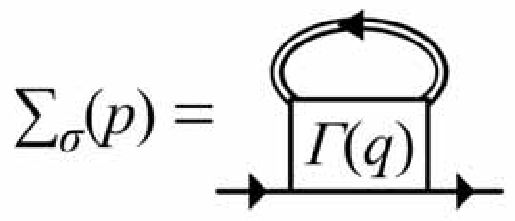
crossover region

The couventional BCS-BEC crossover theories (Gaussian, T-matrix) unphysically give negative spin susceptibility in the BCS-BEC crossover region.

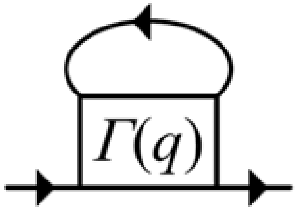


Preformed singlet-pairs in the crossover region

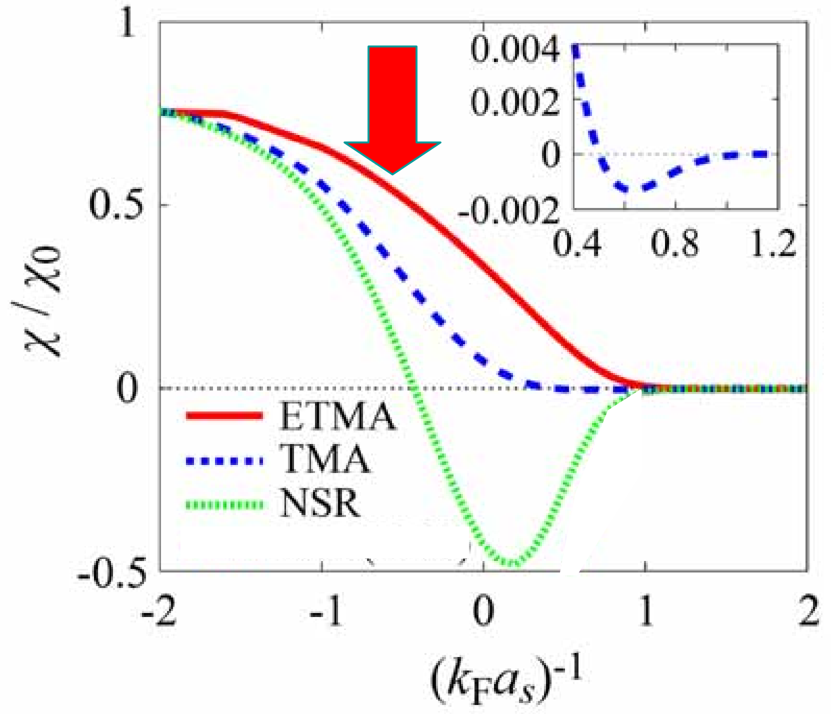
We have succeeded in solving this serious problem by including higher Fluctuations so as to obtain positive χ in the whole crossover region.



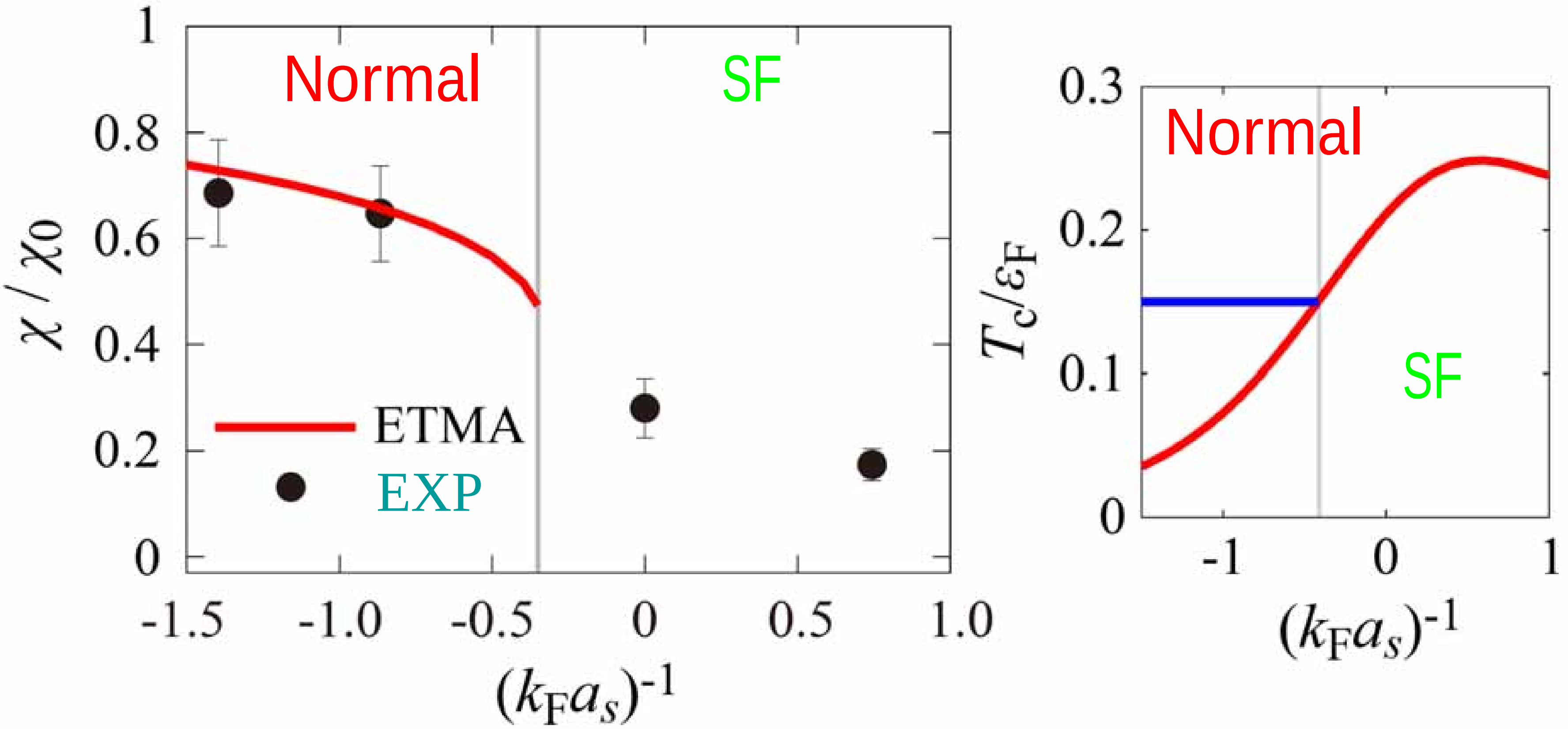
“extended T-matrix (ETMA)



conventional T-matrix approximation (TMA)



spin susceptibility in the crossover region

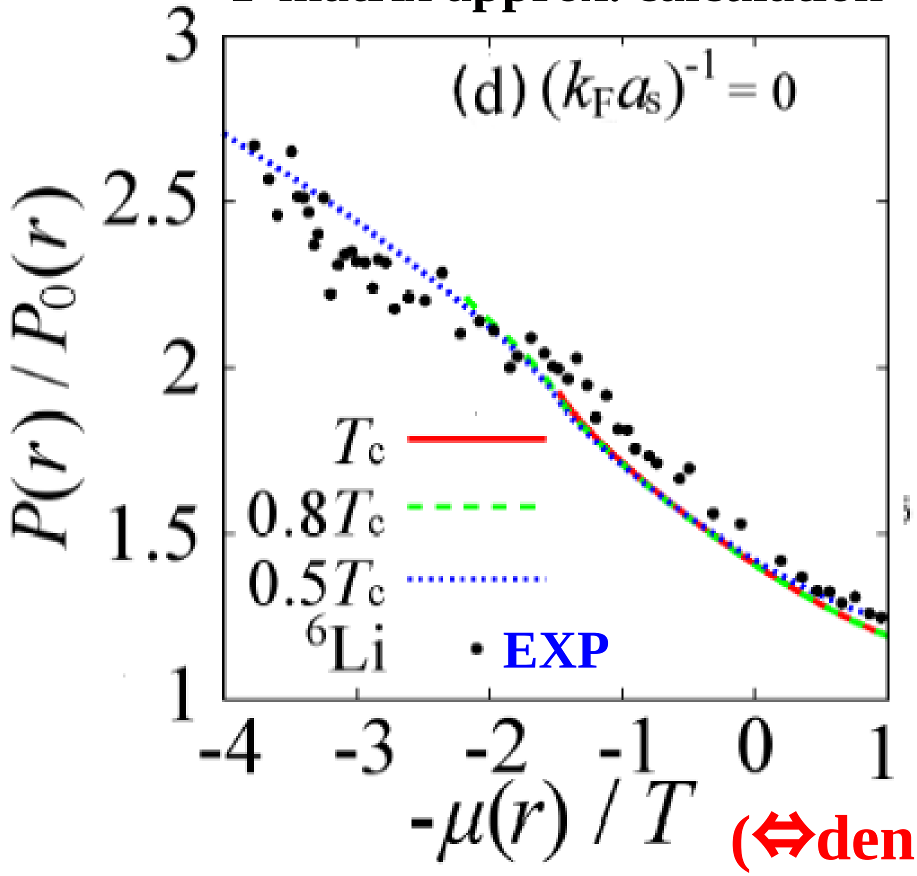


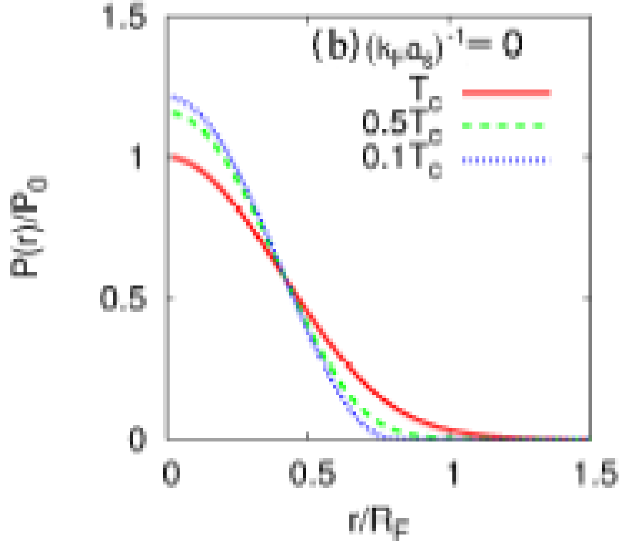
Theory(ETMA) : Kashimura, YO et al., PRA (2012) in press

experimental data: Ketterle et al., PRL **106** (2011) 010402

Local pressure P (EOS)

T-matrix approx. calculation





(⇔density) ⇒ EOS

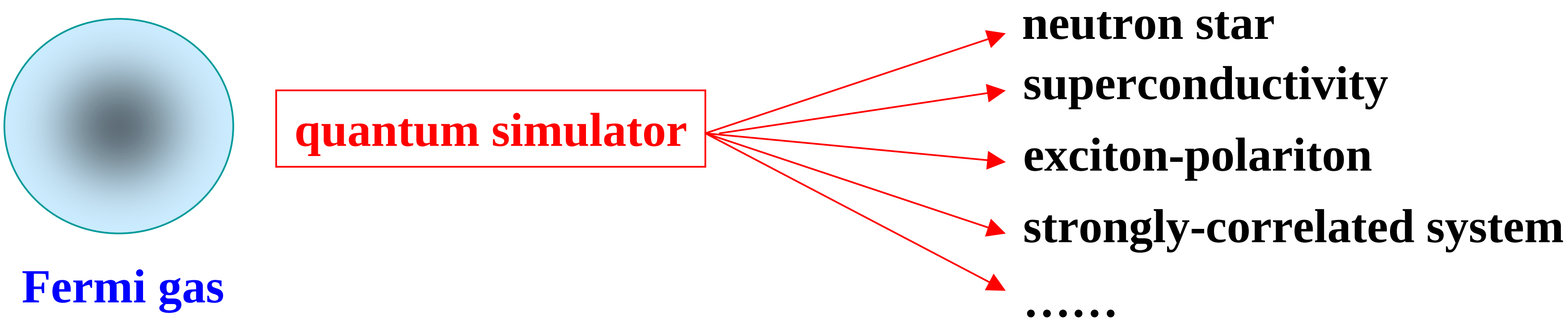
Theory: Watanabe, YO, et al., PRA (2012) in press.

Experimental data: S. Nascimbene, *et al*, NJP **12** (2010), 103026

“Universal thermodynamics”

Summary

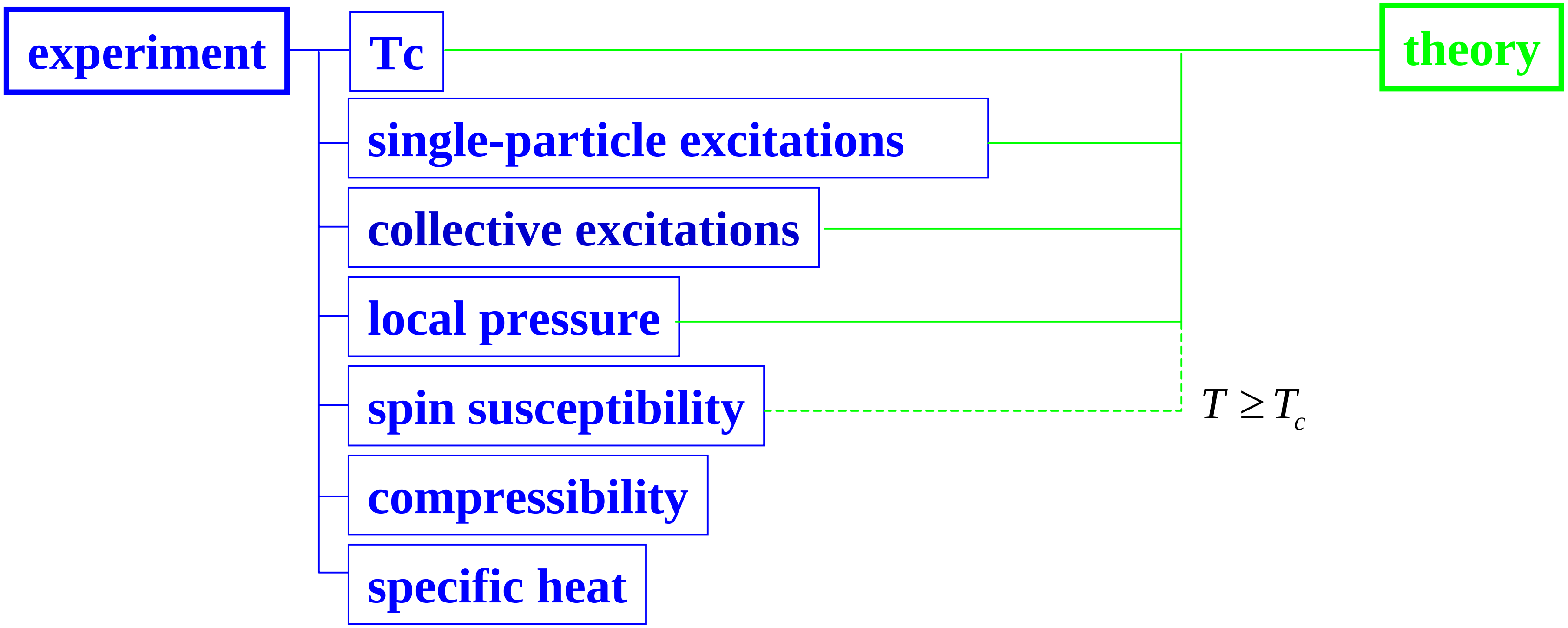
We have discussed strong-coupling effects in the BCS-BEC crossover regime of an ultracold Fermi gas.



Fermi gas

quantum simulator

- neutron star
- superconductivity
- exciton-polariton
- strongly-correlated system
- ……



experiment

- Tc
- single-particle excitations
- collective excitations
- local pressure
- spin susceptibility
- compressibility
- specific heat

theory

$T \geq T_c$