Transverse Spin Structures of the Nucleon

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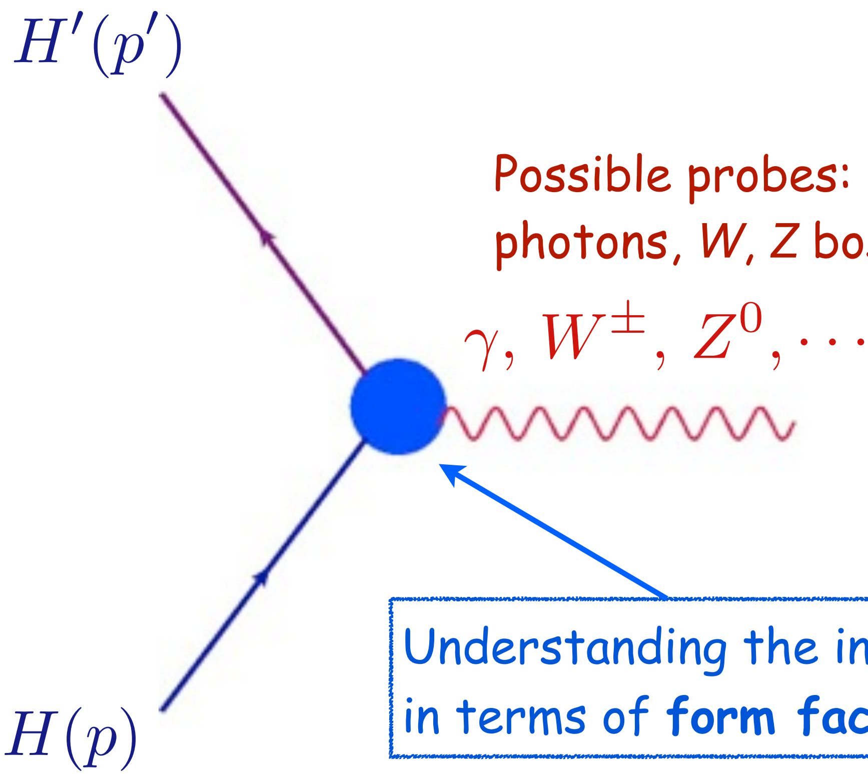
Structure of hadrons

Traditional way of studying structures of hadrons



Possible probes:
photons, *W*, *Z* bosons, mesons, nucleons……

Understanding the internal structures of hadrons *H* in terms of **form factors**.

$$F(q^2) \sim \int \rho(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3\mathbf{r} \Leftarrow \frac{d\sigma}{d\Omega}, A, \cdots$$

M. Burkardt, Phys. Rev. D, 62, 071503 (2000)

Structure of hadrons

1. **Scalar form factors**: Sigma pion-nucleon term
   Quark contribution to the nucleon mass

$$\langle h(p')|\bar{\psi}(0)\psi(0)|h(p)\rangle \sim \Sigma_{\pi N}(t)$$

2. **Vector form factors**: Electromagnetic & weak properties
   Charge, EM radii, EM quark distributions in the nucleon

$$\langle N(p')|\bar{\psi}(0)\gamma_\mu\lambda^a\psi(0)|N(p)\rangle \sim G_E(t),\ G_M(t),\ G_E^s(t),\ G_M^s(t)$$

3. **Axial-vector form factors**: Weak properties, spin content of the nucleon, pion-N couplings (PCAC)......

$$\langle N(p')|\bar{\psi}(0)\gamma_\mu\gamma_5\lambda^a\psi(0)|N(p)\rangle \sim g_A(t),\ g_A^0(t),\ G_A^s(t),\ g_{\pi NN}\cdots$$

Structure of hadrons

4. **Energy-momentum tensor (gravitational) form factors**:
Mass of the nucleon, orbital angular momentum,
D1 term (pressure, shear force)

$$\langle N(p')|T_{\mu\nu}|N(p)\rangle \sim M_2(t),\ J(t),\ d_1(t)$$

5. **Tensor form factors**: Transverse spin structure of the nucleon

$$\langle N(p')|\bar{\psi}(0)\sigma_{\mu\nu}\lambda^a\psi(0)|N(p)\rangle \sim H_T(t),\ E_T(t),\ \tilde{H}_T(t)$$

→ As equally important as vector & axial-vector form factors
but No probes into these EMT and tensor form factors!

Structure of hadrons

Modern approach: Generalized parton distributions make it possible to get access to these EMT & tensor form factors.

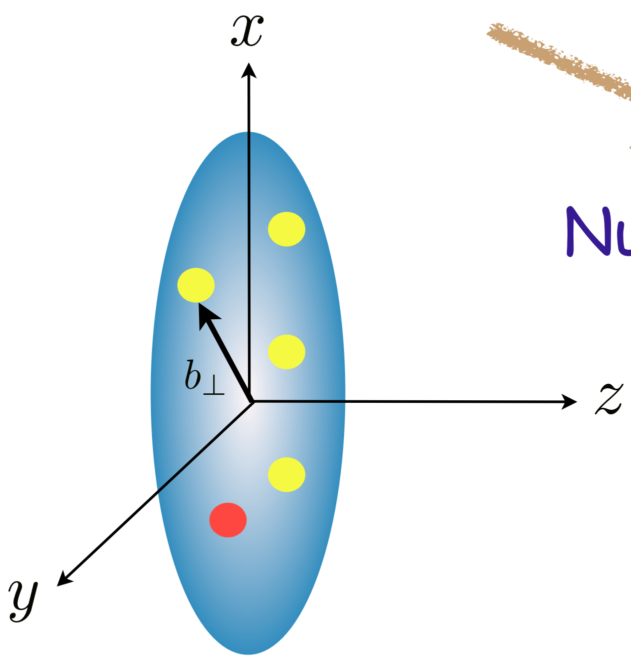
Form factors as Mellin moments of the GPDs

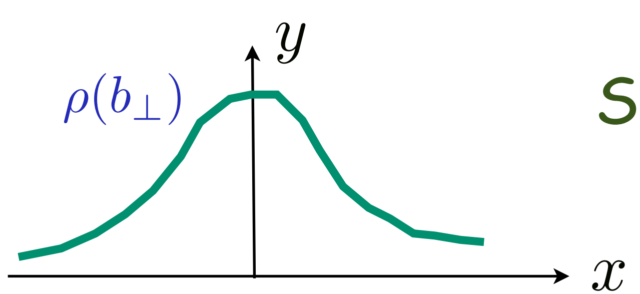
In the present talk, I would like to concentrate on the tensor form factors of the nucleon and their transverse spin structures.

The spin structure of the Nucleon

Axial & Tensor Form factors, Axial-vector charges, Tensor charges

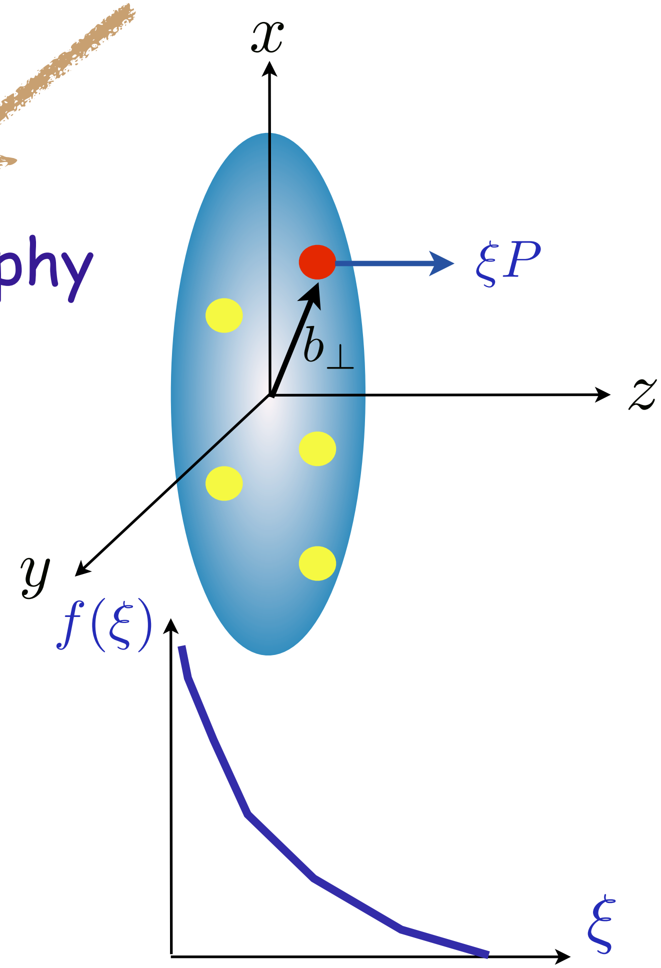
Structure functions



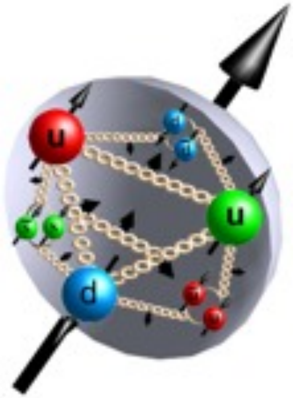


Nucleon Tomography (GPDs)

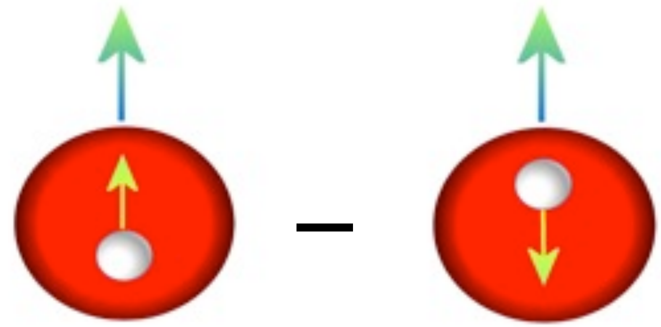
Spin Structure



Transversity: Tensor Charges



$\delta \mathbf{q}(\mathbf{x}) =$



$$\left\langle N \left| \bar{\psi} \sigma_{\mu\nu} \lambda^{\chi} \psi \right| N \right\rangle \sim \text{Tensor charges}$$

- No explicit probe for the tensor charge! Difficult to be measured.
- Chiral-odd Parton Distribution Function can get accessed via the SSA of SIDIS (HERMES and COMPASS).

A. Airapetian et al. (HERMES Coll.), PRL 94, 012002 (2005).

E.S. Ageev et al. (COMPASS Coll.), NPB 765, 31 (2007).

CLAS & CLAS12 Coll.

ppbar Drell-Yan process (PAX Coll.): Technically too difficult for the moment (polarized antiproton: hep-ex/0505054).

Transversity: Tensor Charges

Tensor Charges extracted from the Experimental data:

$$\delta u = 0.60^{+0.10}_{-0.24}\,, \quad \delta d = -0.26^{+0.1}_{-0.18} \text{ at } 0.36\,\mathrm{GeV}^2$$

Based on SIDIS (HERMES, COMPASS) data:
M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

The uncertainty is still large and more data need to be compiled.

Tensor form factors

$$\langle N_{s'}(p')|\overline{\psi}(0)i\sigma^{\mu\nu}\lambda^\chi\psi(0)|N_s(p)\rangle = \overline{u}_{s'}(p')\left[\boxed{H_T^\chi(Q^2)}i\sigma^{\mu\nu} + E_T^\chi(Q^2)\frac{\gamma^\mu q^\nu - q^\mu\gamma^\nu}{2M} + \tilde{H}_T^\chi(Q^2)\frac{(n^\mu q^\nu - q^\mu n^\nu)}{2M^2}\right]u_s(p)$$

$$\boxed{\int_{-1}^{1} dx\, H_T^\chi(x,\xi,t) = H_T^\chi(q^2),}$$

$$\int_{-1}^{1} dx\, E_T^\chi(x,\xi,t) = E_T^\chi(q^2),$$

$$\int_{-1}^{1} dx\, \tilde{H}_T^\chi(x,\xi,t) = \tilde{H}_T^\chi(q^2),$$

$$H_T^0(0) = g_T^0 = \delta u + \delta d + \delta s$$

$$H_T^3(0) = g_T^3 = \delta u - \delta d$$

$$H_T^8(0) = g_T^8 = \frac{1}{\sqrt{3}}(\delta u + \delta d - 2\delta s)$$

$$H_T^{*\chi}(Q^2) = \frac{2M}{\mathbf{q}^2}\int\frac{d\Omega}{4\pi}\langle N_{\frac{1}{2}}(p')|\psi^\dagger\gamma^k q^k\lambda^\chi\psi|N_{\frac{1}{2}}(p)\rangle$$

$$\kappa_T^\chi = -H_T^\chi(0) - H_T^{*\chi}(0)$$

Together with the anomalous magnetic moment, this will allow us to describe the transverse spin quark densities inside the nucleon.

Tensor form factors

Tensor charges and anomalous tensor magnetic moments are scale-dependent.

$$\delta q(\mu^2) = \left(\frac{\alpha_S(\mu^2)}{\alpha_S(\mu_i^2)}\right)^{4/27}\left[1-\frac{337}{486\pi}\left(\alpha_S(\mu_i^2)-\alpha_S(\mu^2)\right)\right]\delta q(\mu_i^2)\,,$$

$$\alpha_S^{NLO}(\mu^2) = \frac{4\pi}{9\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\left[1-\frac{64}{81}\frac{\ln\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}{\ln(\mu^2/\Lambda_{\mathrm{QCD}}^2)}\right]$$

$\Lambda_{\mathrm{QCD}} = 0.248\,\mathrm{GeV}$

M. Gluck, E. Reya, and A. Vogt, Z.Phys. C 67, 433(1995).

Chiral quark-soliton model

Merits of the chiral quark-soliton model

- Fully relativistically field theoretic model.
- Related to QCD via the instanton vacuum.
- Renormalization scale is naturally given.
- All parameters were fixed already.

$$\mathcal{Z}_{\chi\mathrm{QSM}} = \int \mathcal{D}U \exp(-S_{\mathrm{eff}})$$

$$\boxed{H(U) = -i\gamma_4\gamma_i\partial_i + \gamma_4 M U^{\gamma_5}}$$

$$S_{\mathrm{eff}} = -N_c \mathrm{Tr} \ln D(U)$$

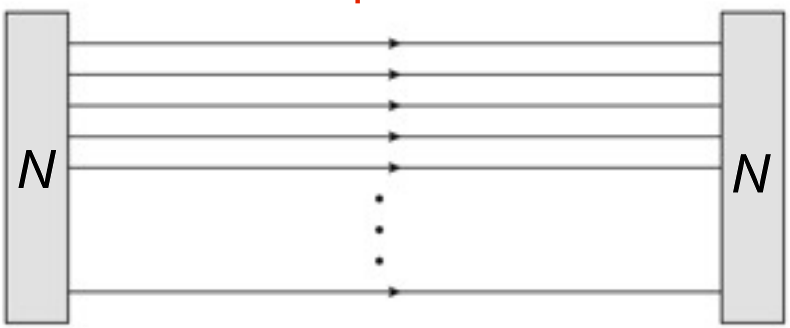
$$\boxed{D(U) = \partial_4 + H(U) + \hat{m}}$$

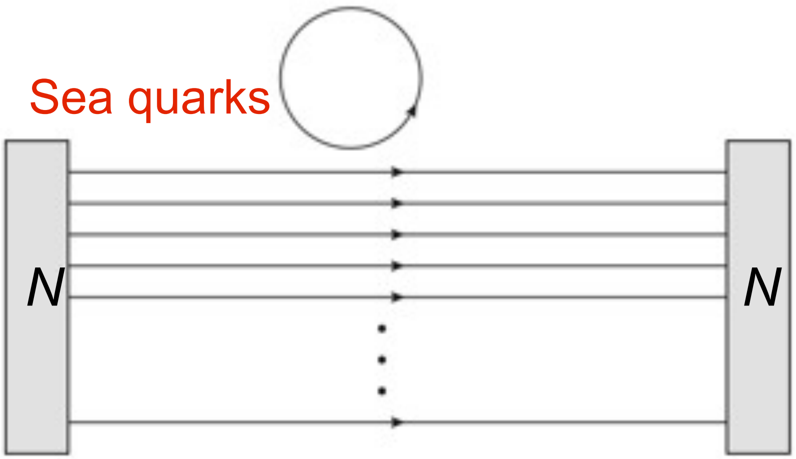
$$\boxed{\hat{m} = \mathrm{diag}(m_u, m_d, m_s)\gamma_4}$$

HChK et al. Prog. Part. Nucl. Phys. Vol.37, 91 (1996)

Chiral quark-soliton model

Nucleon consisting of Nc quarks





$$\Pi_N = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle$$

$$J_N(\vec{x}, t) = \frac{1}{N_c!} \varepsilon^{\beta_1 \cdots \beta_{N_c}} \Gamma^{\{f\}}_{J J_3 Y' T T_3 Y} \psi_{\beta_1 f_1}(\vec{x}, t) \cdots \psi_{\beta_{N_c} f_{N_c}}(\vec{x}, t)$$

$$\lim_{T \to \infty} \Pi_N(T) \simeq e^{-M_N T}$$

$$\Pi_N(\vec{x}, t) = \Gamma_N^{\{f\}} \Gamma_N^{\{g\}*} \frac{1}{Z} \int dU \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle_{f,g} e^{-S_{\rm eff}}$$

$$\lim_{T \to \infty} \frac{1}{Z} \prod_{i=1}^{N_c} \left\langle 0, T/2 \left| \frac{1}{D(U)} \right| 0, -T/2 \right\rangle \sim e^{-(N_c E_{\rm val}(U) + E_{\rm sea}(U))T}$$

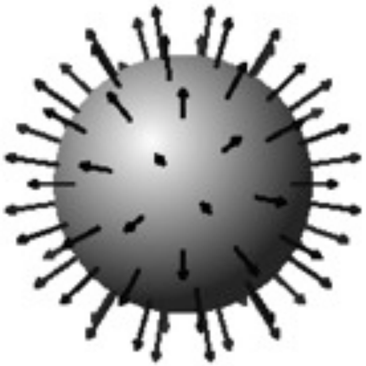
Chiral quark-soliton model

Classical solitons

$$\frac{\delta}{\delta U}(N_c E_{\mathrm{val}} + E_{\mathrm{sea}}) = 0 \rightarrow M_{\mathrm{cl}} = N_c E_{\mathrm{val}}(U_c) + E_{\mathrm{sea}}(U_c)$$

Hedgehog Ansatz:

$$U_{\mathrm{SU}(2)} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$$



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton model

Collective quantization

$$U_0 = \begin{bmatrix} e^{i\vec{n}\cdot\vec{\tau}\,P(r)} & 0 \\ 0 & 1 \end{bmatrix}$$

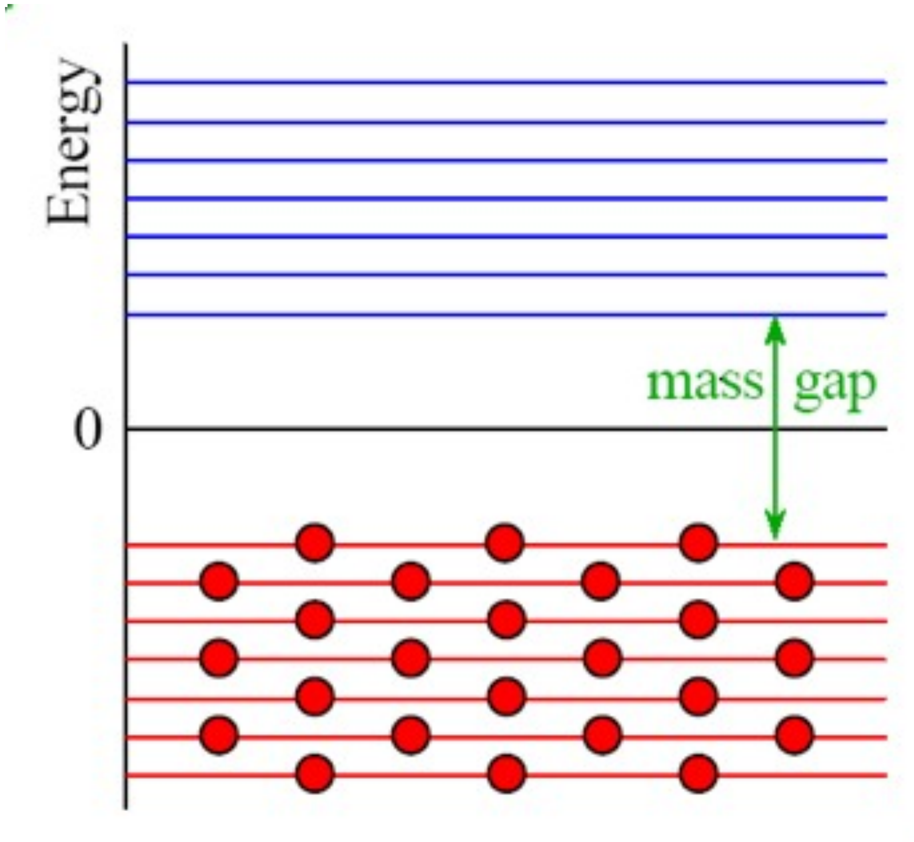
$$U(\boldsymbol{x},t) \;=\; R(t)U_c(\boldsymbol{x}-\boldsymbol{Z}(t))R^{\dagger}(t)$$

$$\int DU[\cdots] \quad\to\quad \int DAD\boldsymbol{Z}[\cdots]$$

$$\mathcal{L} = -M_{sol} + \frac{I_1}{2}\sum_{i=1}^{3}{\Omega_i}^2 + \frac{I_2}{2}\sum_{i=4}^{7}{\Omega_i}^2 + \frac{N_c}{2\sqrt{3}}\Omega_8$$

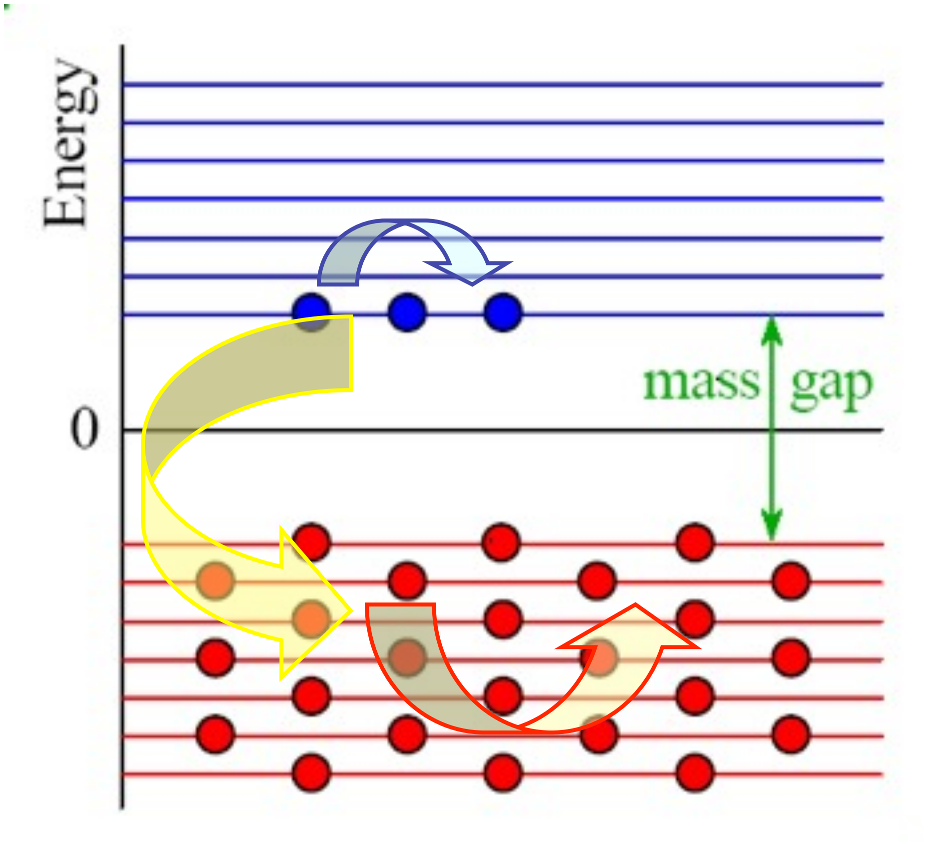
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture



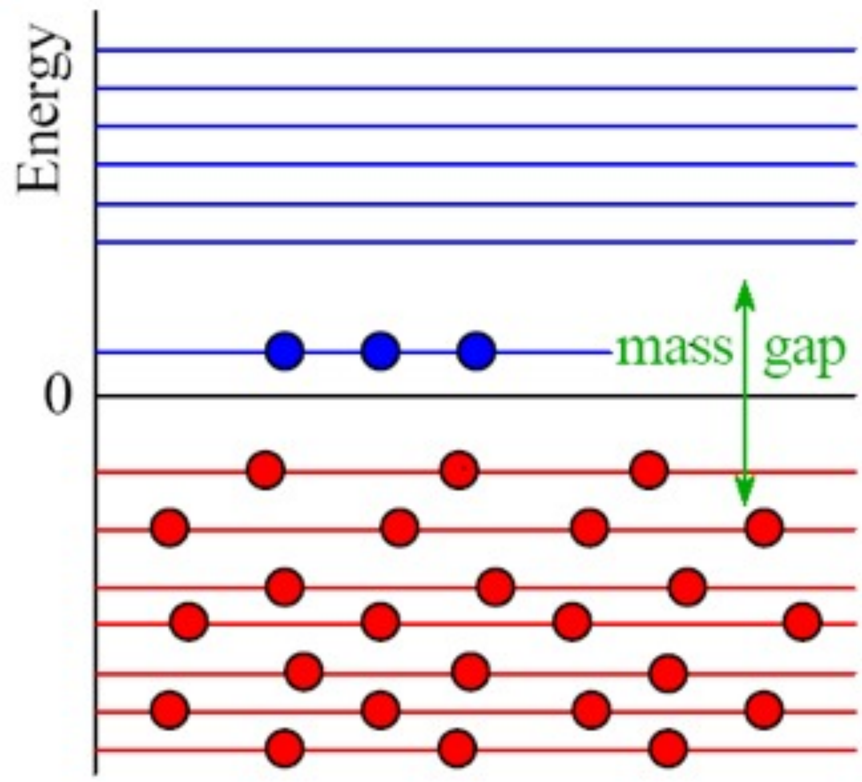
HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture



HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture

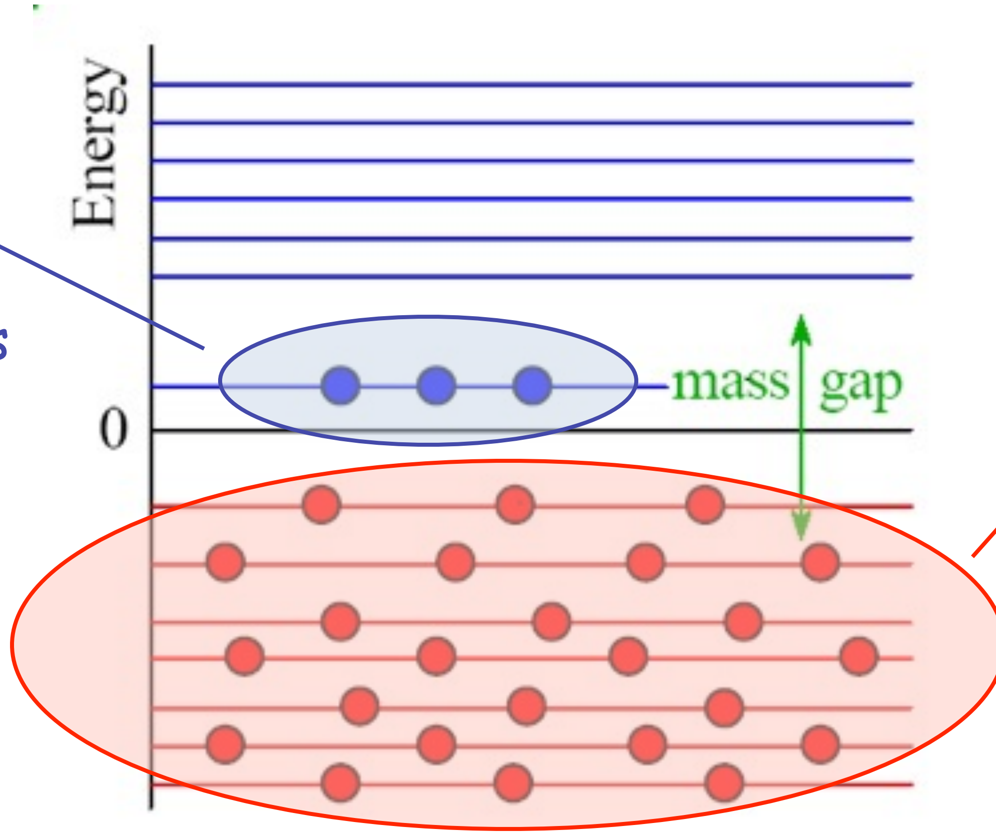


HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Chiral quark-soliton picture

valence
level:

energy
decreases



sea
levels:

energy
increases

system stabilzes

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Results

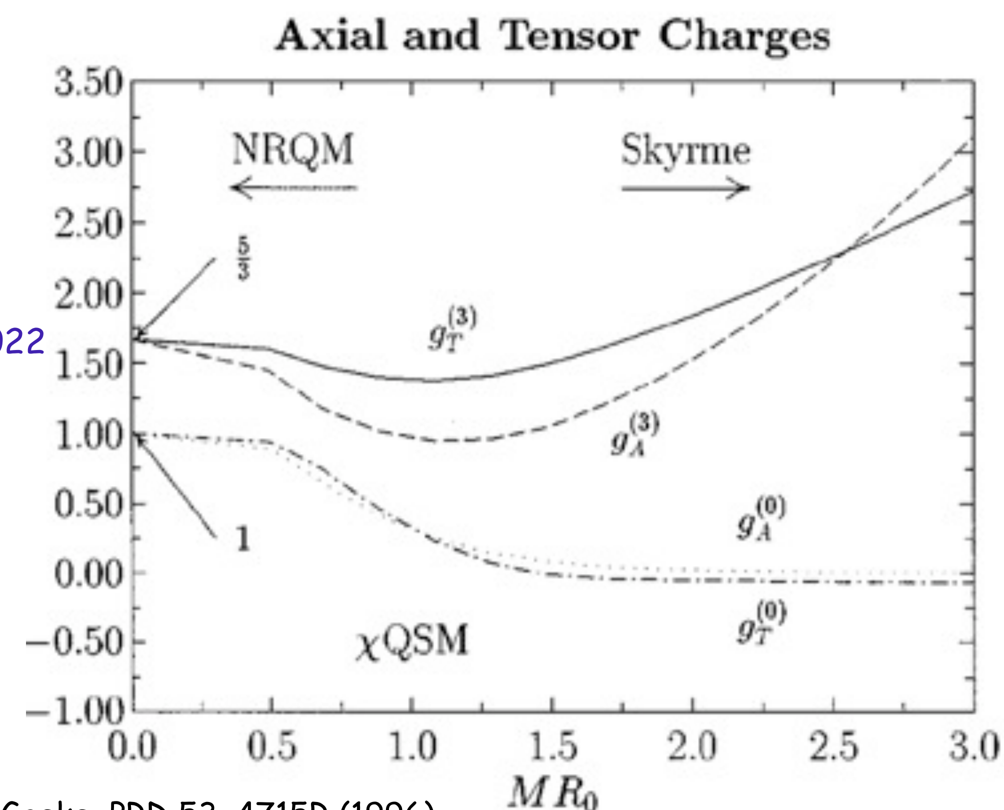
	g_T^0	g_T^3	g_T^8	g_A^0	g_A^3	g_A^8	Δu	δu	Δd	δd	Δs	δs
χQSM SU(3)	0.76	1.40	0.45	0.45	1.18	0.35	0.84	1.08	-0.34	-0.32	-0.05	-0.01
χQSM SU(2)	0.75	1.44	$--$	0.45	1.21	$--$	0.82	1.08	-0.37	-0.32	$--$	$--$
NRQM	1	5/3	$--$	1	5/3	$--$	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$--$	$--$

$$g_A^3 \sim (MR_0)^2 \qquad g_T^3 \sim MR_0$$

$$g_A^0 \sim \frac{1}{(MR_0)^4} \qquad g_T^0 \sim \frac{1}{MR_0}$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

$$g_T^\chi > g_A^\chi$$



HChK, M. Polyakov, K. Goeke, PRD 53, 4715R (1996)

Results

Proton	This work	SU(2)	Lattice	SIDIS	NR
$\lvert\delta d/\delta u\rvert$	0.30	0.36	0.25	$0.42^{+0.0003}_{-0.20}$	0.25

$$
\begin{array}{lll}
\text{SIDIS [16] } (0.80\,\mathrm{GeV}^2): & \delta u = 0.54^{+0.09}_{-0.22}\,, & \delta d = -0.231^{+0.09}_{-0.16}, \\
\text{SIDIS [16] } (0.36\,\mathrm{GeV}^2): & \delta u = 0.60^{+0.10}_{-0.24}\,, & \delta d = -0.26^{+0.1}_{-0.18}, \\
\text{Lattice [21] } (4.00\,\mathrm{GeV}^2): & \delta u = 0.86 \pm 0.13\,, & \delta d = -0.21 \pm 0.005\,, \\
\text{Lattice [21] } (0.36\,\mathrm{GeV}^2): & \delta u = 1.05 \pm 0.16\,, & \delta d = -0.26 \pm 0.01\,, \\
\chi\text{QSM } (0.36\,\mathrm{GeV}^2): & \delta u = 1.08\,, & \delta d = -0.32\,,
\end{array}
$$

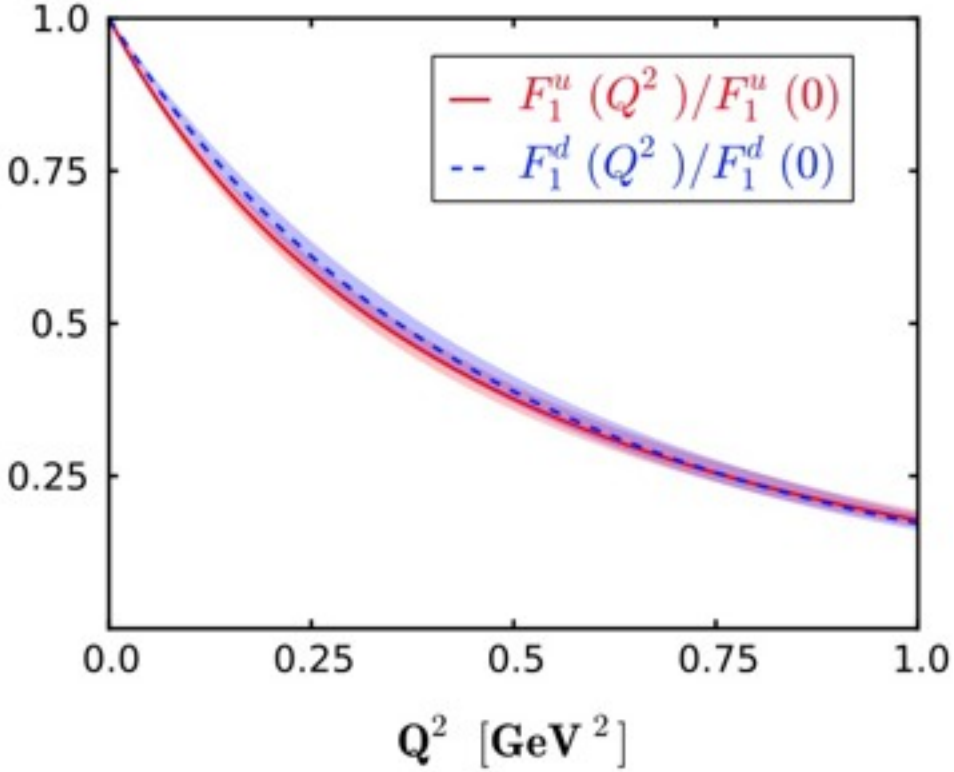
[16] M. Anselmino et al. Nucl. Phys. B, Proc. Suppl. 191, 98 (2009)

[21] M. Goeckeler et al., PLB 627, 113 (2005)

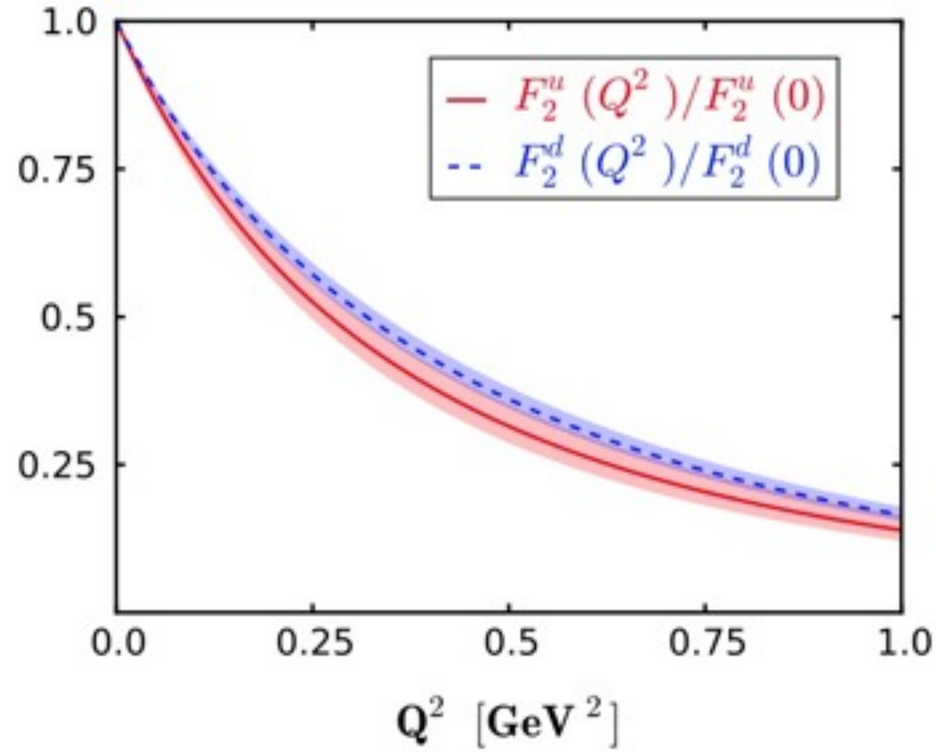
T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Dirac form factor



Pauli form factor



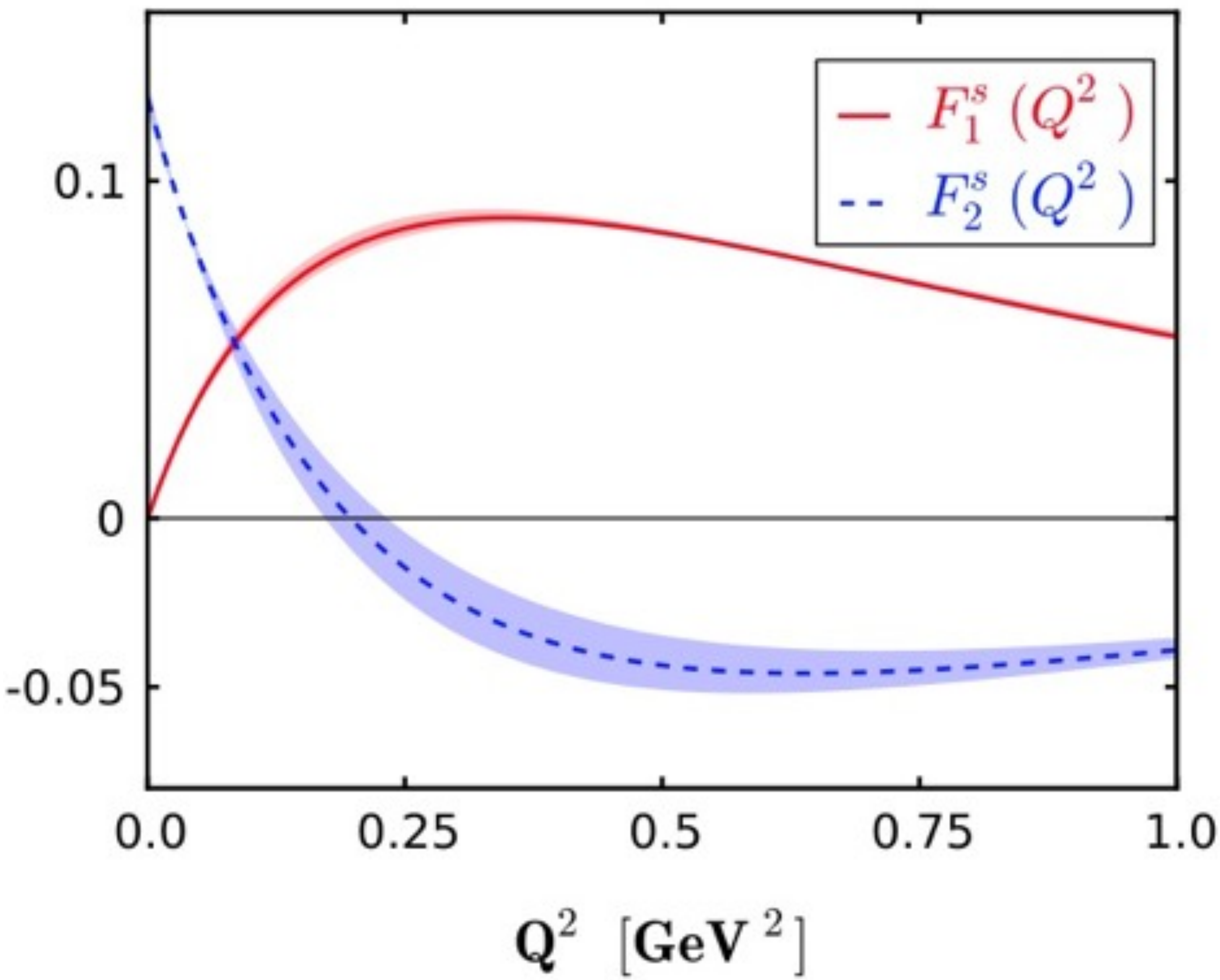
$$\kappa_{\mathrm{u}} = 1.35 \text{ (Exp: } 1.67),$$
$$\kappa_{\mathrm{d}} = -1.80, \text{ (Exp: -2.03)}$$
$$\kappa_{\mathrm{u}}/\kappa_{\mathrm{d}} = 0.75 \text{ (Exp: } 0.82)$$

(SU(2) symmetry assumed)

$$\kappa_p = 1.49 \text{ (Exp: } 1.79)$$
$$\kappa_n = -1.65 \text{ (Exp: -1.91)}$$

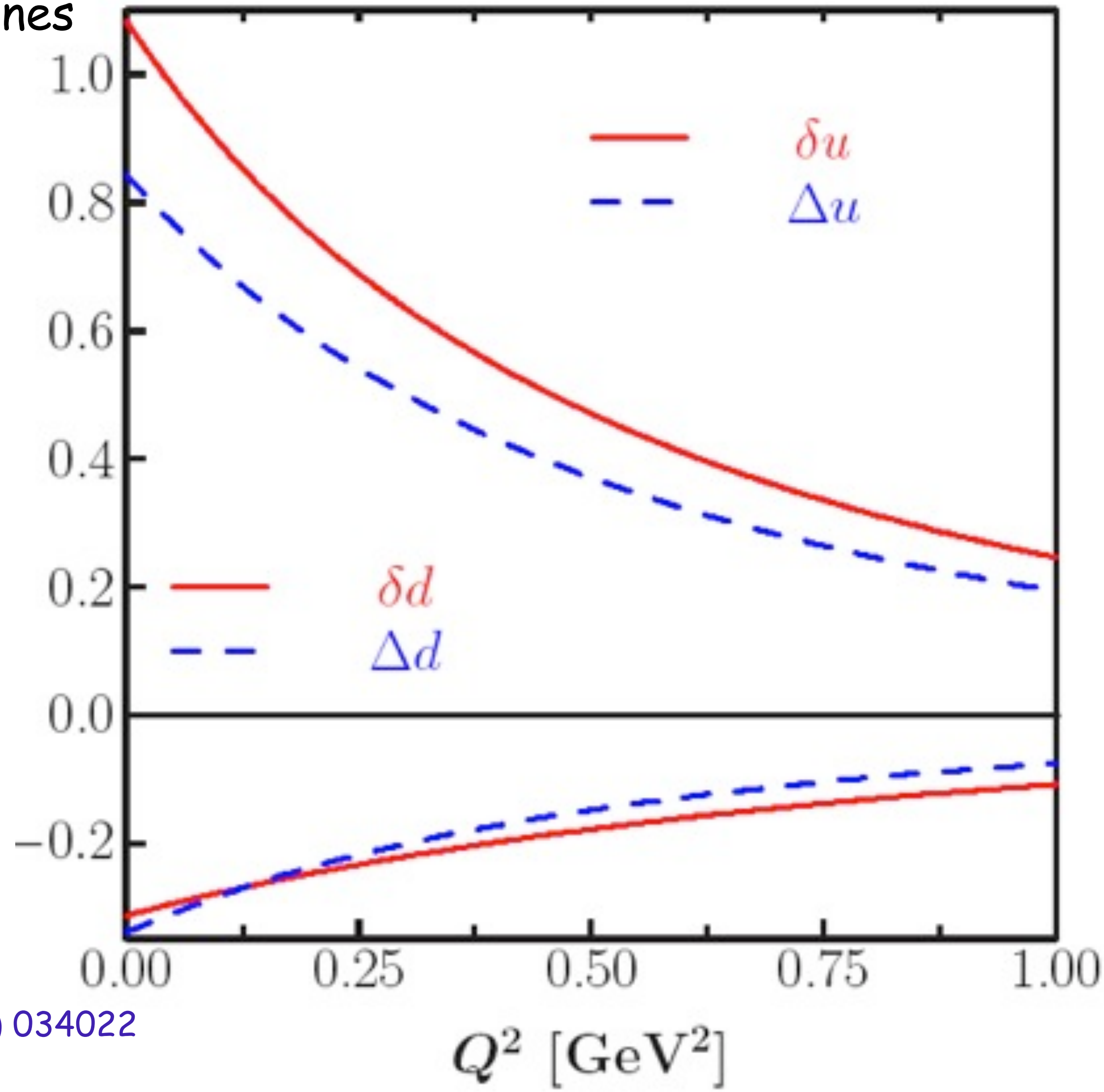
Results

Strange Dirac and Pauli form factors



Results

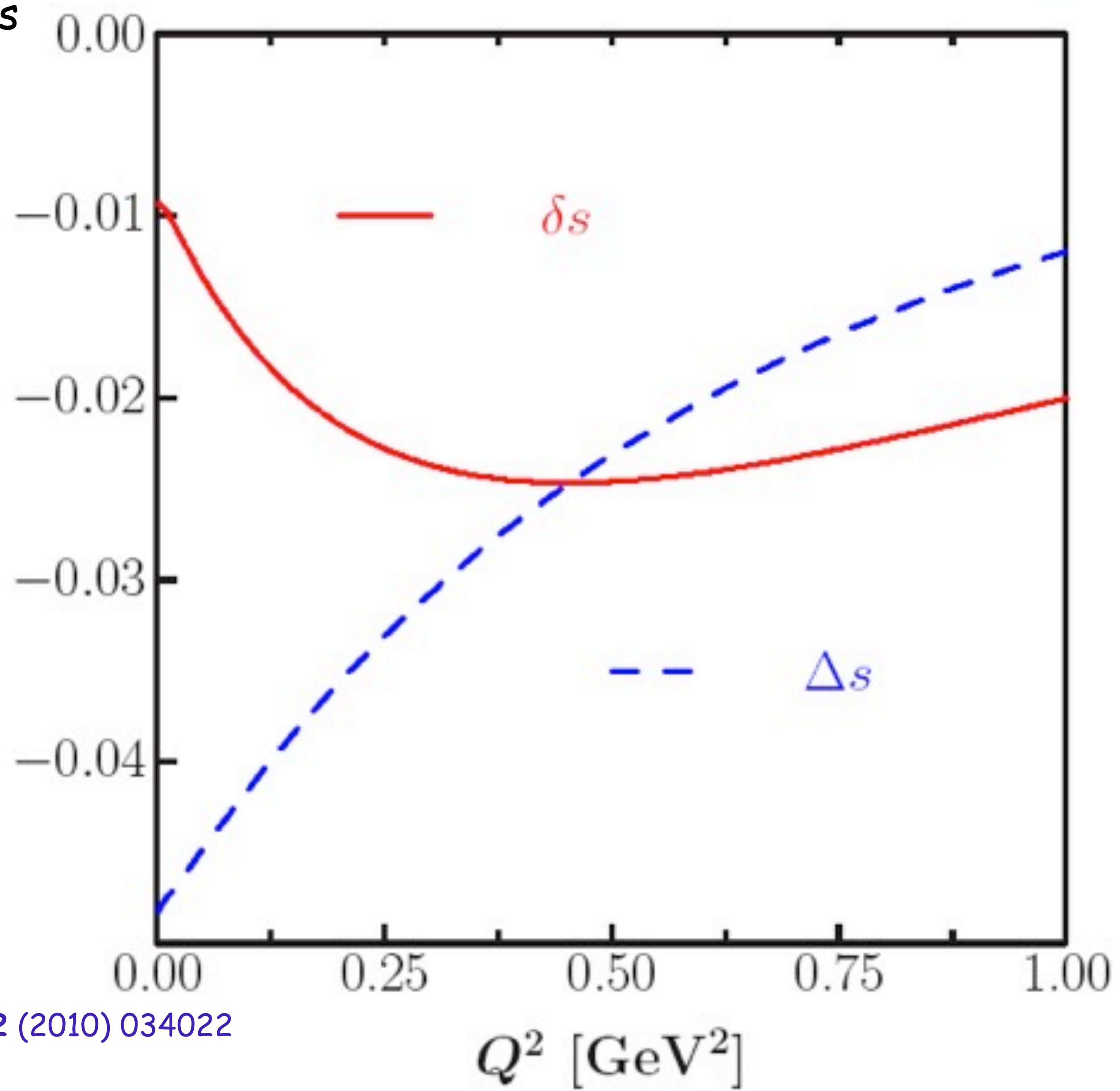
Up and down tensor form factors compared with the axial-vector ones



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Strange tensor form factors
compared with
the axial-vector ones

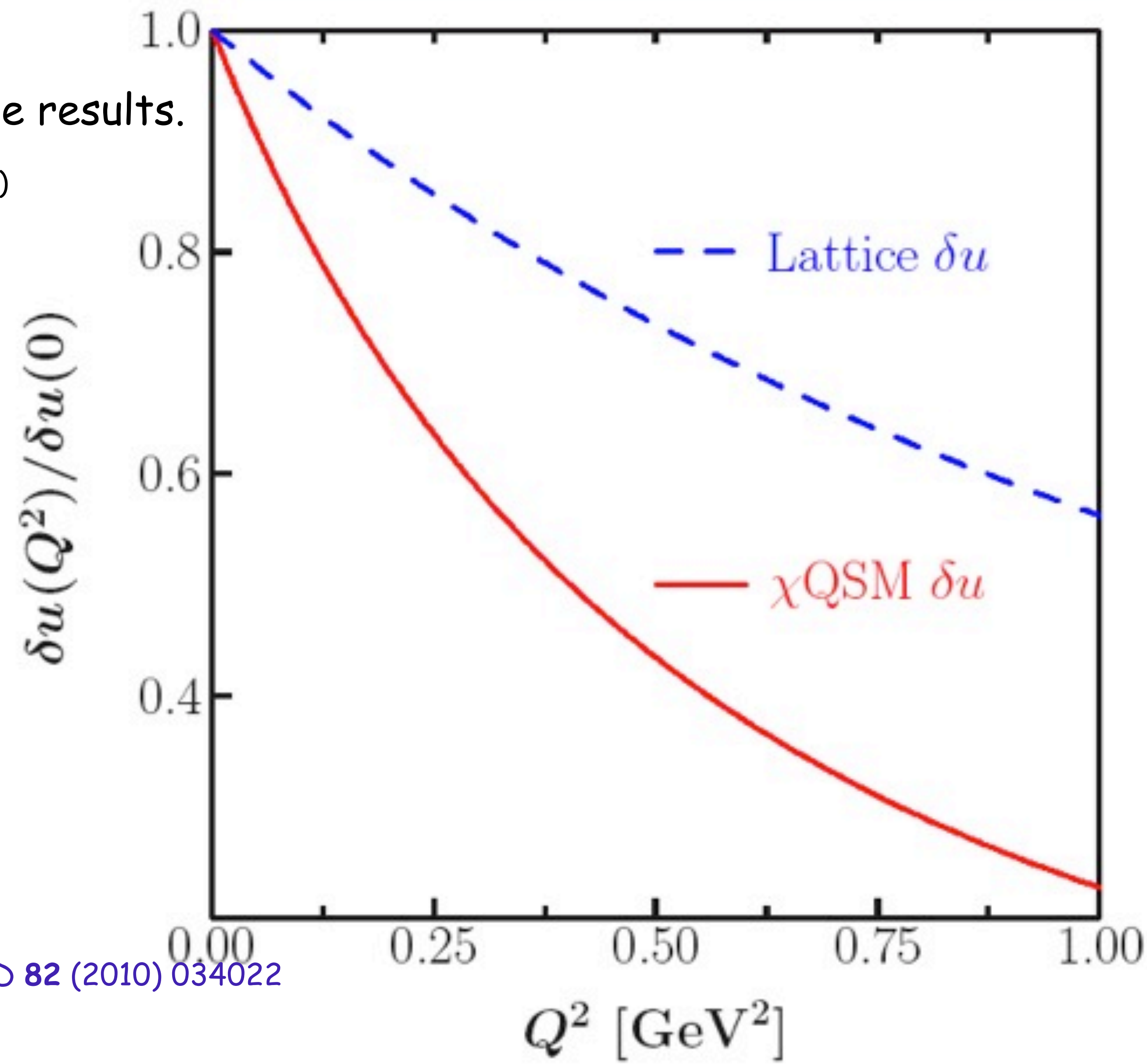


T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Comparison with the lattice results.

M. Goeckeler et al., PLB 627, 113 (2005)



T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

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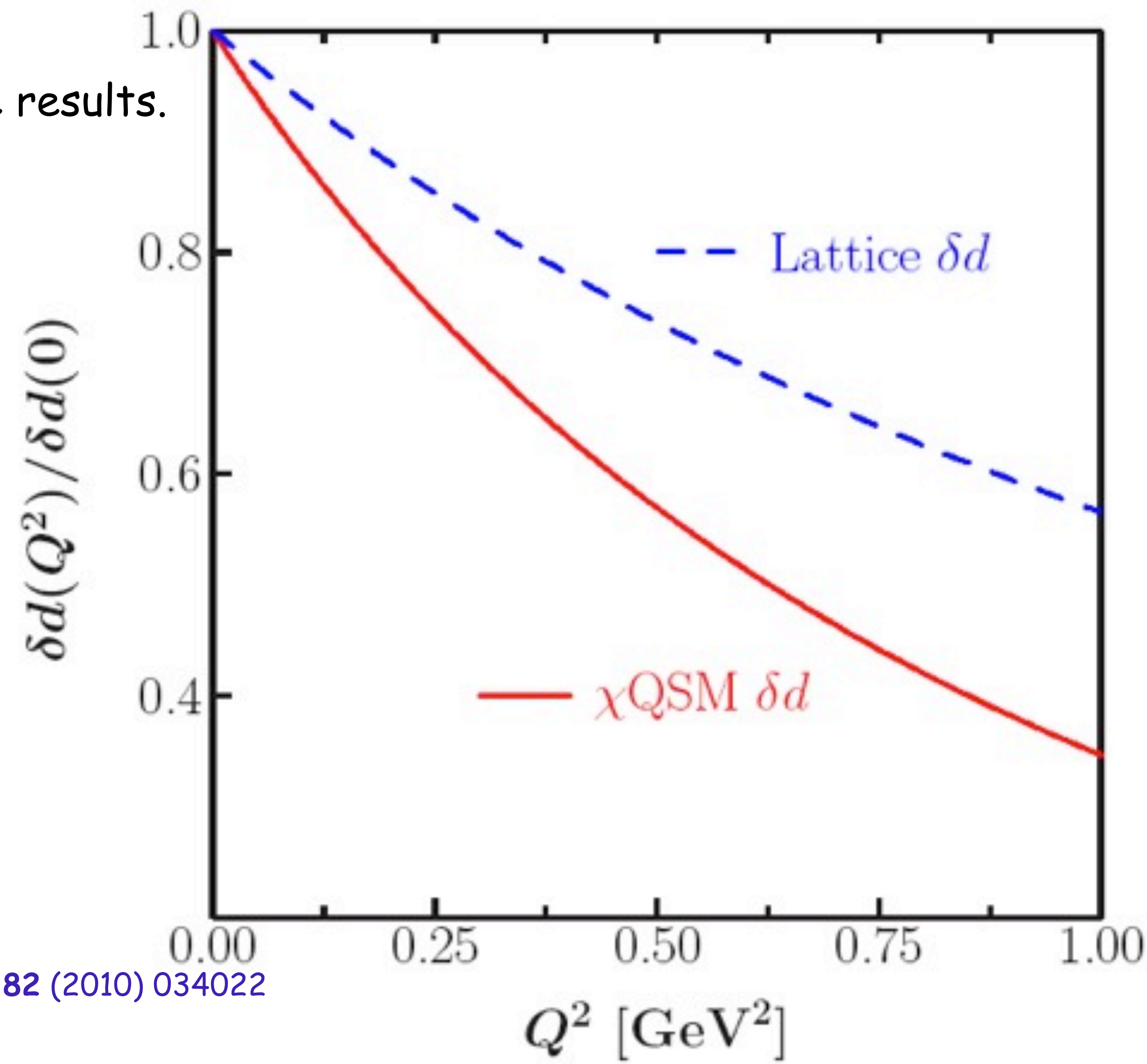

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Results

	$p(uud)$	$n(ddu)$	$\Lambda(uds)$	$\Sigma^+(uus)$	$\Sigma^0(uds)$	$\Sigma^-(dds)$	$\Xi^0(uss)$	$\Xi^-(dss)$
δu	1.08	−0.32	−0.03	1.08	0.53	−0.02	−0.32	−0.02
δd	−0.32	1.08	−0.03	−0.02	0.53	1.08	−0.02	−0.32
δs	−0.01	−0.01	0.79	−0.29	−0.29	−0.29	1.06	1.06

Isospin relations

$$\begin{aligned}
\delta u_p &= \delta d_n, & \delta u_n &= \delta d_p, & \delta u_\Lambda &= \delta d_\Lambda, & \delta u_{\Sigma^+} &= \delta d_{\Sigma^-},\\
\delta u_{\Sigma^0} &= \delta d_{\Sigma^0}, & \delta u_{\Sigma^-} &= \delta d_{\Sigma^+}, & \delta u_{\Xi^0} &= \delta d_{\Xi^-}, & \delta u_{\Xi^-} &= \delta d_{\Xi^0},\\
\delta s_p &= \delta s_n, & \delta s_{\Sigma^\pm} &= \delta s_{\Sigma^0}, & \delta s_{\Xi^0} &= \delta s_{\Xi^-},
\end{aligned}$$

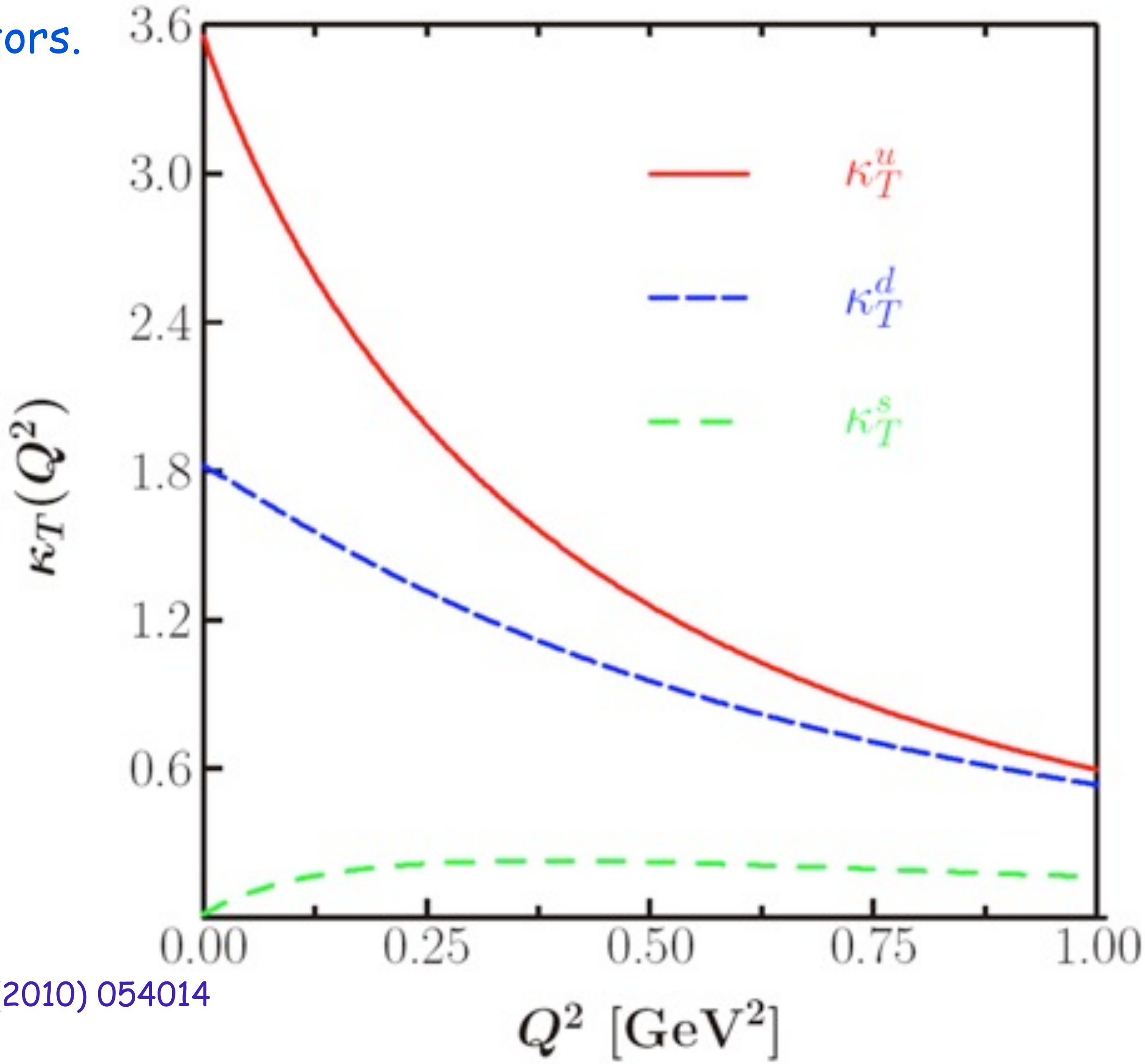
SU(3) relations → Effects of SU(3) symmetry breaking are almost negligible!

$$\begin{aligned}
\delta u_p &= \delta d_n = \delta u_{\Sigma^+} = \delta d_{\Sigma^-} = \delta s_{\Xi^0} = \delta s_{\Xi^-},\\
\delta u_n &= \delta d_p = \delta u_{\Xi^0} = \delta d_{\Xi^-} = \delta s_{\Sigma^\pm} = \delta s_{\Sigma^0}.
\end{aligned}$$

T. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 034022

Results

Flavor decomposition of the anomalous tensor magnetic form factors.

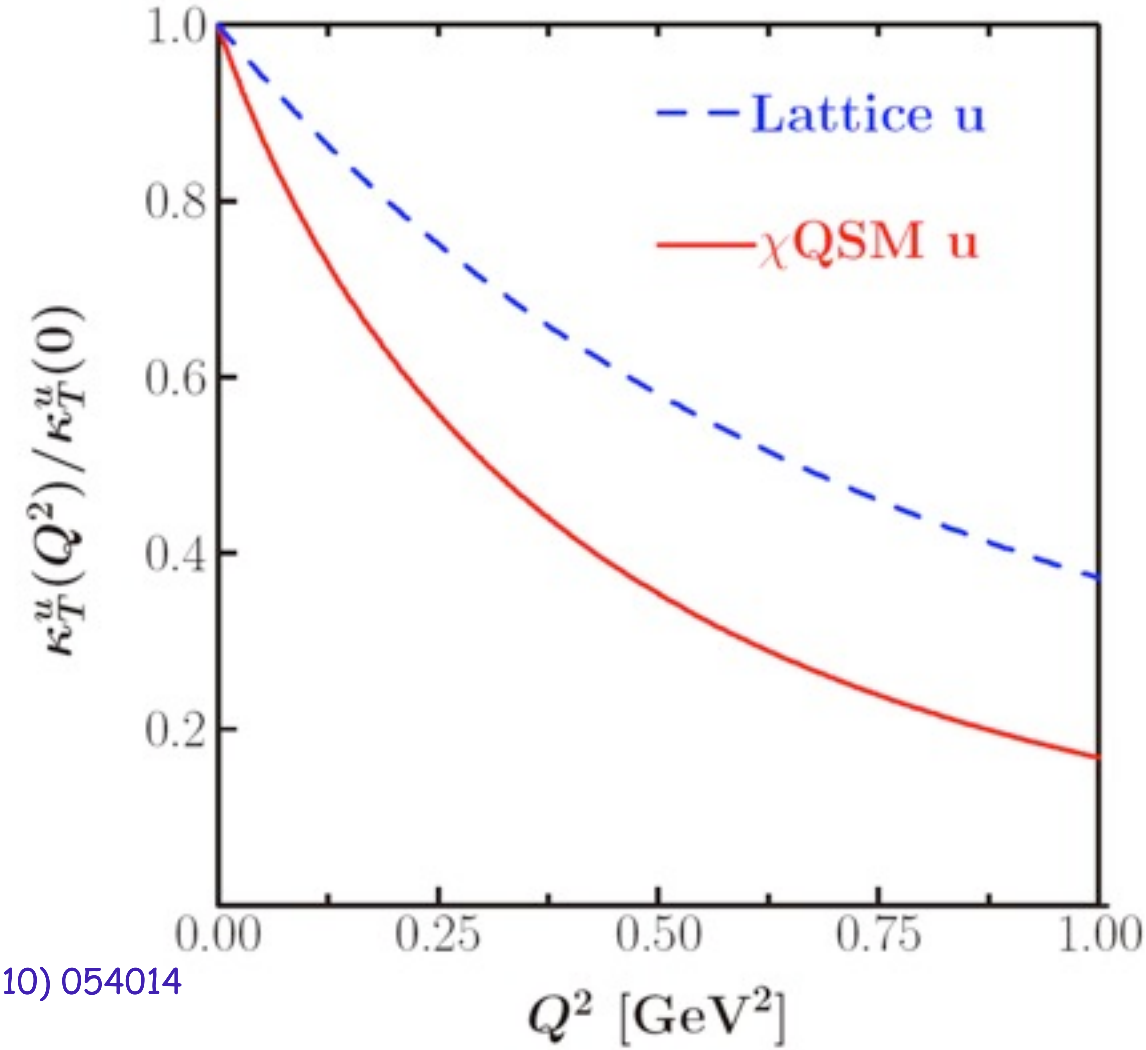


. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Up anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

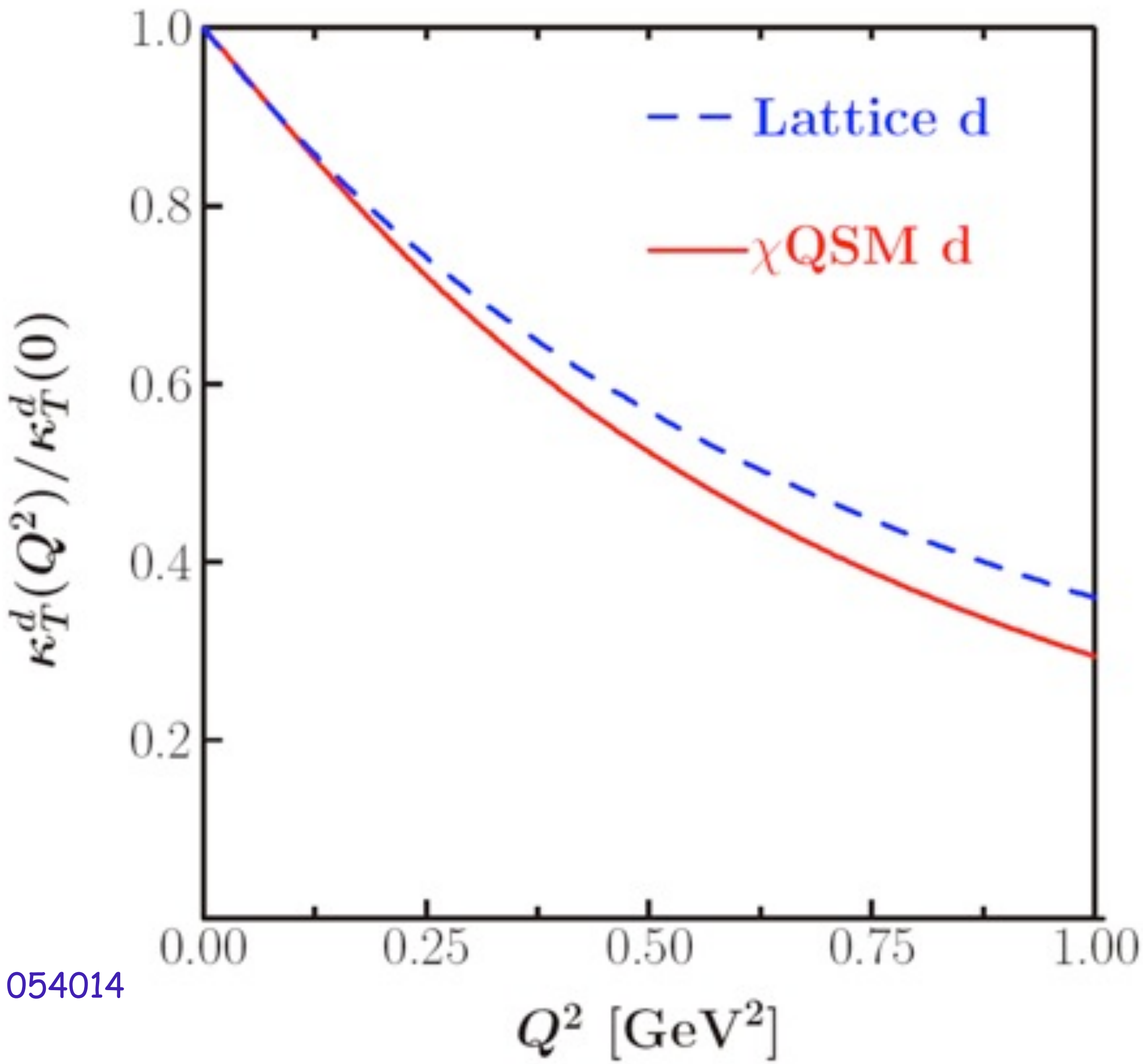


. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

Down anomalous tensor magnetic form factors compared with the lattice one.

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)



. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Results

$\mu^2 = 0.36\,\mathrm{GeV}^2$

	Present work SU(3)	Present work SU(2)	Lattice
κ_T^u	3.56	3.72	3.00 (3.70)
κ_T^d	1.83	1.83	1.90 (2.35)
κ_T^s	$0.2 \sim -0.2$		
κ_T^u/κ_T^d	1.95	2.02	1.58

The present results are comparable with the lattice data!

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

. Ledwig, A. Silva, HChK, Phys. Rev. D **82** (2010) 054014

Transverse spin density

$$\rho(\mathbf{b}, \mathbf{S}, \mathbf{s}) = \boxed{\frac{1}{2}\left[H(b^2) - S^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial E(b^2)}{\partial b^2} - s^i \epsilon^{ij} b^j \frac{1}{M_N} \frac{\partial \kappa_T(b^2)}{\partial b^2} \right]}$$

$$[\mathbf{S}, \mathbf{s}] = [(1,0),(0,0)], \ [\mathbf{S}, \mathbf{s}] = [(0,0),(1,0)]$$

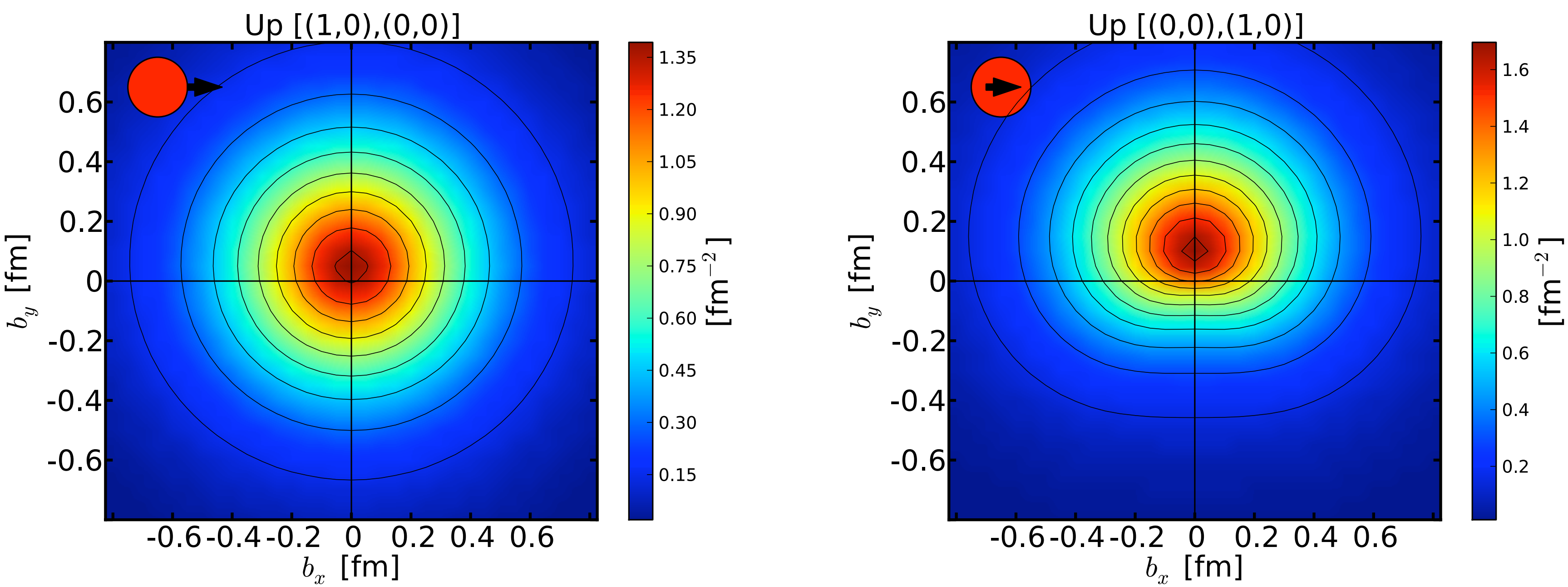
$$\mathcal{F}^{\chi}(b^2) = \int_0^{\infty} \frac{dQ}{2\pi} Q J_0(bQ) F^{\chi}(Q^2)$$

$$H(b^2) = F_1(b^2), \quad E(b^2) = F_2(b^2)$$

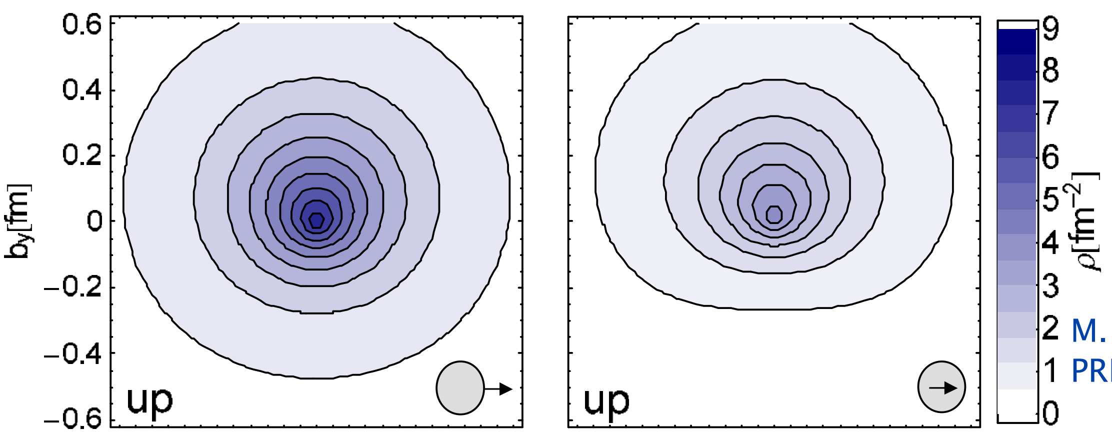
M. Diehl & Ph. Haegler, Euro.Phys. J., C 44 (2005) 87.

Results

Up quark transverse spin density inside a nucleon



T. Ledwig & H−Ch.K, PRD 85, 034041 (2012)

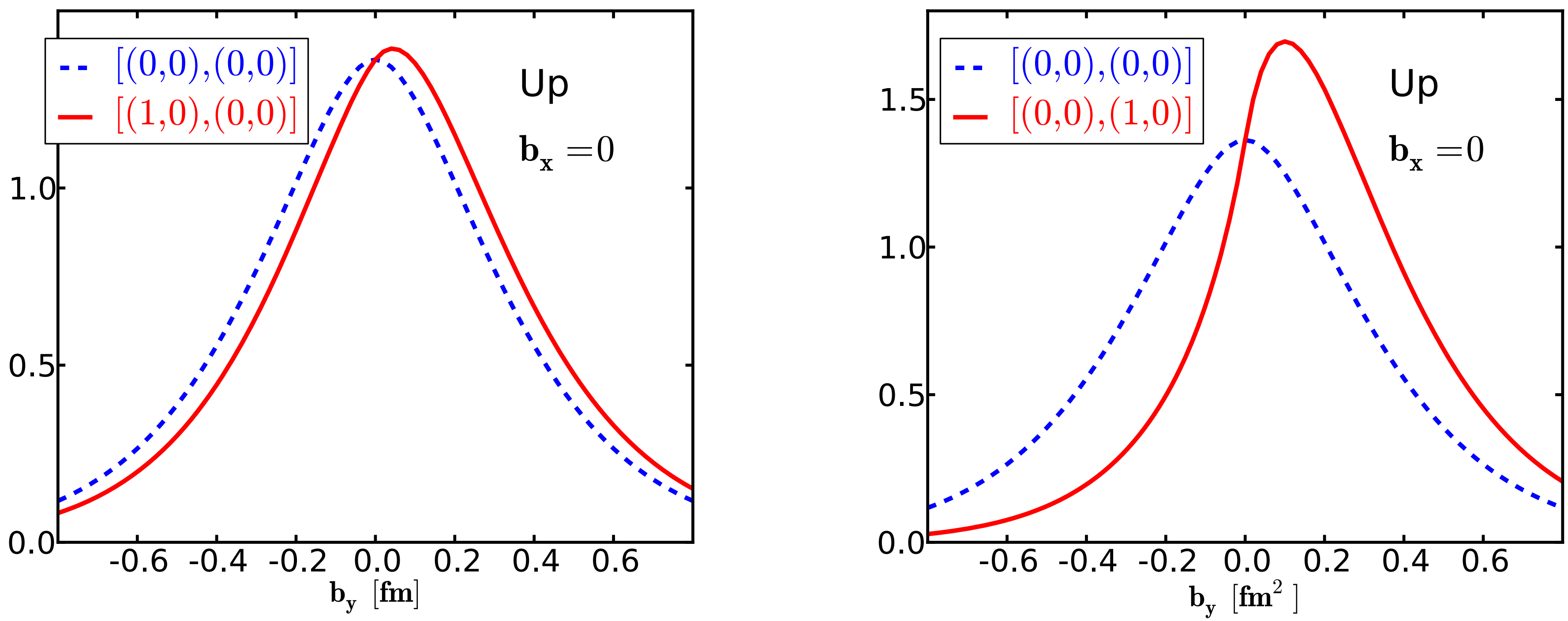


Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.] PRL 98, 222001 (2007)

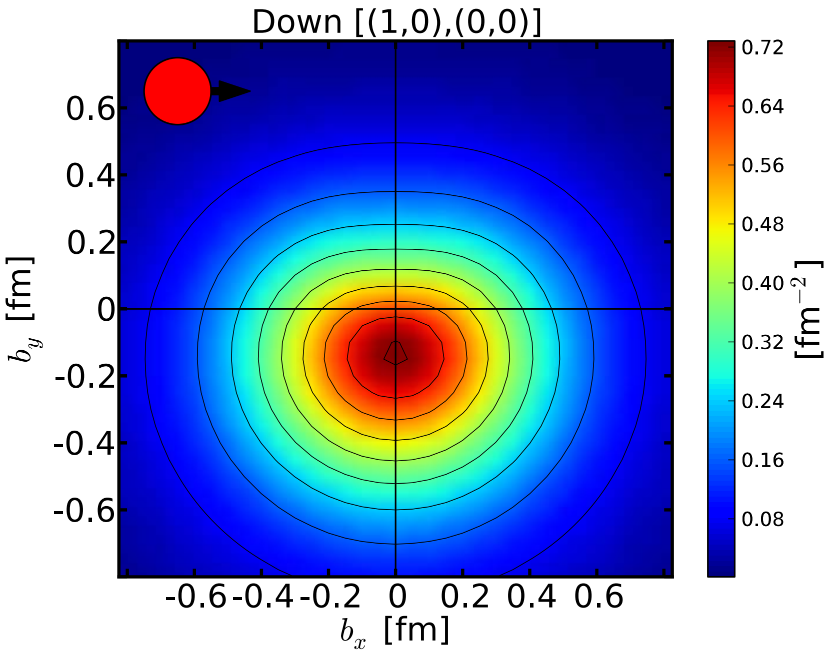
Results

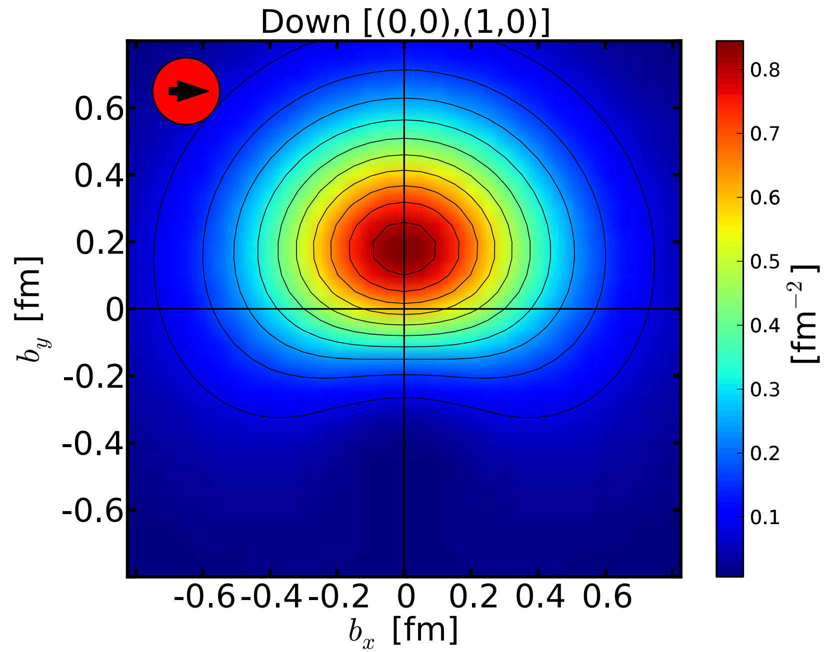
Up quark transverse spin density inside a nucleon



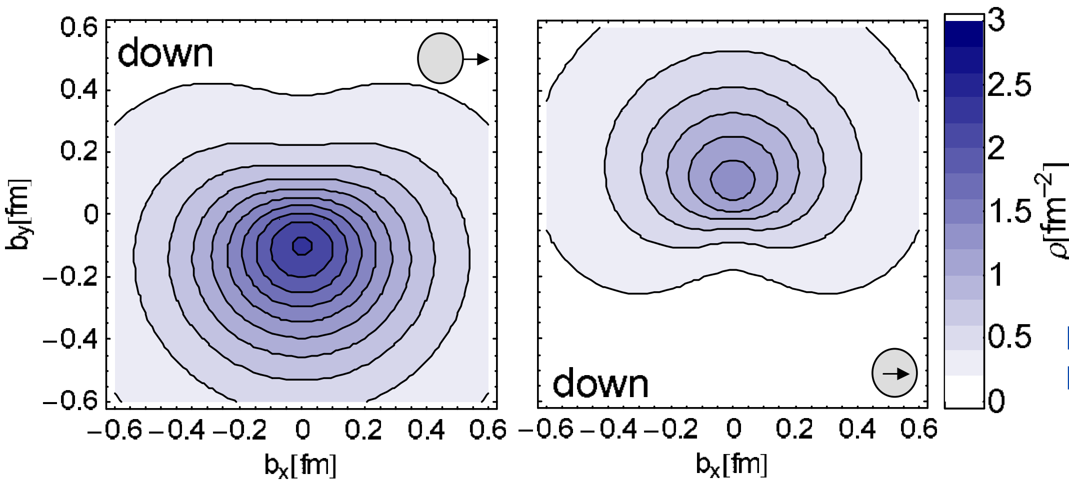
Results

Down quark transverse spin density inside a nucleon





T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

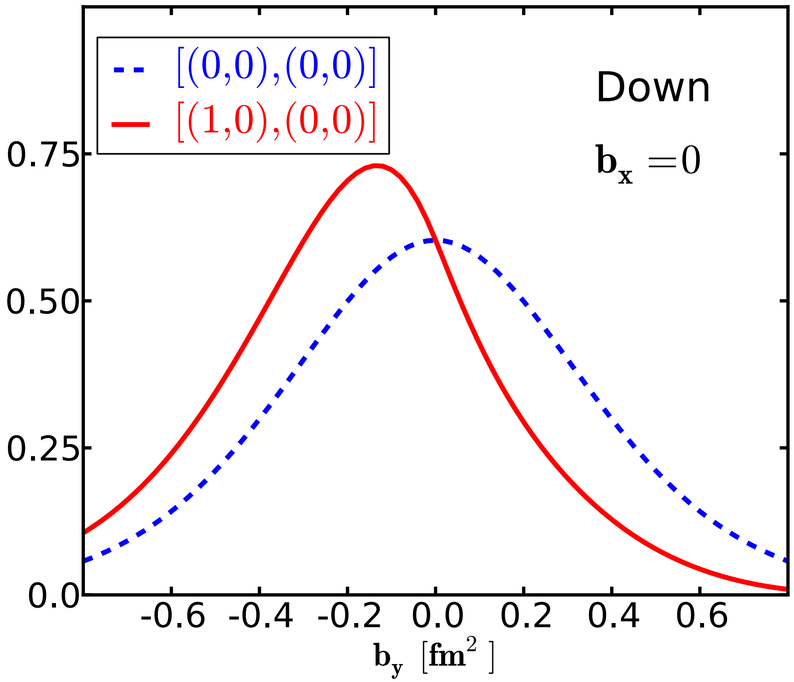


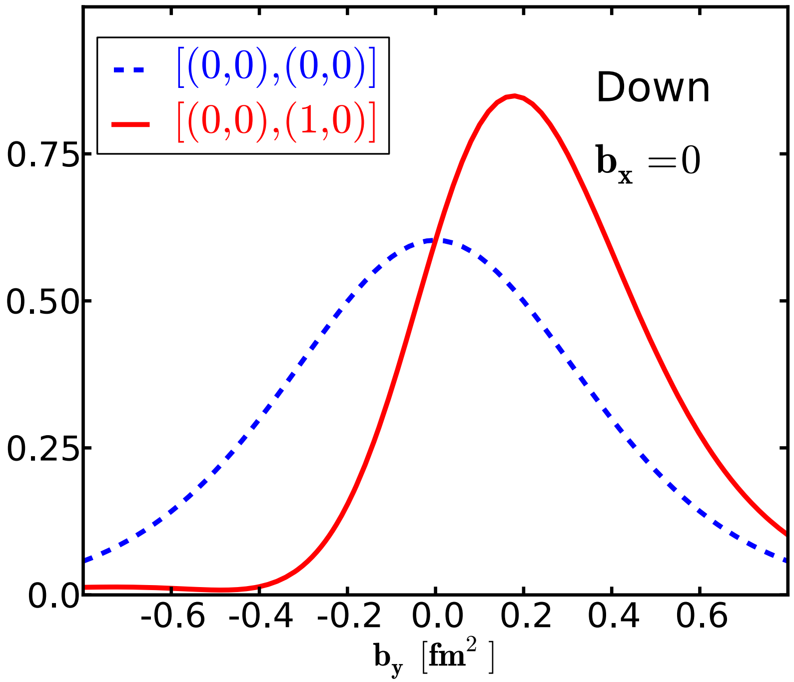
Lattice results

M. Goeckeler et al. [QCDSF Coll. and UKQCD Coll.]
PRL 98, 222001 (2007)

Results

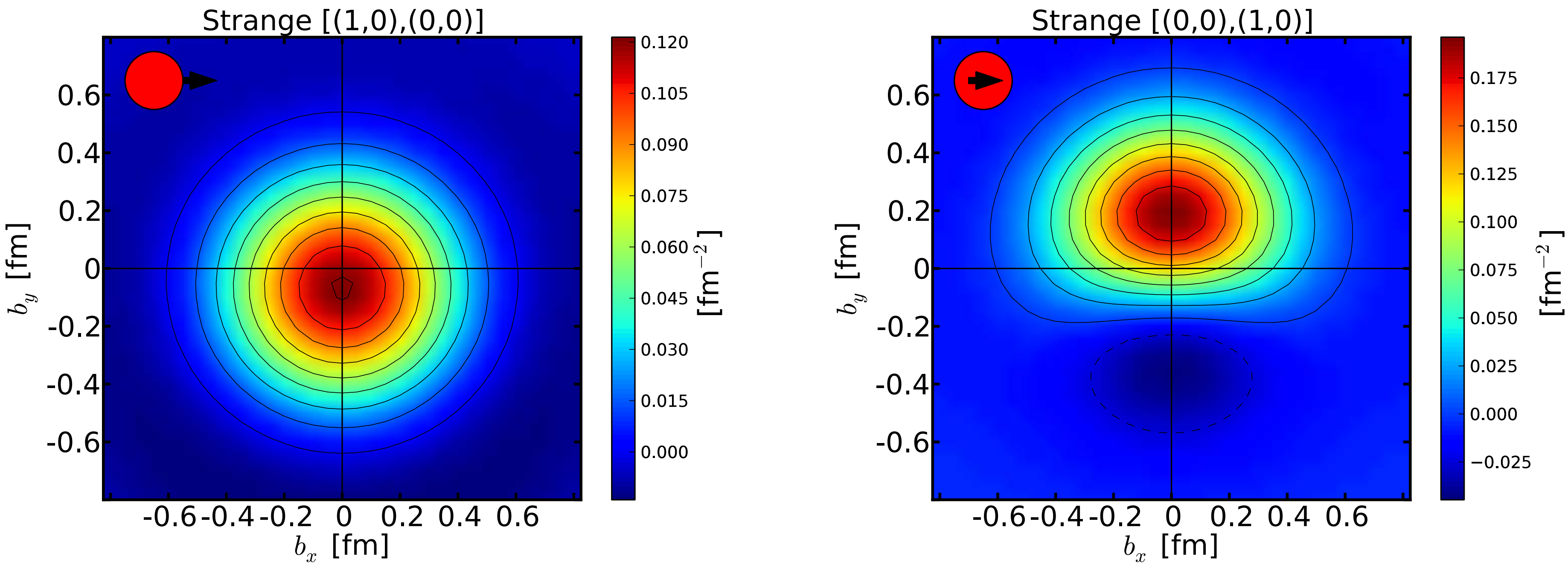
Down quark transverse spin density inside a nucleon





Results

Strange quark transverse spin density inside a nucleon

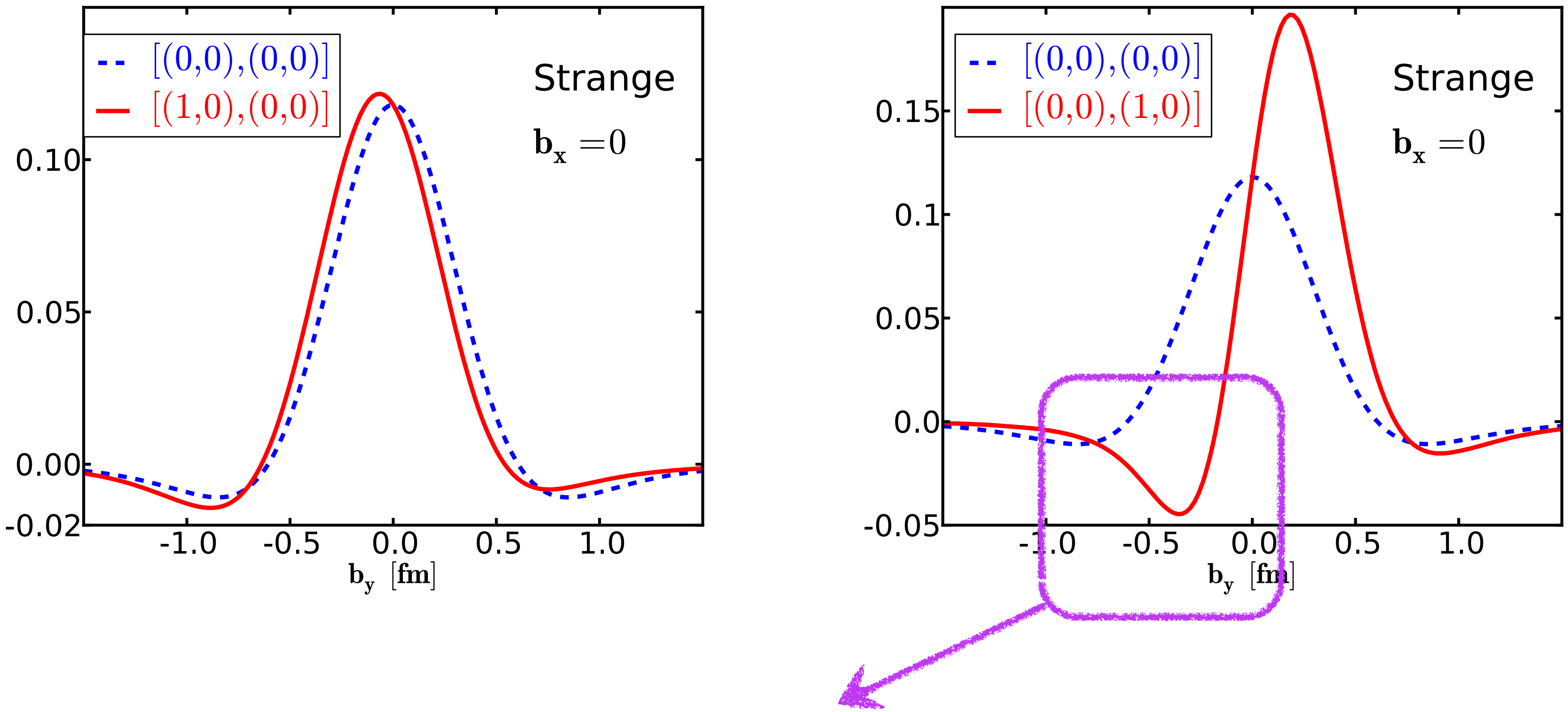


T. Ledwig & H-Ch.K, PRD 85, 034041 (2012)

This is the **first** result of the strange quark transverse spin density inside a nucleon

Results

Strange quark transverse spin density inside a nucleon



Polarized to the negative direction in the b plane.

Summary

- We have reviewed recent investigations on the spin structures of the nucleon, based on the chiral quark-soliton model.

- We have derived the tensor form factors of the nucleon, based on which we have obtained their transverse spin densities. The results are compared with the lattice and "experimental" data.

- The first strange anomalous tensor magnetic moment was obtained, though it is compatible with zero.

- The strange quark transverse spin density was first announced in this work.

Outlook

- The transverse spin structure when both the nucleon and the quarks inside it are polarized: additional form factors are required.
- The FF & GPDs in SU(3): Not much known. Strangeness in the Nucleon at the level of the GPDs should be studied.
- The FF & GPDS for excited hadrons: Hadron tomography in general.

Though this be madness, yet there is method in it.

Hamlet Act 2, Scene 2

Thank you very much!

Baryonic correlation functions

Baryonic observables

$$\lim_{\substack{y_0 \to -\infty \\ x_0 \to +\infty}} \langle 0 | J_N(x) (-i) s^\dagger \gamma_\mu s J_N^\dagger(y) | 0 \rangle \;=\; \lim_{\substack{y_0 \to -\infty \\ x_0 \to +\infty}} \mathcal{K}$$

$$\begin{aligned} \mathcal{K} &= \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU \\ &\times \; J_N(x) \left(-i s^\dagger\right) \gamma_\mu s J_N^\dagger(y) \\ &\times \; \exp\left[\int d^4x \psi^\dagger \left(i\partial\!\!\!/ + iMU^{\gamma^5} + i\hat{m} \right) \psi \right] \end{aligned}$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryonic correlation functions

Baryonic observables

$$
\begin{aligned}
& \langle 0|J_N(x)\bar{\Psi}\hat{\Gamma}\Lambda\Psi J_N^\dagger(y)|0\rangle \\
= \quad & \Gamma^{\alpha_1\alpha_2\cdots\alpha_{N_c}}_{(\frac{1}{2}T_3Y)(\frac{1}{2}J_3Y_R)}\Gamma^{\beta_1\beta_2\cdots\beta_{N_c}*}_{(\frac{1}{2}T_3Y)(\frac{1}{2}J_3Y_R)} \\
\times \quad & \frac{\delta}{\delta\eta^\dagger_{\alpha_1}(\boldsymbol{x},x_0)}\frac{\delta}{\delta\eta^\dagger_{\alpha_2}(\boldsymbol{x},x_0)}\cdots\frac{\delta}{\delta\eta^\dagger_{\alpha_{N_c}}(\boldsymbol{x},x_0)}\frac{\delta}{\delta s_\mu(0)} \\
\times \quad & \mathcal{W}[\eta^\dagger,\eta,s_\mu]\Big|_{\eta^\dagger,\eta,s_\mu=0} \\
\times \quad & \frac{\overleftarrow{\delta}}{\delta\eta_{\beta_1}(\boldsymbol{y},y_0)}\frac{\overleftarrow{\delta}}{\delta\eta_{\beta_2}(\boldsymbol{y},y_0)}\cdots\frac{\overleftarrow{\delta}}{\delta\eta_{\beta_{N_c}}(\boldsymbol{y},y_0)}.
\end{aligned}
$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)

Baryonic correlation functions

Baryonic observables

$$\begin{aligned}
\mathcal{W}[\eta^\dagger, \eta, s_\mu] &= \frac{1}{\mathcal{Z}} \int D\psi D\psi^\dagger DU \\
&\times \exp\left[\int d^4x \left(\psi^\dagger D\psi + i\eta^\dagger\psi + i\psi^\dagger\eta\right.\right. \\
&+ \left.\left. \psi^\dagger i s_\mu \hat{\Gamma}\hat{\Lambda}\psi\right)\right].
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{\mathcal{Z}} \int DU \det\left[D + i s_\mu \hat{\Gamma}\hat{\Lambda}\right] \\
&\times \exp\left[-\int d^4x d^4y \right. \\
&\times \left. \eta^\dagger_\alpha(x)_\alpha \left\langle x \left| \frac{1}{D + i s_\mu \hat{\Gamma}\hat{\Lambda}} \right| y \right\rangle_\beta \eta_\beta(y)\right]
\end{aligned}$$

HChK et al. Prog. Part. Nucl. Phys. Vol.95, (1995)