中性子星新学術キックオフ研究会＠理研（2012年10月）

Symmetry energy and nuclear pasta

Kei Iida (Kochi Univ.)

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科学研究費補助金 新学術領域研究「実験と観測で解き明かす中性子星の核物質」
D01班「中性子星と核物質の理論研究」

―中性子星核物質状態方程式とコンパクト天体現象―

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Introduction

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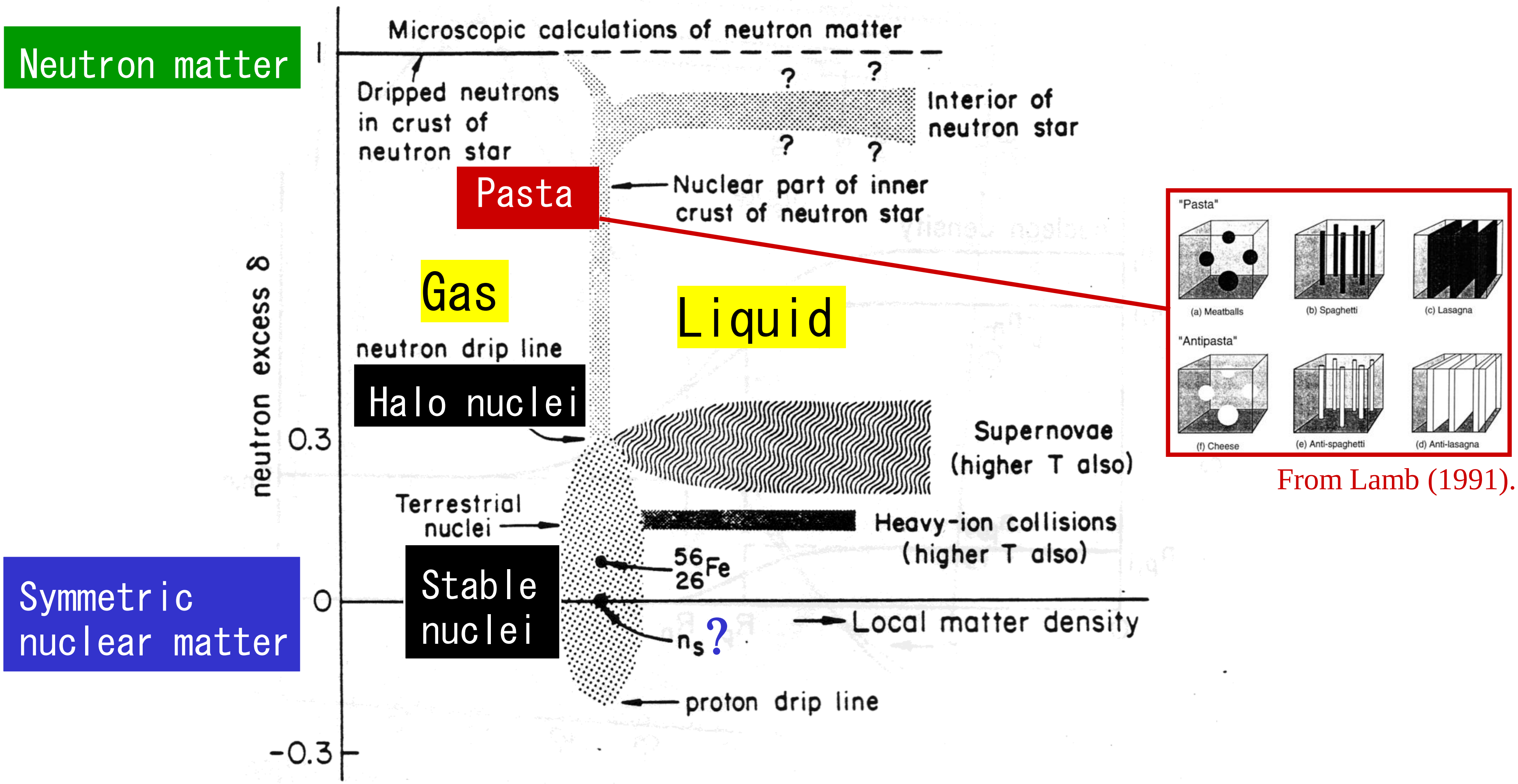
Systems composed of nuclear matter



From Lamb (1991).

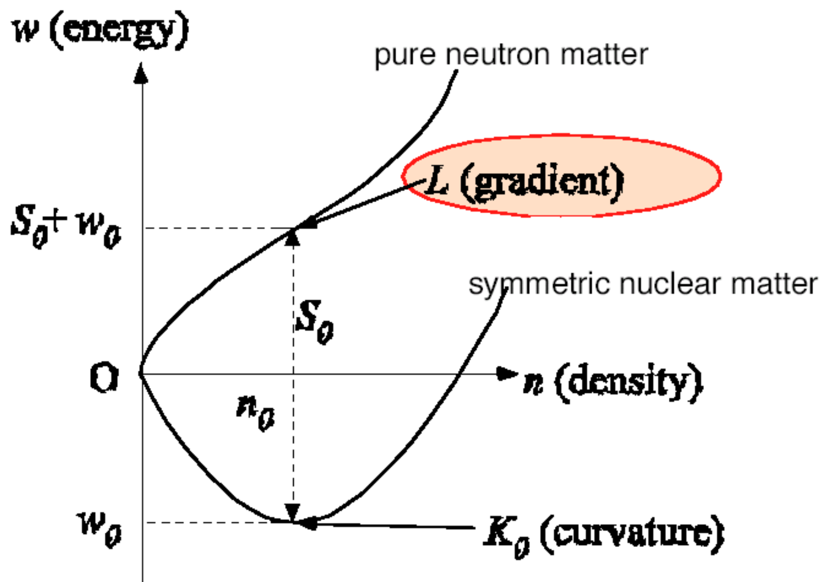
Pethick & Ravenhall, ARNPS **45** (1995) 429.

Phenomenological EOS parameters

Energy per nucleon of bulk nuclear matter near the saturation point (nucleon density n, neutron excess α):

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$

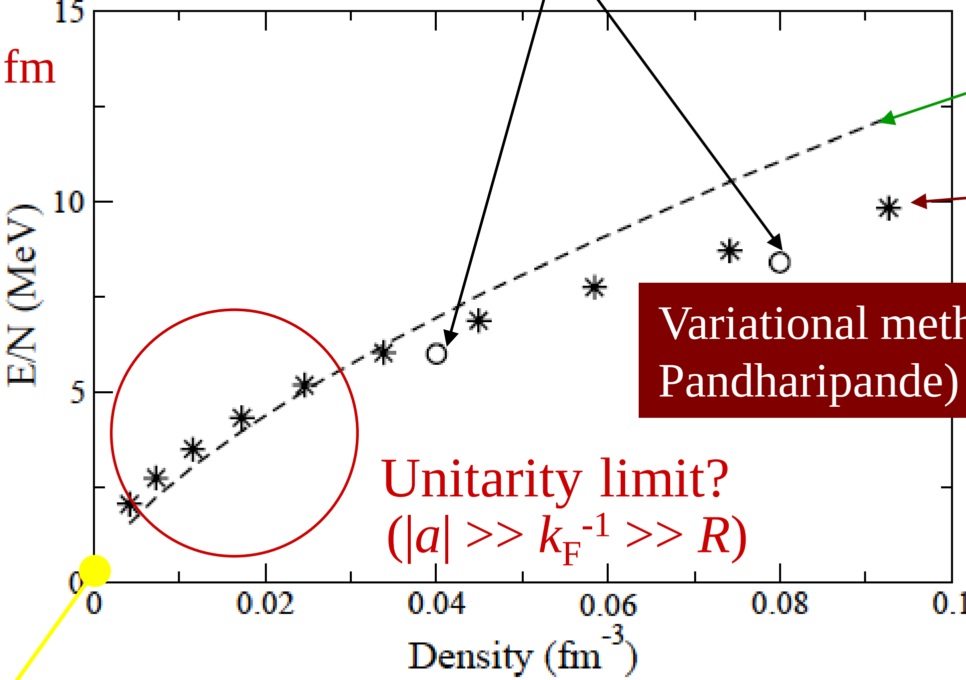
n_0, w_0	saturation density & energy of symmetric nuclear matter
S_0	symmetry energy coefficient
K_0	incompressibility
L	density symmetry coefficient



Microscopic EOS calculations

Pure neutron matter

Scattering length $a \approx$-18 fm
Effective range $R \approx$2 fm



Low-density expansion:

$$E = E_{FG}\left[1 + \frac{10}{9\pi}k_F a + \frac{4}{21\pi^2}(11 - 2\ln 2)(k_F a)^2 + \ldots\right]$$

Ref. Carlson et al., PRC **68** (2003) 025802.

Symmetric nuclear matter

Variational method: Overbinding without phenomenological three-nucleon forces

Nuclear radii and the equation of state of nuclear matter

Question

Can we extract the saturation properties of asymmetric nuclear matter from the size of unstable nuclei?

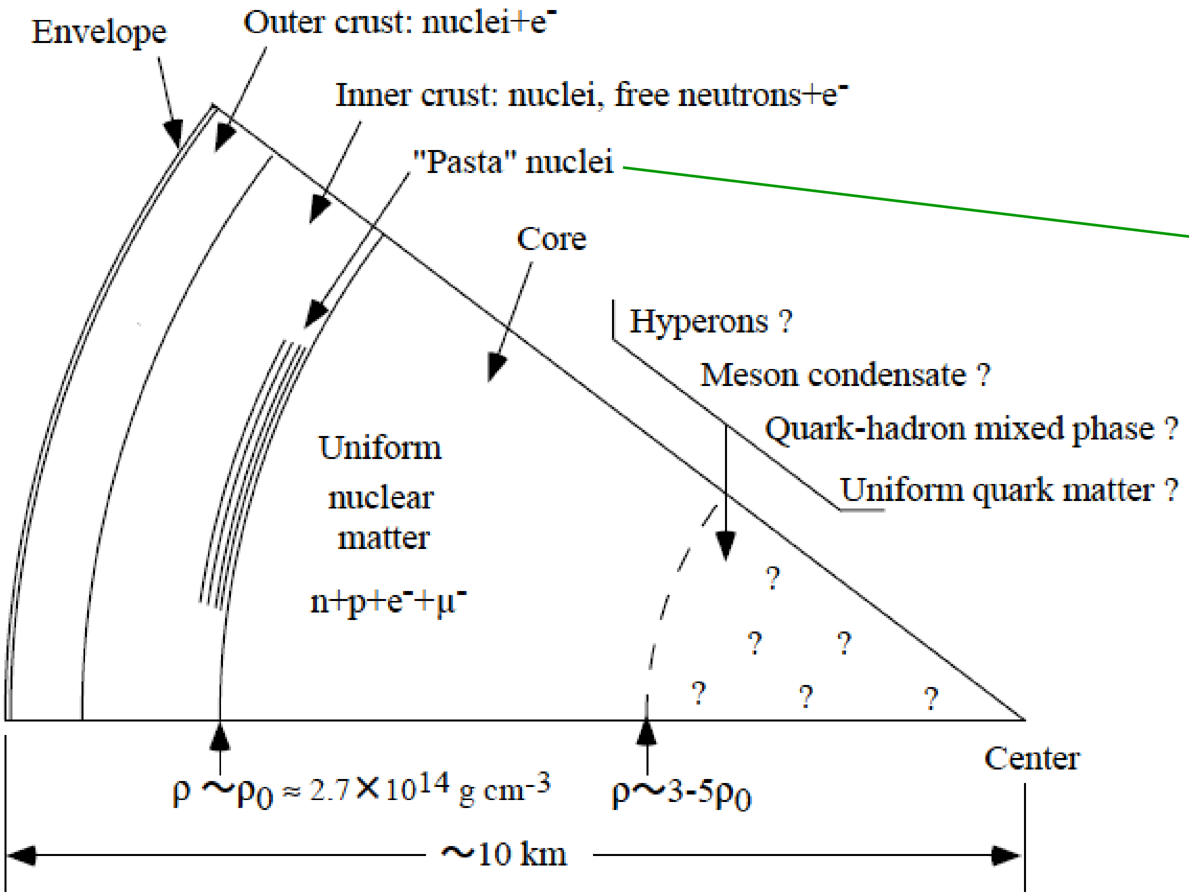
Contents

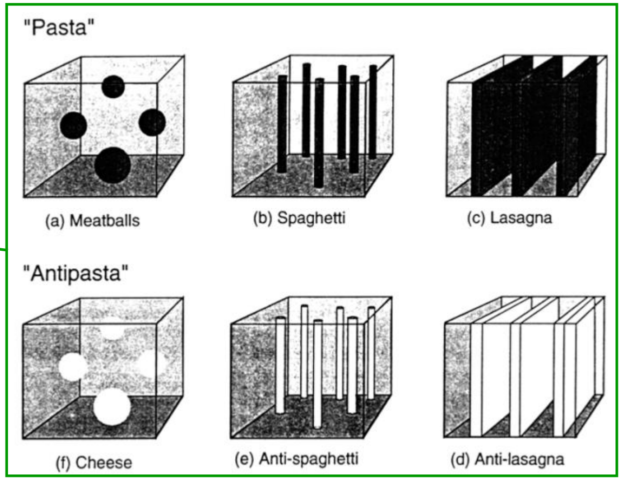
Motivation

Phenomenological EOS model

Macroscopic nuclear model

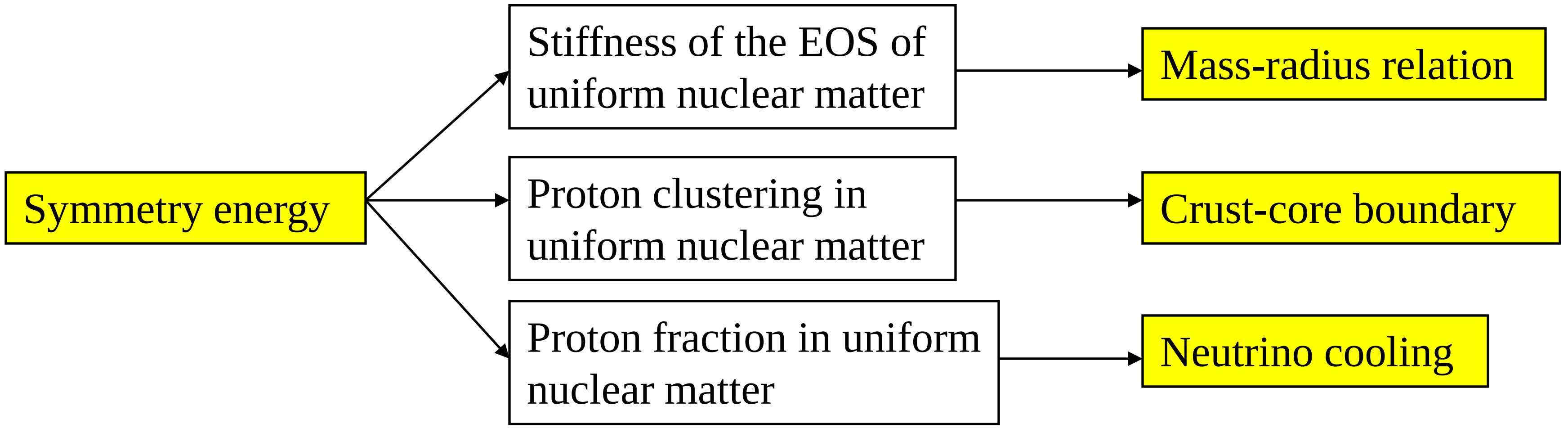
Short summary





From Lamb (1991).

Schematic cross-section of a neutron star.



Motivation

Nuclear matter properties | **Observables**

- Saturation of density and binding energy ························· Masses and radii of stable nuclei

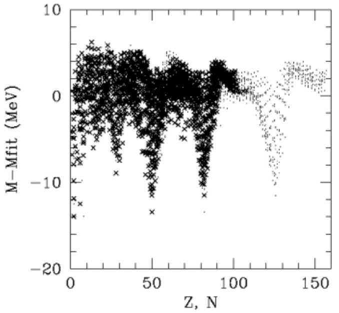
Weizsäcker-Bethe mass formula

$$-E_B = E_{vol} + E_{sym} + E_{sur} + E_{Coul}$$

Saturation energy of symmetric nuclear matter (E_{vol})

Symmetry energy (E_{sym})

Surface energy (E_{sur})



- Incompressibility ··Giant monopole resonances in stable nuclei

$$K_0 = 9(dP/dn)_{n=n_0}$$

Ref. Blaizot, Phys. Rep. **64** (1980) 171.

Caloric curves in nuclear collisions

Ref. Natowitz et al., PRL **89** (2002) 212701.

Our focus:

- Density derivative of the symmetry energy ······························ Radii of *unstable* nuclei

$$L = 3n_0(dS/dn)_{n=n_0}$$

Ref. Oyamatsu & Iida, PTP **109** (2003) 631.

How can one deduce the radii of unstable nuclei from possible future elastic scattering experiments using a radioactive ion beam incident on a proton target?

Ref. Kohama, Iida, & Oyamatsu, PRC **69** (2004) 064316.

Phenomenological EOS model

- Energy per nucleon of bulk nuclear matter (nucleon density n, proton fraction x)

$$w = \frac{3\hbar^2(3\pi^2)^{2/3}}{10mn}\left(n_n^{5/3} + n_p^{5/3}\right) + (1-\alpha^2)v_s(n)/n + \alpha^2 v_n(n)/n$$

$$v_s = a_1 n^2 + \frac{a_2 n^3}{1+a_3 n},\quad v_n = b_1 n^2 + \frac{b_2 n^3}{1+b_3 n},\quad \alpha = 1-2x,\quad n_p = nx, n_n = n(1-x)$$

Near the saturation point of symmetric nuclear matter ($n \to n_0$, $\alpha \to 0$)

$$w = w_0 + \frac{K_0}{18n_0^2}(n-n_0)^2 + \left[S_0 + \frac{L}{3n_0}(n-n_0)\right]\alpha^2$$

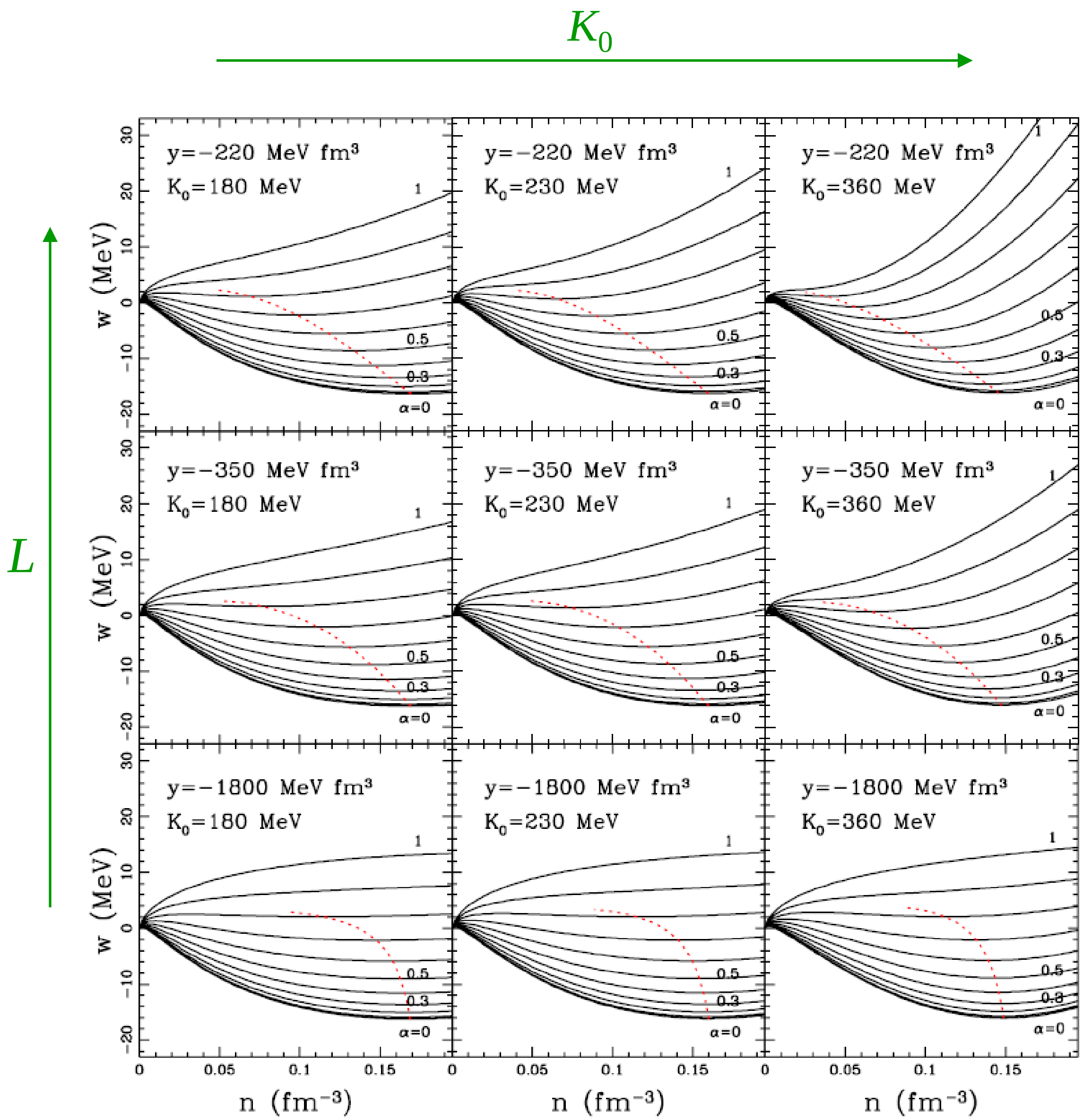
n_0, w_0	saturation density & energy of symmetric nuclear matter
S_0	symmetry energy coefficient
K_0	incompressibility
L	density symmetry coefficient

well known from masses and radii of stable nuclei (n_0, w_0, S_0)

S_0, K_0, L: mainly determine the saturation points at finite neutron excess as

$$n_s = n_0 - \frac{3n_0 L}{K_0}\alpha^2$$

$$w_s = w_0 + S_0\alpha^2$$



saturation line

(y=-$K_0S_0/3n_0L$:   slope at α=0)

Ref. Oyamatsu & Iida, PTP **109**(2003)631.

Macroscopic nuclear model

- Binding energy of a nucleus

$$-E_{\mathrm{B}} = \int d^3r\, n(\mathbf{r}) w\big(n_{\mathrm{n}}(\mathbf{r}), n_{\mathrm{p}}(\mathbf{r})\big) + F_0 \int d^3r\, |\nabla n(\mathbf{r})|^2 + \frac{e^2}{2}\int d^3r \int d^3r' \frac{n_{\mathrm{p}}(\mathbf{r}) n_{\mathrm{p}}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

- Particle distributions (i=n, p)

$$n_i(r) = \begin{cases} n_i^{\mathrm{in}}\left[1-(r/R_i)^{t_i}\right]^3, & r < R_i \\ 0, & r \geq R_i \end{cases}$$

1. Thomas-Fermi approximation

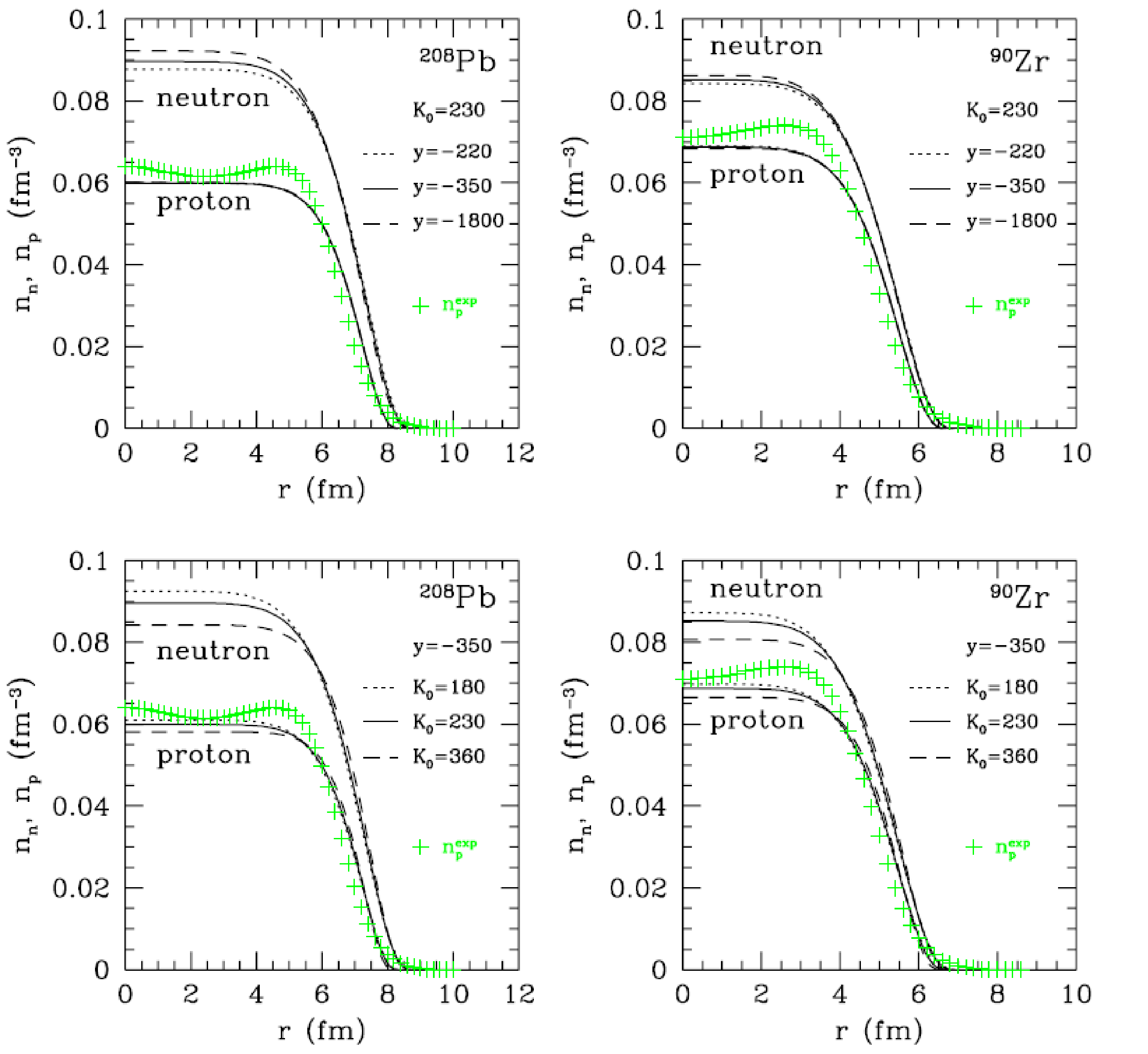
Ref. Oyamatsu, NPA **561** (1993) 431.

extremize the binding energy E_{B} with respect to the particle distributions for fixed mass number A, EOS parameters (n_0, w_0, S_0, K_0, L), and gradient coefficient F_0.

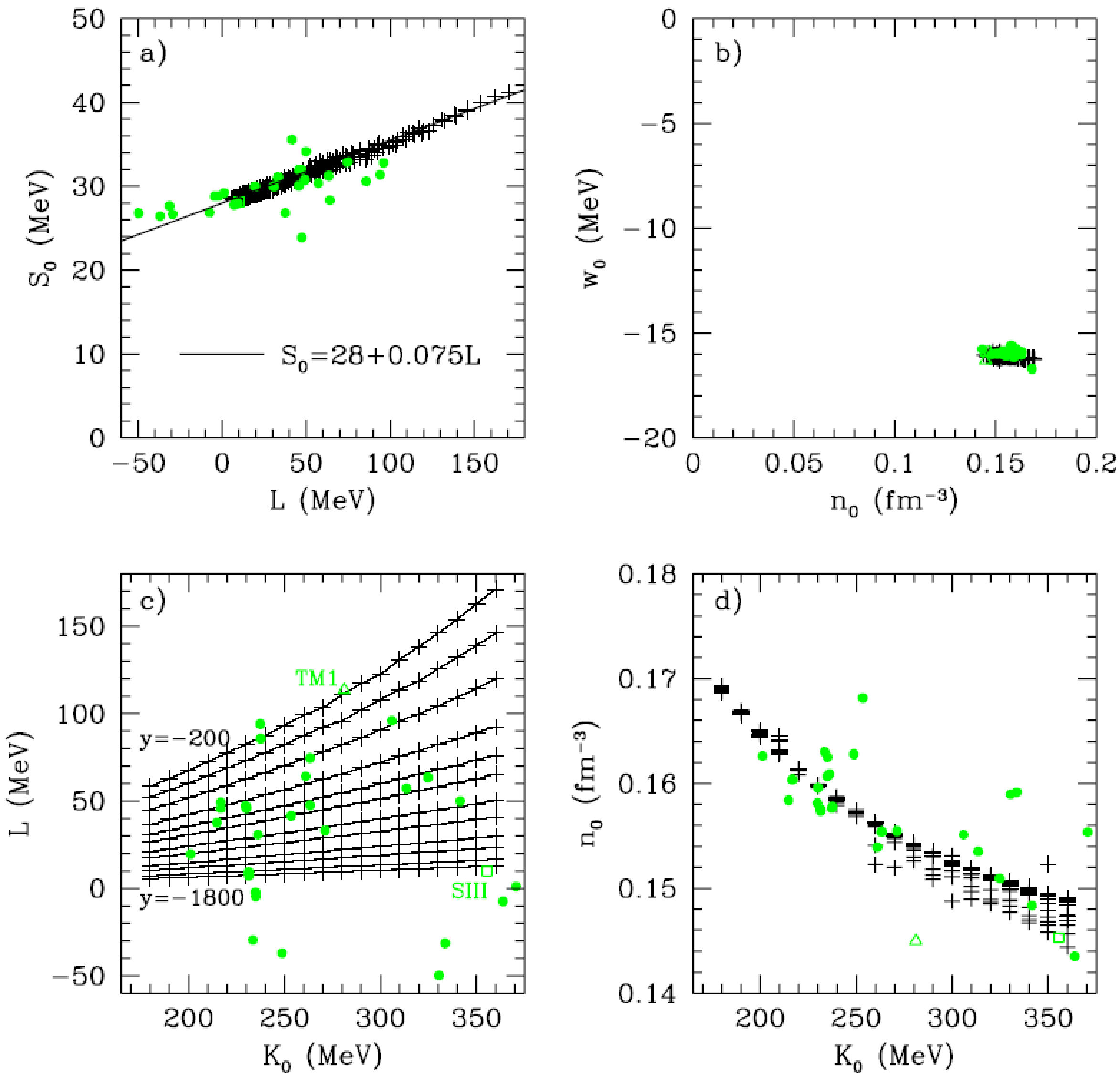
2. Fitting to empirical masses and radii of stable nuclei

Ref. Oyamatsu & Iida, PTP **109** (2003) 631.

obtain the EOS parameters for various sets of (K_0, L) by fitting the calculated optimal values of charge number Z, mass, and rms charge radius R_{c} to empirical data for stable nuclei ($25 \leq A \leq 245$) on the smoothed β stability line.



Ref. Oyamatsu & Iida,
PTP **109**(2003)631.



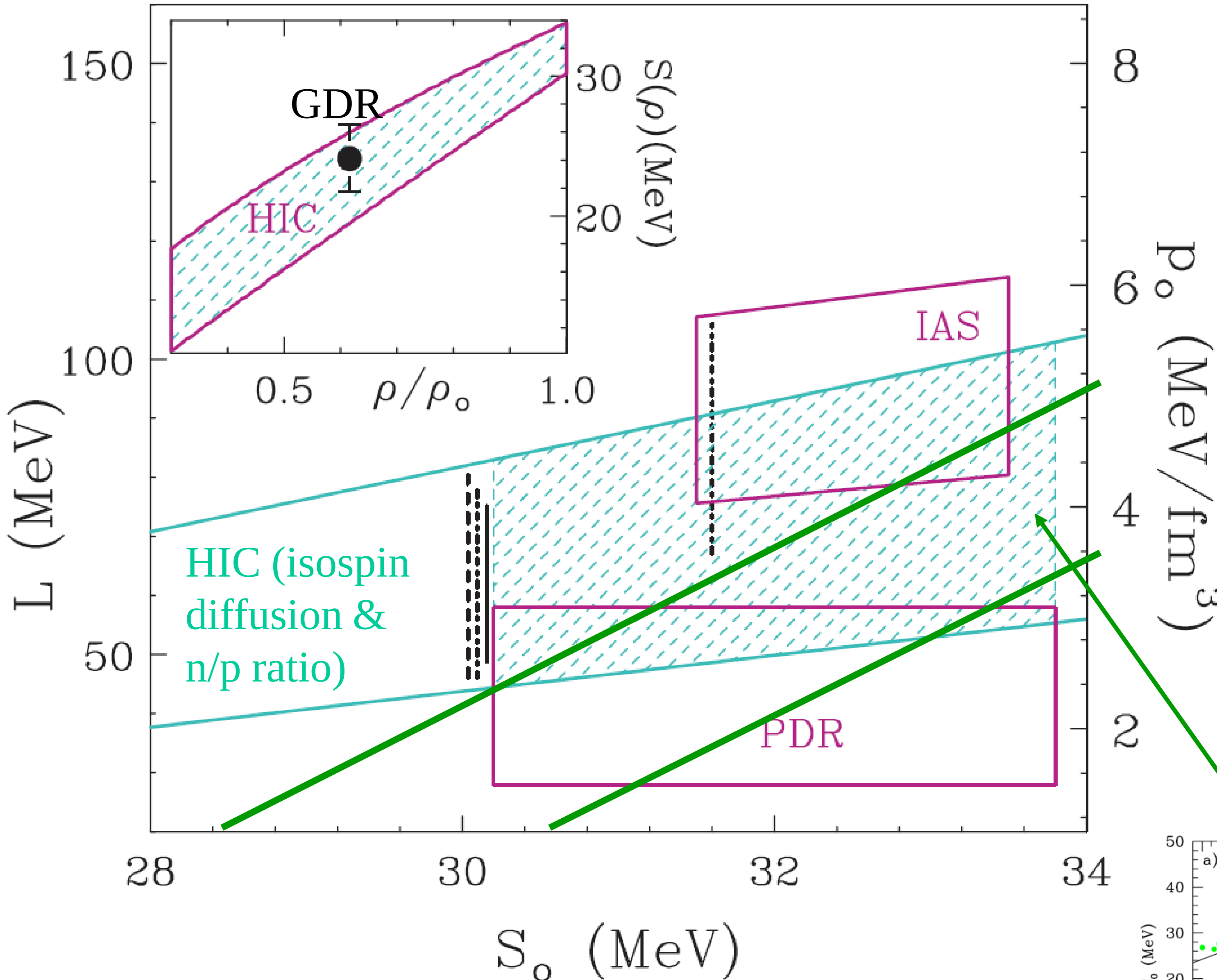
\+ from fitting to empirical masses & radii of stable nuclei

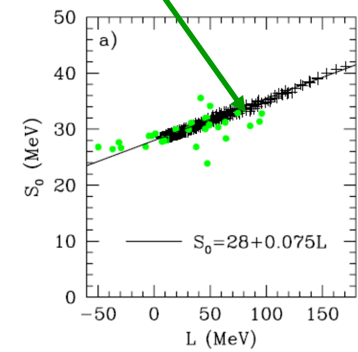
● Skyrme Hartree-Fock models

□ SIII

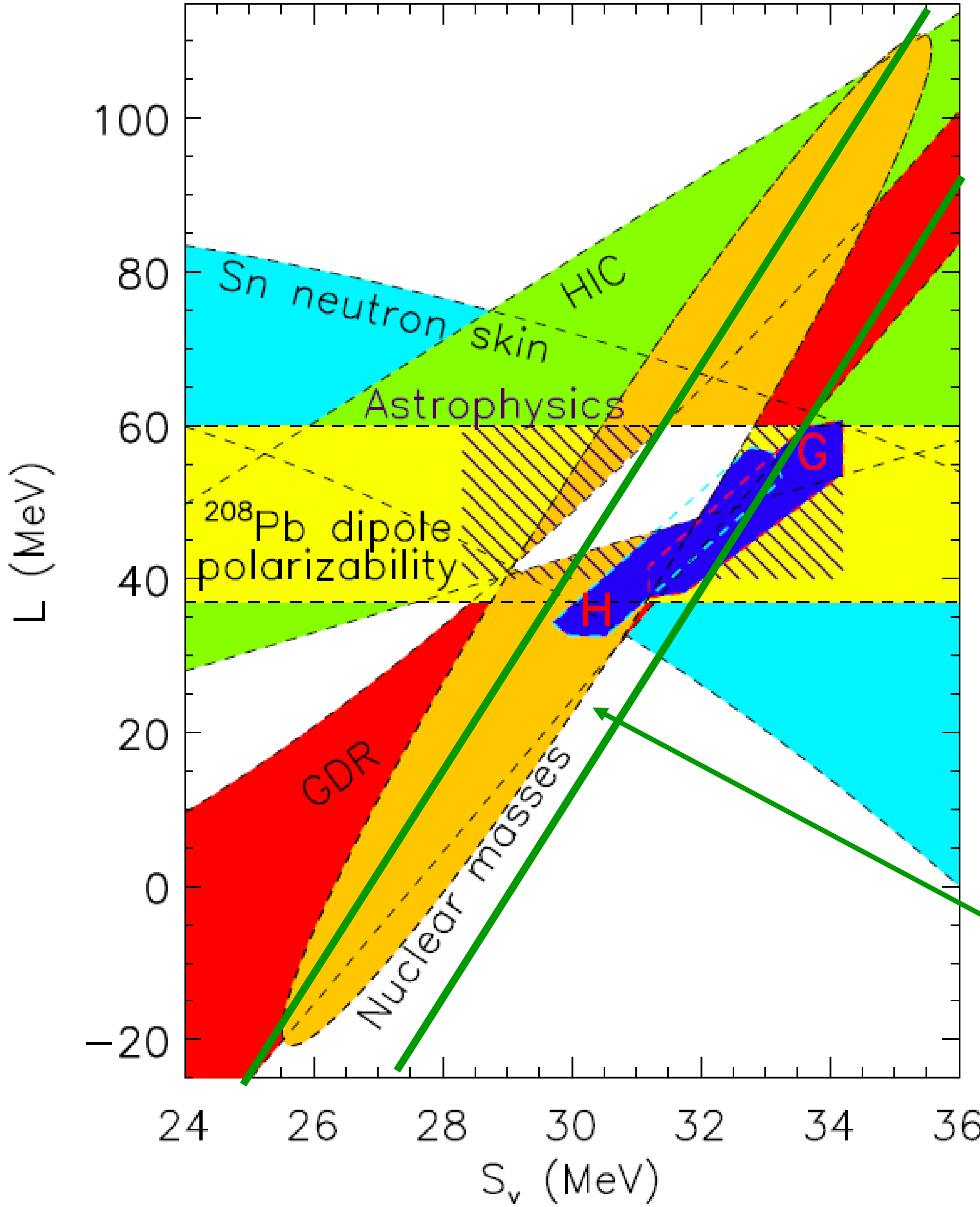
△ TM1

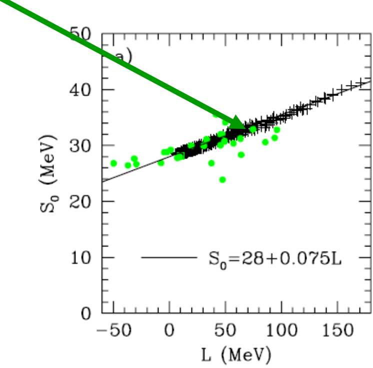
Ref. Oyamatsu & Iida, PTP **109**(2003)631.





Ref. Tsang et al., PRL **102**(2009)122701.





Ref. Lattimer & Lim, arXiv:1203.4286.

Macroscopic nuclear model (contd.)

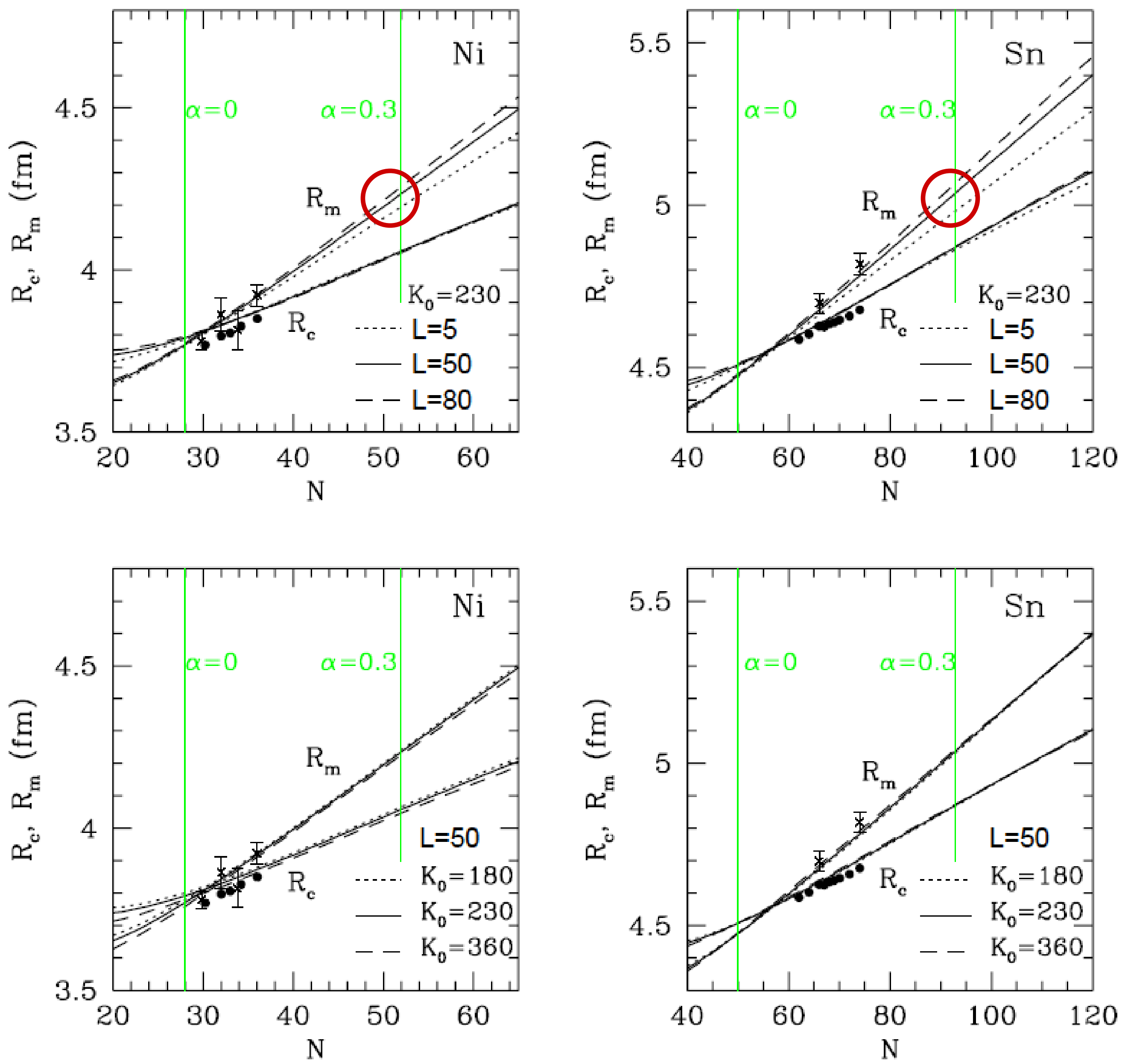
This model

— good at describing global nuclear properties (e.g., masses, rms radii) in a manner that is dependent on the EOS of nuclear matter

— predicts that the matter radii (R_m) depend appreciably on L while being almost independent of K_0

— not so good at describing the nuclear surface (e.g., diffuseness, skin thickness) in the semi-classical Thomas-Fermi approximation adopted here

— does not allow for shell or pairing effects



Ref. Oyamatsu & Iida, PTP **109**(2003)631.

Short summary

1. Derivation of the relations between the EOS parameters from experimental data on radii and masses of stable nuclei

   — L and K_0 are still uncertain.

2. The density-symmetry coefficient L could be determined if a global behavior of matter radii at large neutron excess is obtained from future systematic measurements of matter radii of unstable nuclei.

3. The parameter L characterizing the dependence of the EOS on neutron excess is relevant to the structure and evolution of neutron stars in various ways (mass-radius relation, crust-core boundary, cooling, etc.).

Nuclear masses and the equation of state of nuclear matter

Question

Are existing data for masses of unstable nuclei useful for determination of L?

Contents

Two-proton separation energy

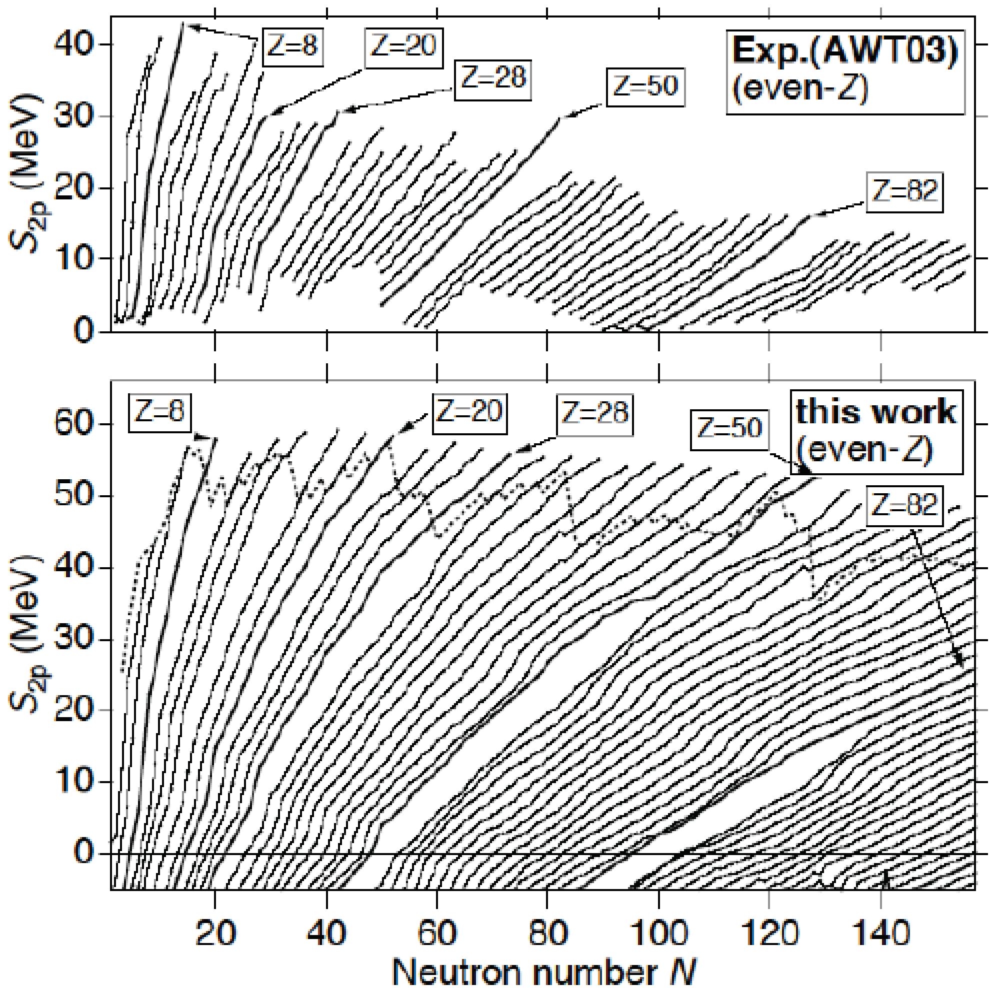
L dependence of nuclear masses

L dependence of the neutron drip line

Two-proton separation energy

Ref. Koura et al., PTP **113** (2005) 305.

Why S_{2p}?



· **Smooth isospin dependence except for shell gaps**

· **Even-odd staggering essentially cancelled out**

Two-proton separation energy (contd.)

Ref. Oyamatsu & Iida, PRC **81** (2010) 054302.

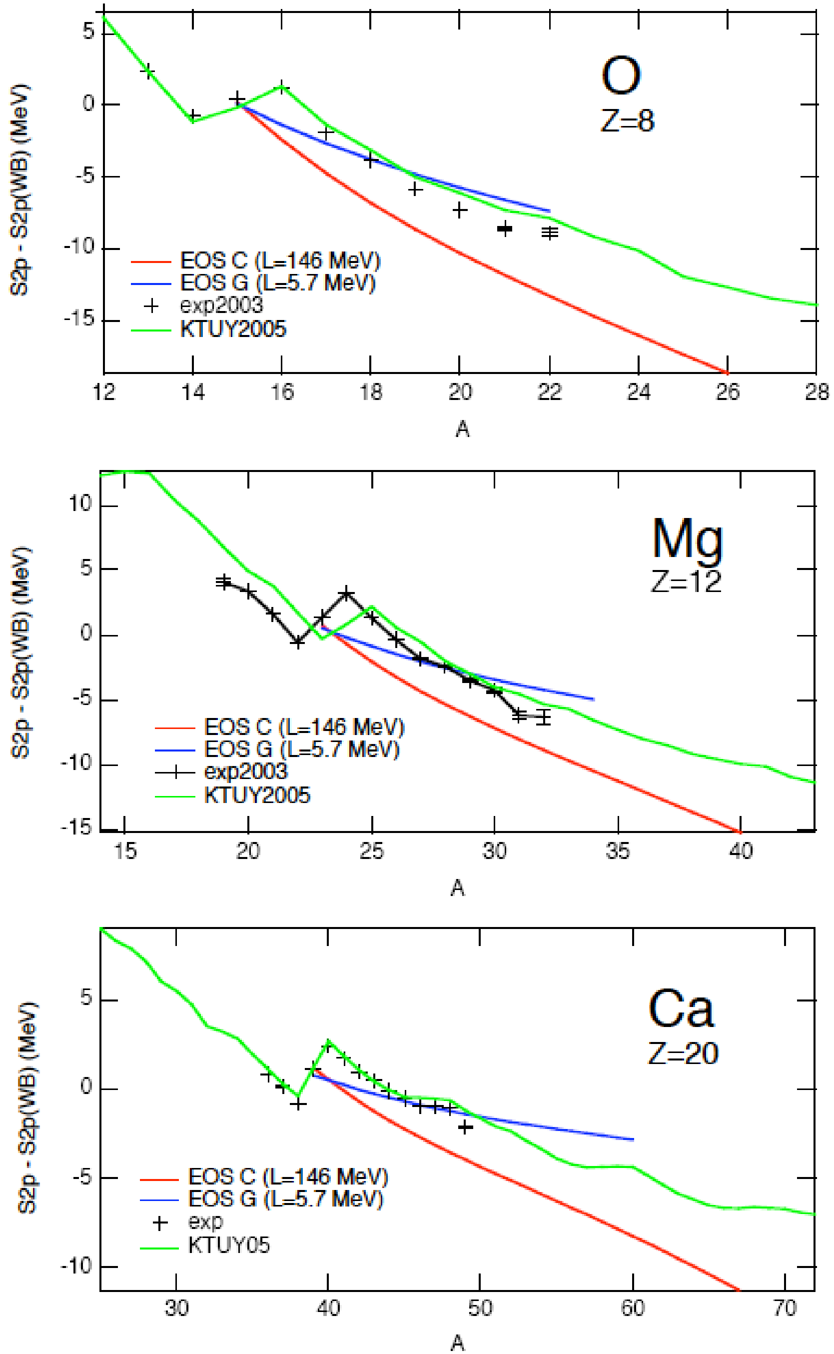
Comparison

· The empirical S_{2p} shows a smooth dependence on neutron excess except for the regimes of $N=Z$, magic N's, and deformation.

· The calculated S_{2p} shows a larger neutron excess dependence for larger L.

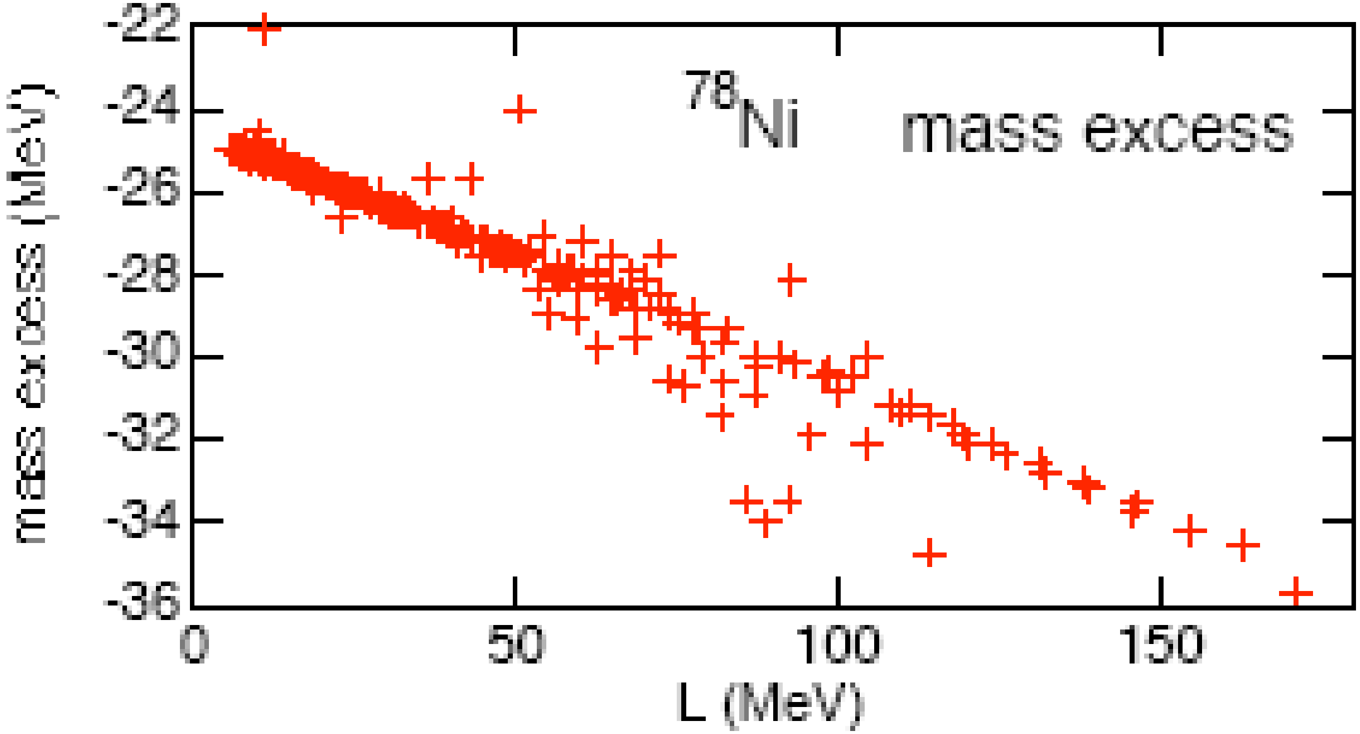
· Comparison between the empirical and calculated S_{2p} looks easier for smaller mass.

· A larger L is more consistent with the empirical data.



L dependence of the calculated nuclear masses

Ref. Oyamatsu & Iida, PRC **81** (2010) 054302.



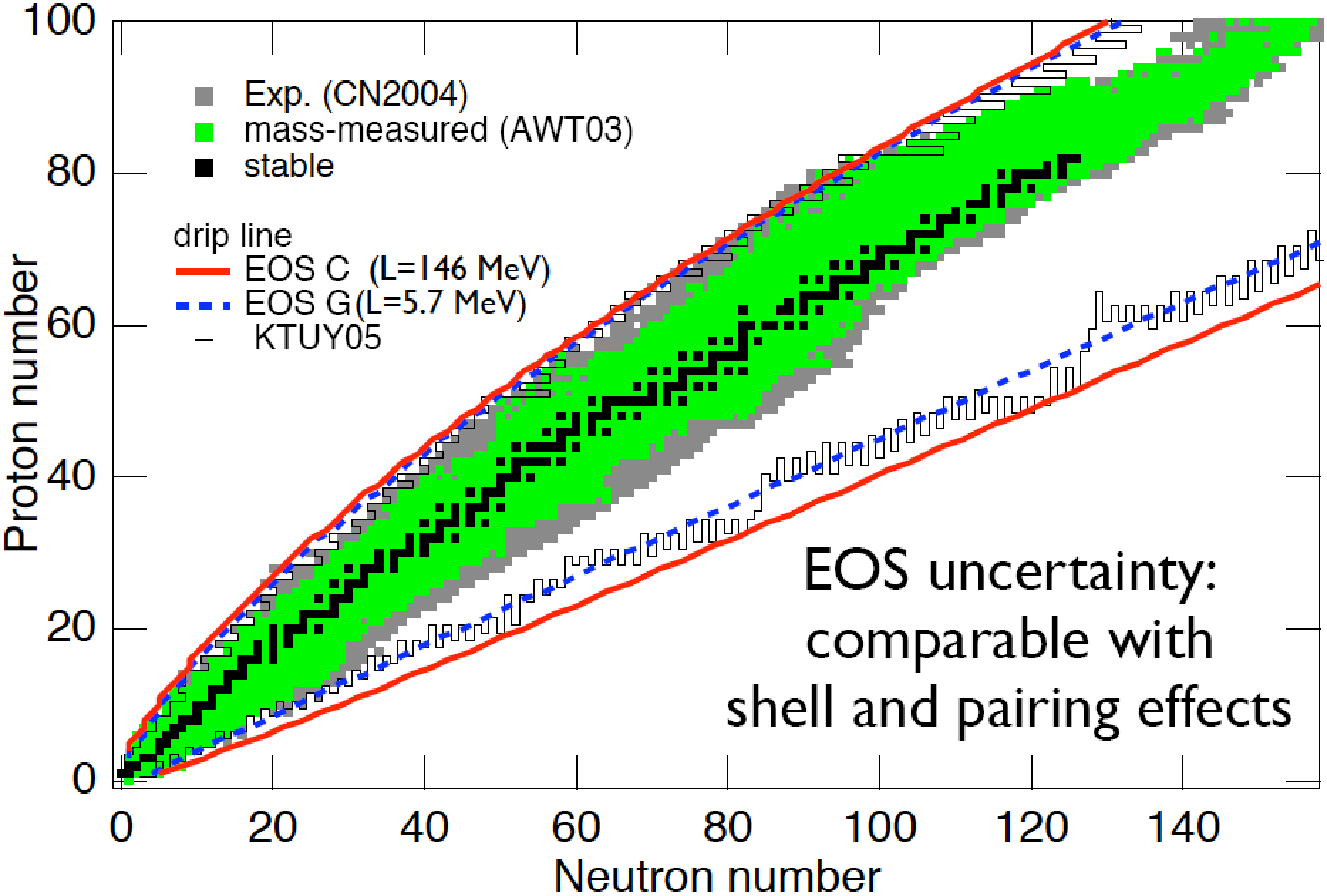
· **Nuclear masses are not always dominated by the bulk properties of nuclear matter.**

· **In fact, the *L* dependence of the calculated mass cannot be explained by the bulk asymmetry term ($L \uparrow \rightarrow S_0 \uparrow \rightarrow$ mass $\uparrow$) .**

· **The surface asymmetry term ($L \uparrow \rightarrow$ surface tension $\downarrow$) is responsible for the *L* dependence of the calculated mass.**

Ref. Iida & Oyamatsu, PRC **69** (2004) 037301.

L dependence of the neutron drip line



Oyamatsu, Iida and H. Koura, PRC 82, 027301, 2010.

Neutron star crusts and the equation of state of nuclear matter

Question

Is the presence of nuclear pasta in neutron stars sensitive to the EOS?

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Motivation

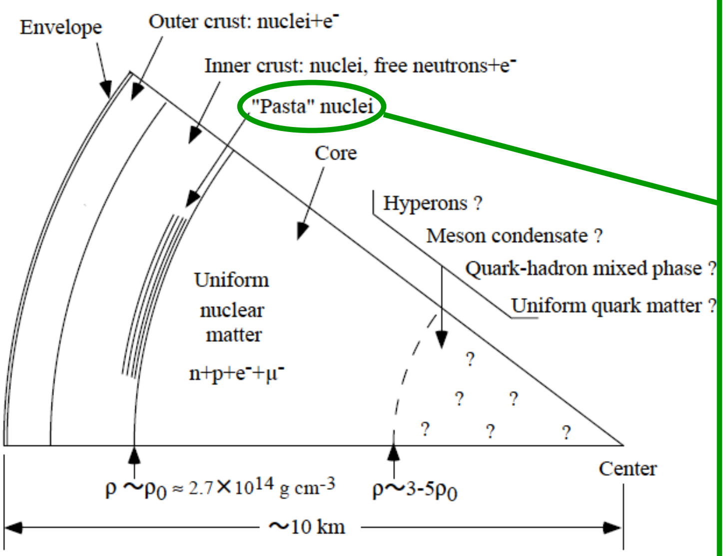
Phase diagrams from macroscopic nuclear model

Gyroid

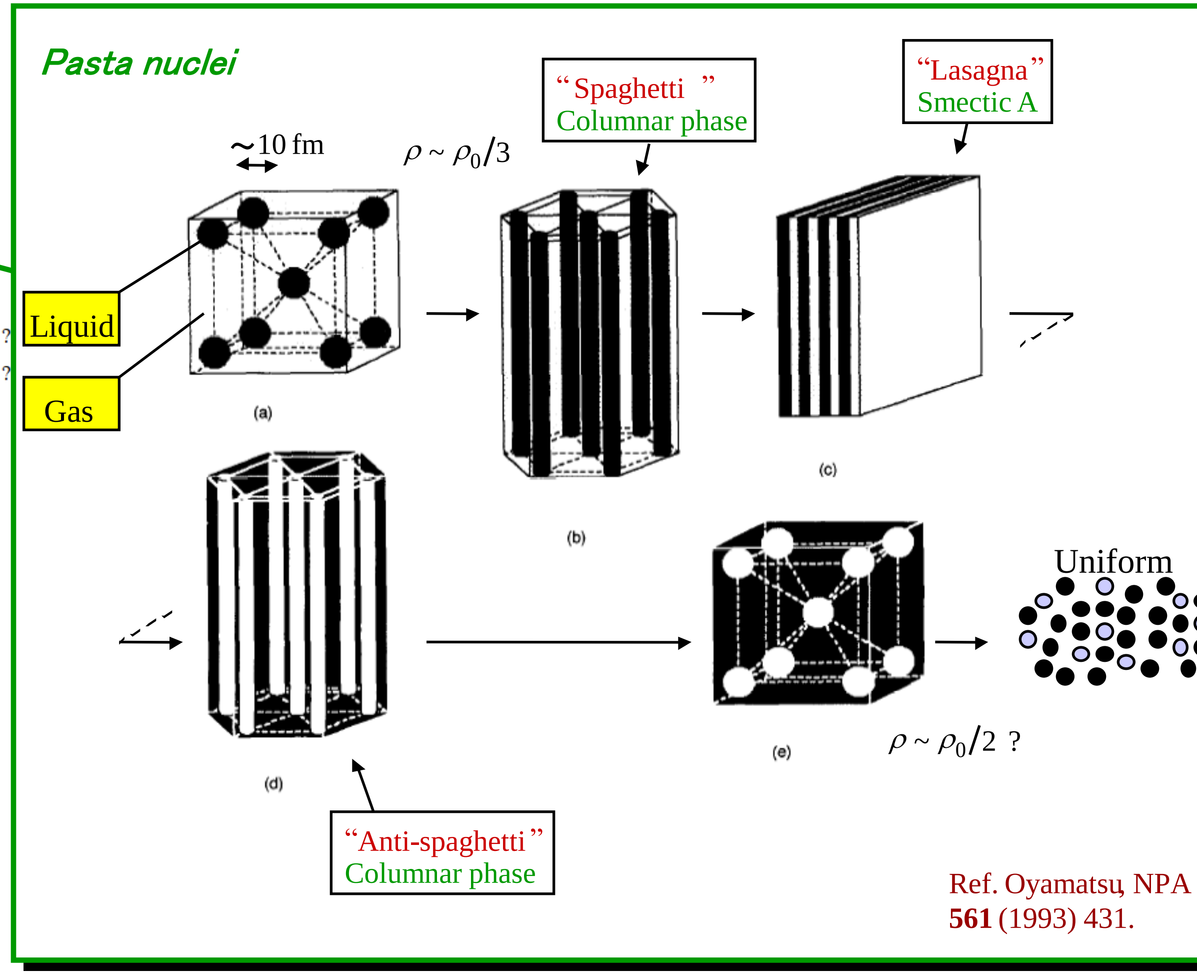
Pastas at finite temperatures

Crustal osillations

Motivation



Schematic cross-section of a neutron star.



Symmetry energy at subnuclear densities → Proton clustering in uniform nuclear matter → Crust-core boundary

Symmetry energy at subnuclear densities → Density of dripped neutrons in the inner crust → Nuclear size

Phase diagrams from macroscopic nuclear model

Macroscopic nuclear model

For a nucleus in vacuum: Ref. Oyamatsu & Iida, PTP **109** (2003) 631.

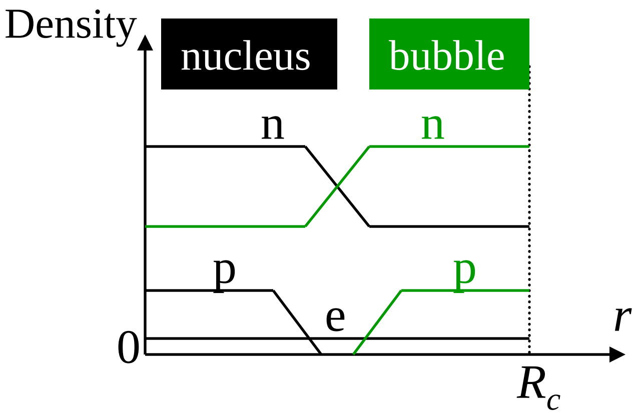
$$-E_B = \int d^3r\, n(\mathbf{r}) w\big(n_n(\mathbf{r}), n_p(\mathbf{r})\big) + F_0 \int d^3r\, |\nabla n(\mathbf{r})|^2 + \frac{e^2}{2}\int d^3r \int d^3r' \frac{n_p(\mathbf{r}) n_p(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$$

with particle distributions (i=n,p): $$n_i(r) = \begin{cases} n_i^{\rm in}\left[1-(r/R_i)^{t_i}\right]^3, & r < R_i \\ 0, & r \ge R_i \end{cases}$$

For a nucleus or bubble in a Wigner-Seitz cell: Ref. Oyamatsu, NPA **561** (1993) 431.

Total energy of a cell: total rest mass -E_B+lattice energy+electron energy

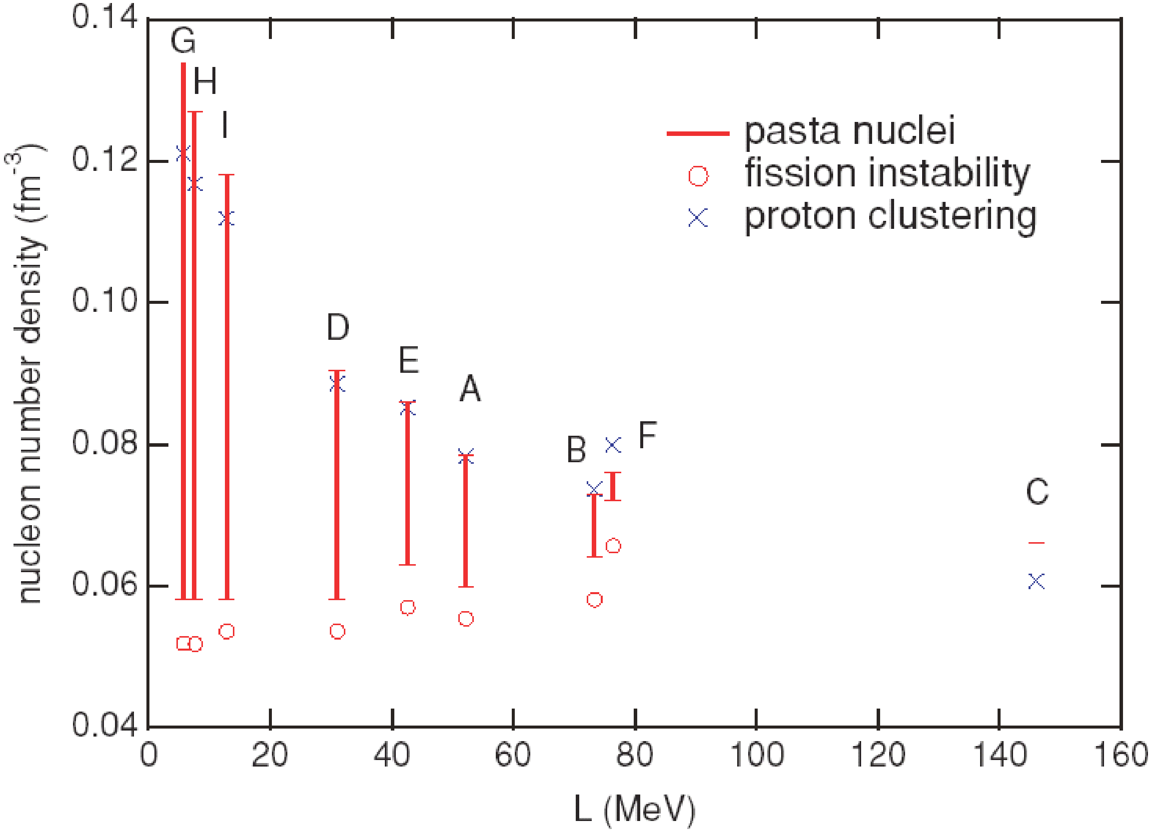
with $$n_i(r) = \begin{cases} \left(n_i^{\rm in} - n_i^{\rm out}\right)\left[1-(r/R_i)^{t_i}\right]^3 + n_i^{\rm out}, & r < R_i \\ n_i^{\rm out}, & r \ge R_i \end{cases}$$



Phase diagrams from macroscopic nuclear model (contd.)

Pasta region

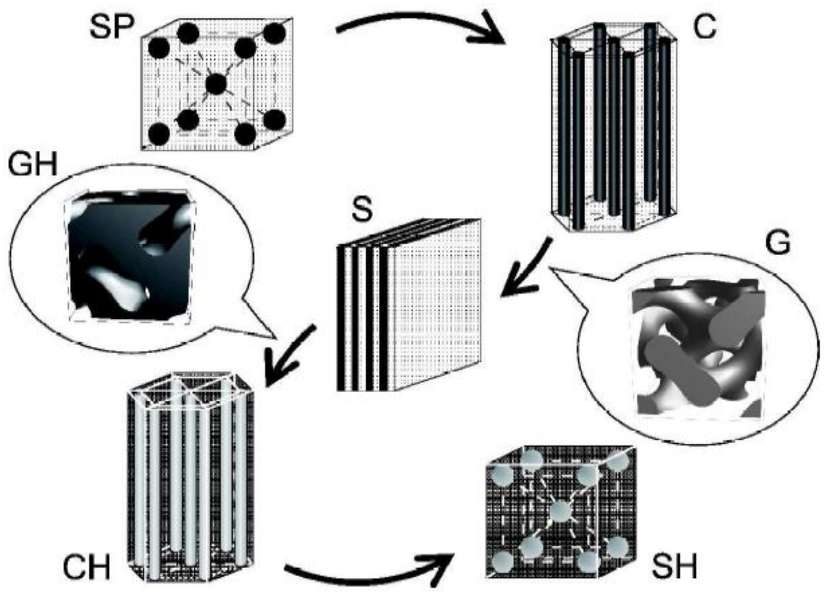
Ref. Oyamatsu & Iida, PRC **75** (2007) 015801.



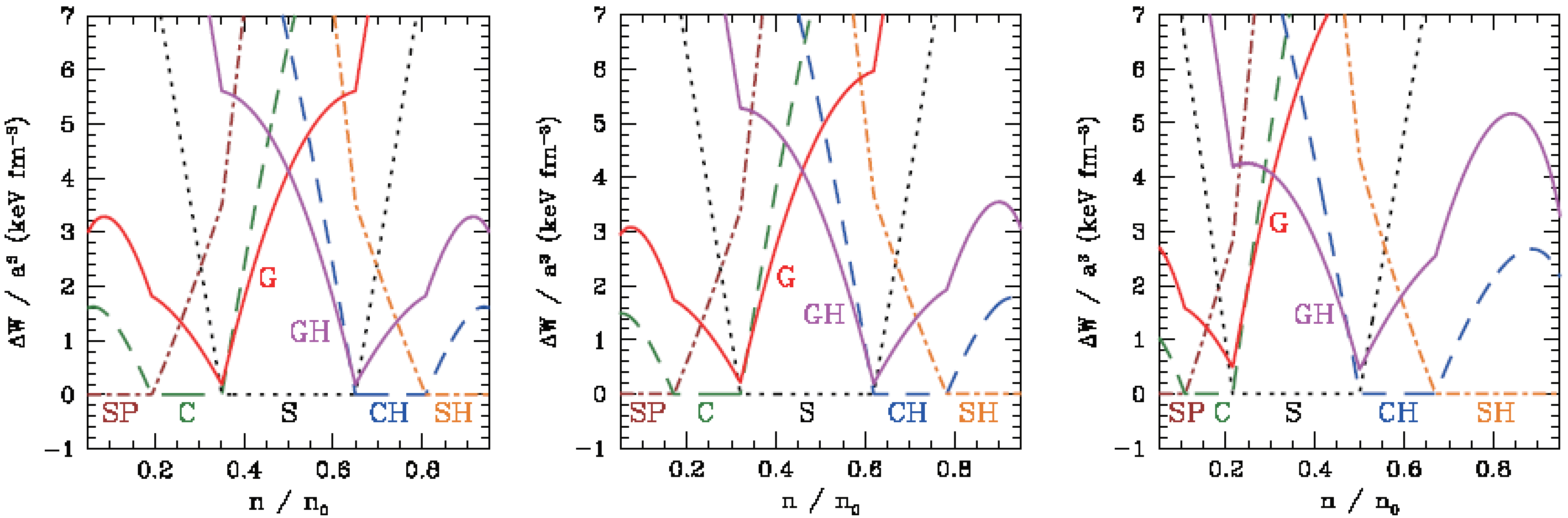
The larger *L*, the narrower pasta region.

Gyroid

Ref. Nakazato, Iida, & Oyamatsu, arXiv:1011.3866.

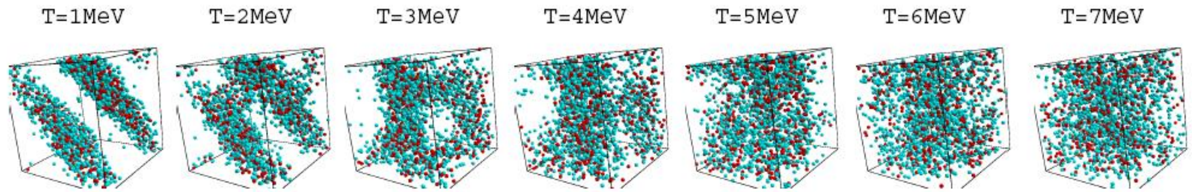


Curvature corrections (x=0.3)

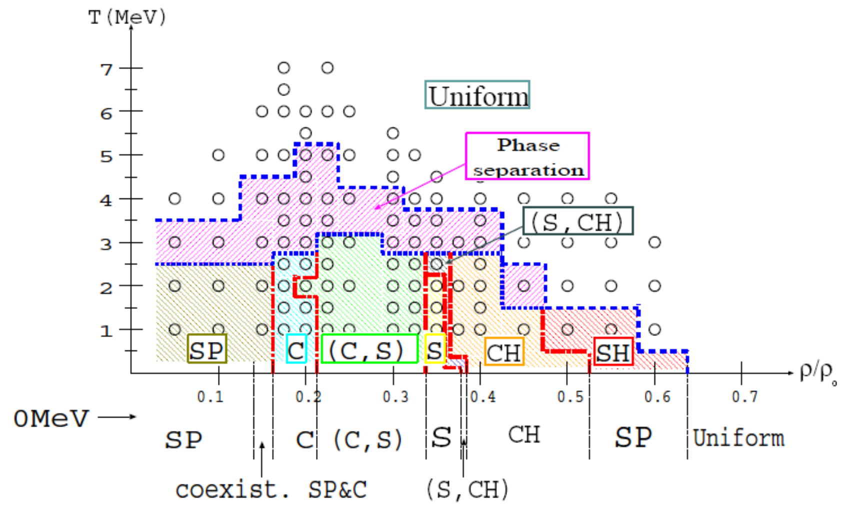


Pastas at finite temperatures

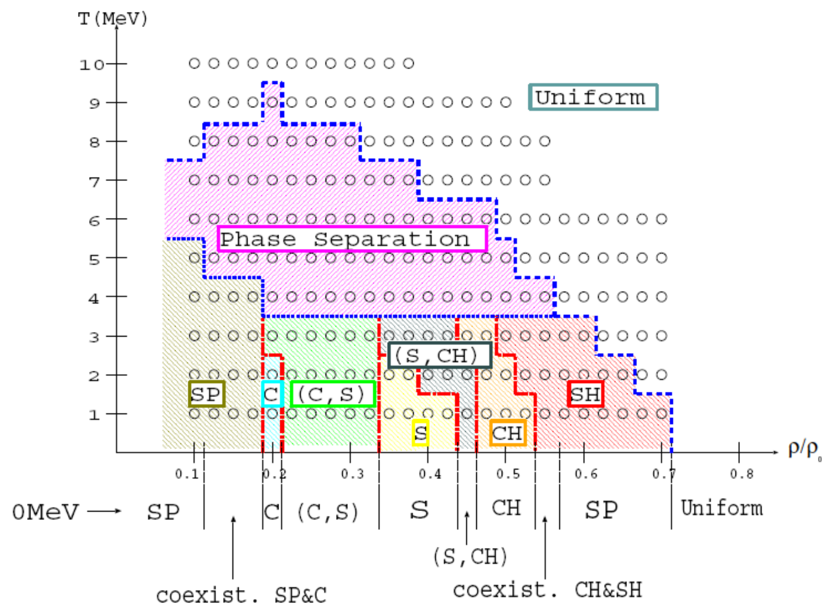
Ref. Sonoda, PhD thesis (2009, U. Tokyo).



QMD model 1
(*L*=93 MeV)



QMD model 2
(*L*=80 MeV)



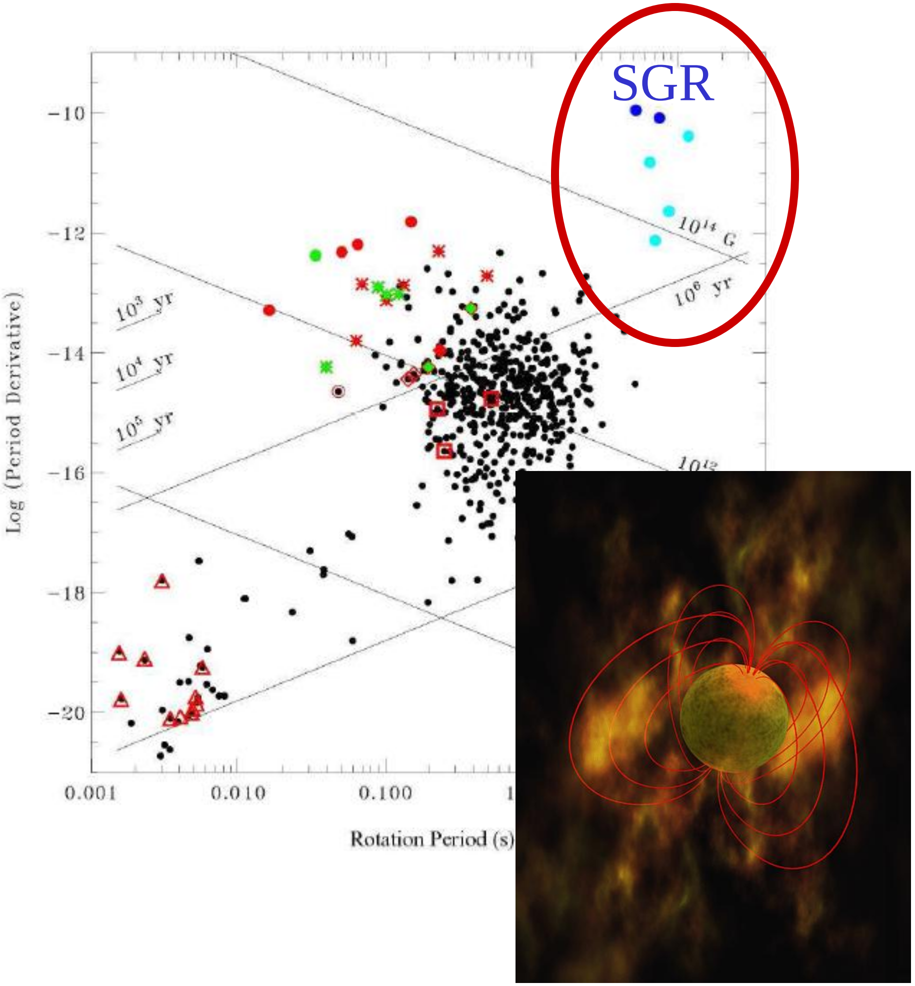
x=0.3

$T < T_c$=15–20 MeV

relevant for collapsing supernova cores

QPOs in giant flares from SGRs

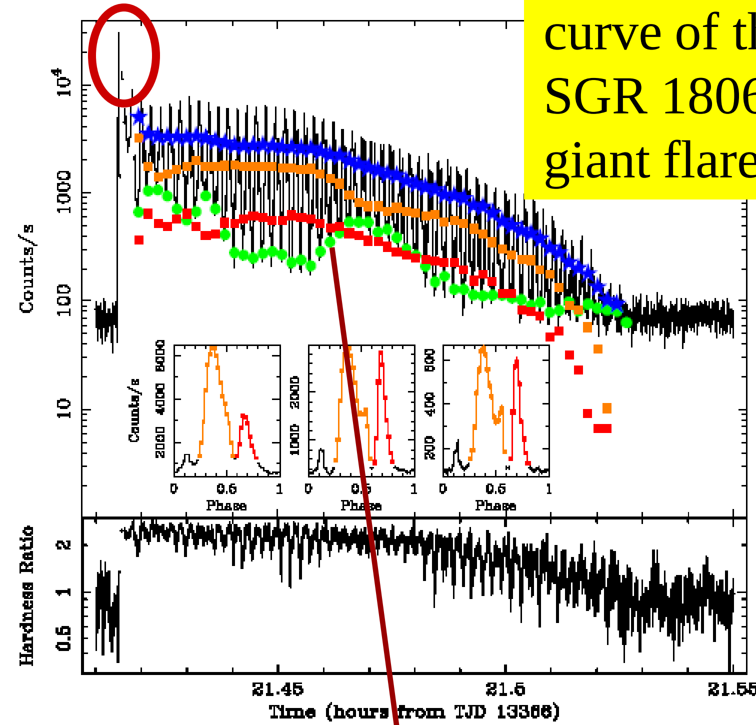
Magnetars



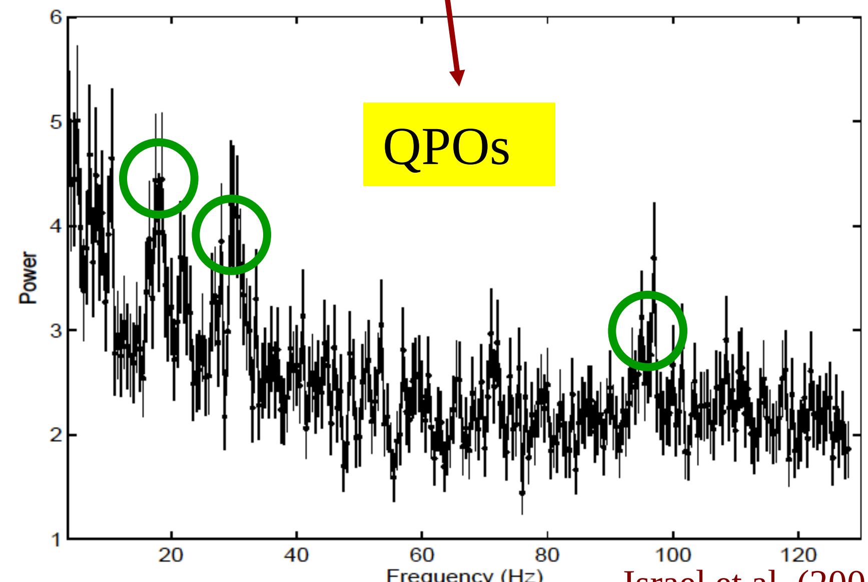
arXiv:astro-ph/0208356

image
by NASA

X-ray light curve of the SGR 1806-20 giant flare



QPOs

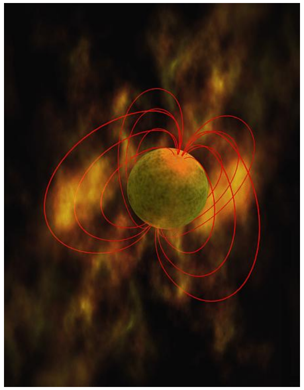


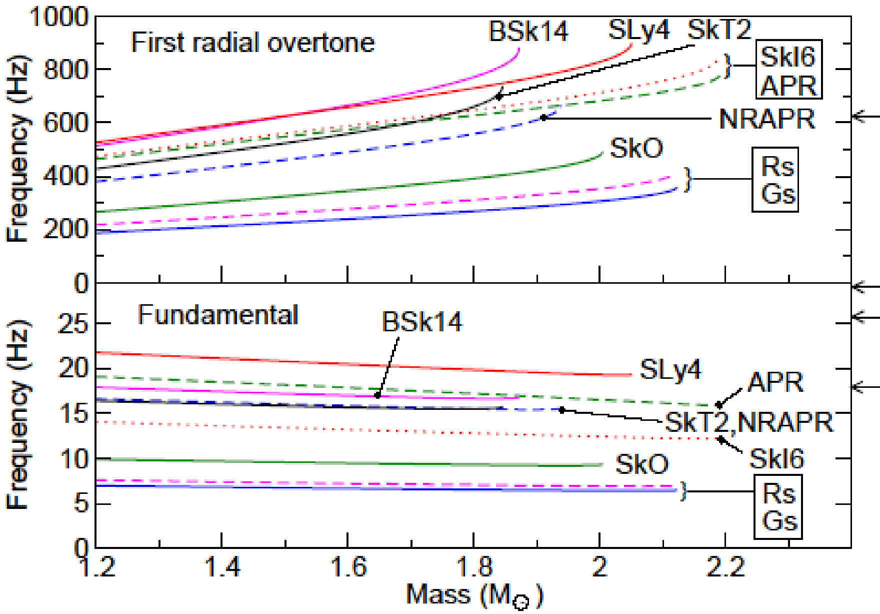
Israel et al. (2005)

QPOs in giant flares from SGRs (contd.)

If unidentified as shear modes, then Alfven modes?

QPOs in the 2004 giant flare can be identified as crustal shear modes?





Observed QPO frequencies

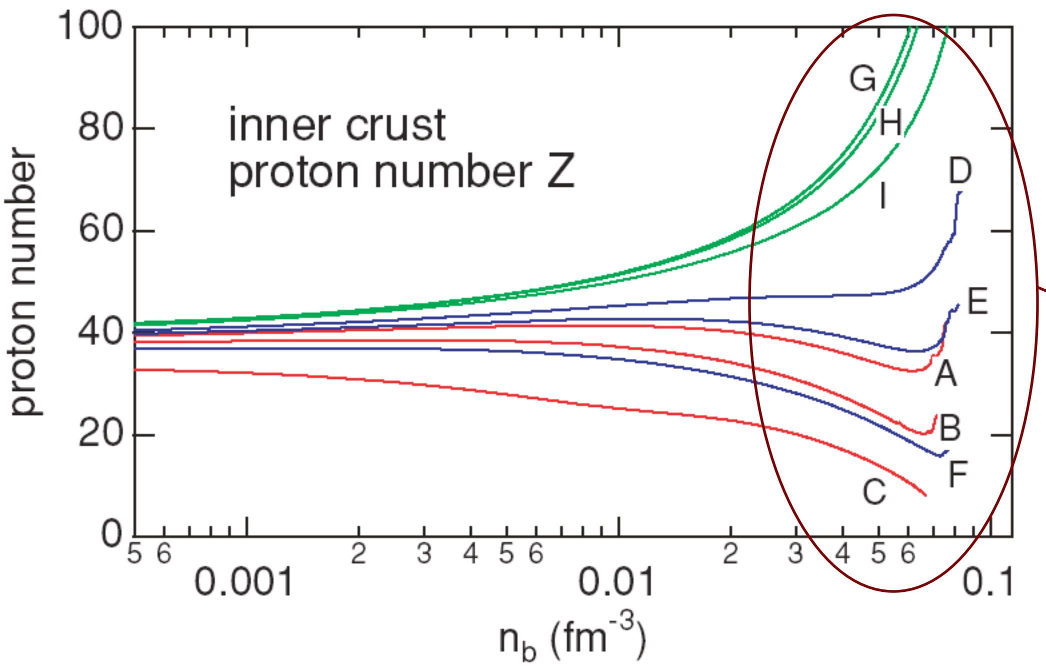
FIG. 3: The crust oscillation frequencies as a function of neutron star mass, for both the fundamental ($n = 0, l = 2$) torsional shear mode and the first radial ($n = 1$) overtone. The curves end at the maximum mass. The arrows on the right indicate QPO frequencies measured during the 2004 hyperflare from SGR 1806-20 [2, 4, 5].

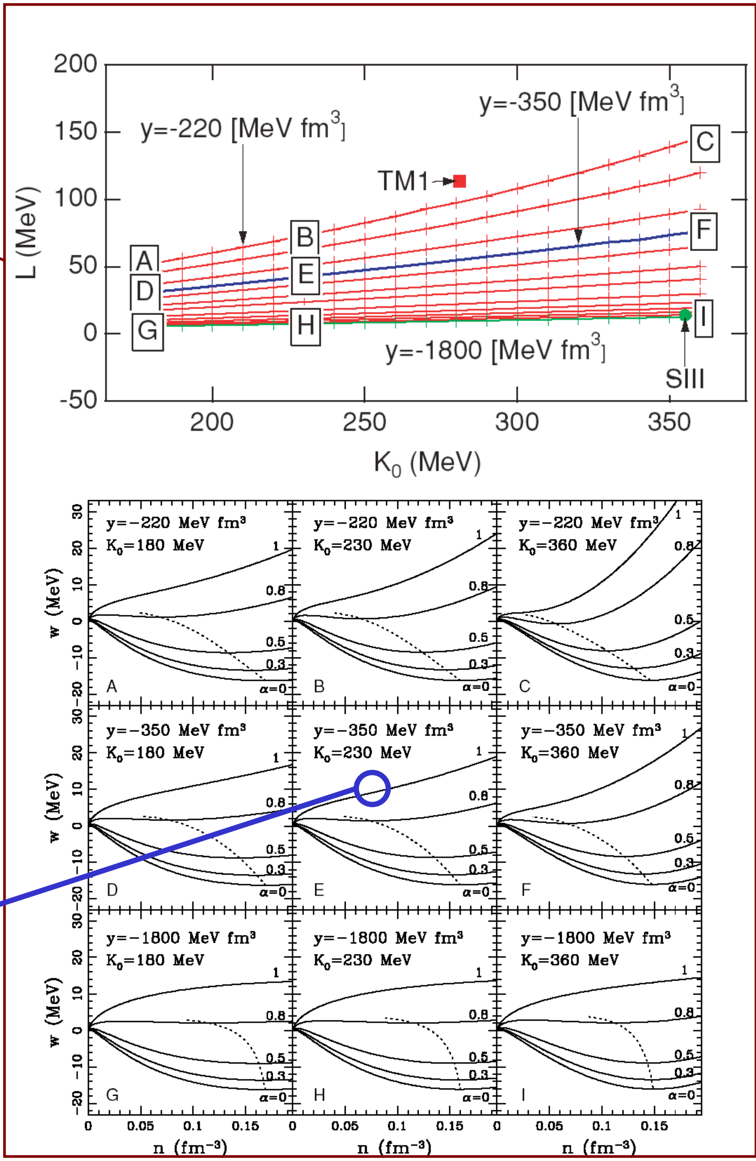
Steiner & Watts (2009)

Possible constraint on L via the relation between the shear modulus ($\propto Z^2$) and L!

Equilibrium nuclear size in the inner crust of a neutron star

Ref. Oyamatsu & Iida, PRC **75** (2007) 015801.



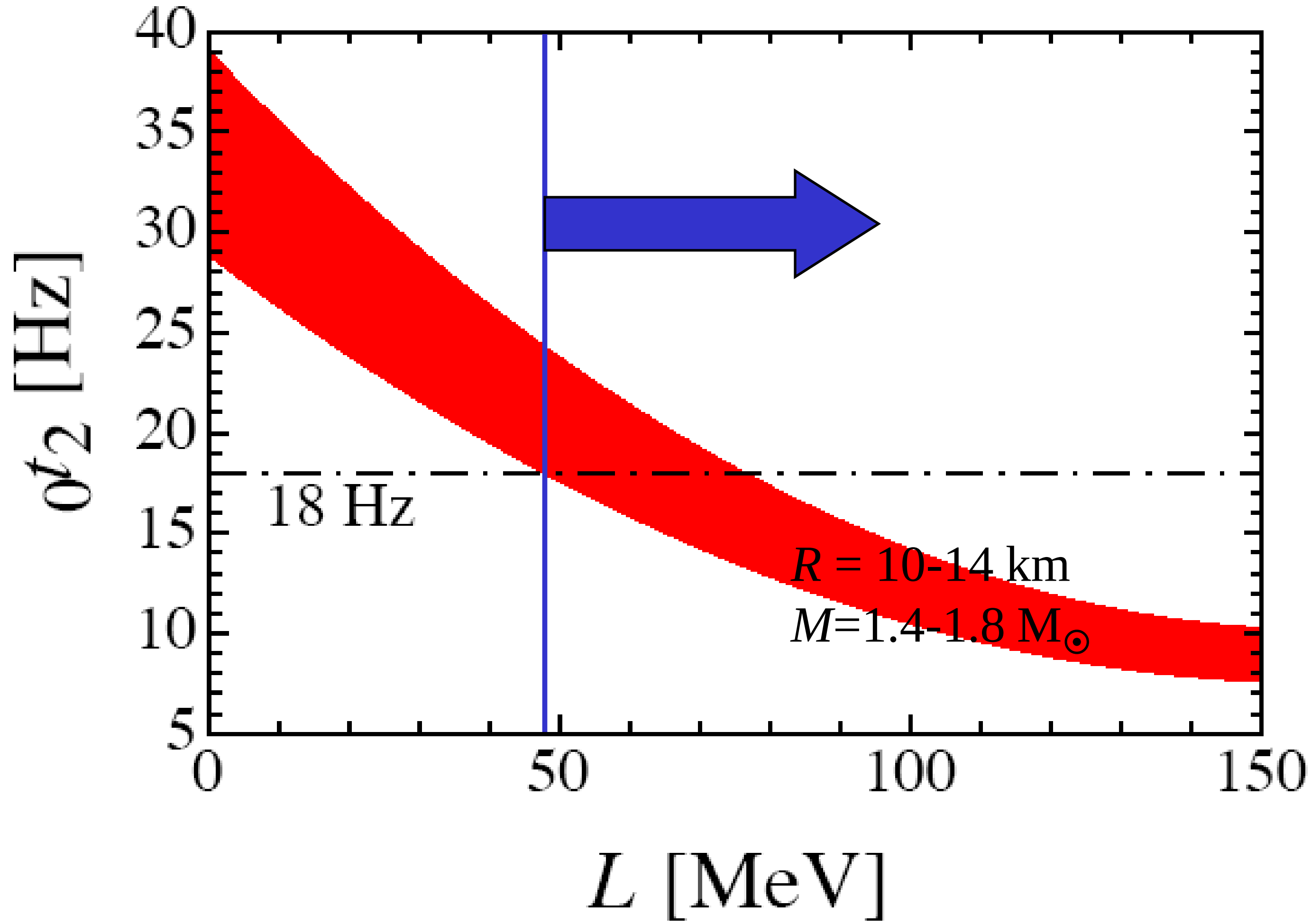


The larger *L*, the smaller size.

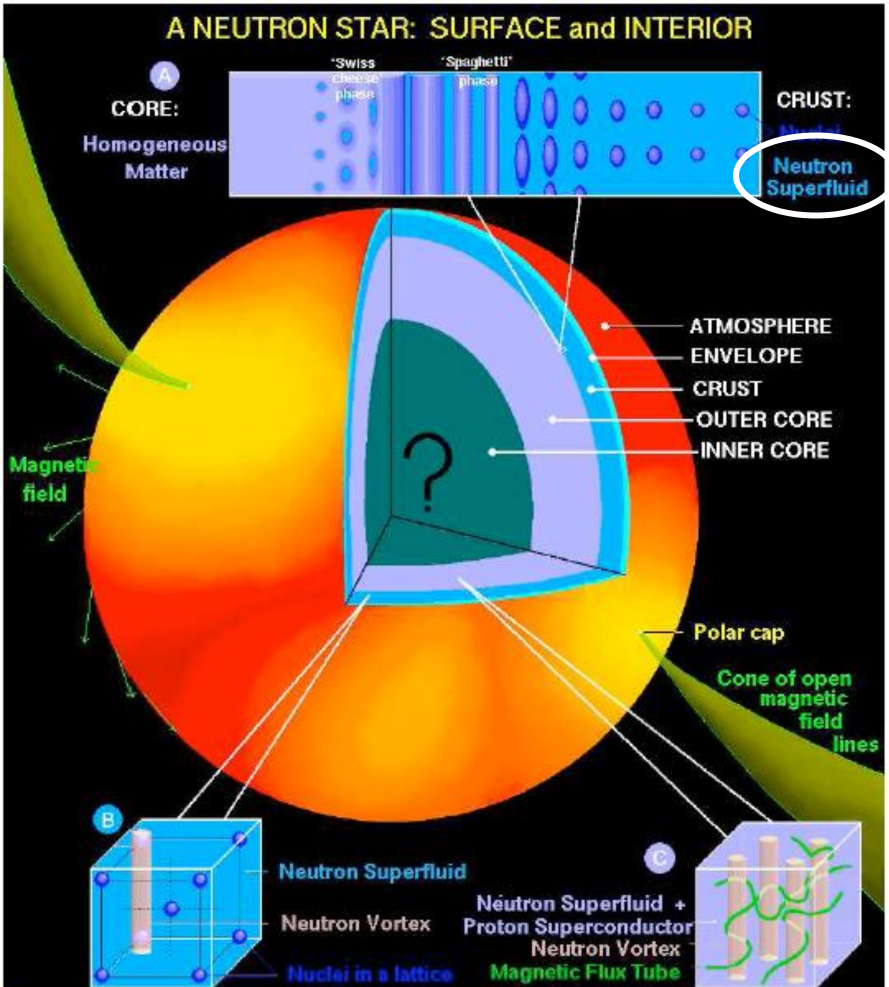
close to the GFMC result with the Argonne v_8' potential

Constraint on L from estimates of crustal torsional oscillation frequencies

Ref. Sotani, Nakazato, Iida, & Oyamatsu, PRL **108** (2012) 201101.



Effects of superfluidity

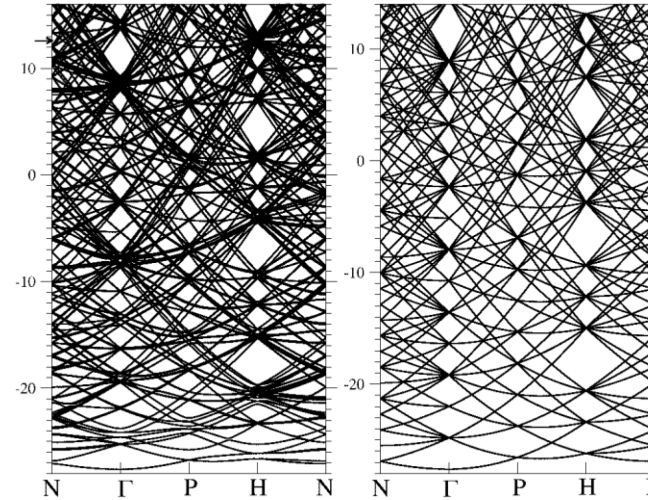


Lattimer and Prakash, astro-ph0405262

Figure by D. Page

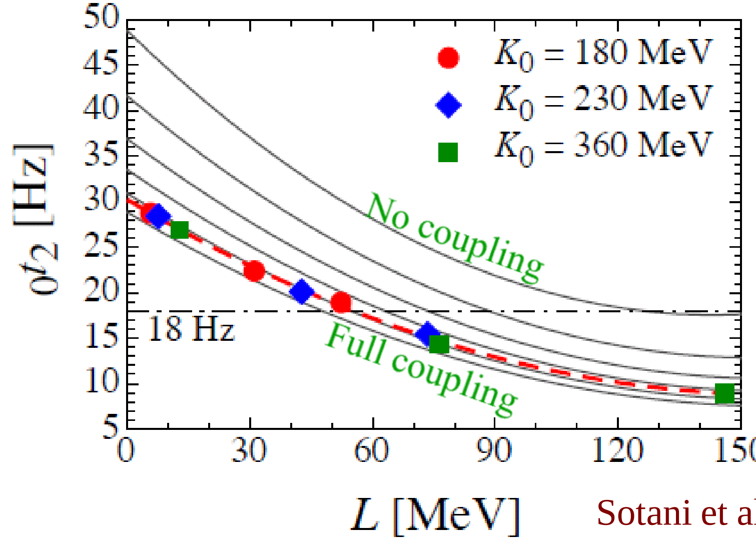
Neutron band structure

crust uniform



Chamel (2012).

Torsional oscillation frequency



Sotani et al. (2012).

Superfluid neutrons are coupled with a lattice of nuclei.

Conclusion

QPOs in giant flares from SGRs

Nuclear size in neutron star crusts

Pasta region in neutron star crusts severely constrained

L **> ~50 MeV**

Confirmation by Hartree-Fock calculations is desired.