A brief introduction to particle accelerators

1. What is “particle accelerator”?
2. How accelerators work? : principles and history
3. A bit about beam dynamics : focusing of beams
4. Application of accelerators
5. RIKEN cyclotrons and RI Beam Factory

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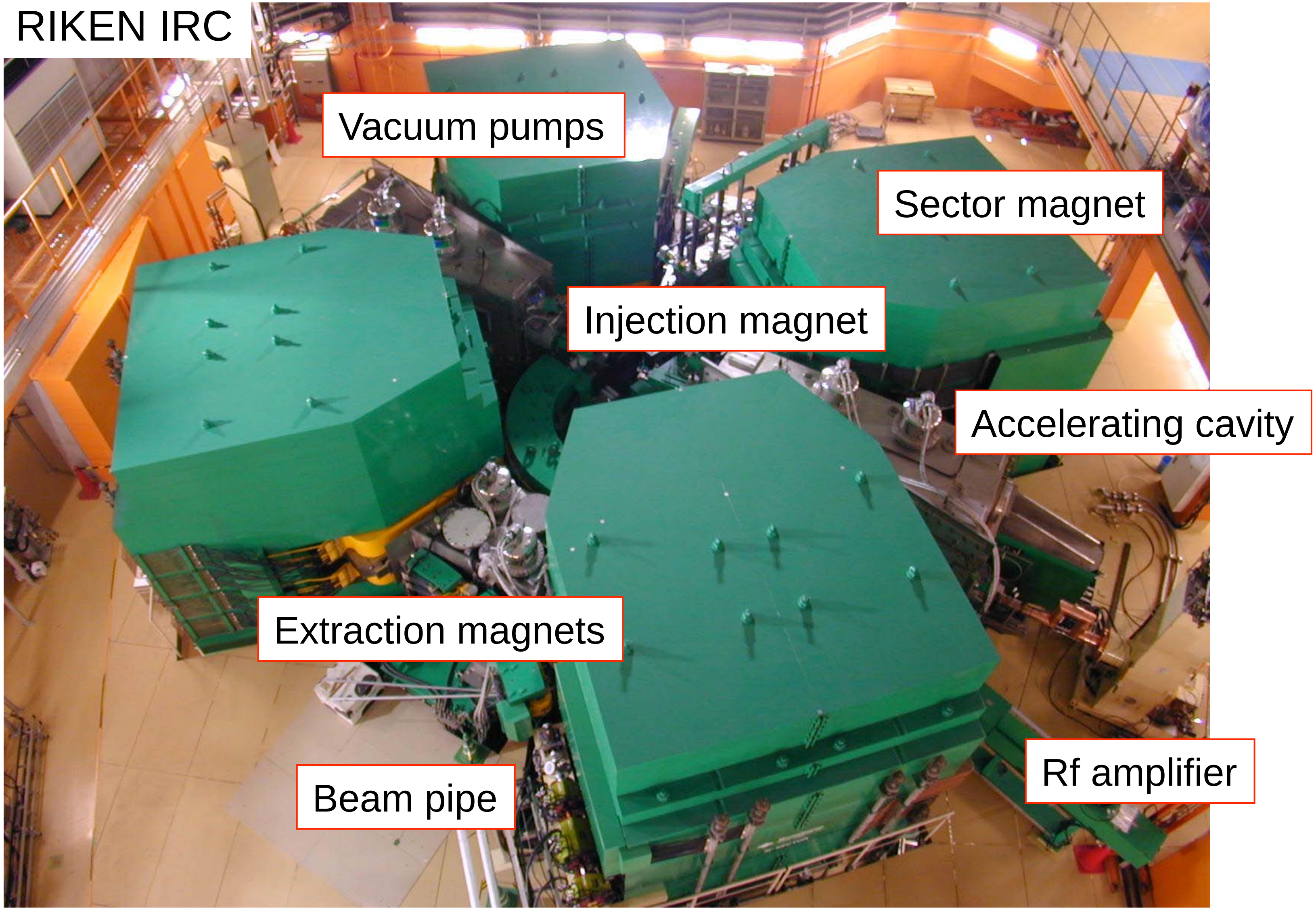
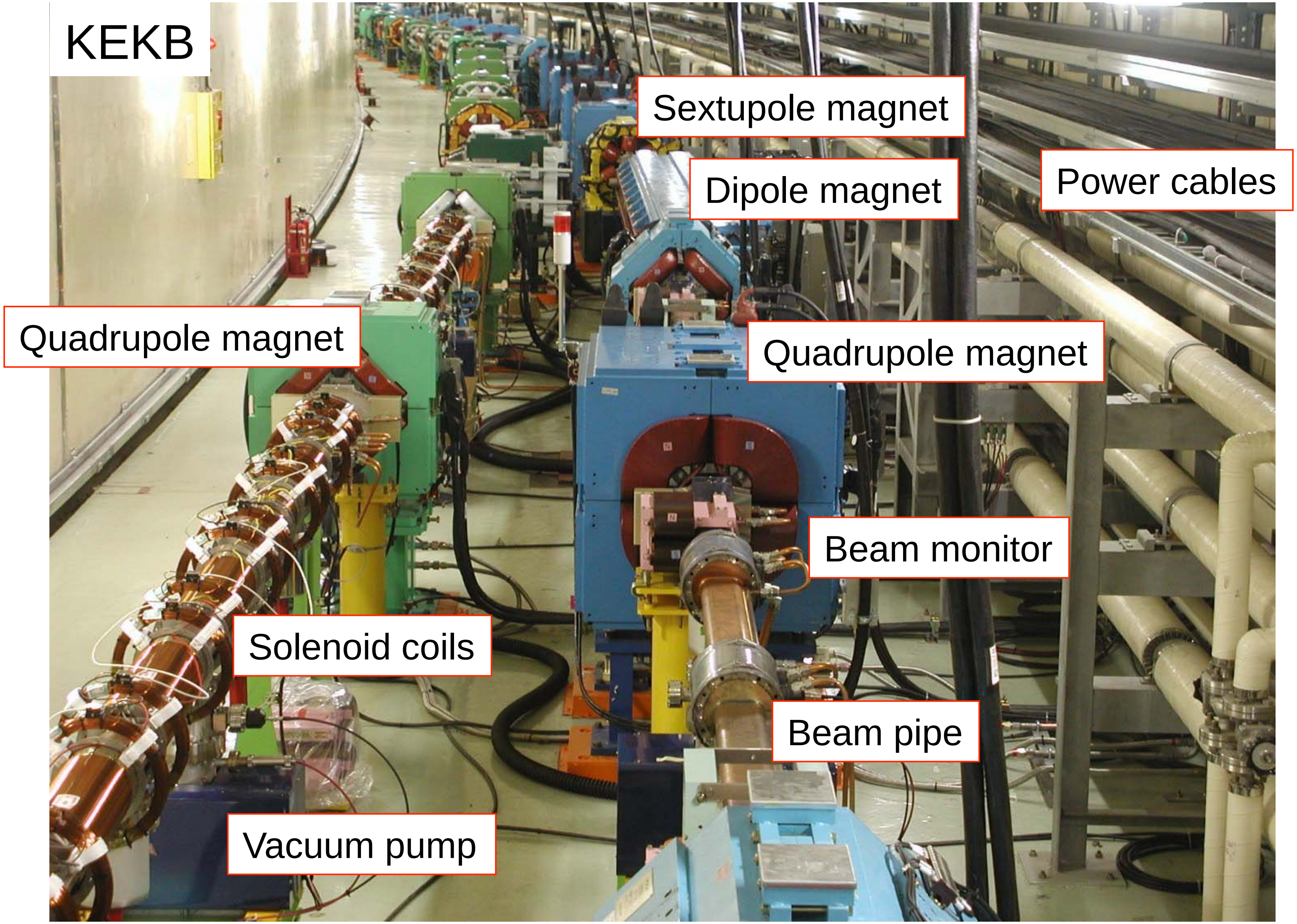
1. What is “particle accelerator” ?

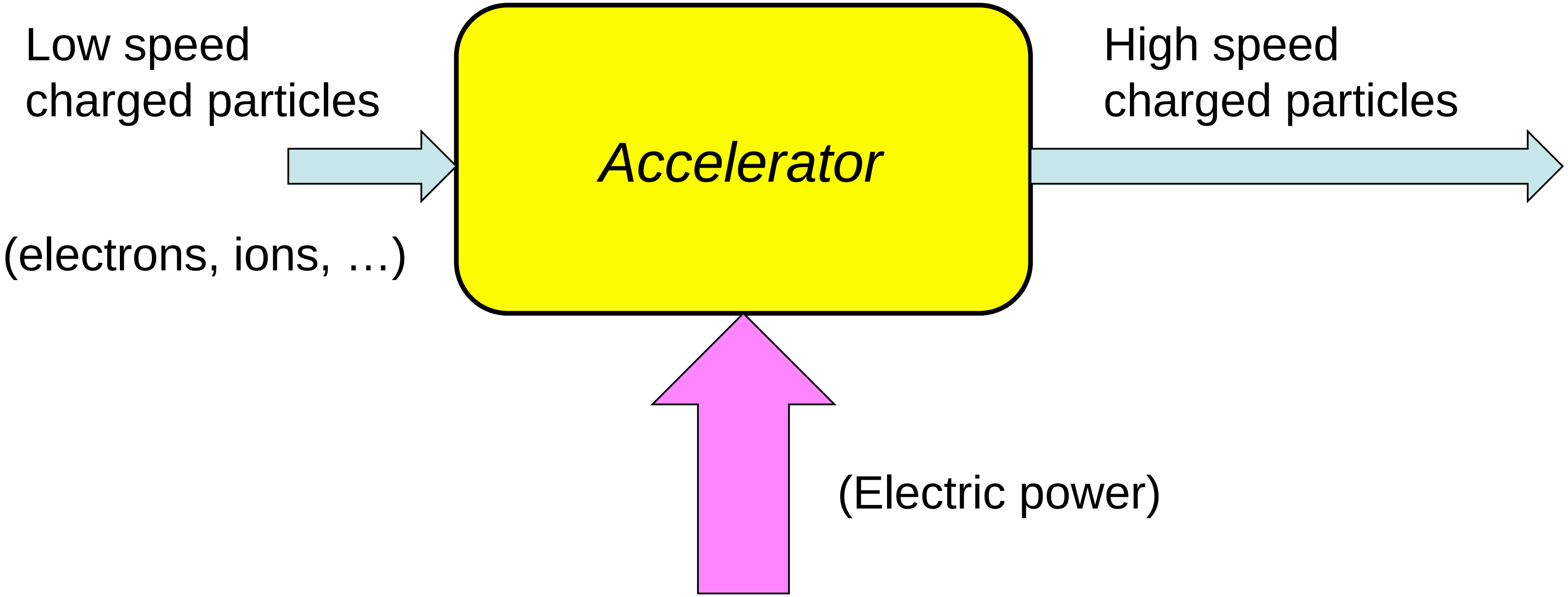
RIKEN IRC





An accelerator consists of many components….

Particle accelerators are, simply speaking…



But, for what?...

Accelerators were invented for exploring nuclear world.

Lecture by Rutherford at the Royal Society（1927.11.30）

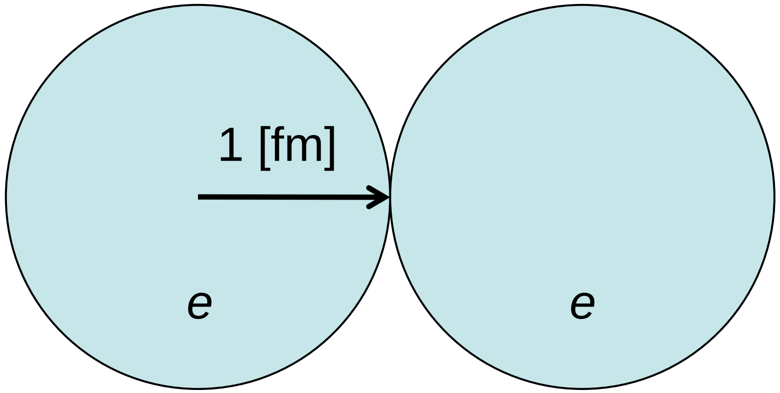
It would be of great scientific interest if it were possible in laboratory experiments to have a supply of electrons and atoms of matter in general, of which the individual energy of motion is greater even than that of the α-particle. This would open up an extraordinarily interesting field of investigation which could not fail to give us information of great value, not only on the constitution and stability of atomic nuclei but in many other directions.

It has long been my ambition to have available for study a copious supply of atoms and electrons which have an individual energy far transcending that of the α and β-particles from radioactive bodies. I am hopeful that I may yet have my wish fulfilled, but it is obvious that many experimental difficulties will have to be surmounted before this can be realised, even on a laboratory scale.

E. Rutherford, Proc. Roy. Soc. (London) A117 (1927) 300

Exercise

What is the potential energy between two identical spheres (radius = 1 [fm] = 10^{-15} [m]) charged uniformly by e (= 1.6×10^{-19} [C]) at contact?

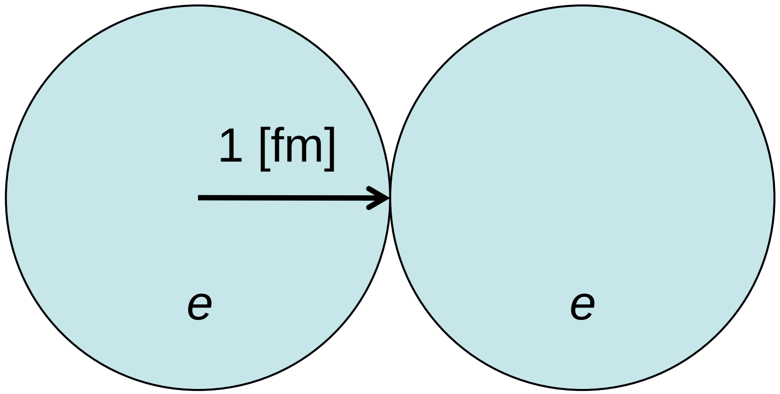


High voltage devices > 1 MV are necessary for nuclear collisions

Electric potential => $V(r) = \dfrac{e}{4\pi\varepsilon_0 r}$ [V]

$$\varepsilon_0 = 8.85 \quad [\text{pF/m}] = 8.85 \times 10^{-12} \quad [\text{F/m}]$$

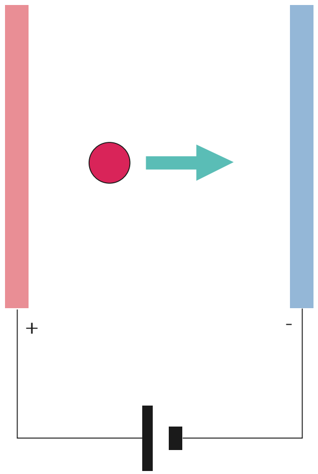
$$V\Big|_{r=2[\text{fm}]} \approx \frac{1.6 \times 10^{-19}}{4 \times 3.14 \times 8.85 \times 10^{-12} \times 2 \times 10^{-15}} \approx 719000\,[\text{V}] \approx 0.7\,[\text{MV}]$$

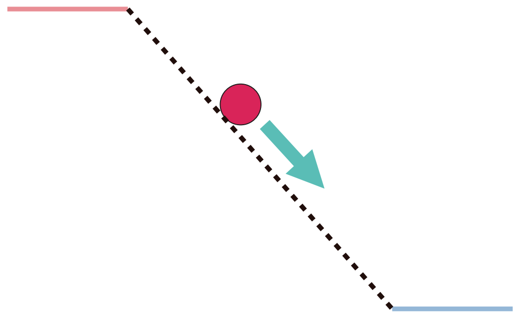


$$eV\Big|_{r=2[\text{fm}]} \approx 719000\,[\text{eV}] \approx 0.7\,[\text{MeV}] \approx 1\,[\text{MeV}]$$

2. How accelerators work?

Acceleration by static electric field





Beam: DC

High voltage

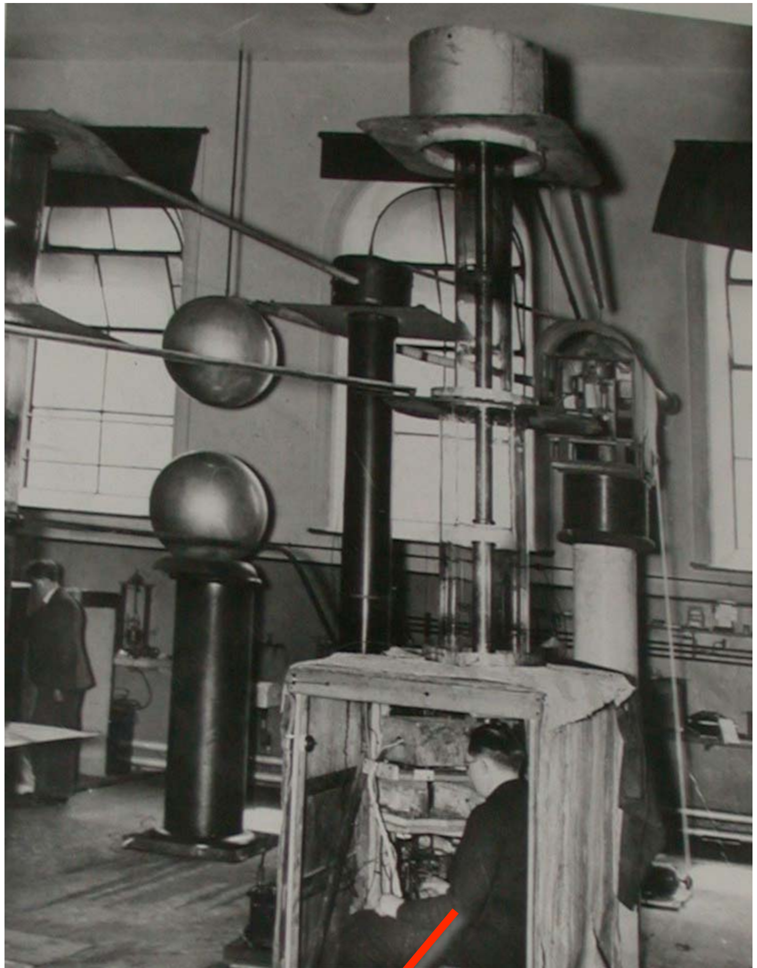
The first man-made nuclear reaction (1932)

J. D. Cockcroft and E. T. S. Walton, Proc. R. Soc. London A137 (1932) 229.

$p(500\ \text{keV}) + {}^{7}\text{Li} \Rightarrow \alpha + \alpha$

J. D. Cockcroft (1897-1967) | E. Rutherford (1871-1937) | E. T. S. Walton (1903-1995)

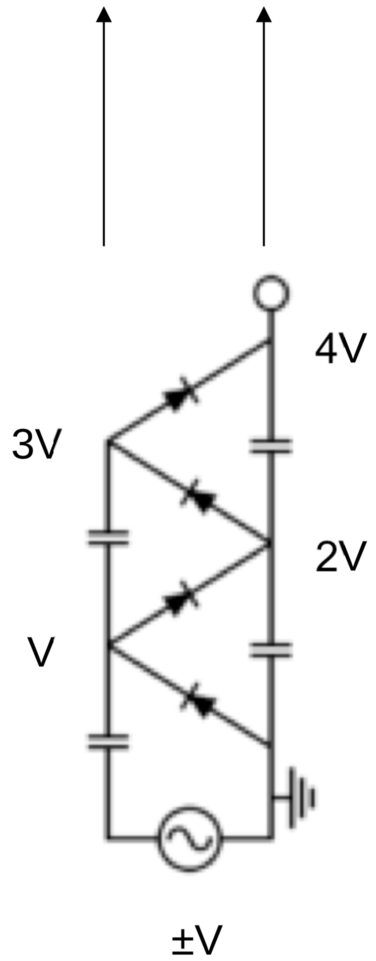
1951 Nobel prize in Physics



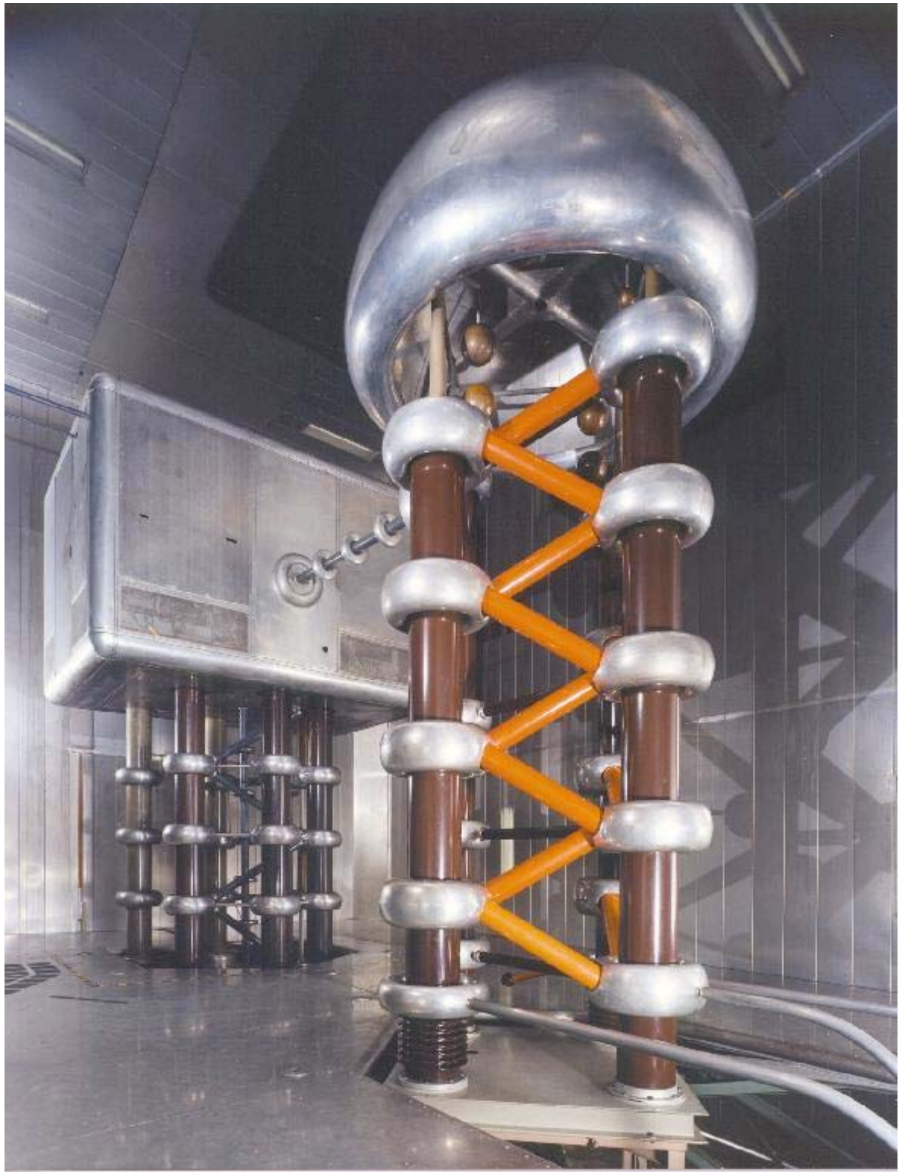
E. T. S. Walton

Dawn of nuclear physics

Cockcroft-Walton Circuit



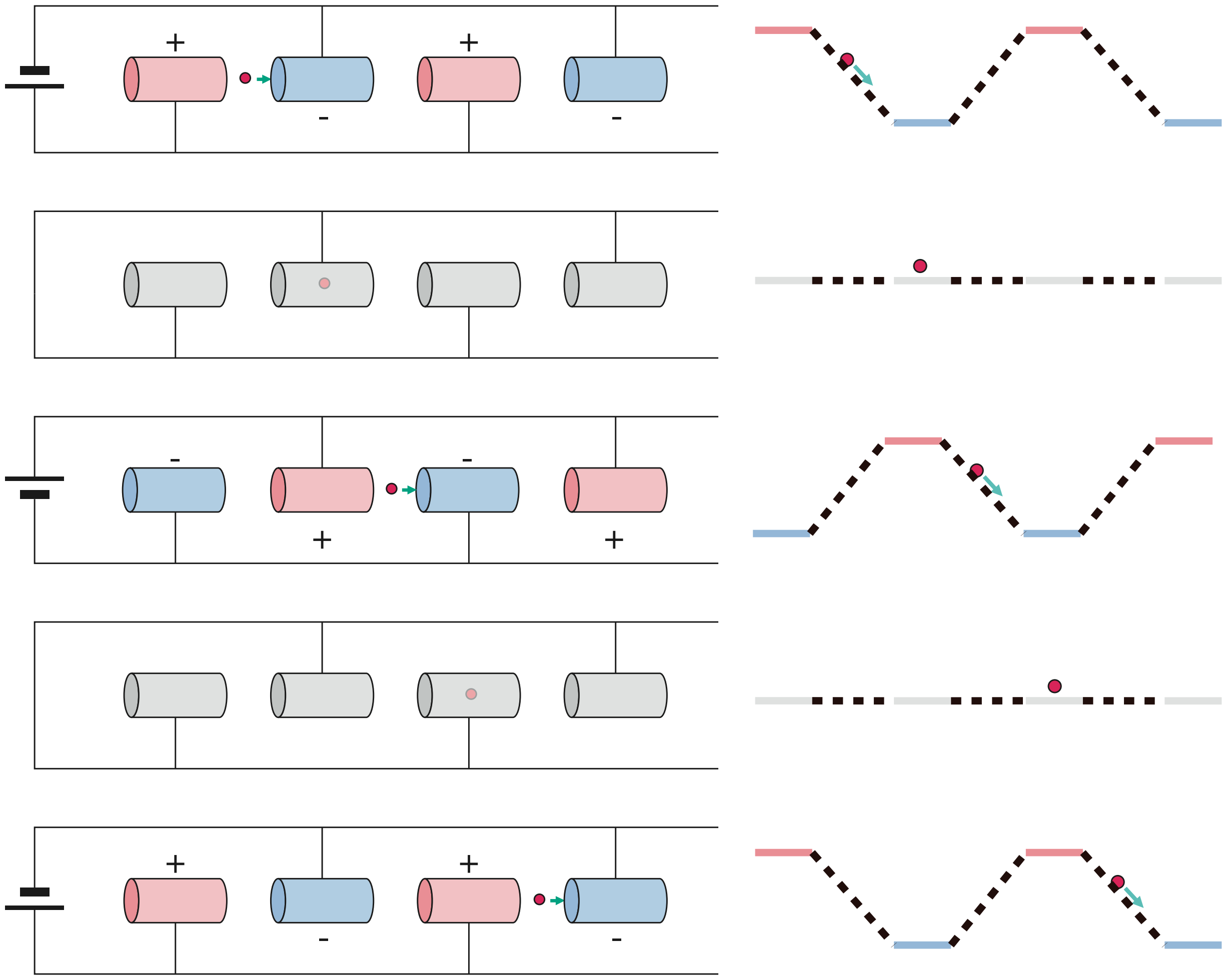
(=> Exercise)

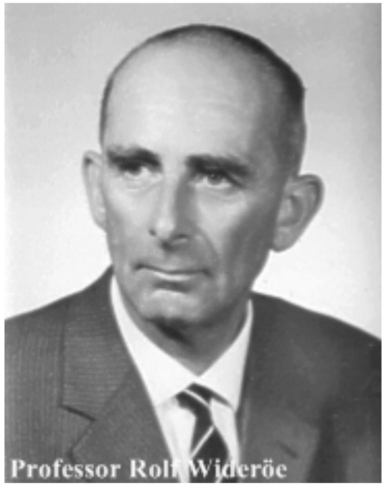


Modern C-W accelerator

RF linear accelerator

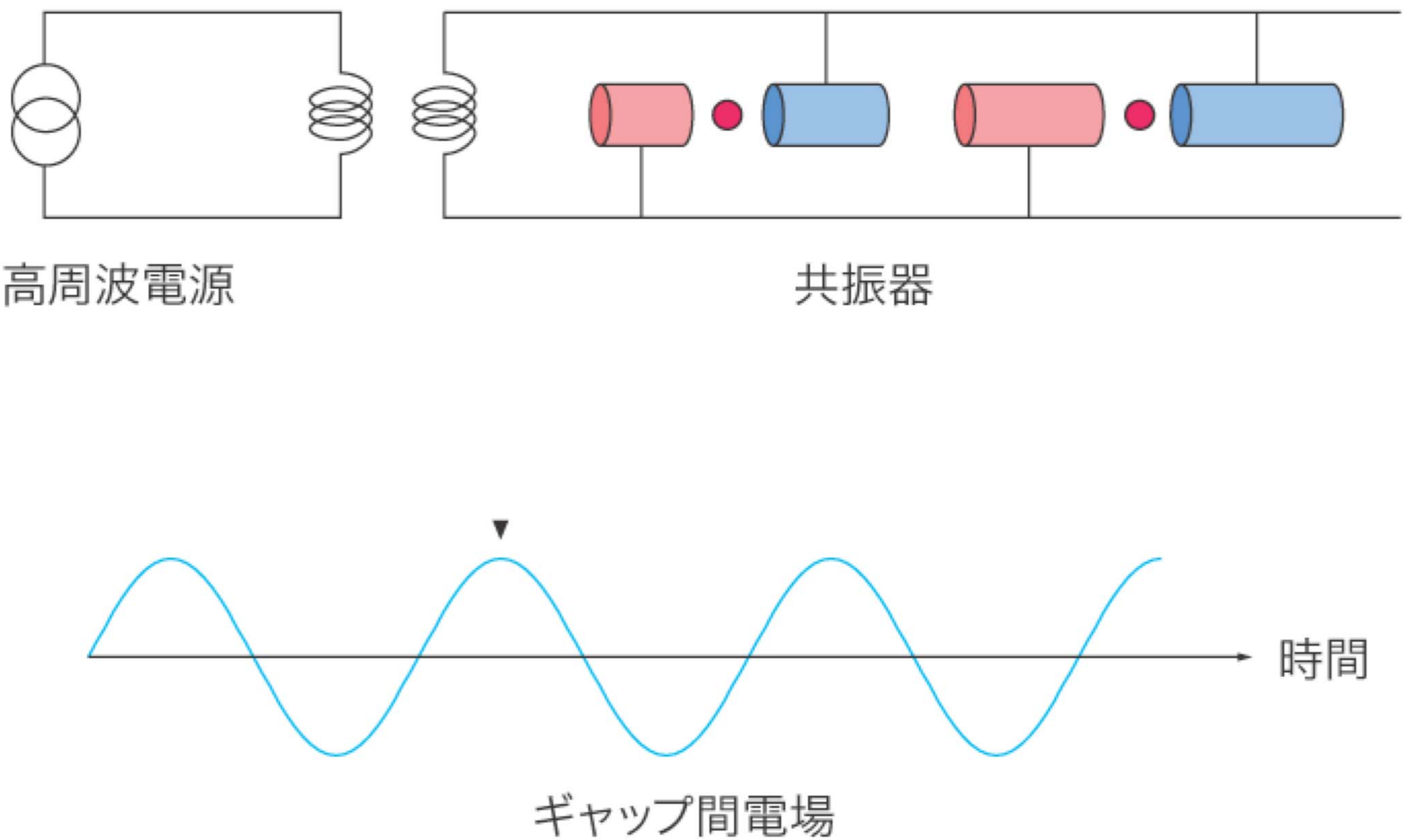
Successive acceleration in rf-gaps





R. Widoroe
(1902-1996)

Became popular after WW-II



Beam: CW (continuous wave)

Modern Linacs

RIKEN Heavy-ion linac
Heavy-ion : 6 MeV/u

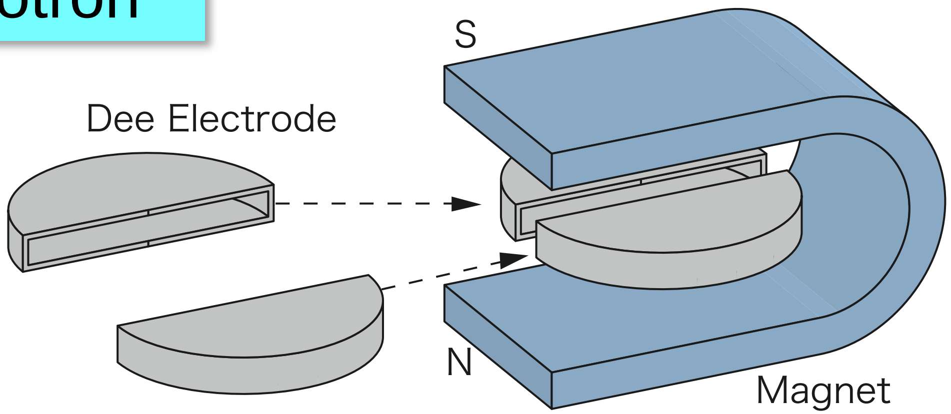
J-PARC proton linac
p: 181 MeV

Lawrence driven by Wideroe's paper (1929)

Re-use of rf-acceleration => compact!

E. O. Lawrence
(1901-1958)

Cyclotron



Principle of cyclotrons: Isochronism

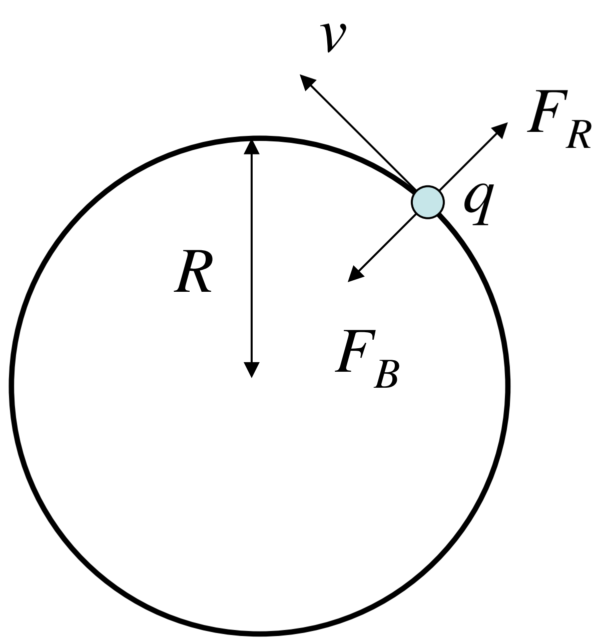
$F_R = mv^2/R$

$F_B = qvB$

$F_R = F_B$ => $mv^2/R = qvB$ =>

$R = mv/qB$

(=>Faster particle has a larger R)

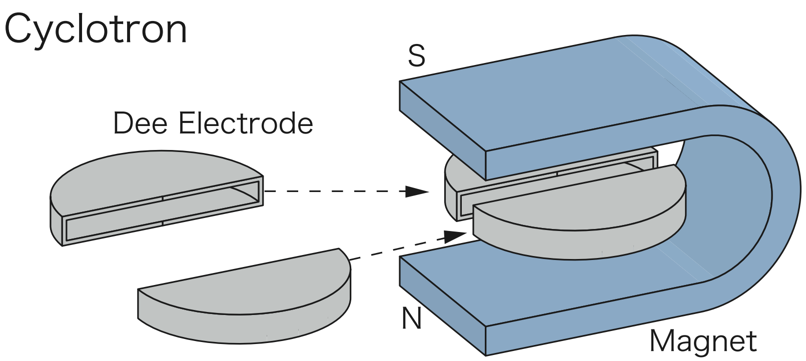


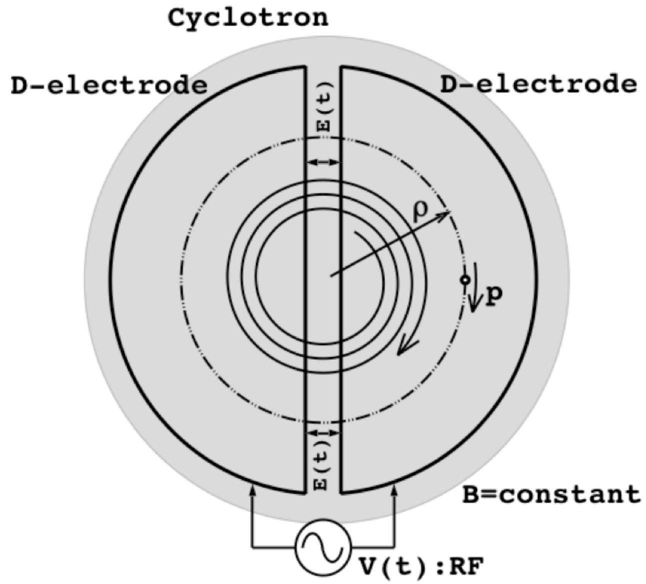
Revolution period:

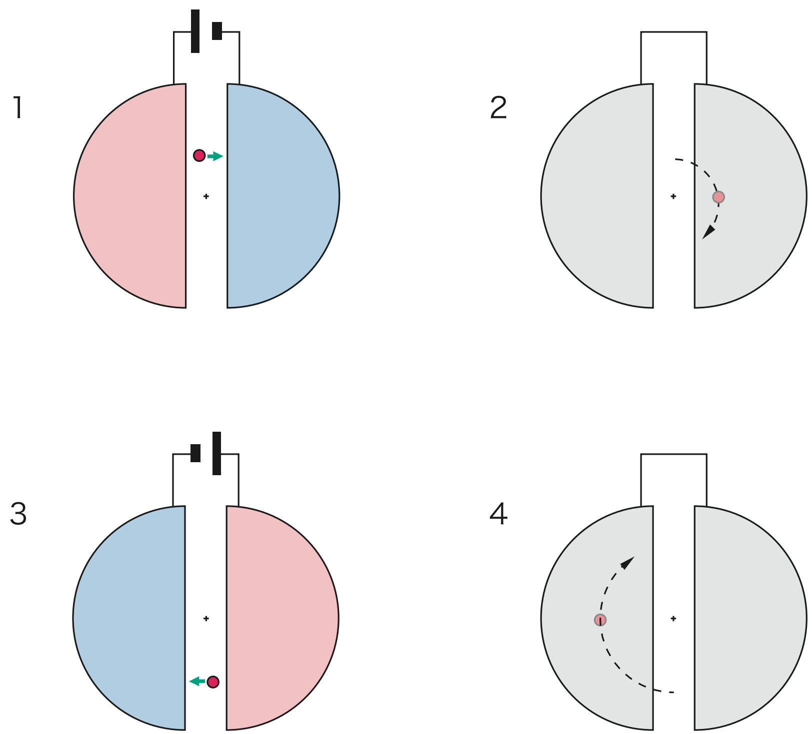
$T=2\pi R/v=2\pi m/qB$

(does not depend on the velocity
as far as the velocity is low)

How cyclotrons work?







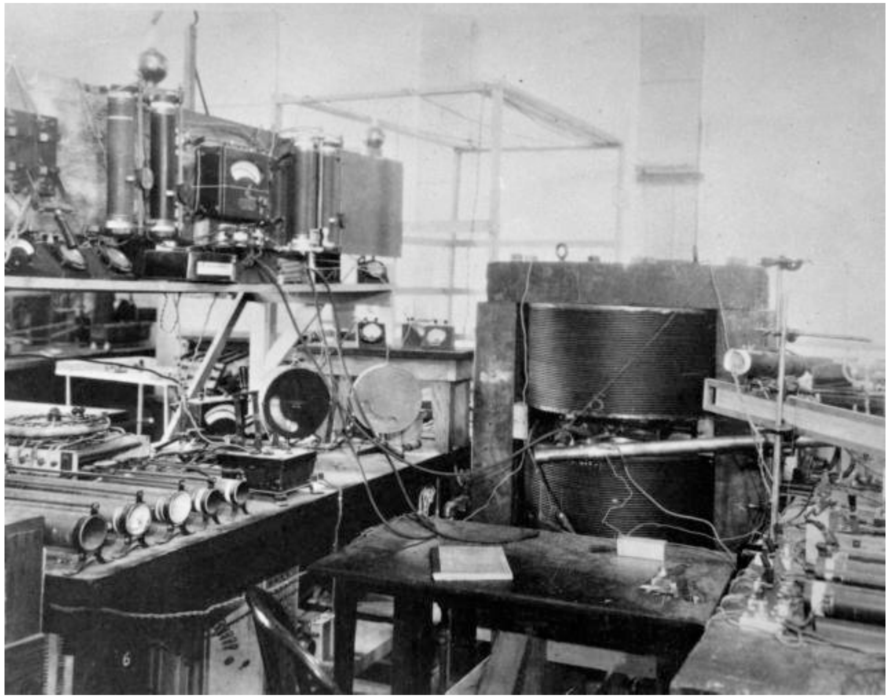
Beam : CW

The 1st cyclotron

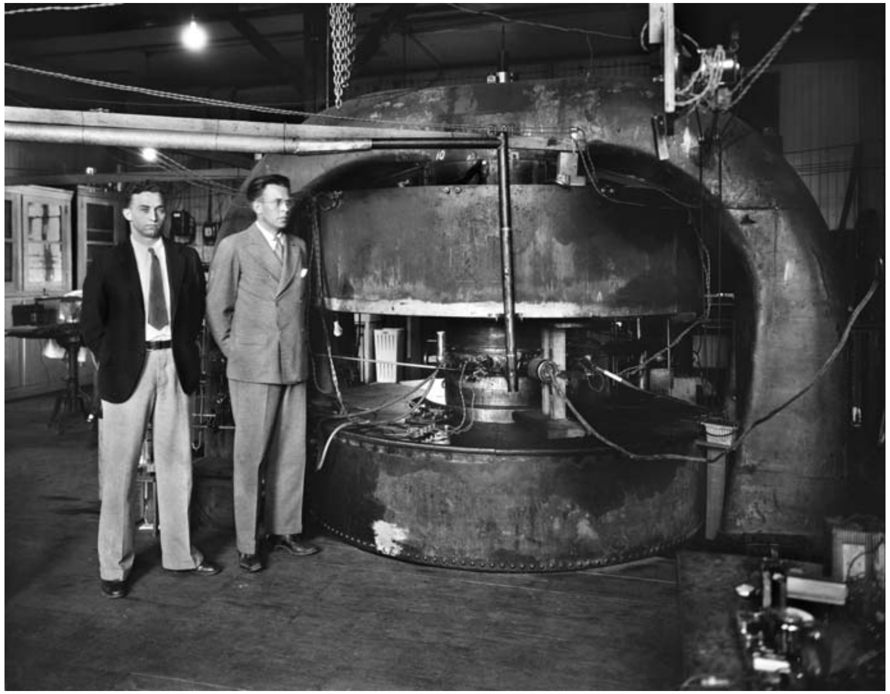
The 1st cyclotron（1931)
Proton 80 keV （v = 0.013 c）
Diameter 10 cm

Ernest O. Lawrence (1901-1958)
1929 Invention of cyclotron
1939 Nobel prize in physics：
「Invention of cyclotron & RI production」

Evolution of cyclotrons-1



Laurence・Livingston (1932)
Proton 1.2 MeV (v = 0.048 c)
Diameter 28 cm



Laurence・Livingston (1932)
Deuteron 5 MeV (v = 0.073 c)
Diameter 69 cm

Production of radioactive isotopes
Medical & biological applications
Promoted nuclear physics experiments

Evolution of cyclotrons-2

Laurence's team (1939)
Deuteron 16 MeV
Diameter 152 cm

(v = 0.13 c)

Nishina's team (1944)
Deuteron 16 MeV
Diameter 152 cm

World's largest before WW-II

Limitation of classical (weak-focusing) cyclotron

Laurence's group (1941)
Intended for proton 100 MeV (v = 0.43 c)
Diameter 470 cm／Weight 4,000 ton
Did not work as a cyclotron due to the relativistic effect

Principle of phase stability (1945) => Birth of synchrotron

V. Veksler, J. Phys. (USSR) 9 (1945) 153
E. M. McMillan Phys. Rev. 68 (1945) 143

- Constant, closed orbit

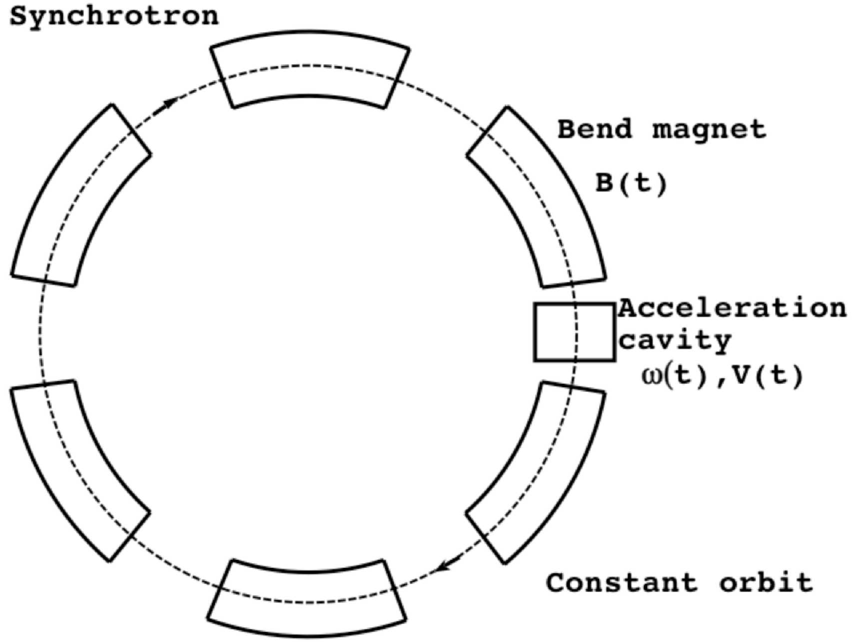
- Two synchronization condition

  1) Momentum and B

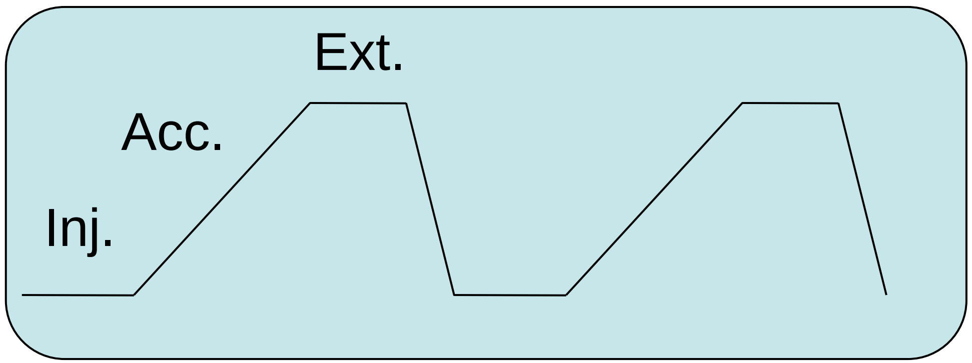
$$p(t) = q\rho B\,(t)$$

  2) Momentum and rf-frequency

$$f_{rev}(t) = \frac{v(t)}{2\pi\rho} = \frac{p(t)}{m_0\gamma(t)}\frac{1}{2\pi\rho}$$

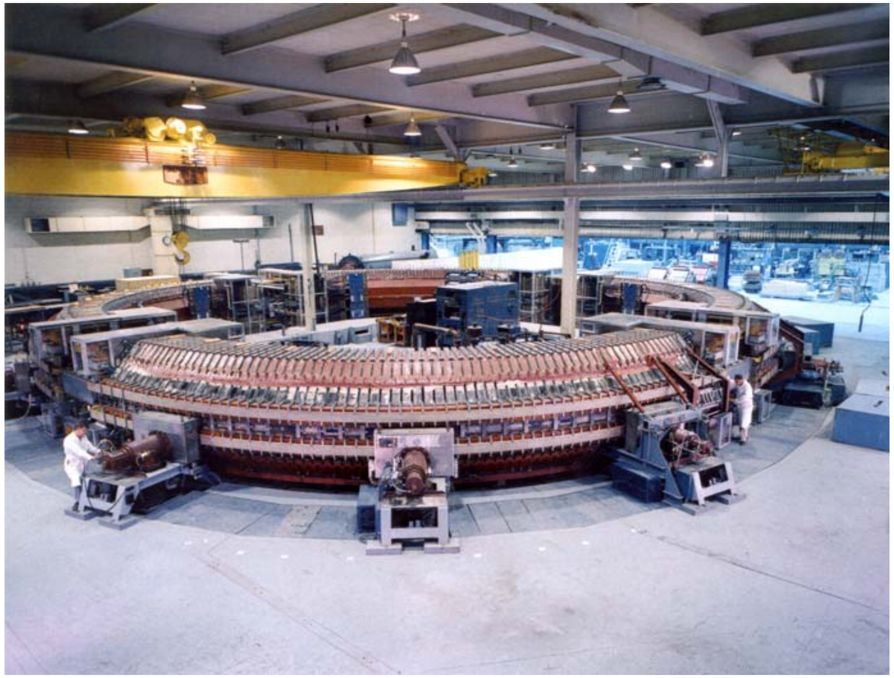


$B\,(t)$



Beam : Pulse

Evolution of synchrotron



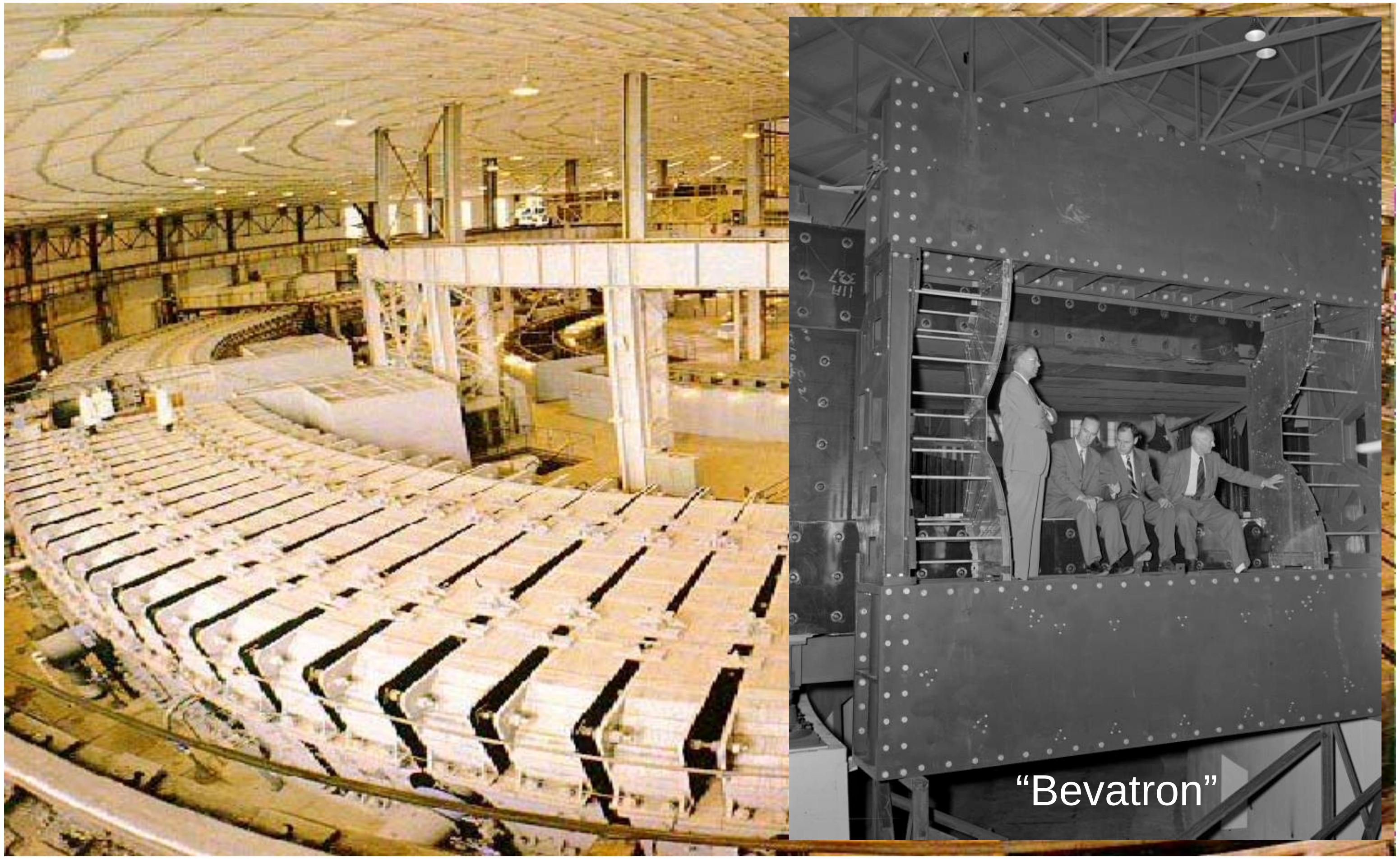
“Cosmotron” (BNL/1952)
Proton 3 GeV (v = 0.971 c)
Diameter 18 m
Weight 2,000 ton

Meson・Baryon

“Bevatron” (LBL/1954)
Proton 6 GeV (v = 0.991 c)
Diameter 39 m
Weight 10,000 ton

Anti-proton(1955)・Meson・Baryon

Limitation of classical (weak-focusing) synchrotron



“Synchrophastron” (Dubna/1957)
Proton10 GeV (v = 0.996 c)
Diameter 56 m／Weight 36,000 ton

Principle of strong focusing (1952)

R. D. Courant, M. S. Livingston, H. S. Snyder, Phys. Rev. 88 (1952) 1190

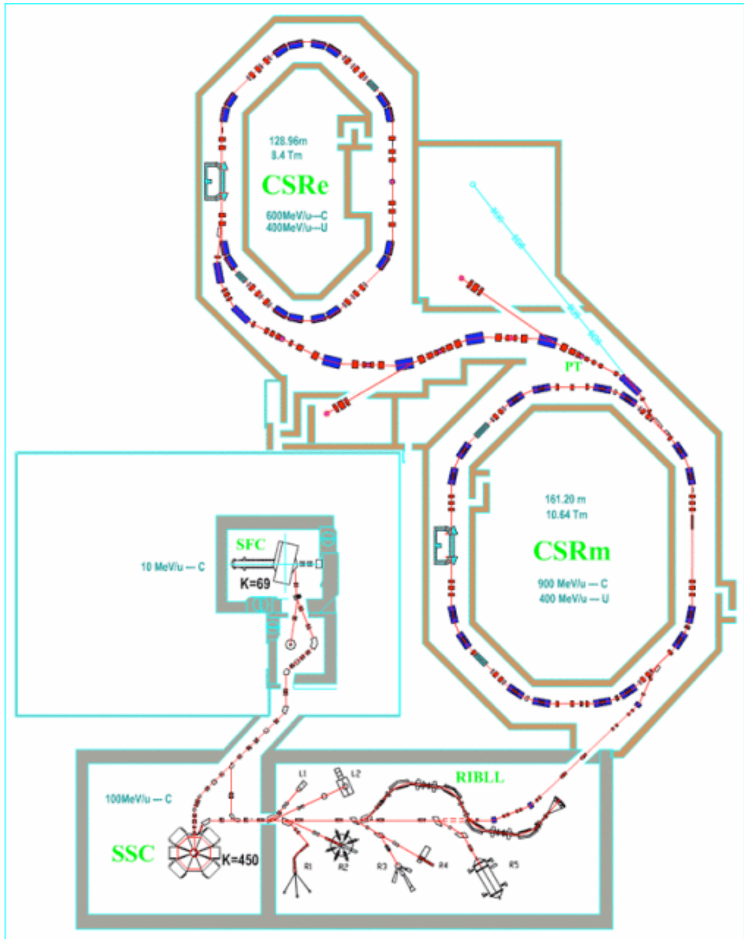
"AGS" (BNL/1960)
Protons 33 GeV (v = 0.9996 c)
Diameter 257 m

Muon-neutrino (1962)
CP-violation (1964)
J-particle (1974)

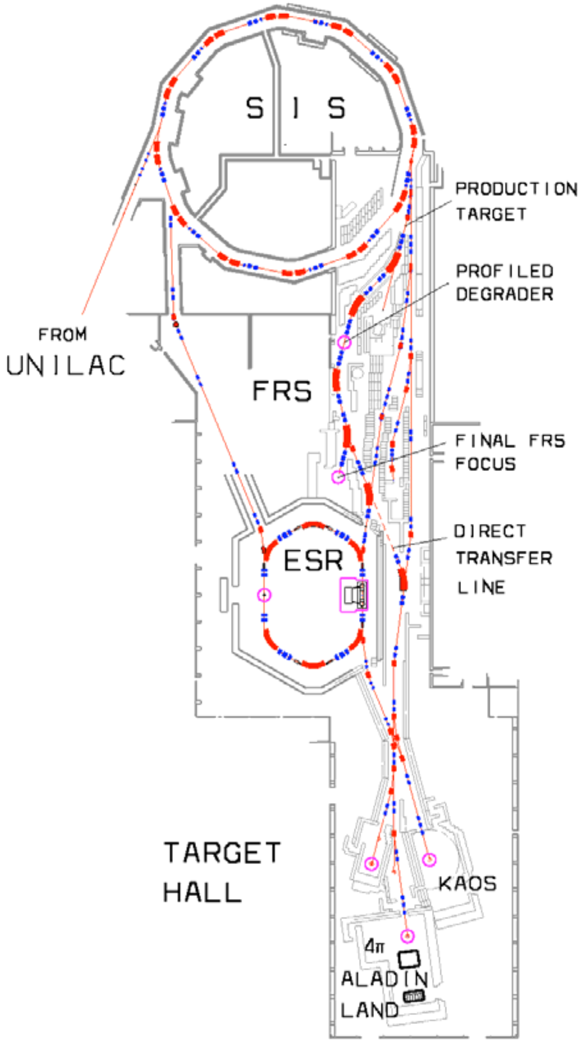
"CERN PS" (CERN/1959)
Proton 28 GeV (v = 0.9995 c)
Diameter 200 m

Heavy-ion synchrotrons

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences



GSI (Germany)



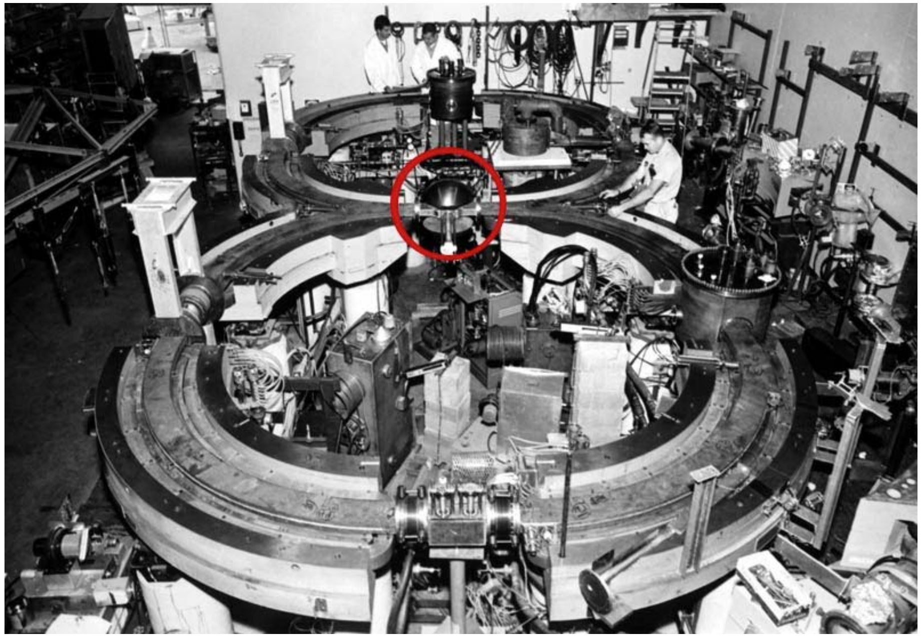
Invention of collider (1959-61)

G. Budker (INP/RU)
B. Touschek (Frascati/IT)
D. Kerst (MURA/US)

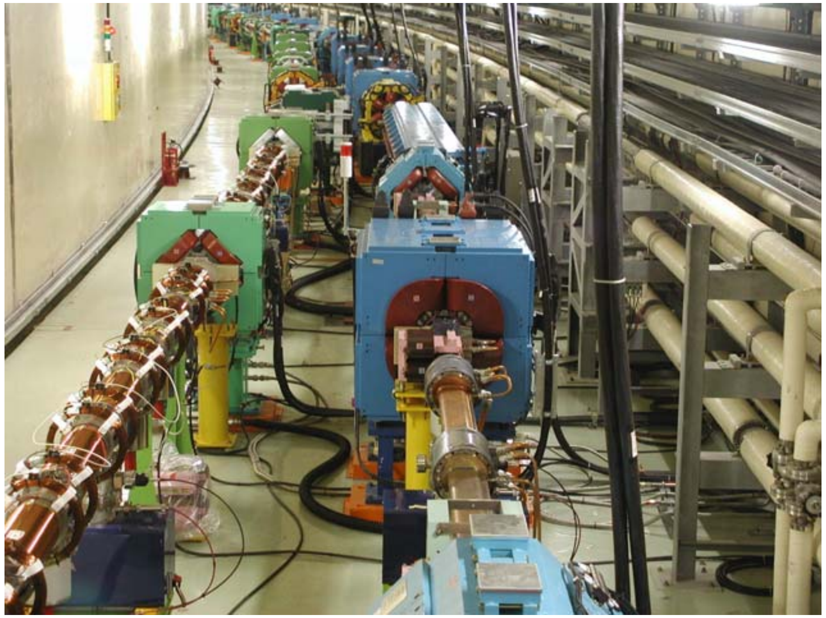
$$2E_{\text{lab}} m_0 c^2 = E_{\text{cm}}^2$$

(=> Prove this!)

e 100 MeV + e 100MeV ⇔ e 40 GeV + e 0 MeV



HEPL(Stanford/1960s)



“KEKB”(KEK)

eletron 8 GeV + positron 3.5 GeV

Origin of CP-violation (2001)

Large Hadron Collider (CERN : 2009~)

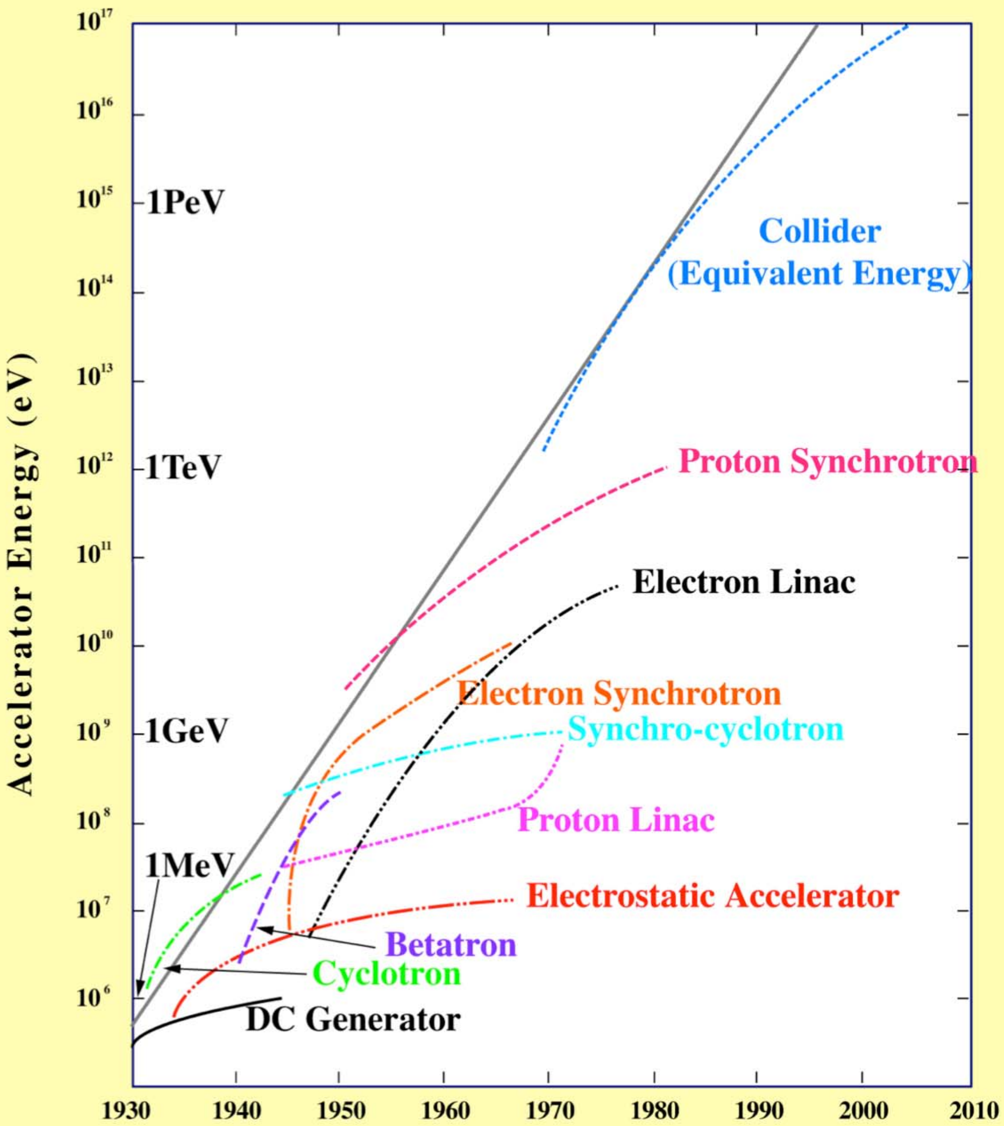
Proton（7 TeV + 7 TeV ）／Diameter 9 km／v = 0.99999999 c

Inside of tunnel（undergrond 100 m）

<=SC-magnet

SC-cavity=>

Evolution of high-energy accelerators



What has driven the evolution?

•The accelerators has provided the answers to the fundamental question: "How are the matters formed?"

•The accelerators has created various research fields and/or applications which could not be covered with the other methods:

e.g. RI production, Synchrotron radiation, Cancer treatment etc..

Nobel prizes related to the Accelerator

1939(P)	Lawrence	(US)	Invention of cyclotron
1951(P)	Cockcroft & Walton	(UK)	First man-made nuclear reaction
1951(C)	Seaborg & McMillan	(US/Cyc)	Transuranium elements
1959(P)	Segre & Chamberlain	(US/Bevatron)	Antiproton
1961(P)	Hofstadter	(US/SLAC)	Electron scattering
1968(P)	Alvarez	(US/Bevatron)	Bubble chamber
1976(P)	Ting & Richter	(US/AGS, SLAC)	J/ψ
1980(P)	Cronin & Fitch	(US/AGS)	Discovery of CP violation
1984(P)	Rubbia & van der Meer	(CERN/SppS)	W/Z boson
1988(P)	Lederman, Schwarts, Steinberger	(US/AGS)	Muon neutrino
1988(C)	Deisenhofer, Huber, Michel	(GR/DESY PF)	Photosynthesis
1990(P)	Friedman, Kendal, Taylor	(US/SLAC)	Quark
1995(P)	Perl	(US/SLAC)	Tau lepton
1997(C)	Boyer & Walker	(UK/SRS Daresbury)	ATP synthesis
2003(C)	Mackinnon	(US/CHESS,NSLS)	Potassium channels
2006(C)	Kornberg	(US/SLAC)	Eukaryotic transcription
2008(P)	Kobayashi & Maskawa	(JP/KEKB,US/SLAC)	Origin of CP violation
2009(C)	Yonath	(JP/KEK-PF,GR/DESY-PF)	Ribosome

Classification of accelerators

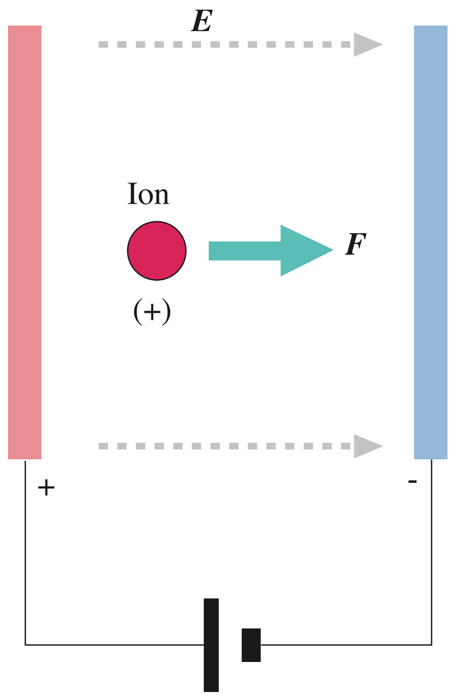
E	B	Linear	Spiral orbit	Closed orbit
Static	Static	Cockcroft-Walton, Van de Graaff		(Impossible!)*
RF(fixed)	Static	(RF) Linac	Cyclotron, Microtron	(e-Storage Ring)
RF(mod)	Static		Synchro-Cyclotron, FFAG	
RF(mod)	Varying			Synchrotron
Induction		Induction Linac		Betatron

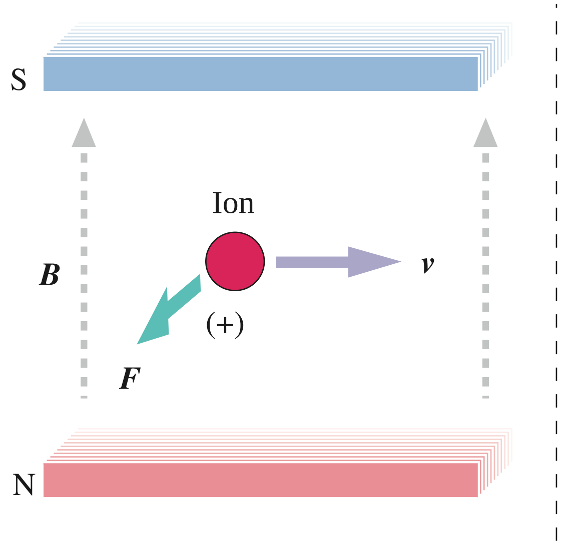
* => Prove this!

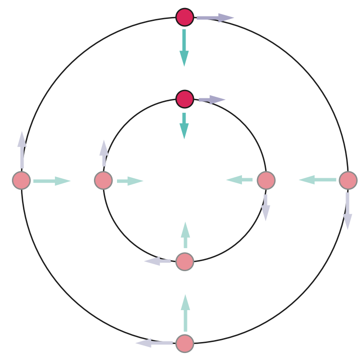
3. A bit about beam dynamics

Particle motion: Lorentz force

$$d\boldsymbol{p}/dt = q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$$







Motion of a charged particle
in a uniform magnetic field

Basics of kinematics-1

$$c \equiv 299\ 792\ 458\ \mathrm{m/s}$$
$$\approx 2.998 \times 10^{8}\,\mathrm{m/s} \approx 3 \times 10^{8}\,\mathrm{m/s}$$

- Kinematical factor (dimension-less)

$$\beta \equiv \frac{|\boldsymbol{v}|}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

- Equivalence

$$\gamma^2 - \beta^2\gamma^2 = 1$$

or

$$\beta\gamma = \sqrt{\gamma^2 - 1}$$

- Total energy of a particle with the rest mass m_0

$$E = m_0 c^2 \big/ \sqrt{1-\beta^2} = m_0 c^2 \gamma \qquad \Rightarrow \gamma = E \big/ m_0 c^2$$

Low-velocity particle.. $E \approx m_0 c^2 + \frac{1}{2} m_0 \boldsymbol{v}^2$

- Kinetic energy

$$T = E - m_0 c^2 = m_0 c^2 (\gamma - 1)$$

$$\Rightarrow \gamma = T \big/ m_0 c^2 + 1$$

Basics of kinematics-2

- Momentum

$$\boldsymbol{p} = m_0 c \boldsymbol{\beta} \gamma = m_0 \gamma \boldsymbol{v}$$

- Energy gain

$$\frac{dT}{dt} = m_0 c^2 \frac{d\gamma}{dt} = \dot{\boldsymbol{x}} \cdot \frac{d\boldsymbol{p}}{dt} = q\boldsymbol{E} \cdot \dot{\boldsymbol{x}}$$ (=> Exercise)

$$\Rightarrow \Delta T = \int q\boldsymbol{E} \cdot \dot{\boldsymbol{x}} dt = q \int \boldsymbol{E} \cdot \boldsymbol{dx}$$ Unit : [eV (electron-volt)]

- Rest mass

Electron $m_e c^2 = 511\,\mathrm{keV} \approx 0.5\,\mathrm{MeV}$

Proton $m_p c^2 = 938\,\mathrm{MeV} \approx 1\,\mathrm{GeV}$

Ion $1\,\mathrm{amu} \Rightarrow 931.494\,\mathrm{MeV}$ $m_A c^2 = 931.494\ A\,\mathrm{MeV}$

Exercise

- Calculate velocity of a proton of T = 400 MeV

$$\gamma = T/m_p c^2 + 1 = 400/938 + 1 = 1.426\ldots$$

$$\beta\gamma = \sqrt{\gamma^2 - 1} = \sqrt{1.426^2 - 1} = 1.017\ldots$$

$$\beta = 1.017\ldots/1.426\ldots \approx 0.713$$

- Calculate velocity of an electron of T = 400 MeV

$$\gamma = T/m_e c^2 + 1 = 400/0.511 + 1 = 783.778865\ldots$$

$$\beta\gamma = \sqrt{\gamma^2 - 1} = 783.778227\ldots$$

$$\beta = 783.778227\ldots/783.778865\ldots \approx 0.999999186$$

Bending of charged particle beams

$$d\boldsymbol{p}/dt = q(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B})$$

$|\boldsymbol{E}| \Rightarrow$ 30 kV/1 cm = 3×10^6 [V/m]

$|\boldsymbol{B}| \Rightarrow$ 1 [T], $|\boldsymbol{v}| \Rightarrow$ 3×10^8 [m/s] (= c) ,

$|\boldsymbol{v}\times\boldsymbol{B}| \Rightarrow 3\times10^8$ [V/m]

=> Magnetic force is more effective for bending and focusing of beams in high-energy accelerators

E : Acceleration
B : Bending and Focusing

Focusing of beam

An accelerators should accelerate not only a single particle,
but also a bunch of particles as well.
=> It is designed so that the eq. of motion have a stable region….

Focusing effects in bending magnets

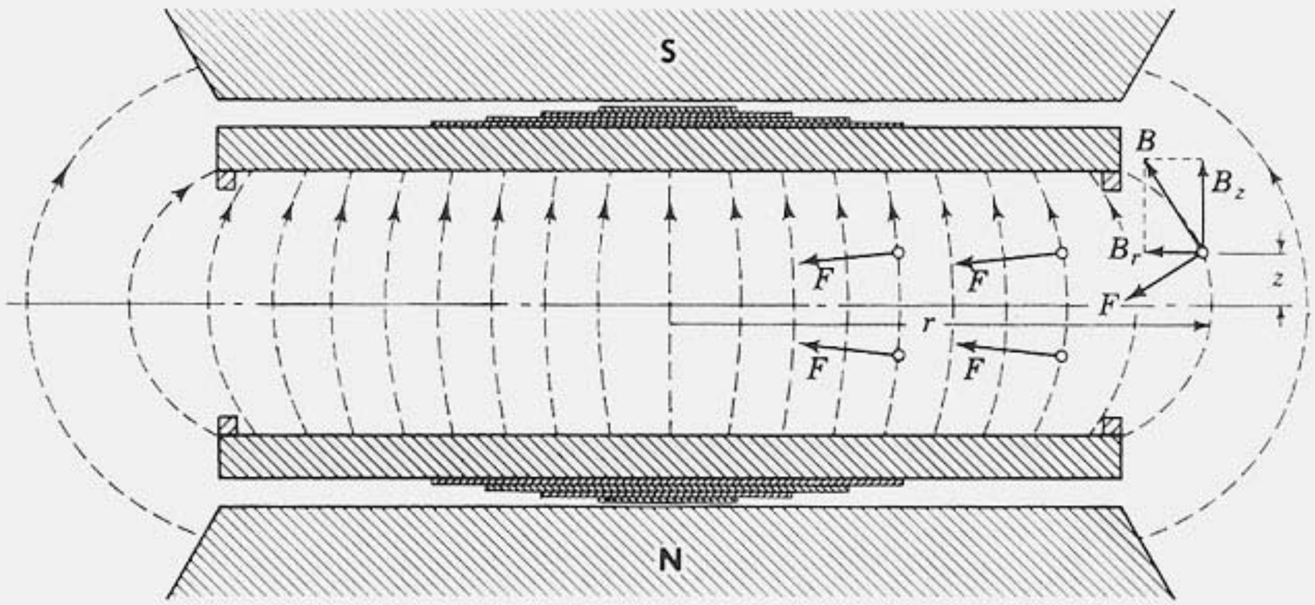
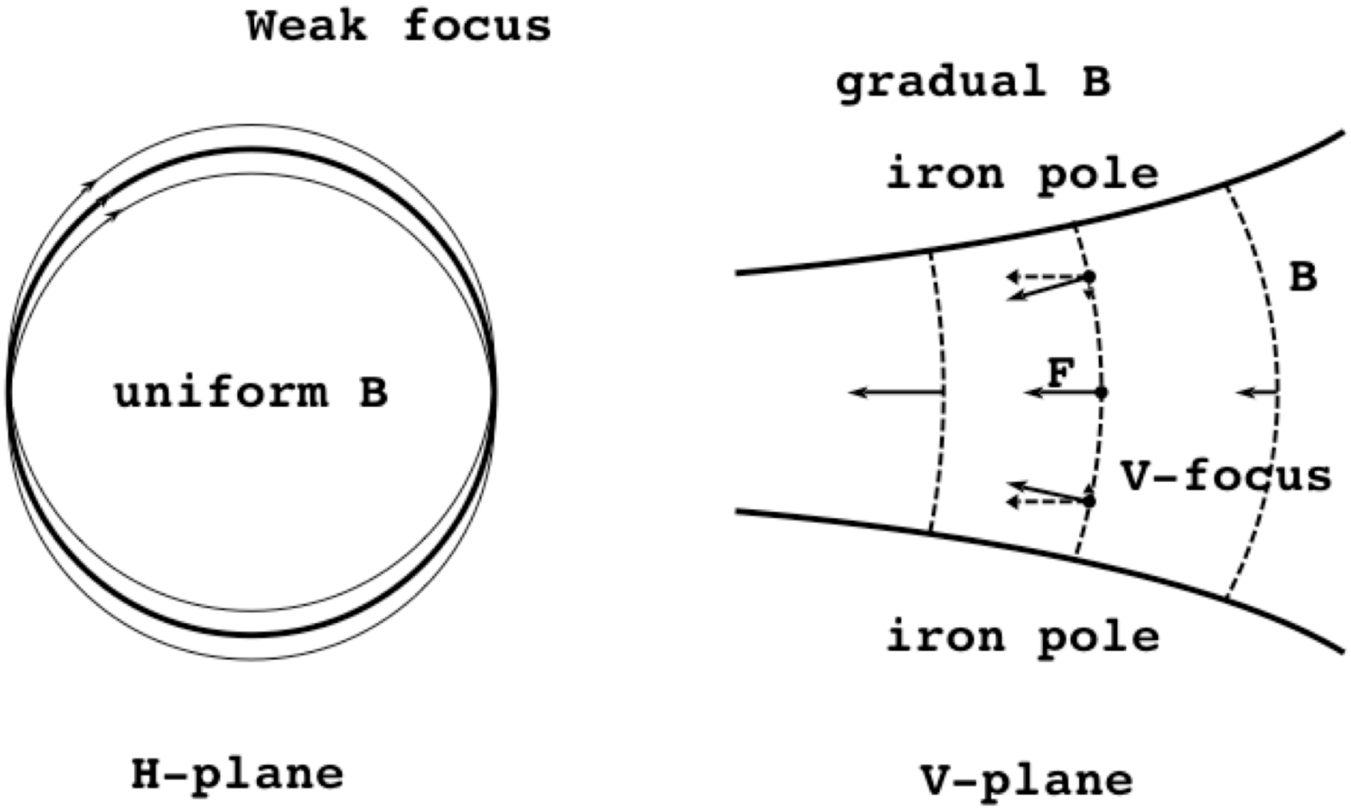
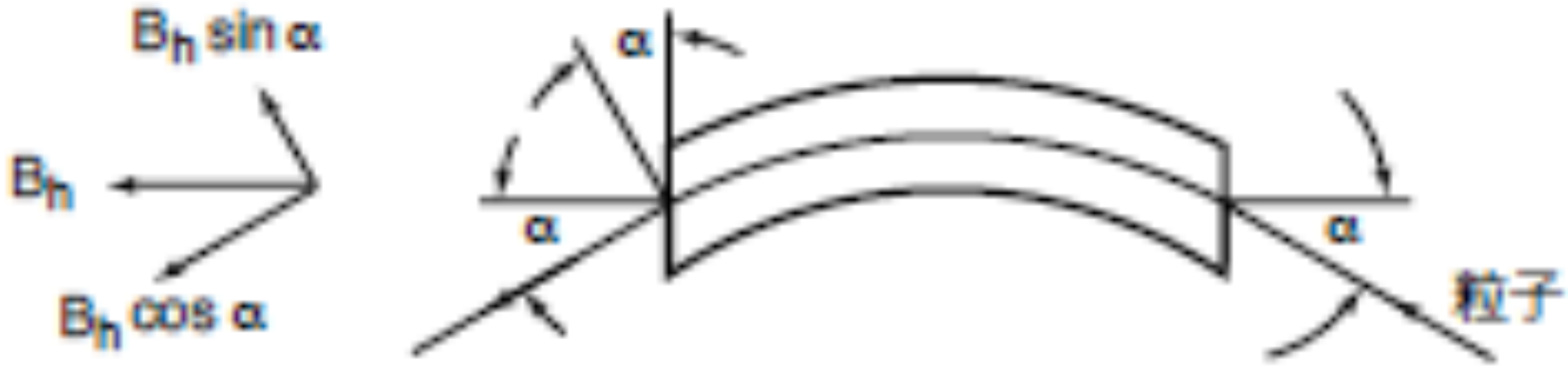


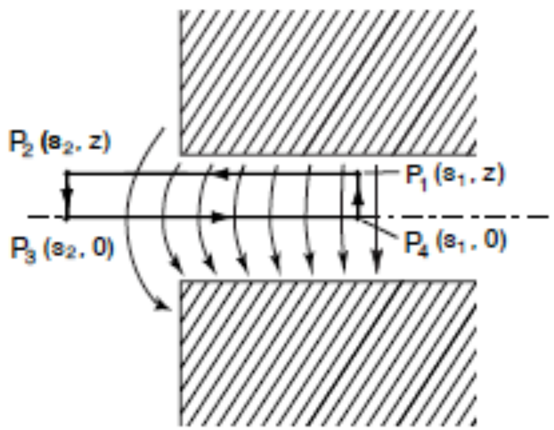
Fig. 6-7. Radially decreasing magnetic field between poles of a cyclotron magnet, showing shims for field correction.

“Weak” focusing



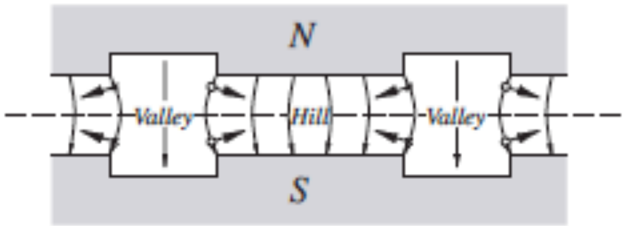
Focusing effects at the edge of BM

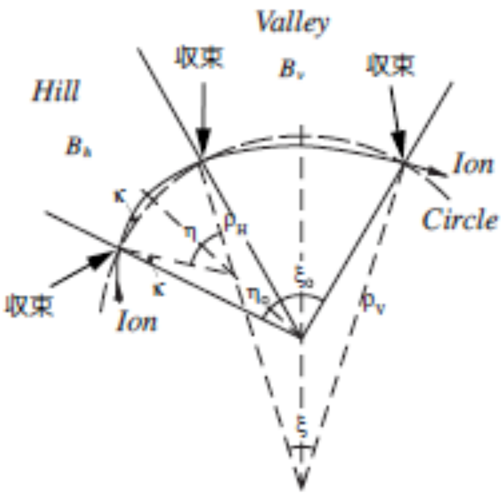


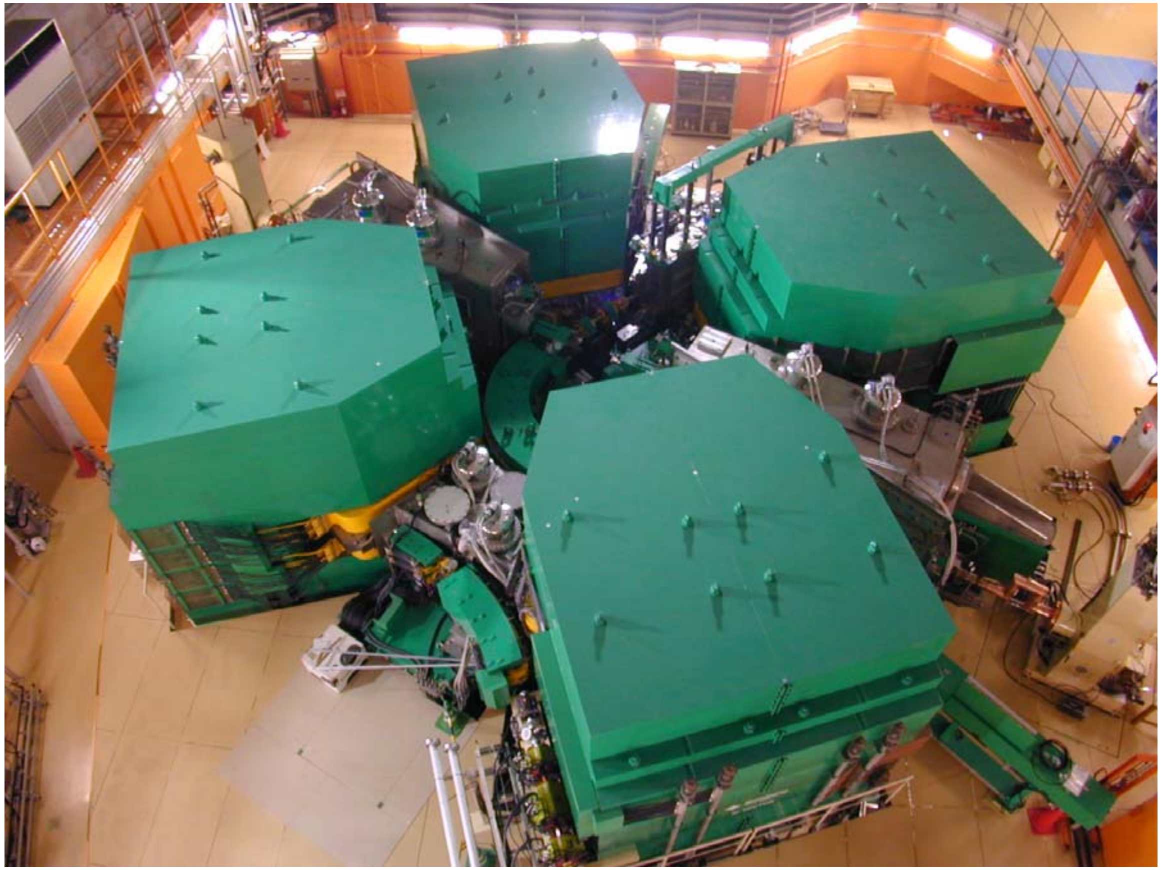


Edge focusing in ring cyclotron

- Consistent with the relativistic effect
- Suitable for high-power ion beams with compact space
- First proposed by Thomas (1938) , constructed later (1972)

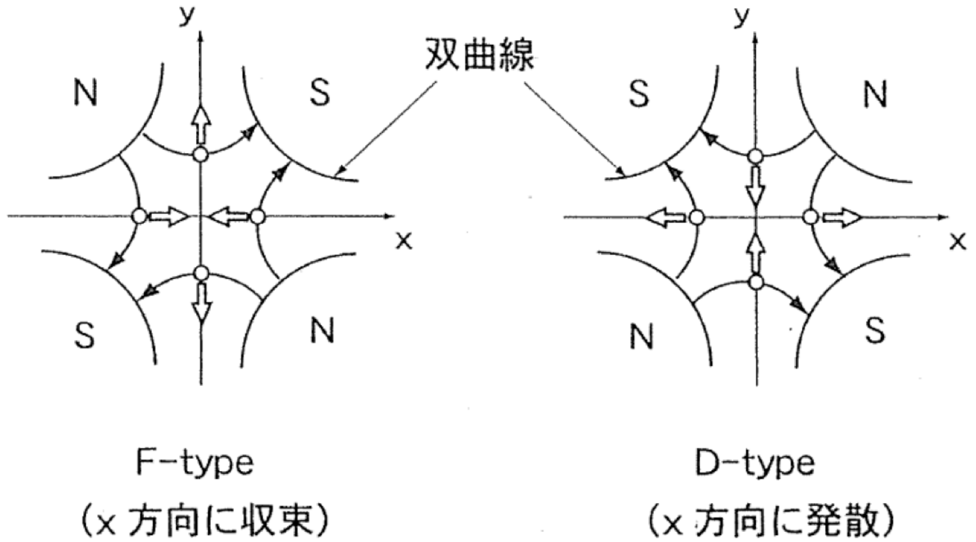


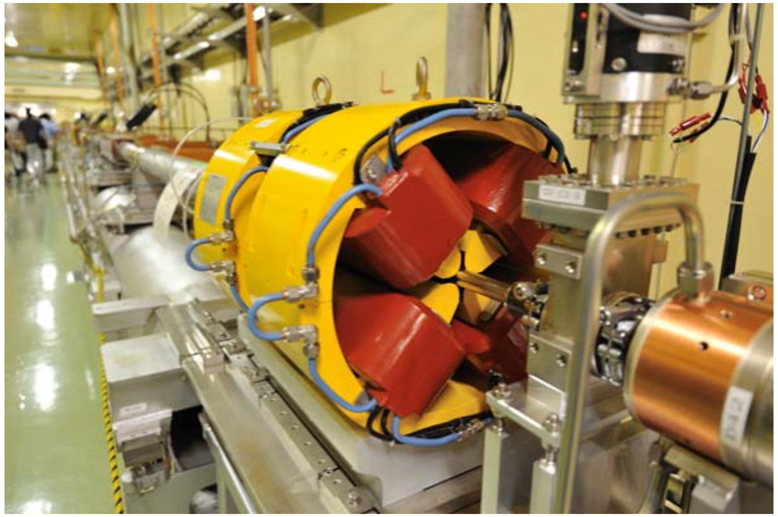




Focusing with quadrupole magnets

- Magnet field in quadrupole





$$\boldsymbol{B} = \left(B_x, B_y, B_s\right) = \left(ay, ax, 0\right)$$

$$a \equiv \left.\frac{\partial B_y}{\partial x}\right|_{x=0, y=0}$$: field gradient

Equation of motion in quadrupole magnets

- Lorentz force

$$\boldsymbol{v} = \left(v_x, v_y, v_s\right) = (0,0,v) \qquad \boldsymbol{B} = \left(B_x, B_y, B_s\right) = (ay, ax, 0)$$

$$\boldsymbol{v} \times \boldsymbol{B} = \left(v_x, v_y, v_s\right) = (-vax, vay, 0)$$

- Equation of motion

$$m\frac{d^2x}{dt^2} = -qvax \qquad m\frac{d^2y}{dt^2} = qvay \qquad \Rightarrow s \equiv vt, \quad B\rho = \frac{mv}{q}$$

$$\Rightarrow \frac{d^2x}{ds^2} = -\frac{qa}{mv}x = -\frac{1}{B\rho}\frac{\partial B_y}{\partial x}x \qquad \frac{d^2y}{ds^2} = \frac{qa}{mv}y = -\frac{1}{B\rho}\frac{\partial B_y}{\partial x}y$$

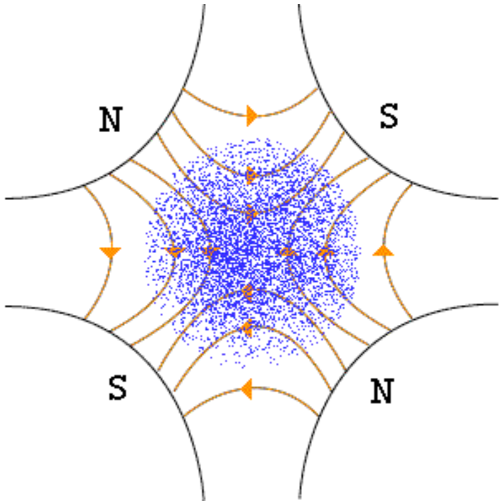
$$\frac{d^2x}{ds^2} + Kx = 0$$: Focusing in x

$$\frac{d^2y}{ds^2} - Ky = 0$$: Defocusing in y

$$K \equiv \frac{1}{B\rho}\frac{\partial B_y}{\partial x}\bigg|_{x=0,y=0}$$

"Strong" focusing

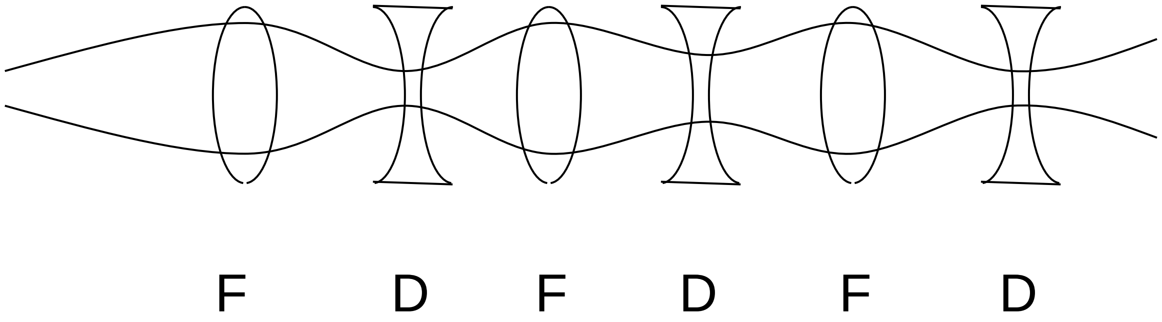
R. D. Courant, M. S. Livingston, H. S. Snyder, Phys. Rev. 88 (1952) 1190



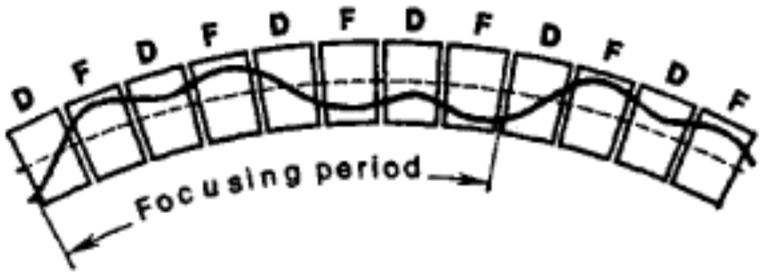
Quadrupole Magnet

Horizontal: Defocus (D)
Vertical : Focus (F)

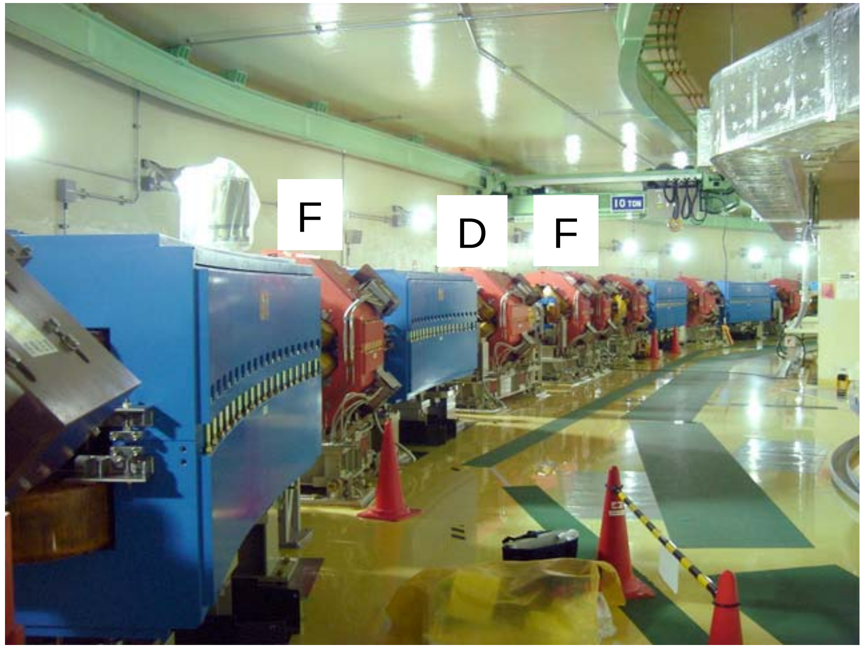
Beam envelope



Particle trajectory



Strong focusing everywhere



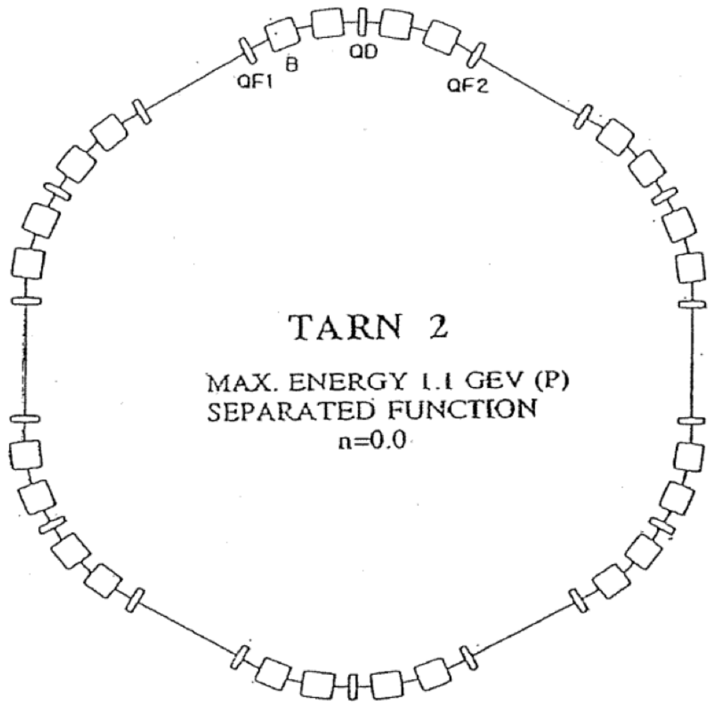
KEK 3GeV RCS

General equation of motion in accelerators

•Equation of motion in a single element

$$\frac{d^2x}{ds^2} + K_x x = 0$$

$$\frac{d^2y}{ds^2} + K_y y = 0$$



•Accelerators are composed of many elements:

$$\frac{d^2x}{ds^2} + K_x(s)x = 0 \qquad K_x(s+C) = K_x(s)$$

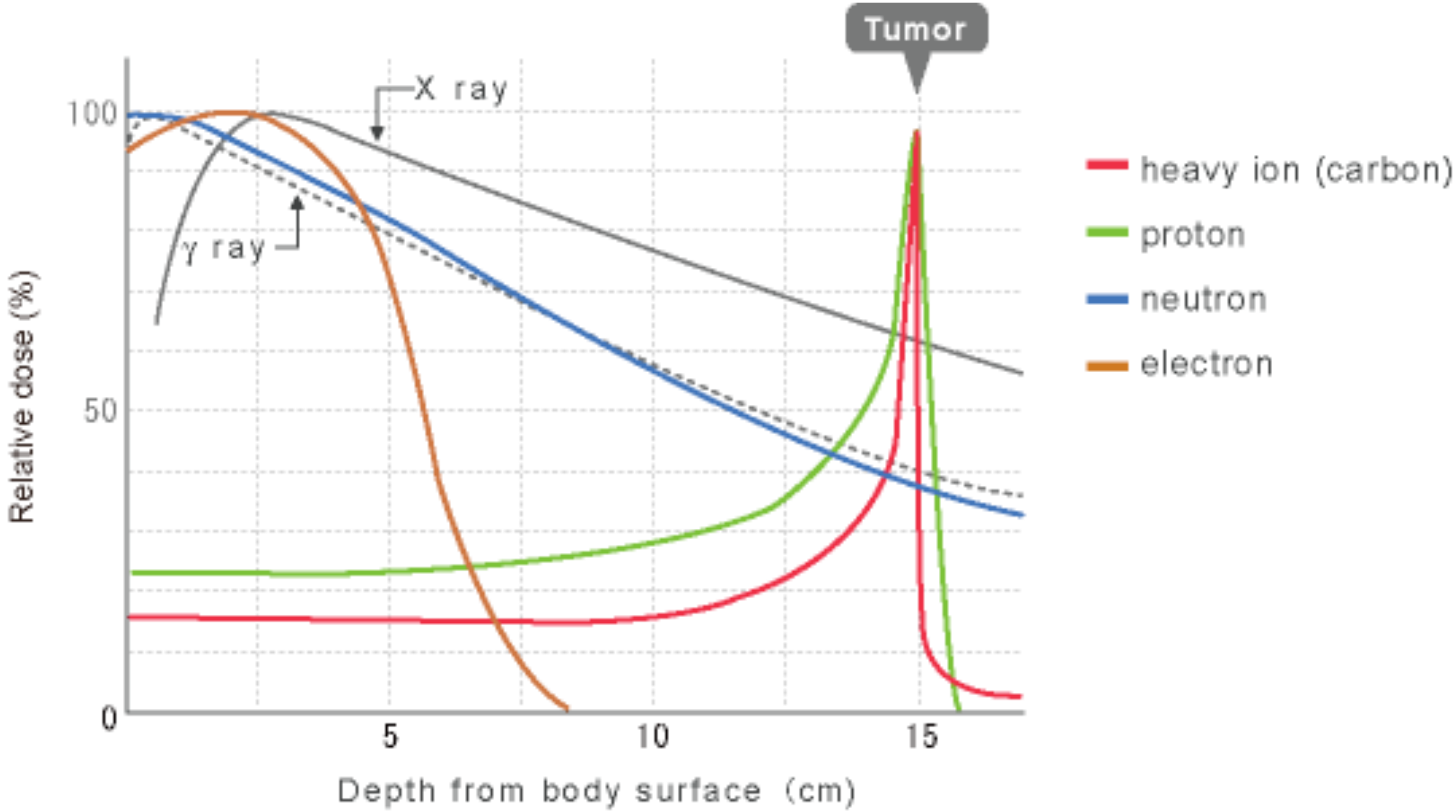
$$\frac{d^2y}{ds^2} + K_y(s)y = 0 \qquad K_y(s+C) = K_y(s)$$

=> Hill's equation

(C : circumference)

4. Application of accelerators

Example: Cancer therapy with heavy-ion beams



HIMAC synchrotron for cancer therapy

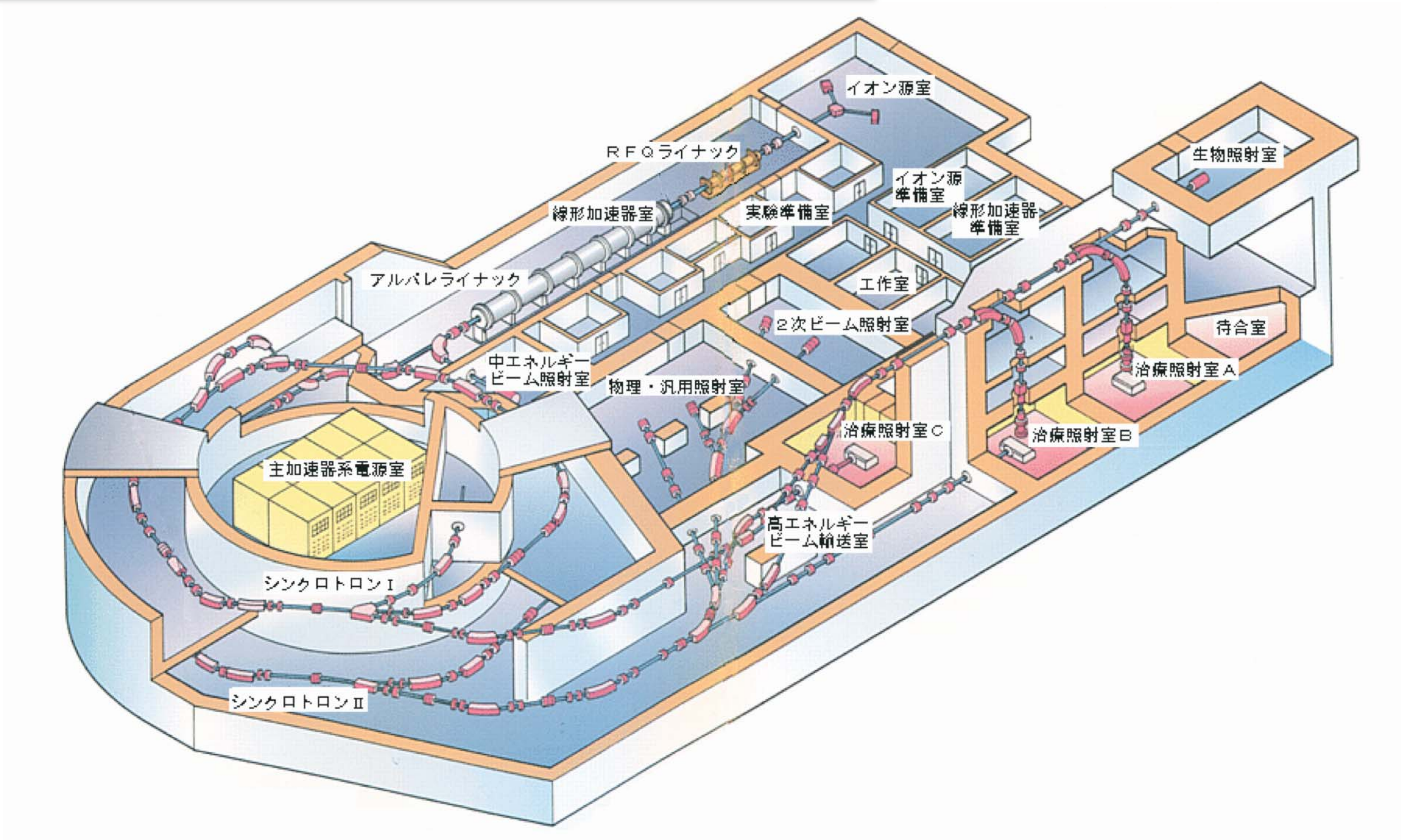
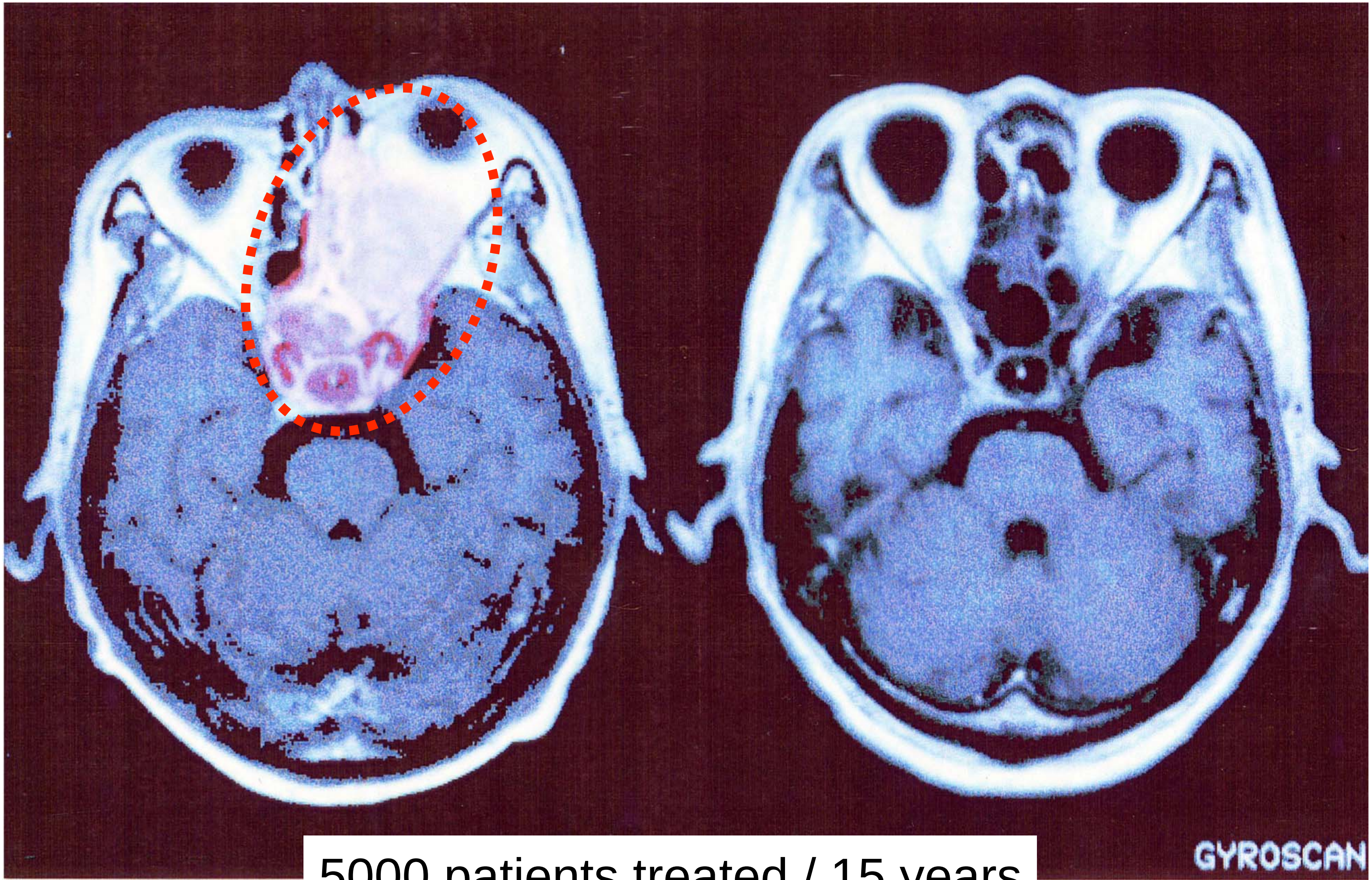


図4　放射線医学総合研究所の重粒子線がん治療装置（HIMAC）

[出典]放射線医学総合研究所：重粒子線がん治療装置HIMAC、1995年8月

治療例@放医研HIMAC

PET(positron-emission tomography) picture



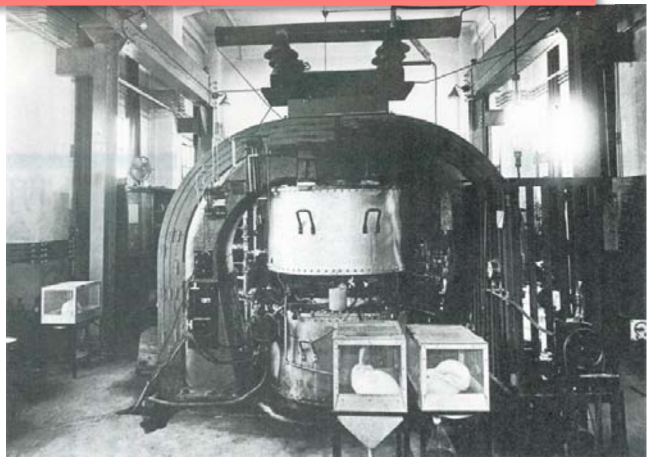
5000 patients treated / 15 years

Other applications

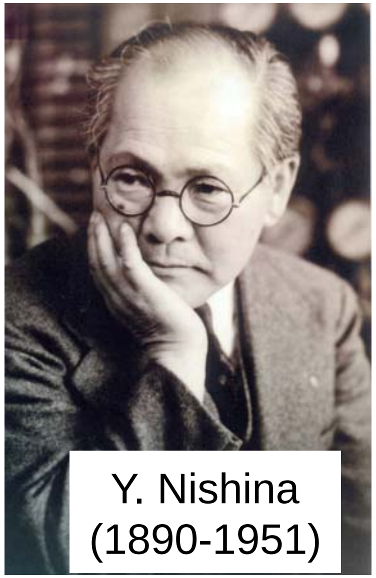
Varian medical systems
X-ray treatment system

Sumitomo Heavy Industries
Cyclotrons for PET 18F

5. RIKEN cyclotrons

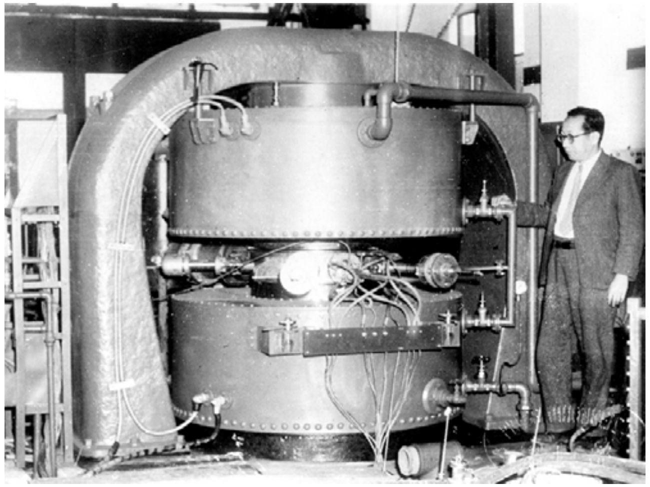


1st (the first Japanese cyclotron)
(Nishina /1937)



Y. Nishina
(1890-1951)

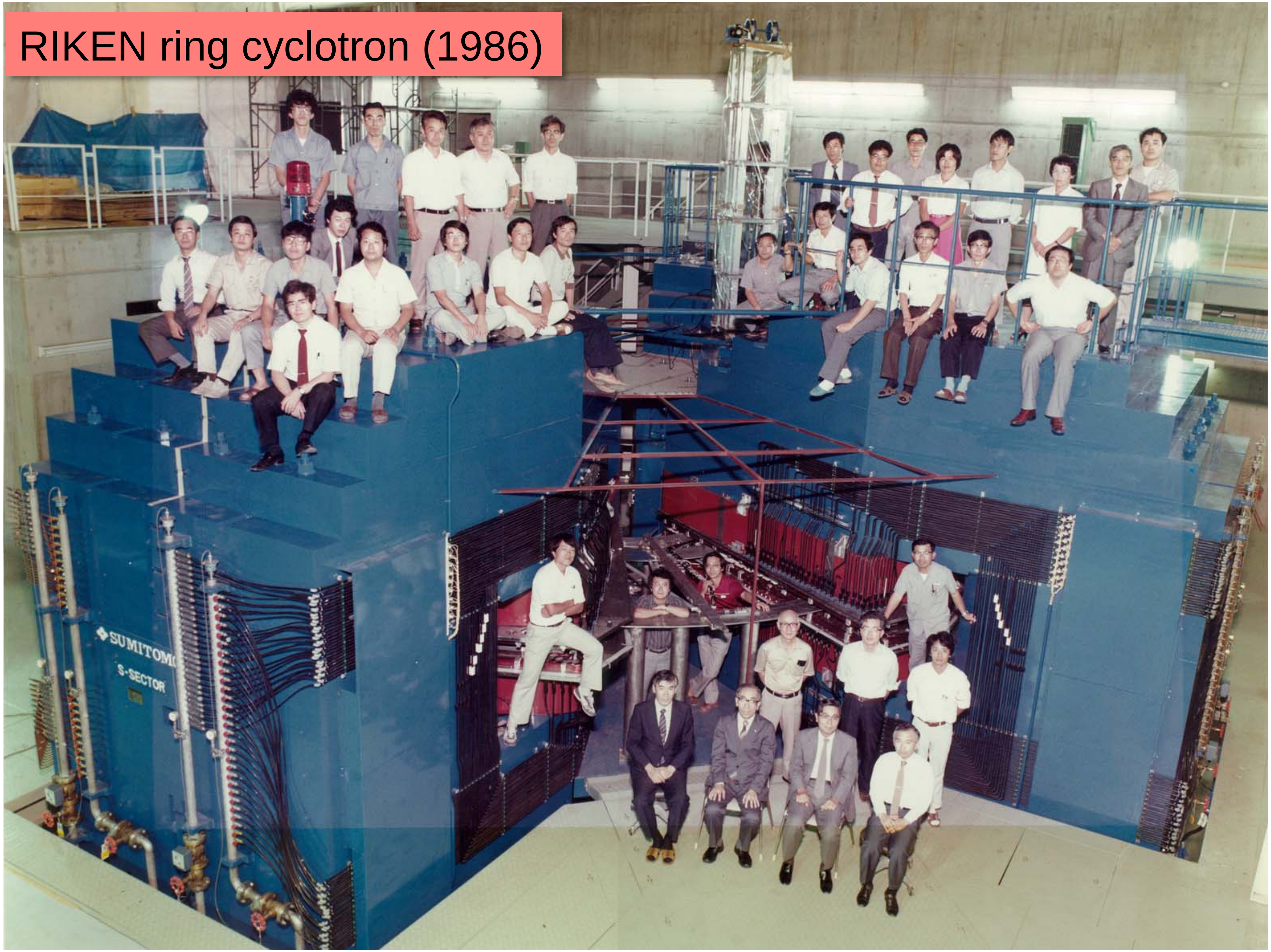
2nd (one of the largest in the world)
(Nishina /1944)



3rd (Sugimoto / 1952)

4th (in Wako campus)
(Kumagai / 1966)

RIKEN ring cyclotron (1986)



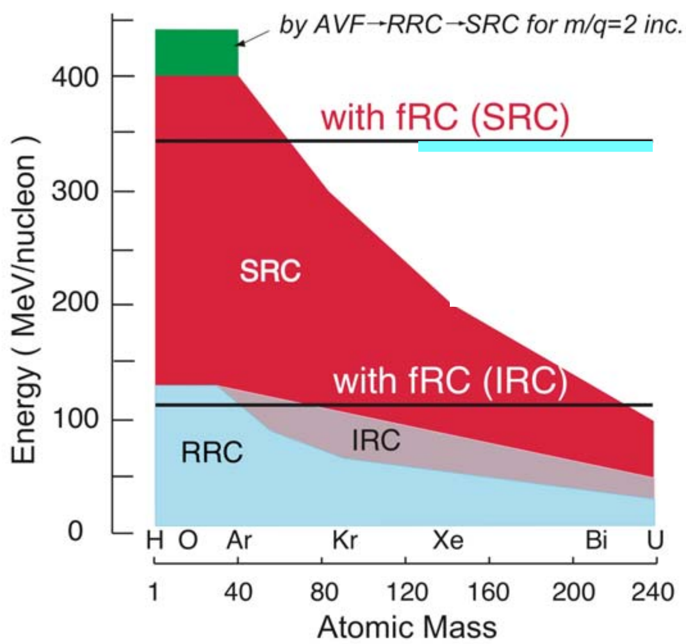
RIKEN SRC (2006)
⇒heaviest & most powerful in the world
⇒Uranium: 345 MeV/u

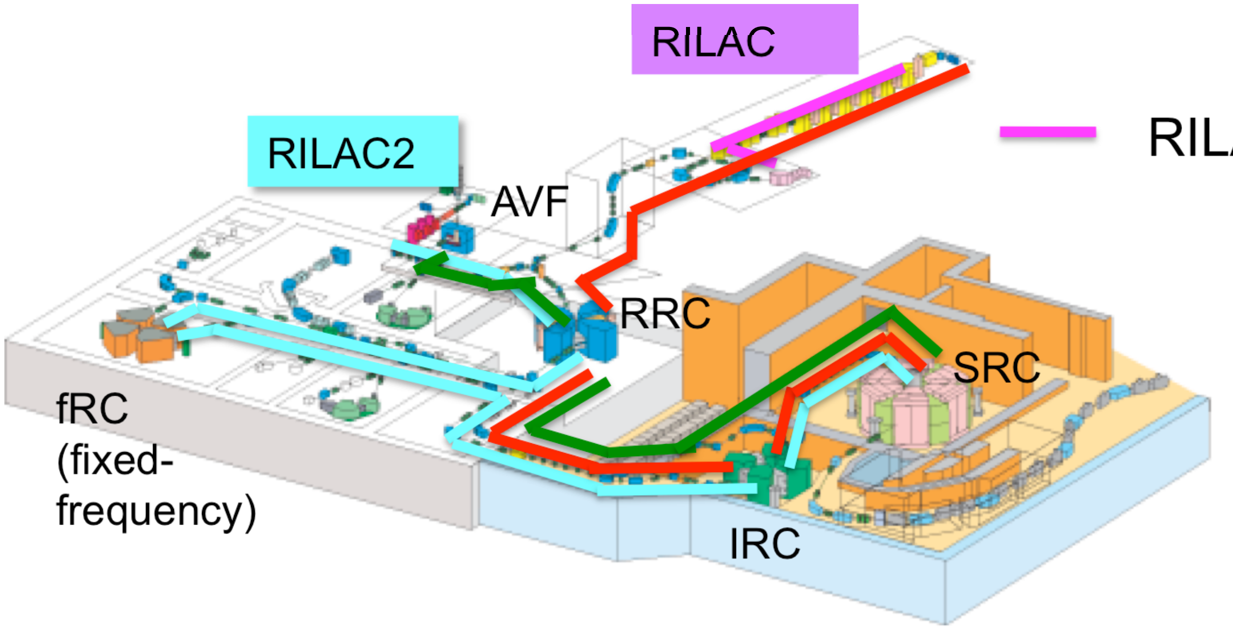
RIBF accelerators

AVF-injection mode (< 440 MeV/u)

Variable-energy mode (< 400 MeV/u)

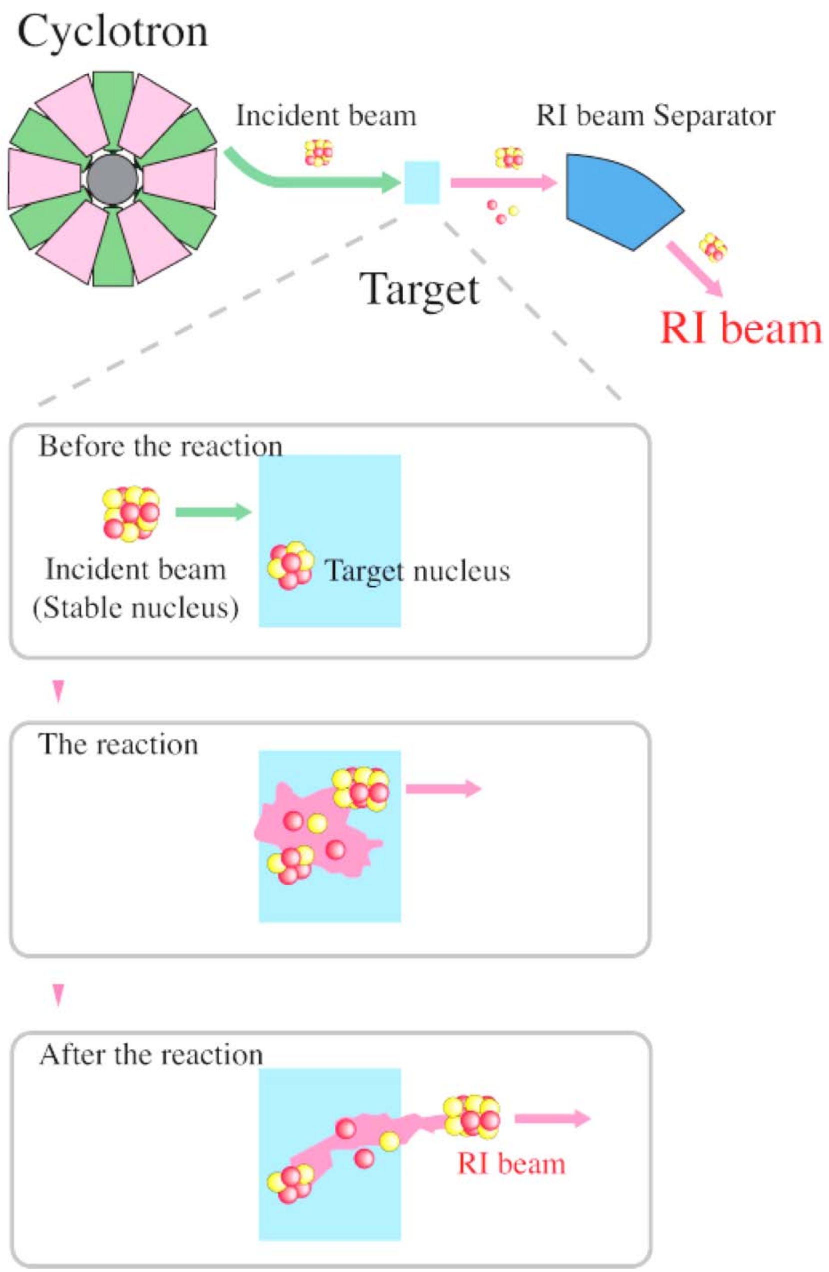
Fixed-energy mode (345 MeV/u)





RILAC – GARIS (SHE)

How to make RI-beams



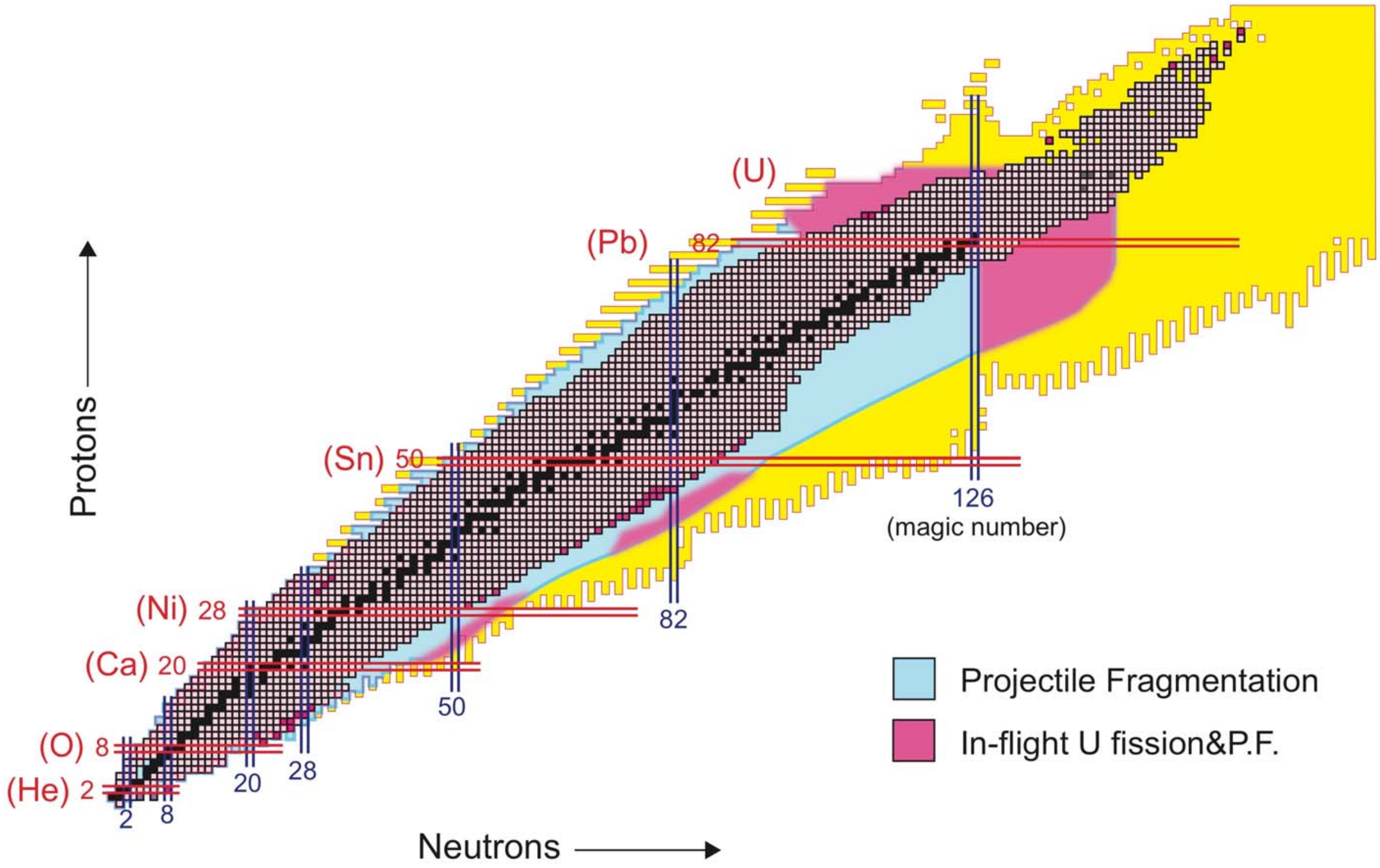
- Beam Energy > 100 MeV/u

  (Speed > 0.4 c)

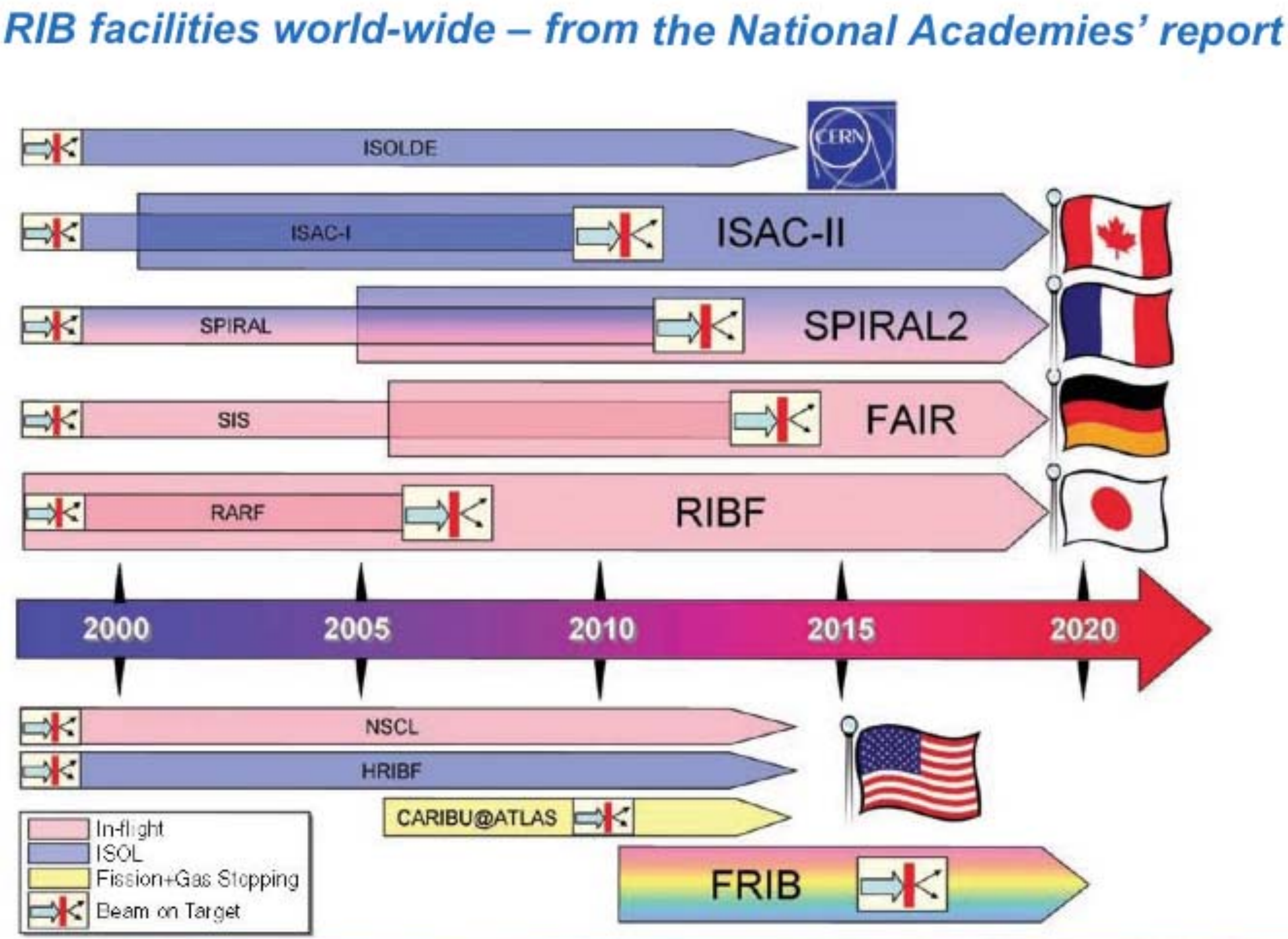
- High Intensity

Ring cyclotrons are suitable for RI-beam production

Nuclear landscape



RIB facilities in the world



Backup

Trajectory in weak focusing machines - 1

$$\frac{d^2x}{ds^2} = -\frac{\nu_r^{\,2}}{\rho^2}x, \quad 0 < \nu_r < 1$$

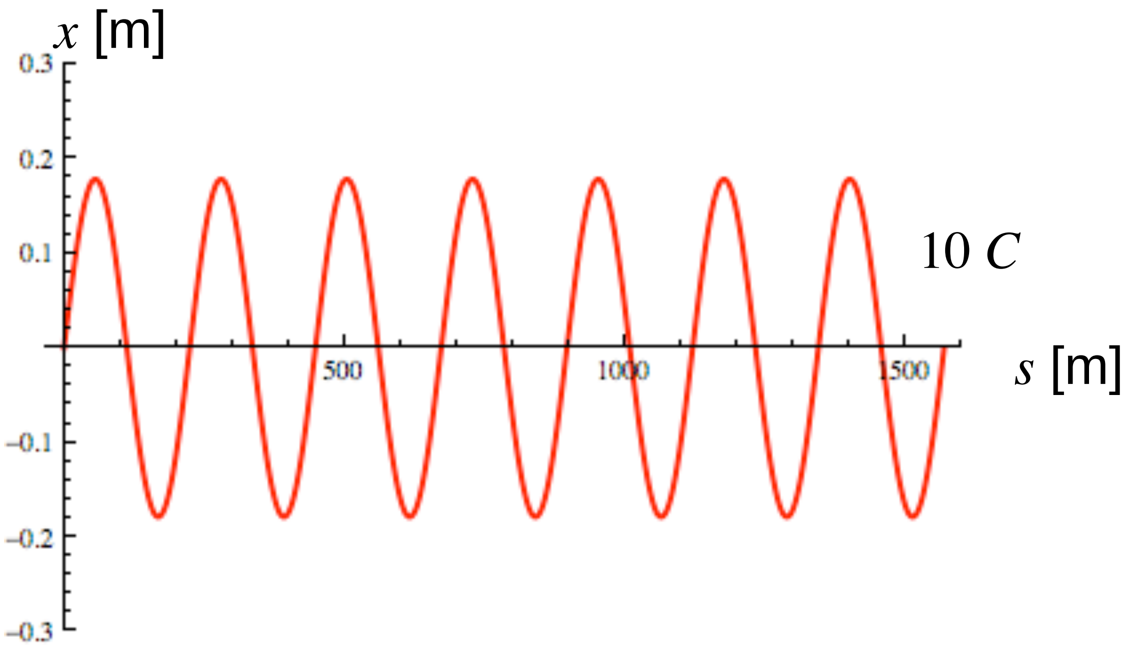
x : Horizontal deviation from the design orbit

s : Orbit length

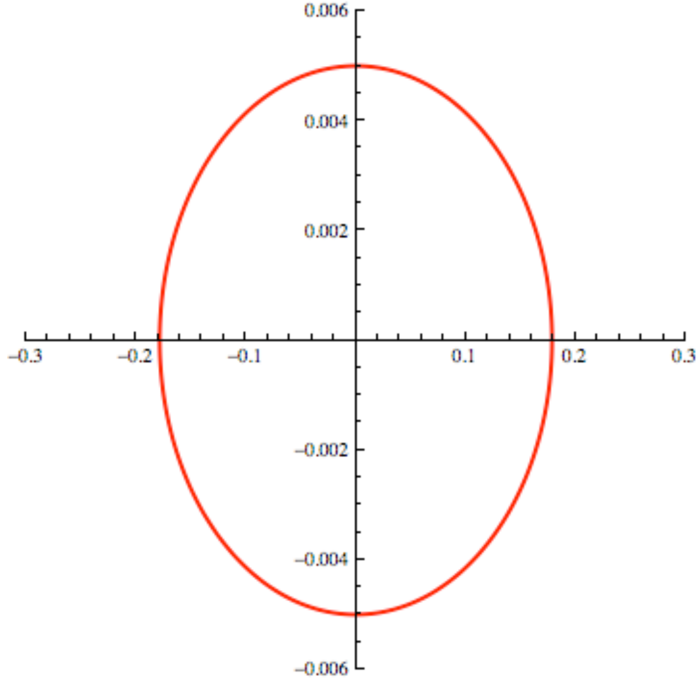
=>Harmonic oscillator

$$\rho = 25\ [\mathrm{m}] \Rightarrow C \approx 157\ [\mathrm{m}], \quad \nu_r = 0.7,$$

$$x(0) = 0, \quad \frac{dx}{ds}(0) = 0.005$$



“Phase curve”



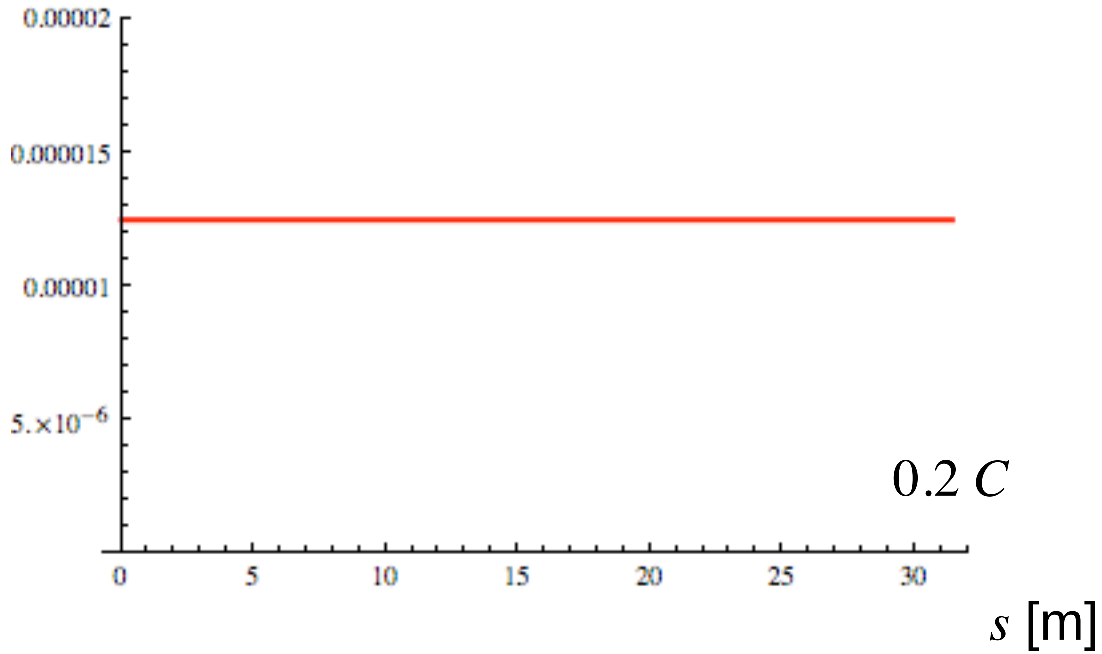
Trajectory in weak focusing machines - 2

- Hamiltonian of harmonic oscillator

$$H(x,p)=\frac{1}{2}\left(p^2+k\,x^2\right)$$

- Hamiltonian is invariant ("first integral" of motion)

$$H\big(x(s),p(s)\big)$$

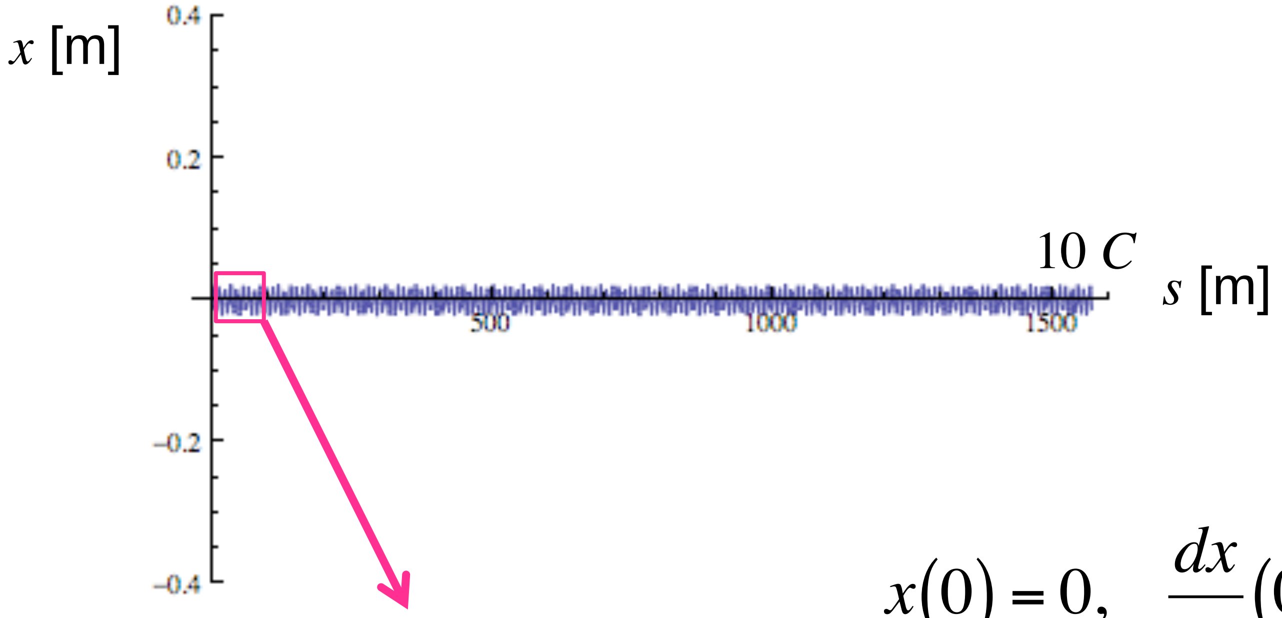


Trajectory in strong focusing machines - 1

$$\frac{d^2x}{ds^2} = -K(s)x, \quad K(s+C) = K(s)$$

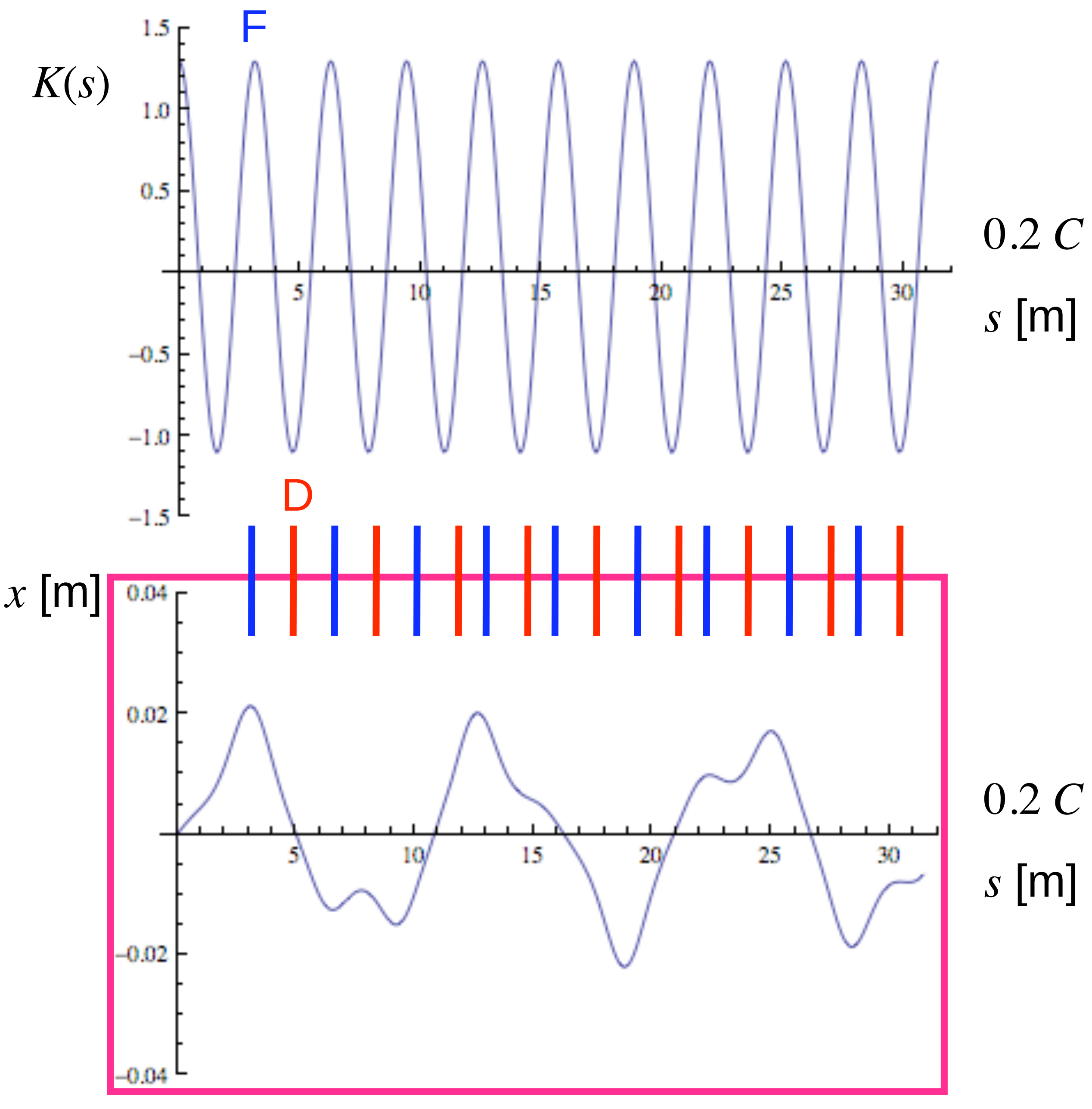
e.g.

$$K(s) = 0.1 + 1.2 \times \cos(50\,s/\rho)$$: ~ 50 Fs + 50 Ds inserted in the ring



$$x(0) = 0, \quad \frac{dx}{ds}(0) = 0.005$$

Trajectory in strong focusing machines - 2

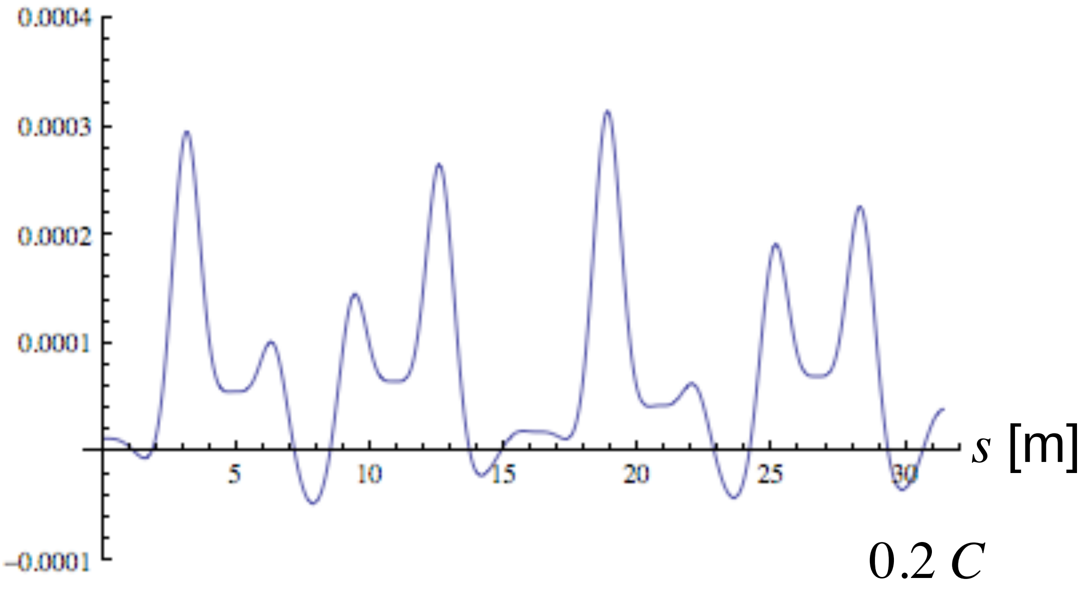


Trajectory in strong focusing machines - 3

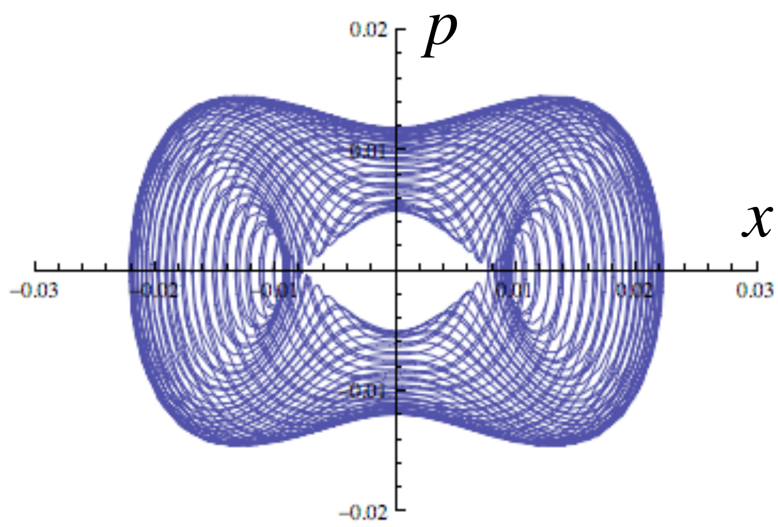
$$H(x,p,s) = \frac{1}{2}\left(p^2 + K(s)\,x^2\right)$$: not conserved (=> exercise)

$$\Rightarrow \frac{dx}{ds} = p, \quad \frac{dp}{ds} = -K(s)x$$

$$H\left(x(s),p(s),s\right)$$

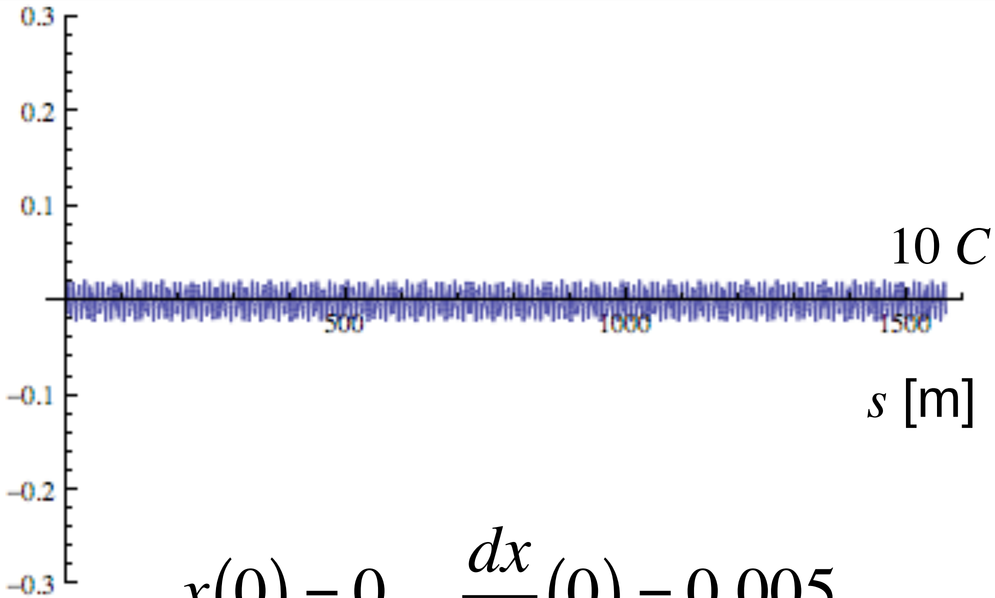


"Phase curve"

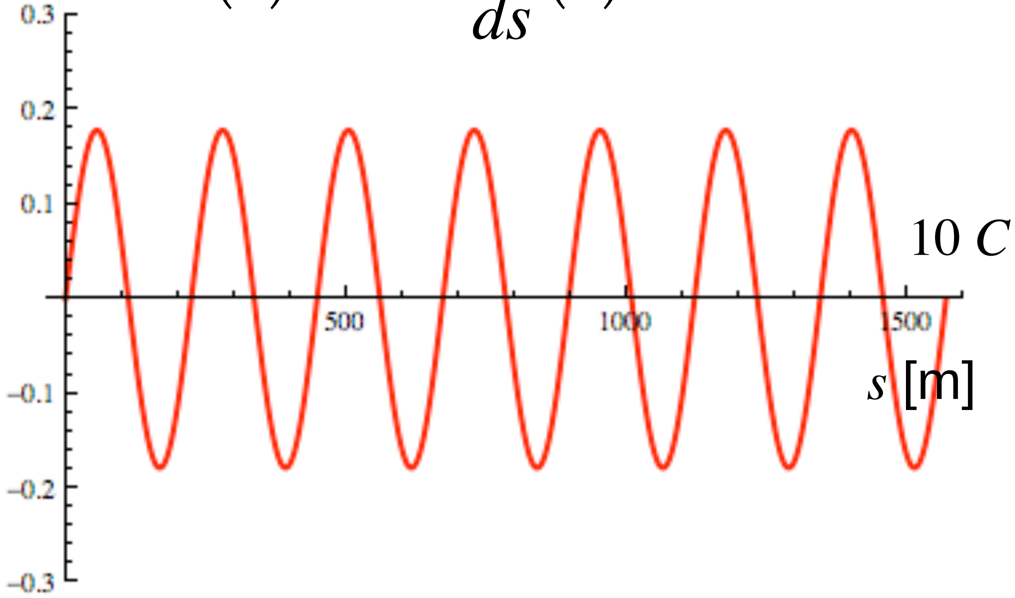


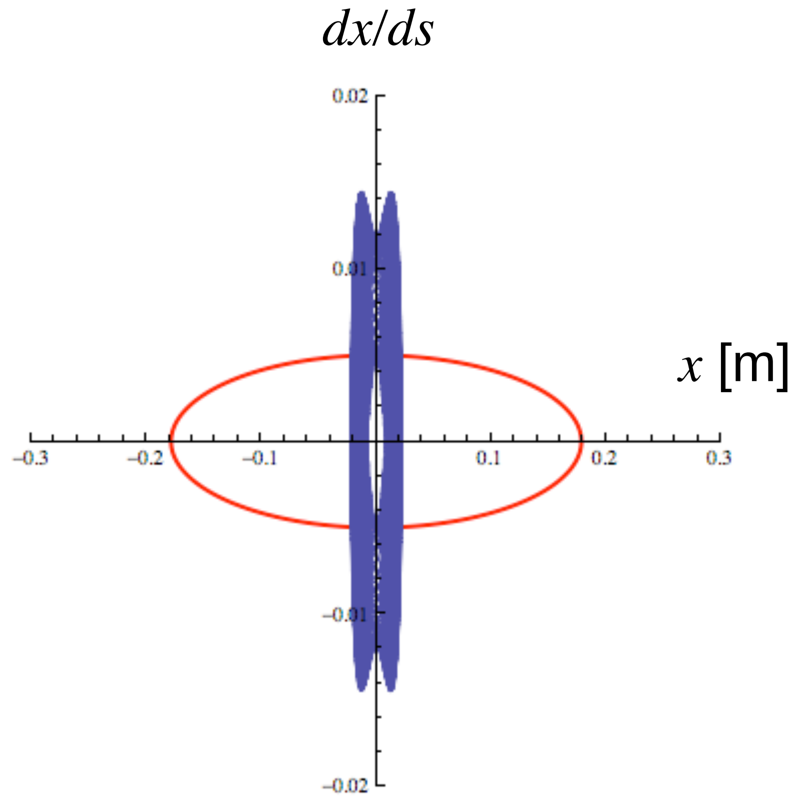
=> But integrable!
(=> See the paper of Courant&Snyder)

Comparison of weak/strong focusing -1



$$x(0) = 0, \quad \frac{dx}{ds}(0) = 0.005$$

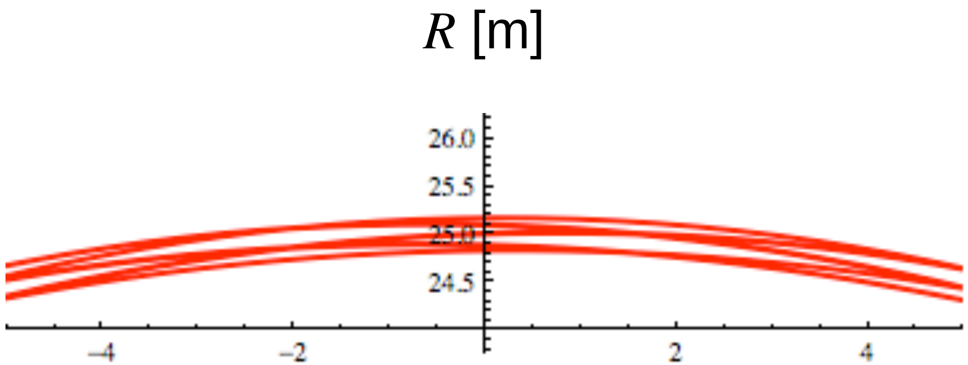




- Phase curve

Comparison of weak/strong focusing -2

Weak



Strong