特推ミーティング

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Contents

- separable potentials for $\Lambda(1405)$ in SU(3) limit from LQCD

  ongoing work with S. Aoki (HAL QCD)

- Justification of our strategy to calculate compositeness

  ongoing work with D. Jido, M. Oka, and Z. Yin

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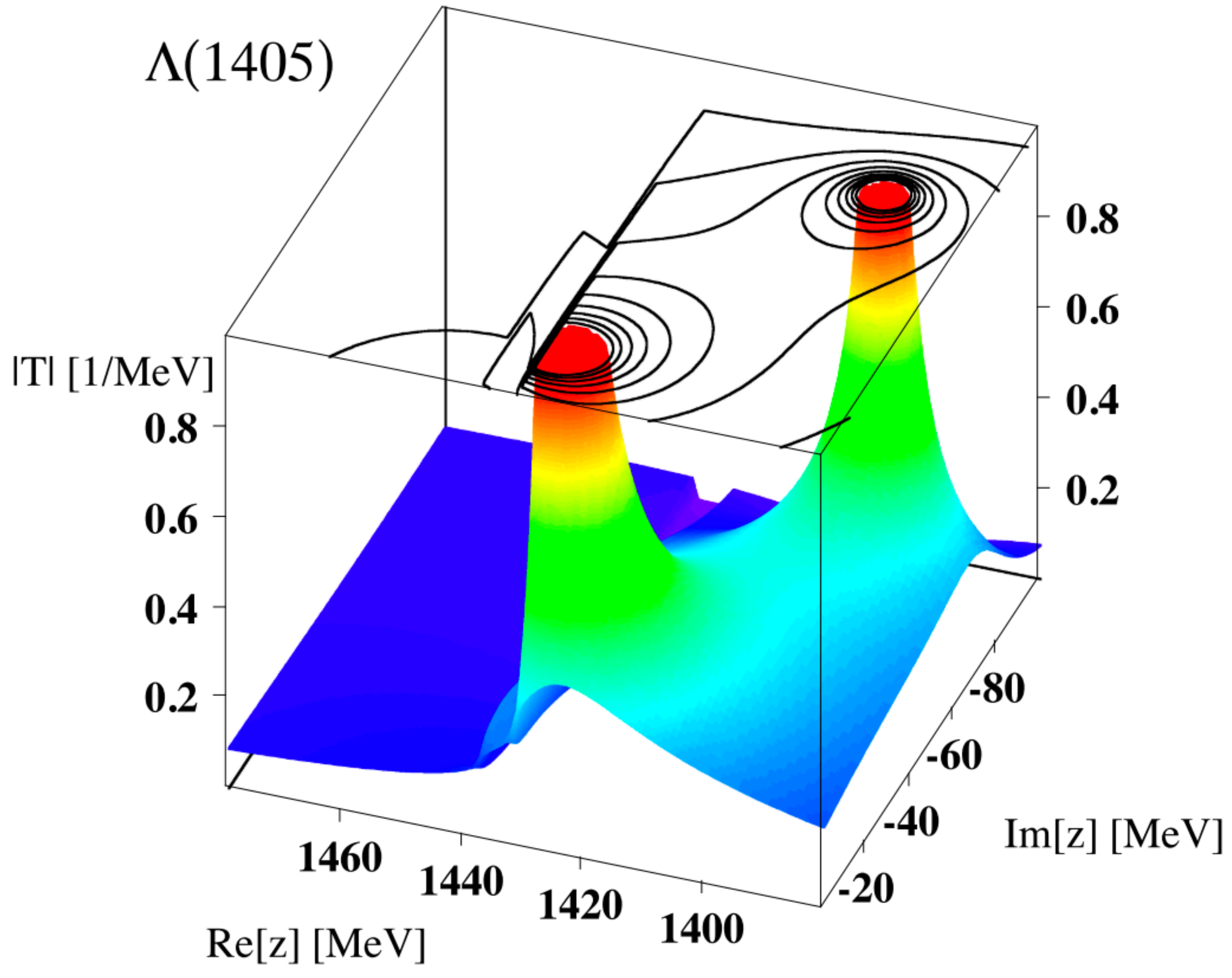
$\Lambda(1405)$ from lattice QCD

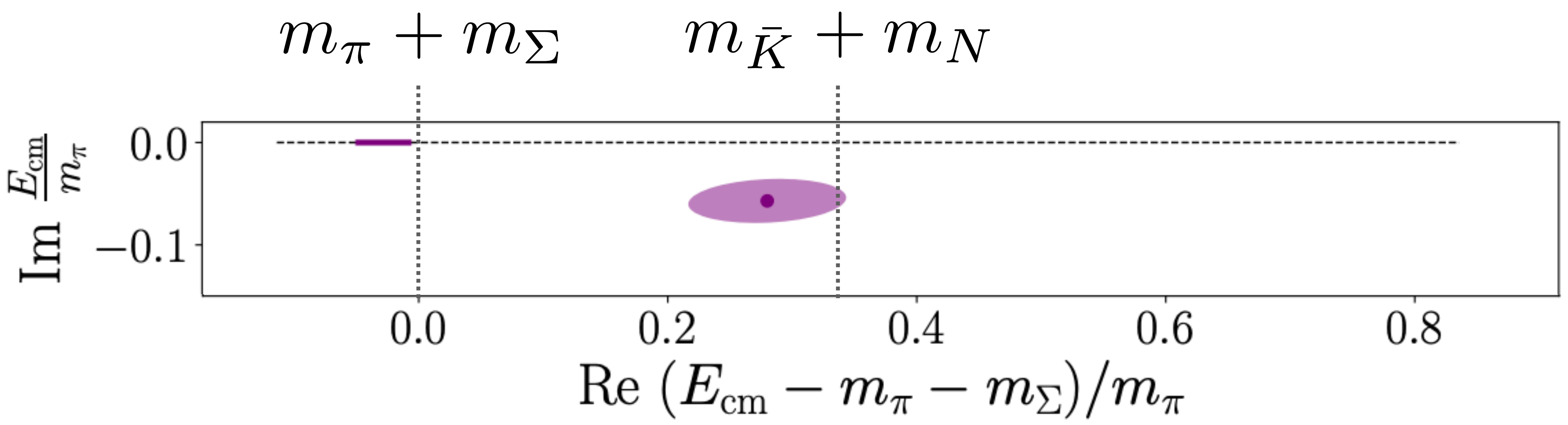
- $\Lambda(1405)$: not a simple Λ baryon (exotic hadron)

- one pole? **two poles?**
  - chiral unitary model [Oller and Meissner, 2001] [Jido, Oller, Oset, Ramos, Meissner, 2003]
  - lattice QCD using finite-volume method at $m_\pi \approx 200$ MeV [Bulava et al. (BaSc Collab.), 2024]

→ virtual state below $\pi\Sigma$ + resonance below $\bar{K}N$



[Hyodo and Jido 2012]



[Bulava et al. (BaSc Collab.), 2024]

- our work: $\Lambda(1405)$ **from HAL QCD approach**

HAL QCD method

[Ishii, Aoki, Hatsuda 2007]
[Ishii et al. 2011]

- R-correlator:

$$R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t) O_2(\mathbf{0},t) \bar{J}(0)\rangle}{\langle O_1(t)\bar{O}_1(0)\rangle\langle O_2(t)\bar{O}_2(0)\rangle} \approx \sum_n C_{\bar{J},n} \underline{\Psi^{W_n}(\mathbf{r})} e^{-(W_n - m_1 + m_2)t}$$

Nambu-Bethe-Salpeter (NBS) wave function

- time-dependent HAL QCD method

(μ: reduced mass)

$$\int d^3r' \, U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

$$\approx V(r)\delta^{(3)}(\mathbf{r}-\mathbf{r}')$$

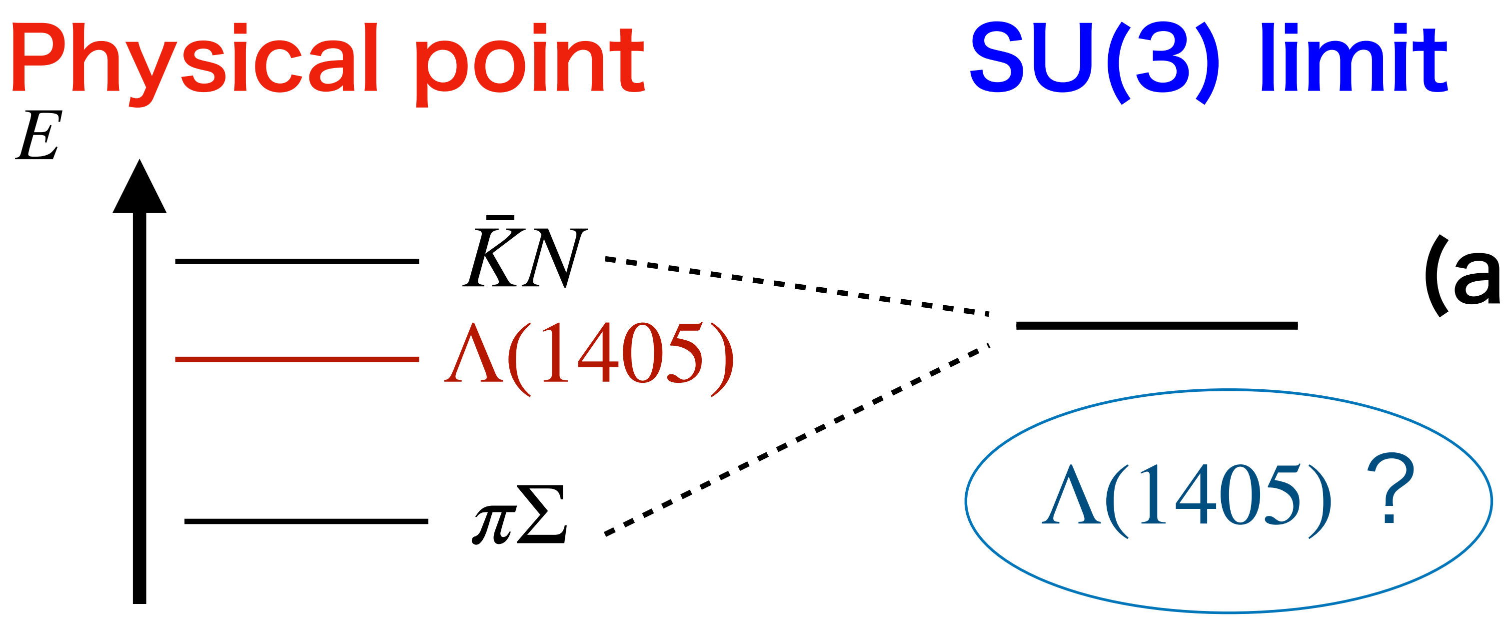
(local (leading-order) approximation)

$$\rightarrow V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right) R(\mathbf{r},t)$$

- **Talks of HAL QCD studies**
  - Ξ_{cc}-Ξ_{cc} (T. Doi, Jul 29)
  - Λ_c-N (L. Zhang, Jul 31)
  - J/ψ-N, η_c-N (Y. Lyu, Jul 31)
  - Left-hand cut (S. Aoki, Aug 1)
  - Neural network (L. Wang, Aug 2)

Setups

- study $\Lambda(1405)$ in **flavor SU(3) limit**



(almost) single-channel analysis

- channels: $\underset{\text{meson}}{\underline{8}} \otimes \underset{\text{baryon}}{\underline{8}} = 27 \oplus 10 \oplus 10^* \oplus \underline{8_s \oplus 8_a \oplus 1}$
  - chiral unitary model: one pole in **singlet channel** & the other in **octet channel**

[Jido, Oller, Oset, Ramos, Meissner, 2003]
[Guo, Kamiya, Mai, Meissner, 2023]

- **neglect coupling between 8_s and 8_a** in this work

$$\begin{pmatrix} V_{8_s 8_s}(r) & V_{8_s 8_a}(r) \\ V_{8_a 8_s}(r) & V_{8_a 8_a}(r) \end{pmatrix} \approx \begin{pmatrix} V_{8_s 8_s}(r) & 0 \\ 0 & V_{8_a 8_a}(r) \end{pmatrix}$$

cf. chiral perturbation theory with Weinberg-Tomozawa term:

- **no coupling between 8_s and 8_a**
- same interactions for 8_s and 8_a

Lattice setups

- $a \approx 0.12$ fm, 32^4 lattices, $m_M = 459.4(1.7)_{\rm stat}$ MeV

$$m_B = 1166.1(4.1)_{\rm stat}\ \mathrm{MeV}$$

- R-correlators $(\mathrm{rep} = 1, 8_s, 8_a)$

$$R^{(\mathrm{rep})}(\mathbf{r}, t) = \frac{\langle M(\mathbf{x}+\mathbf{r}, t) B(\mathbf{x}, t) \bar{\Lambda}^{(X)}(0)\rangle}{\langle M(t)\bar{M}(0)\rangle\langle B(t)\bar{B}(0)\rangle}$$

$\Lambda^{(X)}(t) \sim \sum_{\mathbf{z}} u(\mathbf{z}, t) d(\mathbf{z}, t) s(\mathbf{z}, t)$: 3-quark type

($X = 8$ for $\mathrm{rep} = 8_s, 8_a$
$X = 1$ for $\mathrm{rep} = 1$)

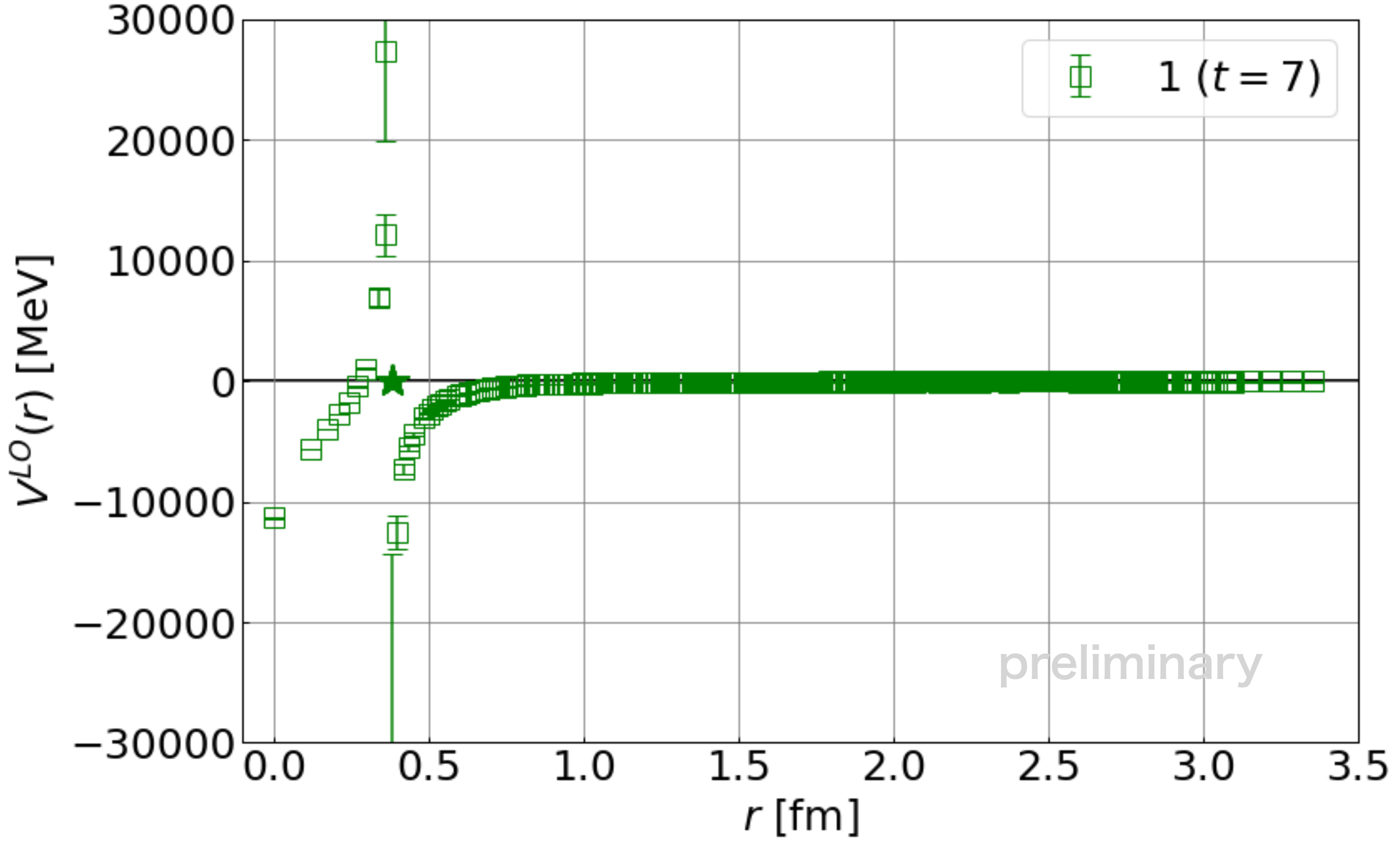
- calculation technique: same as in
[KM, Aoki, PoS **LATTICE2023**, 063 (2024)]

- (at least) **one bound state in each channel**
from $\langle \Lambda^{(8)}(t)\bar{\Lambda}^{(8)}(0)\rangle$ and $\langle \Lambda^{(1)}(t)\bar{\Lambda}^{(1)}(0)\rangle$

Local potentials

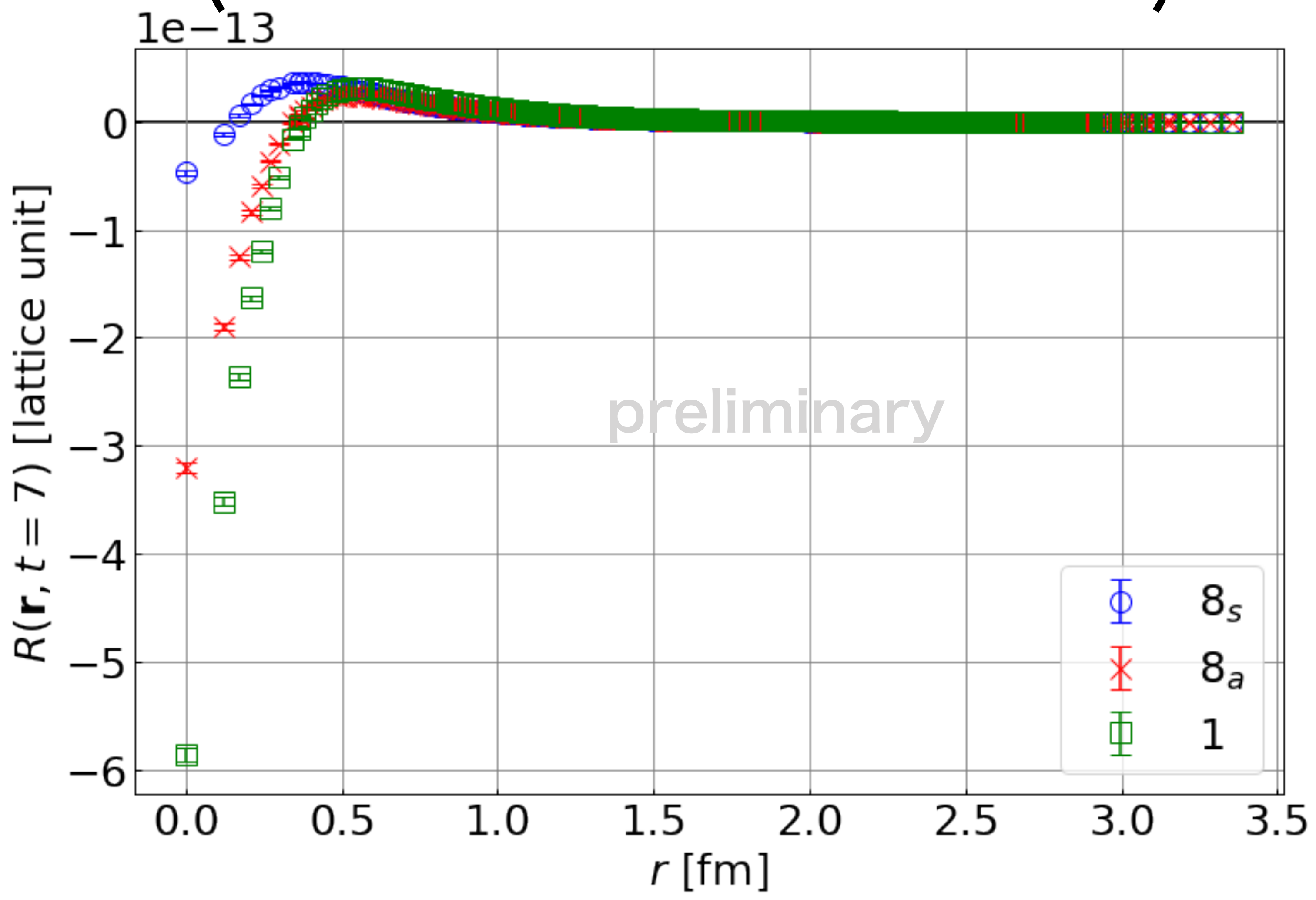
$$V(\mathbf{r}) \simeq \frac{1}{R(\mathbf{r},t)}\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2}\right)R(\mathbf{r},t)$$

- local potential in singlet channel $V_1(r)$



- R-correlators $R(\mathbf{r},t)$

(NBS wave functions)



- **singular behavior in all channels** because of R-correlators crossing zero

no problematic in principle,
but difficult to obtain reliable results

- alternative approach: **separable potential** $U(\mathbf{r},\mathbf{r}') \approx \eta v(\mathbf{r})v(\mathbf{r}')$ $(\eta = \pm 1)$

Separable potentials in the HAL QCD method

- time-dependent equation

$$\left(R(\mathbf{r},t) = \frac{\langle O_1(\mathbf{r},t) O_2(\mathbf{0},t) \bar{J}(0) \rangle}{\langle O_1(t)\bar{O}_1(0)\rangle \langle O_2(t)\bar{O}_2(0)\rangle} \right)$$

$$\int d^3r' \; U(\mathbf{r},\mathbf{r}') R(\mathbf{r}',t) \simeq \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t)$$

$$\approx \eta v(\mathbf{r}) v(\mathbf{r}'), \quad (\eta = \pm 1)$$

(separable potential approximation)

$$\to \quad \eta v(\mathbf{r}) \underline{\int d^3r' \; v(\mathbf{r}') R(\mathbf{r}',t)} \approx \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t)$$

constant (indep. of $\mathbf{r}$)

→ **no singular behavior for** $v(\mathbf{r})$

☑ checked validity of separable potential approx. in KN system

How to extract separable potentials

- time-dependent (TD) equation for separable potential: $(\eta = \pm 1)$

$$\eta v(\mathbf{r}) \underbrace{\int d^3r' \; v(\mathbf{r}')R(\mathbf{r}',t)}_{= A[R,v]\text{: constant (indep. of } \mathbf{r}\text{)}} = \underbrace{\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t)}_{= \mathcal{D}R(\mathbf{r},t)}$$

$$\times \int d^3r \; R(\mathbf{r},t)$$

$$\eta (A[R,v])^2 = \int d^3r \; \underbrace{R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)}_{\text{real}}$$

$$\boxed{\begin{aligned} \eta &= \mathrm{sgn}[\eta(A[R,v])^2] = \mathrm{sgn}\left[\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right] \\ A[R,v] &= \sqrt{|\eta(A[R,v])^2|} = \sqrt{\left|\int d^3r \; R(\mathbf{r},t)\mathcal{D}R(\mathbf{r},t)\right|} \end{aligned}}$$

$$\boxed{v(\mathbf{r}) = \frac{\mathcal{D}R(\mathbf{r},t)}{\eta A[R,v]}}$$

Setups for separable potentials

- neglect coupling between 8_s and 8_a

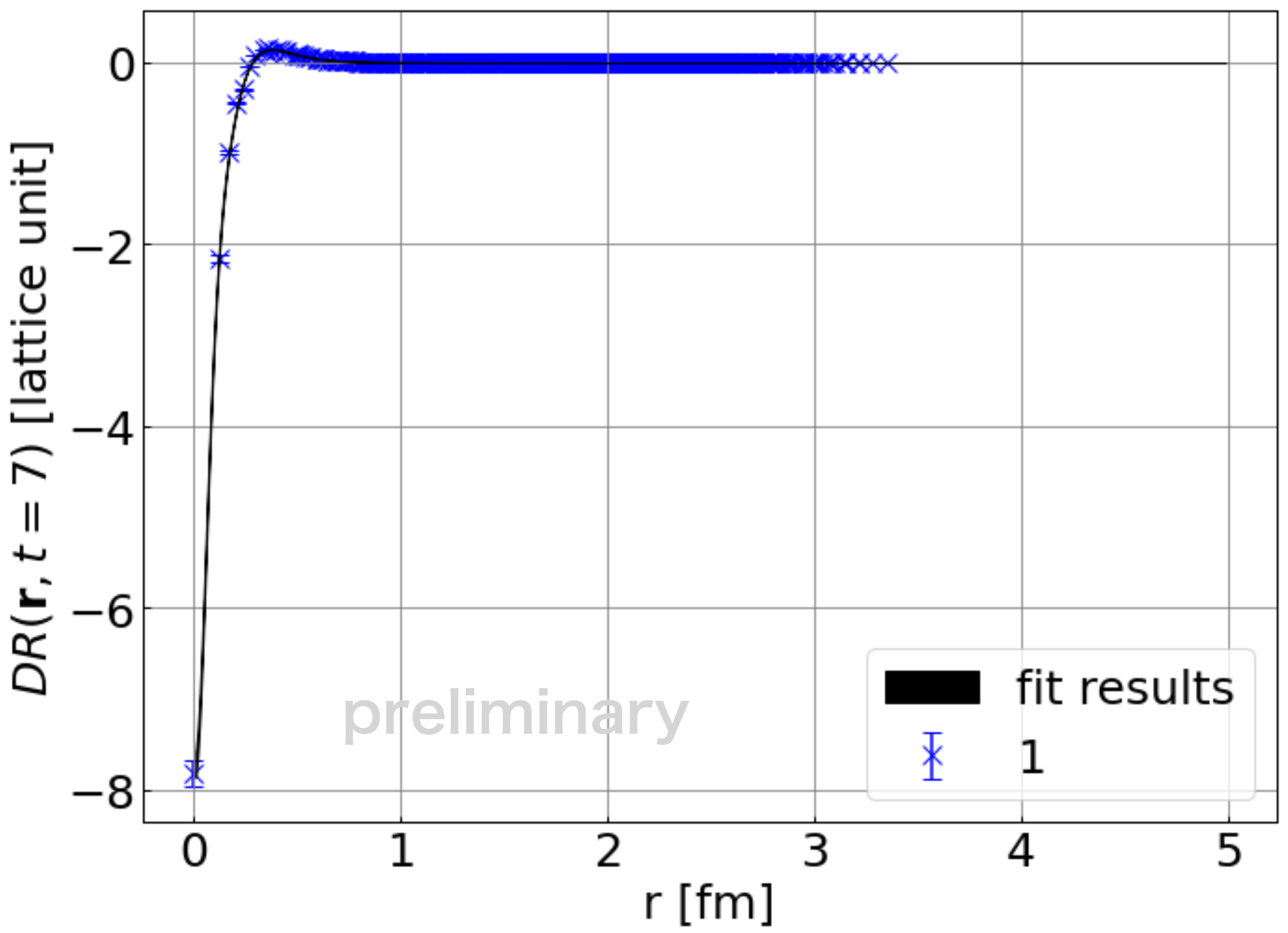
$$U_1(\mathbf{r},\mathbf{r}') \approx \eta_1 v_1(\mathbf{r}) v_1(\mathbf{r}')$$

$$\begin{pmatrix} U_{8_s8_s}(\mathbf{r},\mathbf{r}') & U_{8_s8_a}(\mathbf{r},\mathbf{r}') \\ U_{8_a8_s}(\mathbf{r},\mathbf{r}') & U_{8_a8_a}(\mathbf{r},\mathbf{r}') \end{pmatrix} \approx \begin{pmatrix} \eta_{8_s} v_{8_s}(\mathbf{r}) v_{8_s}(\mathbf{r}') & 0 \\ 0 & \eta_{8_a} v_{8_a}(\mathbf{r}) v_{8_a}(\mathbf{r}') \end{pmatrix}$$

- fitting for $\mathcal{D}R(\mathbf{r},t)$ using multi-Gaussians to obtain potentials in continuum

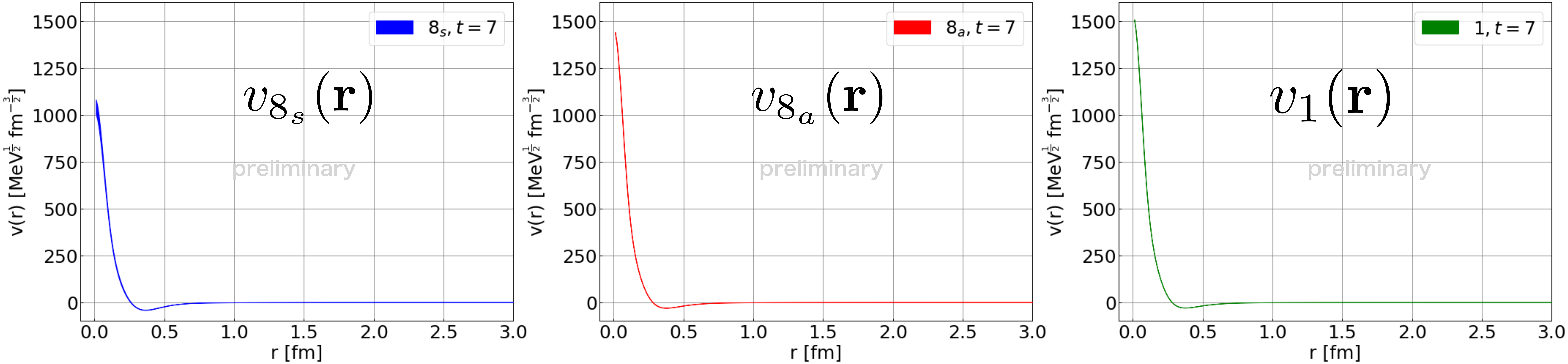
$$\left(\mathcal{D}R(\mathbf{r},t) = \left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t} + \frac{1}{8\mu}\frac{\partial^2}{\partial t^2} \right) R(\mathbf{r},t) \right)$$

$\mathcal{D}R(\mathbf{r},t)$ in singlet channel



Results of separable potentials

- Results of $v(\mathbf{r})$, η



	8_s channel	8_a channel	1 channel
η	-1	-1	-1

- $\eta = -1$ for all three channels ➡ attractive interactions
- magnitude of $v(\mathbf{r})$ in short distance is larger for singlet channel

Binding energies

- solve Schrödinger equation in the Gaussian expansion method with separable potentials

[Hiyama, Kino, Kamimura, 2003]

- systematic error includes
  - timeslice dependence
  - finite-volume effects

- our results (preliminary)

	8_s channel	8_a channel	1 channel
$E_{\rm bind}$ [MeV]	$59.9(5.3)_{\rm stat}(^{+5.6}_{-7.7})_{\rm syst}$	$52.6(3.8)_{\rm stat}(^{+2.1}_{-5.4})_{\rm syst}$	$69.1(6.2)_{\rm stat}(^{+7.7}_{-12.4})_{\rm syst}$

- c.f. estimates from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ $(X=1,8)$:

	$8_s(8_a)$ channel	1 channel
$E_{\rm bind}$ [MeV]	$23.1(28.0)_{\rm stat}$	$78.0(12.3)_{\rm stat}$

  - consistent with the results from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ within (large) errors
  - $E^{8_s}_{\rm bind}, E^{8_a}_{\rm bind} < E^{1}_{\rm bind}$ is satisfied $\Longleftarrow$ same as chiral unitary model

Future work

- more precise consistency check by reducing errors of the estimates from $\langle \Lambda^{(X)}(t)\bar{\Lambda}^{(X)}(0)\rangle$ $(X=1,8)$
    - variational method using $\langle (MB)(t)\bar{\Lambda}(0)\rangle$ and $\langle (MB)(t)(\bar{M}B)(0)\rangle$ additionally
- **coupled-channel analysis** for 8_s and 8_a channel with separable potentials
- studies with more realistic setups
    - **(2+1)-flavor** simulation ⬅ coupled-channel analysis is required
- **more complicated separable form** in the HAL QCD potential
    - a sum of separable terms $U(\mathbf{r},\mathbf{r}') \approx \sum_i \eta^{(i)} v^{(i)}(\mathbf{r}) v^{(i)}(\mathbf{r}')$

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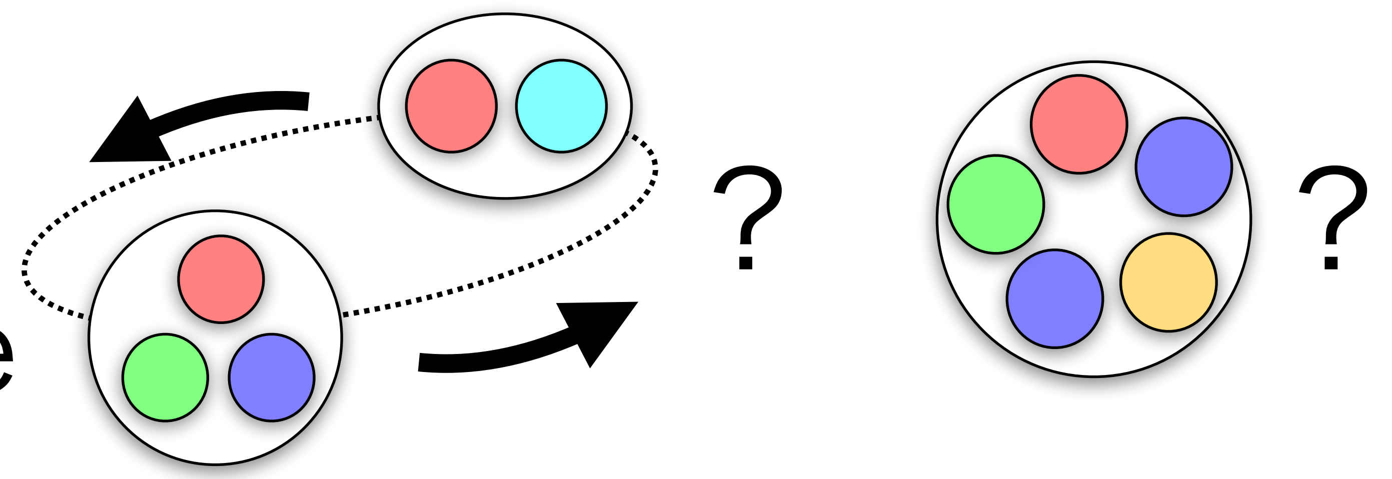
(conventional) Compositeness

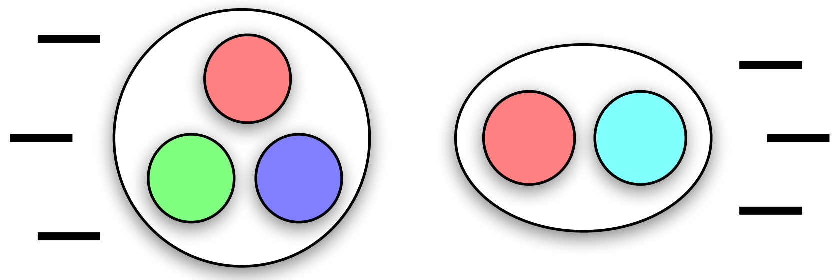


- Compositeness: an attempt to understand the structure of excited hadrons in **model-independent way**

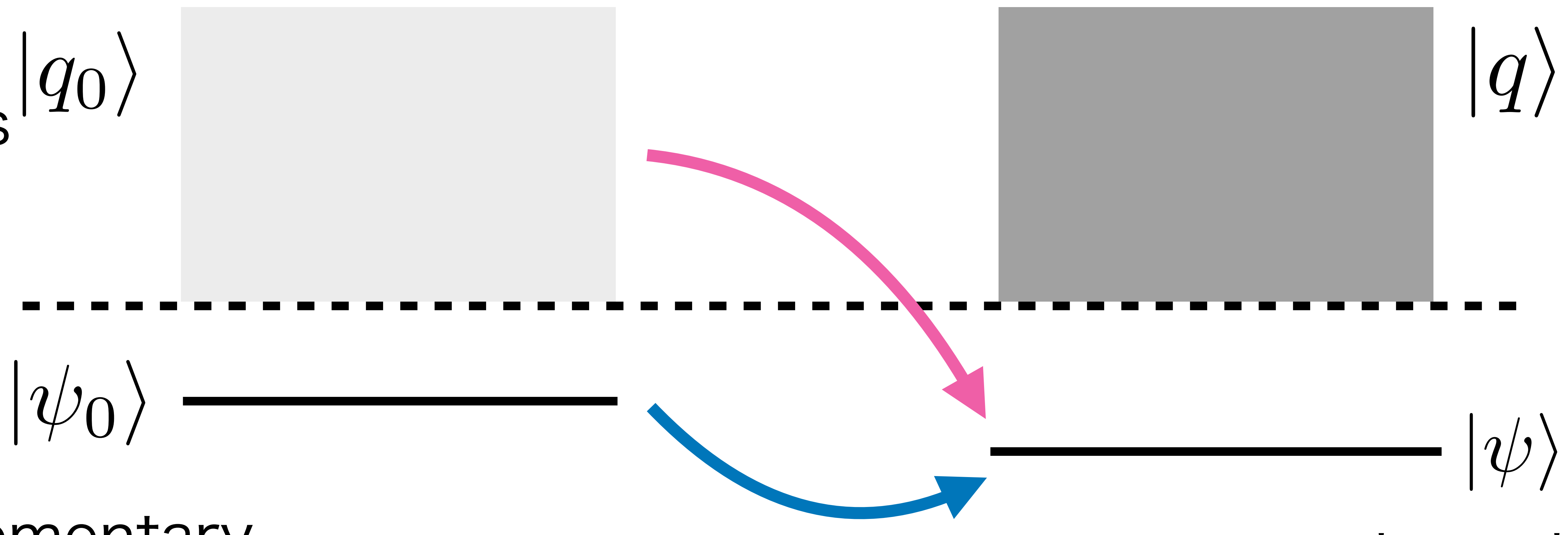
- theory picture: [Weinberg, 1963, 1965]

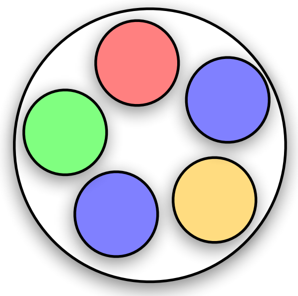
Free Theory **Full Theory (Nature)**

$\hat{H}_0$ $\hat{H} = \hat{H}_0 + \hat{V}$



free scattering states





elementary states (model)

bound state (observed)

Compositeness:

$$X = \int dq_0 \; |\langle \psi | q_0 \rangle|^2$$

Elementariness:

$$Z = |\langle \psi_0 | \psi \rangle|^2$$

$$(X + Z = \langle \psi | \psi \rangle = 1)$$

Approach to Compositeness

- in nature, $|\psi_0\rangle$ cannot be observed directly
- [Sekihara, Hyodo, Jido, 2015]:
  - contribution from $|\psi_0\rangle$ (elementariness) can be detected as an energy dependence of interactions between hadrons
  - in this case, compositeness can be derived from the **residue of bound-state pole** and energy dependence of the **green function (kinematics)**
  - but, deuteron gives $X > 1$ $(Z < 0)$ [Yin, Jido, arXiv:2312.13582]

    ➡ other contributions appear in the energy dependnece

    “interactioness”
  - identifying interactioness requires interaction models (**not model independent**)
- other approach?

  ➡ employ **“Weinberg’s equivalence theorem”** [Weinberg,1963]

Weinberg's equivalence theorem (1/2)

[Weinberg,1963]

Original theory

- free: $\hat{H}_0$
- full: $\hat{H} = \hat{H}_0 + \hat{V}$

Alternative theory

- free: $\hat{H}_0^{\mathrm{alt}}$
- full: $\hat{H}^{\mathrm{alt}} = \hat{H}_0^{\mathrm{alt}} + \hat{V}^{\mathrm{alt}}$

- how does the Alternative theory imitate the Original theory?

Weinberg's equivalence theorem (2/2)

[Weinberg,1963]

- **Theorem:** if Alternative theory has two linear combinations of $|\mathbf{q}_0^a\rangle$, $|\Gamma\rangle$ and $|\bar{\Gamma}\rangle$, such that

Original Theory

$$\langle \mathbf{q}_0 | \hat{V} | \mathbf{q}'_0 \rangle = \langle \mathbf{q}_0^a | \hat{V}^R | \mathbf{q}_0'^a \rangle$$

$$\langle \psi_0 | \hat{V} | \mathbf{q}_0 \rangle = \sqrt{\frac{-E_0}{N}} \langle \bar{\Gamma} | \hat{V}^R | \mathbf{q}_0^a \rangle$$

$$\langle \mathbf{q}_0 | \hat{V} | \psi_0 \rangle = \sqrt{\frac{-E_0}{N}} \langle \mathbf{q}_0^a | \hat{V}^R | \Gamma \rangle$$

$$\langle \psi_0 | \hat{V} | \psi_0 \rangle = \frac{-E_0}{N} \langle \bar{\Gamma} | \hat{V}^R | \Gamma \rangle$$

Alternative theory

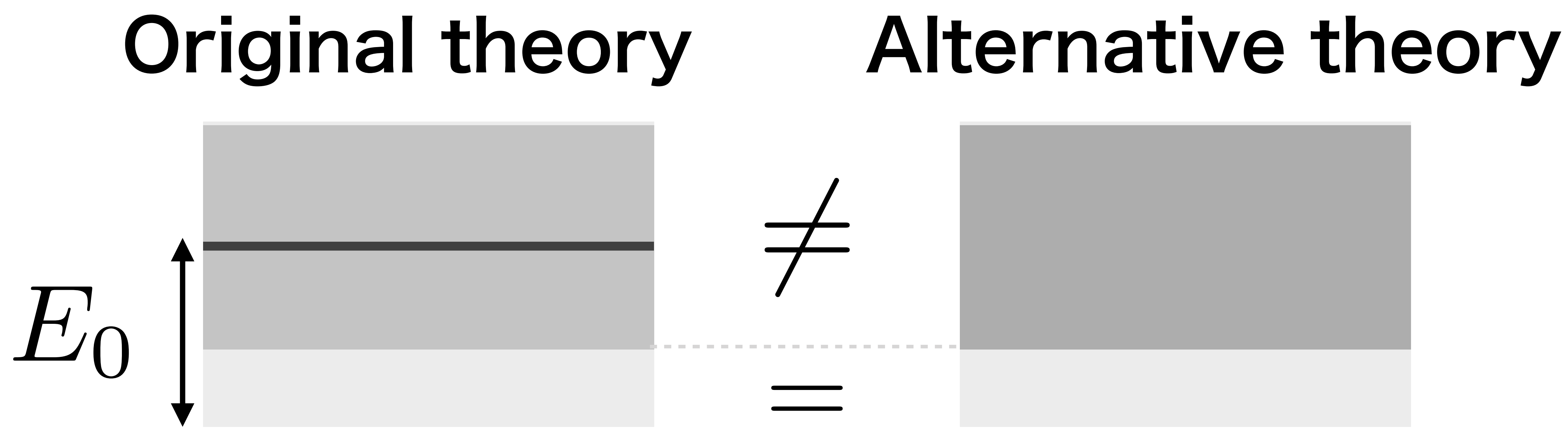
- $N = 1 - \langle \bar{\Gamma} | \hat{V}^{\mathrm{alt}} | \Gamma \rangle$,
- $\hat{V}^R = \hat{V}^{\mathrm{alt}} - \hat{V}^{\mathrm{alt}} |\Gamma\rangle\langle\bar{\Gamma}| \hat{V}^{\mathrm{alt}}$

➡ at $E_0 \to \infty$, Alternative and Original theories share the same phase shifts

$$\delta(q) = \delta^{\mathrm{alt}}(q)$$

Interpretation of Weinberg's equivalence theorem

- Original theory: **bound state, elementary state**, our real world
- Alternative theory: **no bound state or elementary state**, imitating Original theory
- $|\Gamma\rangle$, $|\bar{\Gamma}\rangle$: source of elementary state in Alternative theory
- Alternative and Original theories have the same phase shifts much below E_0



Application to calculation of compositeness

- set cutoff $\Lambda \ll E_0$

➡ (under assumption of existence of $|\Gamma\rangle$ and $|\bar{\Gamma}\rangle$) we can estimate X from Alternative theory **up to the cutoff** Λ

$$\underline{X^\Lambda} = \int^\Lambda \frac{d^3q}{(2\pi)^3} |\langle \underline{\mathbf{q}_0^a} | \underline{\psi\rangle}|^2 = \int^\Lambda \frac{d^3q}{(2\pi)^3} \underline{|\langle \mathbf{q}_0 | \psi\rangle|}^2 \leq \underline{X}$$

Alternative theory

Original theory

- Λ: energy scale under which the elementary state does not contribute

➡ **compositeness**

- natural that $X^\Lambda \to 1$ if $\Lambda \to \infty$

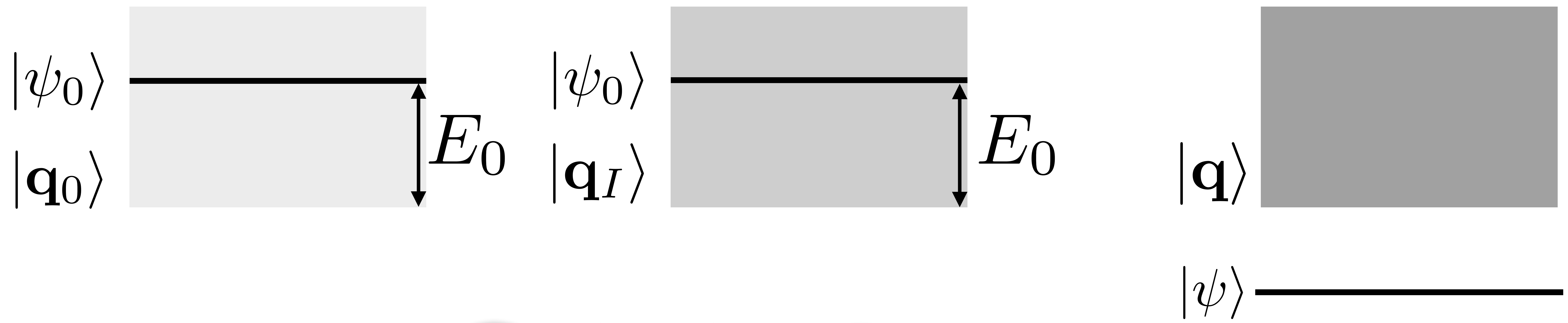
How to determine Λ

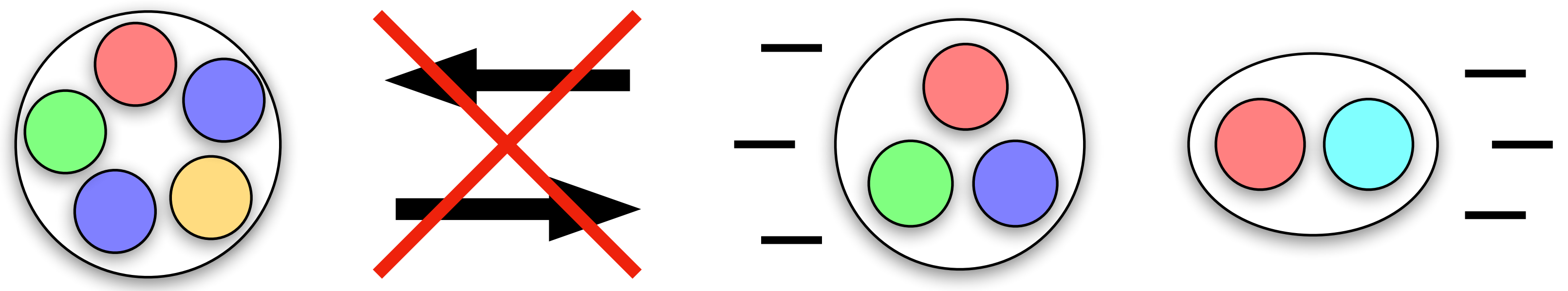
- Note: Λ is not determined uniquely
- what we know: $\delta(q)$ in the Original theory
- assume Λ to be defined such that there **exists a potential** which have the **same $\delta(q)$ as that for Original theory** but **does not generate any bound states**

New definition of compositeness

- define new compositeness in Original theory

- free: $\hat{H}_0$
- **isolated:** $\hat{H}_0 + \hat{V}_{q_0 q_0}$
- full: $\hat{H}_0 + \hat{V}_{q_0 q_0} + \hat{V}_{q_0 \psi_0}$





$$X_{\rm new} = \int \frac{d^3 q}{(2\pi)^3} |\langle \mathbf{q}_I | \psi \rangle|^2 \qquad Z_{\rm new} = Z = |\langle \psi_0 | \psi \rangle|^2$$

Connection to the Alternative theory

- isolated potential in the Original theory = potential without $|\Gamma\rangle$, $|\bar{\Gamma}\rangle$ in Alternative theory

$$\langle \mathbf{q}_0^a | \hat{V}^R | \mathbf{q}_0'^a \rangle = \langle \mathbf{q}_0 | \hat{V} | \mathbf{q}_0' \rangle = \langle \mathbf{q}_0 | \hat{V}_{q_0 q_0} + \hat{V}_{q_0 \psi_0} | \mathbf{q}_0' \rangle = \langle \mathbf{q}_0 | \hat{V}_{q_0 q_0} | \mathbf{q}_0' \rangle$$

➡ $|\mathbf{q}_I\rangle$ is derived by solving Schrödinger equation for $\hat{H}_0^{\mathrm{alt}} + \hat{V}^R$ in the Alternative theory

(in the demonstration using lattice data, we assume $|\mathbf{q}_I\rangle$ is the eigenstate of $\hat{H}_0^{\mathrm{alt}} + \hat{V}^{\mathrm{alt}}$)

$$X_{\mathrm{new}}^{\Lambda} = \int^{\Lambda} \frac{d^3 q}{(2\pi)^3} |\underline{\langle \mathbf{q}_I^a} | \psi \rangle|^2 = \int^{\Lambda} \frac{d^3 q}{(2\pi)^3} |\underline{\langle \mathbf{q}_I} | \psi \rangle|^2 \leq X_{\mathrm{new}}$$

eigenstates of $\hat{H}_0^{\mathrm{alt}} + \hat{V}^R$

eigenstates of $\hat{H}_0 + \hat{V}_{q_0 q_0}$

Example: HAL QCD potentials for bound Δ (1/2)

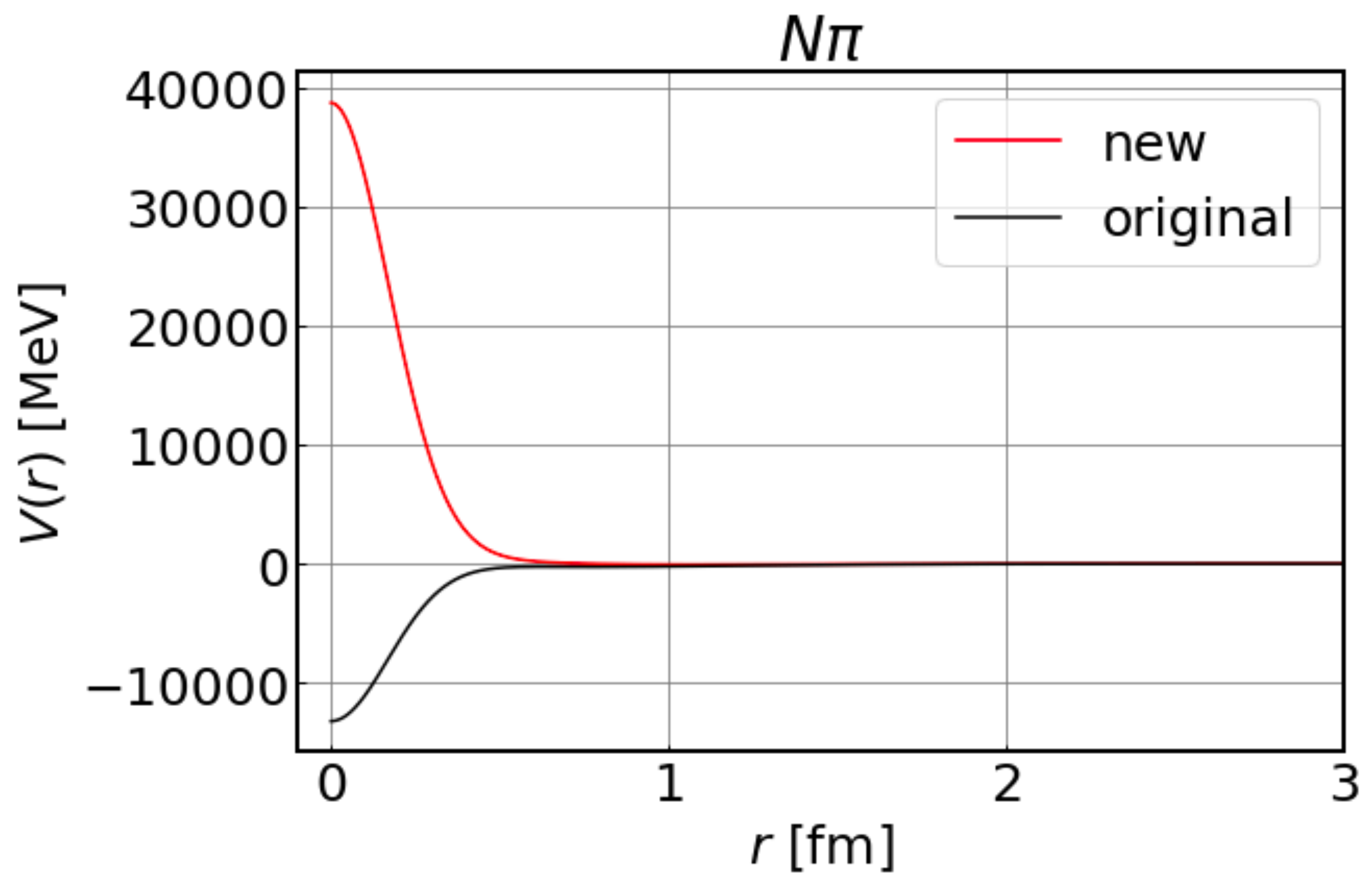
- input: fit data for P-wave $N\pi$ potential at $m_\pi \approx 410\ \mathrm{MeV}$, $m_N \approx 1217\ \mathrm{MeV}$ ➡ $E^{\Delta}_{\mathrm{bind}} \approx 106\ \mathrm{MeV}$
  - fit function: $V^{3G}(r) = \sum_{i=1,2,3} a_i e^{-(r/b_i)^2}$
- potential w/ fit parameters to minimize

$$\chi = \int^{q(E_{\mathrm{cut}})} dq \, \frac{(\delta_{V_{qq}}(q) - \delta_{V^{\mathrm{eff}}_{qq}}(q))^2}{\underline{\sigma(\delta_{V^{\mathrm{eff}}_{qq}}(q))^2}} + \underline{w} E^{V_{qq}}_{\mathrm{bind}}$$

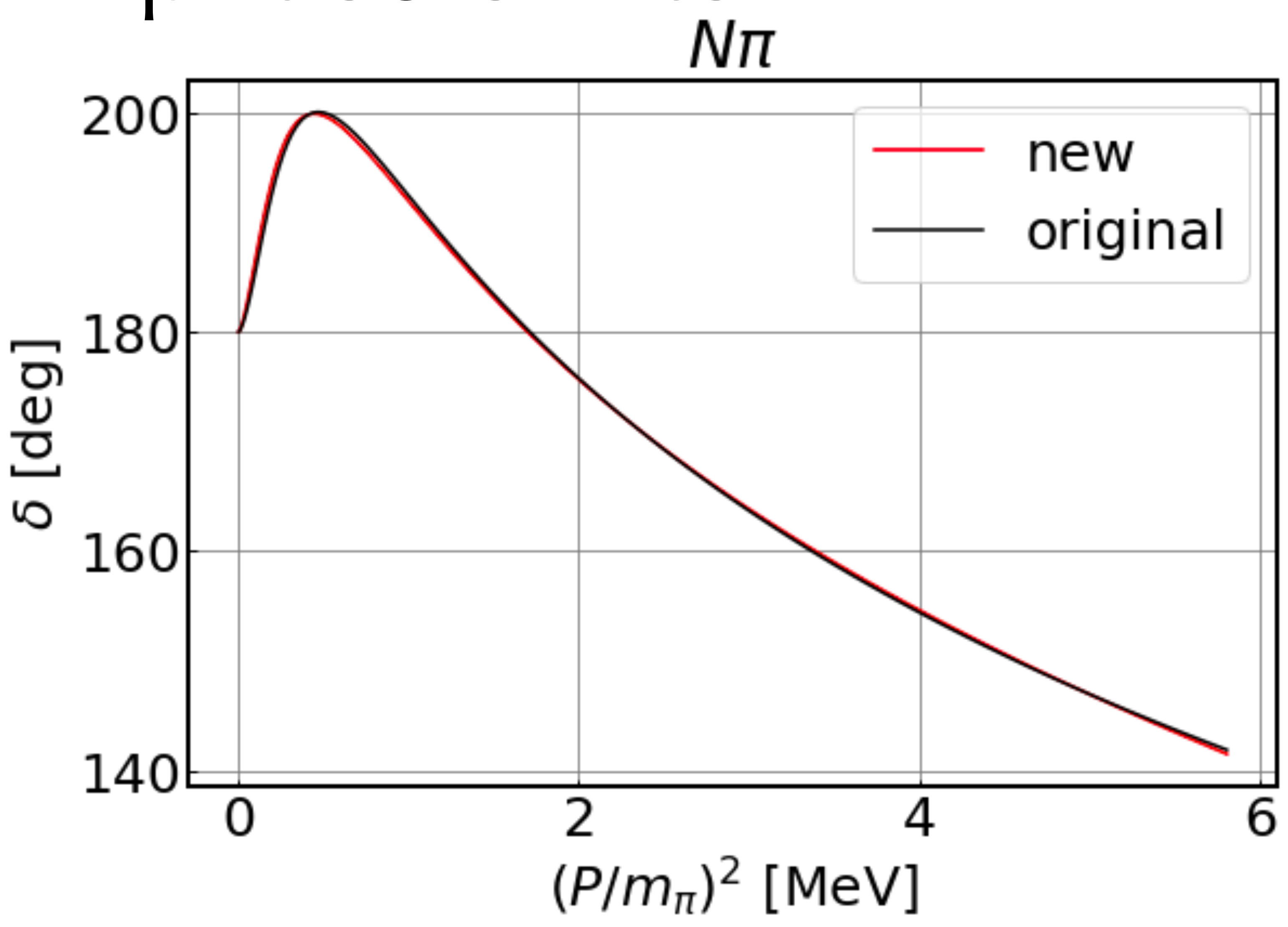
some large positive number

systematic and statistical error

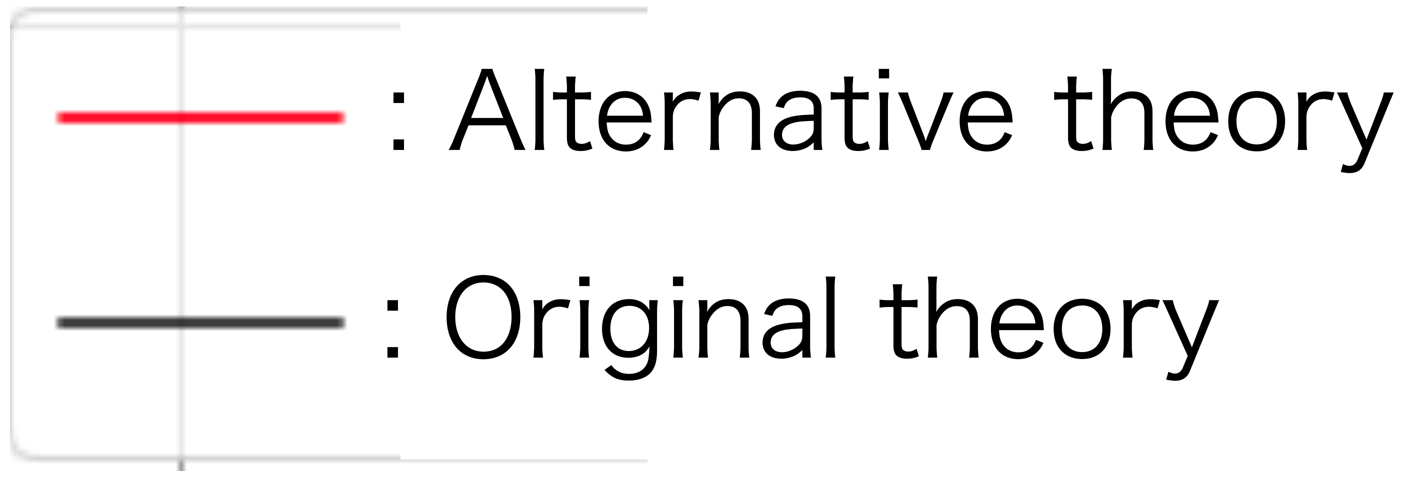
- potentials



- phase shifts

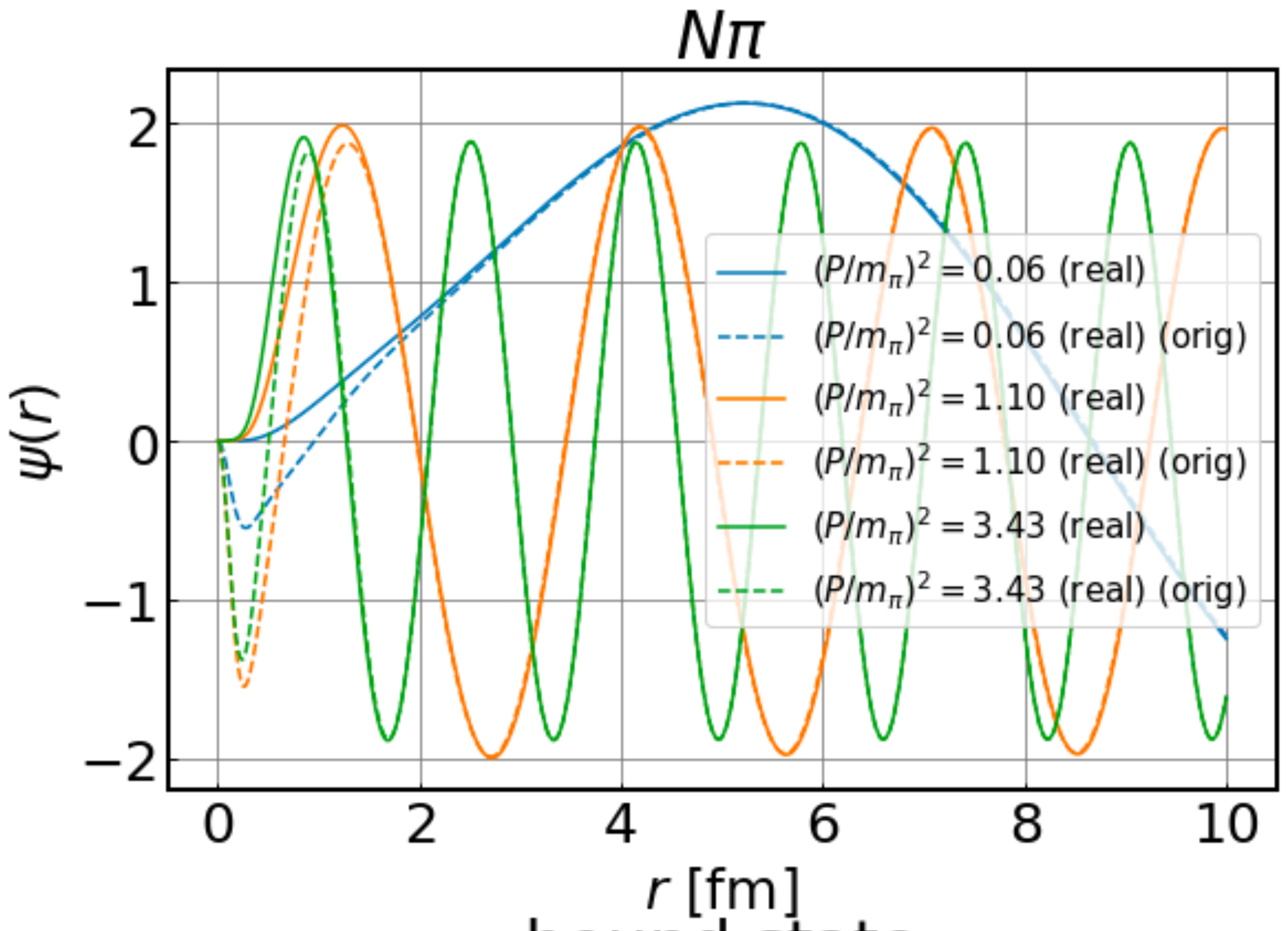


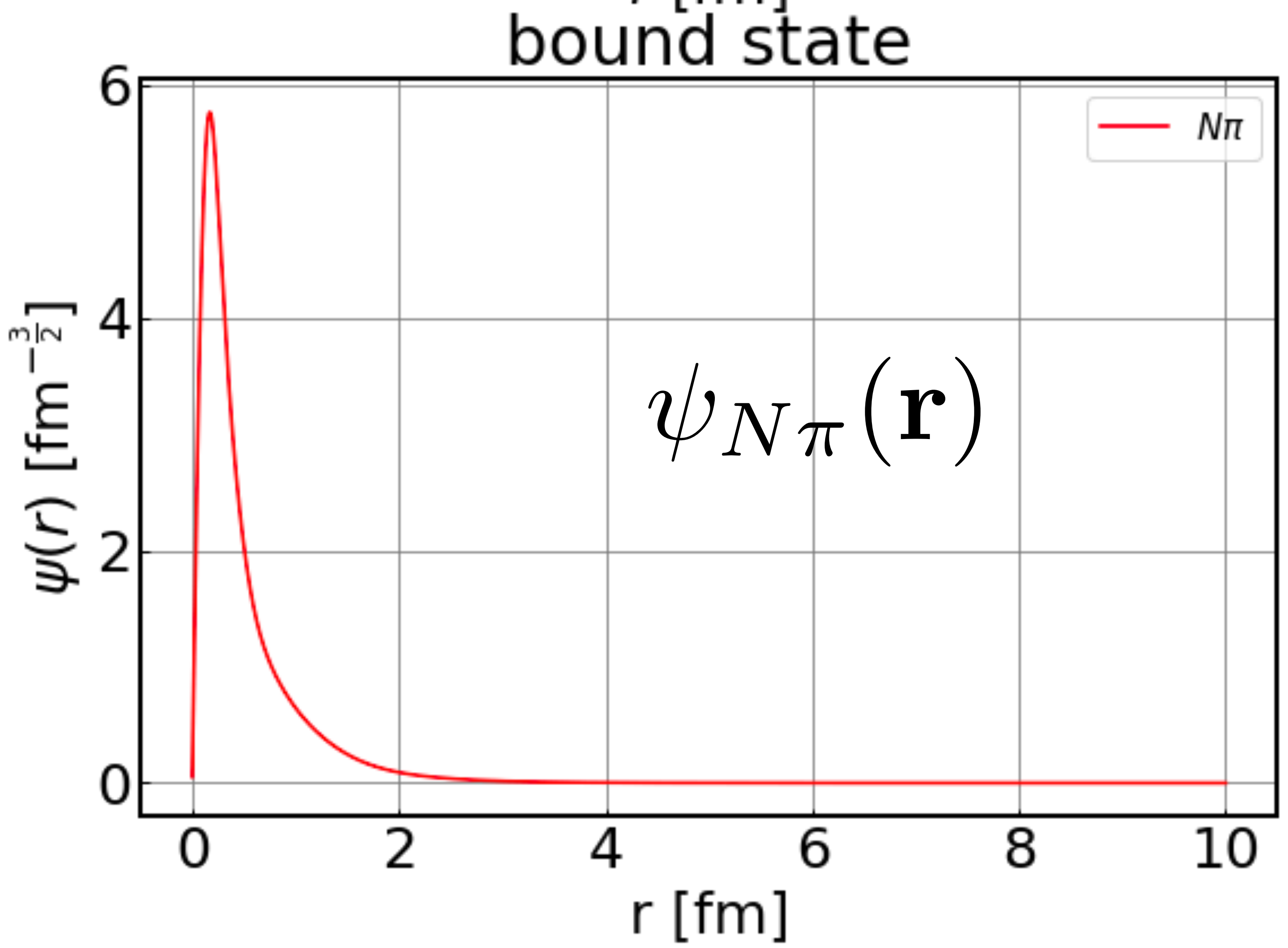
$\Lambda \approx 1000\ \mathrm{MeV}$



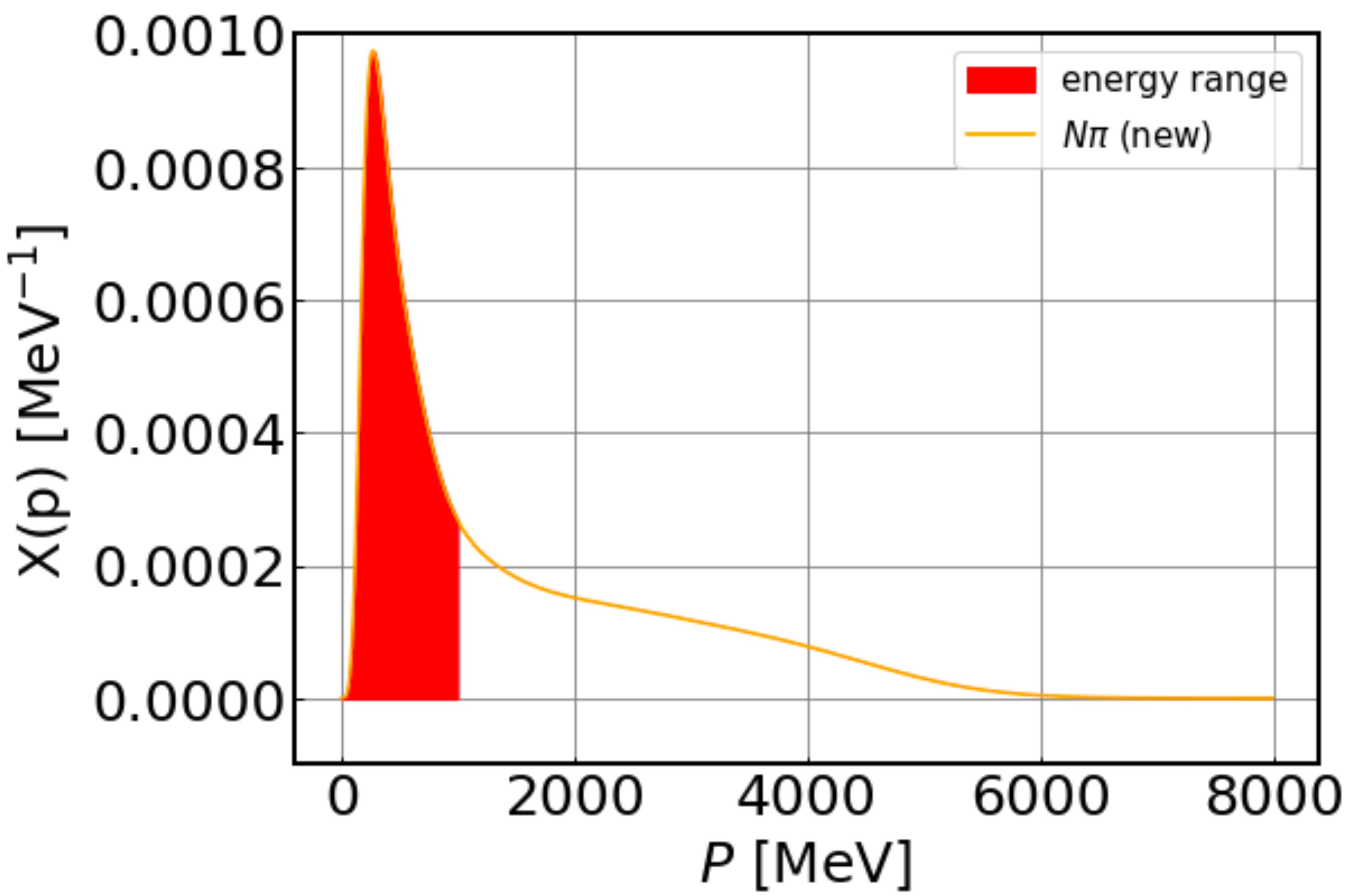
Example: HAL QCD potentials for bound Δ (2/2)

- $|q'\rangle$ (solid lines) and $|q\rangle$ (dashed lines)





- $|\langle \mathbf{q}_I^a|\psi\rangle|$



➡ $X_{\mathrm{new}}^{\Lambda} \approx 0.51$

What to do next

- examine the extreme situations
  - almost “composite”: energy-independent potential in mom. space
  - almost “elementary”: s-channel-diagram potential in mom. space