Lecture 5 Nuclear Theory

Microscopic description of nuclear structure

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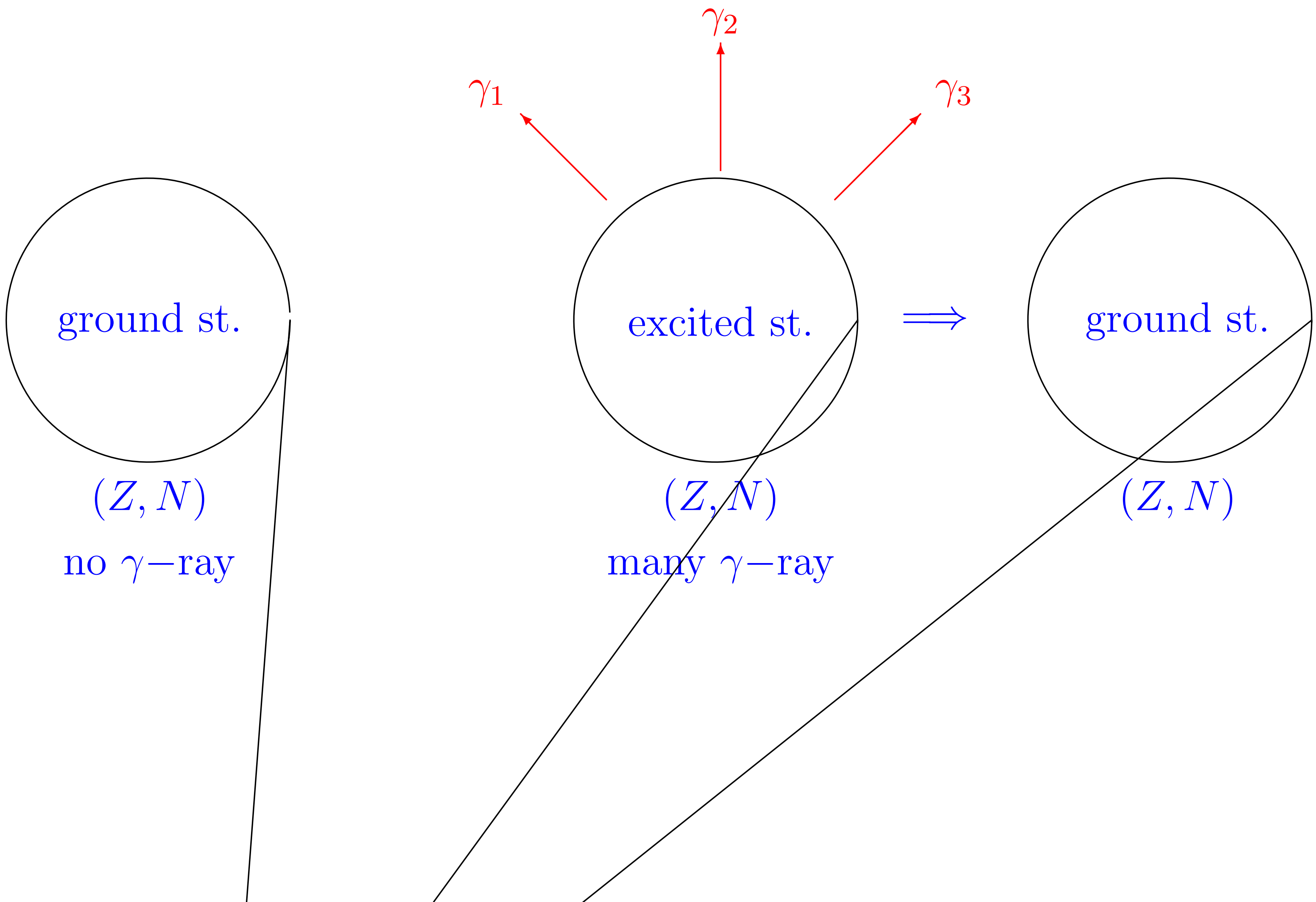
1. Introduction
2. Magic numbers and shell structure in nuclei
3. One- and two-particle nuclei
4. Effective interaction between identical particles
5. Proton-neutron interaction and isomers
6. Summary

1. Introduction – γ spectroscopy –



from γ−ray, we can know the energy levels.

after γ_1

$$\begin{cases} \gamma_2 & \text{no} \\ \gamma_3 & \text{yes} \end{cases}$$

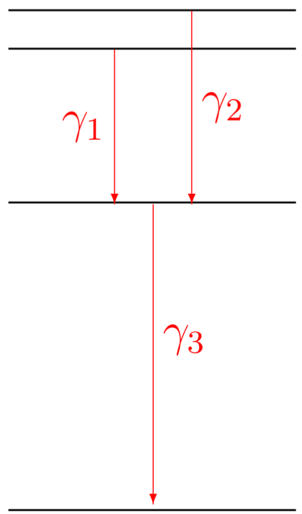
after γ_2

$$\begin{cases} \gamma_1 & \text{no} \\ \gamma_3 & \text{yes} \end{cases}$$

after γ_3

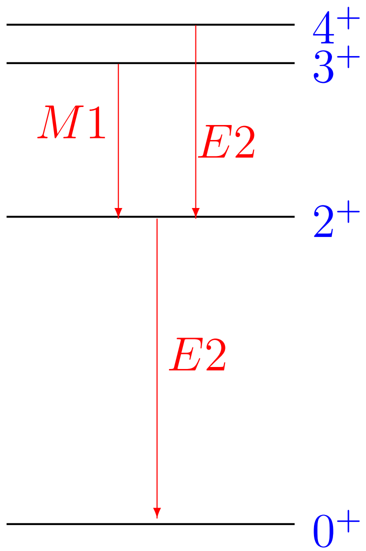
$$\begin{cases} \gamma_1 & \text{no} \\ \gamma_2 & \text{no} \end{cases}$$

$\Longrightarrow$



from coincidence

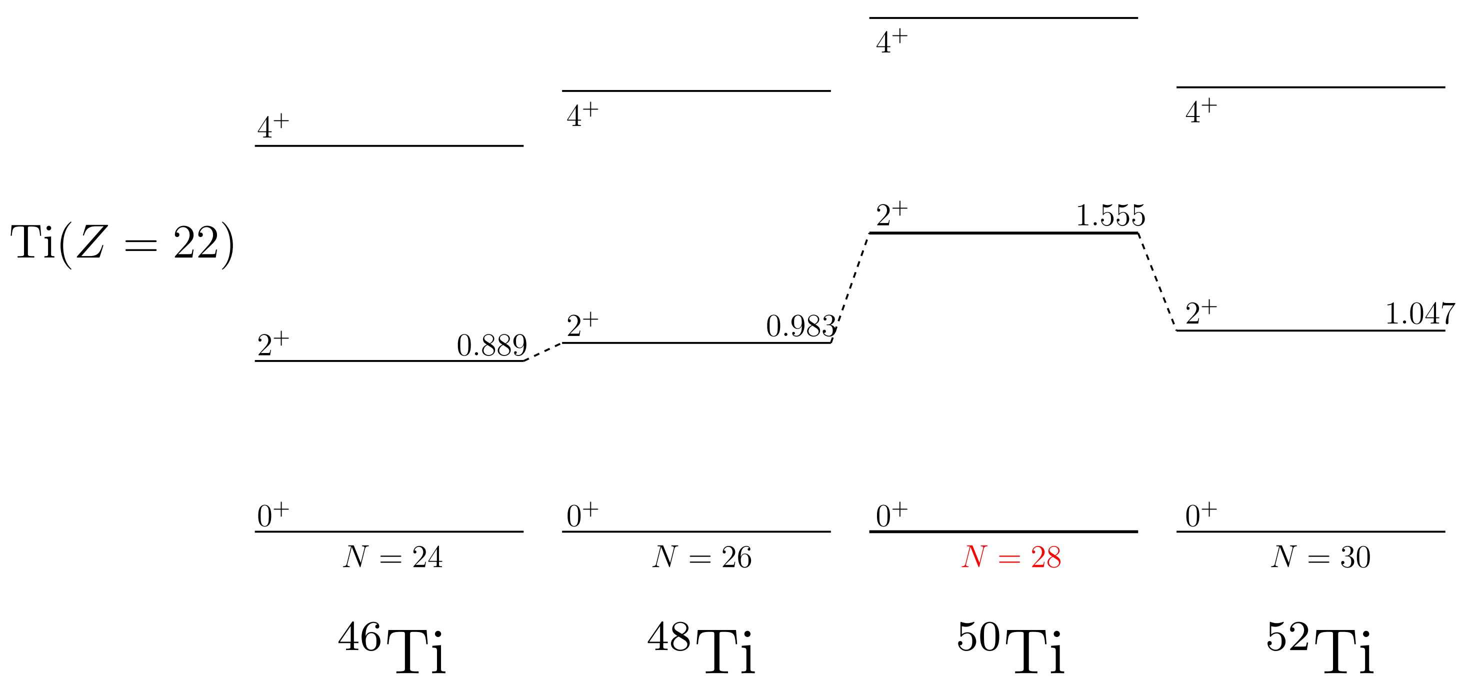
$\Longrightarrow$



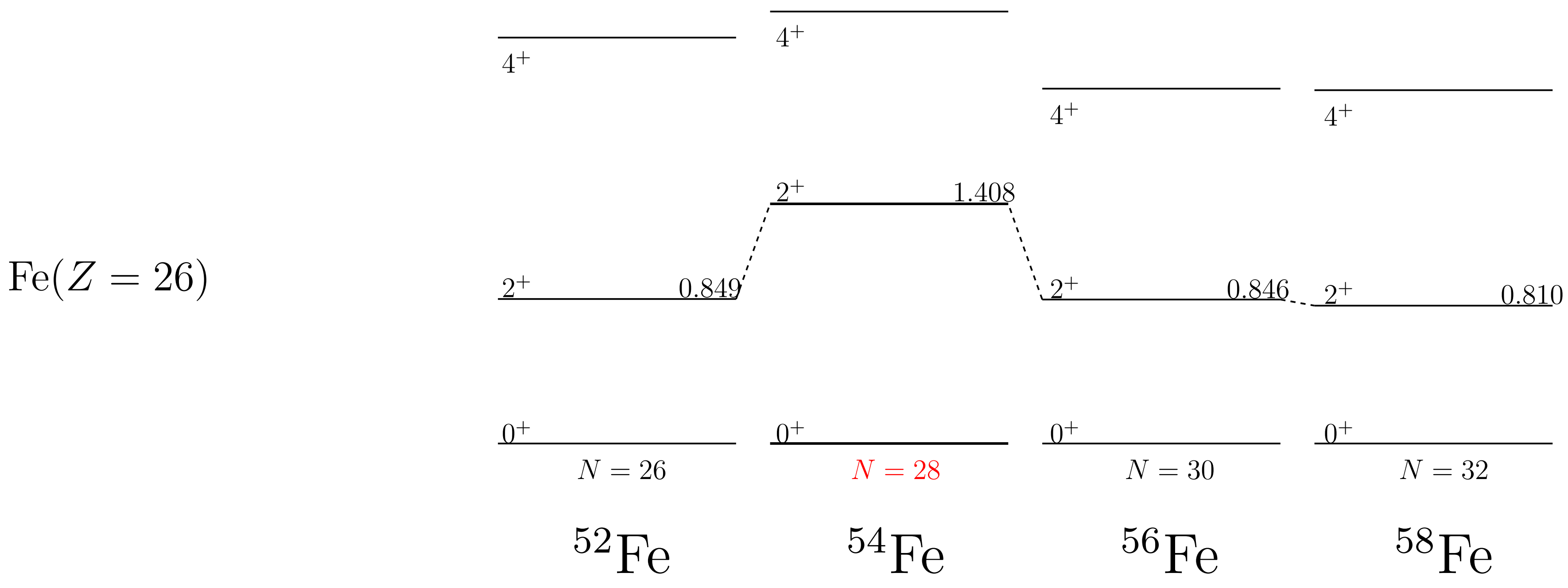
from multipolarity

2. Magic numbers

Energy spectra of Ti-isotopes



Energy spectra of Fe-isotopes



Stability of $N = 28$!!

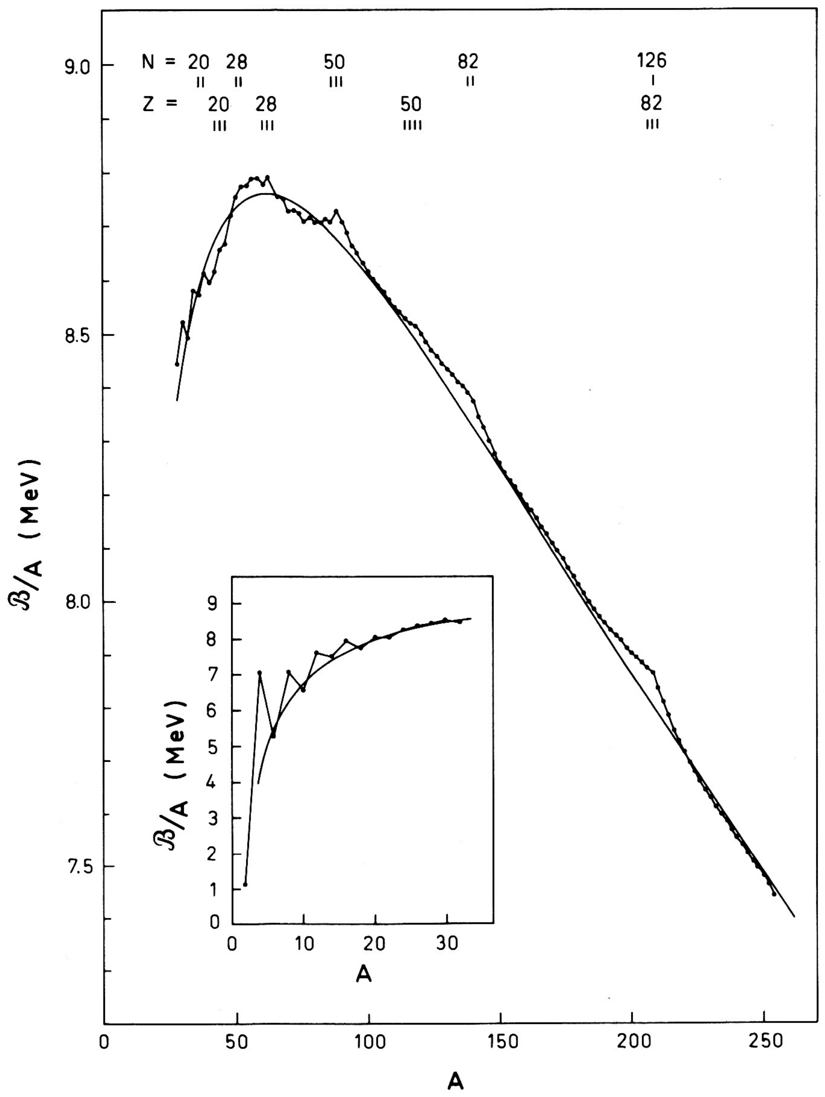
Nuclear Binding Energy

$$\mathcal{B}(Z,N) = ZM_p + NM_n - \mathcal{M}(Z,N)$$

Comparison of $\mathcal{B}_{exp}$ and $\mathcal{B}_{theory}$(liquid dro

$$\Downarrow$$

obvious discrepancy at $N, Z = 20, 28, 50, \cdot$



Magic Numbers

$$Z = 2,\ 8,\ 20,\ 28,\ 50,\ 82$$

$$N = 2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126$$

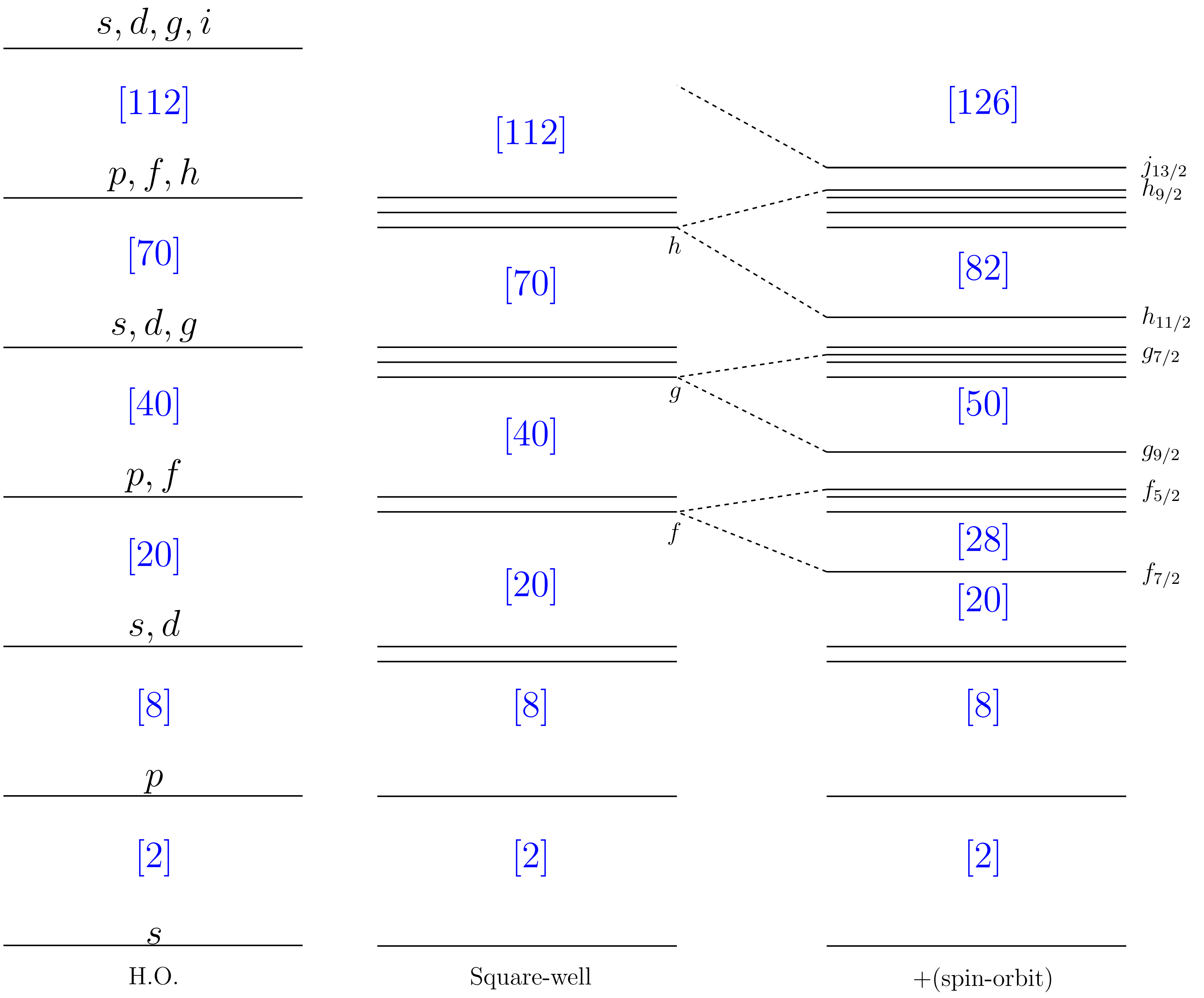
We needed 40 years for understanding these magic numbers !!

Hint: noble gas in atomic system $Z = 2, 10, 18, 36, 54, \cdots$

$$\Downarrow$$

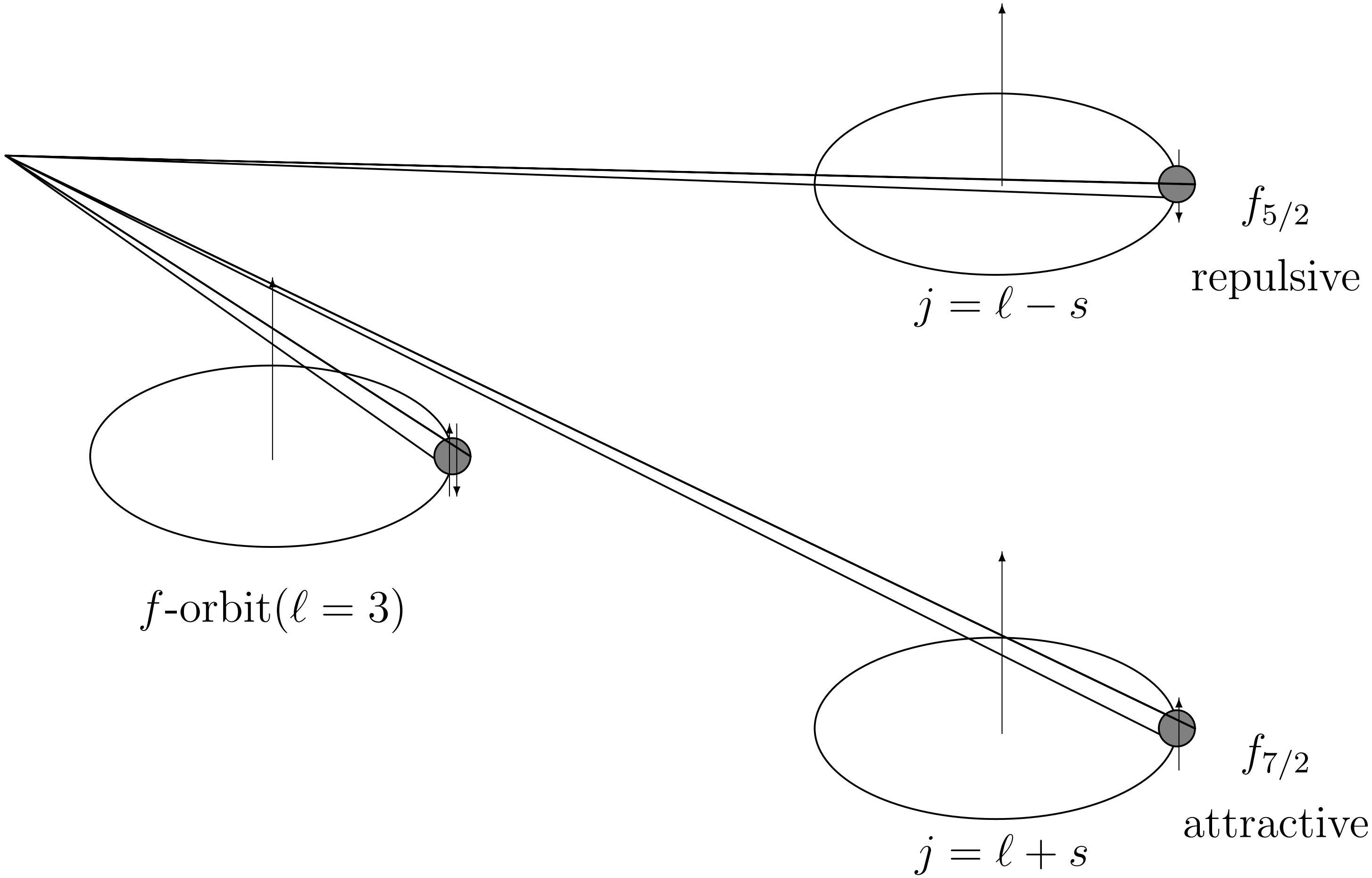
shell structure in nuclei!!(1949 M.G.Mayer and J.H.D.Jensen)

Magic numbers in nuclei



Spin-orbit Splitting

$$-\xi(\boldsymbol{\ell} \cdot \boldsymbol{s})$$



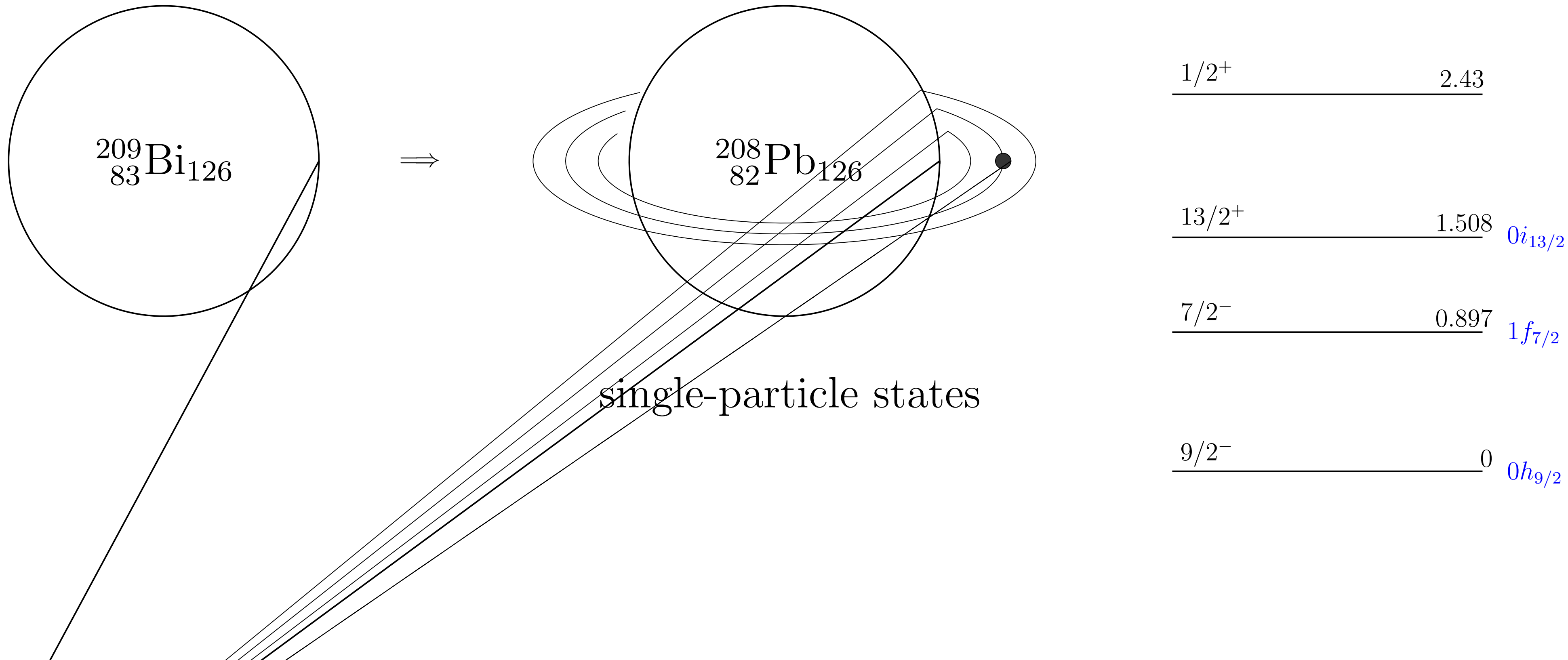
3. One- and Two-particle nuclei

Magic numbers provide us inert core nuclei,

$$\text{e.g. } {}^{4}_{2}\text{He}_{2},\quad {}^{16}_{8}\text{O}_{8},\quad {}^{40}_{20}\text{Ca}_{20},\quad {}^{48}_{20}\text{Ca}_{28},\quad {}^{208}_{82}\text{Pb}_{126}$$

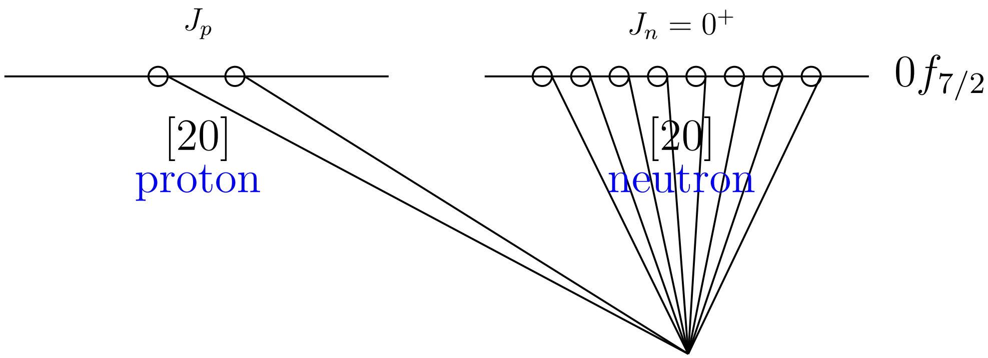
• one-particle nuclei

$${}^{209}_{83}\text{Bi}_{126} \Longrightarrow {}^{208}_{82}\text{Pb}_{126} + p$$



• two-particle nuclei

$$^{50}_{22}\mathrm{Ti}_{28} \Longrightarrow \ ^{48}_{20}\mathrm{Ca}_{28} \ + \ 2p$$



Possible spin-states J in $(0f_{7/2})^2$-configuration $\Longrightarrow$ $J_p = 0^+, \ 2^+, \ 4^+, \ 6^+$

$\vec{J} = \vec{7/2} + \vec{7/2}$

classical mechanic	$J = 0.0 \sim 7.0$
quantum mechanics	$J = 0, \ 1, \ 2, \ 3, \ 4, \ 5, \ 6, \ 7$
fermion statistics	$J = 0, \ 2, \ 4, \ 6$

m-scheme for fermion system

Example $(d_{3/2})^2$

m=3/2	1/2	-1/2	-3/2	M
×	×			2
×		×		1
×			×	0
	×	×		0
	×		×	-1
		×	×	-2

$$M = 2,\ 1,\ (0)^2,\ -1,\ -2 \quad \Longrightarrow \quad J = 2^+$$

$$M = 0 \quad \Longrightarrow \quad J = 0^+$$

$$\Longrightarrow |(d_{3/2})^2 JM\rangle J = 0^+,\ 2^+$$

m-scheme for boson system

Example $(d)^2$

m=2	1	0	-1	-2	M
××					4
×	×				3
×		×			2
×			×		1
×				×	0
	××				2
	×	×			1
	×		×		0
	×			×	-1
		××			0
		×	×		-1
		×		×	-2
			××		-2
			×	×	-3
				××	-4

$$M = \ 4,\ 3,\ (2)^2,\ (1)^2,\ (0)^3, (-1)^2, (-2)^2, -3, -4$$

$$J =$$

Problem: Derive all possible J in the $f^2_{7/2}$-configuration.

m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

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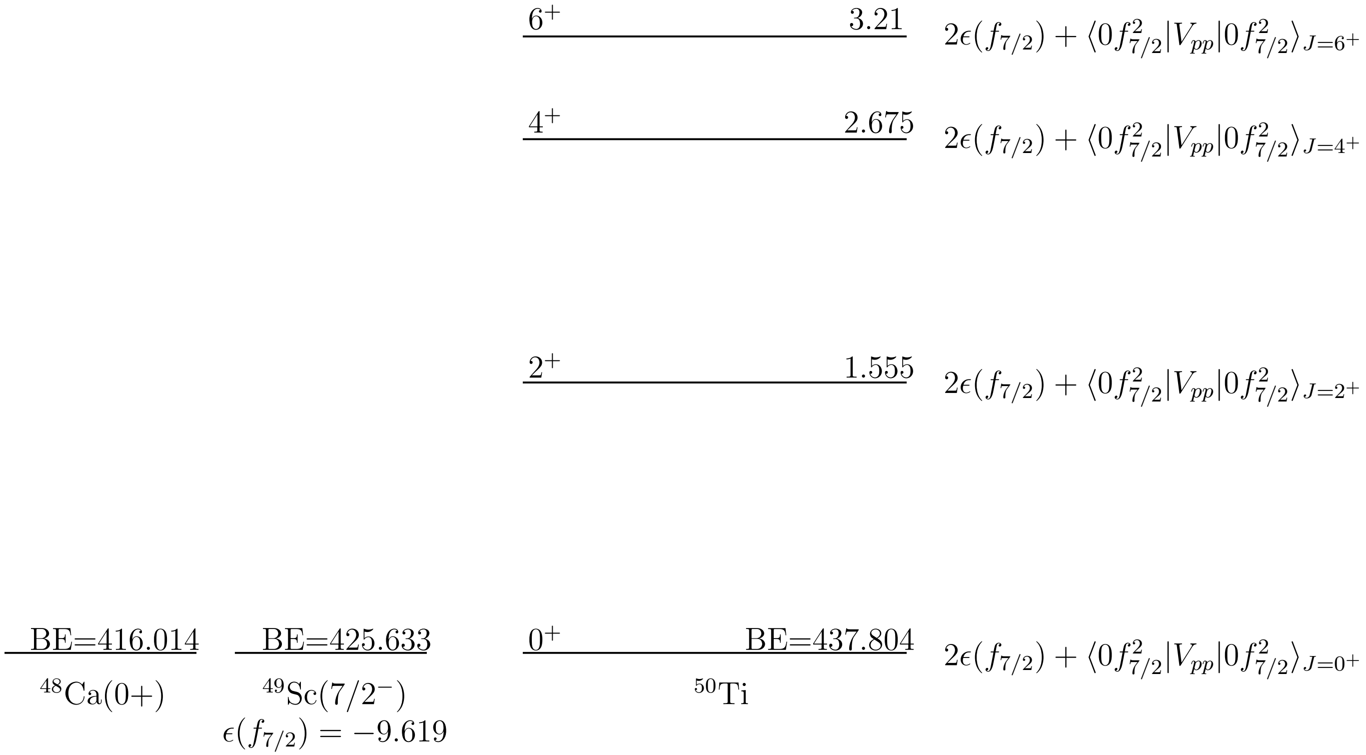
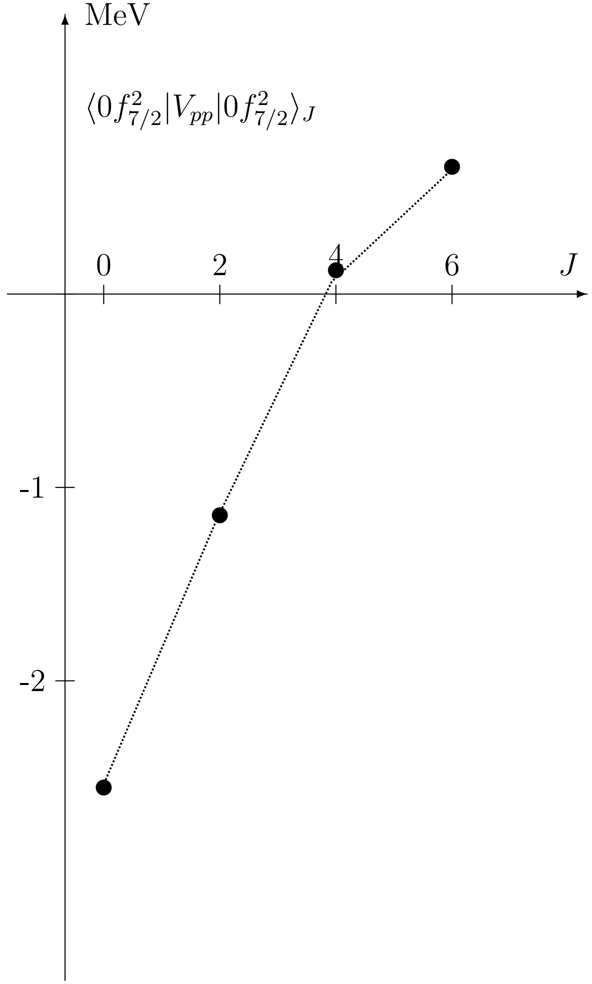
m=7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2	M

$$^{50}_{22}\mathrm{Ti}_{28} \Longrightarrow\ ^{48}_{20}\mathrm{Ca}_{28} + 2p$$





Problem: Derive experimental values of $\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_J$ for $J = 0^+,\ 2^+,\ 4^+,\ 6^+$

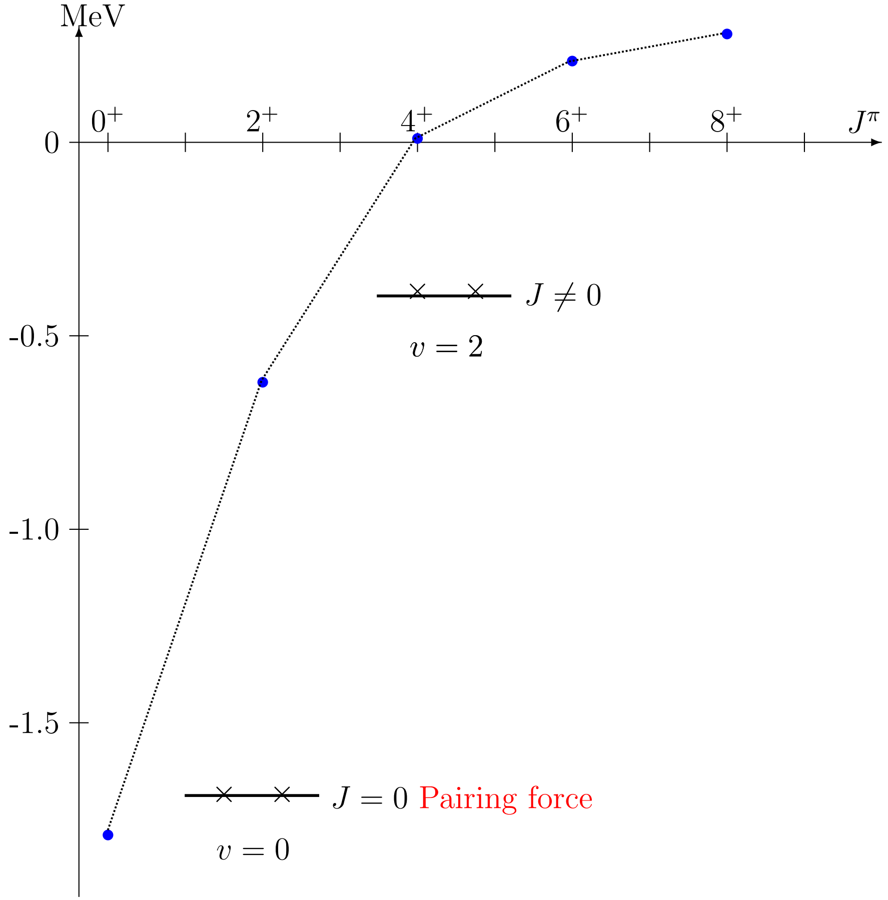
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=0^+} =$$
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=2^+} =$$
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=4^+} =$$
$$\langle 0f^2_{7/2}|V_{pp}|0f^2_{7/2}\rangle_{J=6^+} =$$

4. Effective two-body interaction between identical particles

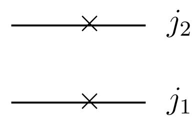
—(proton-proton or neutron-neutron interactions)

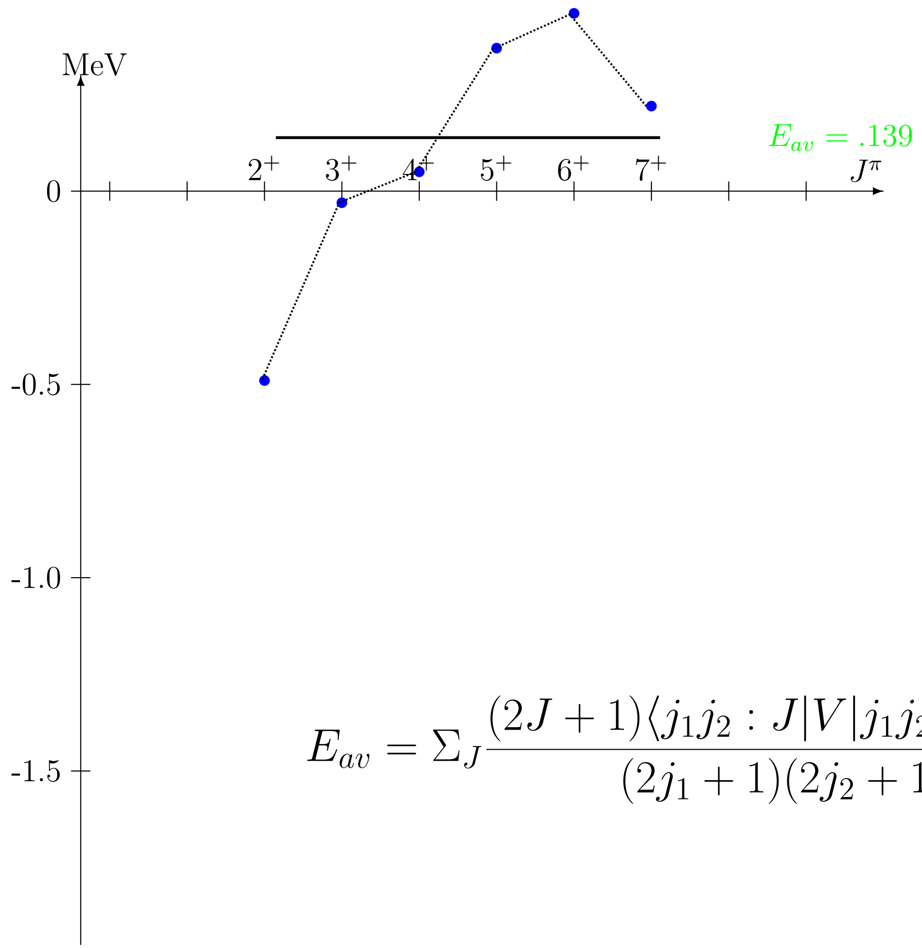
$\langle (0g_{9/2})^2 J | V | (0g_{9/2})^2 J \rangle_{T=1}$

v:seniority



$\langle 0g_{9/2} 1d_{5/2} J | V | 0g_{9/2} 1d_{5/2} J \rangle_{T=1}$





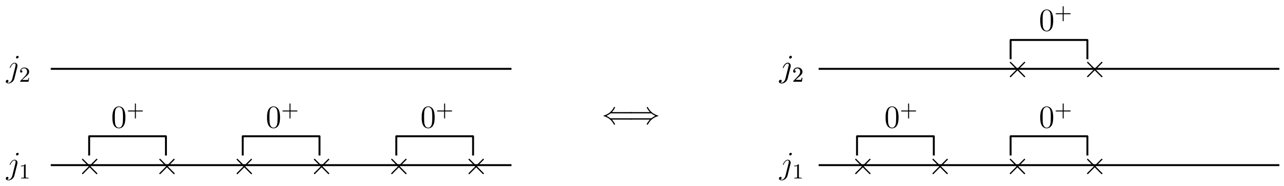
$$E_{av} = \Sigma_J \frac{(2J+1)\langle j_1 j_2 : J | V | j_1 j_2 : J \rangle_{T=1}}{(2j_1+1)(2j_2+1)}$$

Pairing property of effective interaction between identical particles

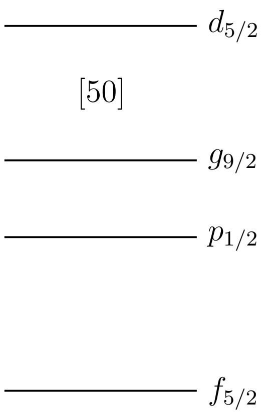
Strong attractive force $\langle j^2 J = 0^+ | V | j^2 J = 0^+ \rangle_{T=1}$

Large matrix element $\langle j^2 J = 0^+ | V | j'^2 J = 0^+ \rangle_{T=1}$

- Such property is reproduced by short-range force like $-V_0 \delta(r)$
- Application of BCS theory to nuclear system



Structure of ^{90}Zr



$${}^{90}_{40}\mathrm{Zr}_{50}$$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \;=\; (g_{9/2}, p_{1/2})_p^2$

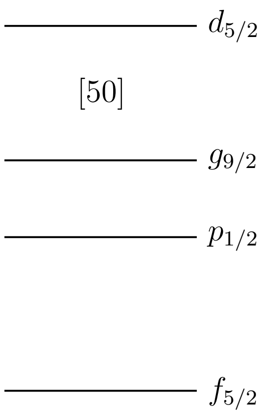
Neutron: $N = 50$ (closed shell) $J_n = 0^+$

$$(g_{9/2})^2 \quad \rightarrow \quad J =$$

$$(g_{9/2}p_{1/2}) \quad \rightarrow \quad J =$$

$$(p_{1/2})^2 \quad \rightarrow \quad J =$$

Structure of ^{90}Zr



$^{90}_{40}\mathrm{Zr}_{50}$

Proton: $Z = 40 \quad (g_{9/2}, p_{1/2})_p^{-10} \quad = \quad (g_{9/2}, p_{1/2})_p^2$

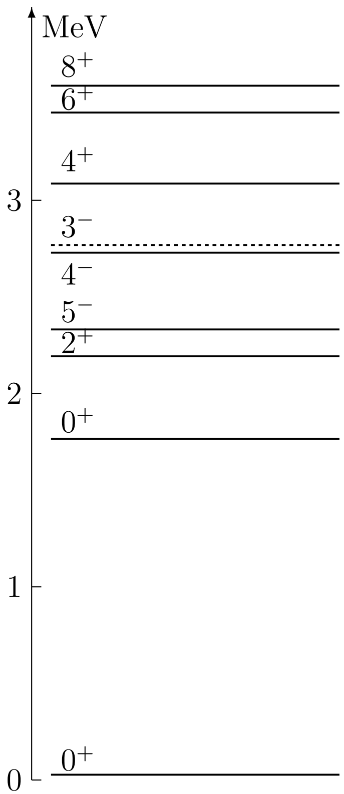
Neutron: $N = 50 \quad$ (closed shell) $\quad J_n = 0^+$

$$(g_{9/2})^2 \quad \rightarrow \quad J = 0^+,\ 2^+,\ 4^+,\ 6^+,\ 8^+$$

$$(g_{9/2}p_{1/2}) \quad \rightarrow \quad J = 4^-,\ 5^-$$

$$(p_{1/2})^2 \quad \rightarrow \quad J = 0^+$$

^{90}Zr



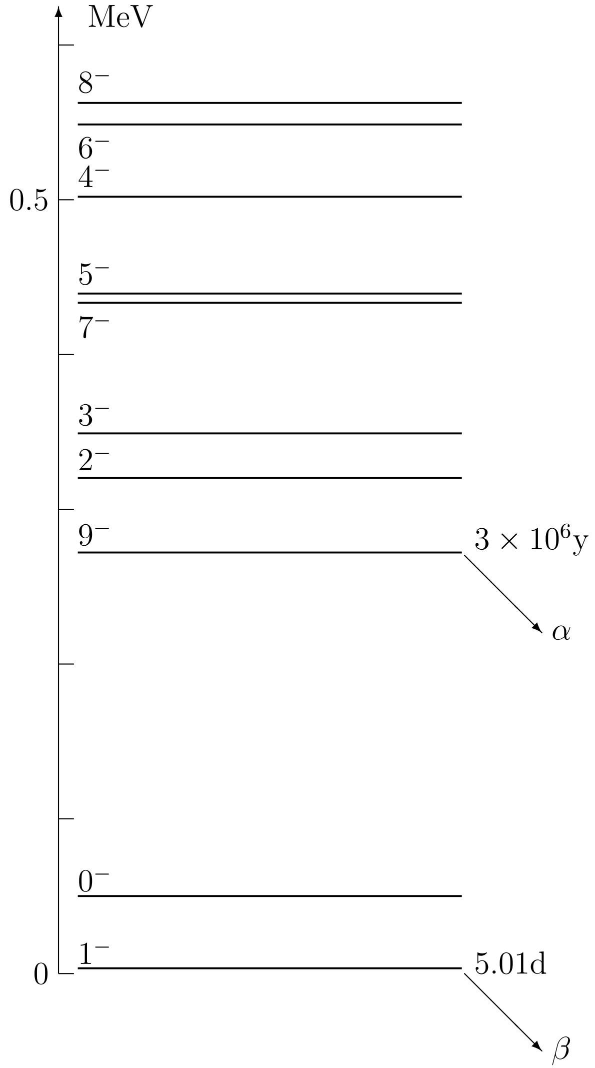
$\leftarrow \quad 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 8^+ | V_{pp} | g_{9/2}^2 J = 8^+ \rangle$

$\leftarrow \quad \epsilon(g_{9/2}) + \epsilon(p_{1/2}) + \langle g_{9/2} p_{1/2} J = 5^- | V_{pp} | g_{9/2} p_{1/2} J = 5^- \rangle$

$$\begin{pmatrix} 2\epsilon(g_{9/2}) + \langle g_{9/2}^2 J = 0^+ | V | g_{9/2}^2 J = 0^+ \rangle & \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \\ \langle g_{9/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle & 2\epsilon(p_{1/2}) + \langle p_{1/2}^2 J = 0^+ | V | p_{1/2}^2 J = 0^+ \rangle \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = E \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

Eigen-value problem

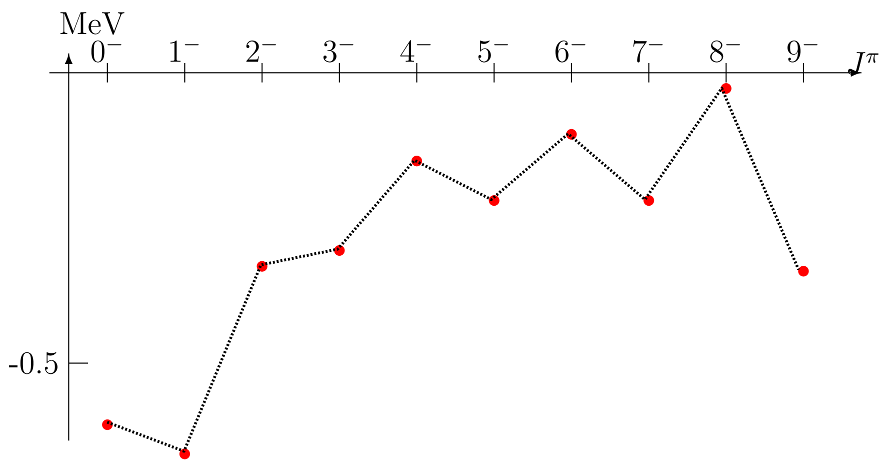
5. Proton-neutron interaction and isomer



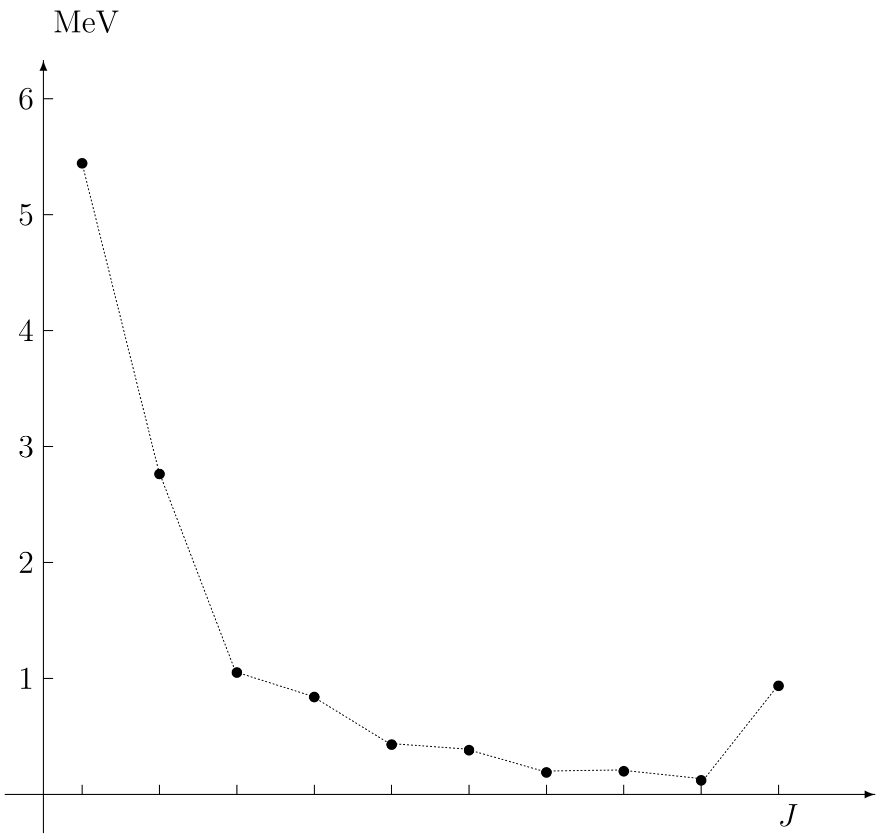
$$^{210}_{83}\mathrm{Bi}_{127} \Longrightarrow {}^{208}_{82}\mathrm{Pb} + \mathrm{p} + \mathrm{n}$$

$$^{210}_{83}\mathrm{Bi}_{127} \qquad (0h_{9/2})_p \times (1g_{9/2})_n \; J = 0^-,\; 1^-,\; \cdots\; , 9^-$$

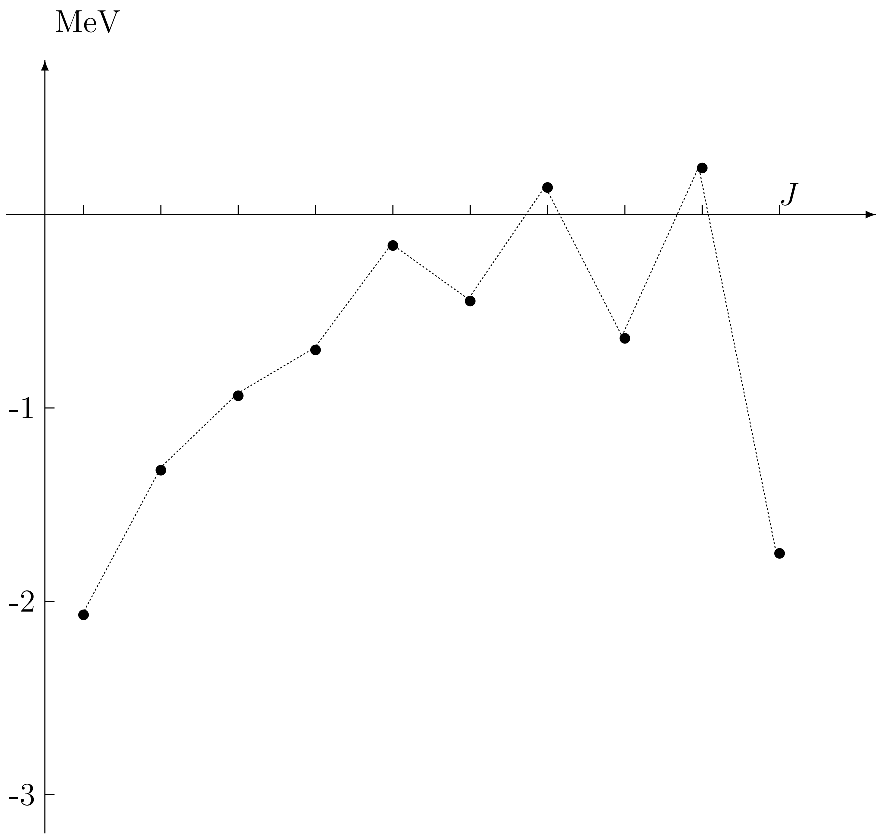
$$\langle (0h_{9/2})_p (1g_{9/2})_n J | V_{pn} | (0h_{9/2})_p (1g_{9/2})_n J \rangle$$



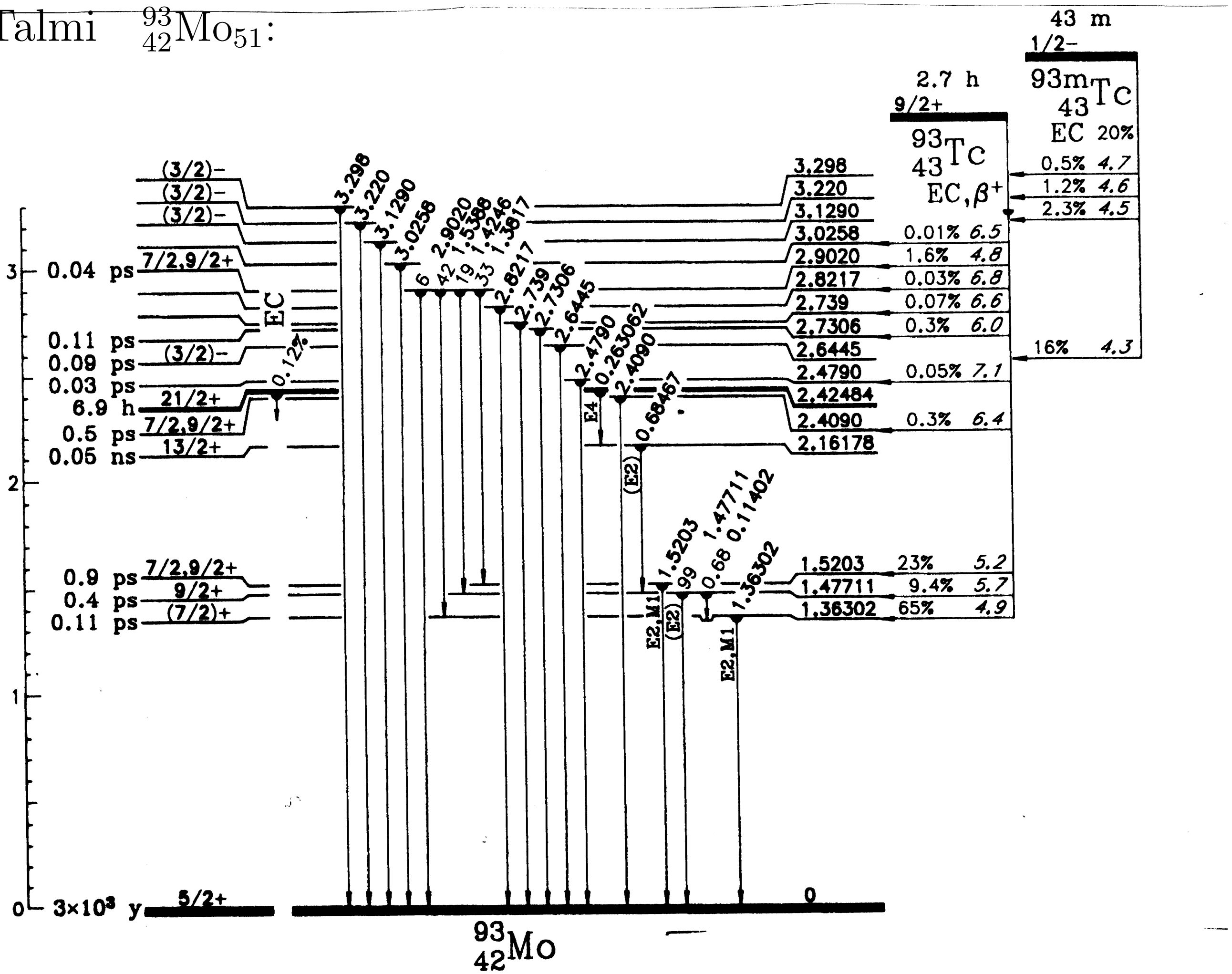
$$\langle (0g_{9/2})_p (0g_{9/2})_n^{-1} : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n^{-1} : J \rangle$$



$$\langle (0g_{9/2})_p (0g_{9/2})_n : J | V_{pn} | (0g_{9/2})_p (0g_{9/2})_n : J \rangle$$



I. Talmi $^{93}_{42}\mathrm{Mo}_{51}$:



$$(0g_{9/2}^2)_p J_p = 8^+ \ \times \ (1d_{5/2})_n J_n = \tfrac{5}{2};\, J = 21/2^+$$

High spin isomers in nuclei near drip-line

Direct One-Proton Emission

First observation: $^{53}_{27}Co_{26}(\frac{19}{2}^{-}) \longrightarrow {}^{52}_{26}Fe_{26}(0^{+})$

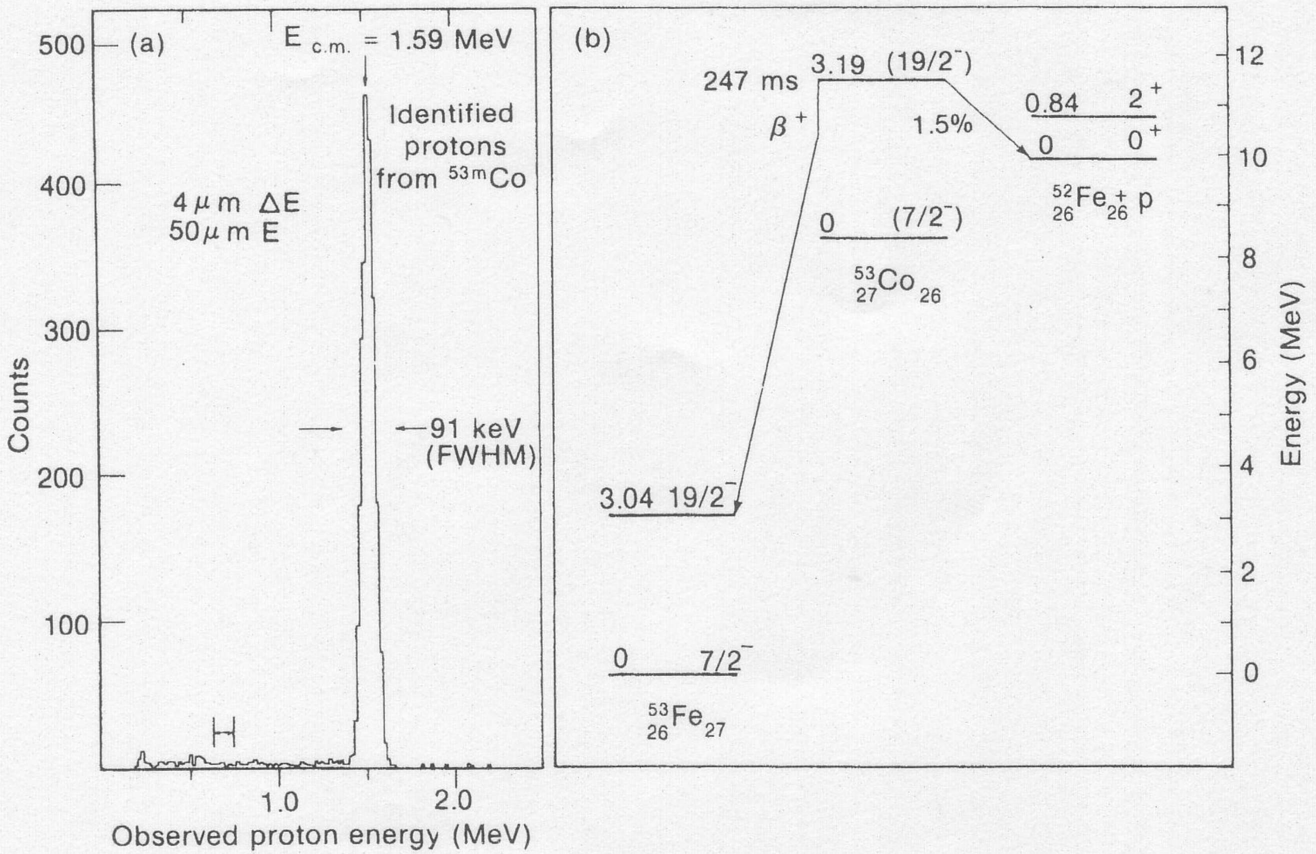


Figure 30. (a) Proton energy spectrum from the decay of ^{53m}Co produced by the ^{54}Fe(p, 2n) reaction. (b) Decay scheme of ^{53m}Co.

Shell-model calculations of high-spin ispmers in neutron-defficient $1g_{9/2}$-shell nuclei
K. Ogawa: Phys. Rev. C28(1983)958

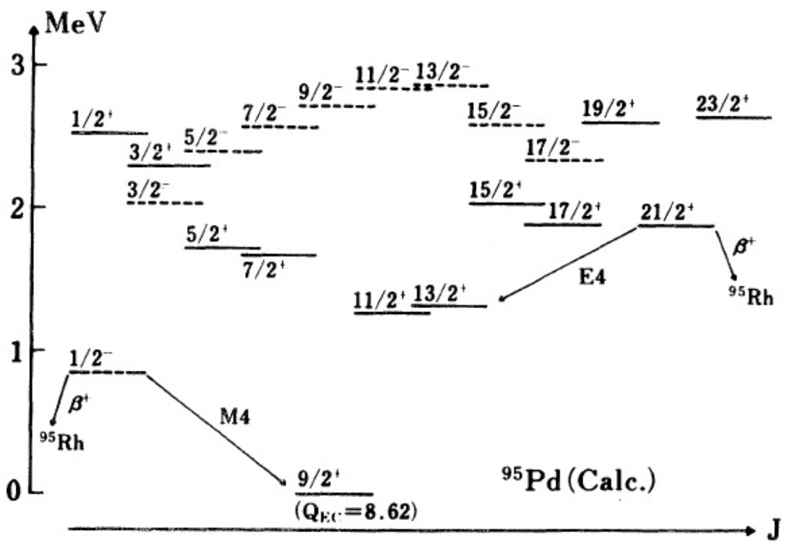


FIG. 1. Calculated energy levels of ^{95}Pd. The lowest levels with each spin-parity state below 3 MeV are shown by solid lines for positive-parity states and by dashed lines for negative-parity states. The transitions from the isomeric states are indicated by arrows.

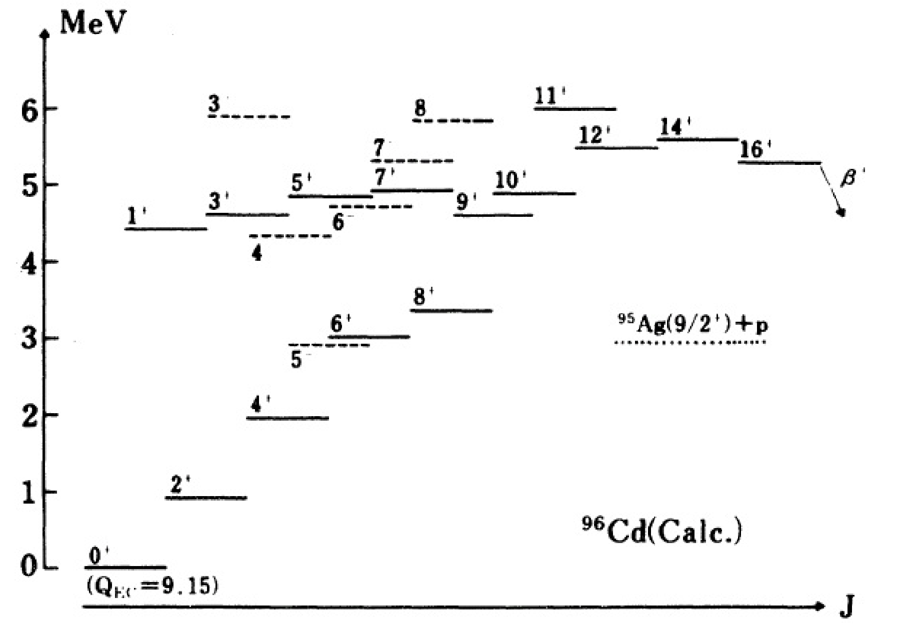


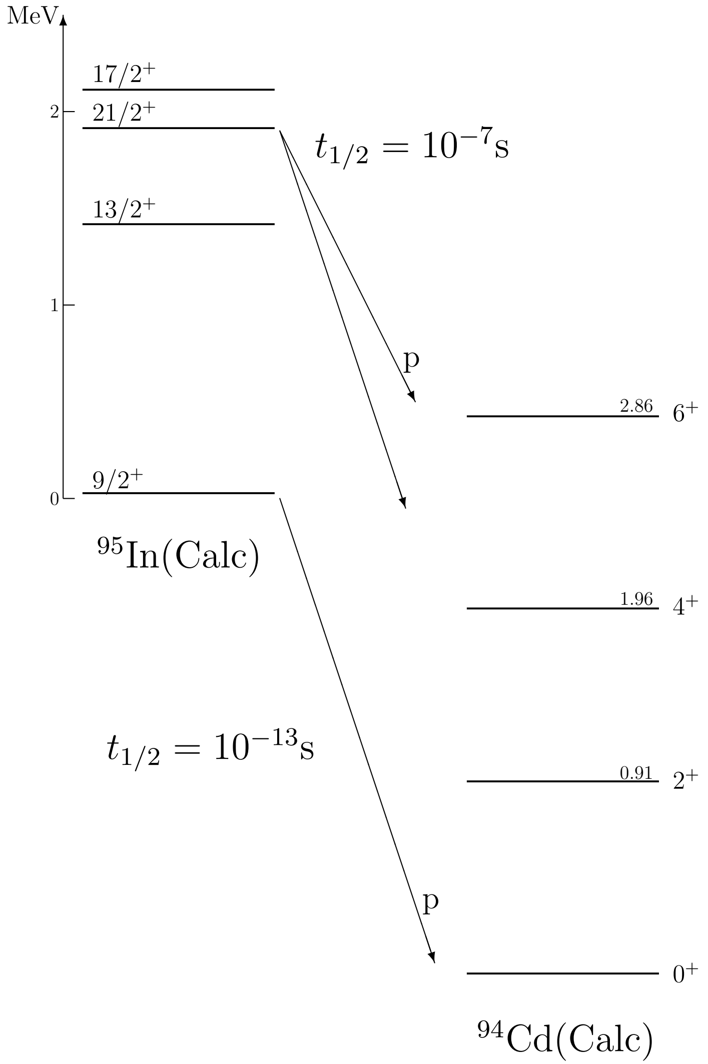
FIG. 3. Calculated energy levels of ^{96}Cd. The lowest levels with each spin-parity state below 6 MeV are shown.

^{95}Pd

^{96}Cd

	^{91}Sn	^{92}Sn	^{93}Sn	^{94}Sn	^{95}Sn	^{96}Sn	^{97}Sn	^{98}Sn	^{99}Sn	^{100}Sn	^{101}Sn
	^{90}In	^{91}In	^{92}In	^{93}In	^{94}In	^{95}In 21/2+	^{96}In	^{97}In 25/2+	^{98}In 9+	^{99}In	^{100}In
	^{89}Cd	^{90}Cd	^{91}Cd	^{92}Cd	^{93}Cd	^{94}Cd 14+	^{95}Cd 23/2+	^{96}Cd 16+	^{97}Cd 25/2+	^{98}Cd	^{99}Cd
	^{88}Ag	^{89}Ag	^{90}Ag	^{91}Ag	^{92}Ag	^{93}Ag	^{94}Ag	^{95}Ag 23/2+	^{96}Ag	^{97}Ag	^{98}Ag
	^{87}Pd	^{88}Pd	^{89}Pd	^{90}Pd	^{91}Pd	^{92}Pd	^{93}Pd	^{94}Pd 14+	^{95}Pd 21/2+	^{96}Pd	^{97}Pd
	^{86}Rh	^{87}Rh	^{88}Rh	^{89}Rh	^{90}Rh	^{91}Rh	^{92}Rh	^{93}Rh	^{94}Rh	^{95}Rh	^{96}Rh
	^{85}Ru	^{86}Ru	^{87}Ru	^{88}Ru	^{89}Ru	^{90}Ru	^{91}Ru	^{92}Ru	^{93}Ru	^{94}Ru	^{95}Ru
	^{84}Tc	^{85}Tc	^{86}Tc	^{87}Tc	^{88}Tc	^{89}Tc	^{90}Tc	^{91}Tc	^{92}Tc	^{93}Tc	^{94}Tc
	^{83}Mo	^{84}Mo	^{85}Mo	^{86}Mo	^{87}Mo	^{88}Mo	^{89}Mo	^{90}Mo	^{91}Mo		^{93}Mo
	^{82}Nb	^{83}Nb	^{84}Nb	^{85}Nb	^{86}Nb	^{87}Nb	^{88}Nb	^{89}Nb	^{90}Nb	^{91}Nb	^{92}Nb
^{80}Zr	^{81}Zr	^{82}Zr	^{83}Zr	^{84}Zr	^{85}Zr	^{86}Zr	^{87}Zr	^{88}Zr	^{89}Zr		

Stability of $^{95}_{49}$In



High-spin isomers in unstable nuclei

6. Summary

- Using $\gamma-$spectroscopy, we know the energy levels of each nucleus.
- Magic numbers provide us a simple description of nuclei.
- From nuclei near magic number, we know the effective interactions between nucleons.
- Effective interaction between identical particles(pp or nn) $\rightarrow$ pairing property
- Effective interaction between proton and neutron $\rightarrow$ high-spin state $\rightarrow$ isomers
- Isomers $\rightarrow$ new stability and new decay modes