Spectrometer (= Magnetic Spectrometer)

Lecture on Nishina School 2012

03-Oct-2012

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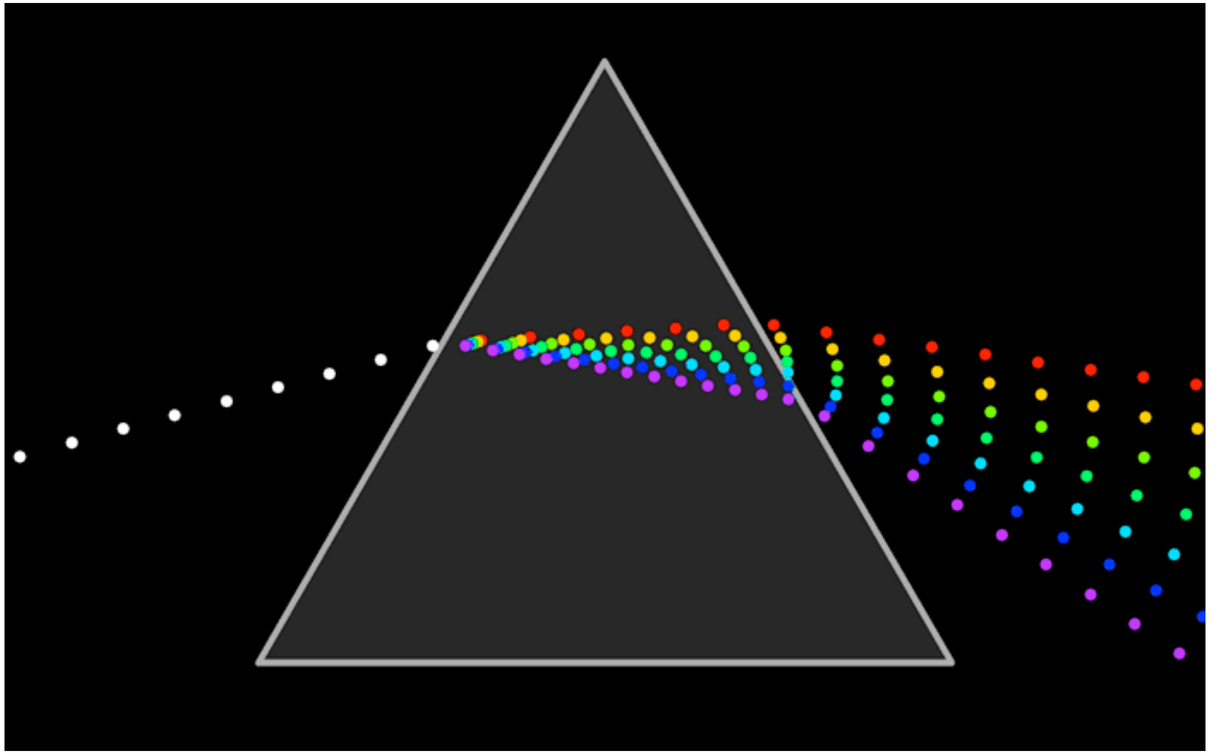
Spectroscopy (:= 分光)

Visible light spectrum
dispersed by optical element(prism)

What is dispersed ?
Color is dispersed

What represents color ?
Frequency := Energy or momentum

So, momentum of light could be measured by this optical element.



Spectroscopy

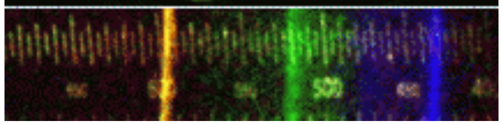
Color spectrum is obtained by
(Optical) Spectrometer

in which optical elements like

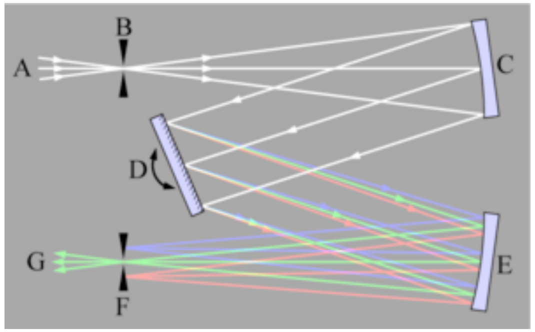
- Lens
- Prism
- Diffraction Cell
- ...

are used in a proper layout.

Alchohol flame



Color Spectrum



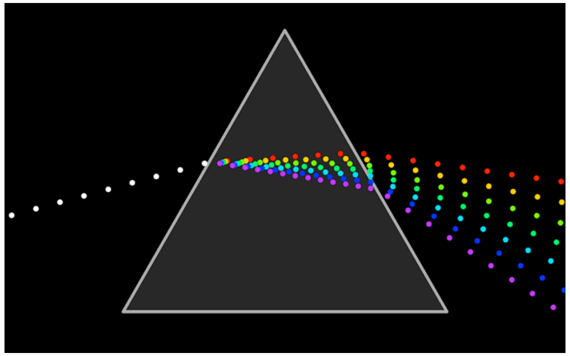
Optical Spectrometer using Diffraction Cell (D)
and other elements

Reference : Wikipedia
ja.wikipedia.org

Spectroscopy on / Spectrometer for ions/electrons

- light
  - lens
  - prism

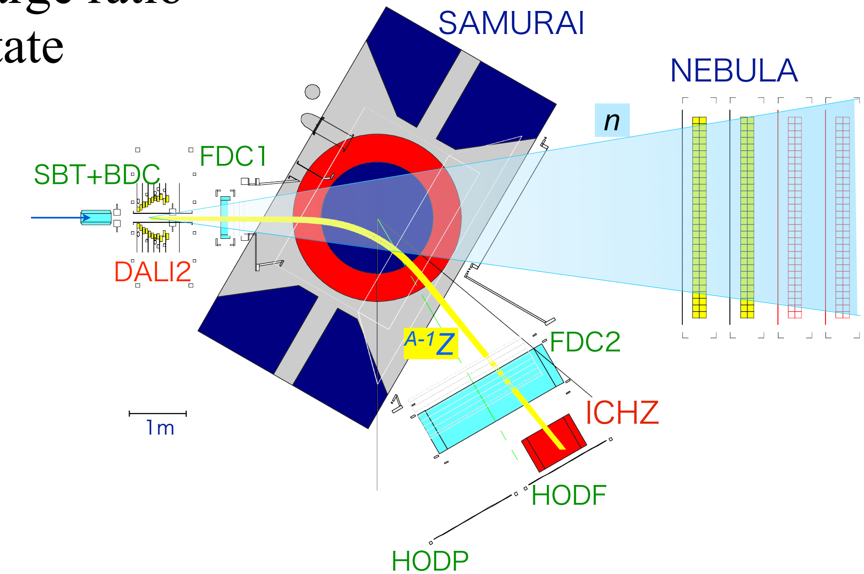
  - wave length shifter
  - ...



- charged particle (electrons/ions)
  - quadrupole field
  - dipole field

  - energy degrader

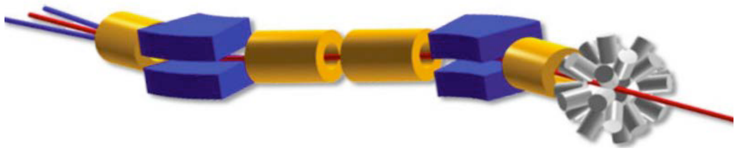
  - mass/charge ratio
  - charge state
  - ...



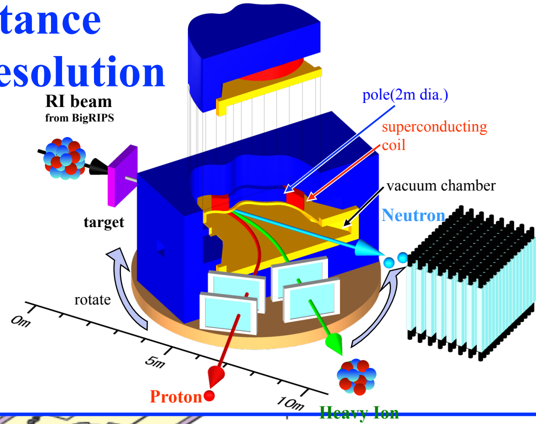
Spectrometer lecture on Nishina School

Magnetic spectrometer @ RIBF

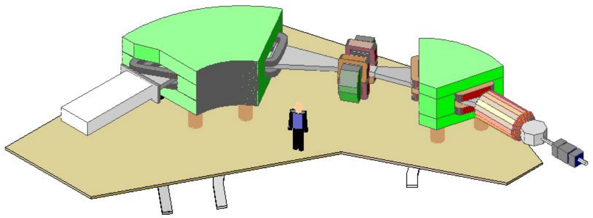
Zero-degree
Multi-purpose
Medium resolution
Medium acceptance

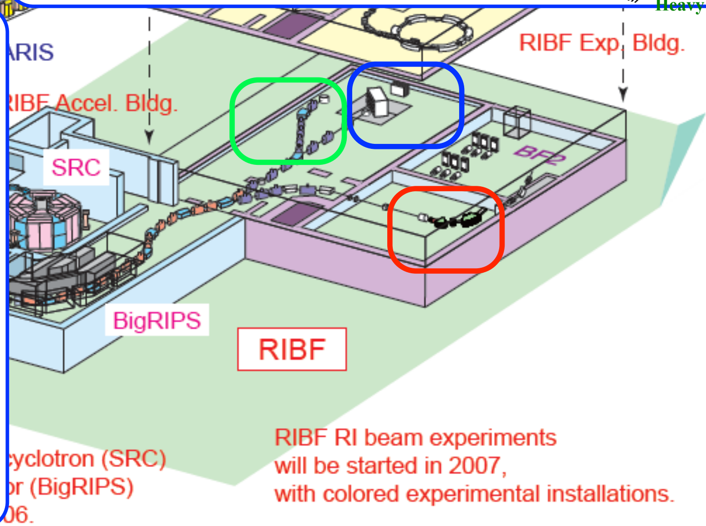


SAMURAI
Large acceptance
Restricted resolution



SHARAQ
High-resolution (p and θ)
Small acceptance

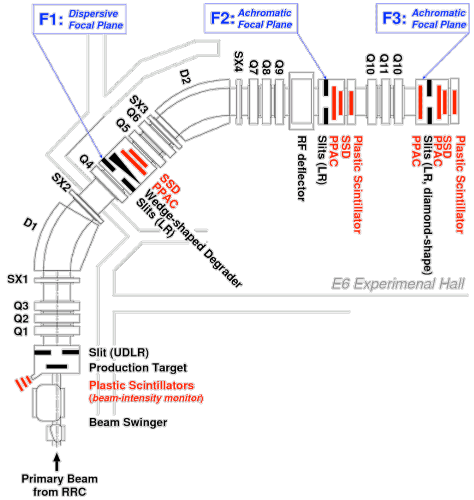




Magnetic spectrometer @ RIBF

RIPS

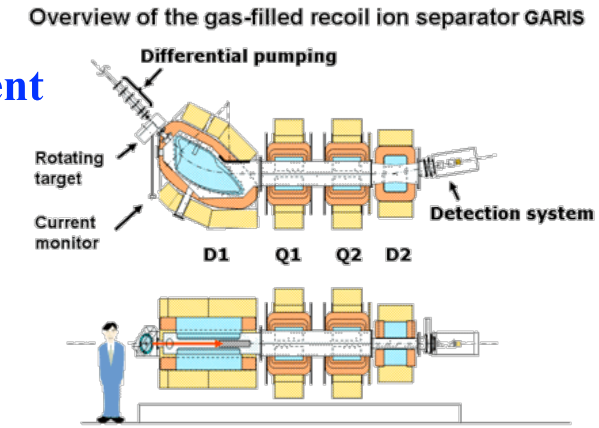
RI Separator
generate high intense secondary beam
E = 60-100 A MeV



GARIS

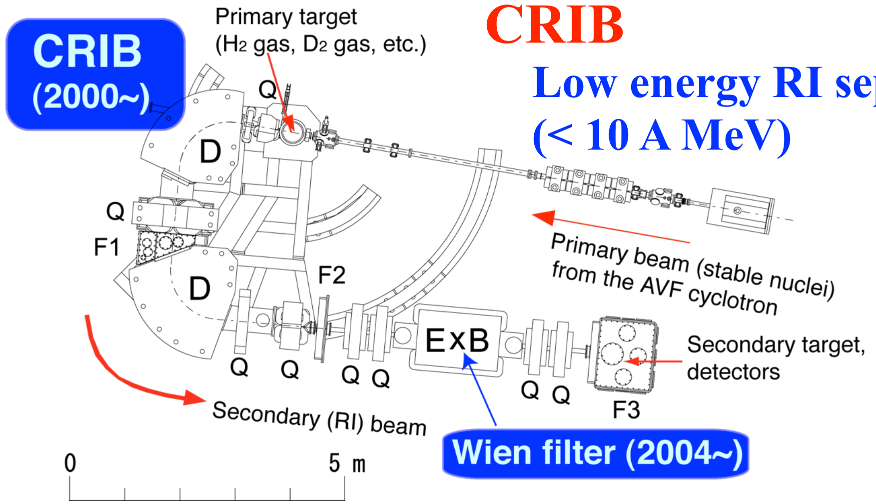
Super Heavy Element

Discovery of Z=113 element



CRIB

Low energy RI separator
(< 10 A MeV)



SMART

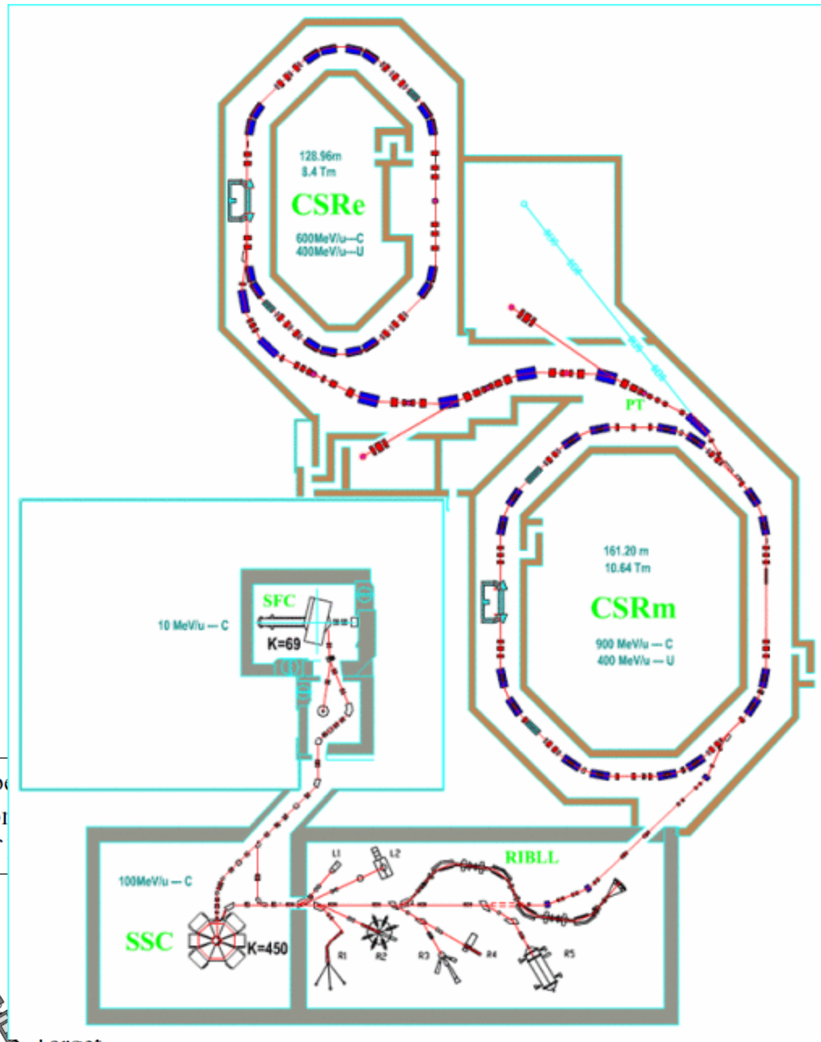
high resolution/
large acceptance

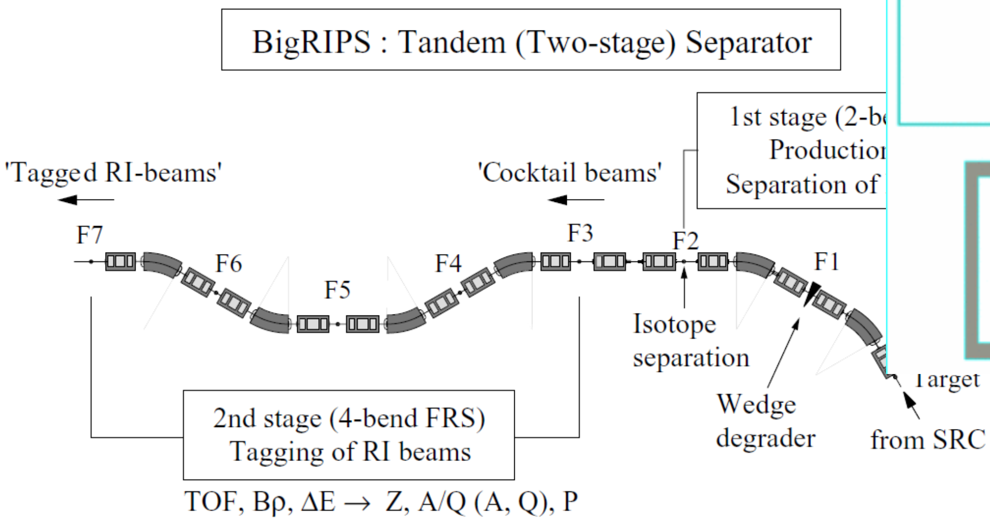
Decommission and Reuse as D1 of SHARAQ

RI-beam separator

A RI-beam separator is a kind of spectrometer.

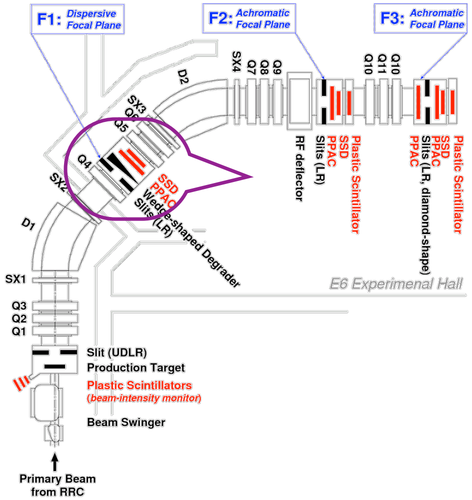
- Beam quality (size, purity)
- Particle identification





RI-beam separator

RIPS
RI Separator
generate high intense secondary beam
E = 60-100 A MeV



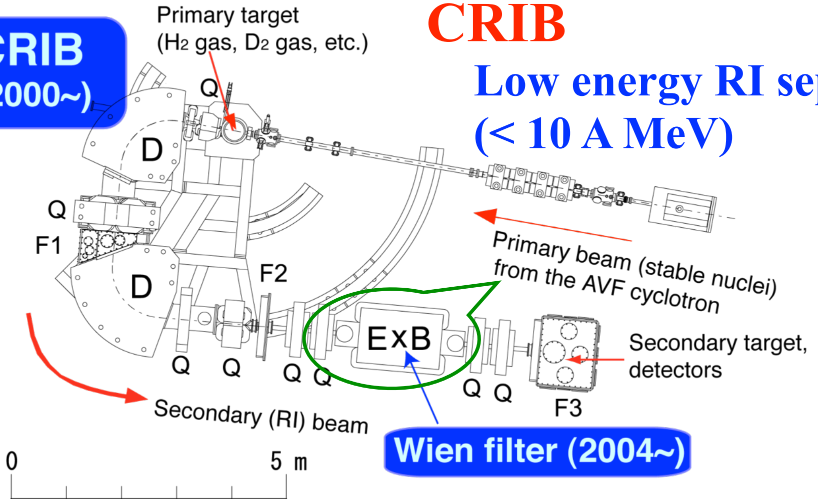
Only A/Q separation by Magnetic field

Higher Energy > 10 MeV/u
Magnetic field + Energy degrader

Lower Energy
Magnetic field + Electric Field

CRIB (2000~)

CRIB
Low energy RI separator
(< 10 A MeV)



Wien filter (2004~)

CARP
RCNP →Kyushu U.
recoil RI separator
E = 0.1 - 1 A MeV

Electric Field

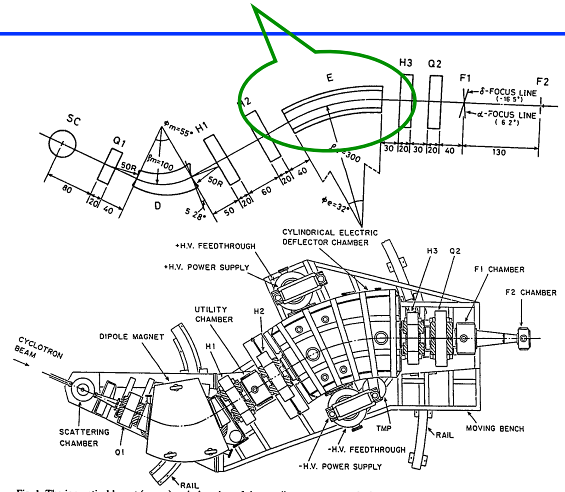
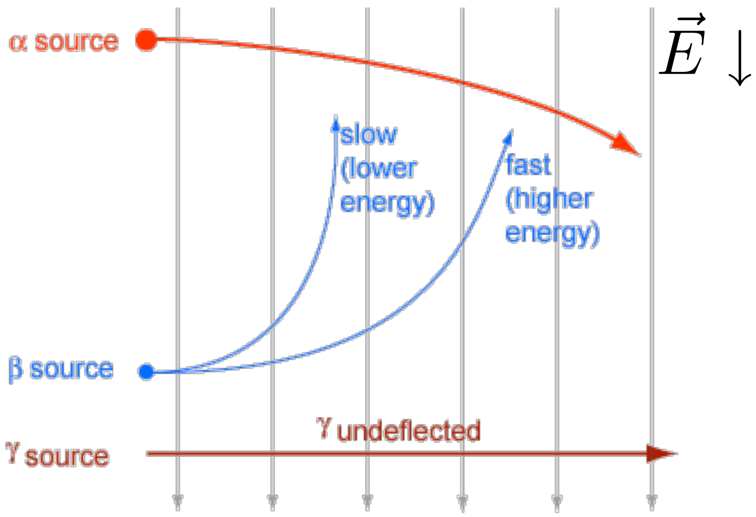


Fig. 1. The ion optical layout (upper) and plan view of the recoil mass separator CARP. The directional and energy focal lines are denoted as α- and δ-focus lines, respectively. All the dimensions are given in cm. SC: scattering chamber; Q1, Q2: magnetic quadrupoles; D: magnetic dipole; H1, H2, H3: magnetic sextupoles, E: electrostatic deflector.

Charged particle motion in magnetic (and electric) field

Motion in electric and magnetic field

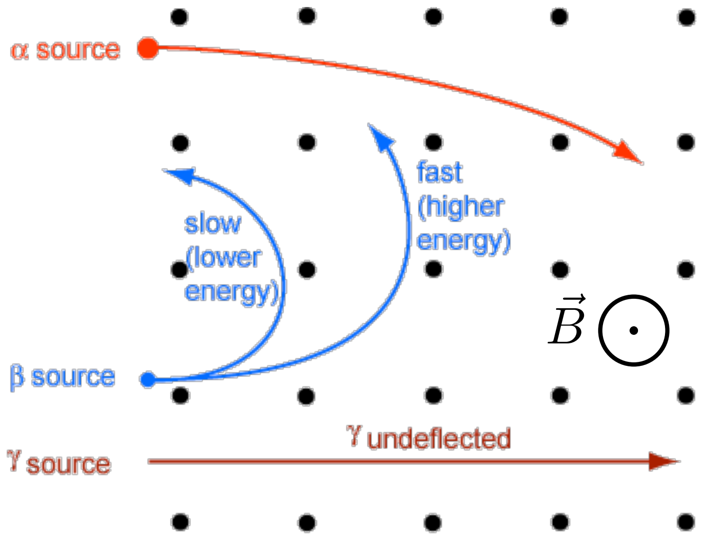
Electric (homogeneous) field



Effect of an electric field on alpha, beta and gamma radiation

$$\vec{F} = Qe\vec{E}$$

Magnetic (homogeneous) field



Effect of a magnetic field on alpha, beta and gamma radiation

$$\vec{F} = Qe\vec{v}\times\vec{B}$$

q=*Qe* [Coulomb]

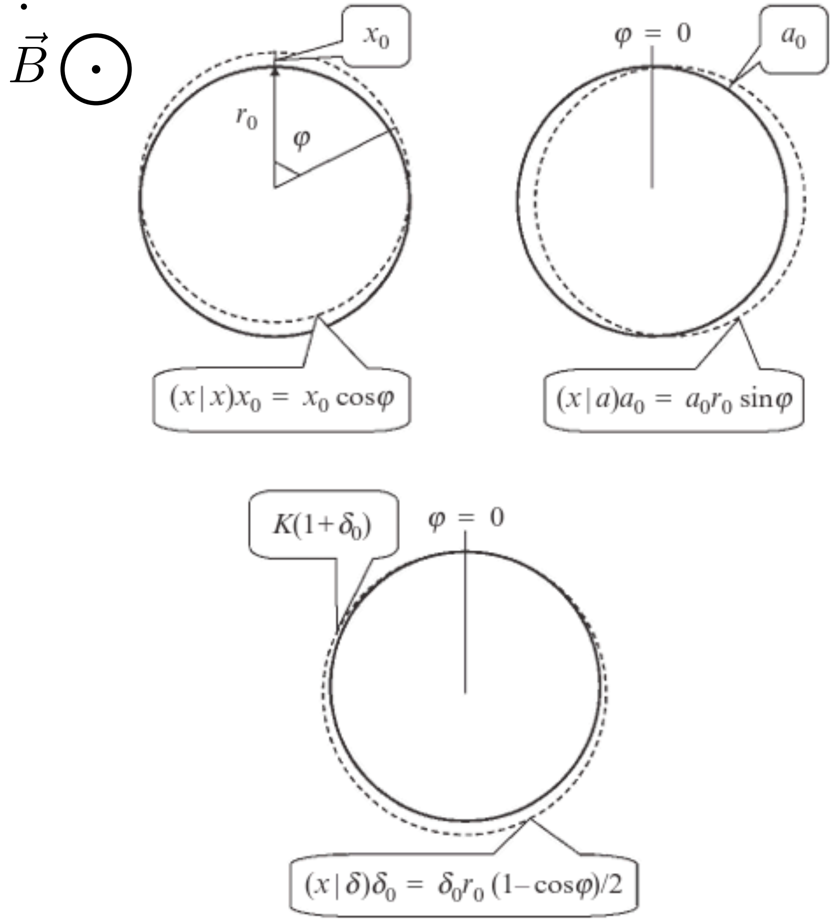
$$\vec{F} = Qe\left(\vec{E} + \vec{v}\times\vec{B}\right)$$

In homogeneous magnetic field

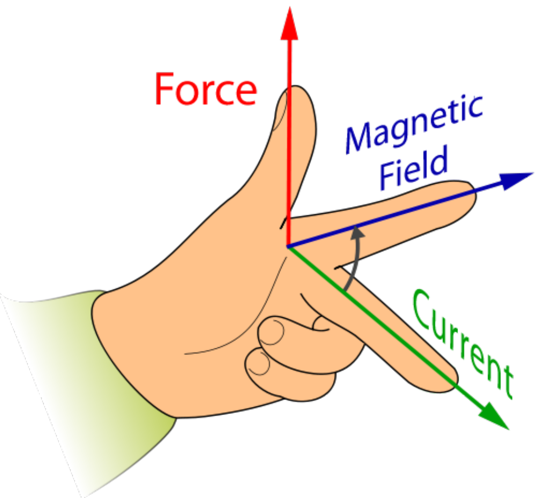
- Motion in homogeneous magnetic field is governed by :

$$\vec{F} = Qe\vec{v} \times \vec{B}$$

which represents the centrifugal force.



- Direction is given by Fleming's left hand rule



- Curvature is given as :

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

$$\left|\frac{d(m\vec{v})}{dt}\right| = mv\frac{d\theta}{dt} = \frac{mv^2}{\rho}$$

$$\rightarrow \frac{mv^2}{\rho} = QevB$$

$$p/q = B\rho$$

In homogeneous magnetic field

- Curvature is given as :

$$\vec{F} = \frac{d(m\vec{v})}{dt}$$

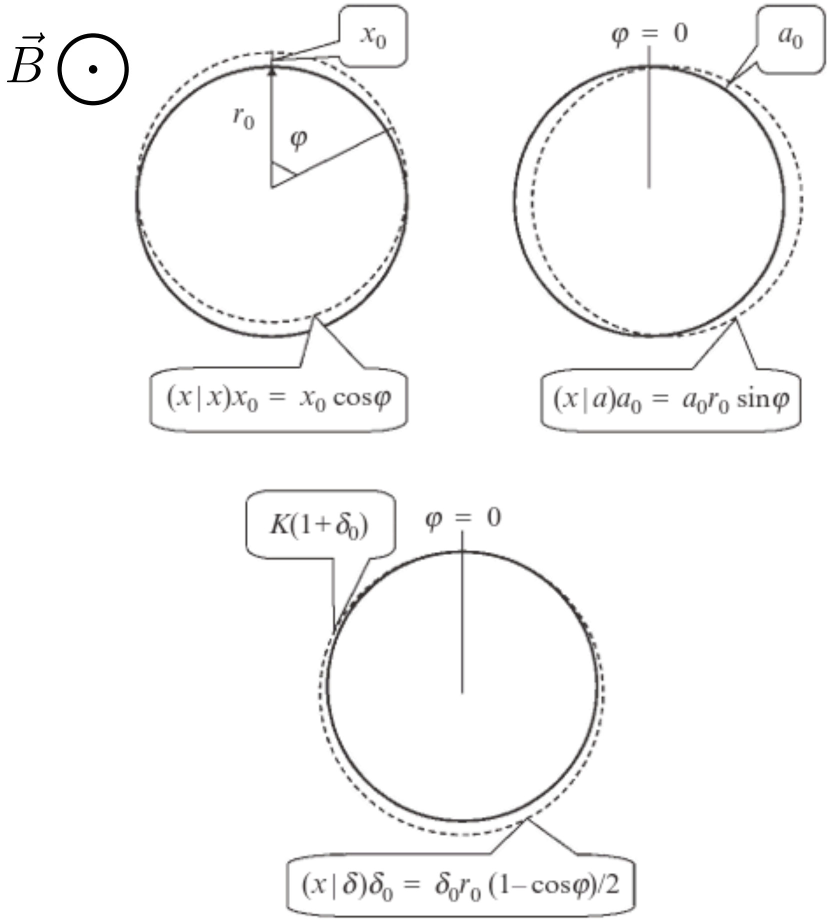
$$\left|\frac{d(m\vec{v})}{dt}\right| = mv\frac{d\theta}{dt} = \frac{mv^2}{\rho}$$

$$\rightarrow \frac{mv^2}{\rho} = QevB$$

$$p/q = B\rho$$

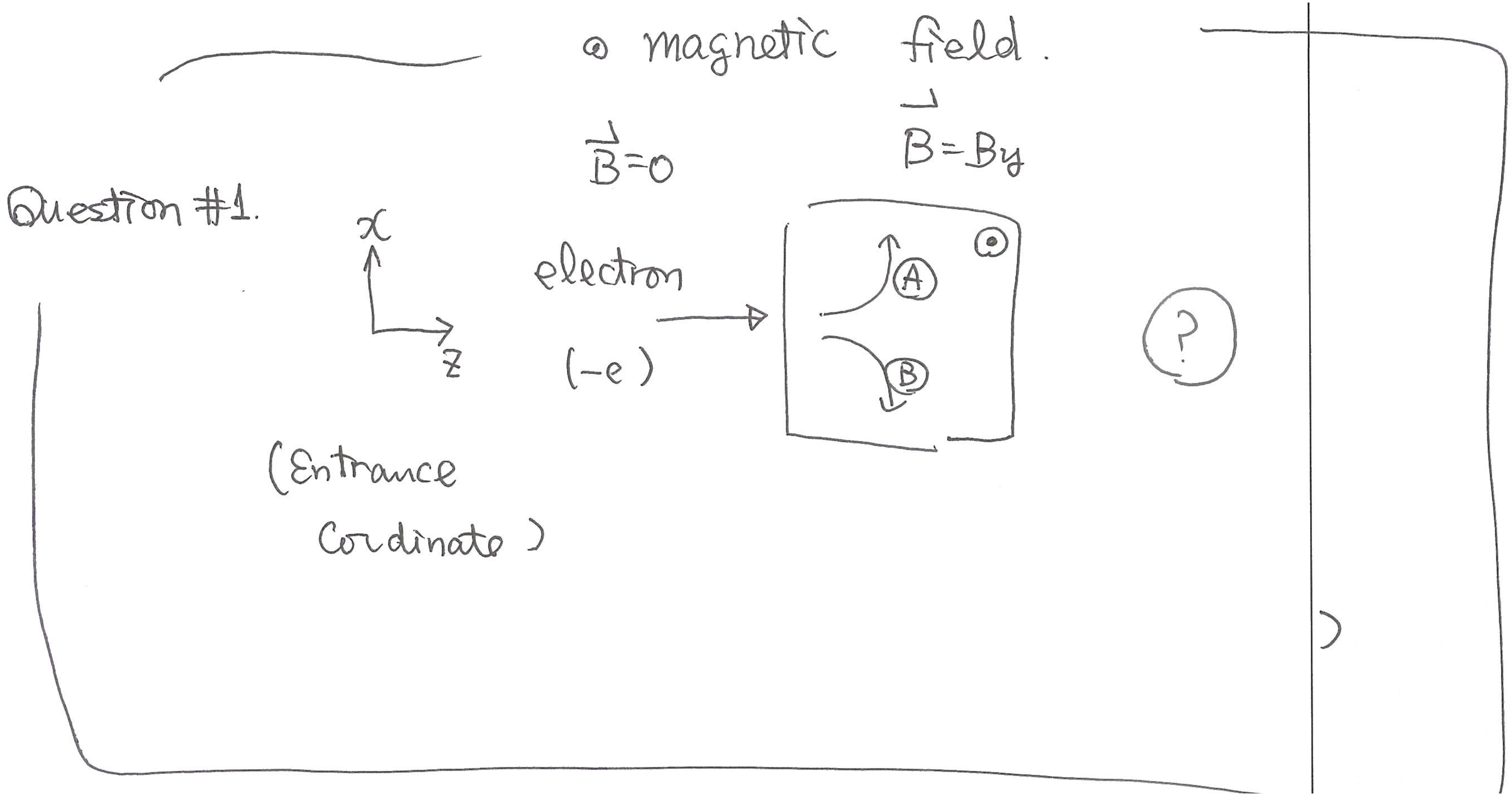
- Description with natural unit notation at $\hbar=c=1$

$$\underset{[\mathrm{eV/c}][\mathrm{charge;\ integer}]}{p/Q} = \underset{[\mathrm{m/s}][\mathrm{T}][\mathrm{m}]}{cB\rho}$$



Direction is important

- Question #1 for home works

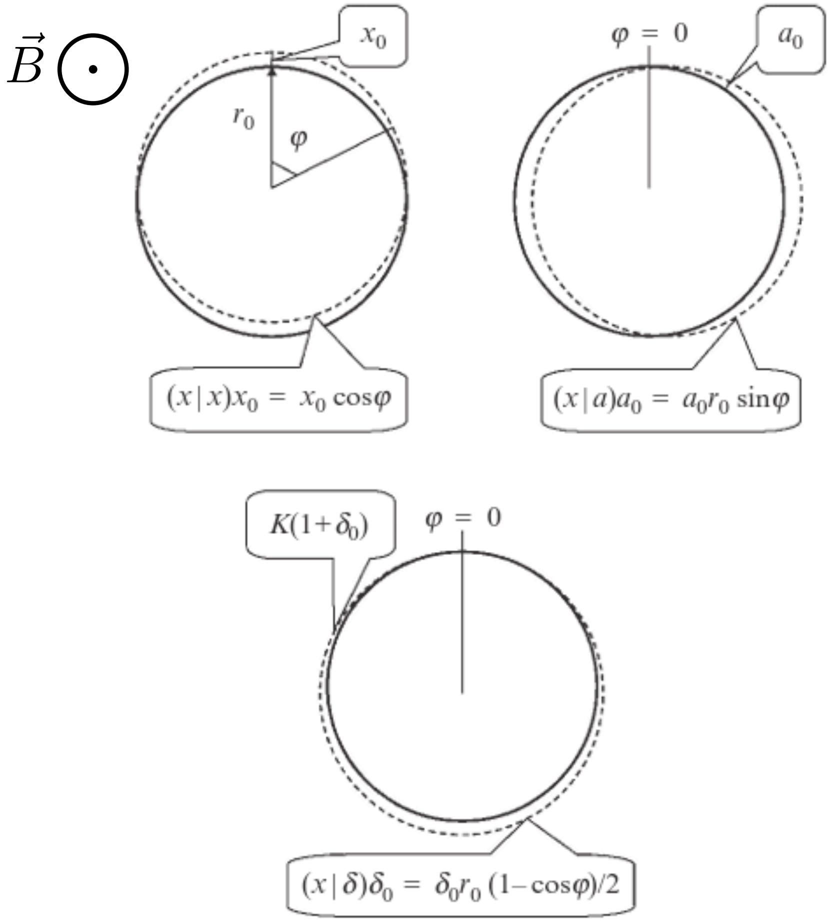


In homogeneous magnetic field

- Description with natural unit notation at $\hbar=c=1$

$$p/Q = cB\rho$$

[eV/c][charge; integer] [m/s][T][m]



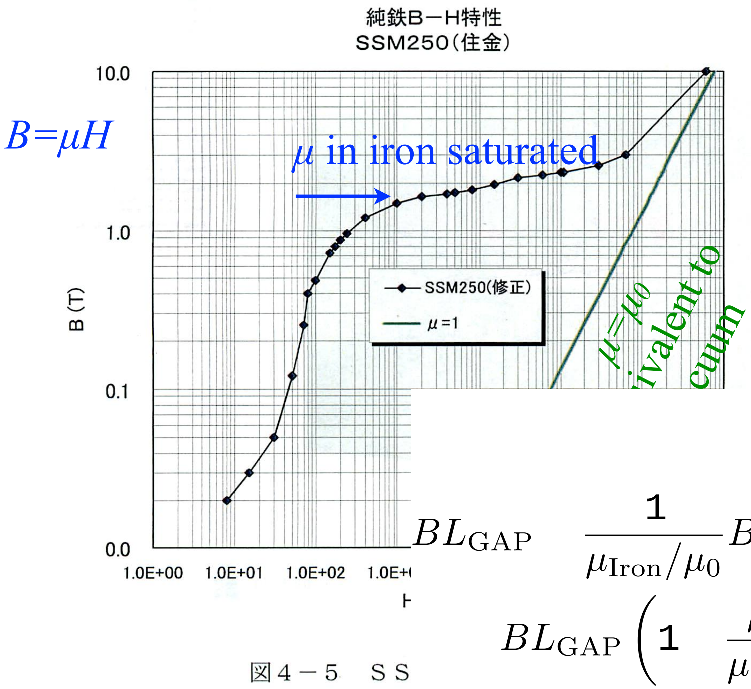
- Question #2
- Let us consider the curvature of
  - 1.5 MeV electrons on 0.1 T
    - electron mass is 0.511 MeV (~ 0.5 MeV/c^2)
  - 250 MeV/A ^{6}He (A=6, Z=Q=2) on 3 T
    - nucleon mass is 940 MeV (~ 1 GeV/c^2)

➡ home works until next week

How to generate magnetic field

High magnetic field by iron yoke

- high permeability (μ) of iron help to generate high magnetic field on GAP region up to B ~1.6 T



$$\oint \vec{H}\cdot d\vec{l} = Ni,$$

$$BL_{GAP} \quad \frac{1}{\mu_{Iron}/\mu_0} BL_{ReturnYoke} = \mu_0 Ni,$$

$$BL_{GAP}\left(1 \quad \frac{\mu_0}{\mu_{Iron}}\frac{L_{RY}}{L_{GAP}}\right) = \mu_0 Ni,$$

$$BL_{GAP} \quad \mu_0 Ni; \text{ if } \frac{\mu_0}{\mu_{Iron}}\frac{L_{RY}}{L_{GAP}} \quad 1.$$

Coil generates mangetic field | high μ generate high magnetic flux | Magnetic field flux guided by iron yoke

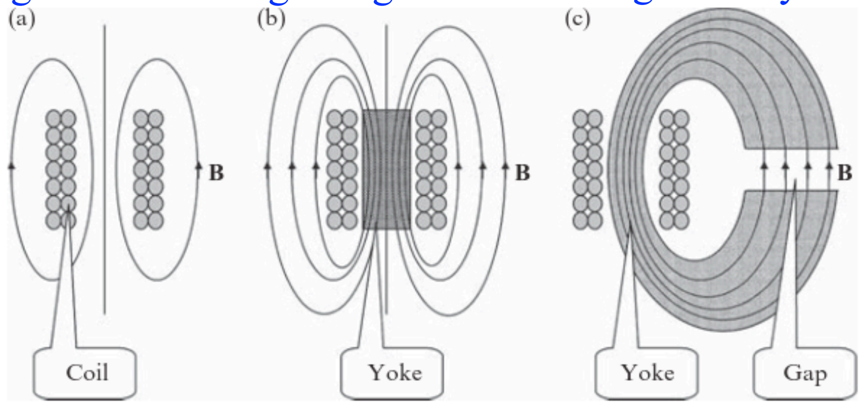


Figure 13 Forming the field of the dipole magnet: (a) creating magnetic field by a coil; (b) amplifying the field by inserting a ferromagnetic material inside the coil, and (c) shaping the magnetic yoke to concentrate the field along circuits with a small magnetic resistivity.

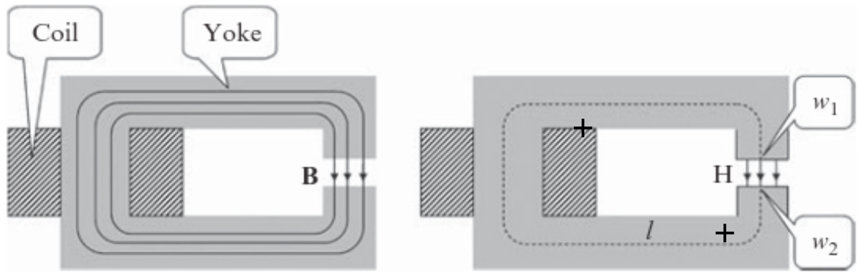
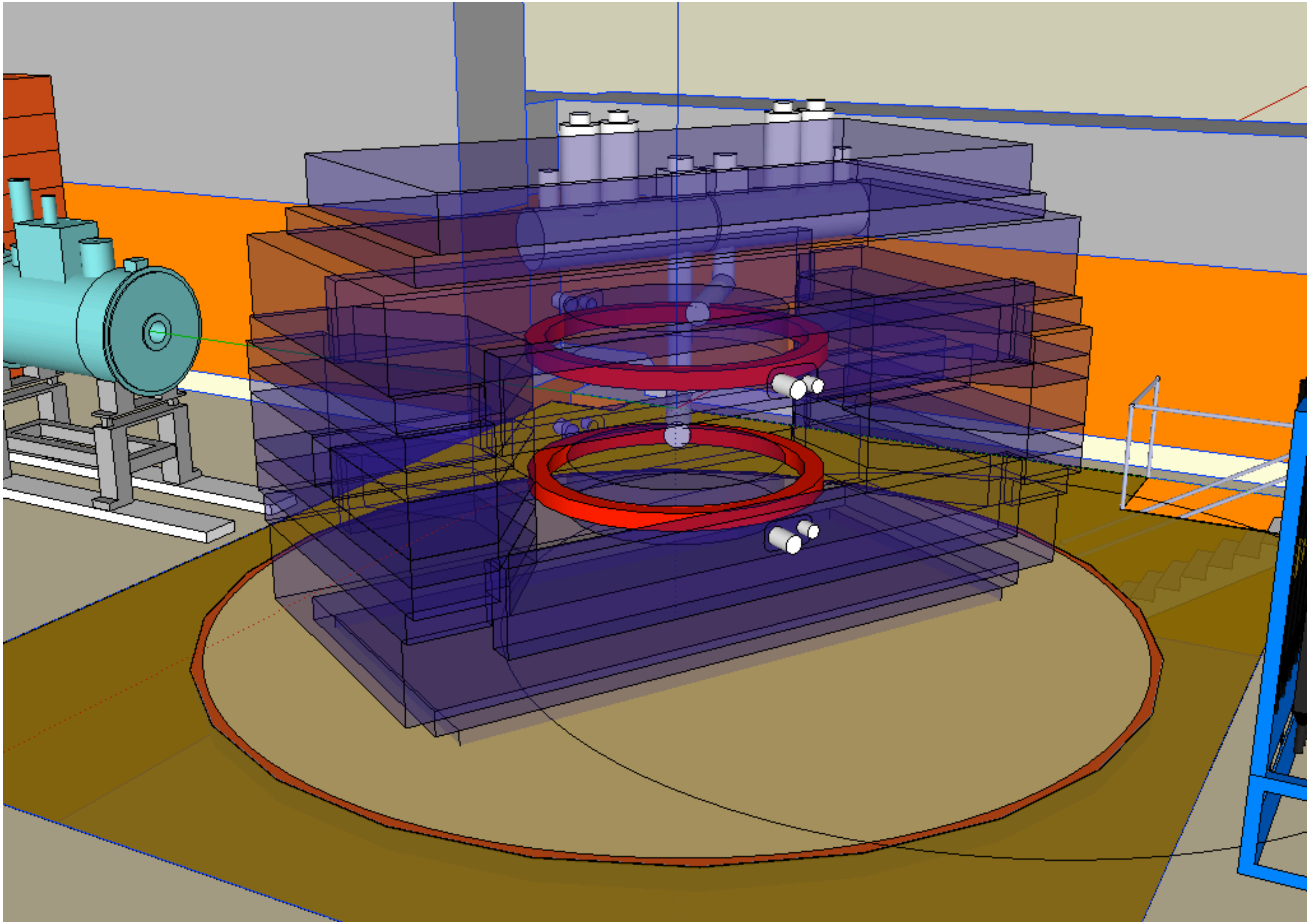


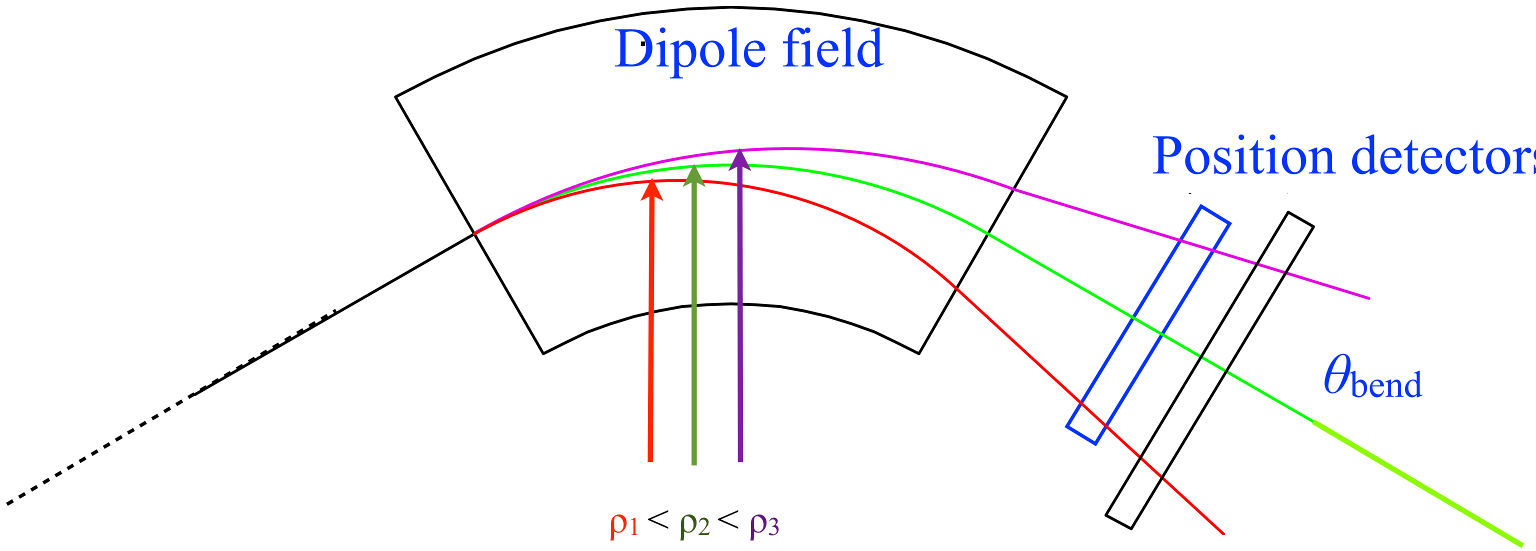
Figure 15 Magnetic flux density lines and magnetic field strength lines in a ferromagnetic yoke with a narrow vacuum gap.



Nishina School

How to obtain the momentum information

Momentum measurement



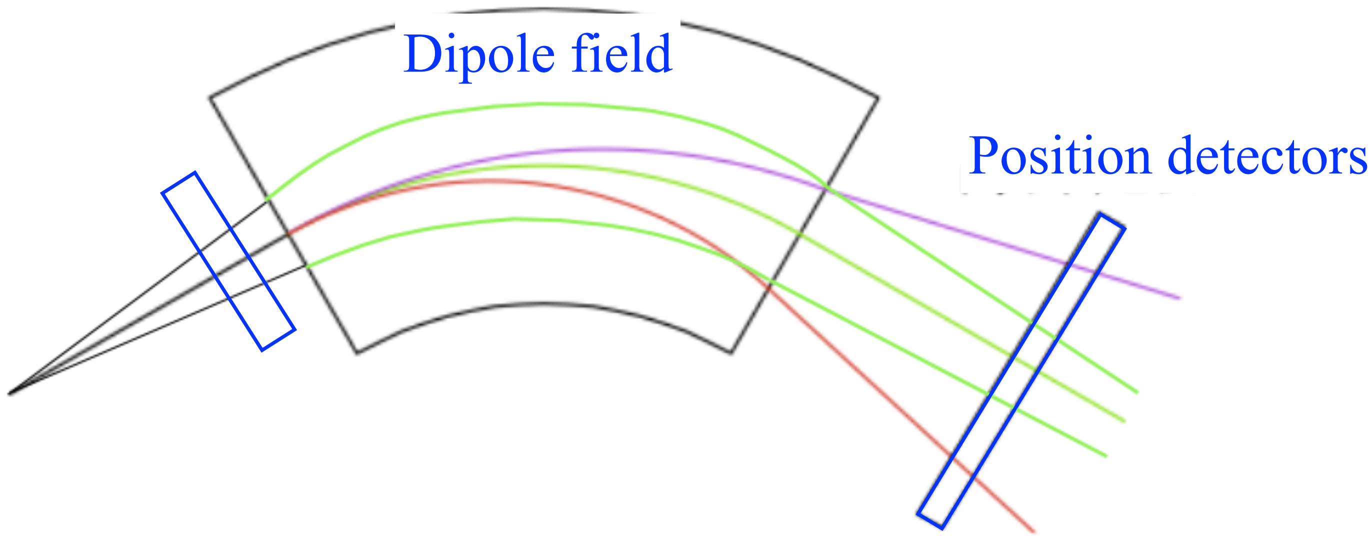
$$\theta_{\mathbf{bend}} \propto q/p = 1/B\rho_{\mathbf{particle}}$$

$$\frac{\Delta\theta}{\theta_{\mathbf{bend}}} = \frac{\Delta p}{p}$$

Ex. θ_{bend} = 60deg ~ 1 radian
$\Delta\theta$ ~ 1 mrad

$$\frac{\Delta p}{p} \sim 10^{-3}$$

Momentum measurement with finite acceptance



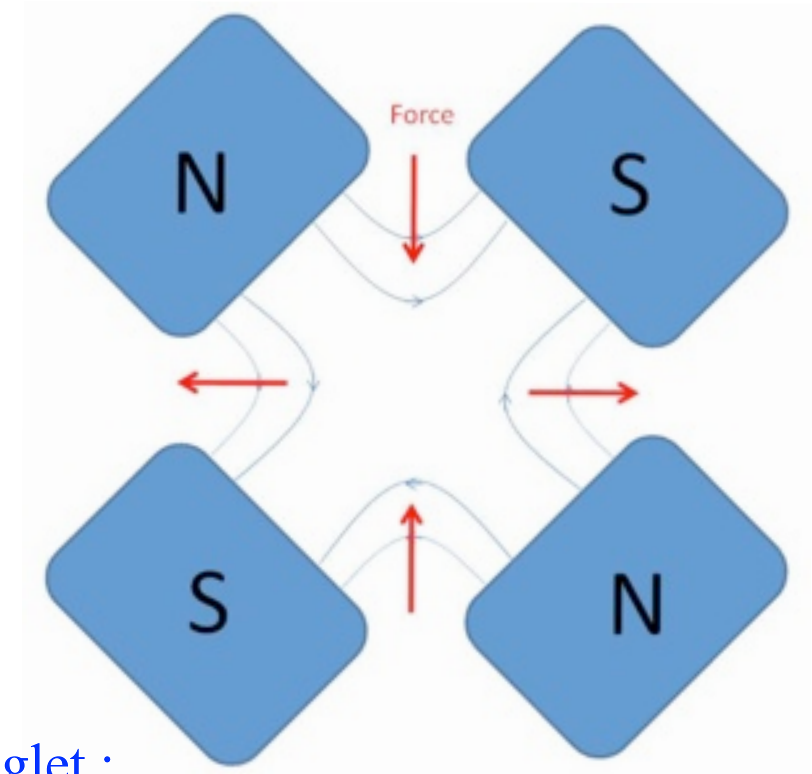
Detector(s) to determine incident angle is necessary

"non-" destructive

The momentum resolution is usually in order of 10^{-3}

Quadrupole magnet

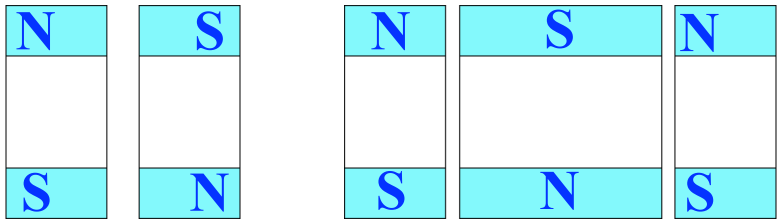
functions as lens
  focus charged particles in horizontal or vertical direction



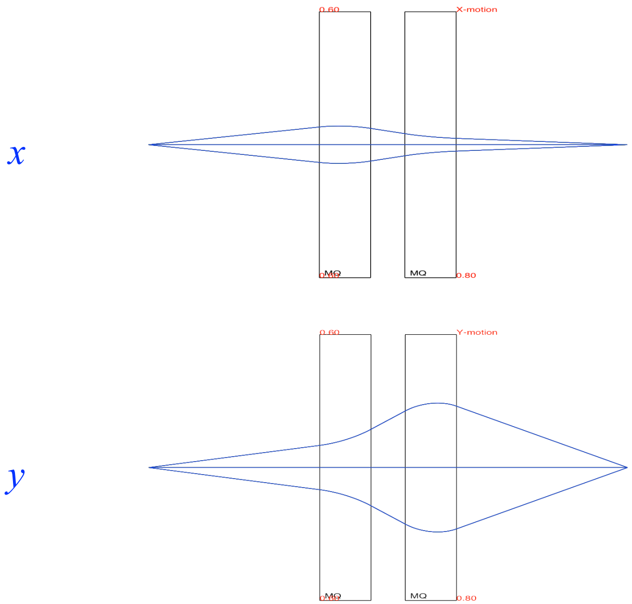
singlet :
  focus on y and defocus on x

Quadrupole magnet as focusing elements

1) Usually used as doublet or triplet :



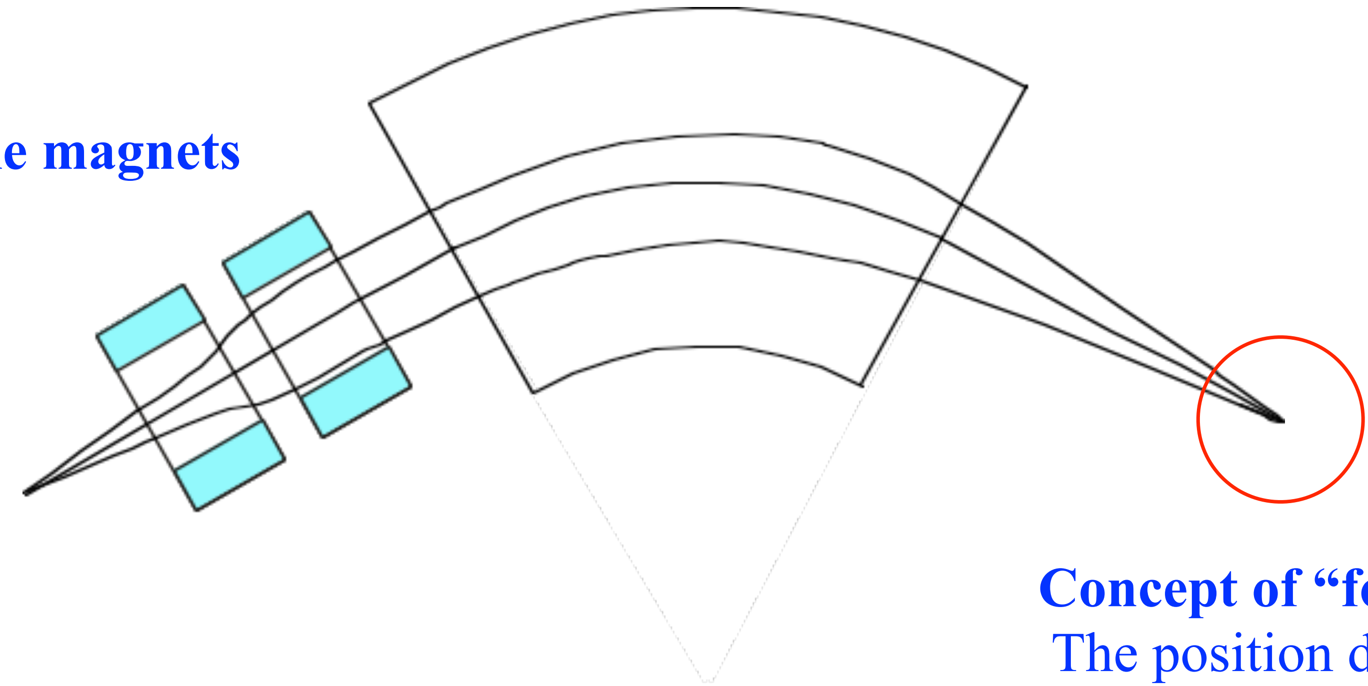
2) Doublet focus for x and y direction simultaneously :



3) Triplet can control the magnification

Momentum measurement conjunction with focusing magnet

Quadrupole magnets

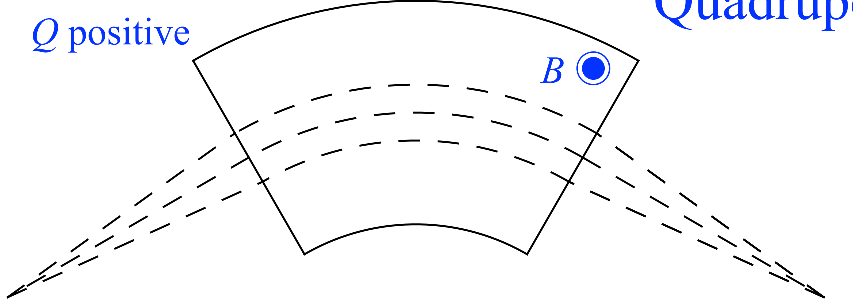


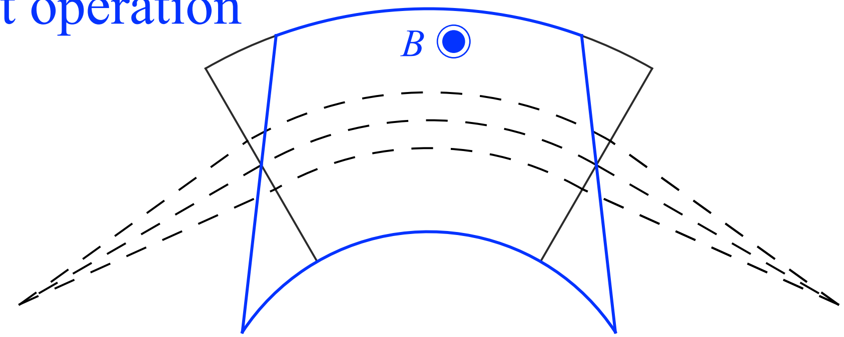
Concept of "focus"
The position doesn't depend on the beam injection angle.

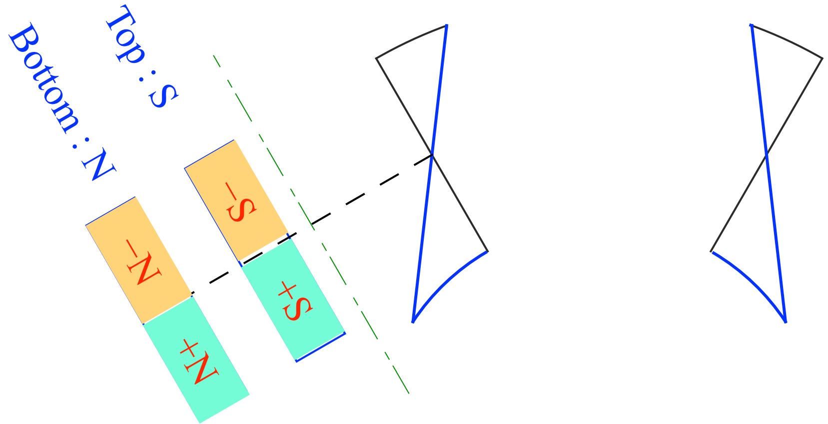
Focusing elements act as acceptance resolving power converter

Momentum measurement conjunction with focusing magnet

Edge effect ≡
Quadrupole magnet operation







H(x) defocus ; V(y) focus

Quantitative analysis of trajectory

and

Paraxial approximation and transfer matrix

Paraxial approximation and transfer matrix

- Paraxial approximation (valid within small cone around central trajectory)
  - in the 1st order description as:

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z1} = \begin{pmatrix} (x|x) & (x|a) & (x|y) & (x|b) & (x|\delta) \\ (a|x) & (a|a) & (a|y) & (a|b) & (a|\delta) \\ (y|x) & (y|a) & (y|y) & (y|b) & (y|\delta) \\ (b|x) & (b|a) & (b|y) & (b|b) & (b|\delta) \\ (\delta|x) & (\delta|a) & (\delta|y) & (\delta|b) & (\delta|\delta) \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z0} \qquad \delta \equiv p/p_0 - 1$$

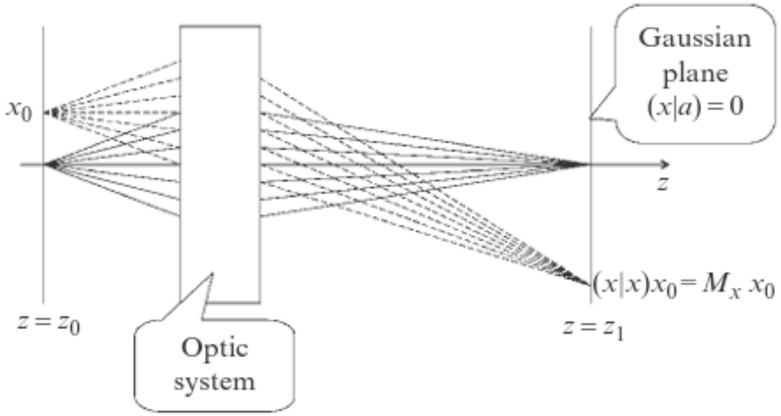
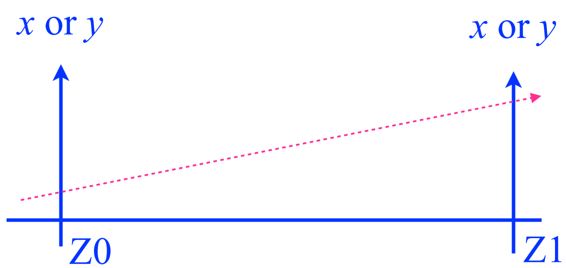


Figure 19 Forming an image in the Gaussian plane in the *x*-direction.

Matrix description in drift space

- Drift space : no optical elements

x=x0+a(Z1-Z0)*
a=a0
δ=δ0



- Matrix description

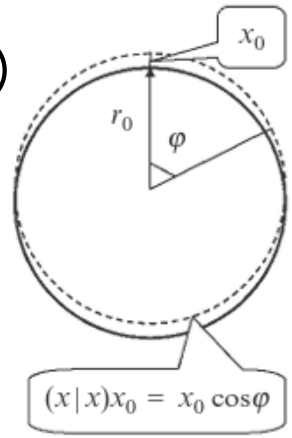
$$\begin{pmatrix} x \\ a \end{pmatrix}_{Z1} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \end{pmatrix}_{Z0}, L \equiv Z1 - Z0$$

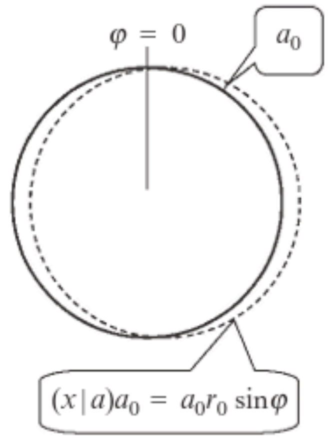
Matrix description in homogeneous magnetic field

- Matrix description of homogeneous magnetic field

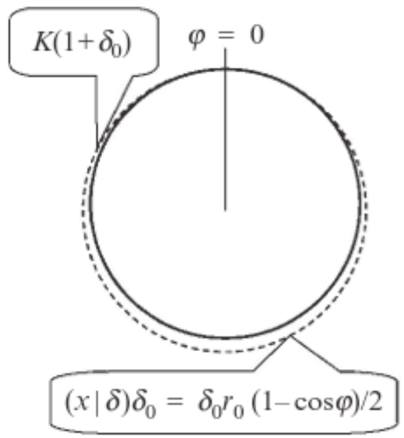
$\vec{B} \odot$

$$\begin{pmatrix} x_2 \\ a_2 \\ \delta_2 \end{pmatrix} = \begin{pmatrix} \cos\phi & \rho_0 \sin\phi & \rho_0(1-\cos\phi) \\ -\sin\phi/\rho_0 & \cos\phi & \sin\phi \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ a_1 \\ \delta_1 \end{pmatrix}$$





- ρ_0 is given by : $p_0/Q = cB\rho_0$

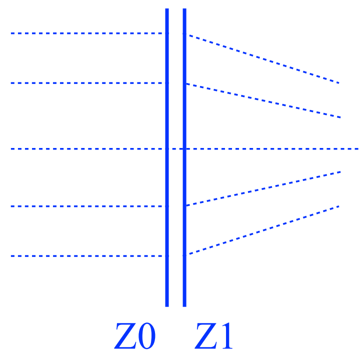


※ Boundary has to be 90 degree cut off

Matrix description in lens element and conbinations

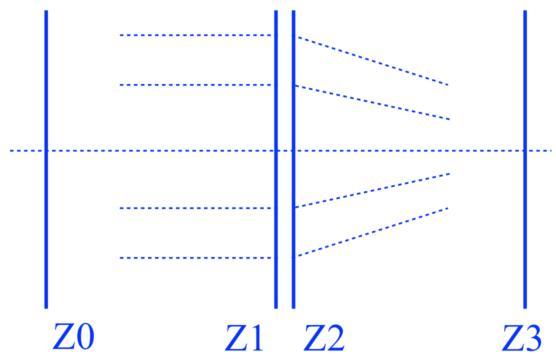
- Matrix description of thin lens element is given as :

$$\begin{pmatrix} x \\ a \end{pmatrix}_{z1} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \begin{pmatrix} x \\ a \end{pmatrix}_{z0}$$



- Matrix can be multiplied with the sequence :

$$\begin{pmatrix} x \\ a \end{pmatrix}_{Z3} = \begin{pmatrix} 1 & Z3 - Z2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \times \begin{pmatrix} 1 & Z1 - Z0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \end{pmatrix}_{Z0}$$



Paraxial approximation and transfer matrix

- Magnetic field is given in *y* direction :
  - (*x*,*a*,*δ*) and (*y*,*b*) is decoupled (in the 1st order) as :

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z1} = \begin{pmatrix} (x|x) & (x|a) & 0 & 0 & (x|\delta) \\ (a|x) & (a|a) & 0 & 0 & (a|\delta) \\ 0 & 0 & (y|y) & (y|b) & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z0}$$

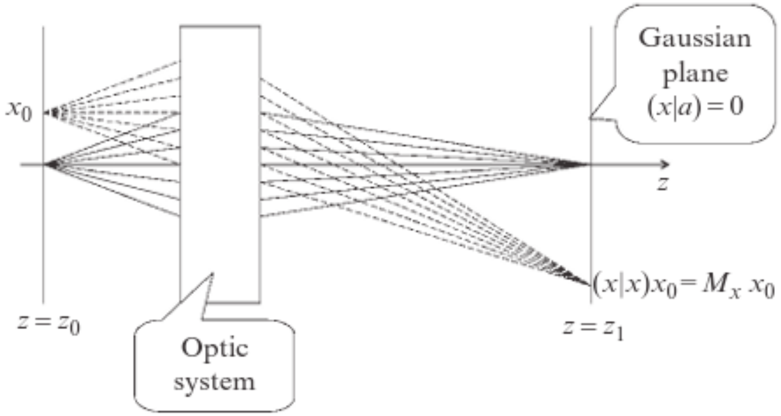


FIGURE 19 Forming an image in the Gaussian plane in the *x*-direction.

Idea of “Focus” in matrix description

- Focus := beam from same position but different direction at Z0
  - come together in the same position at Z1 → $(x|a)=0$

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z1} = \begin{pmatrix} (x|x) & (x|a) & 0 & 0 & (x|\delta) \\ (a|x) & (a|a) & 0 & 0 & (a|\delta) \\ 0 & 0 & (y|y) & (y|b) & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z0}$$

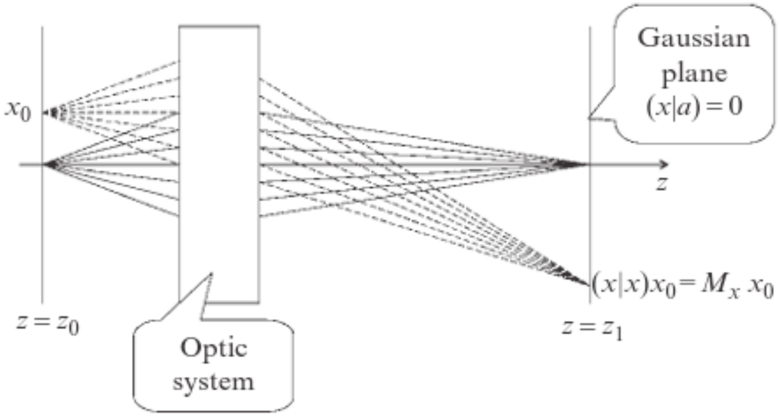
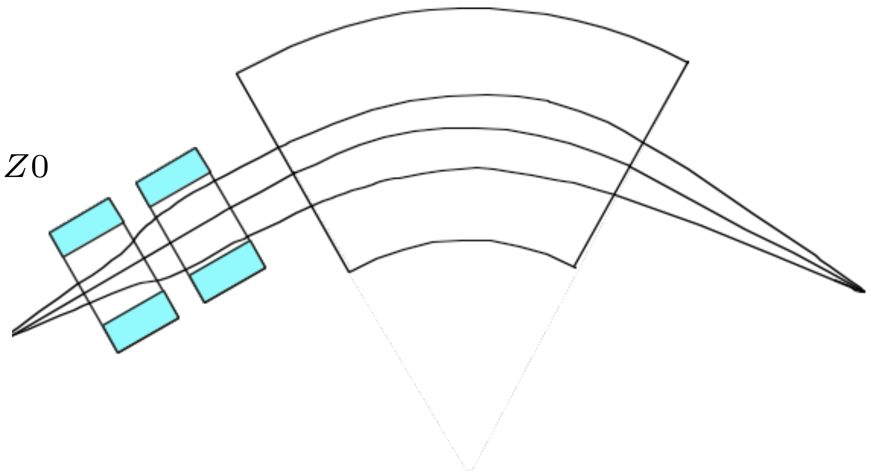


FIGURE 19 Forming an image in the Gaussian plane in the x-direction.



Barber's rule

- From the 1st order transfer matrix, you can prove "Barber's rule"
- Let's try ! → Home work

➡ Important for experiment session in the next week

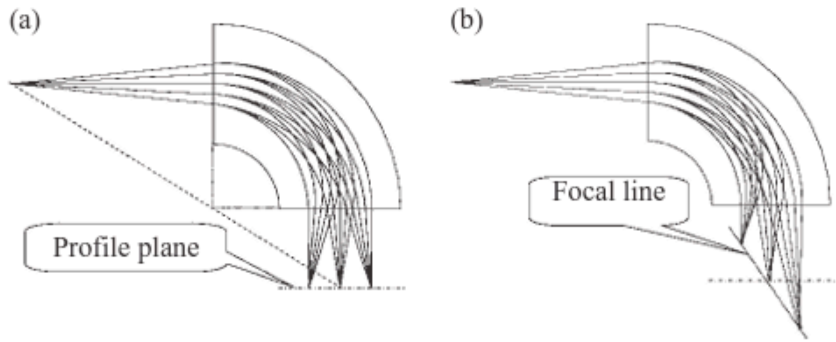


Figure 89 Focusing of a point object by a 90-degree homogeneous magnetic sector field in the linear approximation (a) and taking into account second-order aberrations (b). Shown are trajectories corresponding to three different ion masses and five different starting angles. In the paraxial sharp-cutoff approximation, the object, the image of ions with the nominal mass, and the center of curvature of the optic axis are located at one straight line (Barber's rule). The second-order chromatic aberration $(x \mid a\gamma)a_0\gamma$ leads to inclination of the focal line with respect to the profile plane.

Idea of "Resolving Power" in matrix description

- Resolving power is how optical system could resolve the momenum
  - on focusing condition: $(x|a)=0$; then

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z1} = \begin{pmatrix} (x|x) & \boxed{(x|a)} & 0 & 0 & (x|\delta) \\ (a|x) & (a|a) & 0 & 0 & (a|\delta) \\ 0 & 0 & (y|y) & (y|b) & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z0}$$

$$\delta = \frac{1}{(x|\delta)} \{ x_{Z1} - (x|x) x_{Z0} \}$$

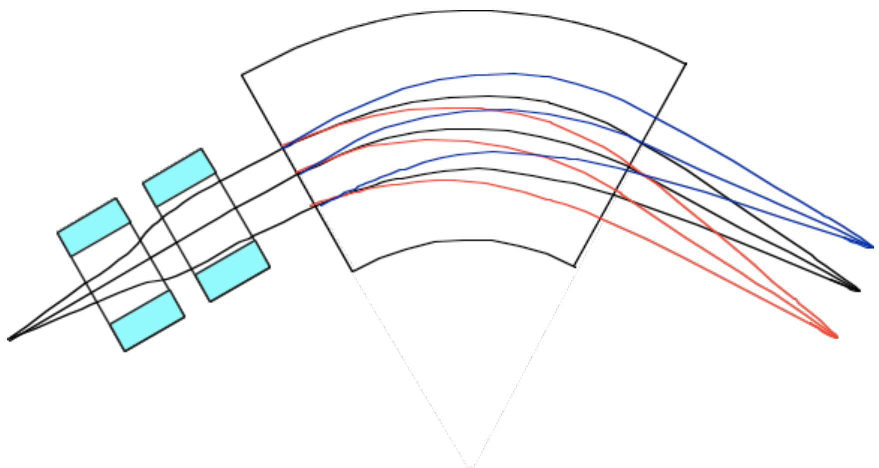
D ; dispersion M_x ; magnification

- Determination accuracy is given as :

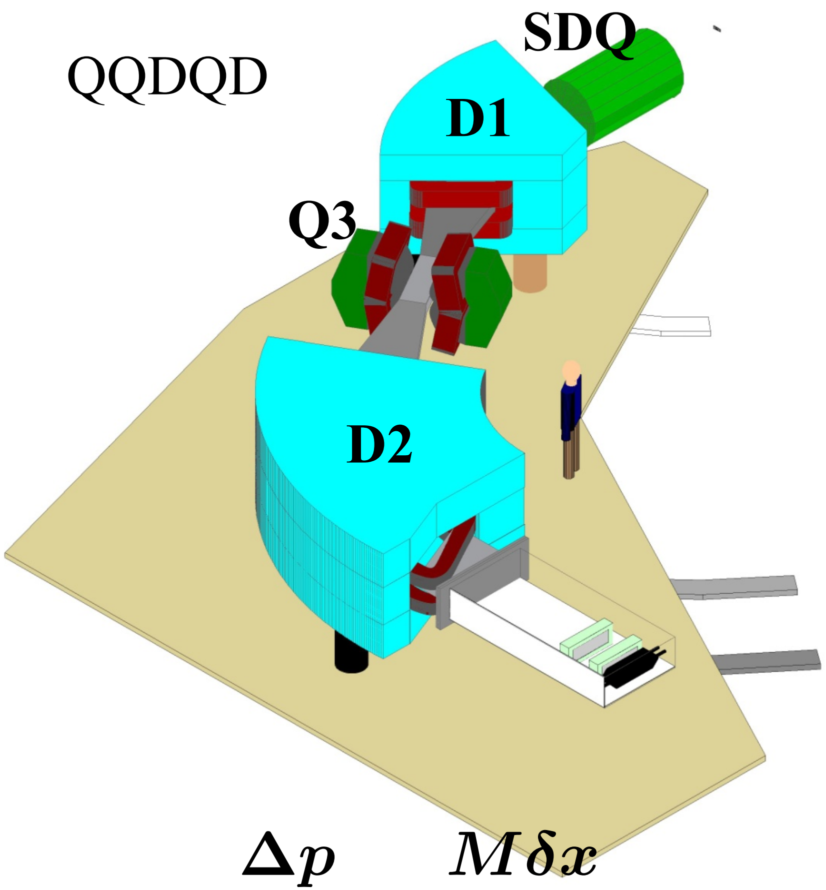
$$\Delta\delta = \frac{1}{D} \sqrt{(\Delta x_{Z1})^2 + M_x^2 (\Delta x_{Z0})^2}$$

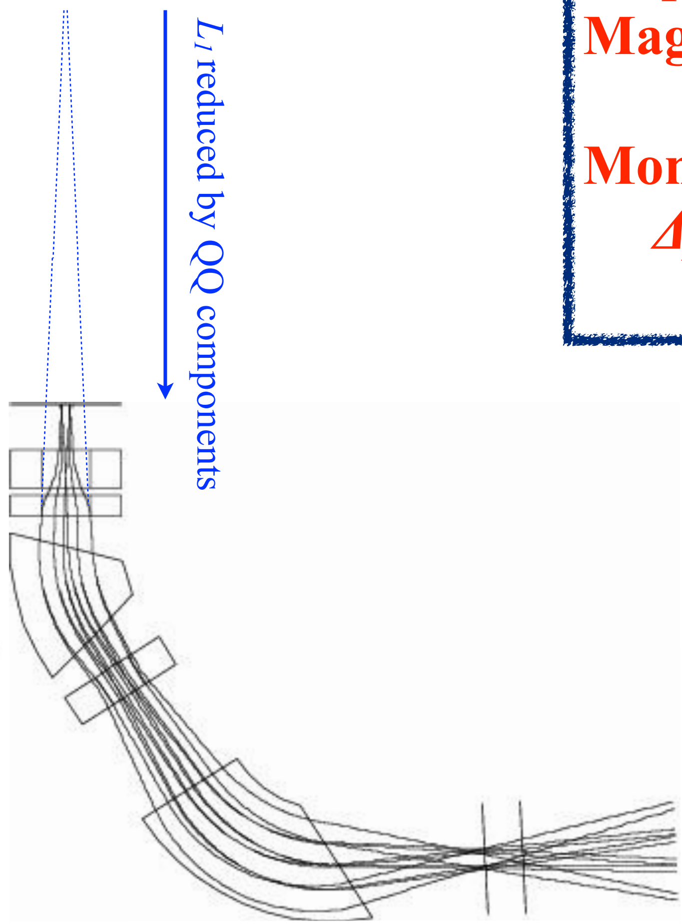
- Resolving power is given as :

$$R = \frac{p}{\Delta p} = \frac{D}{M_x \Delta x_{Z0}}$$



SHARAQ spectrometer designed for pursuing high resolution





Dispersion D**=5.85 m**
Magnification M**=0.397**

Momentum resolution
$\Delta p/p =$ **1/14700**
(δx **= 1 mm ;assumed)**

Momentum acceptance
± 1%
Angular acceptance
~ 5 msr

$$\frac{\Delta p}{p} = \frac{M\delta x}{D}$$

Focusing is necessary ?

- Focusing is not always needed if non-destructive measurement is available (at higher energy)

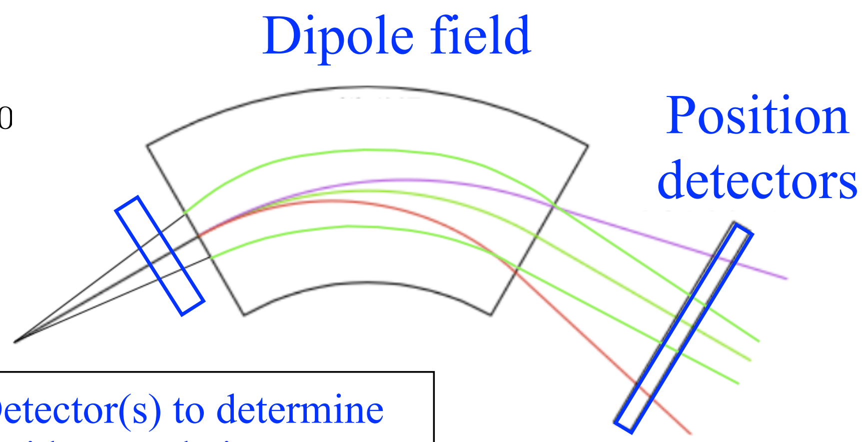
$$\rightarrow \ (x|a) \neq 0$$

$$\begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z1} = \begin{pmatrix} (x|x) & (x|a) & 0 & 0 & (x|\delta) \\ (a|x) & (a|a) & 0 & 0 & (a|\delta) \\ 0 & 0 & (y|y) & (y|b) & 0 \\ 0 & 0 & (b|y) & (b|b) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ a \\ y \\ b \\ \delta \end{pmatrix}_{Z0}$$

- $x_{Z0}, (x,a)_{Z1}$
- $(x,a)_{Z0}, x_{Z1}$

$\rightarrow \ \delta$

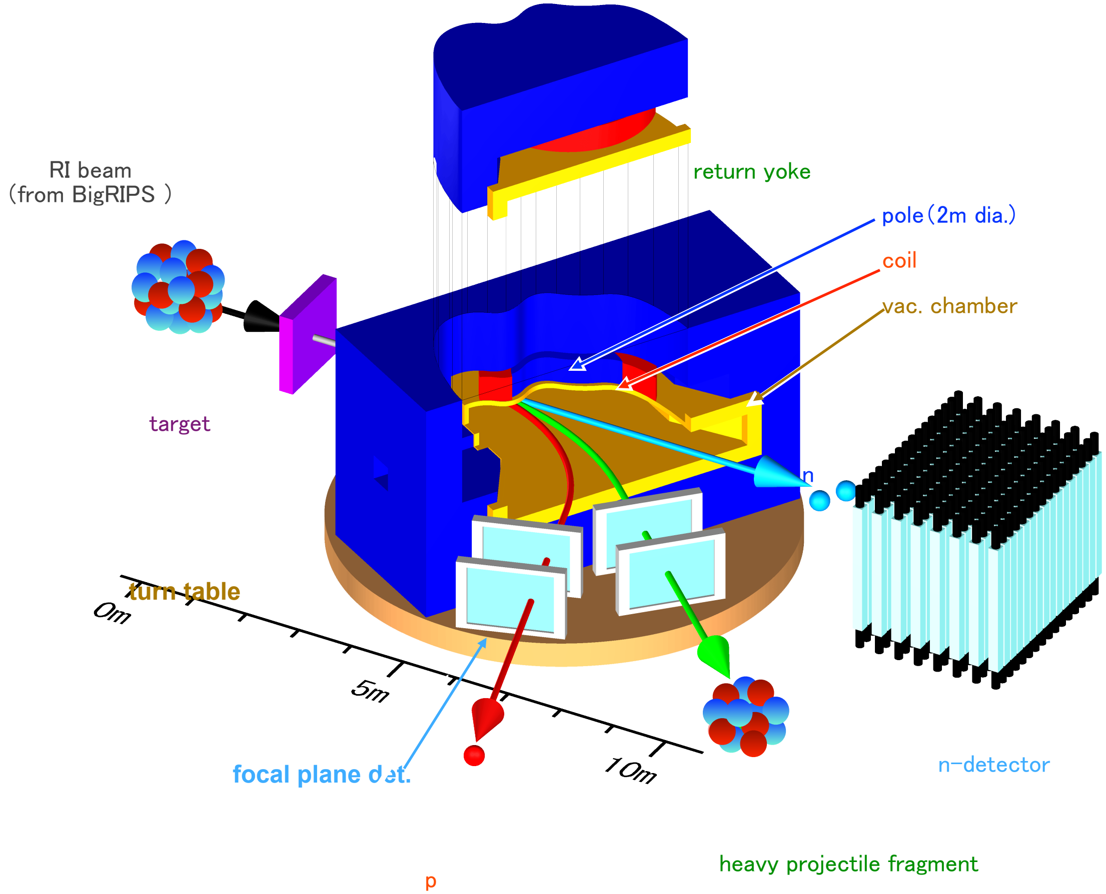
- a_{Z0} *or* a_{Z1} measurement is needed, on the contrary
  "non-" destructive methods are needed



Detector(s) to determine incident angle is necessary

"non-" destructive

SAMURAI spectrometer designed for pursuing large acceptance



Focusing is not always obtained
for higher Bρ particles

Multi particles are detected in coincidence
including neutrons

Momentum resolution
$\Delta p/p = \ 1/700$

$p_{max}/p_{min} \sim 3$
$\Delta\theta_H = \pm 10°$
$\Delta\theta_V = \pm 5°$

Momentum acceptance
± 70 %
Angular acceptance
~ 58 msr

For further studies

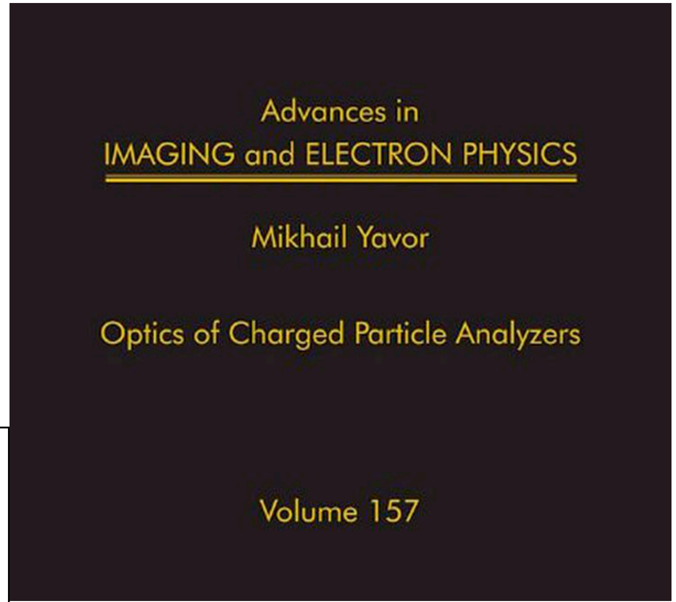
1) References for study motion of ions in electromagnetic field

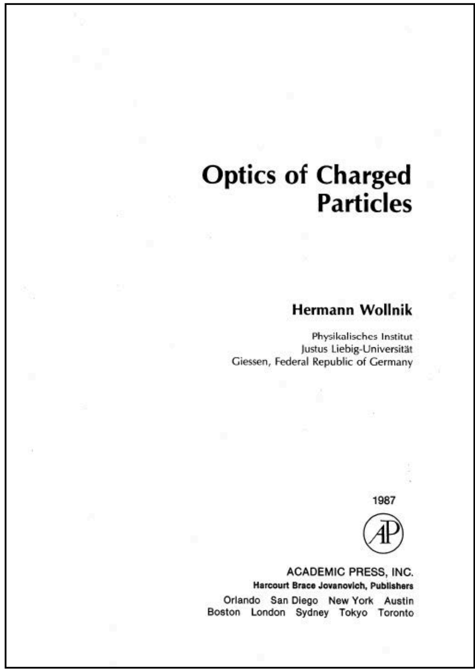
Advances in Imaging and Electron Physics (Chap. 5)
by Mikhail Yavor

Optics of charged particles

by Hermann Wollnik

An ion optical transfer matrix
calculation code,ORBIT2(2002)88. 35.
by S. Morinobu





For further studies

2) References for ion optics and practical computer code packages

GIOS, GICOSY

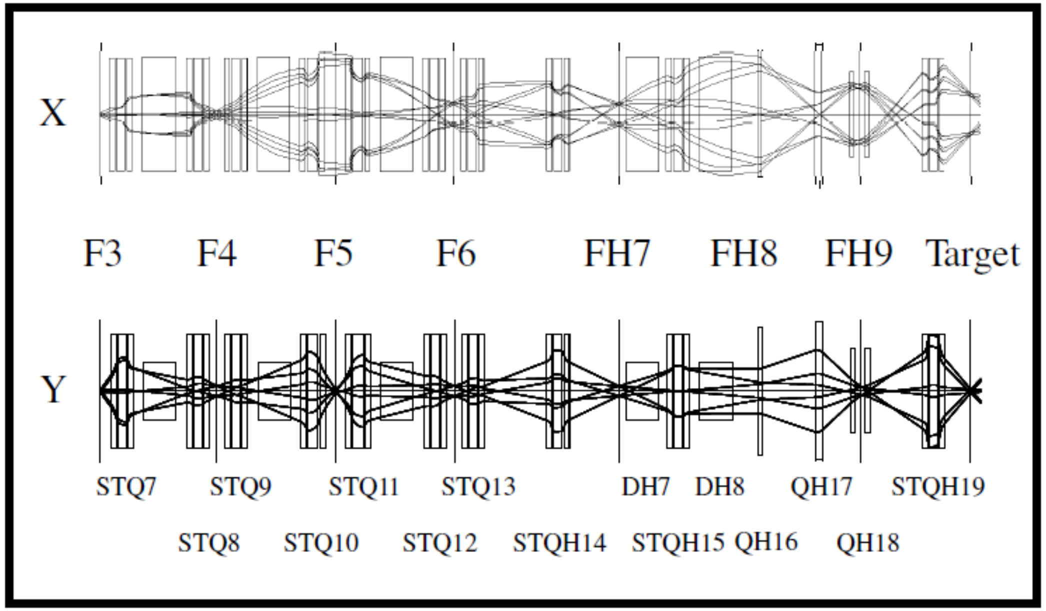
http://www-linux.gsi.de/~weick/gios/

http://www-linux.gsi.de/~weick/gicosy/

COSY Infinity

http://bt.pa.msu.edu/index_cosy.htm

Sample for beam optics for transferring secondary beam to target



Summary

1) Magnetic spectrometer in RIBF
   Various type of magnetic spectrometer
   RI separator

2) Charged particle motion in magnetic field
   Equation of motion
   Generate high magnetic field by iron

3) Ion optics and transfer matrix method
   Idea of focusing
   Idea of resolving power
   Spectrometer specification
      resolving power
      angular acceptance and momentum acceptance

Home works

1) Q1 which direction are electrons precessed ?

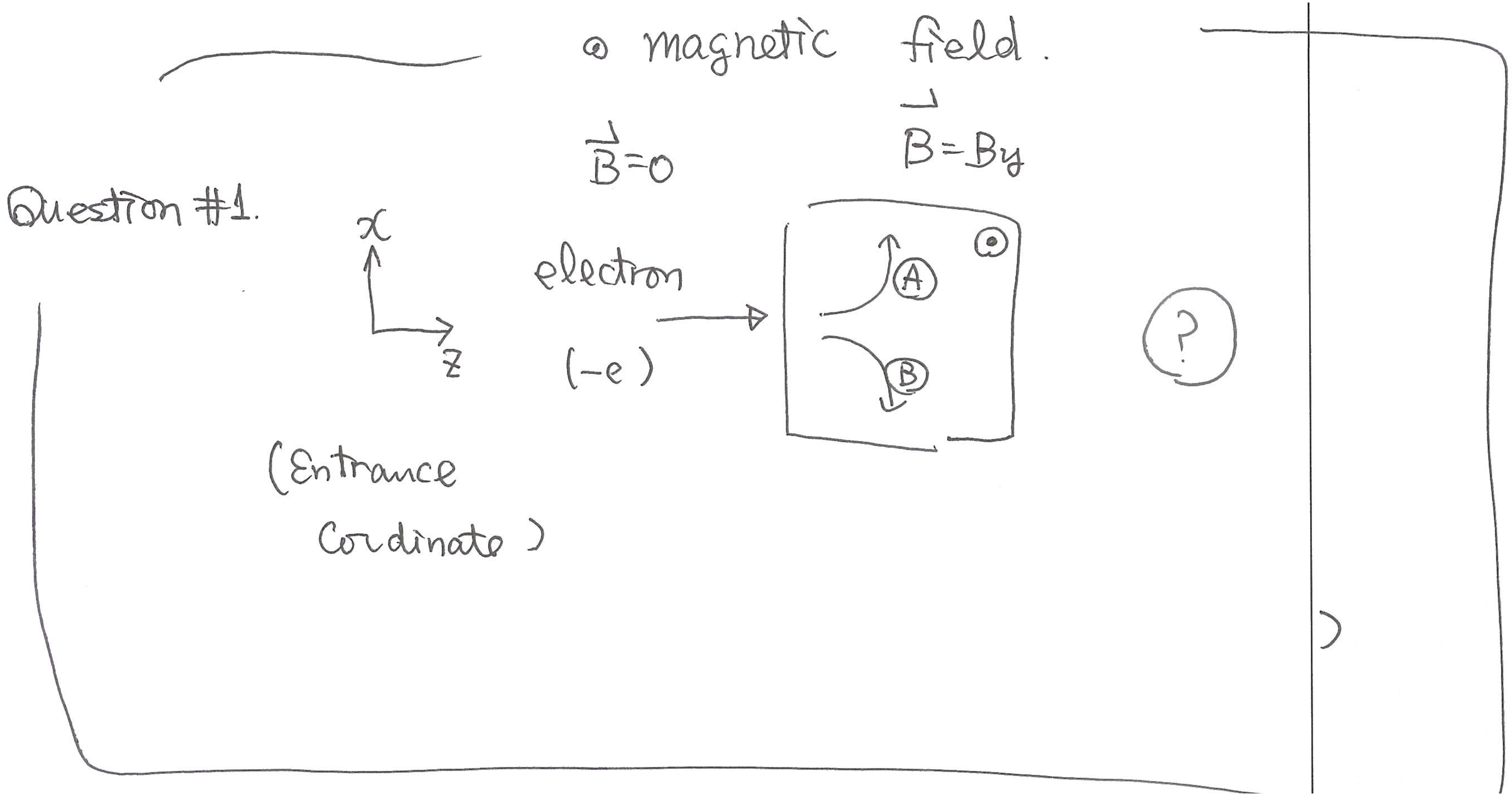
2) Q2 curvature for
   1.5 MeV electrons on 0.1 T
   - electron mass is 0.511 MeV (~ 0.5 MeV/c^2)

- 250 MeV/A ^{6}He (A=6, Z=Q=2) on 1.6 T
   - nucleon mass is 940 MeV (~ 1 GeV/c^2)

3) Barber's rule

Direction is important

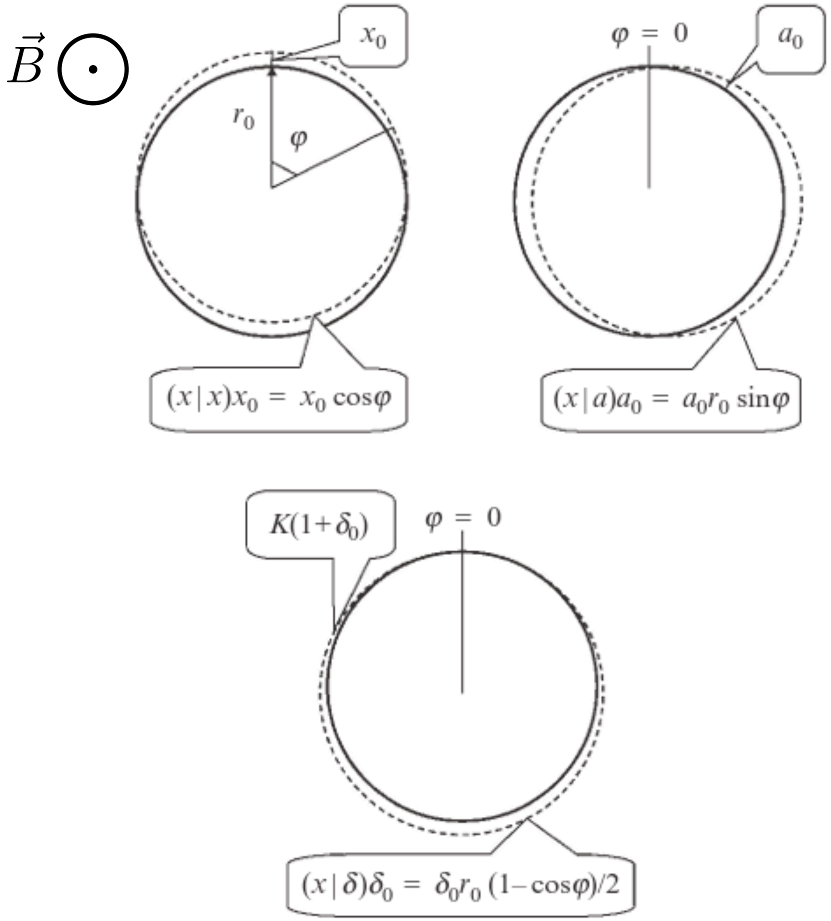
- Question #1 for home works



In homogeneous magnetic field

- Description with natural unit notation at $\hbar$=*c*=*1*

$$\underset{[\mathrm{eV/c}][\mathrm{charge;\ integer}]}{p/Q} = \underset{[\mathrm{m/s}][\mathrm{T}][\mathrm{m}]}{cB\rho}$$



- Question #2
- Let us consider the curvature of
  - 1.5 MeV electrons on 0.1 T
    - electron mass is 0.511 MeV (~ 0.5 MeV/c^2)
  - 250 MeV/A ^{6}He (*A*=6, *Z*=*Q*=2) on 3 T
    - nucleon mass is 940 MeV (~ 1 GeV/c^2)

➡ home works until next week

Barber's rule

- From the 1st order transfer matrix, you can prove "Barber's rule"
- Let's try !   → Home work

➡ Important for experiment session in the next week

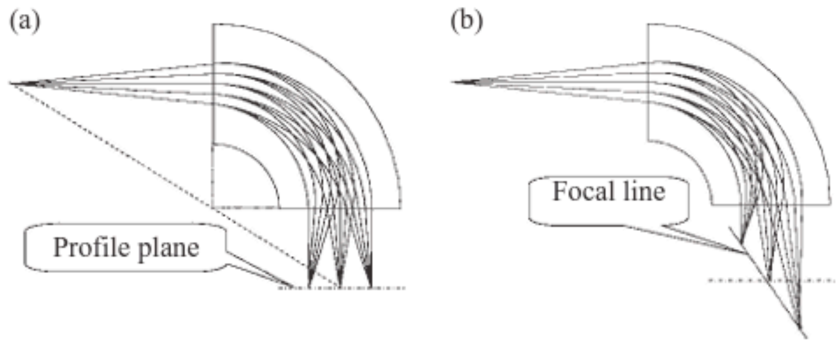


FIGURE 89 Focusing of a point object by a 90-degree homogeneous magnetic sector field in the linear approximation (a) and taking into account second-order aberrations (b). Shown are trajectories corresponding to three different ion masses and five different starting angles. In the paraxial sharp-cutoff approximation, the object, the image of ions with the nominal mass, and the center of curvature of the optic axis are located at one straight line (Barber's rule). The second-order chromatic aberration $(x \mid a\gamma)a_0\gamma$ leads to inclination of the focal line with respect to the profile plane.

Appendix

Reduce for analytical calculation

```
#!/bin/sh

#
# analytical calculation code "reduce" is available from
# http://reduce-algebra.sourceforge.net
#

reduce <<EOREDUCE

clear m1,m2,m3;
matrix m1,m2,m3;
matrix mtotal;

m1:=mat((1,l1,0),(0,1,0),(0,0,1))$
m3:=mat((1,l2,0),(0,1,0),(0,0,1))$

m2:=mat((cos(phi),rho0*sin(phi),rho0*(1-cos(phi))),
(-sin(phi)/rho0,cos(phi),sin(phi)),(0,0,1))$

mtotal := m3*m2*m1;

phi := PI/3 $

mtotal;

phi := PI/2 $

mtotal;

EOREDUCE
```