K ビーム反応計算

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To-do list (ver. Feb. 2024)

■ $K^- d \to \pi\Lambda n, \pi\Sigma n$ reaction calculation. <–> J-PARC E31 Exp.

☐ The nature of $\Lambda(1405)$ in contrast to $\Sigma(1385)$.

☐ Background from πN and YN final-state interactions.

■ $\bar{K}NN$ structure calculation including two-nucleon absorption: $\bar{K}NN \to YN \to \bar{K}NN$.

☐ Shifts of the binding energy and decay width of the $\bar{K}NN$ bound state.

☐ Size of the $\bar{K}NN$ bound state.

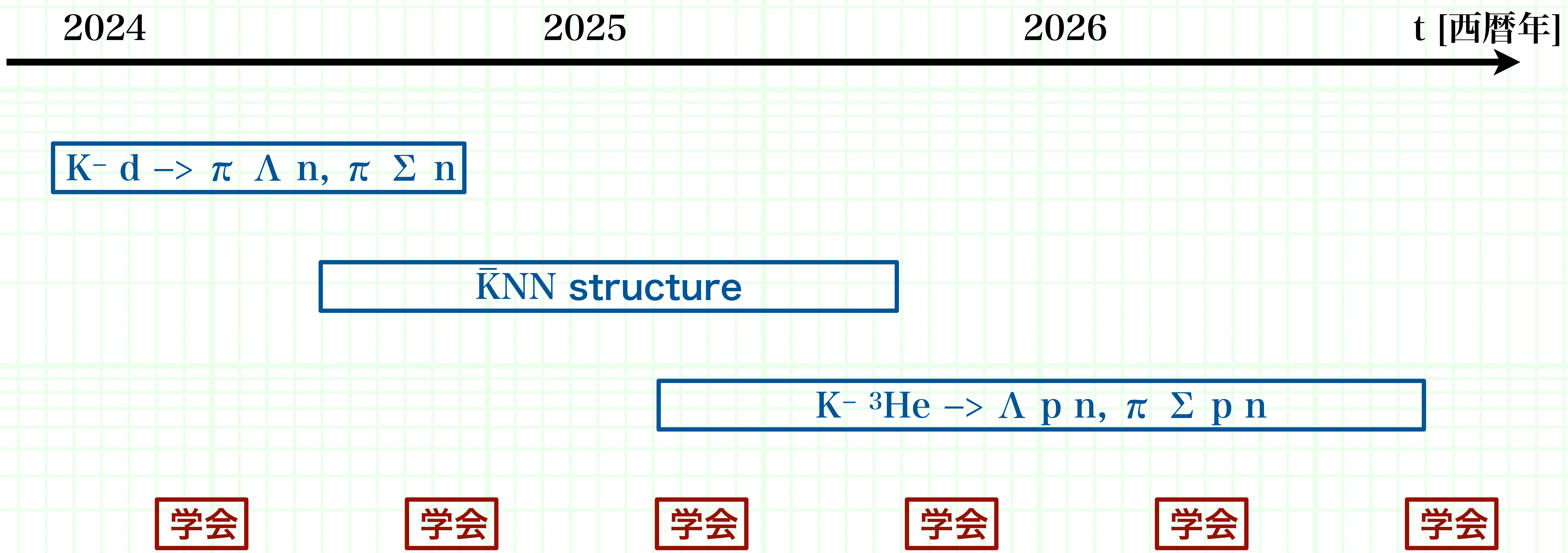
■ $K^- {}^3He \to \Lambda p n, \pi\Sigma p n$ reaction calculation. <–> J-PARC E15 Exp.

☐ How the two-nucleon absorption affects the $\bar{K}NN$ spectrum.

☐ The $\bar{K}NN$ ($I_3 = -1/2$), also known as $\bar{K}^0nn$, bound state.

☐ The nature of $\Lambda(1405)$?

Schedule (ver. Feb. 2024)



The $\bar{K}N$ system in $K^- d \to \pi \Sigma n$

++ Our calculation ++

■ We calculate the cross section of the $K^- d \to \pi \Sigma n$ reaction.

J. Yamagata-Sekihara, T. S. , and D. Jido, under discussion.

□ Angular (= momentum transfer q_{trans}) dependence ?

$\mathbf{q}_{\rm trans} = \mathbf{p}_K - \mathbf{p}_n$ at Lab. frame

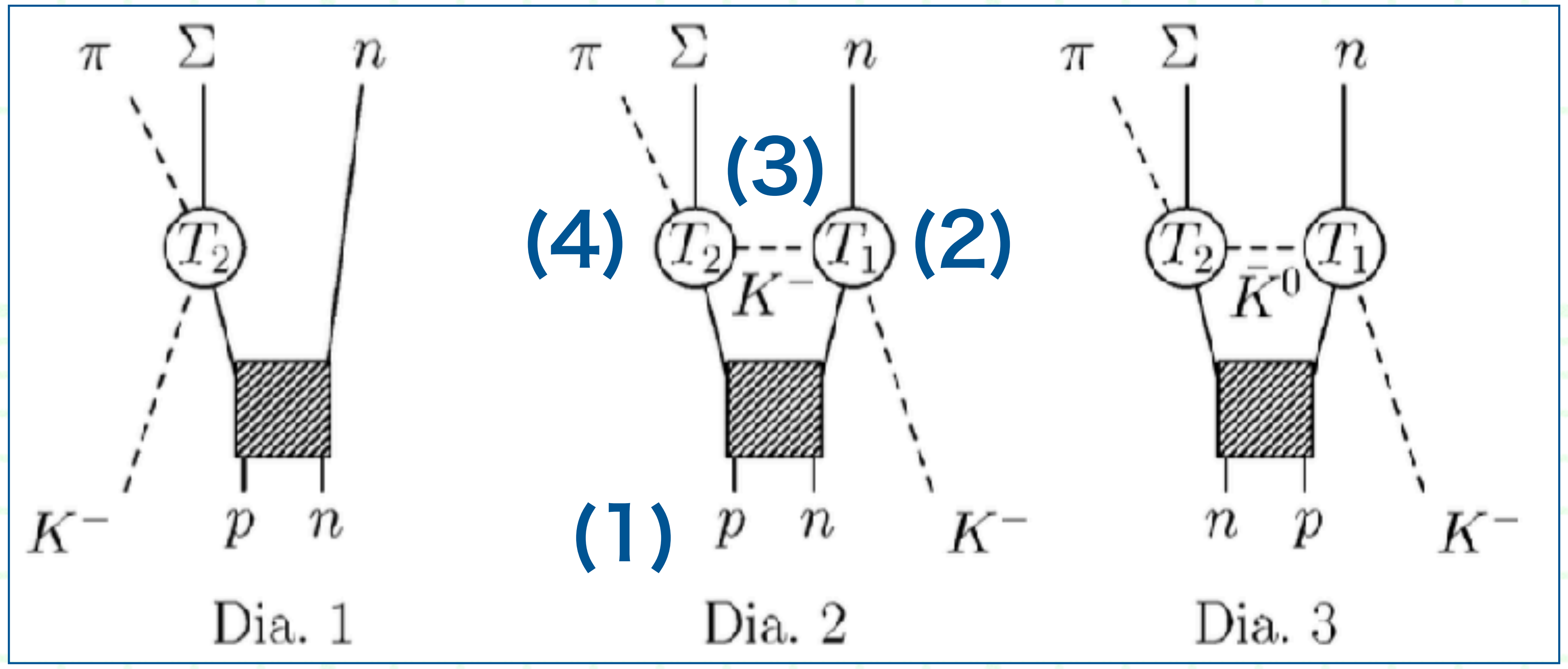
□ Contribution from each component of the reaction diagram ?

(1) Deuteron wave function.

(2) 1st step T_1 ($\bar{K}N \to \bar{K}N$, P_K = 1 GeV/c).

(3) $\bar{K}$ propagator.

(4) 2nd step T_2 ($\bar{K}N \to \pi\Sigma$).



–> We aim to construct a precise model to determine the pole position of the $\Lambda(1405)$ in the complex energy plane.

The $\bar{K}N$ system in $K^- d \to \pi \Sigma n$

++ Deuteron wave function & $\bar{K}$ propagator ++

■ Deuteron wave function.

□ Taken from the CD-Bonn potential (s wave only, ~ 95 %): Small uncertainty.

Machleidt, Phys. Rev. C63 (2011) 024001.

■ Together with the $\bar{K}$ propagator, the scattering Amp. becomes:

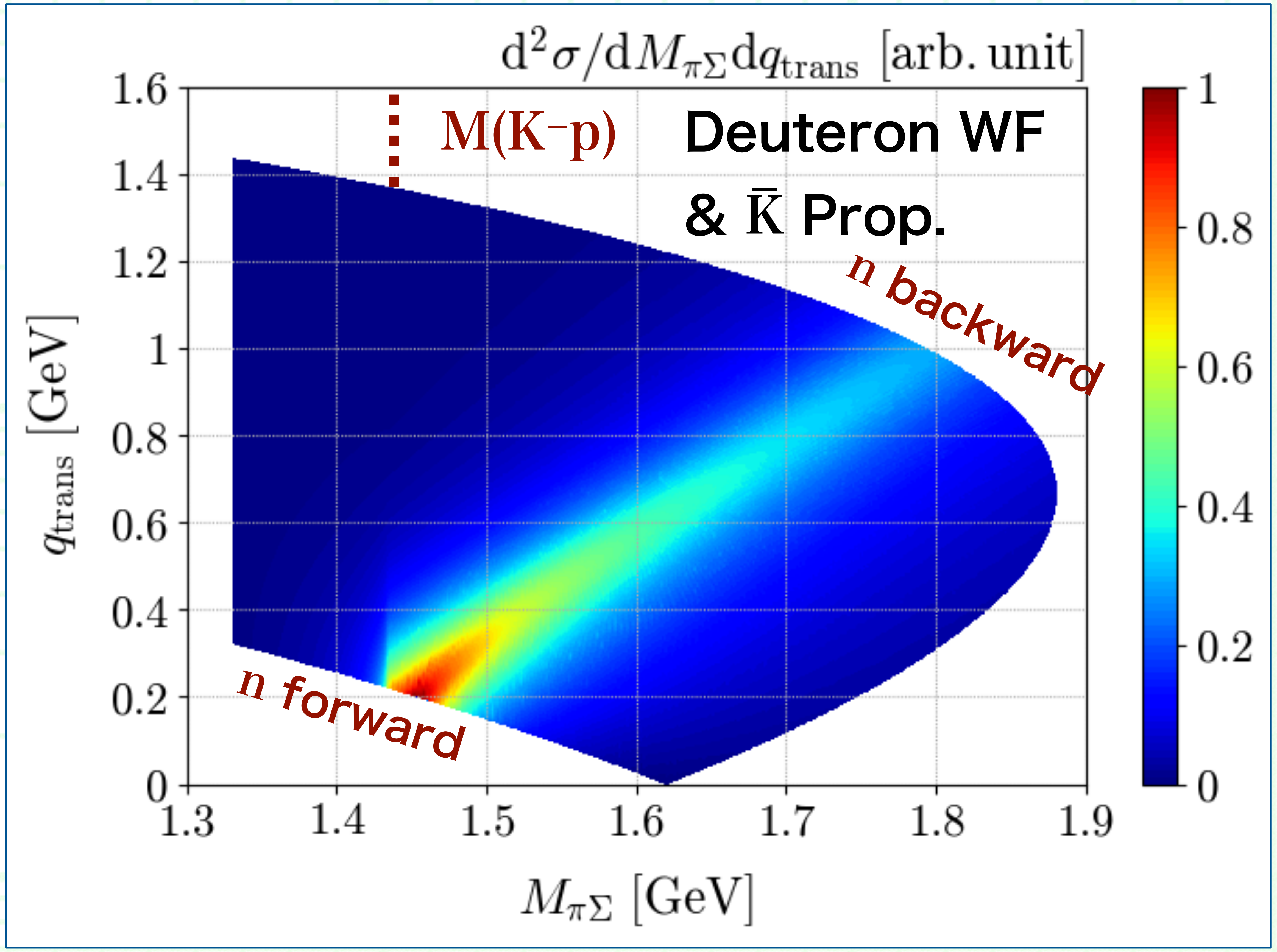
$$\text{Amp.} = \int \frac{d^3 q_{\rm ex}}{(2\pi)^3} \frac{\varphi_{\rm deut}(|\mathbf{q}_{\rm ex} - \mathbf{q}_{\rm trans}|)}{q_{\rm ex}{}^2 - m_K{}^2 + i0}$$

$$\mathbf{q}_{\rm trans} = \mathbf{p}_K - \mathbf{p}_n$$

□ Band width owing to the deuteron WF.

— Off-shell N inside the deuteron.

□ On this band, we may treat the propagating $\bar{K}$ as (almost) on-shell particle.



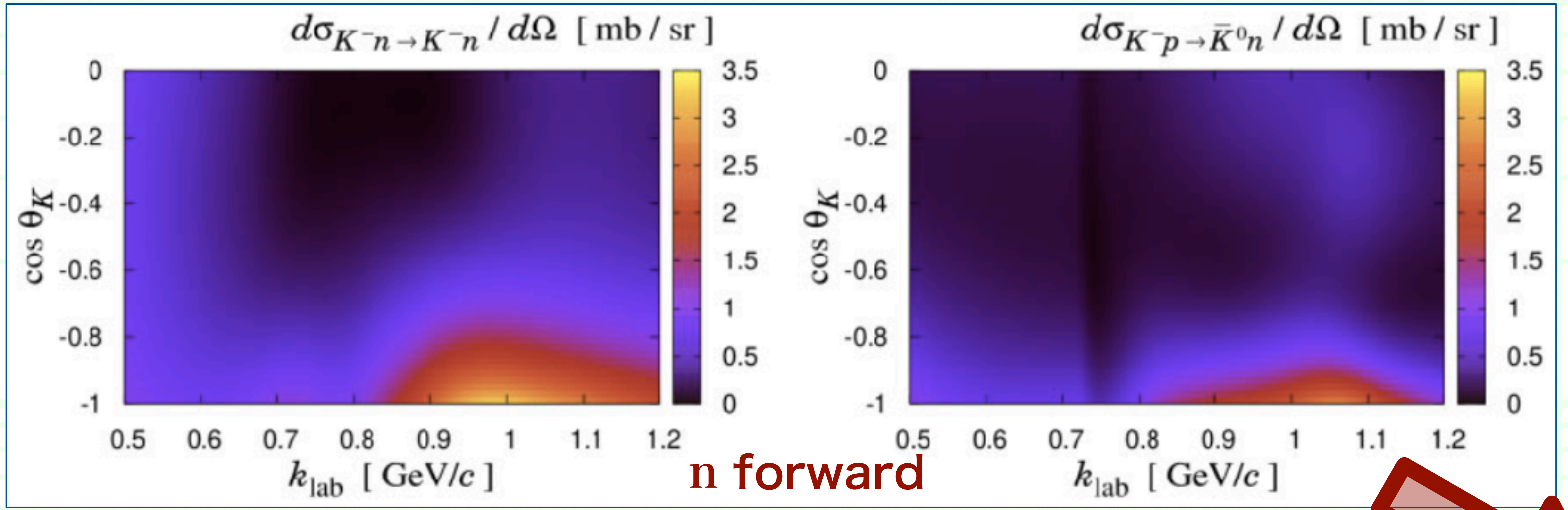
The $\bar{K}N$ system in $K^- d \to \pi \Sigma n$

++ & the 1st step $\bar{K}N \to \bar{K}N$ ++

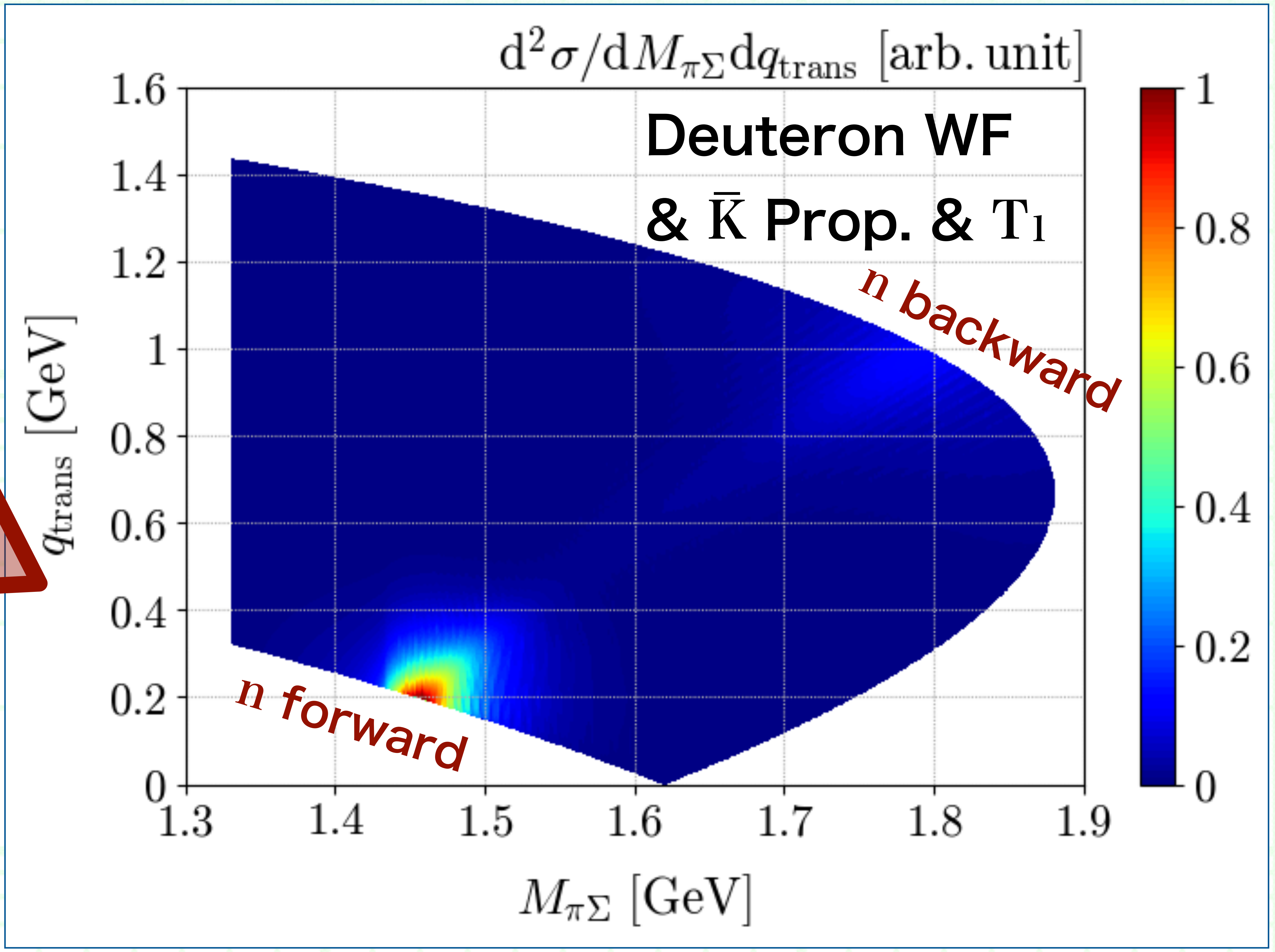
■ Inclusion of the 1st step T_1 ($\bar{K}N \to \bar{K}N$, P_K = 1 GeV/c).

–> Employ the Kamano et al. on-shell amplitude.

Kamano, Nakamura, Lee, and Sato, Phys. Rev. C90 (2014) 065204.



$$\text{Amp.} = \int \frac{d^3 q_{ex}}{(2\pi)^3} \frac{\varphi_{deut}(|\mathbf{q}_{ex} - \mathbf{q}_{trans}|)}{q_{ex}{}^2 - m_K{}^2 + i0} T_1^{(\text{Kamano})}$$



□ Dominated by the small momentum transfer region $q_{trans} < 0.4$ GeV.

The $\bar{K}N$ system in $K^- d \to \pi \Sigma n$

++ & the 2nd step $\bar{K}N \to \pi \Sigma$ ++

■ Inclusion of the 2nd step T_2 ($\bar{K}N \to \pi\Sigma$).

–> Employ the Ikeda-Hyodo-Weise amplitude, which contains the $\Lambda(1405)$.

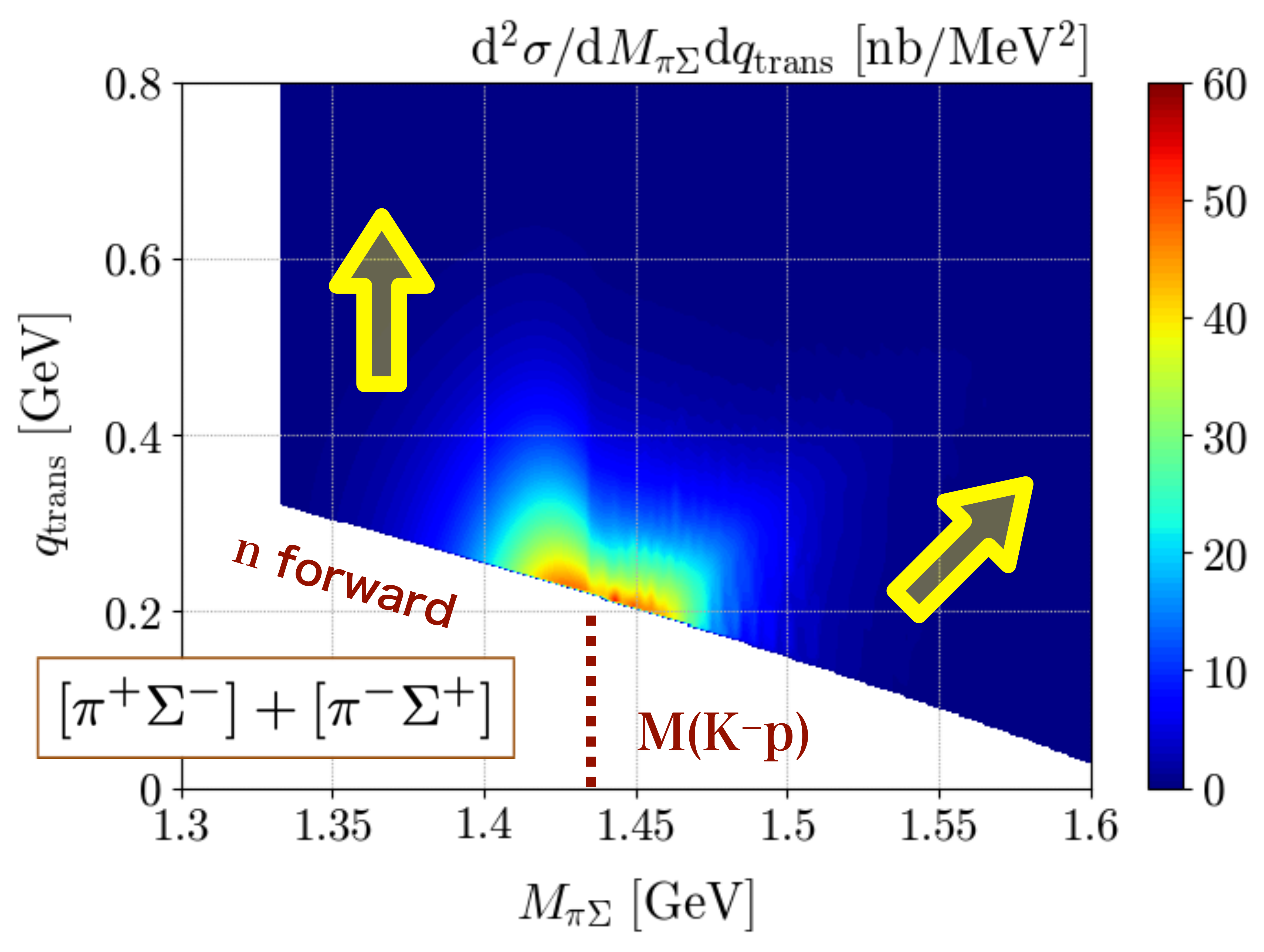
Ikeda, Hyodo, and Weise, Nucl. Phys. A881 (2012) 98.

$$\text{Amp.} = \int \frac{d^3 q_{\rm ex}}{(2\pi)^3} \frac{\varphi_{\rm deut}(|\mathbf{q}_{\rm ex} - \mathbf{q}_{\rm trans}|)}{{q_{\rm ex}}^2 - {m_K}^2 + i0} T_1^{(\rm Kamano)} T_2^{(\rm IHW)}$$

–> Full calculation.

☐ We have two trends.

- Below the $\bar{K}N$ threshold: The $\Lambda(1405)$ signal.
- Above the $\bar{K}N$ threshold: The quasi-free $\bar{K}$ propagation.

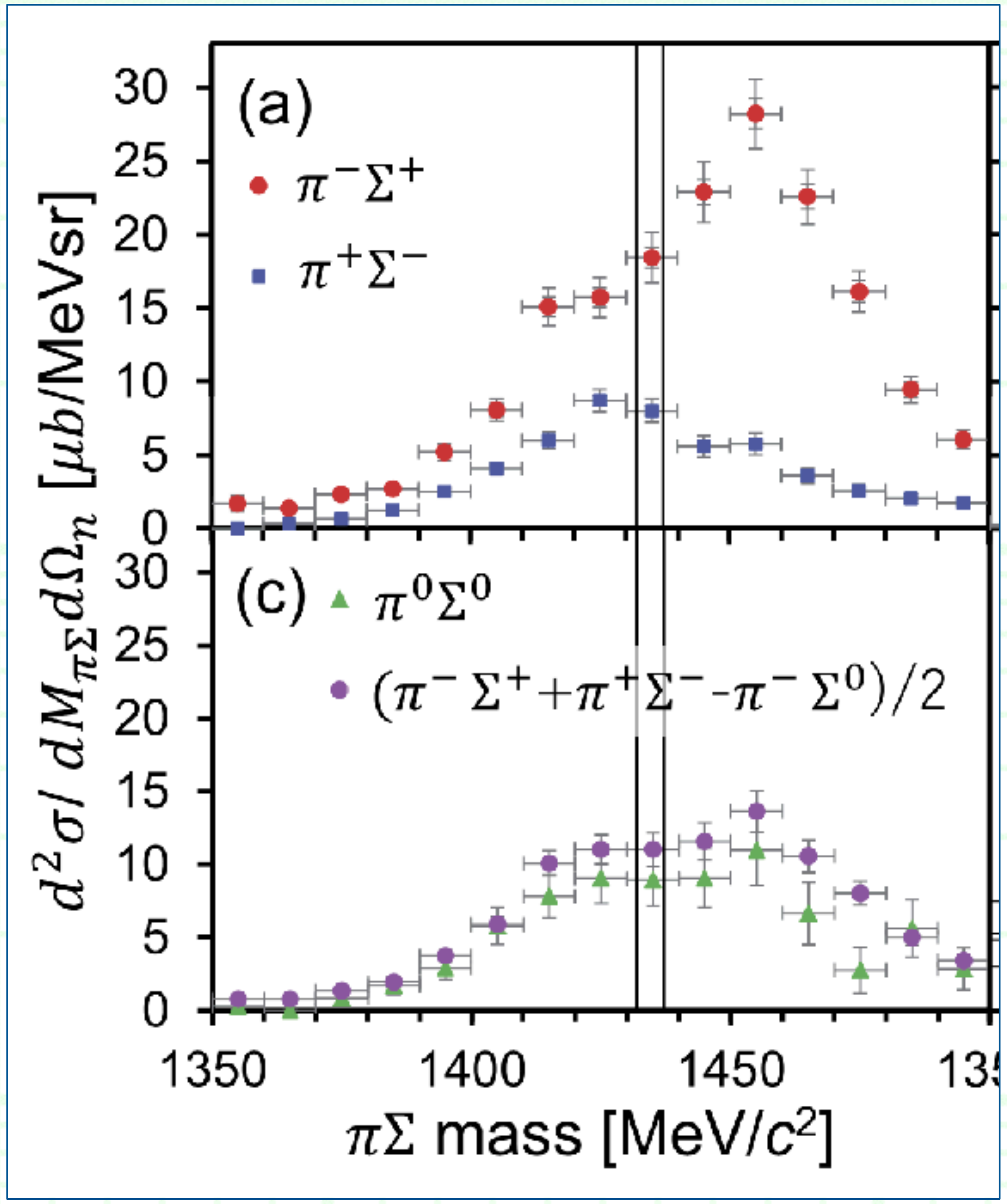


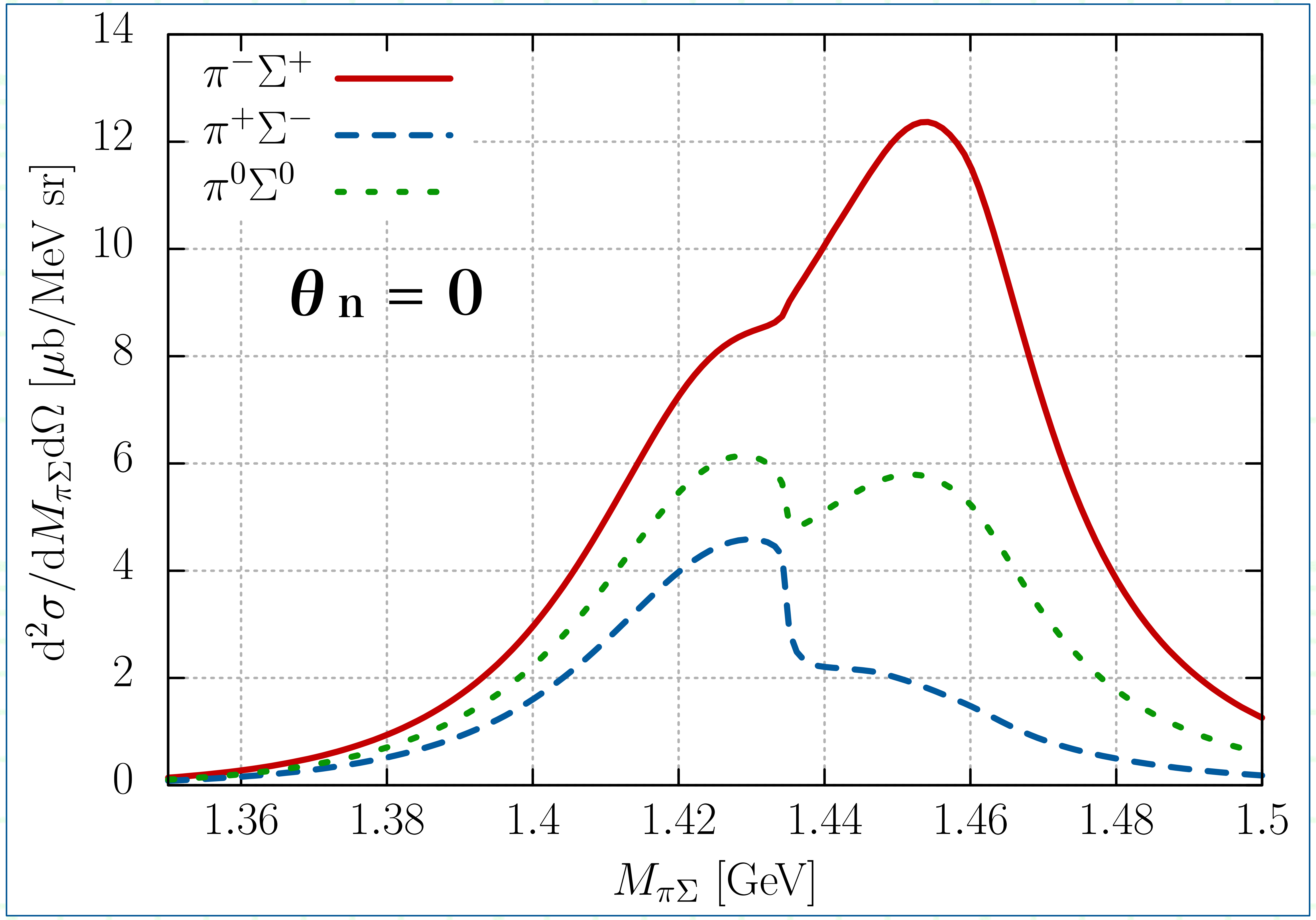
The $\bar{K}N$ system in K^- d –> $\pi\,\Sigma\,n$

++ Spectrum at the forward n ++

■ We can compare the $\pi\,\Sigma$ spectrum at the forward n condition with the Exp. data.

J-PARC E31 Collab., Phys. Lett. B837 (2023) 137637.





□ Quite similar shapes, although the peak heights are quantitatively different.

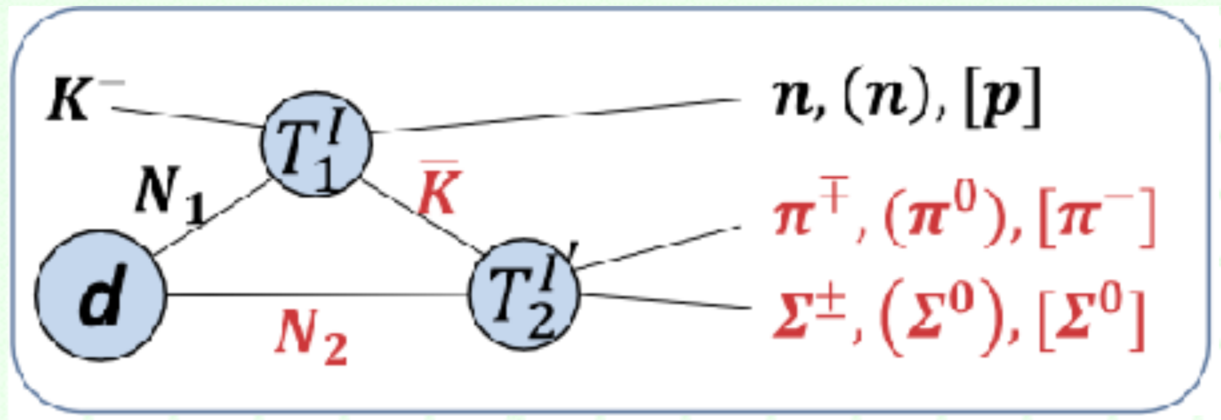
The $\bar{\mathrm{K}}$N system in K$^-$ d –> $\pi\,\Sigma$ n

++ Summary and outlook ++

■ After all, what makes the structure in the K$^-$ d –> $\pi\,\Sigma$ n reaction ?

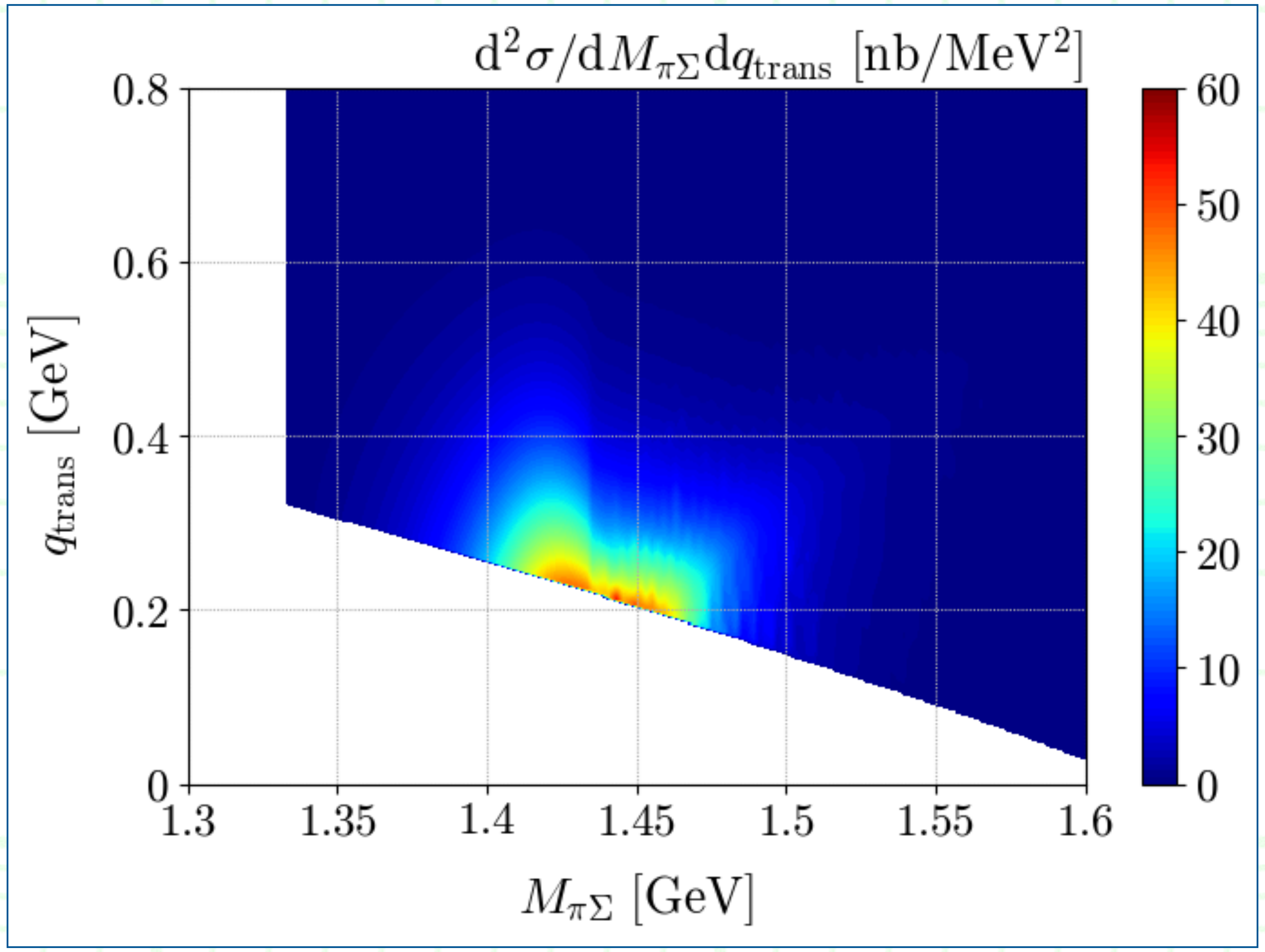
- ☐ Deuteron wave function: Robust.

J-PARC E31 Collab., Phys. Lett. B837 (2023) 137637.



- ☐ $\bar{\mathrm{K}}$ propagator × 1st step $\mathrm{T}_1^{(\mathrm{Kamano})}$ (on-shell) : Fairly robust.
- ☐ 2nd step $\mathrm{T}_2^{(\mathrm{IHW})}$ (contains $\Lambda(1405)$) : Fairly robust.

–> Double-step process makes the structure !
$\bar{\mathrm{K}}$ survives the reaction.



■ Then, we can upgrade the reaction calculation.

- ☐ Final-state interaction ?
- ☐ Difference from the $\Sigma(1385)$ / $\Lambda(1520)$?

–> More precise properties of the $\Lambda(1405)$.

The $\bar{K}NN$ system in $K^- \, {}^3He \to \Lambda pn$

++ Our calculation ++

■ We calculate the cross section of the $K^- \, {}^3He \to \Lambda pn$ reaction.

T. S. , E. Oset, and A. Ramos, PTEP 2016 123D03; under discussion.

☐ Momentum transfer q_{trans} dependence ?

$$\mathbf{q}_{\mathrm{trans}} = \mathbf{p}_K - \mathbf{p}_n \text{ at Lab. frame}$$

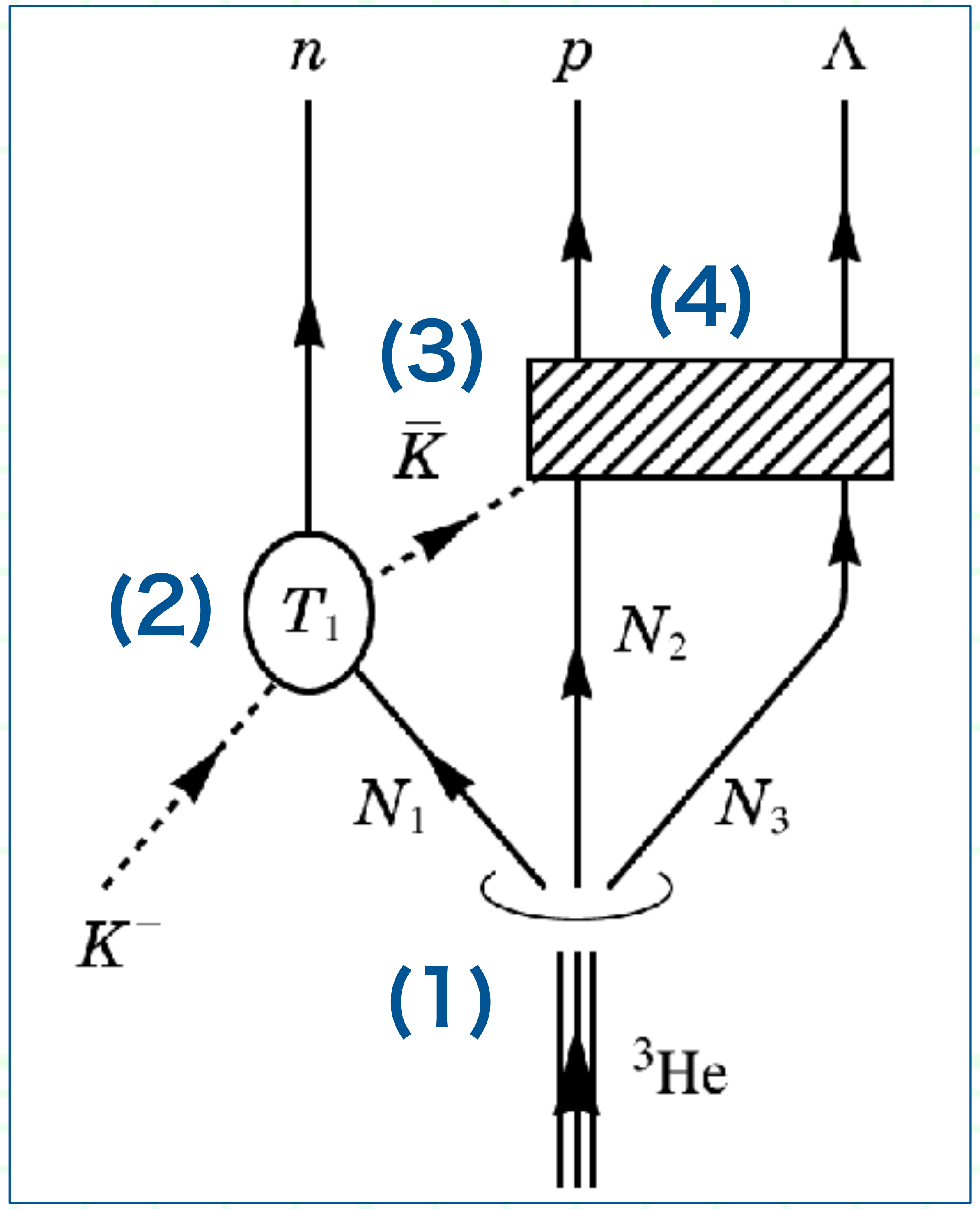
☐ Contribution from each component of the reaction diagram ?

(1) ${}^3He$ wave function.

(2) 1st step T_1 ($\bar{K}N \to \bar{K}N$, $P_K = 1$ GeV/c).

(3) $\bar{K}$ propagator.

(4) Faddeev & $\bar{K}$ absorption $T(\bar{K}NN \to \Lambda p)$.



–> We aim to construct a precise model to search for the $\bar{K}NN$ pole in the complex energy plane.

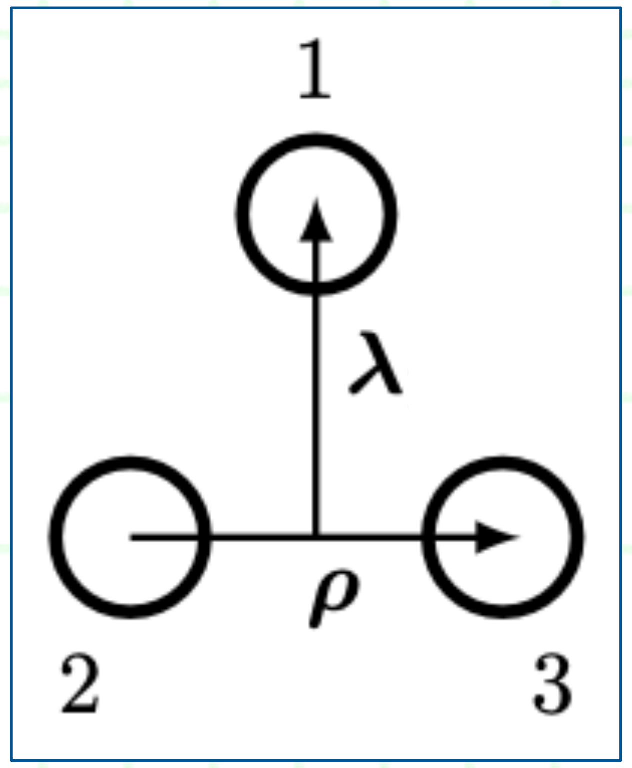
The K̄NN system in $K^- \, {}^3He \to \Lambda pn$

++ ^{3}He wave function ++

■ ^{3}He wave function.

□ A separable parameterization fit for the NNN wave function with the CD-Bonn potential (s wave only, ~ 90 %).

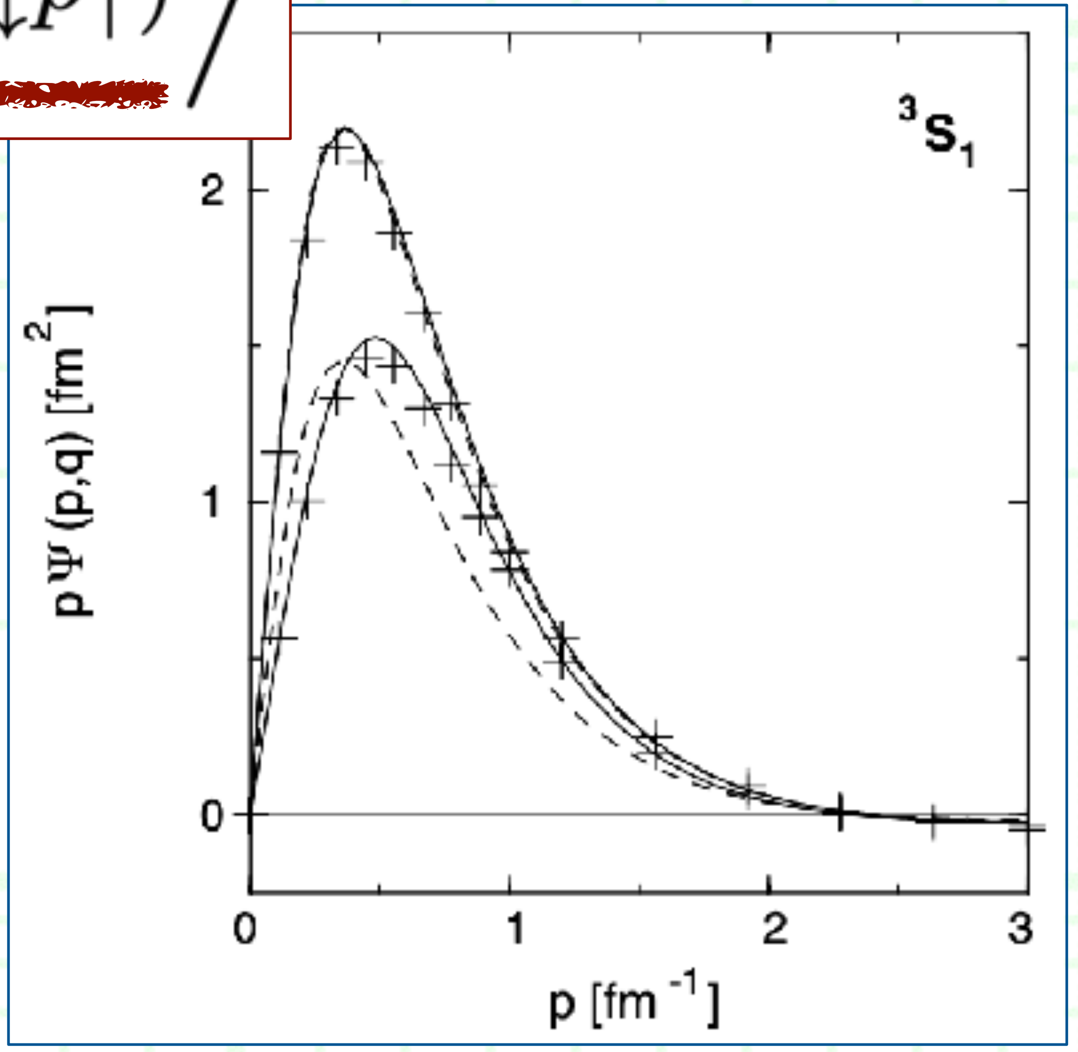
Baru, Haidenbauer, Hanhart, and Niskanen, Eur. Phys. J. A16 (2003) 437.



$^1S_0$ $\quad$ $^1S_0$

$$|^3\mathrm{He}_\uparrow\rangle = v_{^1S_0}(p_\rho)w_{^1S_0}(p_\lambda)\left|-\frac{1}{\sqrt{3}}n_\uparrow(p_\uparrow p_\downarrow - p_\downarrow p_\uparrow) + \frac{1}{2\sqrt{3}}p_\uparrow(p_\uparrow n_\downarrow + n_\uparrow p_\downarrow - p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)\right\rangle$$
$$+v_{^3S_1}(p_\rho)w_{^3S_1}(p_\lambda)\left|-\frac{1}{\sqrt{3}}p_\downarrow(p_\uparrow n_\uparrow - n_\uparrow p_\uparrow) + \frac{1}{2\sqrt{3}}p_\uparrow(p_\uparrow n_\downarrow - n_\uparrow p_\downarrow + p_\downarrow n_\uparrow - n_\downarrow p_\uparrow)\right\rangle$$

$^3S_1$ $\quad$ $^3S_1$



— Functions v and w are given by a five-term expansion:

$$v_\nu(p) = \sum_{n=1}^{5}\frac{a_n^\nu}{p^2+(m_n^\nu)^2}, \quad w_\nu(p) = \sum_{n=1}^{5}\frac{b_n^\nu}{p^2+(M_n^\nu)^2} \quad (\nu = {}^1S_0, {}^3S_1)$$

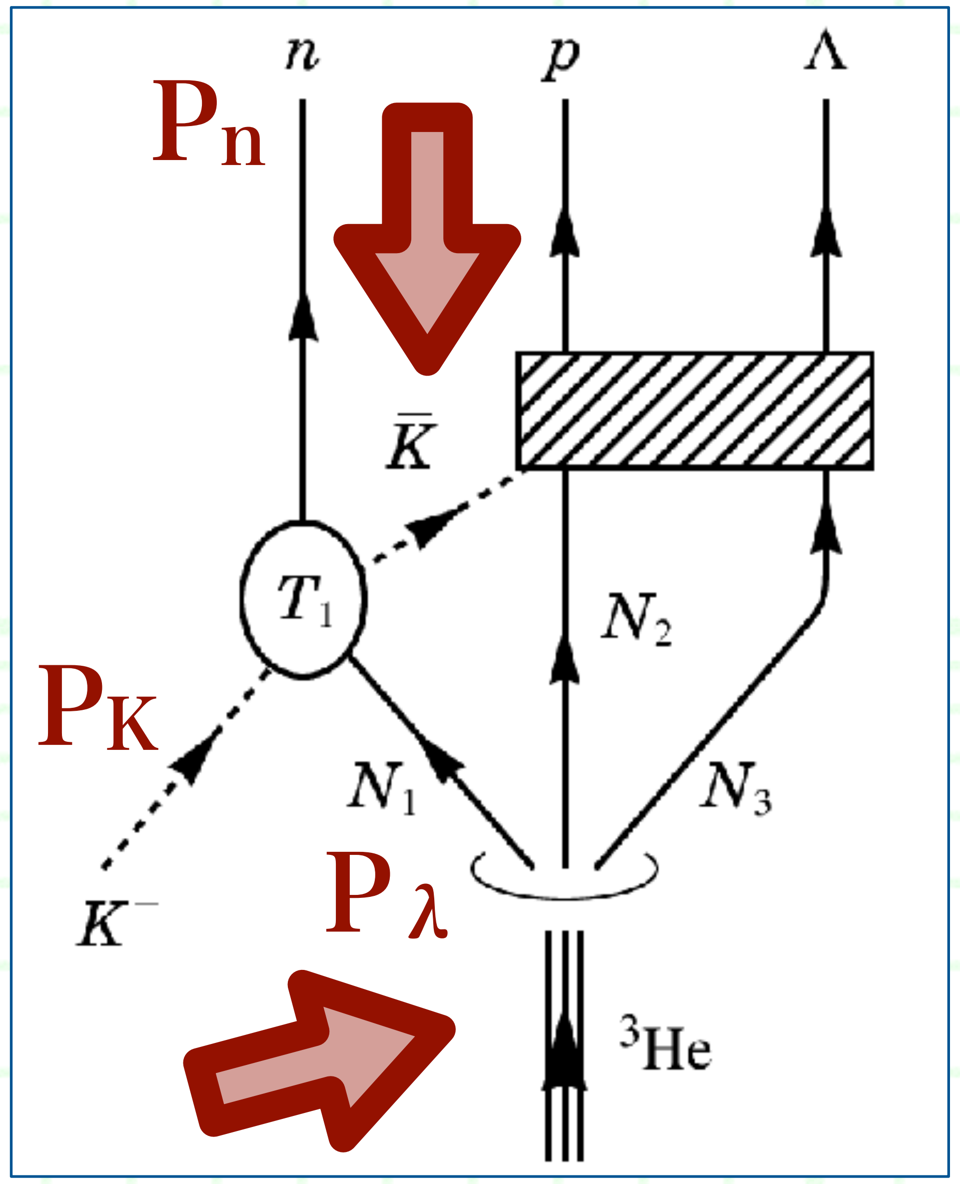
The $\bar{K}NN$ system in $K^- {}^3He \to \Lambda pn$

++ & $\bar{K}$ propagator ++

■ Together with the $\bar{K}$ propagator, the scattering Amp. becomes:

$$\text{Amp.} = \int \frac{d^3p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda)}{(q_{ex}{}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K{}^2 + i0}$$

$$\mathbf{q}_{\text{trans}} = \mathbf{p}_K - \mathbf{p}_n$$

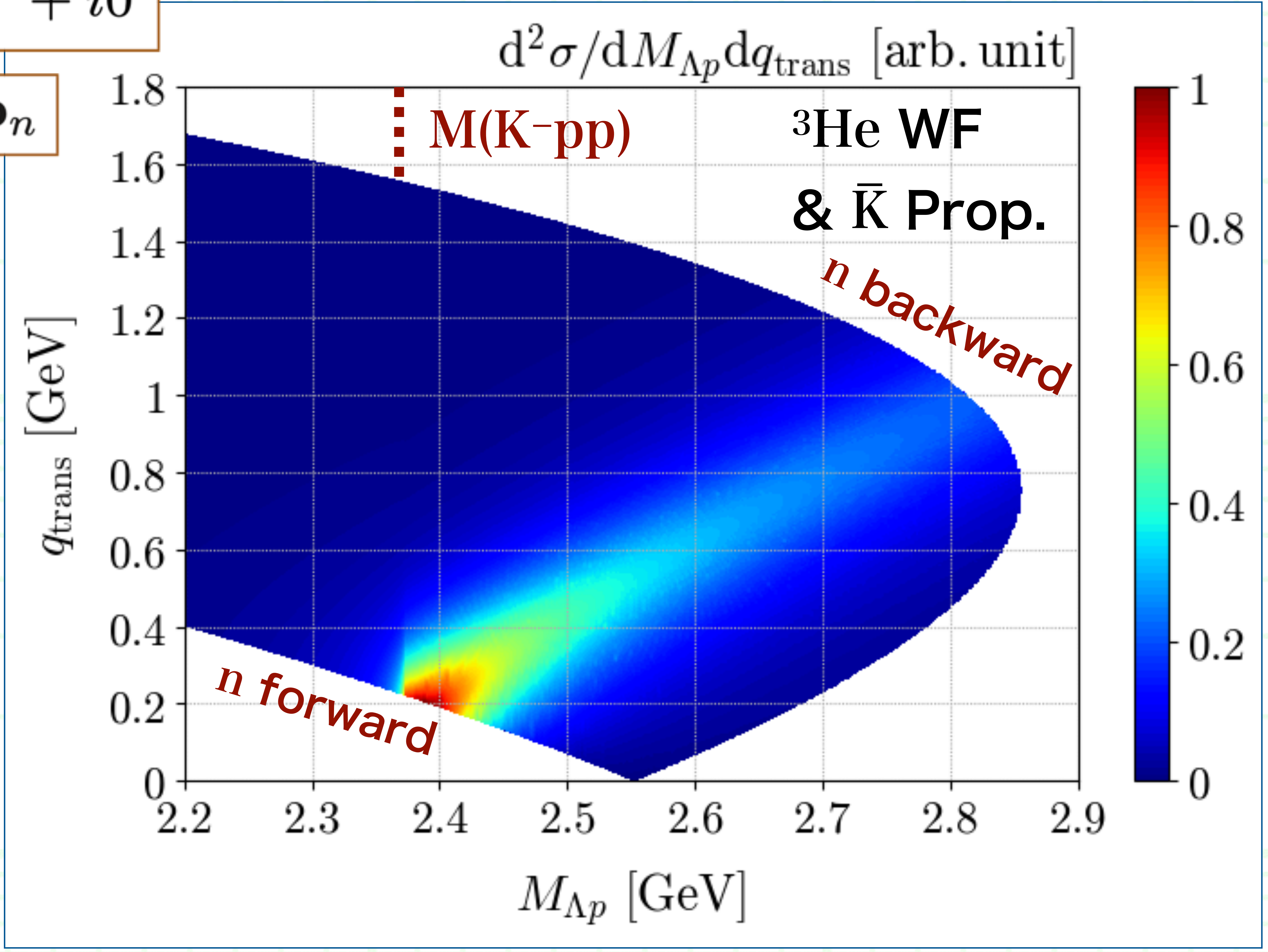


□ Again we have a band structure.

□ Band width owing to the ^{3}He WF.

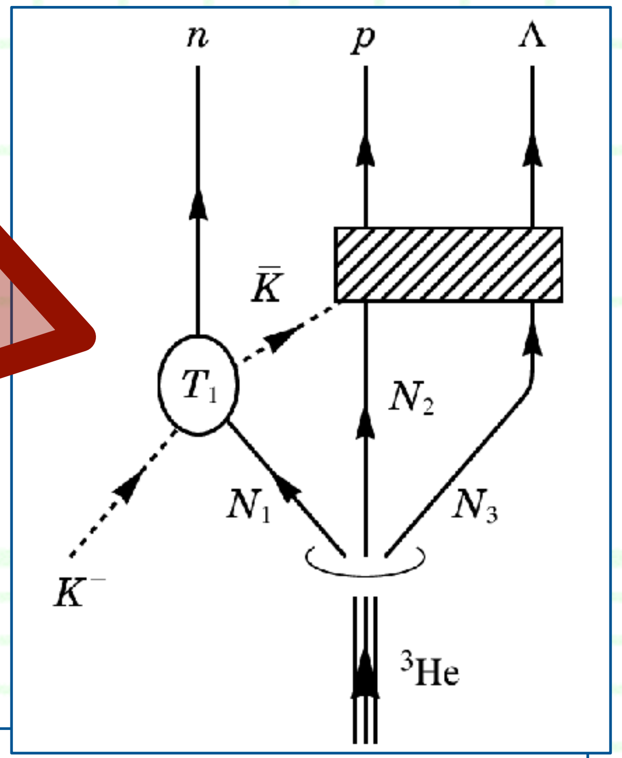
— Off-shell N inside ^{3}He.

□ On this band, we may treat the propagating $\bar{K}$ as (almost) on-shell particle.



The $\bar{K}NN$ system in $K^- \, {}^3He \to \Lambda pn$

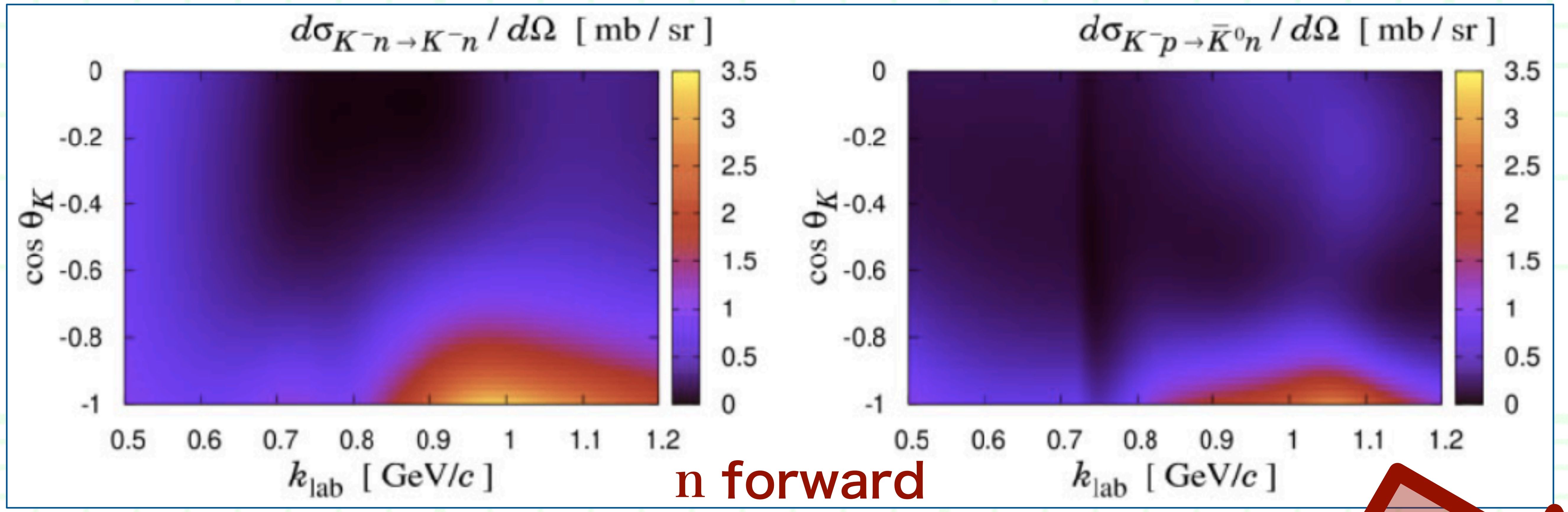
++ & the 1st step $\bar{K}N \to \bar{K}N$ ++



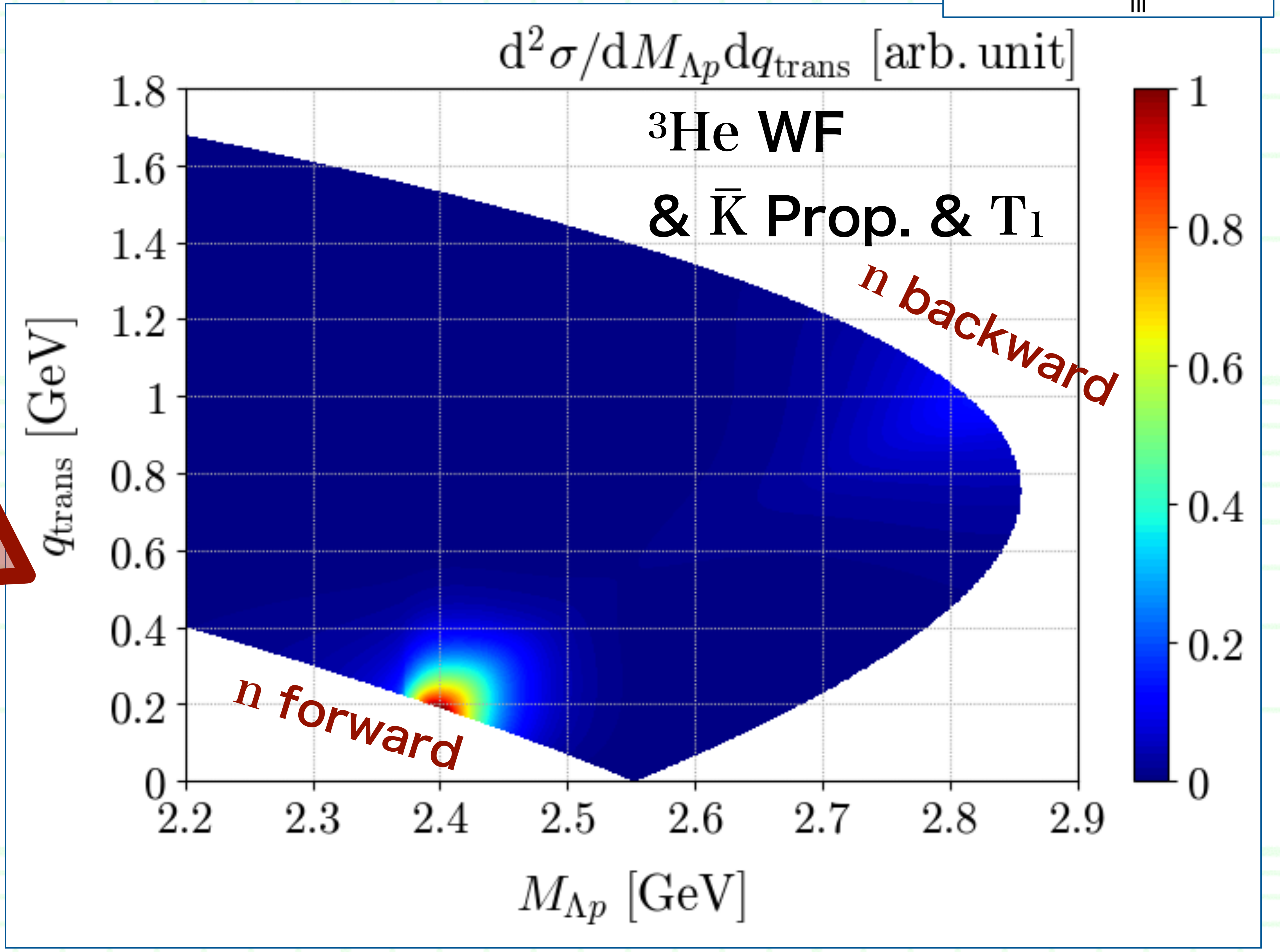
■ Inclusion of the 1st step T_1 ($\bar{K}N \to \bar{K}N$, P_K = 1 GeV/c).

–> Again employ Kamano et al. on-shell amplitude.

Kamano, Nakamura, Lee, and Sato, Phys. Rev. C90 (2014) 065204.



$$\text{Amp.} = \int \frac{d^3p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda) T_1^{(\text{Kamano})}}{(q_{ex}{}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K{}^2 + i0}$$

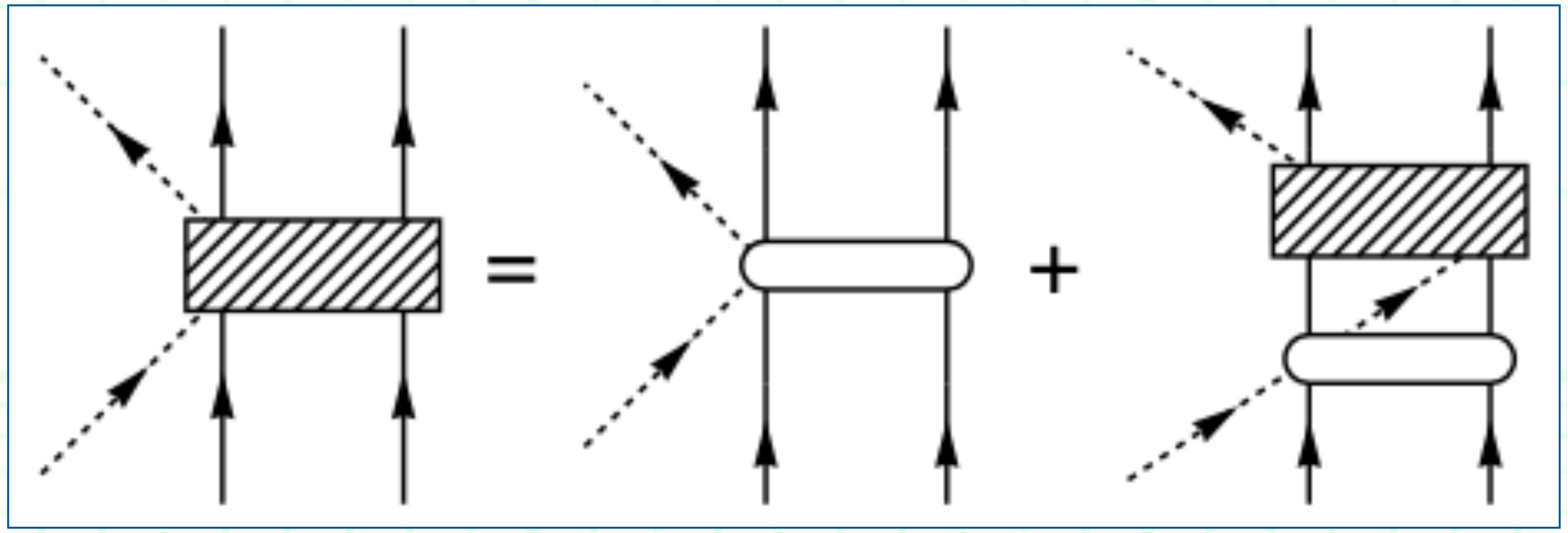


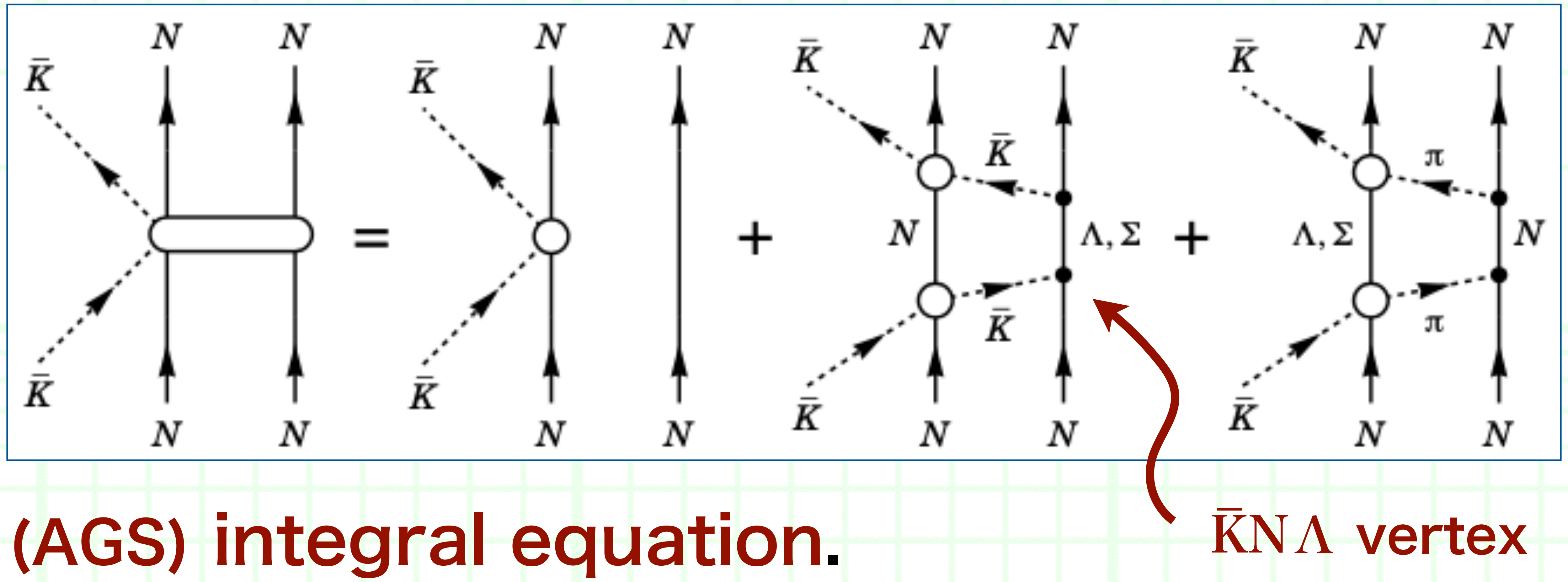
□ Dominated by the small momentum transfer region $q_{trans} < 0.4$ GeV, again.

The K̄NN system in K^- ^{3}He –> Λpn

++ The AGS equation and K̄NN –> Λp ++

- Solve the Faddeev Eq. with the explicit NN interaction as well as K̄N.
  - ☐ NN interaction: Separable form which reproduces the NN($^1S_0$, $^3S_1$) phase shift.
  - ☐ K̄N interaction: K̄N –> K̄N in chiral dynamics & Two-nucleon absorption.





–> Solve the Alt-Grassberger-Sandhas (AGS) integral equation.

$$X_{i,j}(E,p_i,p_j) = Z_{i,j}(E,p_i,p_j) + \sum_n \int_0^\infty \frac{\mathrm{d}k}{2\pi^2} k^2 Z_{i,n}(E,p_i,k) T_n(E,k) X_{n,j}(E,k,p_j)$$

— K̄NN bound state is generated !

The $\bar{\text{K}}$NN system in K^- ^{3}He –> Λpn

++ Reaction cross section ++

■ Inclusion of the $\bar{\text{K}}$NN –> Λp part. –> Full calculation.

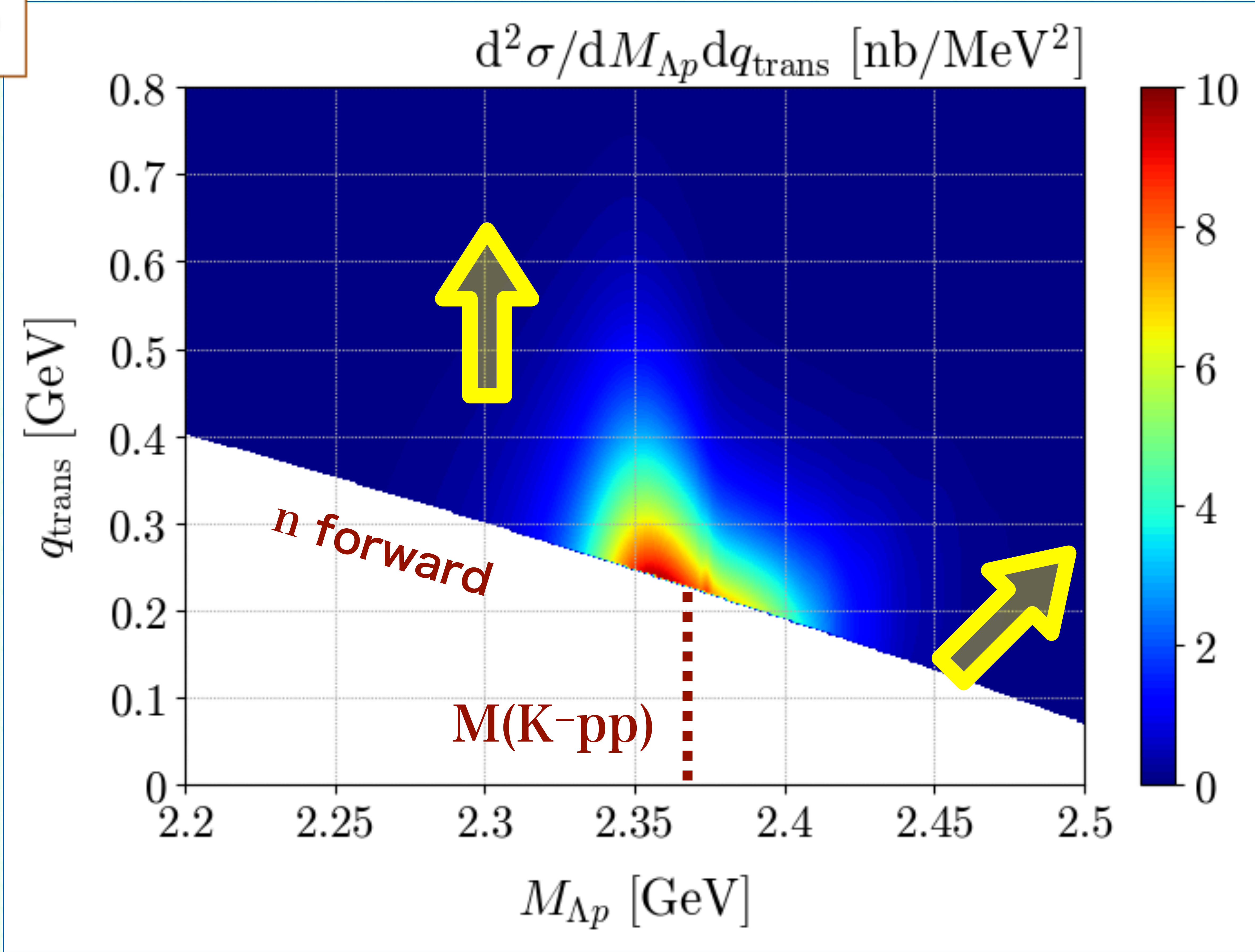
$$\text{Amp.} = \int \frac{d^3 p_\lambda}{(2\pi)^3} \frac{w_\nu(p_\lambda) T_1^{(\text{Kamano})} T_{\bar{K}NN \to \Lambda p}}{(q_{\text{ex}}{}^0)^2 - |\mathbf{p}_\lambda + \mathbf{p}_K - \mathbf{p}_n|^2 - m_K{}^2 + i0}$$

$$T_{\bar{K}NN \to \Lambda p} = \int \frac{d^3 p_\rho}{(2\pi)^3} \frac{v_\nu(p_\rho) V_{\bar{K}N\Lambda} X_{\text{AGS}}}{(q'_{\text{ex}})^2 - m_K{}^2 + i0}$$

□ We have two trends.

- Below the $\bar{\text{K}}$NN threshold:
  The $\bar{\text{K}}$NN bound state signal.
- Above the $\bar{\text{K}}$NN threshold:
  The quasi-free $\bar{\text{K}}$ propagation.

□ These two trends are consistent with the Exp. data.



The $\bar{K}NN$ system in $K^- {}^3He \to \Lambda pn$

++ Summary and outlook ++

■ Consistency of the $K^- {}^3He \to \Lambda pn$ reaction cross section.

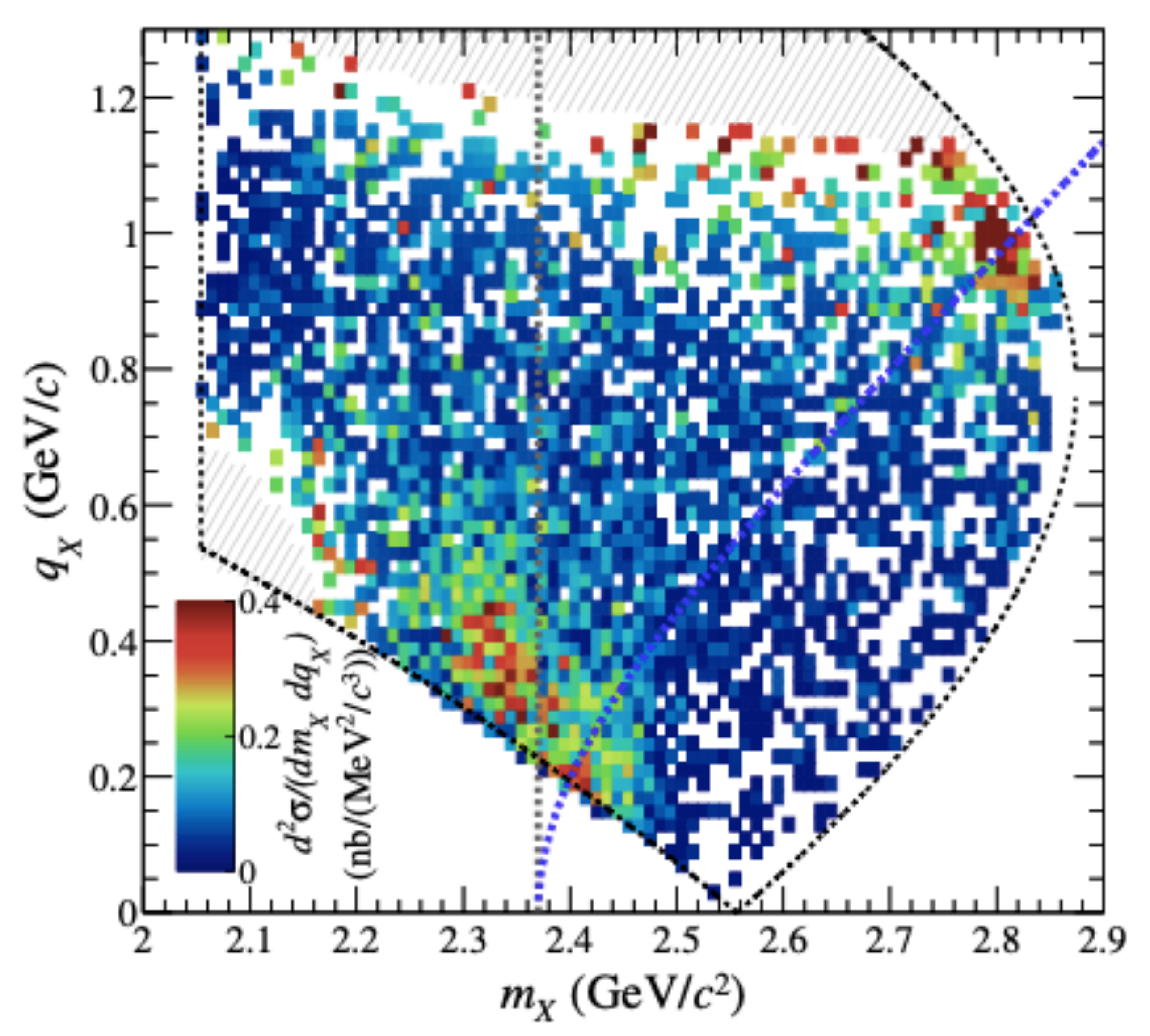
1. Appearance of the quasi-free kaon line.

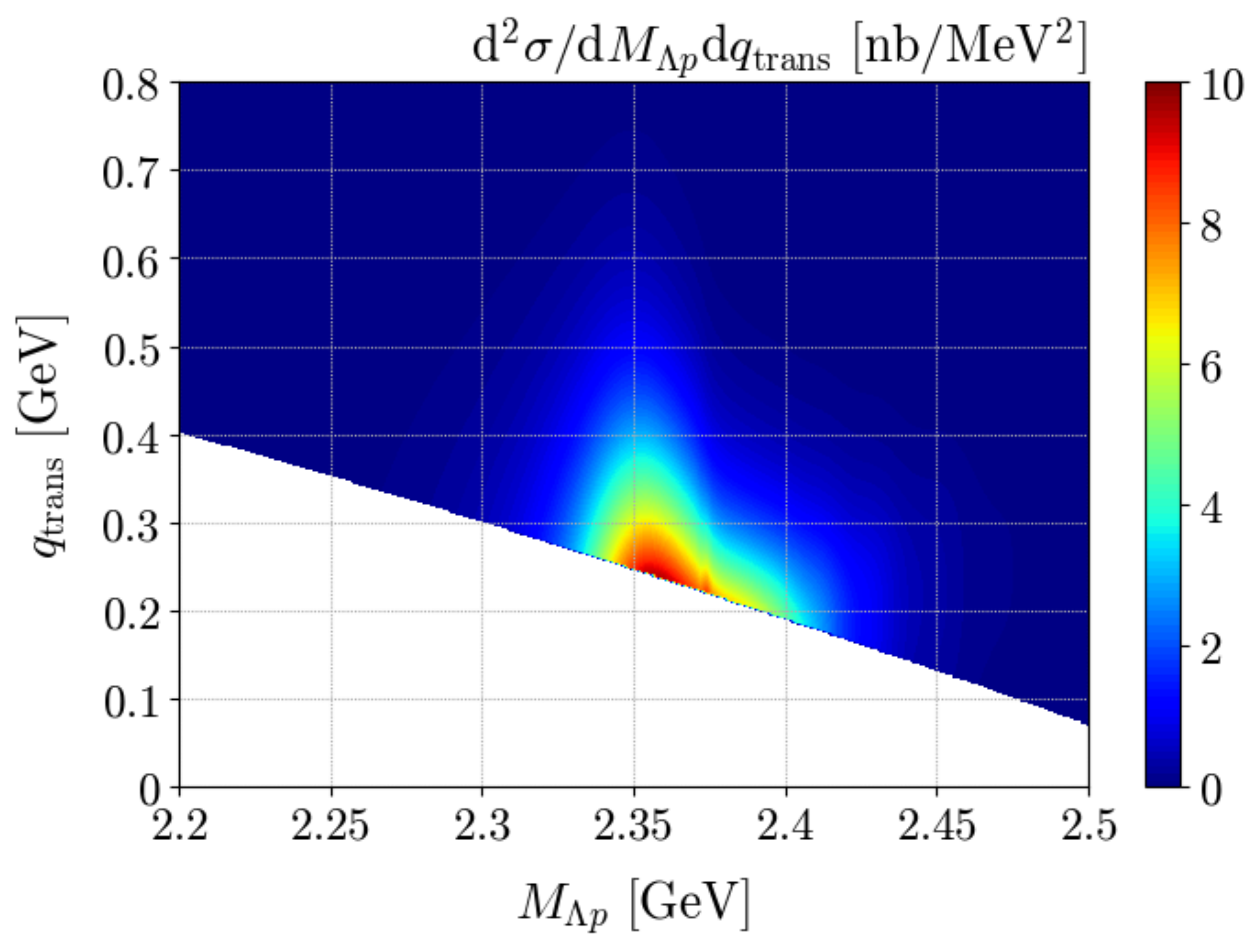
–> $\bar{K}$ is indeed mediated.

2. The q independent signal below the $\bar{K}NN$ threshold.

–> Strongly support the existence of the $\bar{K}NN$ bound state.

Yamaga et al. [J-PARC E15], Phys. Rev. C102 (2020) 044002.





■ Then, we can investigate the scattering amplitude.

☐ Pole position of the $\bar{K}NN$ bound state ?

☐ Spin/parity of the bound state ?

☐ ….

別の話題：

HAL QCD 法と Λ(1405) の有効模型計算

Motivation

Slide by K. Fujiwara.

HAL QCD

Gauge configurations from Lattice QCD $\longrightarrow$ NBS wave function $\Psi_p(r)$ $\longrightarrow$ HAL QCD potential $V_{\mathrm{HAL}}(r)$

$\Omega\Omega$ system in My model

Given potential $V(r)$ $\longrightarrow$ Scattering amplitude $T(E, p', p)$

$\longrightarrow$ NBS wave function $\Psi_p(r)$ $\longrightarrow$ HAL QCD potential $V_{\mathrm{HAL}}(r)$

$\bar{K}N$ system in My model

Given potential $V(r)$ $\longrightarrow$ Scattering amplitude $T(E, p', p)$

Mainly talk

$\longrightarrow$ NBS wave function $\Psi_p(r)$ $\longrightarrow$ HAL QCD potential $V_{\mathrm{HAL}}(r)$

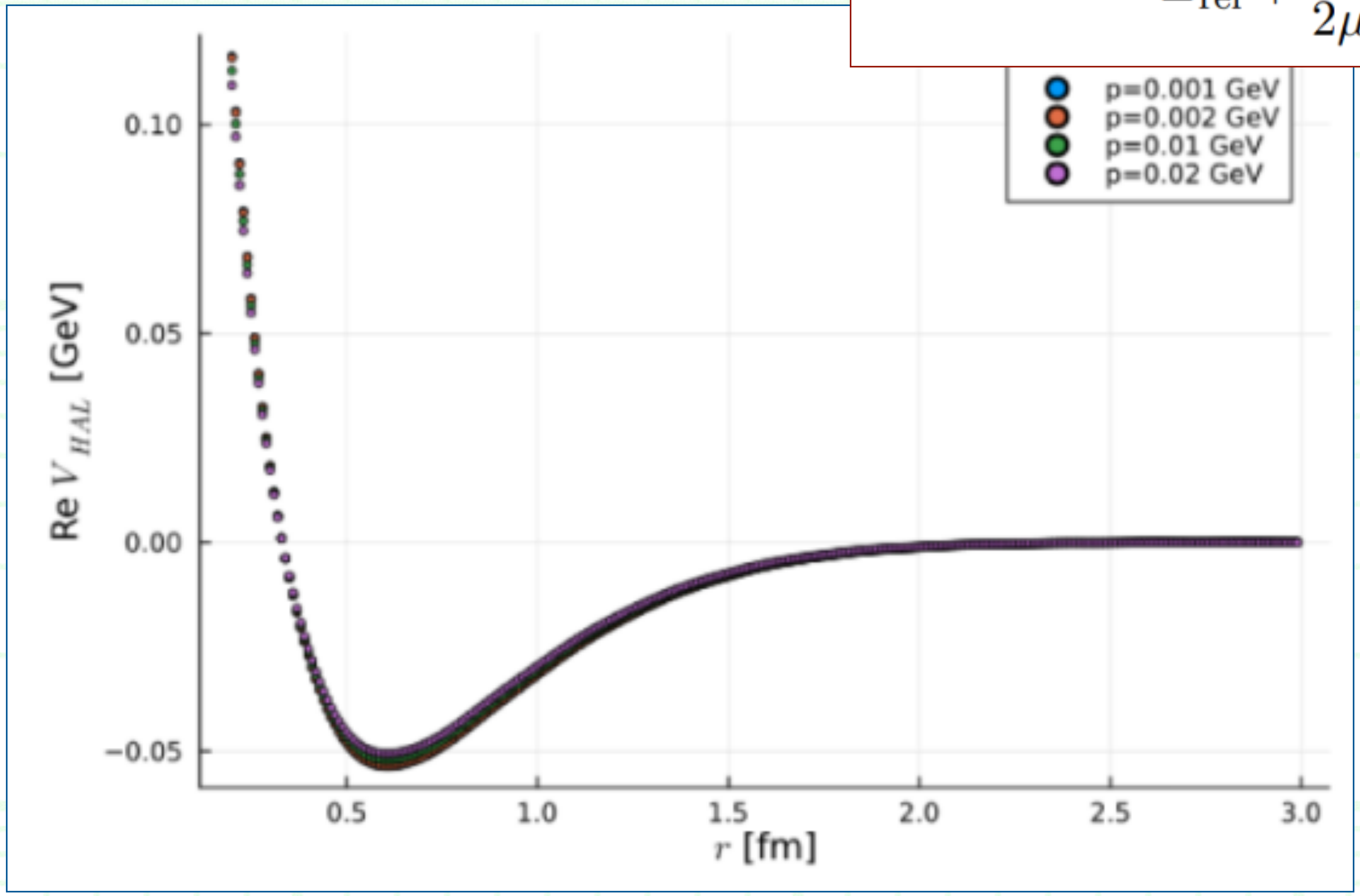
ΩΩ case

$$\Psi_p(r) = j_0(pr) + \int \frac{dk}{2\pi}\, k^2\, j_0(kr)\, \frac{T(E,k,p)}{E - \mathscr{E}(k) + i0}$$
$$= j_0(pr) + \sum_{n'} j_0(k_{n'} r)\, G(E)_{n'}\, T(E)_{n',n_{\rm on}},$$

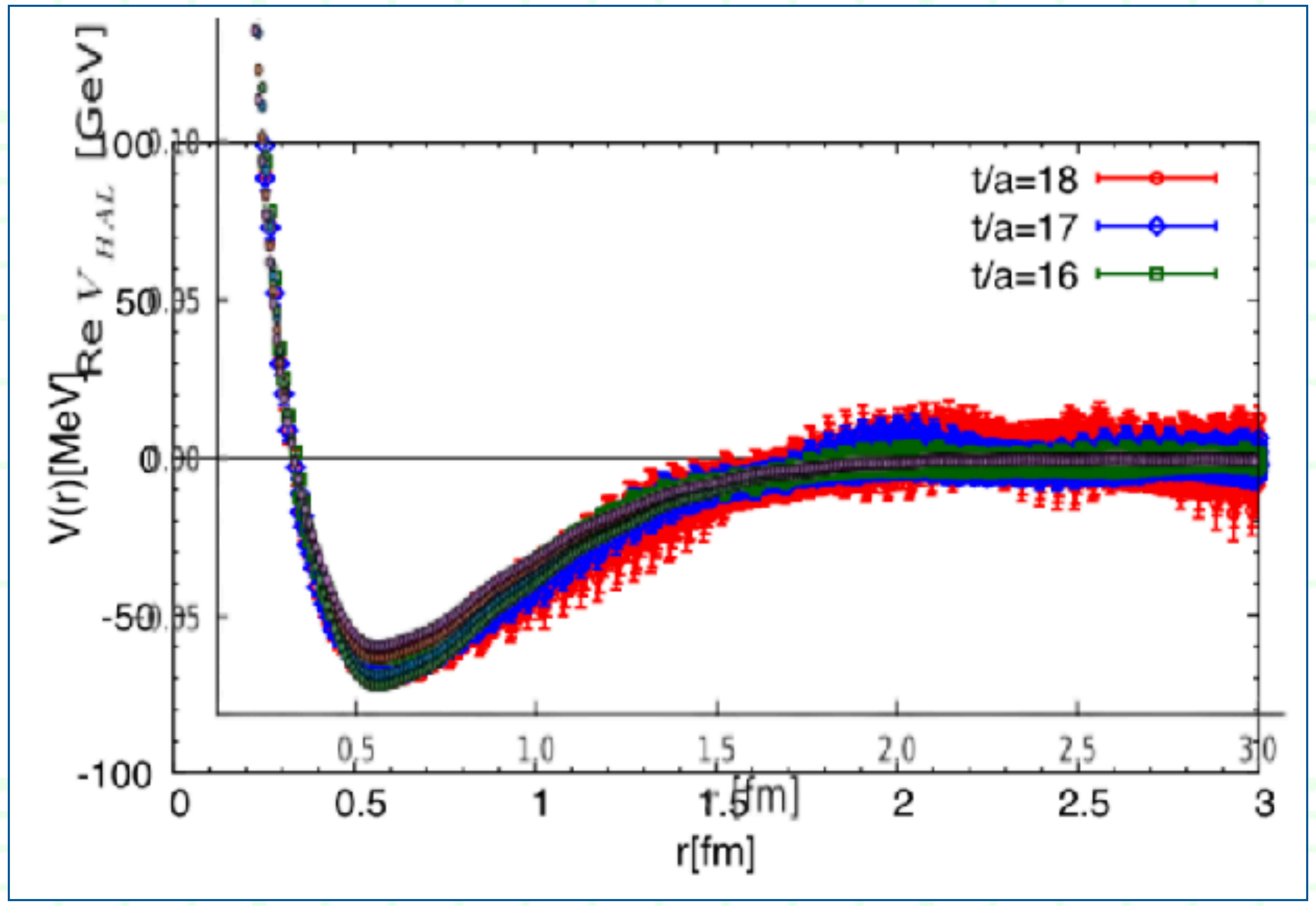
$$T(E,p',p) = V(p',p) + \int_0^\infty \frac{dk}{2\pi^2} k^2 \frac{V(p',k)T(E,k,p)}{E - \mathscr{E}(k) + i0}.$$

$$V_{\rm HAL}(r) = E_{\rm rel} + \frac{1}{2\mu r \Psi_p(r)} \frac{d^2}{dr^2}(r\Psi_p(r))$$
$$\simeq E_{\rm rel} + \frac{1}{2\mu r \Psi_p(r)} \frac{(r+\Delta r)\Psi_p(r+\Delta r) - 2r\Psi_p(r) + (r-\Delta r)\Psi_p(r-\Delta r)}{\Delta r^2}.$$

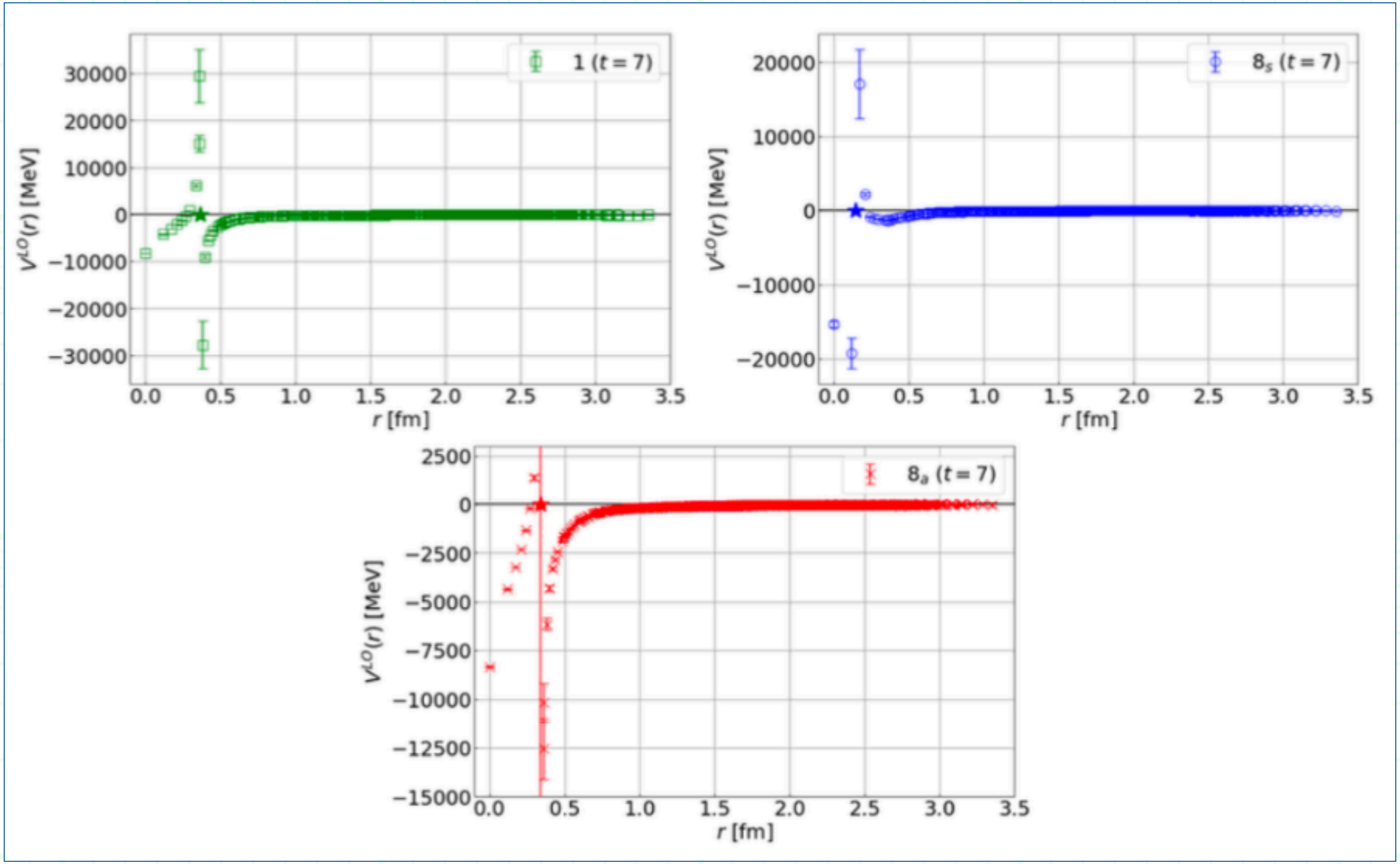
Slide by K. Fujiwara.



Gongyo et al. [HAL QCD],
Phys. Rev. Lett. 120 (2018) 212001.



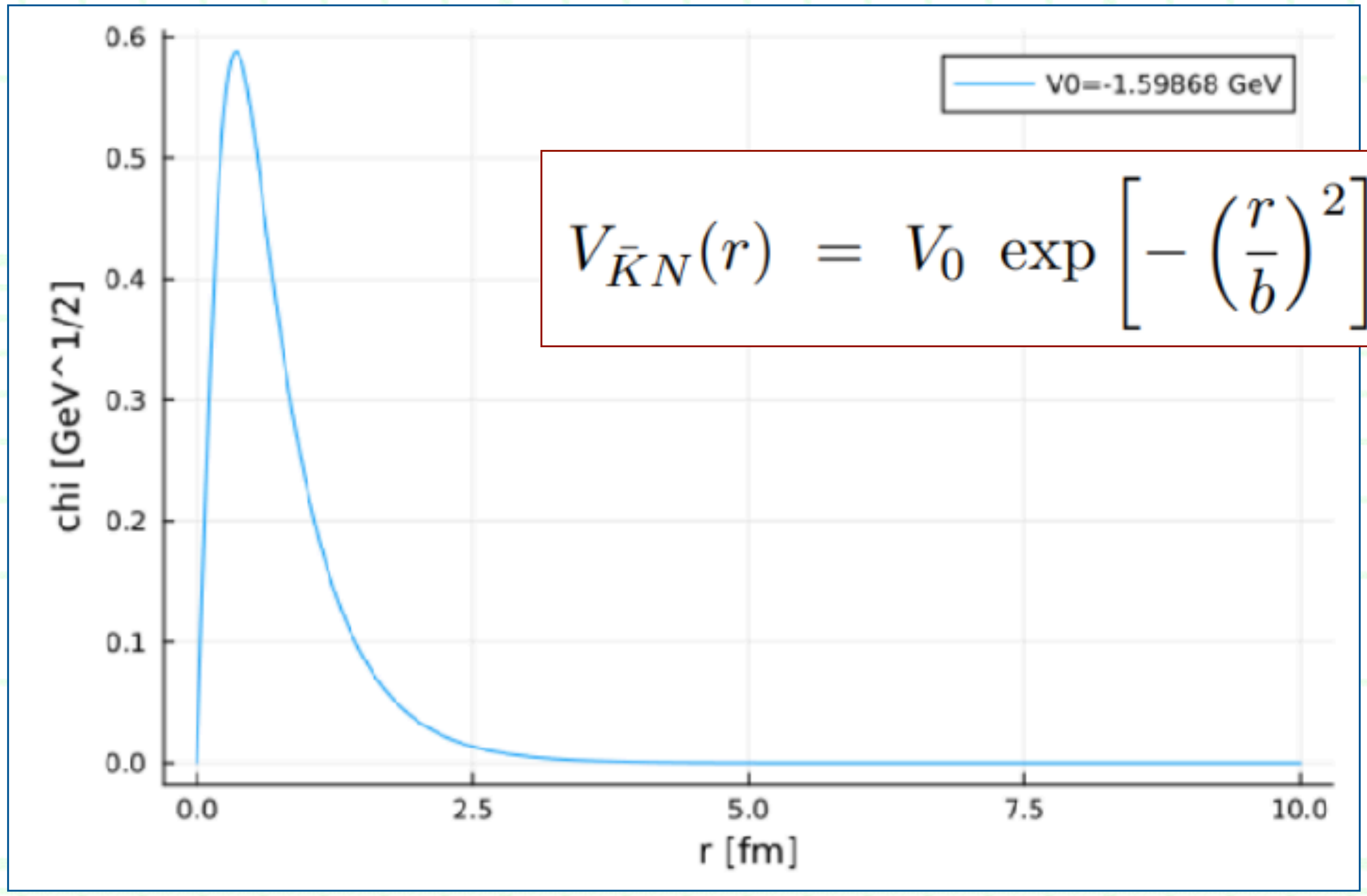
K̄N case



Murakami and Aoki, PoS LATTICE 2023 (2024) 063.

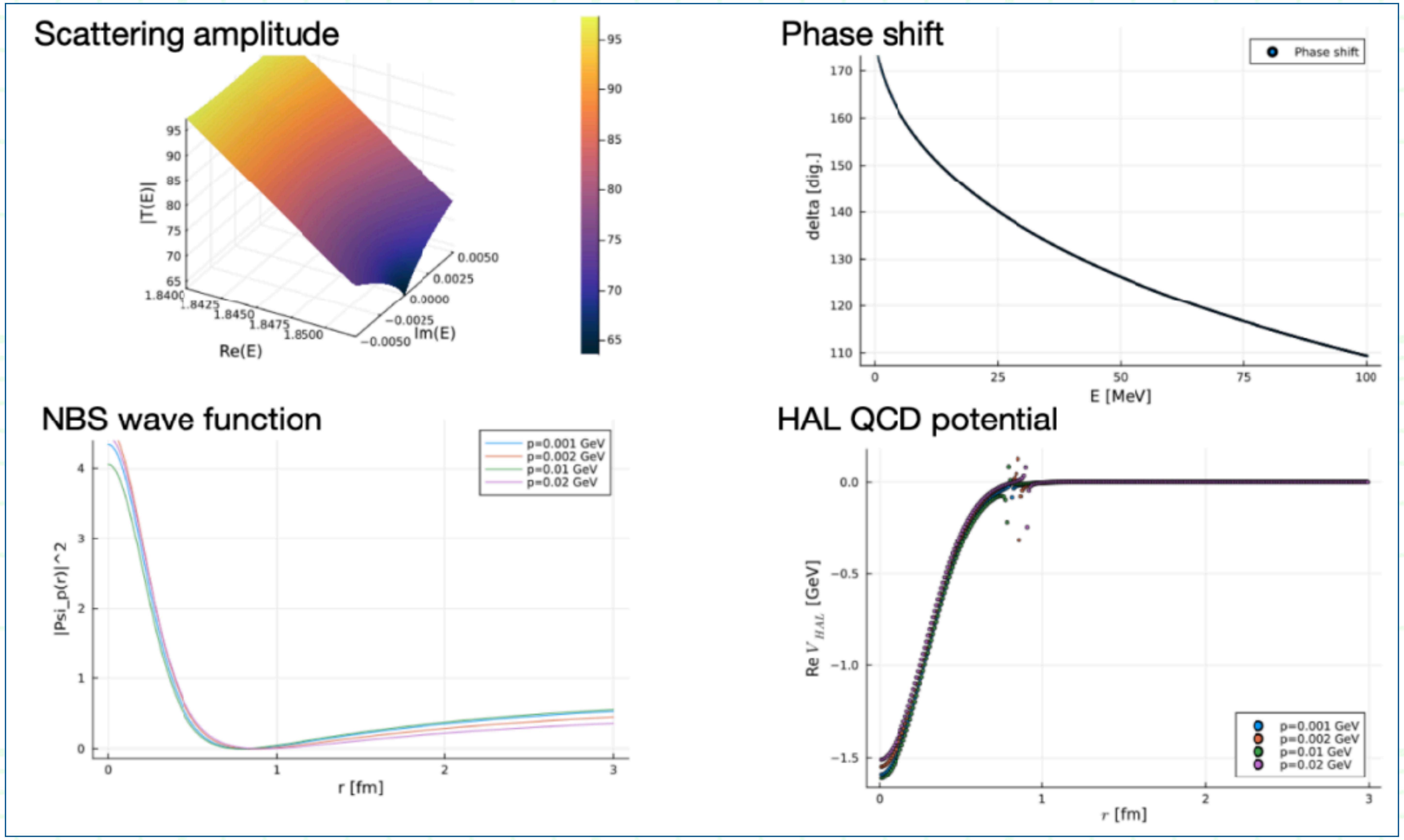
$\bar{\mathrm{K}}$N case

Slide by K. Fujiwara.



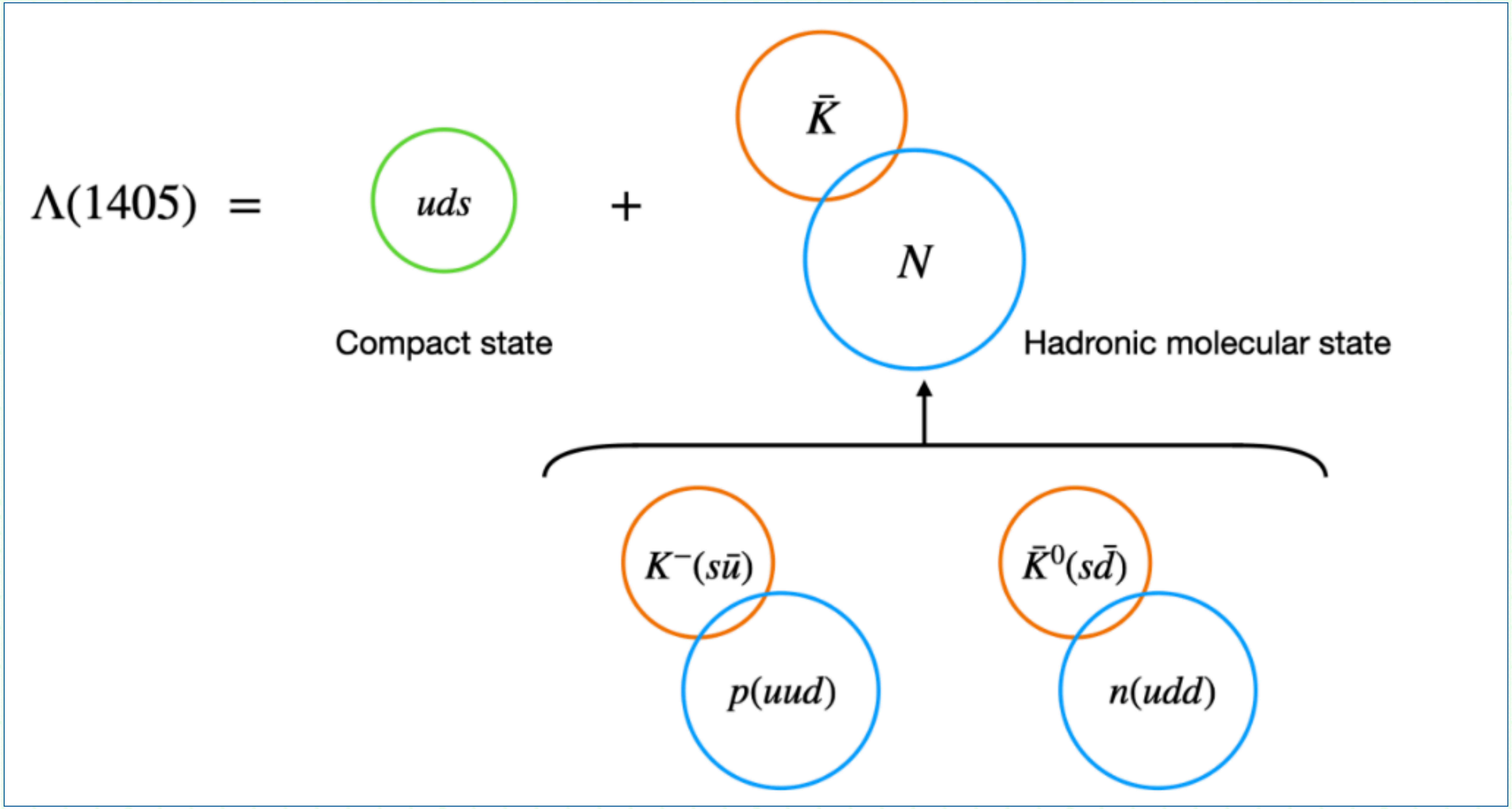
$$V_{\bar{K}N}(r) \;=\; V_0\ \exp\left[-\left(\frac{r}{b}\right)^2\right],$$

$$T(E,p',p) \;=\; V(p',p) + \int_0^\infty \frac{dk}{2\pi^2}k^2\frac{V(p',k)T(E,k,p)}{E-\mathscr{E}(k)+i0}.$$



$$\begin{aligned}\Psi_p(r) \;&=\; j_0(pr) + \int \frac{dk}{2\pi}\,k^2\,j_0(kr)\,\frac{T(E,k,p)}{E-\mathscr{E}(k)+i0}\\ &=\; j_0(pr) + \sum_{n'} j_0(k_{n'}r)\;G(E)_{n'}\;T(E)_{n',n_{\mathrm{on}}},\end{aligned}$$

$\bar{\text{K}}$N case



Slide by K. Fujiwara.